By: Lones Smith &
Tono Carrasco

1. Solutions

1. **Contract Curves**

(i) Computing the MRS for each consumer we get,

$$MRS^A_{x_1x_2} = \frac{ax_2}{x_1} \text{ and } MRS^B_{x_1x_2} = \frac{x_2}{x_1}$$

Making them equal, we get

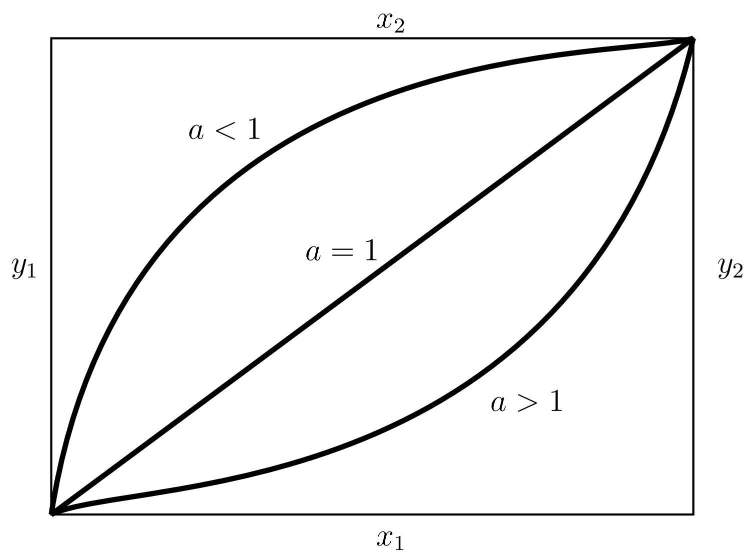
$$\frac{ax_2^A}{x_1^A} = \frac{x_2^B}{x_1^B}$$

We know that $x_2^B = w_2 - x_2^A$ and $x_1^B = w_1 - x_1^A$, so

$$\frac{ax_2^A}{x_1^A} = \frac{w_2 - x_2^A}{w_1 - x_1^A}$$

Solving for x_2^A we get,

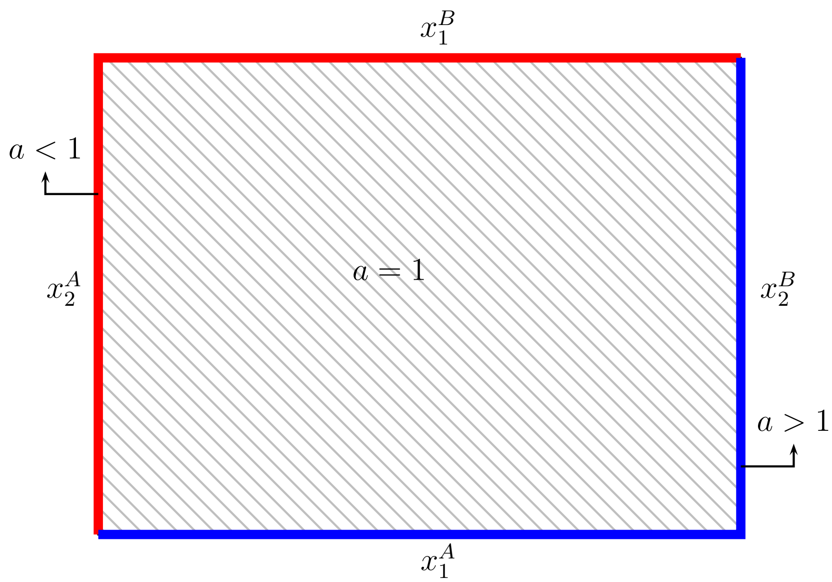
$$x_2^A = \frac{w_2 x_1^A}{a(w_1 - x_1^A) + x_1^A}$$



(ii) Computing the MRS for each consumer we get,

$$MRS^A_{x_1x_2} = a \text{ and } MRS^B_{x_1x_2} = 1$$

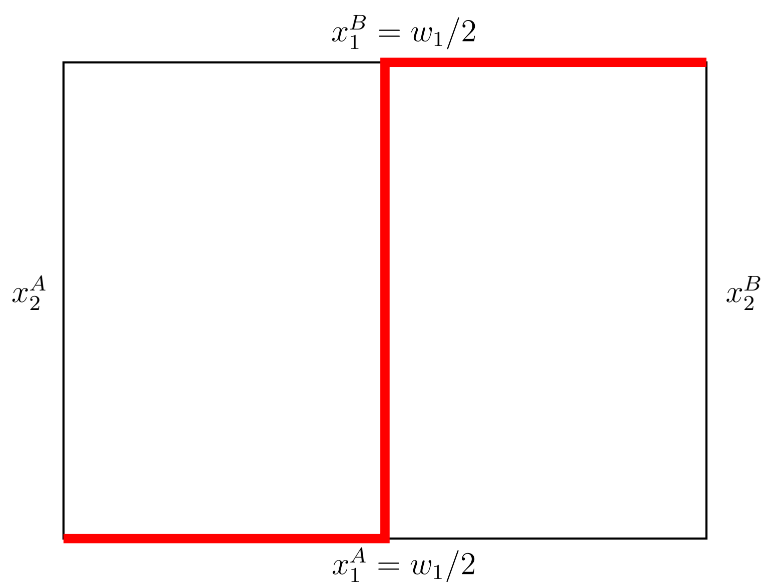
So if $a > 1$ the indifference curves for consumer A are steeper than for consumer B. This imply that the contract curve is the lower right limits of the edge worth box (in black in the figure). If $a < 1$, then the indifference level for consumer A is flatter than for consumer B hence the contract curve is the upper left limits of the edge worth box (in grey in the figure). If $a = 1$, then is all the box.



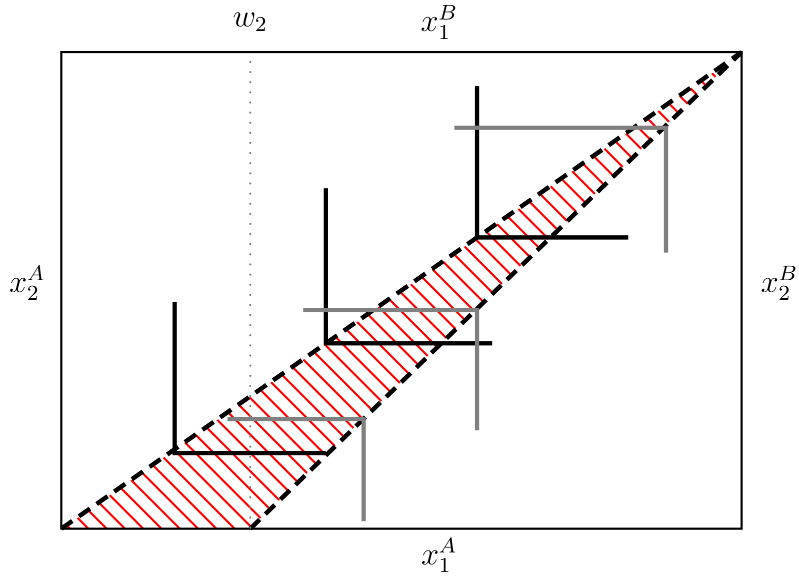
(iii) Preferences are convex, so we can solve equalizing the MRS. Computing the MRS for each consumer we get,

$$MRS^A_{x_1x_2} = \frac{1}{x_1^A} \text{ and } MRS^B_{x_1x_2} = \frac{1}{x_1^B} \Rightarrow x_1^A = x_1^B = w_1/2$$

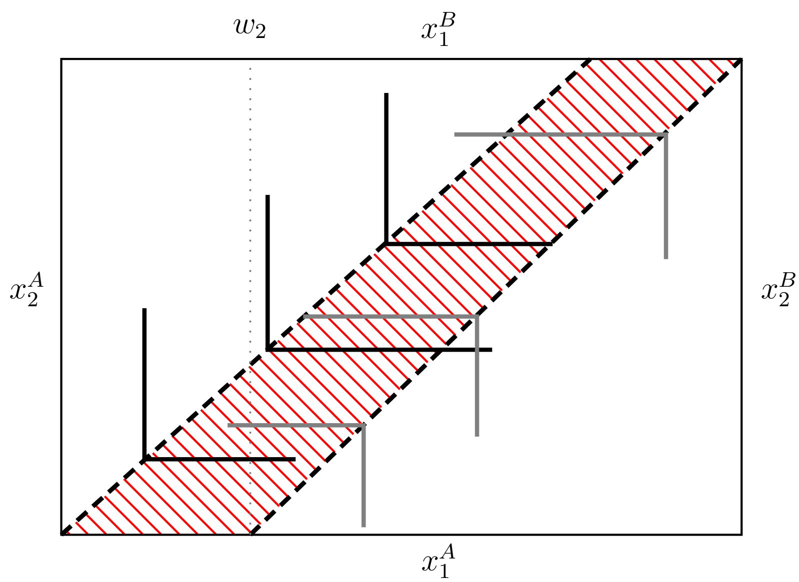
For corner solutions when $x_2^A = 0$ and $x_1^A \leq \frac{w_1}{2}$ or $x_2^B = 0$ and $x_1^A \geq \frac{w_1}{2}$ we will not have tangency, but still this allocations are Pareto efficient.



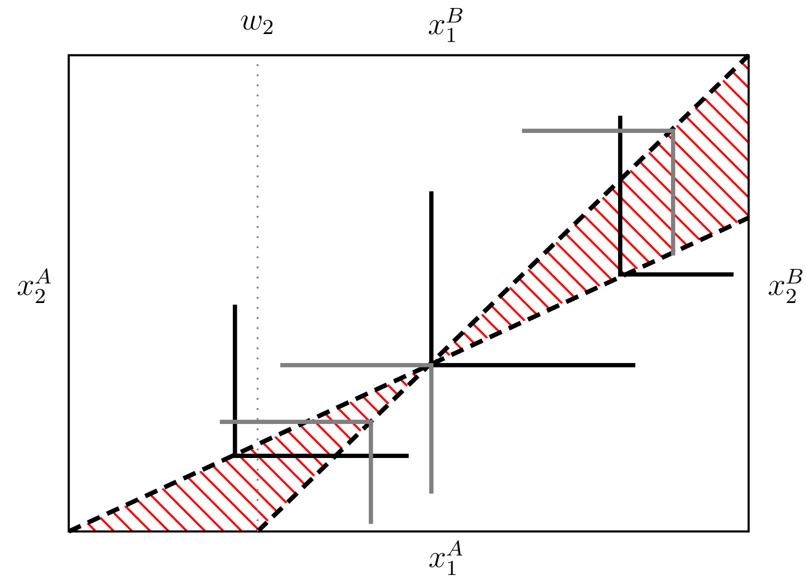
(iv) For consumer B is efficient to have $x_1^B = x_2^B$ and for consumer A to have $ax_1^A = x_2^A$. With our loss of generality consider the case when $w_1 > w_2$ (the case $w_1 < w_2$ is equivalent), then if $a = w_2/w_1$ the contract curve is the highlighted area in the following figure:



If $a > w_2/w_1$, the contract curve is the highlighted area in the following figure:



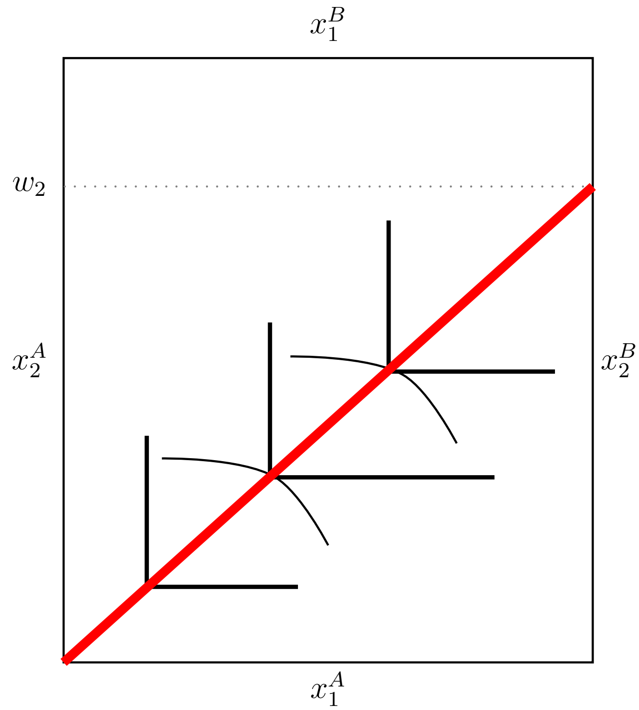
And if $a < w_2/w_1$ then



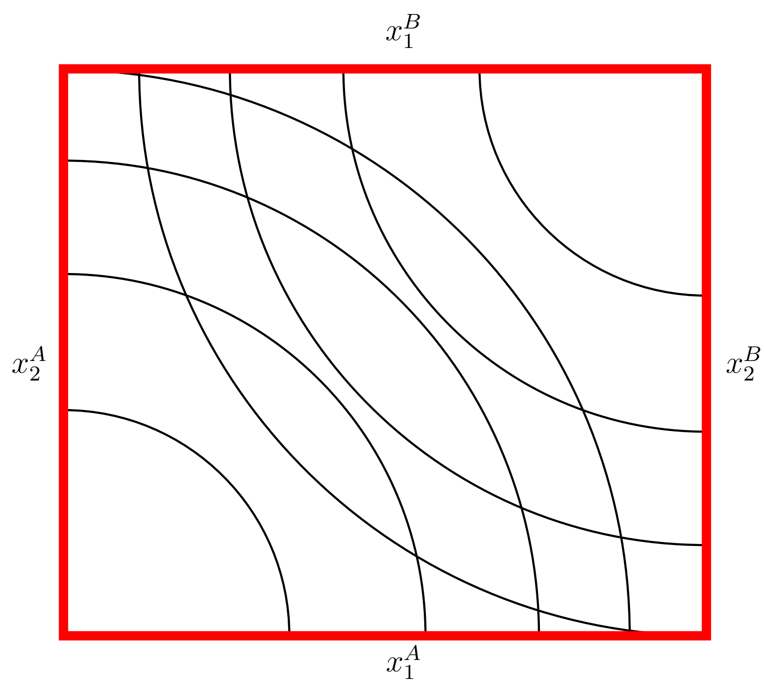
(v) In this case the $MRS_{x_1x_2}$ for consumer B is

$$MRS^B_{x_1x_2} = \left(\frac{x_2}{x_1}\right)^{\frac{1}{2}}$$

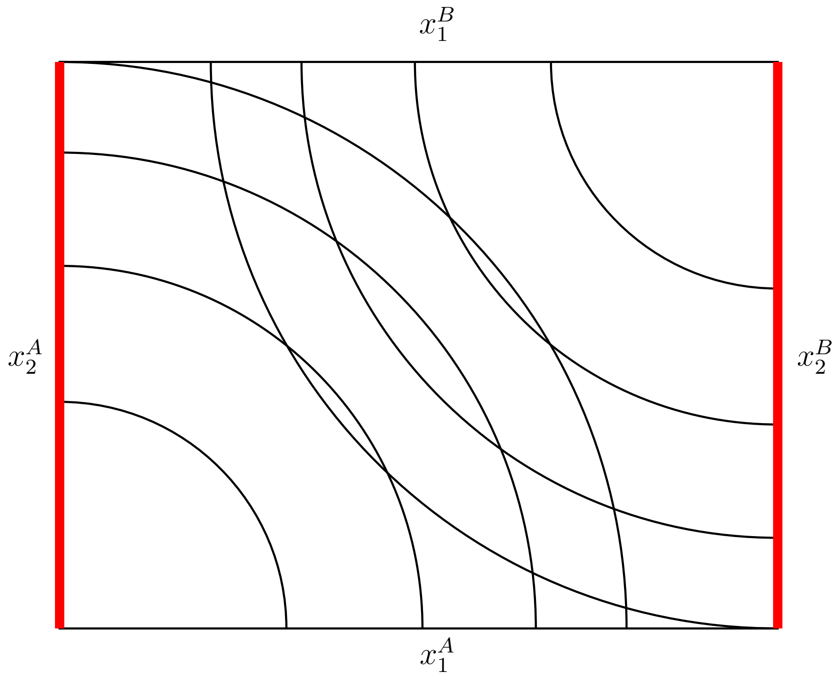
For consumer A we cannot compute this MRS, but we know that the efficient allocations are where $x_1^A = x_2^A$. Then the contract curve is as shown in the following figure. Notice that the contract curve is only the set of interior solutions along A's kink line. The allocations along the right edge above the kink line (i.e. those where A has extra good 2 and B has no good 1) are not efficient because B can be made better off and A no worse off by having A give some good 2 to B.



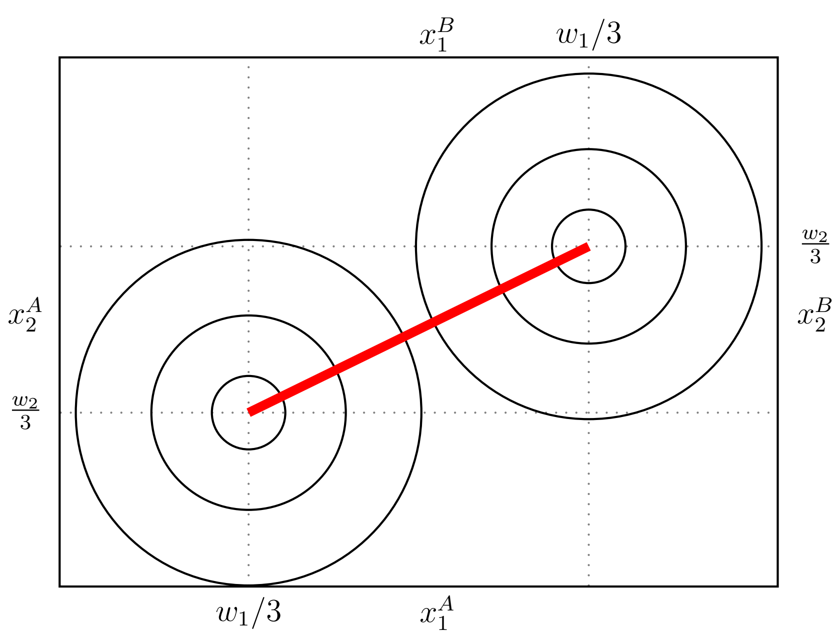
(vi) Notice that utility functions are convex so the lower contour set is convex. So, by equalizing MRS we would not be finding the optimum. If $w_1 = w_2$, then the contract curve are all the allocations in the border of the box. Graphically,



Consider only the case when $w_1 > w_2$ ($w_1 < w_2$ is equivalent). In this case the contract curve are the left and right borders of the box, where good 1 is completely assigned to one consumer and good 2 is divided in any way.



(vii) Both utility functions achieve the highest level at $(w_1/3, w_2/3)$ so the contract curve are all the allocations in the line that connects both bliss points.



2. **Monopolist in a Box**

(i) Equilibrium price $p = 1$ for interior solution since Mikey has $MRS = 1$. Collin would optimally choose where $MRS = p$ which imply $9 - 2x = 1$ hence $x^C = 4$ and $m^C = 16$. Mikey would have the rest, namely $x^M = 16$ and $m^M = 4$. The outcome is Pareto efficient.

(ii) Collin's demand for good x from part (i) is $x^C = (9 - p)/$ and thus $m^C = 20 - p(9 - p)/3$. Then Mikey sets p to maximize

$$U = (20 - x^C) + (20 - m^C) = 40 - (x^C + m^C)$$

Plugging the demands for Collin, we get

$$U = 20 + \frac{(9 - p)(p - 1)}{2}$$

Maximizing in p we get, $p^M = 5$, so $x^C = 2, m^C = 10$. The rest if for Mikey, namely $x^M = 18$ and $m^M = 10$. The allocation is not efficient, Mikey's $MRS = 1$, Collin's $MRS = 5$. Notice how the utility levels went from $V = 36$ to $V = 24$ for Collin, whereas from $U = 20$ to $U = 28$ for Mikey.

(iii) Mikey will find it optimal to induce the allocation on the contract curve (so efficient) that is also on Collin's indifference curve going through his endowment. The indirect utility function for Collin is,

$$V = \left(\frac{9 - p}{2}\right)\left(9 - \left(\frac{9 - p}{2}\right)\right) + 20 - p\frac{(9 - p)}{2}$$

Which is,

$$V = 20 + \frac{(9 - p)^2}{4}$$

So Mikey maximizes,

$$U = 20 + \frac{(9 - p)(p - 1)}{2} + A(p)$$

Where $A(p)$ is the transfer from Collin to Mikey. Of course, the best thing to of for Mikey is set $A(p) = (9 - p)^2/4$ so he chooses p to maximize,

$$U = 20 + \frac{(9 - p)(p - 1)}{2} + \frac{(9 - p)^2}{4}$$

The solution to this is $p^* = 1$ and so efficiency is restored, which yields $x^C = 4$, $m^C = 16 - A(p) = 16 - 16 = 0$. For Mikey $x^M = 16$ and $m^M = 4 + A(p) = 20$.

3. **War Coupons and Walras law** By assumption, more is better and coupons are not needed for all goods. Thus, all money will be spent. However, it could be that coupons are not all used up, because the money runs out. Maybe coupons are just needed for a subset of goods that one does not wish to buy.

4. **Economics** 603 **- Final Exam** 2009 **-** *Q#6*

   (a) A has income valued at $I(p) = 12 + 6p$. His demands are $x^A(p) = I(p)/3 = 4 + 2p$ and $y^A(p) = 2I(p)/(3p) = 4(2 + p)/p$.

   (b) Vito gets $x^V(p) = 16 - x^A(p) = 12 - 2p$ and $y^V(p) = 18 - 4(2+p)/p = 14 - 8p$.

   (c) Maximizing $U^V(x^V, y^V) = (12 - 2p)(14 - 8/p) = 168 + 16 - 28p - 96/p$ in p yields $28 = 96/p^2$, or $p = 2\sqrt{6/7} \approx 1.85$.

   (d) Vito's MRS differs from the price ratio $p \approx 1.85$, since:

$$MRS = \frac{U_x}{U_y} = \frac{x^V}{y^V} = \frac{12 - 2p}{14 - 8/p} \approx \frac{12 - 2 \cdot 1.85}{14 - 8/1.85} \approx 8.3/9.68 = 0.86$$

5. **Economics** 603 **- Practice Final -** *Q#1* The excess demand is,

$$z_x(p) = \begin{cases} -Nw_x & \text{if } p > a \\ \in [-Nw_x, 0] & \text{if } p = a \\ 0 & \text{if } 1 < p < a \\ \in [0, Mw_y] & \text{if } p = 1 \\ Mw_y/p & \text{if } p < 1 \end{cases}$$

6. **Economics** 711 **- Final** 2010**-** *Q#3*

   (a) $(x^A, y^A) = (\frac{1}{1+p}, \frac{1}{1+p})$ and $(x^B, y^B) = (\frac{p}{2}, \frac{1}{2})$. Equilibrium price is $p = 1$. If the Walrasian auctioneer calls out q as the relative price of y, then the value of the excess demand at p is $ED_x(p) + qED_y(p) = \frac{1}{1+p} + \frac{p}{2} - 1 + q(\frac{1}{1+p} + \frac{1}{2} - 1) = \frac{1}{1+p} + \frac{p}{2} - 1 + q(\frac{1-p}{2(1+p)})$. In order to maximize the value of excess demand, the auctioneer will set $q(p) = 0$ when $p > 1$ and $q(p) = \infty$ when $p < 1$. $q(1)$ could be any non-negative number.

   (b) $p_0 = 2, p_1 = 0, p_2 = \infty, p_3 = 0, p_4 = \infty$,....It won't converge.

   (c) $p_{t+1} = o_t + c\dot{z}_y(p_t) = p_t + 6(\frac{1}{1+p_t} - \frac{1}{2})$. Hence $p_0 = 2 + \sqrt{3}$, $p_1 = 2$, $p_2 = 1$. It converges to equilibrium price.

7. **Economics** 603 **- Midterm** 2010**-** *Q#7* The box is 40 by 20, Ann's kink line has a slope of 1, Kate's a slope of 1/3 so they intersect. The contract curve is a correspondence; it is the two areas between the two lines: One where both have to many wheels and one where both have too many frames.

8. **Economics** 603 **- Practice Midterm -** $Q\#7$ (a) follows graphically. For (b), if there are extra left shoes then markets will only clear if the price of left shoes is zero, if there are no extras then any price will clear the markets; he will be weakly worse off accepting the extra shoe.