Assignment 6 Report

MAE 438

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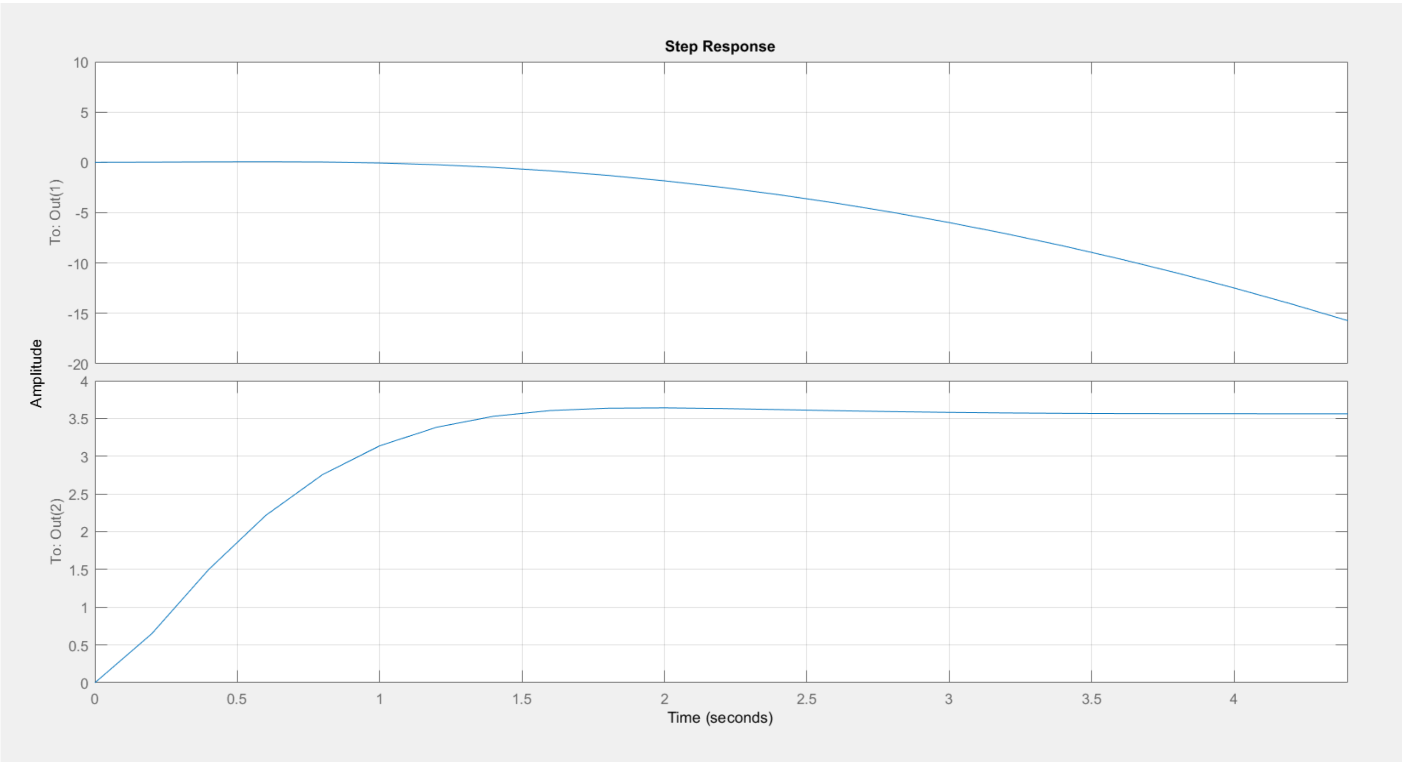
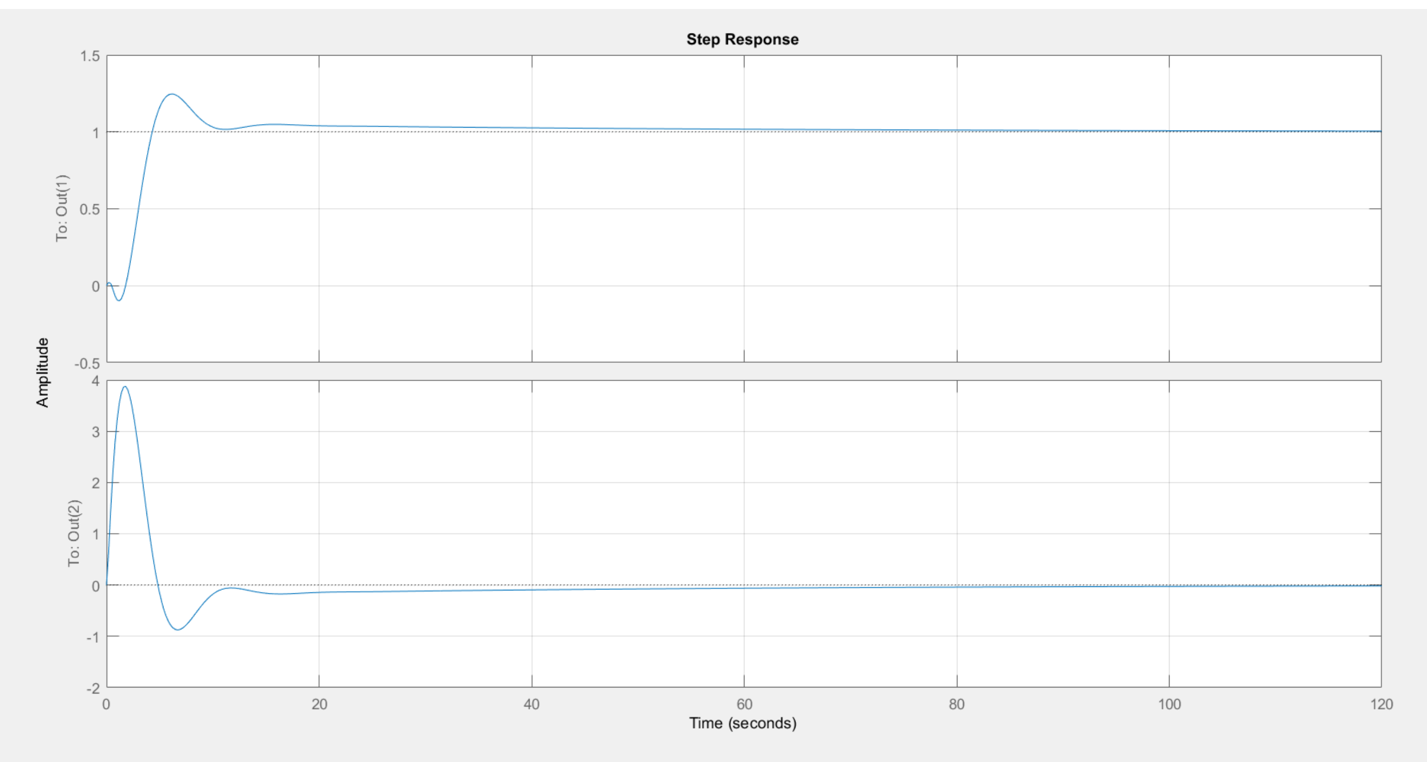
Compensator on theta system only

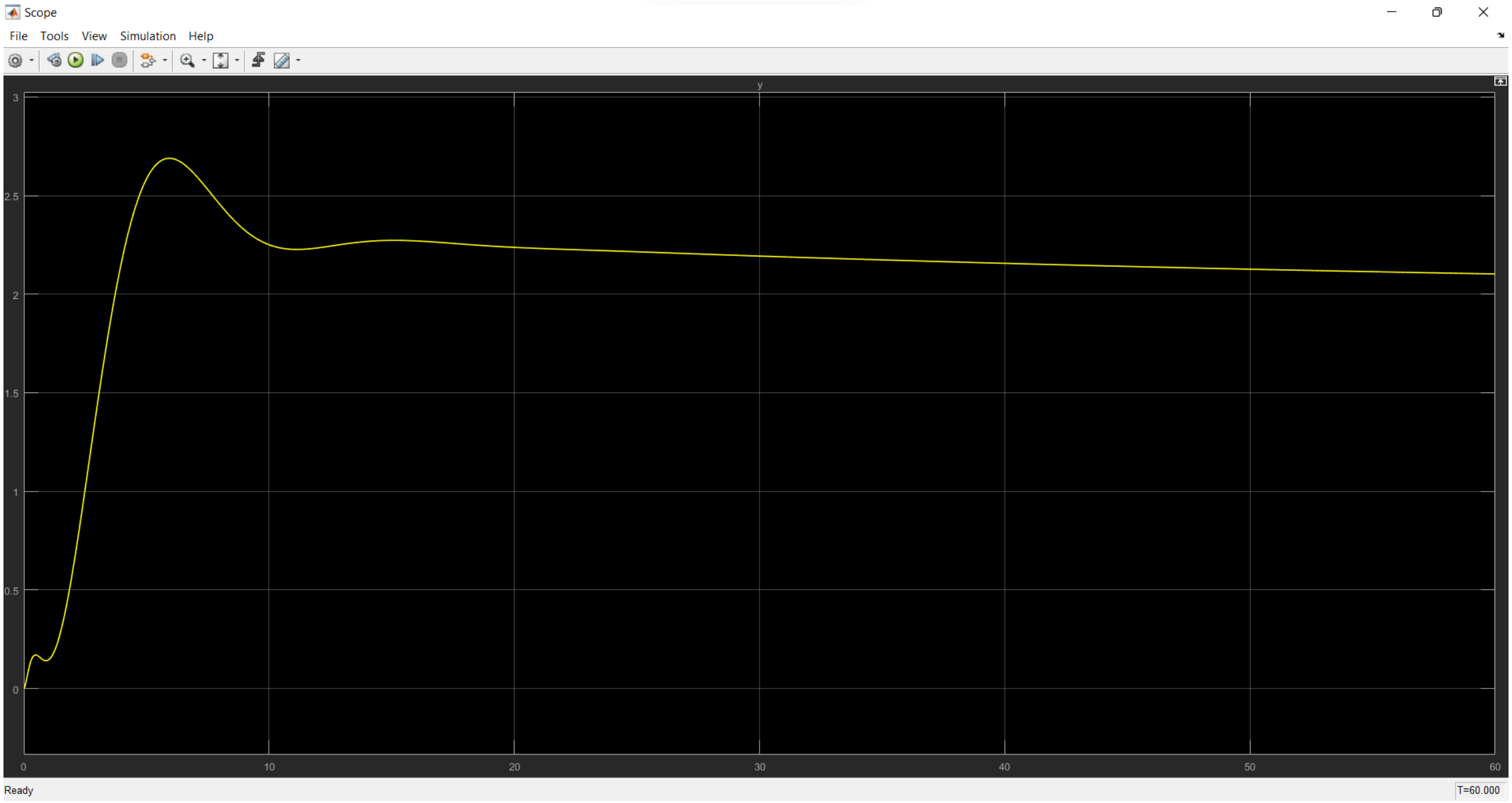
$$C_2(s) = -\frac{75s+300}{s+20}$$ stabilize theta first

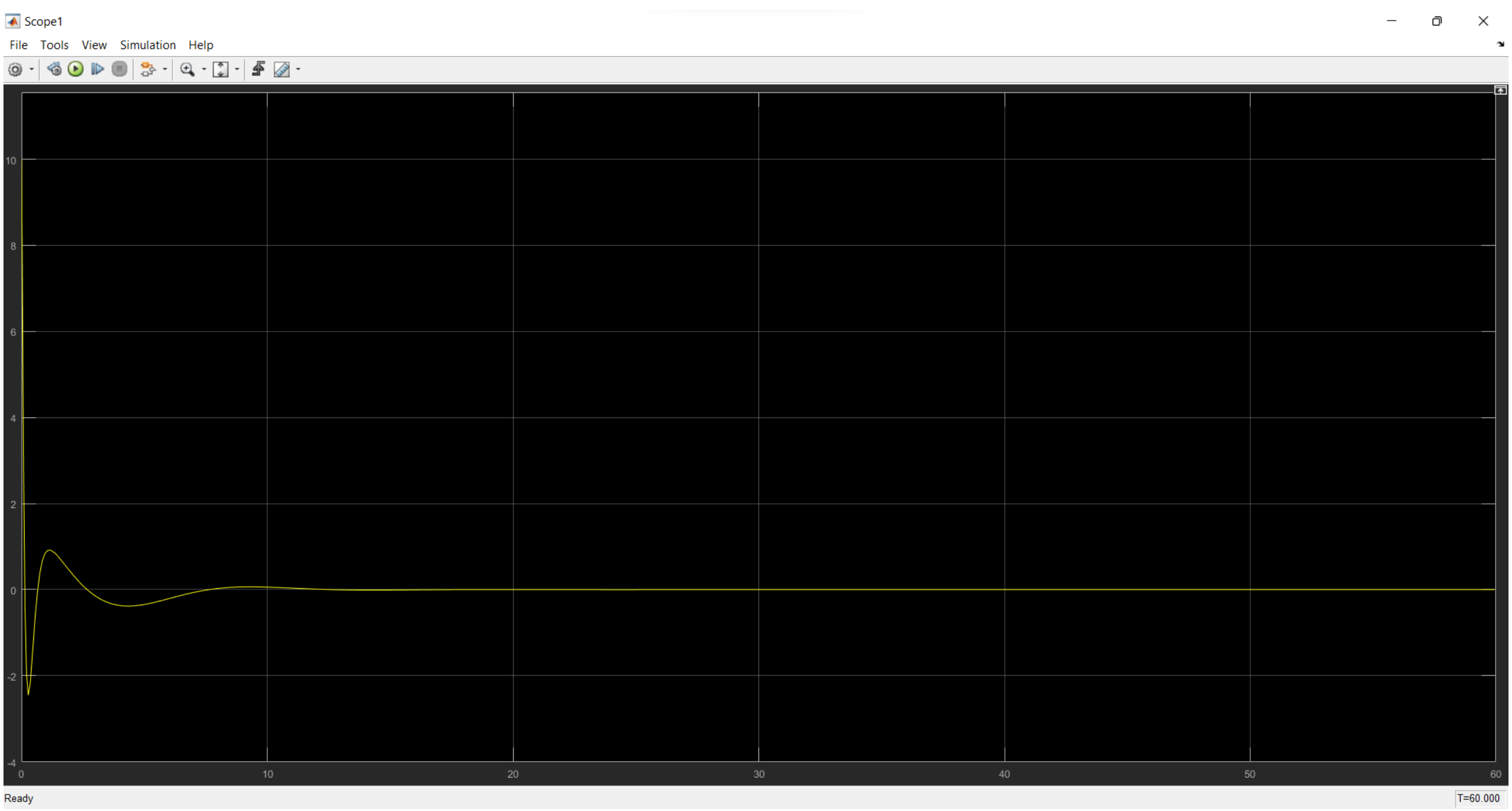


$$C_1(s) = \frac{3(s-1.5)(s+0.02)}{(s+7)(s+4)}$$

Compensator on both y and theta systems




Scope
File Tools View Simulation Help
y
3
2.5
2
1.5
1
0.5
0
0
10
20
30
40
50
60
Ready
T=60.000


Scope1
File Tools View Simulation Help
10
8
6
4
2
0
-2
-4
0
10
20
30
40
50
60
Ready
T=60.000

Problem 1 (Fredrick & Chow 10.1)
Calculate the controllability test matrix Q and evaluate the system for controllability. (Please don't use the Matlab `ctrb` command.)

$$A = \begin{bmatrix} -4 & 1 & 2 \\ 1 & -5 & 3 \\ 2 & 0 & -6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 0.5 \\ 2 \end{bmatrix}$$

```
% Q = [B A*B A^2*B ... A^(n-1)*B]
A1 = [-4 1 2;1 -5 3;2 0 -6];
B1 = [1;0.5;2];
Q1 = [B1,A1*B1,A1^2*B1];
uncontrollability1 = length(A1)-rank(Q1)
```

```
>> Q1

Q1 =

    1.0000    0.5000  -17.5000
    0.5000    4.5000  -52.0000
    2.0000  -10.0000   61.0000

>> uncontrollability1

uncontrollability1 =

     0
```

∴ **the system is controllable**

Problem 2
Calculate the controllability test matrix Q and evaluate the system for controllability. (Please don't use the Matlab `ctrb` command.)

$$A = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \\ 10 & 5 \end{bmatrix}$$

```
% Q = [B A*B A^2*B ... A^(n-1)*B]
A2 = [-2 0 0 0;
    0 -4 0 0;
    0 0 -5 0;
    0 0 0 0];
B2 = [1 0;0 1;2 0;10 5];
B22 = [1;0;2;10];
Q2 = [B2,A2*B2,A2^2*B2,A2^3*B2];
Q22 = [B22,A2*B22,A2^2*B22,A2^3*B22];
uncontrollability2 = length(A2)-rank(Q2)
uncontrollability22 = length(A2)-rank(Q22)
```

```
>> Q2

Q2 =

     1     0    -2     0     4     0    -8     0
     0     1     0    -4     0    16     0   -64
     2     0   -10     0    50     0  -250     0
    10     5     0     0     0     0     0     0

>> uncontrollability2

uncontrollability2 =

     0
```

```
>> Q22

Q22 =

     1    -2     4    -8
     0     0     0     0
     2   -10    50  -250
    10     0     0     0

>> uncontrollability22

uncontrollability22 =

     1
```

∴ **system 2 is controllable**

∴ **system 22 is uncontrollable**

Problem 3 (Fredrick & Chow 10.6)
Calculate the observability test matrix O and evaluate the system for observability. (Please don't use the Matlab `obsv` command.)

$$A = \begin{bmatrix} -4 & 1 & 2 \\ 1 & -5 & 3 \\ 2 & 0 & -6 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

```
% find observability test matrix O and
evaluate system for observability
A3 = [-4 1 2;1 -5 3;2 0 -6];
C3 = [0 1 0];
O3 = [C3;(A3'*C3')';(A3'^2*C3')'];
unobservable3 = length(A3)-rank(O3)
```

```
>> O3

O3 =

     0     1     0
     1    -5     3
    -3    26   -31

>> unobservable3

unobservable3 =

     0
```

∴ system 3 is observable

Problem 4 (Fredrick & Chow 10.7)
Calculate the observability test matrix O and evaluate the system for observability. (Please don't use the Matlab `obsv` command.)

$$A = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 2 & 0 & 1 \end{bmatrix}$$

```
A4 = [-2 0 0 0;0 -4 0 0;0 0 -5 0;0 0 0 0];
C4 = [1 2 0 1];
O4 =
[C4;(A4'*C4')';(A4'^2*C4')';(A4'^3*C4')'];
unobservable4 = length(A4)-rank(O4)
```

```
>> O4

O4 =

     1     2     0     1
    -2    -8     0     0
     4    32     0     0
    -8  -128     0     0

>> unobservable4

unobservable4 =

     1
```

∴ system 4 is unobservable

Problem 5 (Fredrick & Chow 10.3)

Use the Matlab `place` command to determine a state-feedback gain K to place the poles of the system of problem 1 at $s = -4, -8$, and -10. Verify the result by computing $A_c = A - BK$ and finding its Eigenvalues.

```
>> K1 = place(A1,B1,[-4 -8 -10])

K1 =

   -1.9852    3.4170    3.6384

>> A_c = A1-B1*K1

A_c =

   -2.0148   -2.4170   -1.6384
    1.9926   -6.7085    1.1808
    5.9705   -6.8339  -13.2768

>> eig(A_c)

ans =

   -4.0000
  -10.0000
   -8.0000
```

Problem 6 (Fredrick & Chow 10.4)

Use the Matlab `place` command to determine a state-feedback gain K to place the poles of the system at $s = -1, -2.5, -4.5$and -5.5.

$$A = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 10 \end{bmatrix}$$

Verify the result by computing $A_c = A - BK$ and finding its Eigenvalues.

```
>> K2 = place(A2,[1;1;2;10],[-1,-2.5,-4.5,-5.5])

K2 =

    0.3646    0.4219    0.0833    0.1547

>> A_c = A2-[1;1;2;10]*K2

A_c =

   -2.3646   -0.4219   -0.0833   -0.1547
   -0.3646   -4.4219   -0.0833   -0.1547
   -0.7292   -0.8437   -5.1667   -0.3094
   -3.6458   -4.2187   -0.8333   -1.5469
```

```
>> eig(A_c)

ans =

   -1.0000
   -2.5000
   -5.5000
   -4.5000
```

Problem 7

Use a state-transformation to convert to upper-companion form. Use a hand calculation to determine the state-feedback gain that places the poles at -9, -10, -11, -12. Transform the gain such that it will place the poles of the original system as prescribed. Verify the result.

$$A = \begin{bmatrix} 0 & 20 & -13 & 1 \\ 0 & 1 & 0 & 1 \\ -2 & -5 & -7 & 0 \\ 0 & 0 & 0 & 20 \end{bmatrix} \text{ and } B = \begin{bmatrix} -5 \\ 0 \\ 30 \\ -1 \end{bmatrix}$$

Upper companion form

```
A_u =

   14.0000  153.0000 -686.0000  520.0000
    1.0000    0.0000    0.0000   -0.0000
         0    1.0000    0.0000   -0.0000
   -0.0000    0.0000    1.0000    0.0000


B_u =

    1.0000
         0
   -0.0000
         0
```

Finding the gain

poles = -9,-10,-11,-12; $\quad polynomial = (s+9)(s+10)(s+11)(s+12)$

$$expanded\ polynomial = s^4 + 42s^3 + 659s^2 + 4578s + 11880$$

Coefficients = [1, 42, 659, 4578, 11880]

Code for problem 7

```
A = [0 20 -13 1;0 1 0 1;-2 -5 -7 0;0 0 0 20];
B = [-5;0;30;-1];
 p = poly(A);
 for i = 1:length(p)-1
     p2(i) = p(i);
 end
 % p = [1 6 11];
 k = rank(A);
 W = zeros(k);
 n = p2;
 for i = 1:k
     m = zeros(size(n));
     m(i:end)=n(1:k-(i-1));
     W(i,:) = m;
 end
 Q = ctrb(A,B);
 T = Q*W;
 A_u = inv(T)*A*T
 B_u = inv(T)*B

 a = A_u(1,:)';
 ac = poly([-9 -10 -11 -12])'; % poles: -9 -10 -11 -12
 ac = ac(2:end);
 K = (ac-a)';
 A_c = A_u-B_u*K
```

Poles : $-9, -10, -11, -12$

Polynomial : $(s+9)(s+10)(s+11)(s+12)$

$$= s^4 + 42 s^3 + 659 s^2 + 4578 s + 11880$$

~ Gain Equation: $K^T = (\vec{a}_c - \vec{a})^T$

$$\vec{a}_c = \begin{bmatrix} 42 \\ 659 \\ 4578 \\ 11880 \end{bmatrix} ; \quad \vec{a} = \begin{bmatrix} 14 \\ 153 \\ -686 \\ 520 \end{bmatrix}$$

$K = [28, 506, 5264, 11360]$