MAE 438 Hybrid Maglev Project
Part 3
Peter DeTeresa

TASK 1 – Evaluate the system for controllability and observability

TASK 2 – Design state-feedback controller based on pole placement.

TASK 3 – Simulate time response of closed-loop linear state-space system using same type of input

TASK 4 – Evaluate time response of non-linear Simulink model under state feedback control

TASK 5 – Compare performance and control effort of state feedback and classical designs with both linear and nonlinear plants.

TASK 1 – Evaluate the system for controllability and observability

State space representation of system:

$$A = \begin{bmatrix} -52.936 & -1218.9 & -33566 & -46318 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad C = [0 \quad 65823 \quad 1219 \quad 46320] \qquad D = 0$$

The controllability matrix for the system is:

$$\text{Controllability matrix} = \begin{bmatrix} 1 & -52.936 & 1583.3 & -52855 \\ 0 & 1 & -52.936 & 1583.3 \\ 0 & 0 & 1 & -52.936 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Determine the number of uncontrollable states by comparing the rank of the controllability matrix to the size of matrix A

$$uncontrollable\ states = 4 - 4 = 0$$

$$\therefore \text{ the system is controllable}$$

The observability matrix for the system is:

$$\text{Observability matrix} = \begin{bmatrix} 0 & 65823 & 1219 & 46320 \\ 65823 & 1219 & 46320 & 0 \\ -3.4832e+06 & -8.0186e+07 & -2.2094e+09 & -3.0488e+09 \\ 1.042e+08 & 2.0363e+09 & 1.1387e+11 & 1.6134e+11 \end{bmatrix}$$

Determine the number of unobservable states by comparing the rank of the observability matrix to the size of matrix A

$$unobservable\ states = 4 - 4 = 0$$

$$\therefore all\ states\ are\ observable$$

TASK 2 – Design state-feedback controller based on pole placement.

Equations for feedback controller design using pole placement

$$T = (ControllabilityMatrix) * (W)$$

$$\vec{\vec{a}} = T^{-1} * a * T \qquad \vec{\vec{b}} = T^{-1} * b$$

$$\vec{a} = \begin{bmatrix} 52.9358000000001 \\ 1218.90000000000 \\ 33565.6000000000 \\ 1218.90000000000 \end{bmatrix} \qquad \vec{a}_c = \begin{bmatrix} -36056.3358000000 \\ -36056.3358000000 \\ -54279173612.7977 \\ 2639854858819.44 \end{bmatrix}$$

$$\underline{K}^T = \overrightarrow{a_c} - \vec{a} = \begin{bmatrix} -36109.2716000000 \\ 85216586.7717200 \\ -54279207178.3977 \\ 2639854857600.54 \end{bmatrix}$$

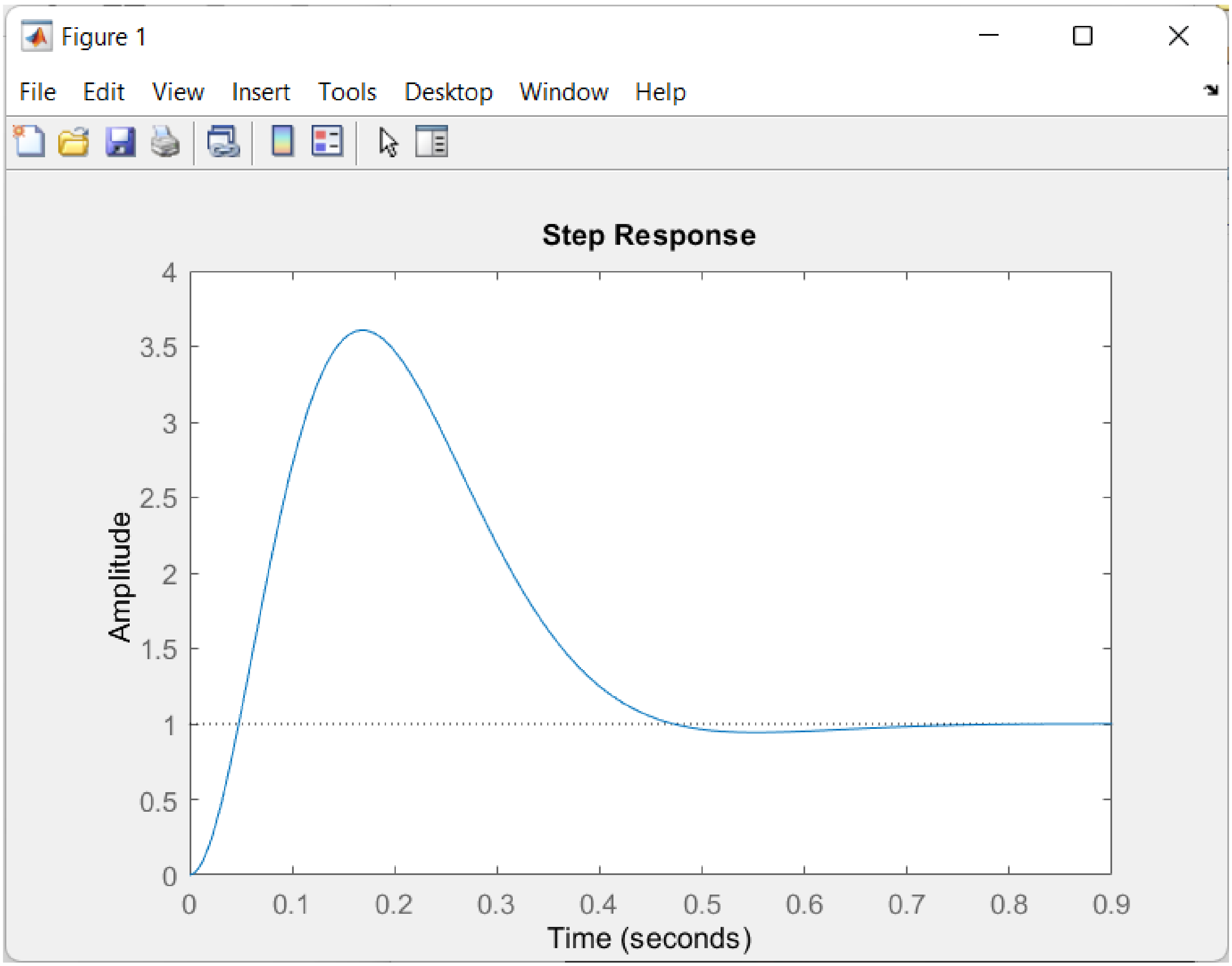
The closed loop plant matrix is now:

$$\underline{A} - \underline{B} * \underline{K} = \begin{bmatrix} 36056.335800 & -85217805 & 54279173612 & -2639854903918 \\ 1 & 0 & 0 & 1.3824e-10 \\ 0 & 1 & 0 & -5.0022e-12 \\ 0 & 0 & 1 & 9.9476e-14 \end{bmatrix}$$

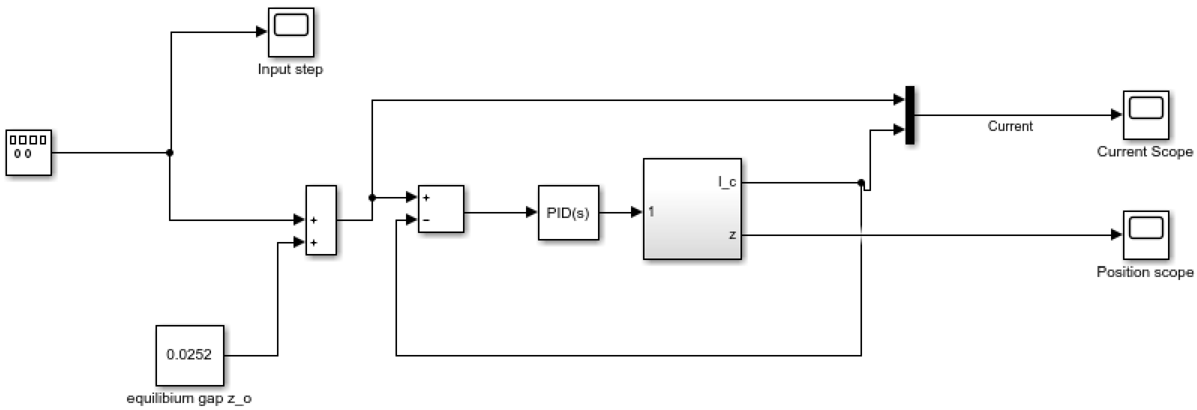
Eigenvalues of system:

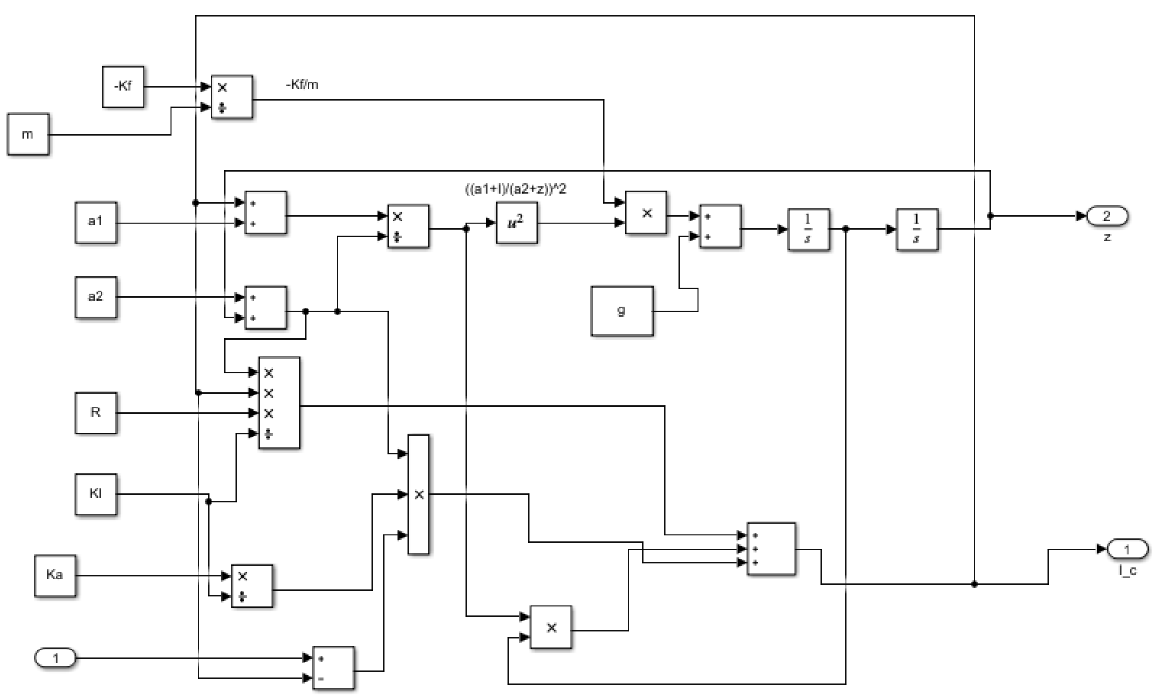
$$\lambda = -42.272, \qquad -4.606 \pm 27.084i, -1.4518$$

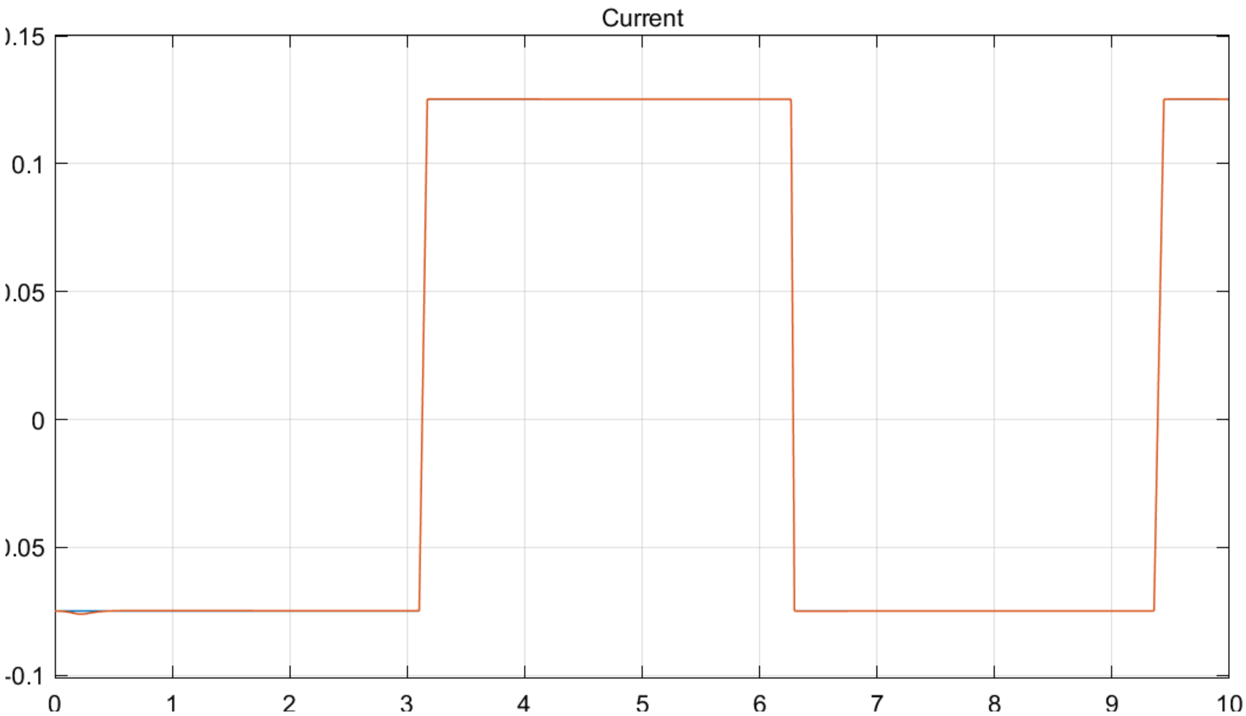
TASK 3 – Simulate time response of closed-loop linear state-space system using same type of input

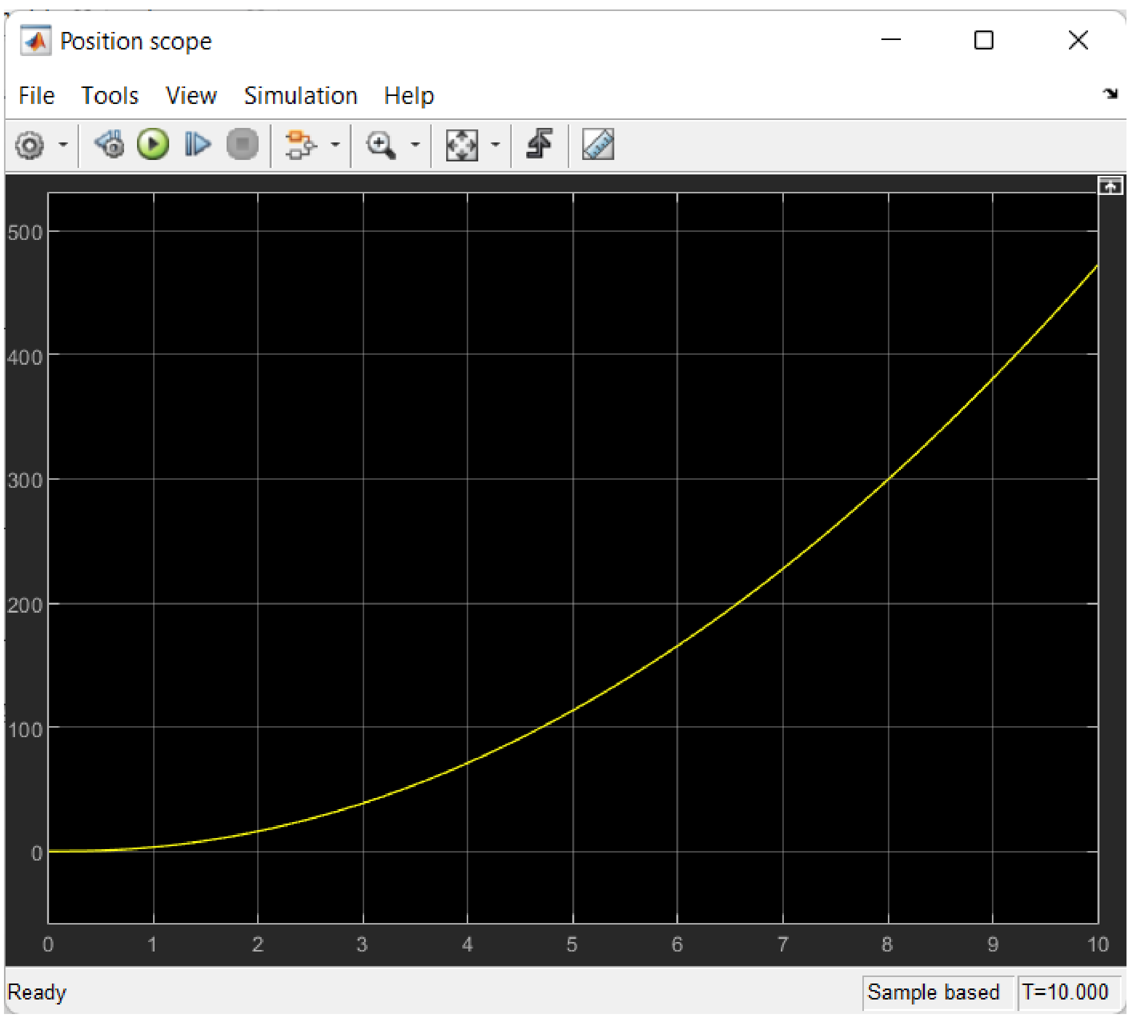


TASK 4 – Evaluate time response of non-linear Simulink model under state feedback control



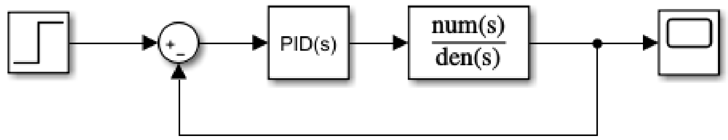



Current
).15
0.1
).05
0
).05
-0.1
0
1
2
3
4
5
6
7
8
9
10

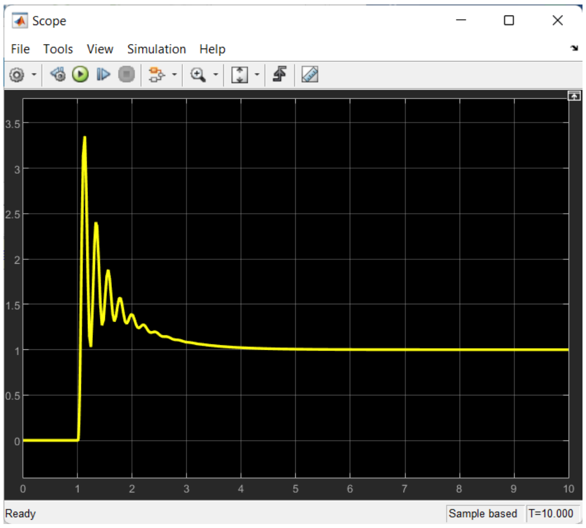

Position scope
File
Tools
View
Simulation
Help
500
400
300
200
100
0
0
1
2
3
4
5
6
7
8
9
10
Ready
Sample based
T=10.000

TASK 5 – Compare performance and control effort of state feedback and classical designs with both linear and nonlinear plants.

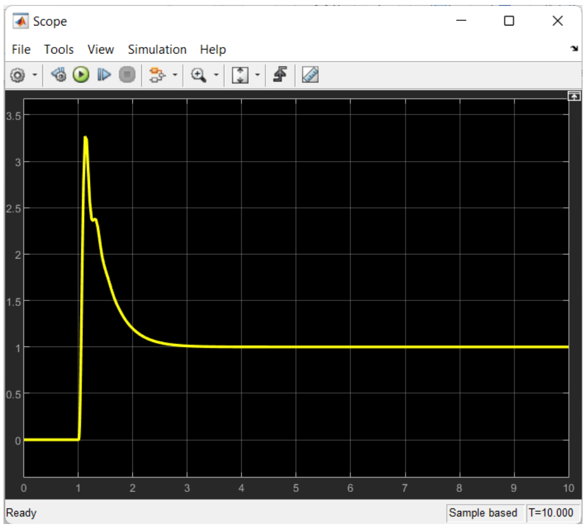
Linear Plant



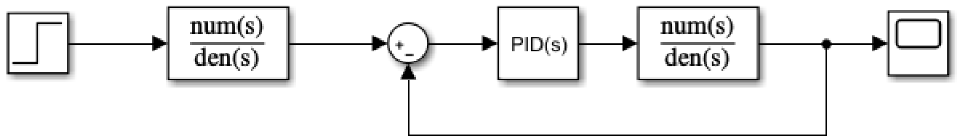
Kp = -2500, Ki = 1500, Kd = -50



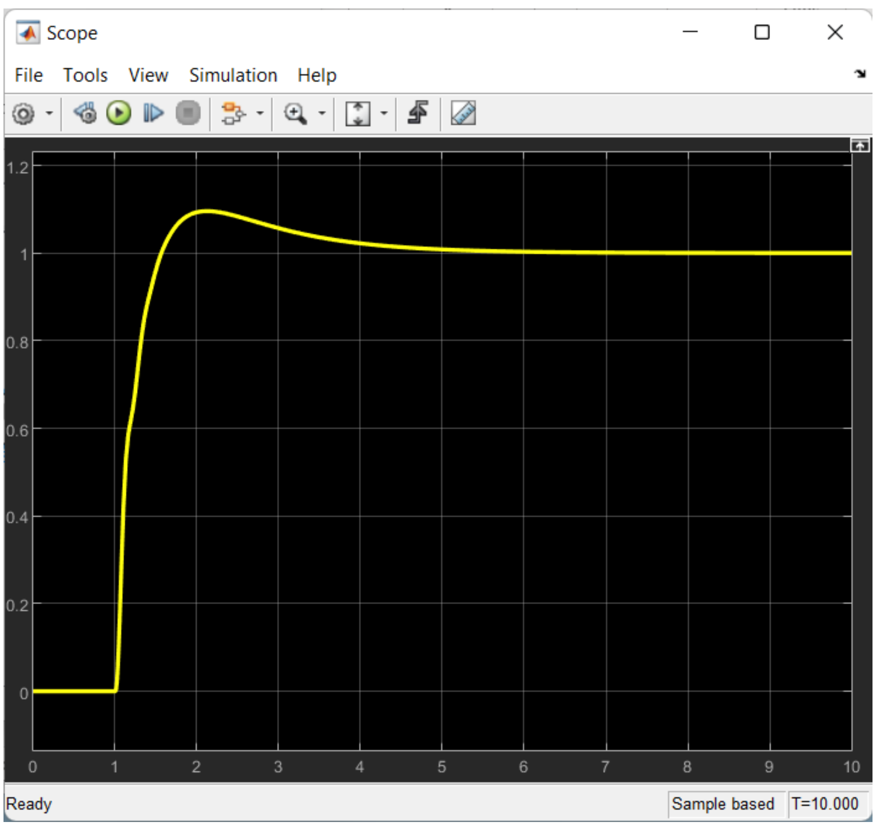
Kp = -2000, Ki = -1500, Kd = -60



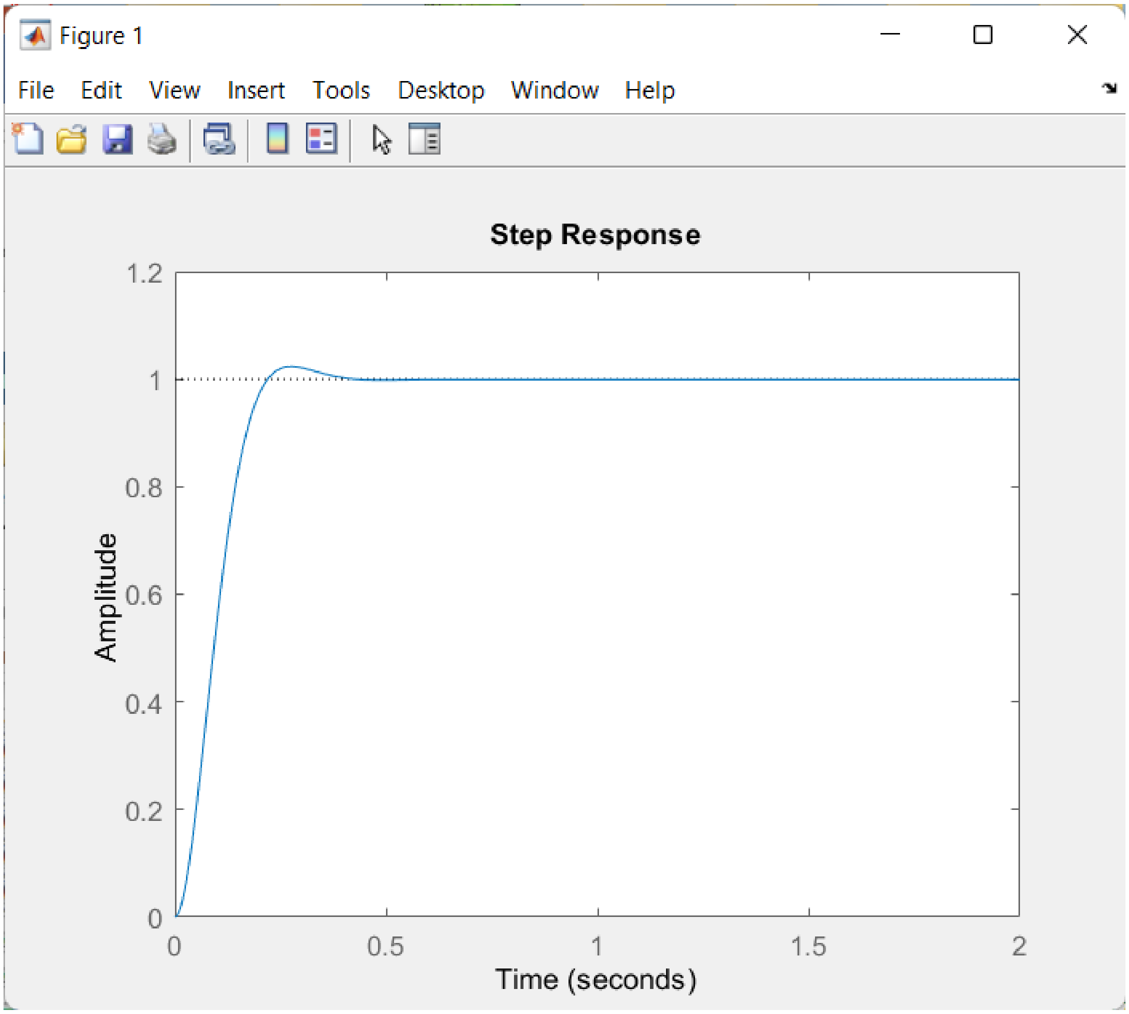
Introducing a prefilter from part 2 of the project leads to an improved linear step response. For a step input = 1:



Employ prefilter transfer function $G(s) = \frac{0.1s+1}{s+1}$



Comparing the Simulink results with a MATLAB script, we can see the improvement in our step response:

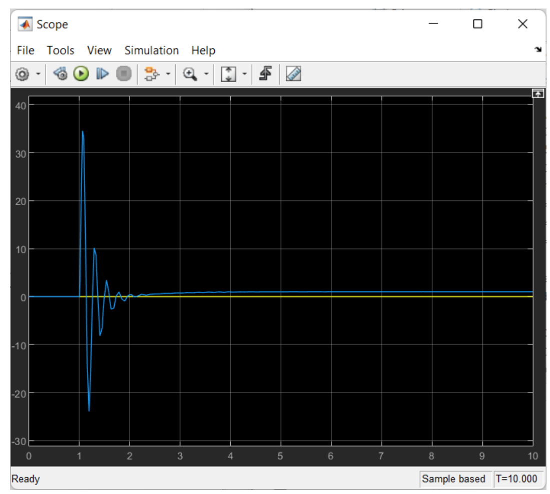


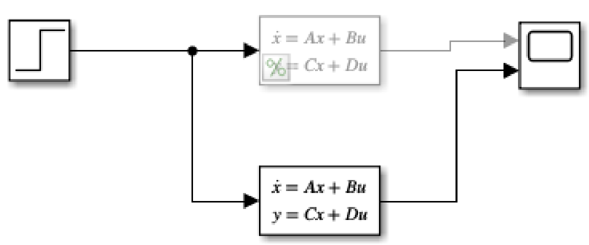
RiseTime: 0.13559
SettlingTime: 0.30665
SettlingMin: 0.90322
SettlingMax: 1.0235
Overshoot: 2.3459
Undershoot: 0
Peak: 1.0235
PeakTime: 0.27511

Fdafdafda

From the linear simulation, the settling time is much improved using the PID controller with a prefilter design. The derivative control was increased slightly to smooth out some of the oscillations early on in the step response. However, the nonlinear simulation did not take well to the prefilter and so I left it out of the nonlinear response. The plot took on a sort of shark-tooth shape which is not at all the desired effect. The goal is to reduce the oscillations to reduce settling time and improve stability, so judging by the linear simulation the prefilter is desirable but the values must be altered in order to achieve optimal performance.

The state-space model unfortunately was not successful for me. I tried using the place command in MATLAB to add a pole in the A matrix to improve the response of the linear system, but the oscillations in the beginning could not be rectified. This is why for part 4 of the project I will recommend that the designer stick with the classical PID control design.





```
% find controllability test matrix Q without using MATLAB
ctrb function
% Q = [B A*B A^2*B ... A^(n-1)*B]
A1 = [-4 1 2;1 -5 3;2 0 -6];
B1 = [1;0.5;2];
Q1 = [B1,A1*B1,A1^2*B1];
uncontrollability1 = length(A1)-rank(Q1)

A2 = [-2 0 0 0;
    0 -4 0 0;
    0 0 -5 0;
    0 0 0 0];
B2 = [1 0;0 1;2 0;10 5];
B22 = [1;0;2;10];
Q2 = [B2,A2*B2,A2^2*B2,A2^3*B2];
Q22 = [B22,A2*B22,A2^2*B22,A2^3*B22];
uncontrollability2 = length(A2)-rank(Q2)
uncontrollability22 = length(A2)-rank(Q22)

% find observability test matrix O and evaluate system for
observability
A3 = [-4 1 2;1 -5 3;2 0 -6];
C3 = [0 1 0];
O3 = [C3;(A3'*C3')';(A3'^2*C3')'];
%      0     1     0
%      1    -5     3
%     -3    26   -31
unobservable3 = length(A3)-rank(O3)

A4 = [-2 0 0 0;0 -4 0 0;0 0 -5 0;0 0 0 0];
C4 = [1 2 0 1];
O4 = [C4;(A4'*C4')';(A4'^2*C4')';(A4'^3*C4')'];
unobservable4 = length(A4)-rank(O4)
```

```
% define state space A,B,C,D
clear;clc;
Kp = -2700;Ki = -1900;Kd = -50;
g = tf(-24.379,[1 52.93 -0.047 -32249]);
num = -24.379*[Kp Kd Ki];
den = [1 52.9358 -24.378*Kd -(32255+24.378*Kp) -24.378*Ki];
[a,b,c,d]=tf2ss(num,den);
s = ss(a,b,c,d);
Mc = ctrb(a,b);
Mo = obsv(a,c);
W = [1,52.9358000000001,1218.90000000000,33565.6000000000;
    0,1,52.9358000000001,1218.90000000000;
0,0,1,52.9358000000001;
0,0,0,1]; % w matrix
T = Mc*W;
a_bar = inv(T)*a*T;
b_bar = inv(T)*b;
a_vec =
[52.9358000000001,1218.90000000000,33565.6000000000,1218.90
000000000];
ac_vec = [-36056.3358000000,85217805.6717200,-
54279173612.7977,2639854858819.44];
K_T = ac_vec-a_vec;
q = a_bar-b_bar*K_T
```