

## 1. Exercises

1. **Contract Curves** For each of the following two person  $(A, B)$  two good  $(x_1, x_2)$  exchange economies with a total economy-wide endowment of  $(w_1, w_2)$ , solve for and draw the contract curve. (Don't forget corner solutions and do case analysis for different parameters if necessary!)
  - (i)  $U^A = x_1^a x_2$  and  $U^B = x_1 x_2$  with  $a > 0$ .
  - (ii)  $U^A = ax_1 + x_2$  and  $U^B = x_1 + x_2$  with  $a > 0$ .
  - (iii)  $U^A = U^B = \ln x_1 + x_2$ .
  - (iv)  $U^A = \min\{ax_1, x_2\}$  and  $U^B = \min\{x_1, x_2\}$  with  $a > 0$ .
  - (v)  $U^A = \min\{x_1, x_2\}$  and  $U^B = \sqrt{x_1} + \sqrt{x_2}$  with  $w_2 > w_1$ .
  - (vi)  $U^A = U^B = x_1^2 + x_2^2$
  - (vii)  $U^A = U^B = -(x_1 - \frac{1}{3}w_1)^2 - (x_2 - \frac{1}{3}w_2)^2$
2. **Monopolist in a Box** Mikey the monopolist has utility  $U = x + m$  and is endowed with 20 units of good  $x$  and none of good  $m$ . Collin the consumer has utility  $V = x(9 - x) + m$  and is endowed with 20 units of good  $m$  and none of  $x$ .
  - (i) If Mikey and Collin are in a pure exchange economy, what is the equilibrium price ratio  $p_x/p_m$  and allocation? Is the allocation efficient?
  - (ii) Now suppose the following protocol is followed: Mikey sets a price ratio  $p_x/p_m$ , Collin chooses optimally from his ensuing budget set, then Mikey consumes whatever is left over in the economy. What will be the price ratio and allocation? Is the allocation efficient?
  - (iii) Now Mikey sets both the price ratio  $p_x/p_m$  and the size of a transfer of good  $m$  that Collin must make, then Collin must decide whether to participate (or just consume his endowment) and if to participate which affordable bundle to consume. Mikey then consumes whatever is left over in the economy. What will be the price ratio, transfer, and allocation? Is the allocation efficient?
3. **War Coupons and Walras law** During a war, consumers are issued generic (not good-specific) ration coupons. So consumers have to have sufficient currency and coupons to buy some goods. Assume preferences are strictly monotonic, and that for some good that all consumers desire, coupons are not required. True or false: Walras Law holds for both the coupon-valued excess demand and currency-valued excess demand.

4. **Economics 603 - Final Exam 2009 - Q#6** There are two goods  $x$  and  $y$ . Angelo and Vito have utility functions  $U^V(x, y) = xy$  and  $U^A(x, y) = xy^2$ . Also, Vito has endowment  $w_x^V = 4$  and  $w_y^V = 12$ , and Angelo has endowment:  $w_x^A = 12$  and  $w_y^A = 6$ . Vito murders the auctioneer and inserts his brother, who offers him a price ratio he can't refute:  $p = p_y/p_x$ . Angelo chooses how much of each good to buy.
- (a) Given a price ratio  $p$ , how much of each good  $x^A(p)$  and  $y^A(p)$  will Angelo choose to purchase?
  - (b) Given  $x^A(p)$  and  $y^A(p)$ , how much of each good will Vito end up consuming for any price?
  - (c) Determine what price Vito optimally charges.
  - (d) Show that the resulting allocation is inefficient.
5. **Economics 603 - Practice Final - Q#1** Assume that  $N$  people have utility  $U = ax + y$  where  $a > 1$ , and each is endowed with  $w_x$  and no good  $y$ .  $M$  people have utility  $U = x + y$  and each is endowed with  $w_y$  and no good  $x$ . Derive the market excess demand for good  $x$  as a function of price ratio  $p = p_x/p_y$ .
6. **Economics 711 - Final 2010- Q#3** Assume utility functions  $u^A(x, y) = \min\{x, y\}$  and  $u^B(x, y) = xy$ , and an endowment  $(1, 0)$  for  $A$  and  $(0, 1)$  for  $B$ . Let the price ratio of  $y$  to  $x$  be  $p$ .
- (a) Define the function that maximizes the value of net excess demand as a function  $p$ , and show that it has a fixed point.
  - (b) If we started out with  $p = 1$ , what sequence of prices would be called out by an auctioneer who uses this rule? Does it converge to equilibrium price?
  - (c) If we started out with  $p = 2 + \sqrt{3}$ , what sequence of prices would be generated by the Walrasian tatonnement process? Let the excess demand of good  $i$  at  $p$  be  $z_i(p)$ . Walrasian tatonnement process is the process  $p_t$  such that  $p_{t+1} = p_t + c \cdot z_y(p)$ . Assume  $c = 6$
7. **Economics 603 - Midterm 2010- Q#7** Ann and Kate are bicycle enthusiasts; Ann enjoys unicycles (1 wheel and 1 frame) and Kate only tricycles (3 wheels and 1 frame). Each is endowed with 20 wheels and 10 frames. Draw the contract curve with wheels on the  $x$ -axis and frames on the  $y$ -axis.
8. **Economics 603 - Practice Midterm - Q#7**
- (a) Assume that Trader Ike and Trader Jose have increasing, convex and homothetic preferences over goods  $X$  and  $Y$  in a pure exchange economy. Show that

if the contract curve touches the diagonal of the Edgeworth box at the same point, it necessarily includes the whole diagonal.

- (b) Now assume that Trader Ike and Trader Joe consume left and right shoes only in pairs. Ike is endowed with five rights shoes while Joe is endowed with five left shoes. If Joe is offered an extra left shoe prior to a Walrasian auctioneer clearing the market, should he take it?