MAE 438/538
Hybrid ODF Maglev Ball system
Part 1

Peter DeTeresa

Email: pdete001@odu.edu

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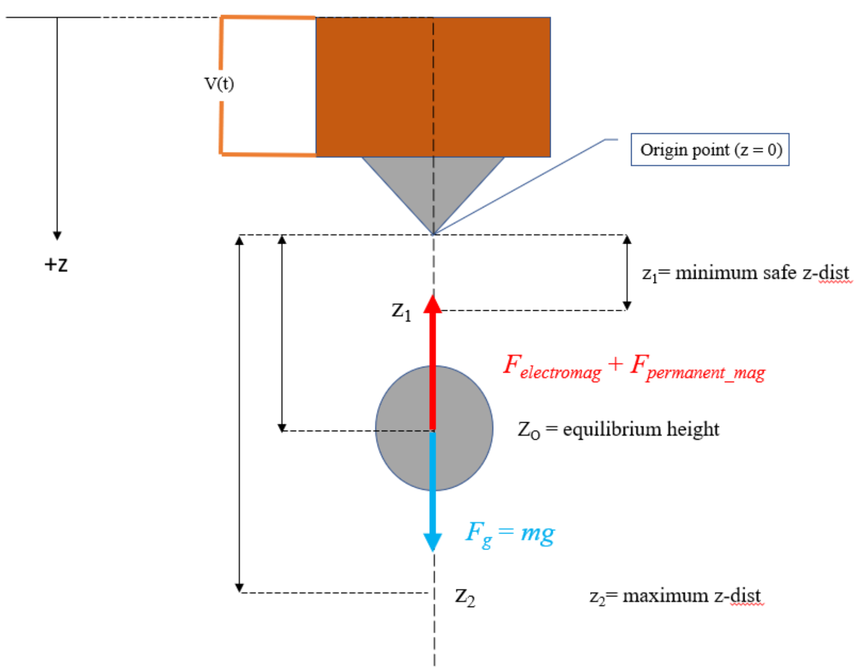
TASK 1 – SYSTEM CHOICE

The purpose of the system is to levitate a steel ball using magnetic forces. It is called a hybrid system because it consists of a permanent magnet and an electromagnet; the use of a permanent magnet means that less power is required for the electromagnet to raise and maintain the height of the ball, but it also means that there is a minimum nonzero distance between the ball and the top of the apparatus where the magnets are positioned, which is referred to as the minimum air gap. The height of the ball is controlled by the current through the coil of wire of the electromagnet (which is induced by applying a voltage across the terminals of the coil).

The two measured parameters are the command current and the height of the ball, which employs the use of two laser sensors. The nonlinear equations that describe the physical motion and the electrical behavior of the system are as follows:

$$\ddot{z} = -\frac{K_f}{m}\left[\frac{a_1 + I}{a_2 + z}\right]^2 + g$$

$$K_a(I_c - I) = -RI + \dot{I}\frac{K_l}{(a_2 + z)} - (a_1 + I)\frac{K_l \dot{z}}{(a_2 + z)^2}$$



PARAMETER VALUES

$$a_1 = \mathbf{46.5894}\ \boldsymbol{Amps}$$

$$a_2 = 0.007\ m$$

$$K_f = 5.0678e - 6$$

$$\mathrm{K_a} = 11.7858$$

$$\mathrm{K_l} = 0.0065$$

$$\mathrm{m} = 1.1\ \mathrm{kg}$$

$$\mathrm{R} = 1.1\ ohms$$

$$\mathrm{g} = 9.81\ \mathrm{m/s^2}$$

TASK 2 – Linearized System Model

z_o – equilibrium height

I_o – equilibrium current

$$\ddot{z} = \frac{\partial \ddot{z}}{\partial z} * z + \frac{\partial \ddot{z}}{\partial I} * I$$

$$\boxed{\ddot{z} = \frac{2K_f}{m} \frac{(I_o + a_1)}{(z_o + a_2)^2} \left[\frac{(I_o + a_1)}{(z_o + a_2)} * z - I \right]}$$

$$\dot{I} = \frac{\partial \dot{I}}{\partial z} z + \frac{\partial \dot{I}}{\partial \dot{z}} \dot{z} + \frac{\partial \dot{I}}{\partial I} I + \frac{\partial \dot{I}}{\partial I_c} I_c$$

$$\frac{\partial \dot{I}}{\partial z} z = \frac{K_a(I_c - I_o) + I_o R - \frac{K_l(I_o + a_1)}{(a_2 + z_o)^2}}{K_l} z \qquad \frac{\partial \dot{I}}{\partial \dot{z}} \dot{z} = \frac{(I_o + a_1)}{(z_o + a_2)} \dot{z}$$

$$\frac{\partial \dot{I}}{\partial I} I = \left[\frac{R}{K_l} (z_{o+} a_2) + \frac{\dot{z}_o}{(z_{o+} a_2)} - \frac{K_a}{K_l} (z_{o+} a_2) \right] I \qquad \frac{\partial \dot{I}}{\partial I_c} I_c = \frac{K_a}{K_l} (z_o + a_2) I_c$$

State space representation:

$\dot{\vec{x}} = A\vec{x} + Bu$ where the transfer function is $H(s) = \frac{Y(s)}{X(s)} = C(sI - A)^{-1}B + D$

$y = C\vec{x} + Du$

$$x = \begin{pmatrix} z \\ \dot{z} \\ I \end{pmatrix}, \quad \dot{x} = \begin{pmatrix} \dot{z} \\ \ddot{z} \\ \dot{I} \end{pmatrix}, \quad u = I_c, y = z$$

$$A = \begin{vmatrix} 0 & 1 & 0 \\ -\frac{1}{m}\frac{\partial \ddot{z}}{\partial z} & 0 & -\frac{1}{m}\frac{\partial \ddot{z}}{\partial I} \\ \frac{\partial \dot{I}}{\partial z} & \frac{\partial \dot{I}}{\partial \dot{z}} & \frac{\partial \dot{I}}{\partial I} \end{vmatrix}, \quad B = \begin{vmatrix} 0 \\ 0 \\ \frac{\partial \dot{I}}{\partial I_c} \end{vmatrix}, \quad C = |1 \quad 0 \quad 0|, \quad D = [0]$$

z_o is related to I_o through the first equation. Since they are both equilibrium constants, we have:

$$\dot{z_o} = 0\,, \qquad \ddot{z_o} = 0$$

Rearranging equation 1 for I_o yields the following expression:

$$\mathrm{I_o} = \sqrt{\frac{mg}{K_f}(\mathrm{z_o} + \mathrm{a_2})} - \mathrm{a_1}$$

From measurement documentation, z_o is selected to be 0.0252 m [3]. Therefore:

$$\mathrm{I_o} = 0.3975$$

The transfer function is the ratio of the output to the input. For this system, the input is the *command* current (not the excitation current) and the output is the height of the ball. Both are functions of time, so they can be transformed into the Laplace domain.

Since the partial derivatives for the linear model are with respect to the equilibrium conditions, they are constant values ($z_o, I_o, a_1, a_2, I_{c_o}$). Therefore to make the equations more easier to read, the symbol ‘k’ will be used to refer to a particular linear constant, with an identifying subscript linking it to a partial derivative

$$k_z = \frac{\partial \ddot{z}}{\partial \mathrm{z}}, k_I = \frac{\partial \ddot{z}}{\partial I}$$

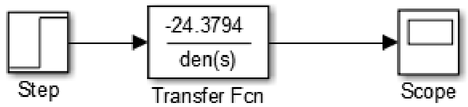
$$k_{i_z} = \frac{\partial \dot{I}}{\partial \mathrm{z}},\; k_{i_{\dot{z}}} = \frac{\partial \dot{I}}{\partial \dot{z}}, k_{i_c} = \frac{\partial \dot{I}}{\partial I_c}, k_{i_I} = \frac{\partial \dot{I}}{\partial I}$$

$$\frac{output}{input} = \frac{z}{I_c} = \frac{\dfrac{-k_i}{m} * k_{i_{i_c}}}{s^3 - k_{i_i} * s^2 + \left(\dfrac{k_i}{m} * k_{i_{\dot{z}}} + \dfrac{k_z}{m}\right) * s - k_{i_i} * \dfrac{k_z}{m}}$$

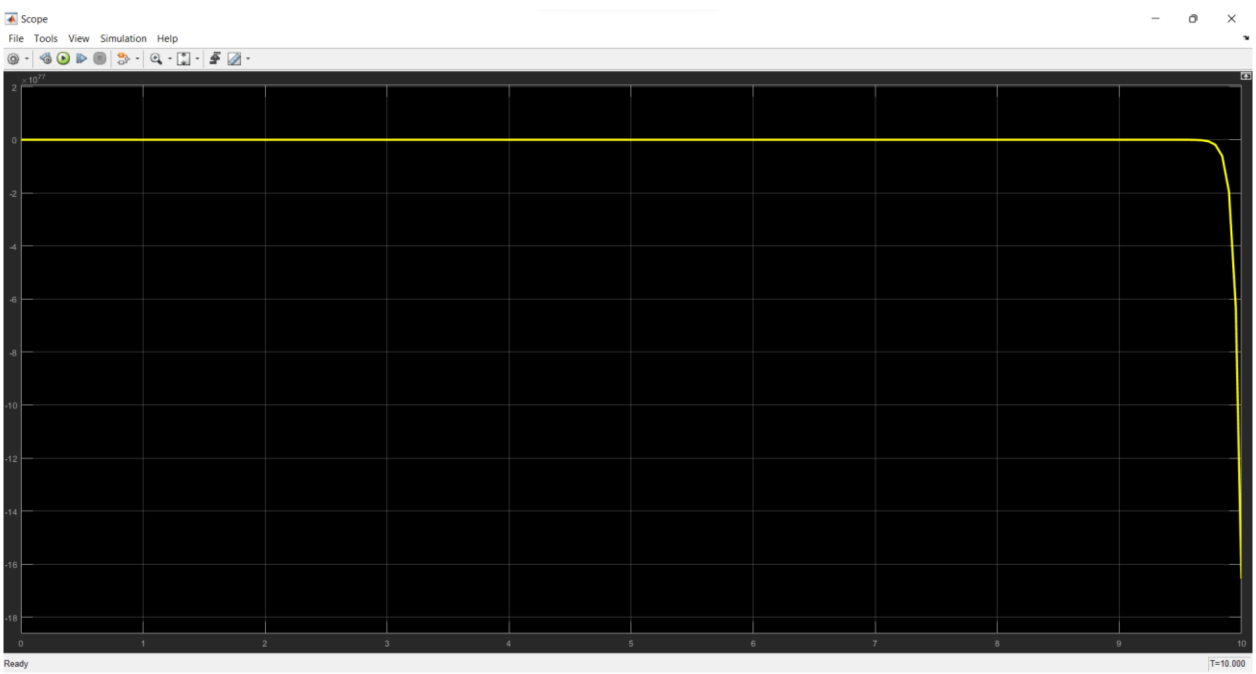
$$\frac{z}{I_c} = \frac{-\dfrac{2K_f}{m}\dfrac{K_a}{K_l}\dfrac{(I_o + a_1)}{(z_o + a_2)^2}(z_o + a_2)}{s^3 - \dfrac{1}{K_l}[(K_a - R)(z_o + a_2)]s^2 - \dfrac{2K_f}{mK_l}\dfrac{(I_o + a_1)^2}{(z_o + a_2)^3}[(K_a - R)(z_o + a_2)]}$$

$$\frac{z}{I_c} = \frac{-24.3794}{s^3 + 52.9258s^2 - 32254.6763}$$

Linear Simulink Model



Linear Response



TASK 3 – Expected performance

Zeros: None

Poles:

-36.9139 +13.4354i

-36.9139 -13.4354i

20.9019

Gain: -24.379

Zero-Pole-Gain form:

$$\text{tf} = \frac{-24.379}{(s - 20.9)(s^2 + 73.83s + 1543)}$$

$$\omega_n = \sqrt{1543} = 39.2810$$

$$\zeta = \frac{73.83}{\omega_n} = 1.8795$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{1 - \zeta^2}}{\zeta}\right) = 1.2444j$$

$$\omega_d = \omega_n * \sqrt{1 - \zeta^2} = 62.5114j$$

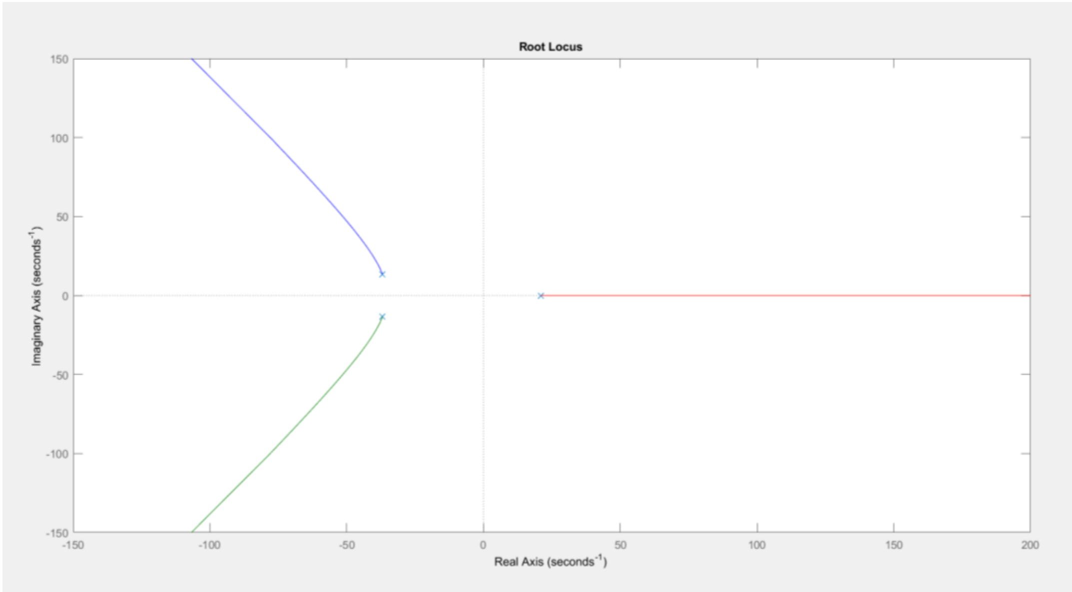
$$Risetime: \; t_r = \frac{\pi - \phi}{\omega_d} = -0.0199 \; - \; 0.0503j$$

$$\%OS = 100 * e^{\left(\frac{-\zeta pi}{\sqrt{1-\zeta^2}}\right)} = -84.2567 \; - 53.8592j$$

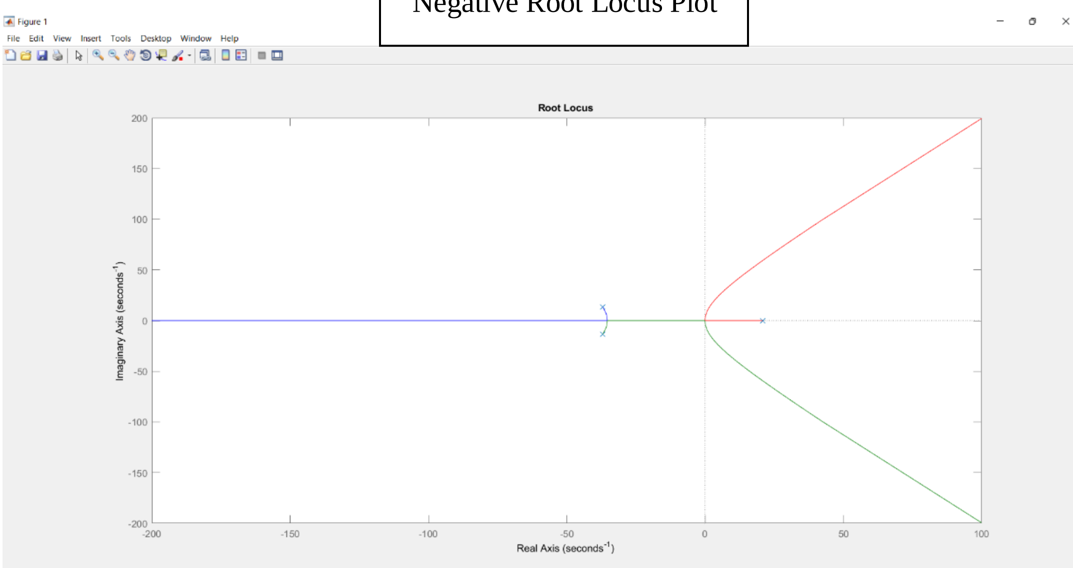
*It is important to note that using the ‘stepinfo()’ MATLab function for the transfer function, all pole-zero specifications returned either NaN or inf

TASK 4 – Preliminary Design Evaluation

Positive Root Locus Plot

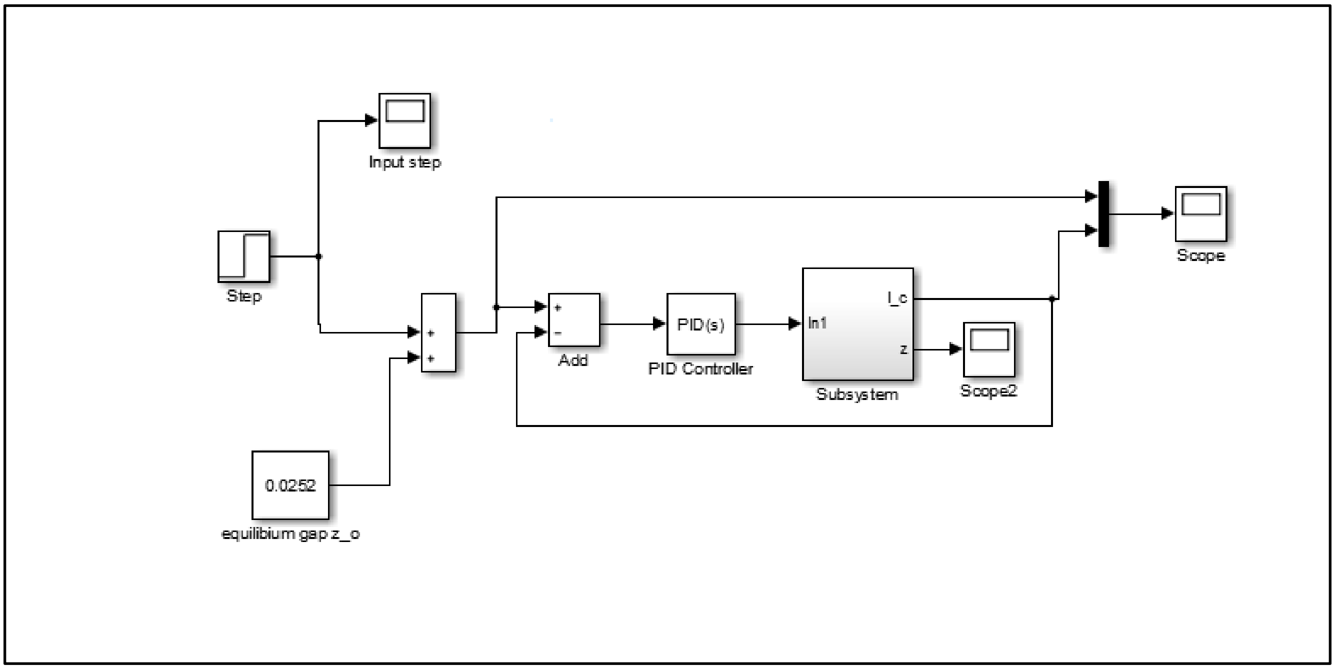


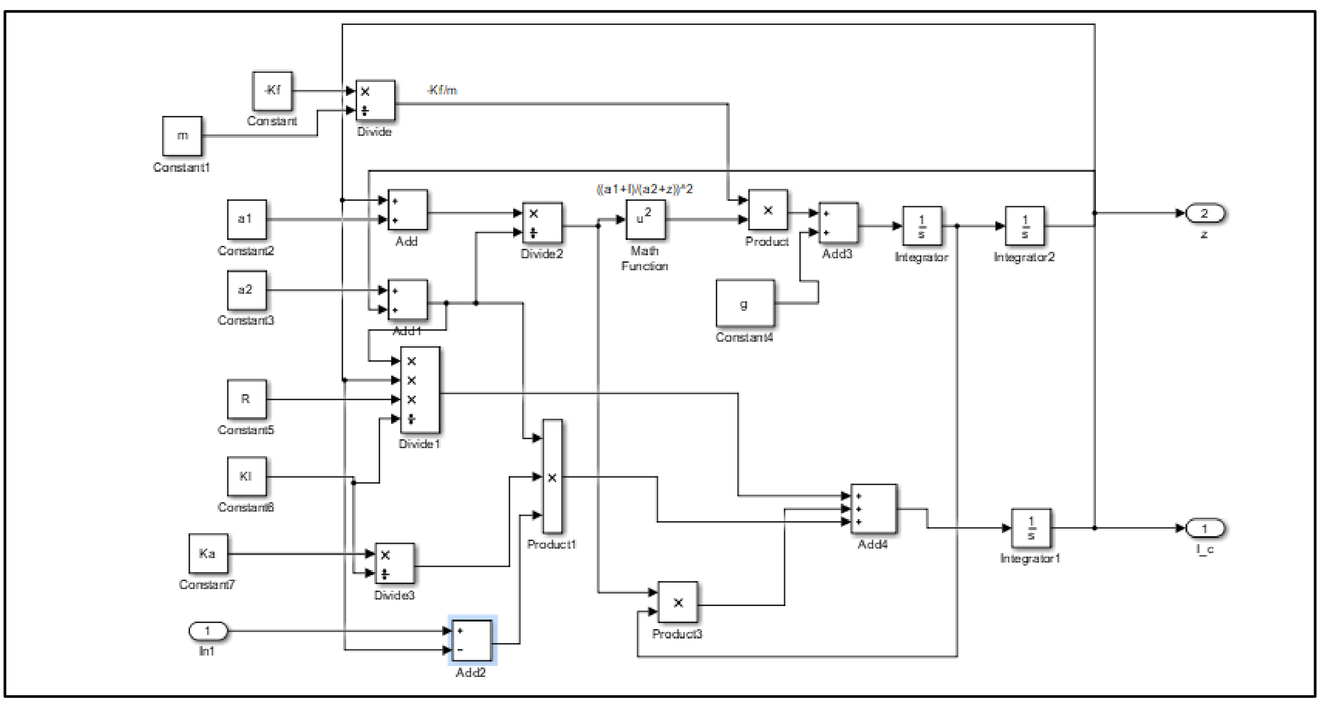
Negative Root Locus Plot

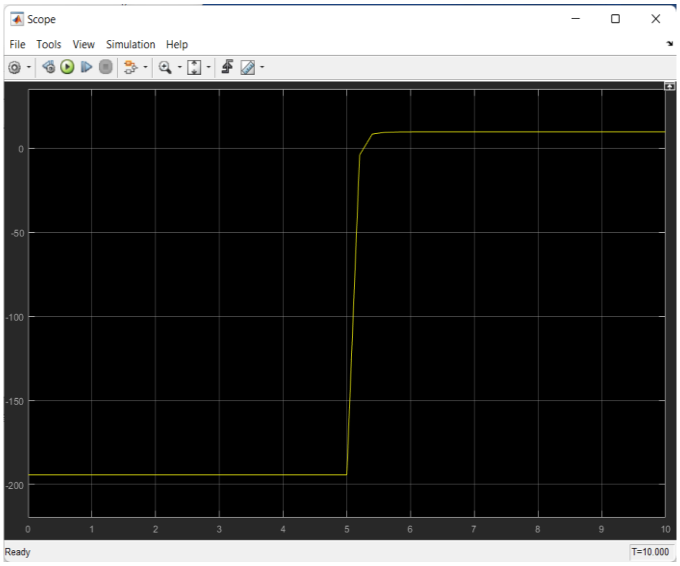


Since the system is unstable, P-control alone will not be enough to achieve the desired design parameters. The pole in the right-hand plane shows that Integral and derivative control are necessary as well. With P-control alone, the system will oscillate undesirably for an extended period of time when one position is required.

TASK 5 – Nonlinear Simulink model



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-100
-150
-200
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Ready
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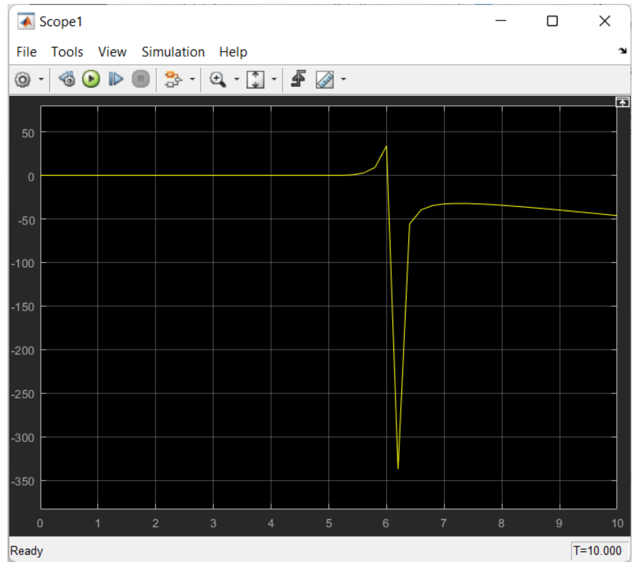
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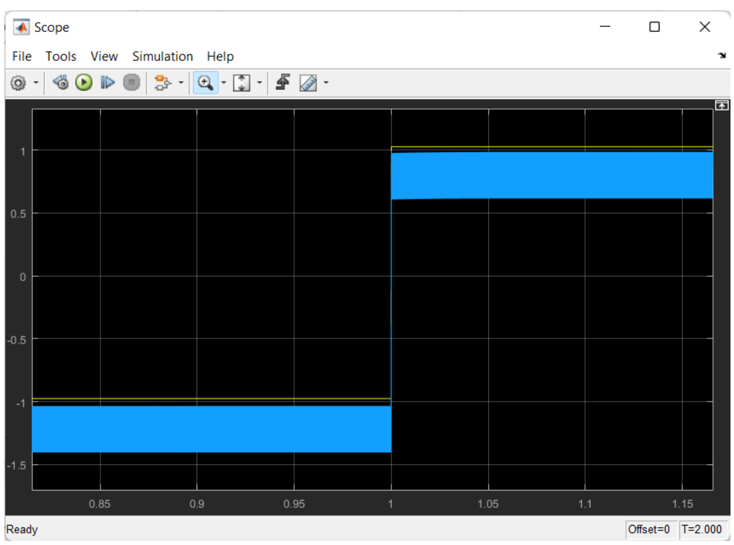
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-0.5
-1
-1.5
0.85 0.9 0.95 1 1.05 1.1 1.15
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References

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