Unit IV

fuzzy logic

→ fuzzy refers to things that are not clear or are vague.
→ In situation when we can't determine whether the state is true or false, their fuzzy logic provides flexibility for reasoning.

→ fuzzy logic is a form of many-valued logic in which the truth values of variables may be any real no. betn 0 and 1, instead of just the traditional values of true or false.

→ used to deal with uncertain info and is a mathematical method for representing vagueness and uncertainty in decision-making.

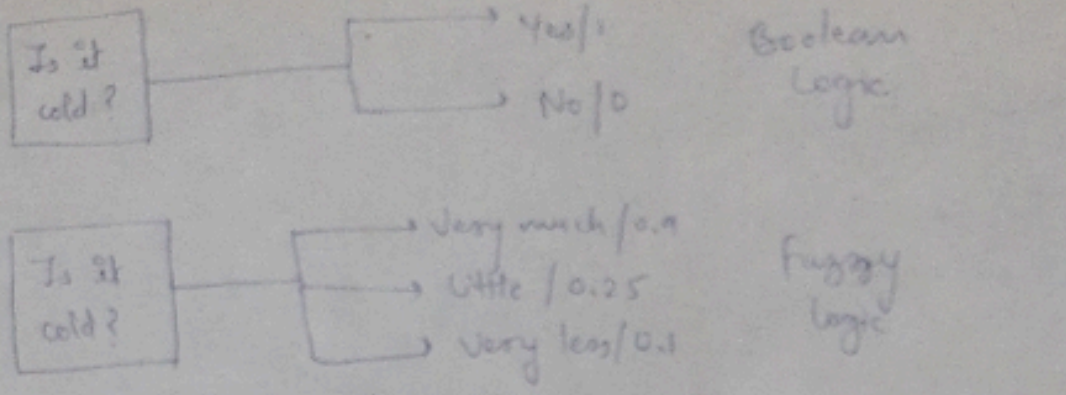
→ based on idea that in many cases, concept of true or false is too restrictive, and there are many shades of gray in betn. It allows for partial truths, where a statement can be partially true or false, rather than fully true or false.

Applications - control systems, image processing, medical diagnosis, AI

→ fundamental concept is membership func", which defines the degree of membership of an input value to a certain set or category. The membership func" is a mapping from an input value to a membership degree betn 0 & 1, where 0 represents non-membership & 1 represents full membership.

→ fuzzy logic is implemented using fuzzy Rules, which are if-then statements that express the relationship betn input variables and output variables in a fuzzy way.
→ The output of a fuzzy logic sytem is a fuzzy set, which is a set of membership degrees for each possible output value.

→ In Boolean system truth value 1.0 represents the absolute truth value & 0 represents the absolute false value. But in fuzzy system, there is no logic for absolute truth and absolute false value. Here there is an intermediate value too present which is partially true & partially false.



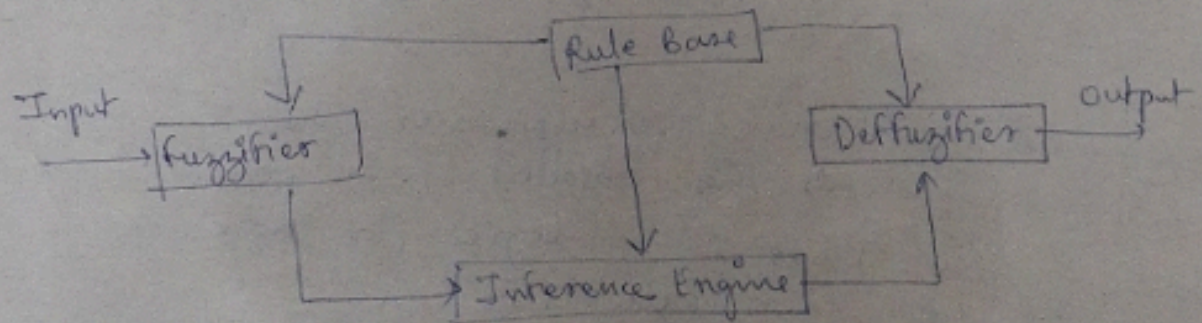
Architecture → has 4 parts

★ Rule Base – contains set of rules and IF-THEN conditions provided by the experts to govern the decision-making system, on the basis of linguistic info. Recent developments in fuzzy theory offer several effective methods for the design and tuning of fuzzy controllers. Most of these developments reduce the no. of fuzzy rules.

★ Fuzzification – used to convert inputs i.e. crisp numbers into fuzzy sets. Crisp inputs are basically the exact inputs measured by sensors and passed into the control system for processing, such as temp, pressure, rpm's.

★ Inference Engine – determines the matching degree of the current fuzzy input w.r.t each rule and decides which rules are to be fired according to the input field. Next, the fired rules are combined to form the control actions.

★ Defuzzification – used to convert the fuzzy sets obtained by the Inference Engine into a crisp value.



Fuzzy logic Arch

Membership fun^{n}

→ A graph that defines how each point in the input ~~phase~~ space is mapped to membership value betn 0 and 1.
→ Input space is often referred to as the universe of discourse or universal set (u), which contains all the possible elements of concern in each particular app^{n}.

There are 3 types of fuzzifiers -
* Singleton fuzzifier
* Gaussian fuzzifier
* Trapezoidal or triangular fuzzifier

What is fuzzy Control?

* technique to embody human-like thinkings into a control system
* It may not be designed to give accurate reasoning but it is designed to give acceptable reasoning.
* can emulate human deductive thinking, that is, the process people use to infer conclusions from what they know.
* Any uncertainties can be easily dealt with the help of fuzzy logic.

Advantages (fuzzy logic System)

* can work with any type of inputs whether it is imprecise, noisy input info
* ~~Con~~ constructions is easy & understandable
* comes with mathematical concepts of set theory & reasoning of that is quite simple.
* provides very efficient soln to complex problems in all fields of life as it resembles human reasoning & decision-making.
* little memory is req as algos can be described with little data.

Disadvantages

* researchers proposed diff ways to solve a given problem through fuzzy logic which leads to ambiguity. No systematic approach to solve a given problem.
* Proof of its characteristics is difficult or impossible in most cases because every time we do not get a mathematical description of our approach.
* works on precise as well as imprecise data so most of the time accuracy is compromised.

Application

* used in aerospace field for altitude control of spacecraft & satellites.
* used in automotive system for speed control, traffic control.
* used for decision-making support systems & personal evaluation in large company business.

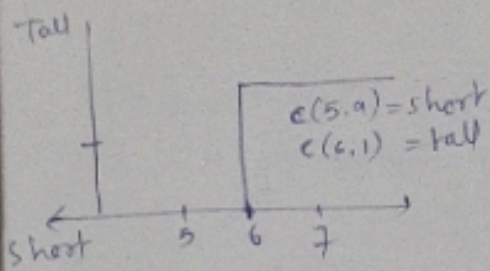
* chemical industry for controlling pH, drying, chemical distillation process.
* Natural language processing & various intensive applications in AI.
* modern control systems such as expert systems.
* Neural networks as it mimics how a person would make decisions, only much faster. It is done by Aggregation of data & changing it into more meaningful data by forming partial truths as fuzzy sets.

Crisp Set — Countability and finiteness are identical properties which are the collection objects of crisp set. 'X' is a crisp set defined as the group of elements present over the universal set i.e. U. In this case a random element is present that may be a part of X or not that means two ways are possible to define the set. These are first element would become from set X, or It does not come from X.

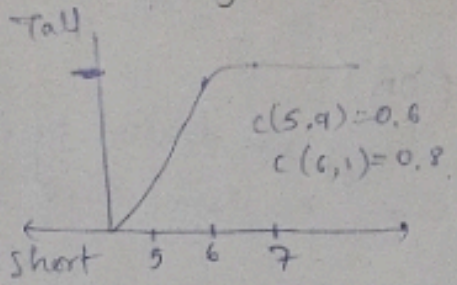
Fuzzy Set — The integration of the elements having a changing degree of membership in the set is called as fuzzy set. The word "fuzzy" indicates vagueness. On the other hand, we can say that the replacement among various degrees of the membership implies that the vagale and ambiguity of the fuzzy set Hence the measurement of the membership of the elements from the universe in the set against a function for detecting the uncertainty & ambiguity.

Crisp Set	Fuzzy Set
* defines the value is either 0 or 1	* defines the value bet 0 & 1 including both 0 & 1.
* also called a classical set.	* specifies the degree to which something is true.
* shows full membership.	* shows partial membership
* Ex → she is 18 yrs old	* Ex → she is about 18 yrs old
* used for digital design.	* used in fuzzy controller.
* bi-valued function logic	* multi valued function logic
* full membership means totally true/false, yes/no, 0/1.	* Partial membership means true to false, yes to no, 0 to 1.

Crisp Set



Fuzzy Set



Fuzzy Set Theory

→ mathematical framework that allows for dealing with uncertainty & imprecision in data.

→ In classical set theory, a set is defined by a membership func^n that assigns a value of either 0 or 1 to each element of set indicating whether the element belongs to the set or not.

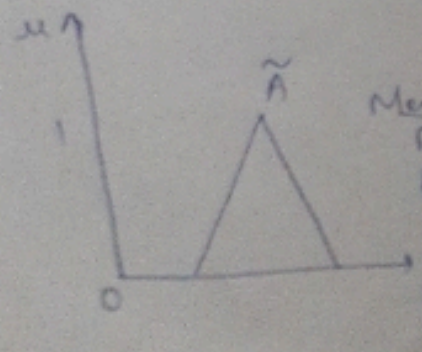
→ In fuzzy set theory, membership functions assigns a degree of membership to each element of set, ranging from 0 to 1. This allows for a more flexible represn of uncertainty and imprecision.

→ Applications — AI, control systems, decision making, pattern recognition.

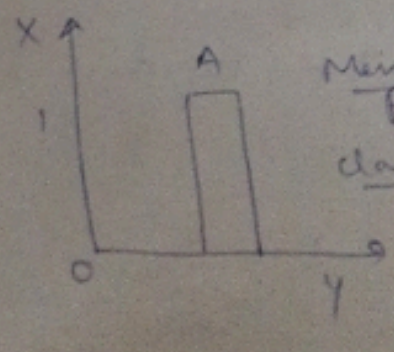
→ Fuzzy logic is used to create fuzzy rules, which are statements that relate fuzzy sets to each other. These rules can be used to build fuzzy inference systems, which take fuzzy inputs and produce fuzzy outputs.

→ Fuzzy inference system used to control complex systems that are difficult to model using traditional methods.

→ Fuzzy set theory is used in data analysis eg clustering. Fuzzy clustering is a technique that partitions a data set into grps or clusters based on degree of membership of each data point to the different clusters.



Membership fun^n of fuzzy set Ã



Membership funcn of classical set A

Common Operations on Fuzzy Set

(1) Union (OR)

– Consider 2 fuzzy sets denoted by A and B, then let's consider Y be the union of them, then for every member of A and B, Y will be:

$$\text{degree of membership}(Y) = \max(\text{degree of membership}(A), \text{degree of membership}(B))$$

(2) Intersection ('AND')

$$\text{degree of membership (DOM)}(Y) = \min(\text{DOM}(A), \text{DOM}(B))$$

(3) Complement ('NOT')

$$\text{DOM}(Y) = 1 - \text{DOM}(A)$$

(4) Difference

$$\text{DOM}(Y) = \min(\text{DOM}(A), 1 - \text{DOM}(B))$$

Properties of Fuzzy Sets

(1) Commutative Property ⟶ if order of operands does not alter result

→ 2 fuzzy sets $\tilde{A}$ and $\tilde{B}$

$$\tilde{A} \cup \tilde{B} = \tilde{B} \cup \tilde{A} \quad \text{or} \quad \tilde{A} \cap \tilde{B} = \tilde{B} \cap \tilde{A}$$

(2) Associative Property ⟶ allow change in order of ~~operands~~ operations performed on operand, but relative order of operand cannot be changed

→ 3 fuzzy sets $\tilde{A}$, $\tilde{B}$ & $\tilde{C}$

$$\tilde{A} \cup (\tilde{B} \cup \tilde{C}) = (\tilde{A} \cup \tilde{B}) \cup \tilde{C}$$

$$\tilde{A} \cap (\tilde{B} \cap \tilde{C}) = (\tilde{A} \cap \tilde{B}) \cap \tilde{C}$$

(3) Distributive Property

$$\tilde{A} \cup (\tilde{B} \cap \tilde{C}) = (\tilde{A} \cup \tilde{B}) \cap (\tilde{A} \cup \tilde{C})$$

or

$$\tilde{A} \cap (\tilde{B} \cap \tilde{C}) = (\tilde{A} \cap \tilde{B}) \cup (\tilde{A} \cap \tilde{C})$$

(4) Idempotenty Property (or Tautology) → does not alter the element or membership value of elements in set

$\tilde{A} \cup \tilde{A} = \tilde{A}$

$\tilde{A} \cap \tilde{A} = \tilde{A}$

(5) Identity Property — fuzzy set $\tilde{A}$ and universal set U.

$\tilde{A} \cup \varphi = \tilde{A}$ $\quad$ $\tilde{A} \cap \varphi = \varphi$

$\tilde{A} \cap U = \tilde{A}$ $\quad$ $\tilde{A} \cup U = U$

(6) Transitive Property

If $\tilde{A} \subseteq \tilde{B} \subseteq \tilde{C}$, then

$\tilde{A} \subseteq \tilde{C}$

(7) Involution Property → complement of complement of set itself

$\overline{\overline{\tilde{A}}} = \tilde{A}$

(8) De Morgan's law

$\overline{\tilde{A} \cap \tilde{B}} = \overline{\tilde{A}} \cup \overline{\tilde{B}}$

or

$\overline{\tilde{A} \cup \tilde{B}} = \overline{\tilde{A}} \cap \overline{\tilde{B}}$

in proving tautologies & contradiction

Fuzzy & Crisp Relations

→ 2 types of mathematical relations

→ Crisp rel^n is a binary rel^n that maps elements from one set to elements of another set. It is a well-defined, precise relationship bet^n elements that is either true or false.

Ex → rel^n "greater than" bet^n 2 integers is a crisp rel^n. If one integer is greater than the other, the rel^n is true; otherwise it is false.

→ fuzzy rel^n is a binary rel^n that allows for degrees of truth. It can be thought of as a more flexible and nuanced version of crisp rel^n, where the relationship bet^n elements is not just a simple true or false, but can be any value in the range [0, 1].

This means that fuzzy rel^{n}s can represent relationships that are not-well-defined or are difficult to quantify precisely.

Ex → rel^n bet^n temp & outside & level of comfort

fuzzy rel^{n}s → used in fuzzy logic & handle uncertain info.

Crisp rel^{n}s → used in traditional mathematical models & algos

~~Fuzzy to Crisp Conversion~~

Fuzzification — method of transforming a crisp qty (set) into a fuzzy qty (set).

→ can be achieved by identifying the various known crisp & deterministic quantities as completely non-deterministic and quite uncertain in nature.

→ uncertainty emerged due to ~~vagueness~~ vagueness of linguistic variables which lead variables to be represented by a membership func^n as they can be fuzzy in nature.

Ex → when I say temp is 45°C never converts this input value to a linguistic variable like favourable temp for human body, hot or cold.

Defuzzification — Inverse of fuzzification.

→ mapping done to convert ~~crisp~~ fuzzy results into crisp results.

→ process is capable of generating a non-fuzzy control action which illustrates the possibility distribution of an inferred fuzzy control action.

→ this process can also be treated as rounding off process, where fuzzy set having a grp of membership values on the unit interval reduced to a single scalar qty.

Fuzzification	Defuzzification
→ Precise data converted into imprecise data	→ Imprecise data converted into precise data
→ method of converting crisp qty into fuzzy qty	→ mapping done to convert fuzzy results into crisp results
→ ~~Like~~ Ex → like Voltmeter	→ Ex → like D/A converter
→ Methods → Intuition, Inference, Rank ordering, angular fuzzy sets, neural network	→ Methods → Max Membership Principle, Centroid Method, Weighted Avg Method, Center of Sums
→ Complexity → simple	→ Complexity → complicated
→ use If-Then rules	→ use center of gravity methods to find centroid of sets.

Membership functions (MF)

→ used to map input values to a degree of membership in a fuzzy set.

→ A fuzzy set is a set with continuum of membership degrees ranging from 0 to 1, 0 means the element is not a member of the set, and 1 means it is a full member.

→ A membership func" takes an input value & returns a degree of membership value bet" 0 and 1.

→ shape of MF determines how the input value is mapped to the degree of membership.

Types of MFs used in fuzzy logic –

① Triangular MF – has triangular shape, peak representing max degree of membership.
func" defined by 3 parameters : lower bound
upper bound
peak value

② Trapezoidal MF – has trapezoidal shape, flat top representing max degree of membership.
4 parameters : lower bound, upper bound,
2 peak values

③ Gaussian MF – has a bell-shaped curve, peak representing max degree of membership.
2 parameters : mean & standard deviation

④ Sigmoidal MF – has S-shaped curve, midpoint of curve representing max degree of membership.
2 parameters : midpoint & slope.

→ Choice of MF depends on specific app" & characteristics of input variable being modeled.

→ Triangular & trapezoidal functions → used for inputs with clear boundaries

→ Gaussian & Sigmoidal functions → used for inputs with gradual transitions.

Fuzzy Inference System

→ key unit of a fuzzy logic system having decision making as its primary work.

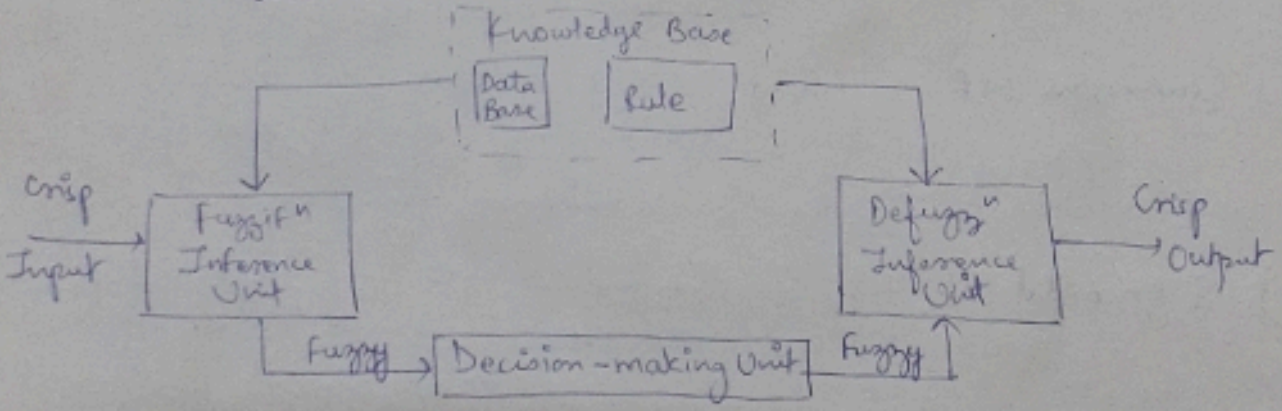
→ uses "If, Then" rules along with connectors "OR" ~~and~~ or "AND" for drawing essential decision rules.

Characteristics → * here output, from FIS is always a fuzzy set irrespective of its input which can be fuzzy or crisp.

* necessary to have fuzzy output when it is used as a controller
* A defuzzifⁿ unit would be there with FIS to convert fuzzy variables into crisp variables.

Functional Blocks of FIS →

* Rule Base — contain fuzzy If-then rules.
* Database — defines membership functions of fuzzy sets used in fuzzy rules.
* Decision-making Unit — performs operation on rules.
* Fuzzifⁿ Inference Unit — converts crisp quantities into fuzzy quantities.
* Defuzzyⁿ Inference Unit — converts fuzzy quantities into crisp quantities.



Working → * A fuzzyⁿ unit supports applⁿ of numerous fuzzyⁿ methods, and converts the crisp input into fuzzy input

* A knowledge base — collecⁿ of rule base & database is formed upon the conversion of crisp input into fuzzy input.
* defuzzyⁿ unit fuzzy input is finally converted into crisp output.

Methods of FIS →

① Mamdani Fuzzy I.S → anticipated to control a steam engine and boiler combⁿ by synthesizing a set of fuzzy rules obtained from people working on system.

② Takagi Sugeno Fuzzy Model → Rule format
(TS Method)

IF x is A and y is B THEN Z = f(x,y)

AB are fuzzy sets in antecedants and Z = f(x,y) is a crisp funcⁿ in the consequent.

Fuzzy IF-THEN Rules

→ type of rule-based system in fuzzy logic

→ traditional if-then rules → input variables are precise & output is also precise

→ fuzzy if-then rules → input & output variables are fuzzy sets, hence can have degrees of membership rather being just true or false.

→ fuzzy if-then rules consist of 2 parts: antecedant, consequent

→ antecedant → "if" part of rule → specifies conditions that must be met for rule to be applied
↳ expressed as fuzzy set over one/more input variables

→ consequent → "then" part of rule → specifies output that should be produced if antecedant is true.
↳ expressed as a fuzzy set over one/more output variables

→ When fuzzy if-then rule applied to a set of input variables, the degree of membership of each input variable in the fuzzy set is determined, and degree of membership of output variable in consequent fuzzy set is computed.
The resulting fuzzy set represents the output of the rule.

→ used in temperature control systems
traffic signal control systems
financial forecasting systems

Fuzzy Implications

→ type of logic that allows for degrees of truth rather than just true or false values.

→ Fuzzy Implication is a binary operation that takes 2 fuzzy sets as input and produces a fuzzy set as output.

→ In fuzzy logic, degree of truth of a statement is represented by a value betn 0 & 1 with 0 indicating falsehood and 1 indicating absolute truth.

→ Fuzzy Implications determine degree of truth of a conclusion based on degree of truth of one / more premises.

Types →

(1) Lukasiewicz I. — most commonly used

↳ defined as $\max(0, x+y-1)$

x, y → truth values of antecedant & consequent, resp.

(2) Goguen I. — $x \rightarrow y = \min(1, (1-x)+y)$

(3) Zadeh I. — $x \rightarrow y = \min(x, y)$

(4) Mamdani I. — simplest

↳ $x \rightarrow y = \min(\mu x, \mu y)$

→ Fuzzy Implications are used to represent relⁿ betⁿ 2 fuzzy sets.

→ Represented by (→) symbol

→ FI represented by t-norm Funcⁿ, which maps the degree of membership of the consequent.

→ Commonly used t-norm Funcⁿs includes minimum, product, etc

→ Each type of Implication uses a diff t-norm funcⁿ & has its own strengths & weaknesses.

Fuzzy Algorithms

→ computational methods that use fuzzy logic principles to process and analyze data that is uncertain or imprecise.

→ these algos are designed to handle data that cannot be easily modeled using traditional binary logic, & are often used in decision making, pattern recognition & control systems.

→ typically involve 4 main steps

① Fuzzific"

② Rule Eval" — here fuzzy sets are combined using fuzzy implications to produce a set of fuzzy outputs. Fuzzy Implications represent - - -

③ Aggregation — here fuzzy outputs ~~are~~ from all rules are combined to produce a single fuzzy output that represents the overall result of the algo. This is done using weighted avg or some other aggregation method.

④ Defuzz"

→ Some common fuzzy algos include fuzzy clustering, f. decision trees, f. neural networks, and f. control systems.
These algo used in predictive modeling, image and signal processing, and robotics.

→ Fz Algos has advantages over traditional binary logic methods as they can ~~bound~~ model complex relationships betn variables.

→ But can be more computationally intensive & difficult to implement than traditional methods & req specialized knowledge & expertise.

Fuzzy Controller

→ type of controller that uses fuzzy logic principles to control a systems in real time.

→ used in systems too complex or too difficult to model using traditional control methods, or in systems where input/output variables are uncertain or imprecise.

→ Components

① Fuzzifier — converts crisp input variables into fuzzy sets. Input variables may be physical parameters such as temp, pressure, speed or abstract variables (error/deviation)

② Inference Engine — evaluates set of fuzzy rules that describe rel^n betn input & output variables.

Rule, represented in "if-then" statements such as "If temp is high, then reduce speed".

The IE evaluates these rules to determine appropriate control action to take based on current state of system.

③ Defuzzifier — converts fuzzy output generated by IE into a crisp control signal that can be used to control the system.

The defuzz^n process involve aggregating fuzzy output from all other rules & then determining centroid or max of resulting fuzzy set.

→ Applications — temp control, speed control, robot control

Industrial App's of Fuzzy logic

1. Control Systems — temp control, press control, speed control
   → able to make control decisions based on uncertain data, resulting in more accurate control.
2. Robotics — for path planning, obstacle avoidance, robot control
   → allows robots to make decisions based on uncertain ~~data~~ sensor data.
3. Automotive Industry — engine control, transmission control
   → allows more precise & efficient control of these systems resulting in improved vehicle performance & safety
4. Consumer Electronics — washing machines, ~~air~~ ACs, refrigerators.
   → allows these devices to make decisions based on uncertain data, resulting in more effective operation.
5. Medical Diagnosis — for decision support systems
   → allows medical professionals to make decisions based on uncertain data ~~for~~ resulting in ~~accurate~~ accurate diagnosis.
6. Financial Analysis — for credit scoring, risk analysis, investment analysis.
   → more informed financial decisions.

Fuzzy to Crisp Conversions

2 Common methods are —

① Centroid Method — calculates center of gravity of fuzzy set. It calculates weighted avg of values in ~~fuzzy~~ fuzzy set, where the weights are given by degree of membership of each value in the set. The resulting value is crisp output.

Ex → fuzzy set of temp values → from 20°C to 40°C with a degree of membership from 0.2 to 1.0.

Centroid of this set ⇒ $\frac{(20*0.2 + 21*0.3 + \ldots\ldots + 40*1.0)}{(0.2 + 0.3 + \ldots\ldots + 1.0)}$

resulting value is crisp output. (35°C & 36°C here)

② Maximum Method — calculates max degree of membership in fuzzy set. Resulting value is crisp output.

Ex → fuzzy set of temp values → from 20°C to 40°C with degree of membership from 0.2 to 1.0

⇒ Max degree of membership in this set is 1.0 which corresponds to a temp value of 40°C.

∴ crisp output would be 40°C.