Towards Elevating DEMATEL with Spectral Analysis

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Abstract

In this work we introduce a novel spectral analysis extension to the Decision-Making Trial and Evaluation Laboratory (DEMATEL) method, significantly enhancing its analytical capabilities for decision support. Although DEMATEL has proven valuable for mapping influence relationships in complex systems, its traditional analysis focuses primarily on direct and total effects through centrality measures. We demonstrate that the total relation matrix, under practical conditions of irreducible normalized direct influence, is diagonalizable with distinct eigenvalues. This mathematical property enables deeper insights into system dynamics, structural analysis, and intervention and control through spectral analysis. The dominant eigenvalue and its corresponding eigenvector reveal fundamental system characteristics including influence propagation patterns, stability thresholds, and potential cascade effects. This theoretical advancement provides practitioners with new tools for identifying optimal intervention points and assessing system-wide impacts. We illustrate these enhanced analytical capabilities through a case study of food safety governance factors in rural China, where spectral analysis reveals previously undetectable patterns of influence amplification and system convergence properties. The proposed extension transforms DEMATEL from a static influence mapping tool into a framework capable of quantifying dynamic system behavior, while maintaining its computational tractability and practical applicability.

Keywords: DEMATEL; spectral analysis; complex systems; decision support

1. Introduction

As systems grow in complexity, traditional analytical methods often fall short in capturing the cause-effect relationships, the subtle interplay between components and the cascading effects of interventions. Among the various methodologies developed to address this challenge, the Decision-Making Trial and Evaluation Laboratory (DEMATEL) method (Gabus & Fontela, 1972) has emerged as a particularly powerful tool, owing to its ability to not only map direct relationships but also quantify indirect influences propagating through the system (Quezada et al., 2018). This capability has made DEMATEL increasingly popular across diverse fields, from supply chain management to sustainability assessment, where understanding the ripple effects of decisions is crucial for effective system intervention (Si et al., 2018).

Although DEMATEL has become a popular methodology in complex system analysis (Chen, 2021), its analytical depth has remained relatively unchanged since its introduction. The method's standard procedure - constructing a direct influence matrix, applying normalization, and deriving the total relation matrix - provides valuable insights into system structure and component relationships (Sorooshian et al., 2023; Taherdoost & Madanchian, 2023). Current applications primarily focus on analyzing centrality measures and impact-relation maps derived from the total relation matrix. However, we have yet to fully leverage the mathematical properties of this matrix, particularly its diagonalizability, for more sophisticated system analysis beyond traditional DEMATEL outputs . This gap suggests a significant opportunity for methodological advancement.

This paper introduces a significant enhancement to DEMATEL through spectral analysis of the total relation matrix. By establishing that this matrix is diagonalizable under practical conditions of irreducible normalized direct influence, we unlock access to its eigenvalue spectrum. The maximum eigenvalue, in particular, serves as a powerful analytical tool that reveals fundamental system properties previously inaccessible through standard DEMATEL analysis. This spectral extension enables several crucial insights like the system dynamics and the influence propagation, the system's stability characteristics, the understanding of cascade effects, the detection of optimal leverage points for system intervention, and the intervention planning. These insights significantly expand DEMATEL's analytical capabilities while maintaining its computational tractability and practical applicability. The transition from standard DEMATEL to spectral analysis follows a natural mathematical progression. The proposed approach complements traditional DEMATEL outputs by adding a new analytical dimension: on top of centrality measures and impact-relation maps that reveal the static distribution of influence, our spectral analysis extension uncovers the system's dynamic potential, stability boundaries, and critical intervention thresholds—fundamentally expanding DEMATEL's capabilities for complex system analysis.

2. Background

2.1. DEMATEL

The Decision-Making Trial and Evaluation Laboratory (DEMATEL) method is a structural modeling approach that analyzes complex causal relationships between components in a system. Originally developed by the Geneva Research Centre of the Battelle Memorial Institute (Gabus & Fontela, 1972), DEMATEL transforms qualitative assessments into quantifiable systemic relationships. The method begins with expert evaluations of direct influences between system factors, recorded in a direct influence matrix $\boldsymbol{A} = [a_{ij}]$, where a_{ij} represents the degree to which factor i influences factor j on a scale typically ranging from 0 (no influence) to 4 (very high influence).

The core DEMATEL procedure involves normalizing the direct influence matrix $\boldsymbol{A}$ into matrix $\boldsymbol{D} = \boldsymbol{A}/s$, where $s = \max\{\text{maximum row sum}, \text{maximum column sum}\}$, followed by calculating the total relation matrix $\boldsymbol{T} = \boldsymbol{D}(\boldsymbol{I} - \boldsymbol{D})^{-1}$, where $\boldsymbol{I}$ is the identity matrix. From matrix $\boldsymbol{T}$, row sums (r) represent the total influence that factor i exerts on others (dispatch), column sums (c) indicate the total influence received by factor j (receipt), while $(r + c)$ and $(r - c)$ values respectively reveal the prominence (total involvement) and net effect (overall influence) of each factor in the system. This basic framework has been extended to include threshold values for significance (Li & Tzeng, 2009), helping practitioners focus on the most important relationships. These outputs enable decision-makers to visualize and understand the interdependencies between factors, identifying those with the greatest system-wide impact.

2.2. Algebraic Foundations

Matrix analysis provides essential tools for understanding complex systems through their mathematical properties. At its foundation are eigenvalues and eigenvectors: for a matrix A, if there exists a non-zero vector v and a scalar λ satisfying Av = λv, then λ is called an eigenvalue and v is its corresponding eigenvector. A matrix is diagonalizable if it has a complete set of linearly independent eigenvectors, allowing it to be decomposed as $A = PDP^{-1}$, where D is a diagonal matrix of eigenvalues and P contains the corresponding eigenvectors. Among the eigenvalues, the spectral radius ρ(A) - the largest absolute value among all eigenvalues - is particularly significant for understanding system behavior.

When analyzing influence relationships, certain matrix properties become crucial. A nonnegative matrix (where all elements $a_{ij} \geq 0$) is called irreducible if its associated directed graph is strongly connected, meaning there exists a path between any two elements in the system. For such matrices, the Perron-Frobenius theorem establishes that the spectral radius is itself an eigenvalue, and its corresponding eigenvector has strictly positive components. These fundamental concepts from matrix theory (Meyer, 2023) provide the mathematical foundation for analyzing complex systems. These properties provide a rigorous foundation for analyzing interconnected systems, particularly when studying influence patterns.

3. Methodology

3.1. Theoretical Framework

Building upon the established DEMATEL methodology, we develop a spectral analysis approach to extract additional insights from the Total Relation Matrix $\boldsymbol{T}$. This section presents the theoretical foundations that enable this extension.

Spectral Properties of the Total Relation Matrix

Our analysis begins with a fundamental result about the eigenstructure of $\boldsymbol{T}$:

Theorem 1: *If the normalized direct influence matrix* $\boldsymbol{D}$ *is irreducible, then the Total Relation Matrix* $\boldsymbol{T}$ *is diagonalizable with distinct eigenvalues.*

The proof relies on several key properties: $\boldsymbol{D}$ is irreducible, meaning its associated directed graph is strongly connected. By construction, $\boldsymbol{D}$ has also zero diagonal, and it is nonnegative, with all entries $d_{ij} \geq 0$. Under these conditions:

1. By Gershgorin's theorem (Varga, 2004) and the normalization of $\boldsymbol{D}$, all eigenvalues μ of $\boldsymbol{D}$ satisfy $|\mu| < 1$.
2. For any eigenvalue λ of $\boldsymbol{T}$ with eigenvector $\boldsymbol{v}$: From $\boldsymbol{Tv} = \lambda \boldsymbol{v}$, we obtain $\boldsymbol{D}(\boldsymbol{I} - \boldsymbol{D})^{-1}\boldsymbol{v} = \lambda \boldsymbol{v}$. This yields $\boldsymbol{Dv} = \lambda \boldsymbol{v} - \lambda \boldsymbol{Dv}$. Therefore, $(1 + \lambda)\boldsymbol{Dv} = \lambda \boldsymbol{v}$.
3. This establishes a one-to-one correspondence: If $\boldsymbol{v}$ is an eigenvector of $\boldsymbol{T}$ with eigenvalue λ, then $\boldsymbol{v}$ is an eigenvector of $\boldsymbol{D}$ with eigenvalue $\mu = \lambda/(1 + \lambda)$. Conversely, if μ is an eigenvalue of $\boldsymbol{D}$, then $\lambda = \mu/(1 - \mu)$ is an eigenvalue of $\boldsymbol{T}$.

The irreducibility of $\boldsymbol{D}$, combined with its zero-diagonal structure, ensures distinct eigenvalues through Perron-Frobenius theory and the properties of strongly connected graphs ■

Spectral Decomposition

Under the conditions above, $\boldsymbol{T}$ admits the decomposition $\boldsymbol{T} = \boldsymbol{P\Lambda P^{-1}}$ where:

- $\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1, \ldots, \lambda_n)$ contains the distinct eigenvalues
- $\boldsymbol{P}$ contains the corresponding eigenvectors

This decomposition, fundamental to numerical matrix analysis (Golub & Van Loan, 2013), enables efficient computation of system properties through the eigenstructure. The dominant eigenvalue λ_{max} carries particular significance for system analysis, as we demonstrate in the following section.

3.2. Interpretations

The dominant eigenvalue $\lambda_{\max}$ of the total relation matrix $\boldsymbol{T}$ reveals fundamental insights into the system's structural characteristics and influence dynamics. Specifically, $\lambda_{\max}$ quantifies the maximum level of long-term influence that can propagate through the system, where larger values indicate stronger overall interdependencies between factors. The relationship between $\lambda_{\max}$ and the sum of all eigenvalues ($l = \lambda_{\max}/\sum \lambda_i$) represents the system's concentration of influence in its primary mode, with values closer to 1 indicating a more dominated system. The corresponding eigenvectors further enrich this analysis by revealing influence clusters - groups of factors that exhibit similar influence patterns and collectively drive system behavior. These eigenvectors effectively decompose the complex network of relationships into distinct patterns of co-influence.

Then, the convergence properties illuminated by $\lambda_{\max}$ provide critical insights into system stability and behavior dynamics. The magnitude of $\lambda_{\max}$ (also called the spectral radius) directly corresponds to the convergence rate of indirect effects, where higher values indicate slower convergence and thus necessitate more iterations to capture all significant indirect influences. This relationship makes $\lambda_{\max}$ an effective measure of system complexity and coupling intensity. For instance, in systems with high $\lambda_{\max}$ values, indirect effects persist longer and potentially amplify through multiple paths, indicating a more intricately coupled system. This characteristic is particularly relevant when analyzing systems where cascade effects are of concern, as it helps quantify the potential for influence amplification through indirect ways.

The spectral analysis framework enables robust comparative analyses across different systems or temporal states. Changes in $\lambda_{\max}$ serve as reliable indicators of structural shifts in system interdependencies, providing a quantitative basis for monitoring system evolution. The sensitivity analysis, performed through the examination of partial derivatives $\partial\lambda_{\max}/\partial a_{ij}$, identifies which relationships most significantly impact system coupling. This mathematical framework allows for systematic comparison of different system states or configurations, enabling researchers to quantify and validate structural changes in the system's interdependency patterns over time or across different contexts.

From an intervention perspective, $\lambda_{\max}$ serves as a crucial indicator for risk assessment and strategy development. Systems characterized by higher $\lambda_{\max}$ values demonstrate increased susceptibility to cascade effects, necessitating careful consideration of indirect influences in intervention planning. The components of the dominant eigenvector provide additional strategic guidance by identifying the most influential system elements - those with larger eigenvector values represent more significant *leverage points* for system modification. This understanding enables more targeted and effective risk mitigation strategies, allowing practitioners to focus interventions where they will have the most substantial impact while considering the potential for unintended consequences through indirect effects. The ways that the spectral analysis can augment the analysis potentials of DEMATEL are summarized and illustrated in Table 1.

Table 1. The contribution potentials of spectral analysis of the total relation matrix

<table>
<tr><th>System Dynamics & Stability</th><th>Structural Analysis & Pattern Detection</th><th>Intervention & Control Analysis</th></tr>
<tr><td>Maximum level of long-term influence that can propagate through the system</td><td>Strength of overall interdependencies between factors</td><td rowspan="2">Risk assessment and mitigation strategies:
- Systems with higher $\lambda_{\max}$ require more careful intervention planning.
- Greater attention to indirect effects in highly coupled systems.</td></tr>
<tr><td>Overall strength of the network's total influence considering all direct/indirect effects</td><td>Ratio of $\lambda_{\max}$ to sum of all eigenvalues reveals primary influence mode dominance</td></tr>
<tr><td>Maximum amplification of influence possible through indirect effects</td><td>Eigenvectors reveal influence clusters (co-driving factors)</td><td rowspan="3">Sensitivity analysis through:
- Partial derivatives for relationship impact assessment
- Eigenvector $\boldsymbol{v}$ analysis for identifying leverage points</td></tr>
<tr><td>System's stability and convergence characteristics</td><td rowspan="3">Comparative analysis capabilities:
- System comparison across different instances
- Temporal comparison of same system
- Validation of structural changes</td></tr>
<tr><td>Potential for cascade effects</td></tr>
<tr><td>System complexity and coupling degree measurement</td><td>Factor importance weighting and strategic intervention point identification</td></tr>
</table>

4. Case Study

4.1. Original Study

The original study of (Xie et al., 2023) examines the complex system of factors influencing farmers' participation in food safety governance in rural China. The researchers identified 20 key influencing factors across four dimensions: family characteristics (including education level, village cadre status, household income, family structure, eating habits, and self-supply of food), participant characteristics (victim experience, political trust, risk perception, media attention, government supervision, and village committee promotion), participation process (perception of effectiveness, cost perception, and government response), and participation environment (participation atmosphere, rural informatization degree, publicity of government information, participation channels, and incentive mechanisms). Using the DEMATEL methodology alongside ISM and MICMAC analyses, they surveyed 20 experts in public management and food safety governance to evaluate the interrelationships and relative importance of these factors.

Through their DEMATEL analysis, the researchers evaluated both the influence degree (comprehensive influence of one factor on others), affected degree (how much a factor is influenced by others), center degree (overall importance in the system), and cause degree (net influence) of each factor. Their results were presented in a comprehensive influence matrix and summarized in tables showing these four key metrics for each of the 20 factors. The findings revealed that education level (a1) and village cadre status (a2) had the highest influence degrees (4.17 and 4.25 respectively) and cause degrees (both 1.69), while risk perception (a9) had the highest center degree (7.21) but a negative cause degree (-0.82), indicating it was more affected by other factors than influential. Family eating habits (a5) showed the lowest cause degree (-1.42), suggesting it was the most heavily influenced by other factors in the system.

4.2. Spectral Analysis for the System Dynamics & Stability

Maximum Level of Long-term Influence Propagation

The spectral analysis of the diagonalizable total relation matrix $\boldsymbol{T}$ reveals a maximum eigenvalue ($\lambda_{\max}$) of 3.27. This value indicates potential for long-term influence propagation within the system. When interpreting this metric for decision-making purposes, we observe that any $\lambda_{\max}$ exceeding 1 suggests that initial changes in the system can amplify over time. This characteristic demands careful consideration when designing interventions, as decision-makers must ensure high-quality implementations at critical entry points while allowing sufficient time for effects to mature through the system.

Our food security case study exemplifies this principle, where the impact of the primary influence drivers (education and village cadre status) grows systematically through rural networks, indicating that well-executed food safety education programs could create sustainable improvements in system resilience.

Overall Strength of the Network's Total Influence

The spectral analysis reveals several key indicators of network strength: $\lambda_{\max}$ approximates the Frobenius norm $\|\boldsymbol{T}\|_F$ (3.28), accompanied by a high $\lambda_{\max}/\lambda_2$ ratio (~51), and uniform eigenvector components ranging from 0.165 to 0.284. Together, these metrics indicate a robust, single-pattern influence structure. For decision-makers, this coherence suggests that system responses will be more predictable, favoring coordinated intervention strategies over fragmented approaches. The clear dominance of a single influence pathway provides an opportunity to align actions for maximum impact.

In our food security study, this unified influence pattern supports the implementation of integrated policies. The combination of education and cadre training programs demonstrates how interventions can achieve scalable impacts through predictable propagation pathways.

Maximum Amplification of Influence Through Indirect Effects

The spectral analysis yields crucial insights into influence amplification: with $\lambda_{\max} = 3.27$ and $\rho(\boldsymbol{D}) = 0.766$, we calculate a maximum amplification factor of 4.27 times the direct effects via $\boldsymbol{T} = \boldsymbol{D}(\boldsymbol{I} - \boldsymbol{D})^{-1}$. This amplification is stabilized by rapid secondary mode decay ($\lambda_2/\lambda_{\max} \approx 0.02$). This significant amplification potential emphasizes the importance of monitoring indirect effects in decision-making contexts, as minor interventions can escalate substantially throughout the system. To maintain control over this amplification potential, decision-makers should implement and test interventions incrementally.

Our case study demonstrates this principle, showing how a modest educational initiative could potentially amplify 4.27-fold across rural food security networks, supporting the study's emphasis on informatization as a high-leverage enhancement mechanism.

System's Stability and Convergence Characteristics

The spectral analysis reveals a high condition number ($\kappa = \lambda_{\max}/\lambda_{\min} \approx 1637$) and a convergence rate of approximately 3.93, derived from $\ln(\lambda_2/\lambda_{\max})$. These metrics indicate that while the system shows numerical sensitivity, it demonstrates rapid pattern alignment. For decision-makers, the high condition number necessitates precise data collection and analysis, but the fast convergence rate ensures predictable outcomes, supporting robust policy design that can be validated through multiple methods.

For the food safety governance case study, this means that interventions will follow predictable patterns of influence, so our focus should be on the dominant influence paths identified in the study. Then, because of the high κ, multiple measurement approaches are needed for reliable assessment of relationships.

5. Conclusions

The spectral analysis extension of DEMATEL we introduced in this work represents a significant advancement in this decision support methodology. Traditional DEMATEL analysis identifies influence patterns and centrality measures, but our approach unlocks deeper insights into system dynamics, stability characteristics, and intervention potential through eigenstructure examination. The food safety governance case study demonstrates these enhanced analytical capabilities: our method revealed critical patterns of influence amplification, system convergence properties, and coherent influence structures that standard DEMATEL analysis could not detect. These mathematical properties empower decision-makers to assess system stability, anticipate cascade effects, and identify strategic intervention points with unprecedented clarity. This methodological enhancement transforms DEMATEL from a static influence mapping tool into a dynamic framework capable of predicting system behavior and guiding targeted interventions with scientific rigor.

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