Generalization of Newton's second law

The force is equal to the time derivative of the momentum :

$$\vec{F} = \frac{d\vec{p}}{dt}$$

where

$$\vec{p} = m\vec{v}$$

This formulation is important in case m does not remain constant (rocket equation for instance, or any leaking system)

Validity of Newton's laws

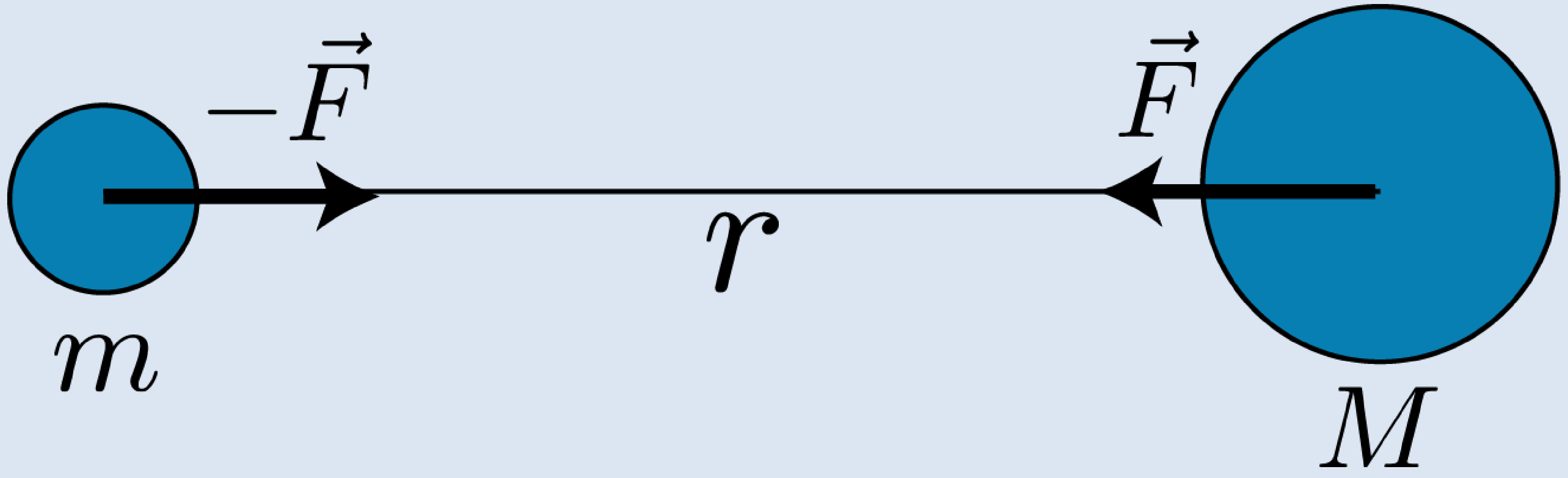
- Newton's laws are valid to describe the motion of celestial bodies and man-made spacecraft, except for very small general relativity effects (rotation of line of apsides and very slow orbital decay).

- The laws are well verified in the vicinity of the Earth and in the solar system.

- All velocities in consideration are smaller than 10^{-3} c → Lorentz factor negligible.

$$\text{Lorentz factor} = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

The Law of gravitation



$$F = G\frac{Mm}{r^2}$$

$$\frac{F}{m} = \frac{\mu}{r^2}$$

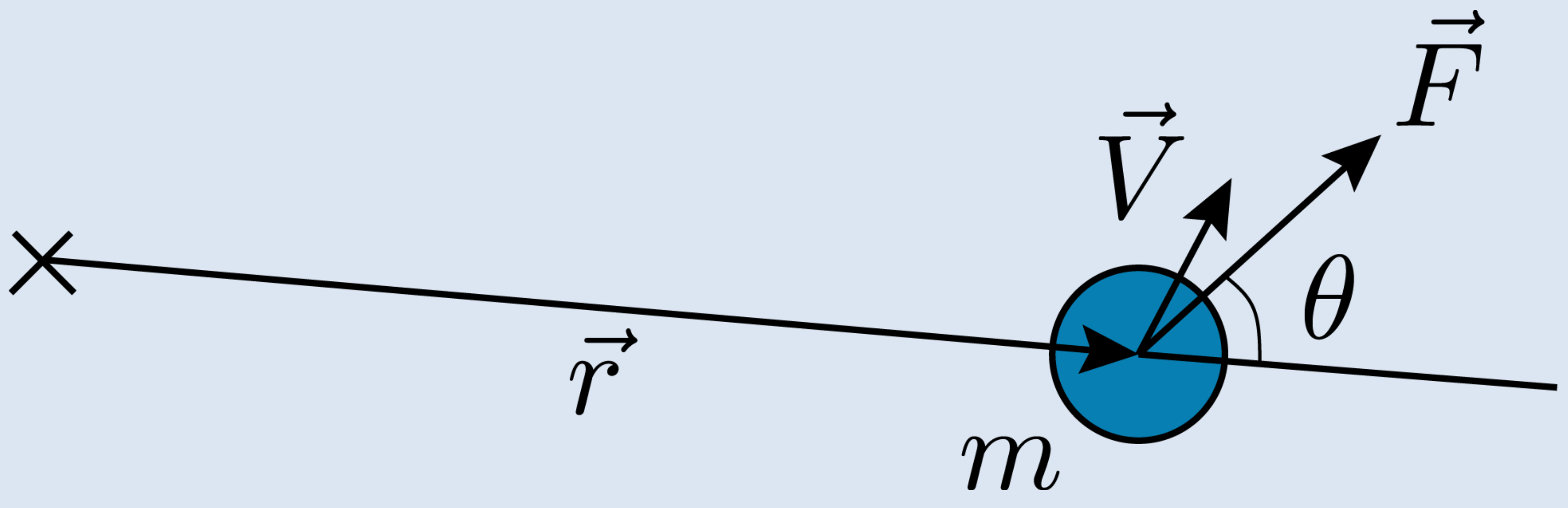
$\mu = GM$ $\quad$ M is the mass of the attracting celestial body

m the mass of the spacecraft

$$G = 6.673 \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$$

Newton's second law for rotations

Fixed point or center of mass of a set of point masses or of a solid body



- Torque = moment of force: $\vec{T} = \vec{r} \times \vec{F}$
- Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$
- Newton's second law for rotations:

$$\vec{T} = \frac{d\vec{L}}{dt}$$

Angular momentum and Moment of inertia

EPFL

- $\vec{L}$ for a solid body, with a rotation axis Δ:

$$\vec{L} = I_{\Delta}\vec{\omega}$$

with the moment of inertia I_{Δ} around the axis Δ.

- Moment of inertia:

$$I_{\Delta} = \sum_i m_i r_i^2$$

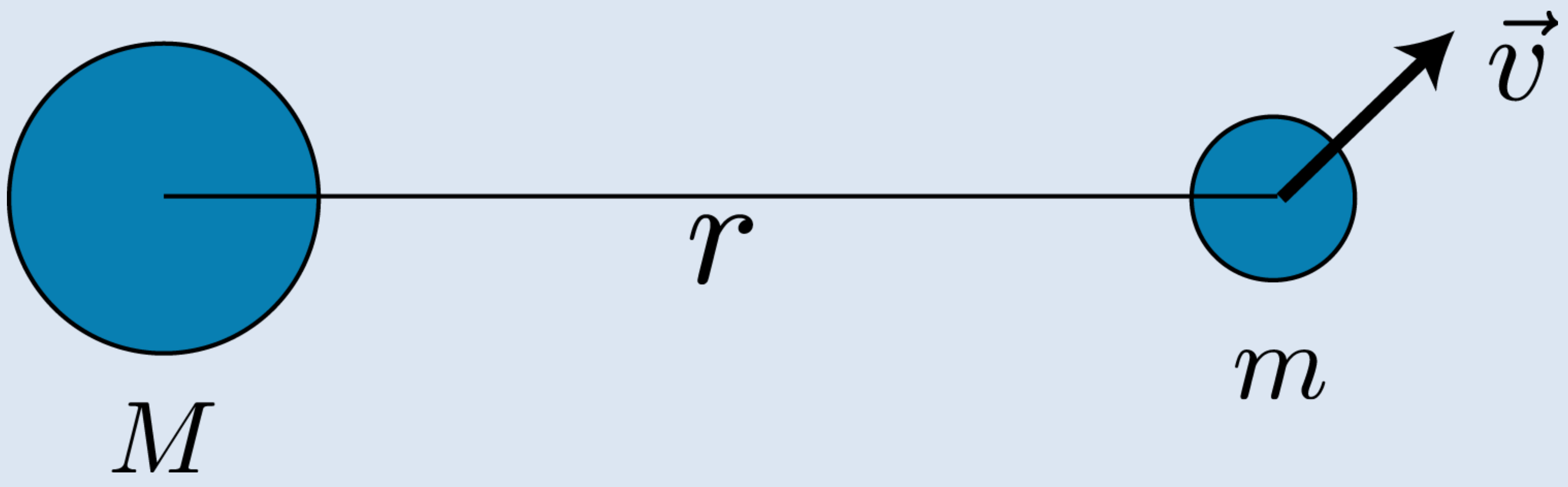
where the r_i are the distances of mass elements to the axis of rotation Δ.

$$I_{\Delta} = \iiint_V r^2 \rho\left(r\right) dV$$

where r is the distance of the mass element to the axis of rotation Δ.

Potential energy of a spacecraft

- Potential energy of a spacecraft of mass m in the gravitational field of a much larger mass M:



- Potential energy (m << M $\longrightarrow$ the center of mass of the two bodies is at the center of the large mass M):

$$E_{\text{pot}} = -GM\frac{m}{r} \quad \rightarrow \quad E_{\text{pot}} = -\frac{\mu}{r}$$

In the rest of the course, E_{pot} will always be understood "per unit mass"

Conservation's laws

- **Conservation of momentum** for translations**:**
  in an isolated system (absence of forces).

- **Conservation of angular momentum** for rotations:
  in an isolated system (absence of torques).

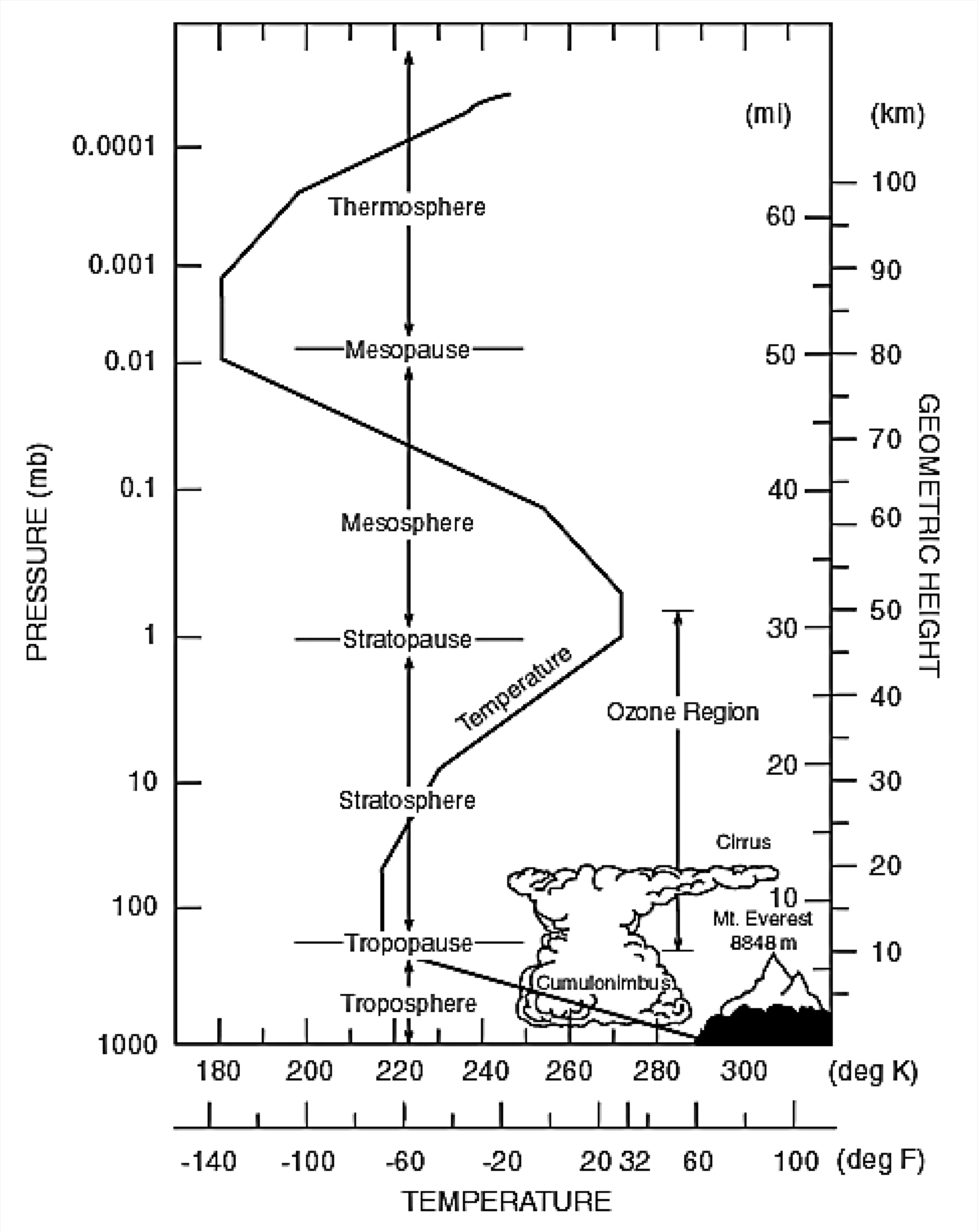
- **Conservation of mechanical energy:**
  Mechanical energy is the sum of potential and kinetic energies, and is conserved in a conservative force field (a gravitational force field is conservative in the absence of dissipative forces).

Chemical composition of the atmosphere

- 78 % of nitrogen, N_2
- 21 % of oxygen, 0_2
- 1 % of argon, Ar
- < 0.1 % of CO_2, H_20 and others

- Atomic oxygen is the most abundant element in the thermosphere and beyond (> 80 km altitude).

Variation in pressure and temperature with altitude



On the ground, the standard pressure is 1013 mb (or hectopascal or hPa) with a temperature of 288 K or 15° C, which is the average temperature on the surface of the Earth.

In the first layer of the atmosphere, the troposphere, the temperature decreases with about 6.5° C per 1000 meters elevation.

At the tropopause, approximately, 9 to 15 km above the surface of the Earth, depending on the latitude, the temperature is approximately 218 K, or -55° C.

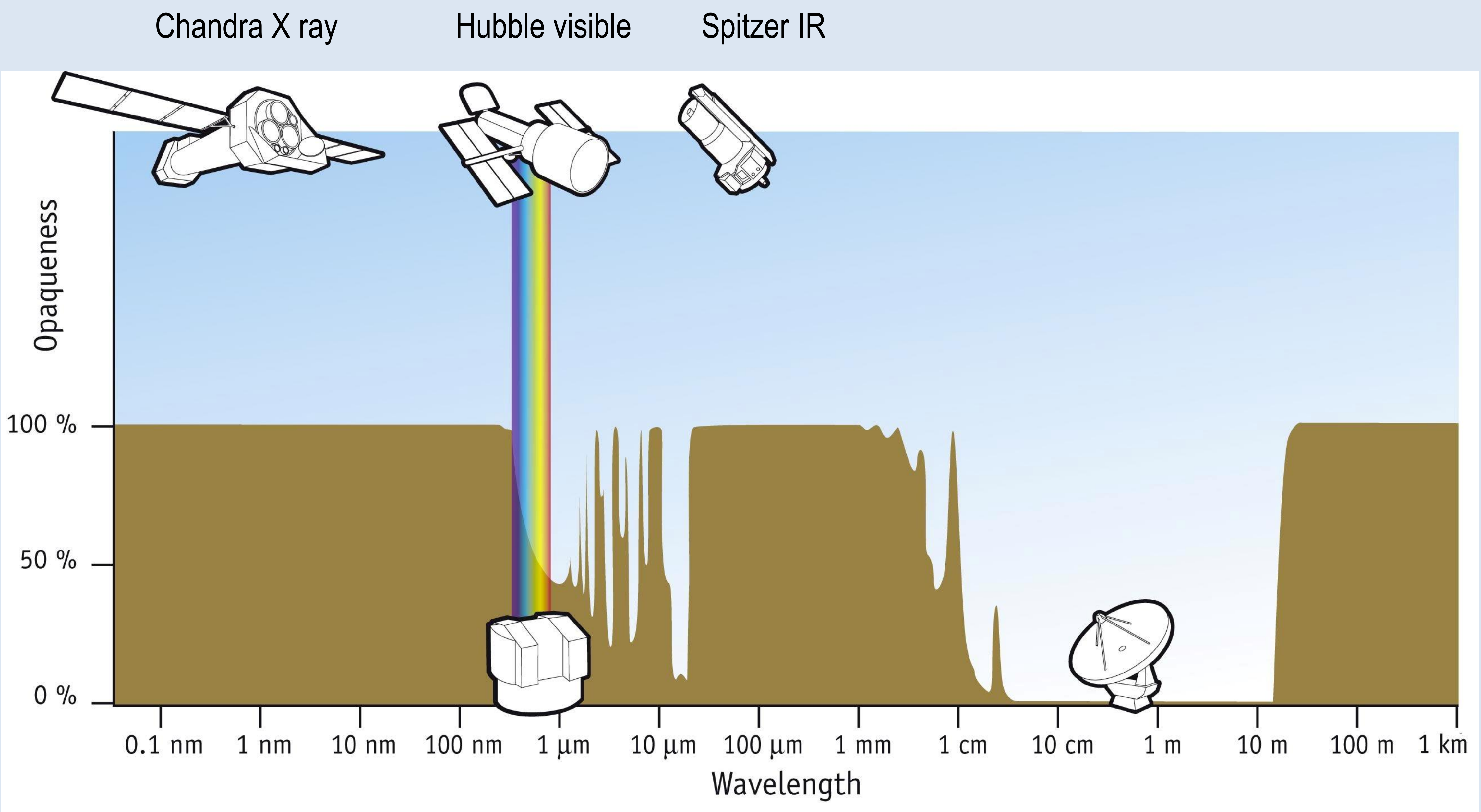
The temperature increases in the layer 40-50 km altitude due to the formation of ozone. The ozone is created by a dissociation of the oxygen molecule and combination with another oxygen atom to form ozone (O_3).

In the mesosphere, about 50 km altitude, the temperature decreases again. Above 80-90 km altitude, in the thermosphere, the temperature increases because of the ionization of mainly oxygen and some nitrogen atoms.

100 km altitude is what the Fédération Aéronautique Internationale is considering as the limit to space.

Credits: Oxford University Press, 1999

Transparency windows in the atmosphere



The atmosphere is opaque to radiations except a window in the visible part of the spectrum, around 0.5 microns, or 5,000 angstroms. Telescopes on the ground observe and study objects in the sky in this visible light window. There is a large radio window, for radio telescopes studying objects in the large wavelength range, from about 1 cm to 10 m wavelength.
Celestial objects emitting in ultraviolet, X-ray, gamma ray and infrared can be observed only from space with facilities like the Chandra X-Ray Observatory or Infrared Spitzer Telescope.

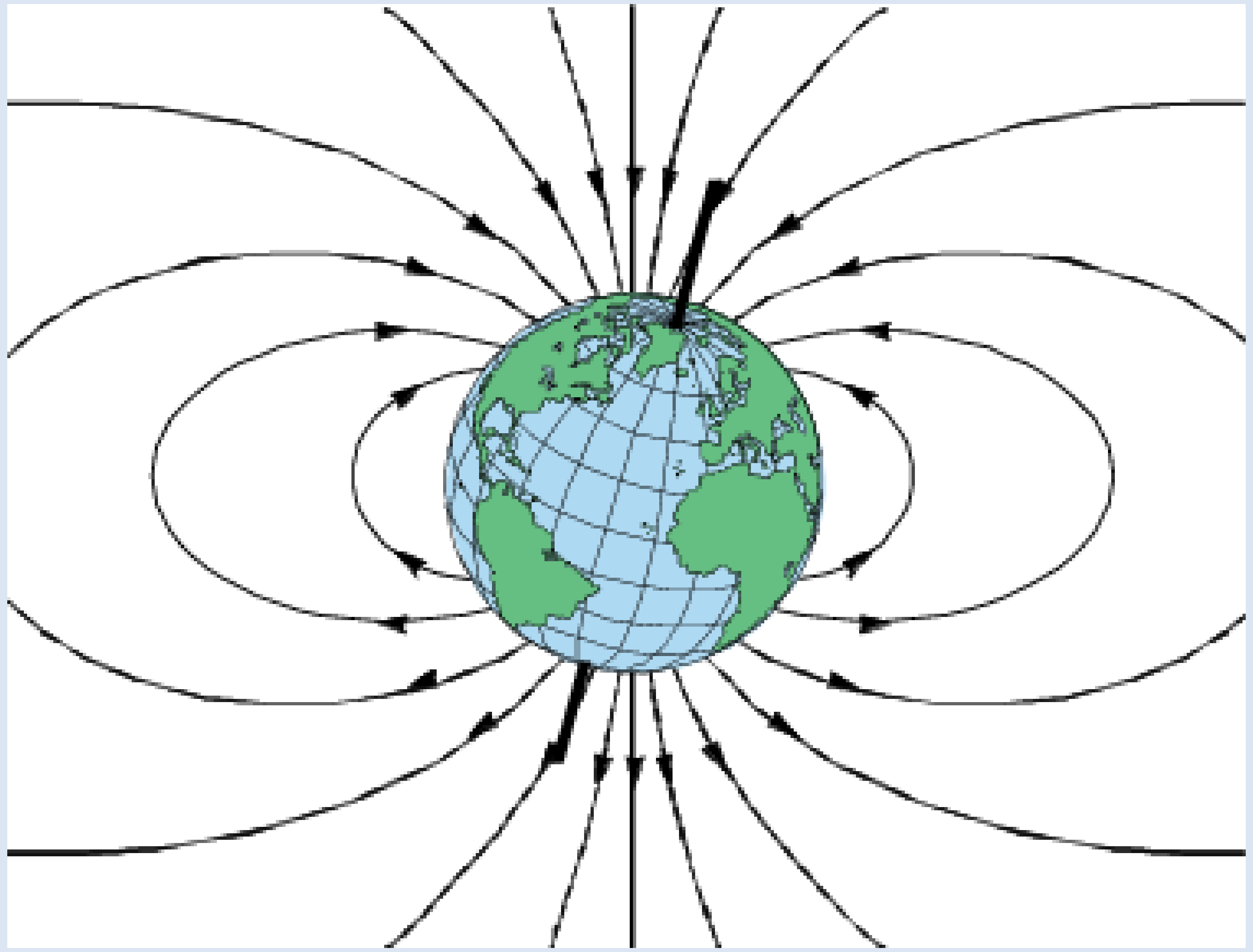
The Hubble Space Telescope is covering the visible part of the spectrum, plus the near ultraviolet and the near infrared. Being outside of the Earth atmosphere, HST has a higher resolution than ground-based telescopes with the same primary mirror size.

Credits: ESA, Hubble, F. Granato

The geomagnetic field – magnetic dipole model

The amplitude of the magnetic field is a function of the distance to the center of the Earth and the magnetic latitude (zero degrees at the Equator, +90 at the North Magnetic Pole, and -90 at the South Magnetic Pole).

B_0 is the magnetic field on the Equator.



- Close to the Earth's surface, the geomagnetic field is essentially a bipolar field slightly offset from the center of the Earth.

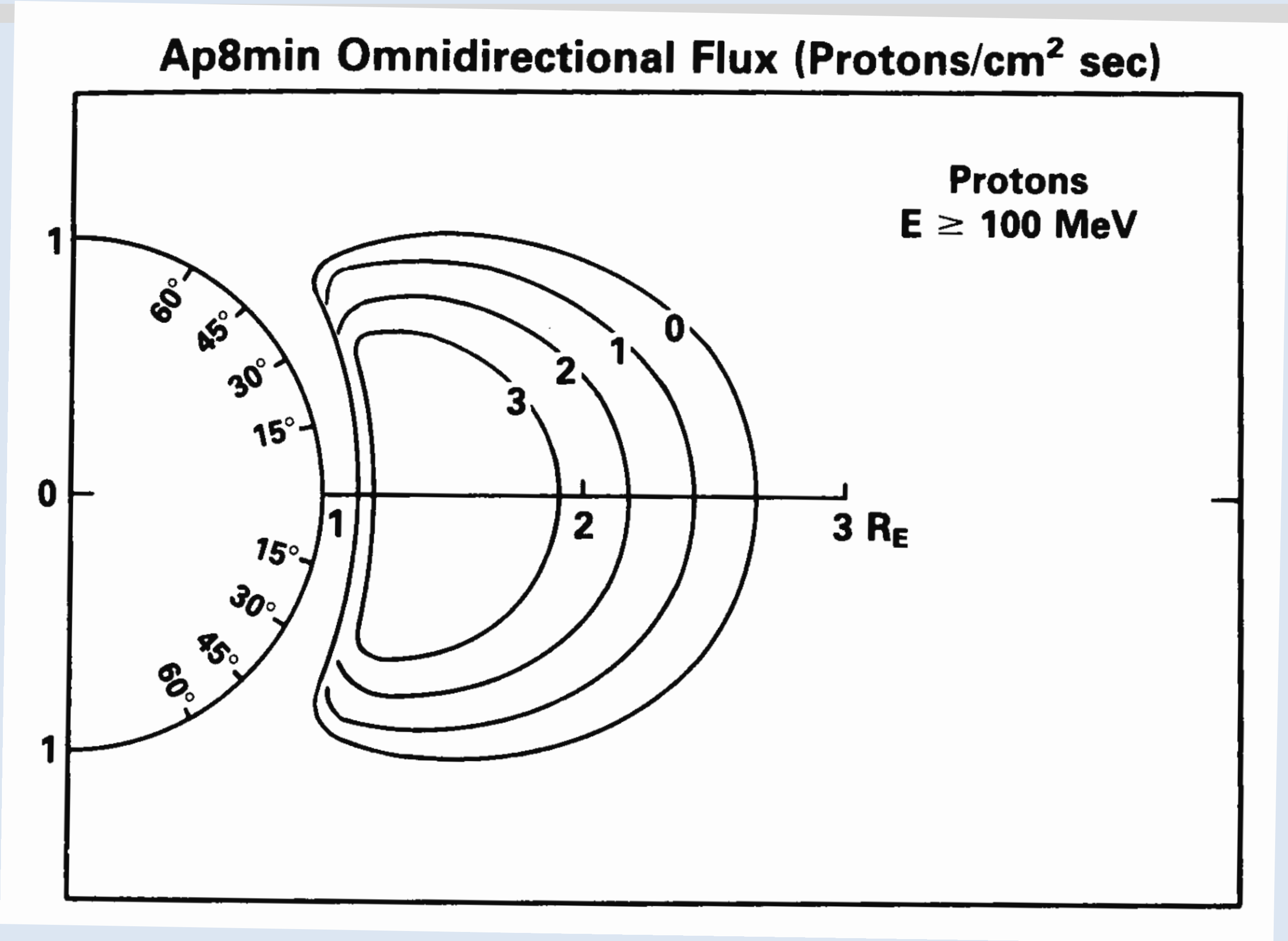
$$B(R,\lambda) = \left(1 + 3\sin^2\lambda\right)^{\frac{1}{2}} \frac{B_0}{R^3}$$

1

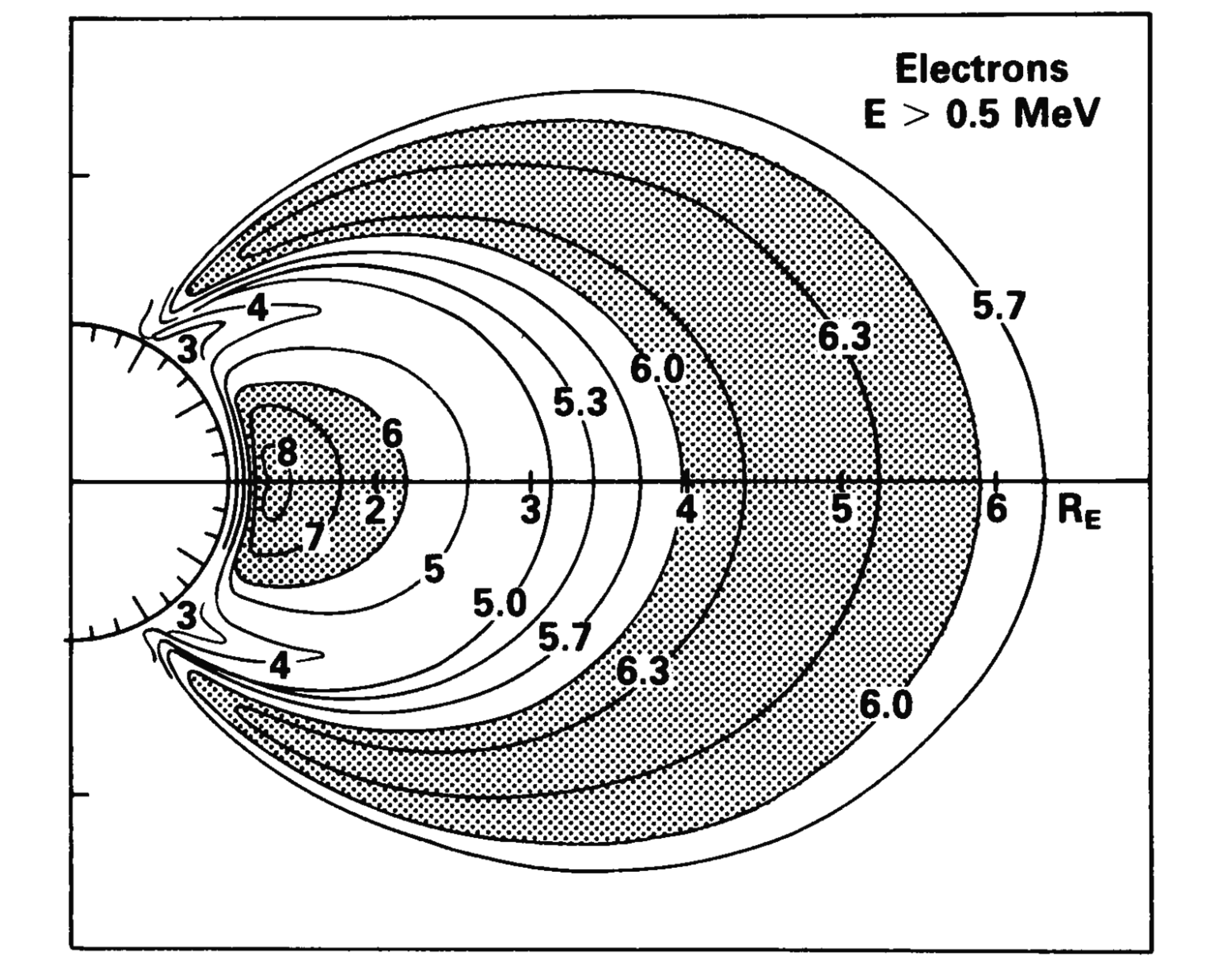
$$\begin{aligned} B_0 &= B(R = 1, \lambda = 0) \\ &= 0.30 \text{ gauss} \\ &= 3.12 \times 10^{-5} \text{ Tesla} \end{aligned}$$

B: Local magnetic field.
λ: Magnetic latitude.
R: Distance to the Earth's center measured in Earth radius unit R_E.

Radiation (or van Allen) Belts – RB



units = $\log_{10}$ omnidirectional particle flux/cm^{2}sec



- High energy protons and electrons trapped in two regions of the magnetosphere.
- Protons and electrons in the inner RB, electrons only in the outer RB.
- Energy of the RB particles is bigger than 30 keV, up to 100 MeV.
- The outer radiation belt is not as harmful to electronic systems as the inner radiation belt.

Solar Cycle (11 years)

This table represents the solar cycle from 1979 until 2040, the solar cycle maximum being the time with the maximum number of sunspots.

The Solar activity decreases within six years and increase in five years.

Solar Cycle	21	22	23	24	25	26
Sunspot Maximum	1979	1990	2001	2012	2023	2034
Sunspot Minimum	1985	1996	2007	2018	2029	2040

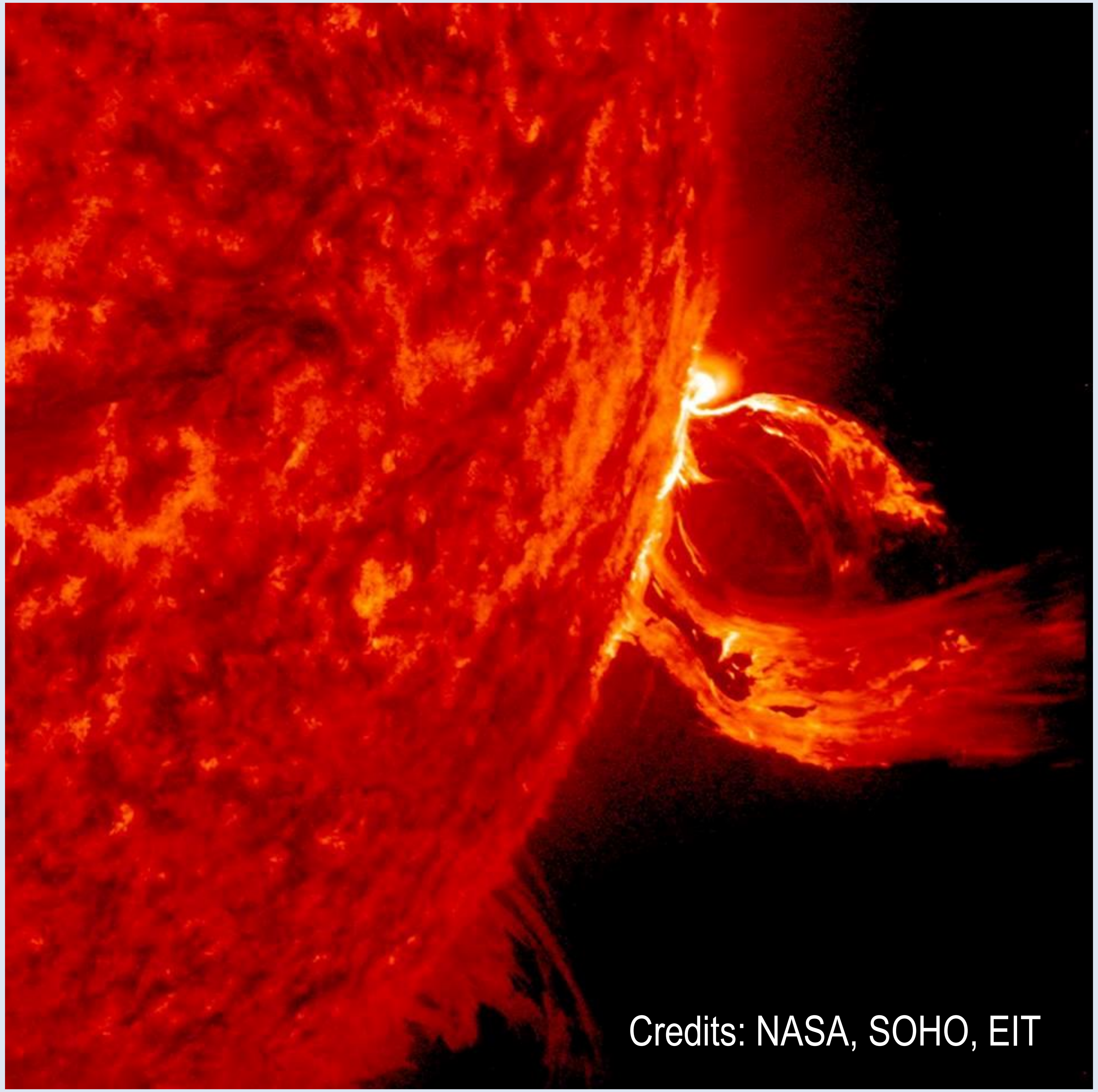
Coronal Mass Ejection (CME) – June 17-18, 2015

- A Coronal Mass Ejection (CME) is a massive burst of solar wind, other light isotope plasma, and magnetic fields rising above the solar corona and being released into space. CME is mostly observed in short wavelength part of the spectrum, typically in UV, extreme UV, or even X-rays.

- The Solar and Heliospheric Observatory (SOHO) is a ESA/NASA Sun-observing satellite located on the Lagrange L1 point of the Sun-Earth system.

- SOHO always remains in the same position versus the Earth about 1.5 million km towards the Sun.

- SOHO-EIT image in resonance lines of eight and nine times ionized iron (Fe IX/X) at 171 Angstroms in the extreme ultraviolet, showing the lower solar corona at a temperature of about 1 million K.



Physiological effect of radiation and typical doses

- RAD = Radiation Absorbed Dose = Amount of energy absorbed = 0.01 J/kg (100 erg/g)
- REM = Roentgen Equivalent Man = RAD x Q
- Q = quality factor = function of type of radiation

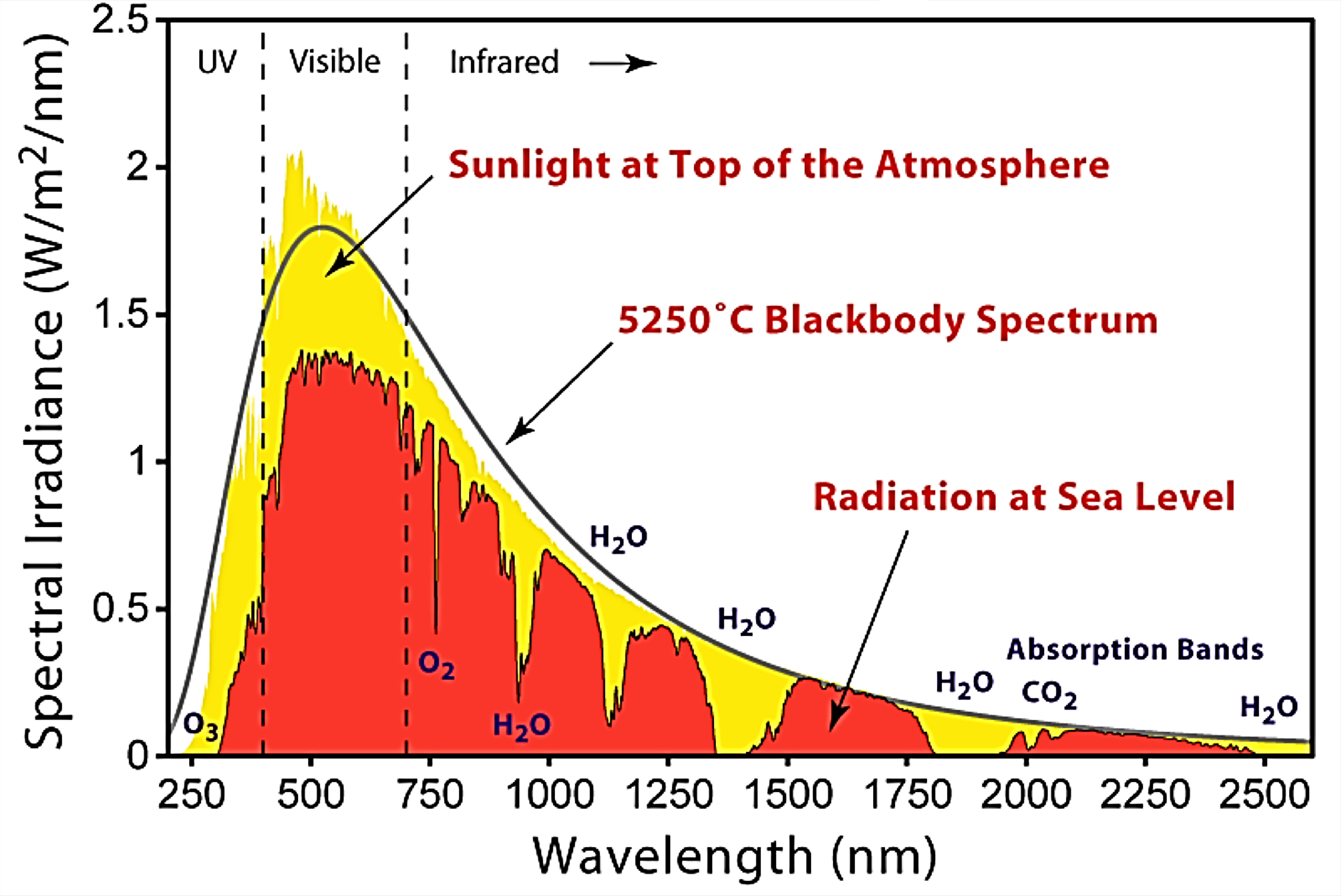
  = 1 (x-ray, gamma ray, electrons, beta)
  = 2-20 (neutrons)
  = 20 (alphas)
  = 20+ (iron ions)
- Sievert = Sv = 100 REM.

Effect	Dosage (REM)
Blood count changes in population	15-20
Vomiting "effective threshold"*	100
Mortality "effective threshold"*	150
LD_{50}** with minimal supportive care	320-360
LD_{50}** with full supportive medical treatment required	480-540

Effect	Dosage (REM)
Transcontinental round trip in jet	0.004
Chest X-ray (lung dose)	0.01
Living one year in Houston (sea level)	0.1
Living one year in Denver (elev. 1600 m)	0.2
Skylab 3 for 84 days (skin)	17.85
Space shuttle Mission (STS-41D)	0.65

- *Lowest dosage causing effects in at least one member of exposed population.
- **LD_{50} is the lethal dosage in 50 % of the exposed population.

Solar irradiance spectrum



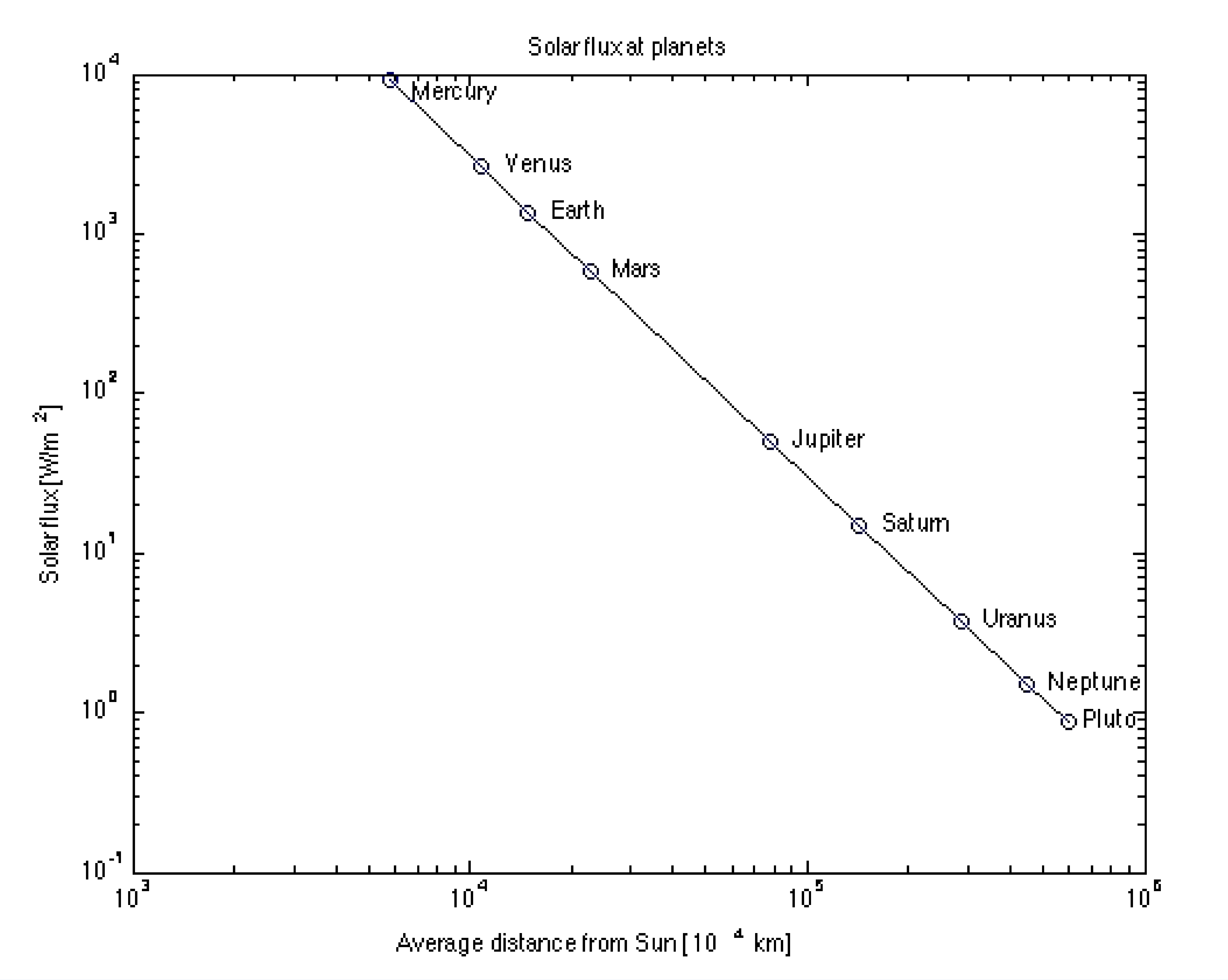
Credits: Wikipedia, prepared by Robert A. Rohde

Solar irradiance or flux in the solar system

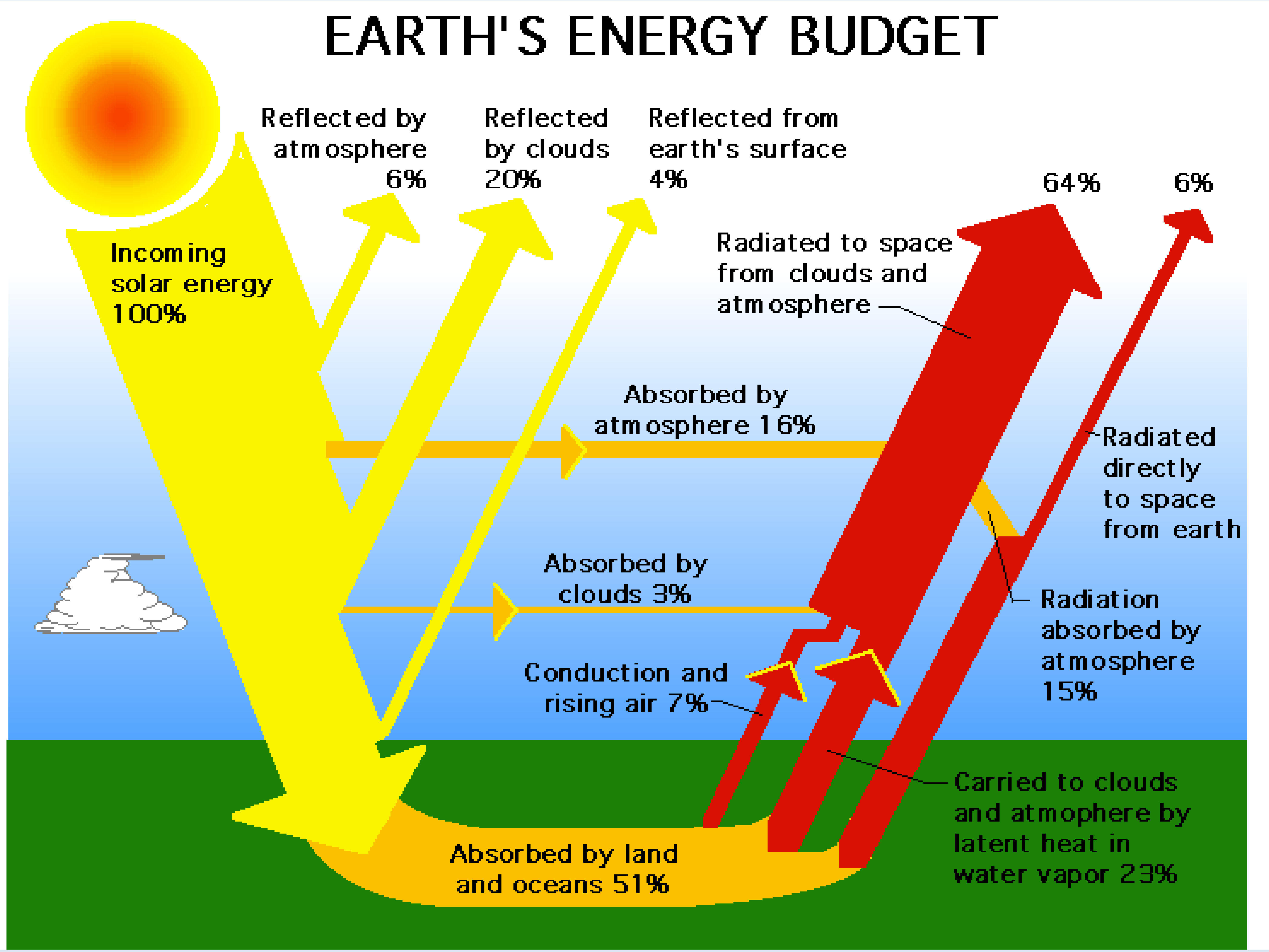
$$Solar\ flux \cong 1368\ w/m^2 \frac{R^2_{Earth}}{R^2_{Planet}}$$

Note: 1366 to 1370 W/m^2 are values of the Solar Constant that you find in many references.

According to Werner Schmutz from the World Radiation Center in Davos Switzerland, the Solar Constant is slowly decreasing and was at around 1361 W/m^2 in April 2015.



Earth’s energy budget



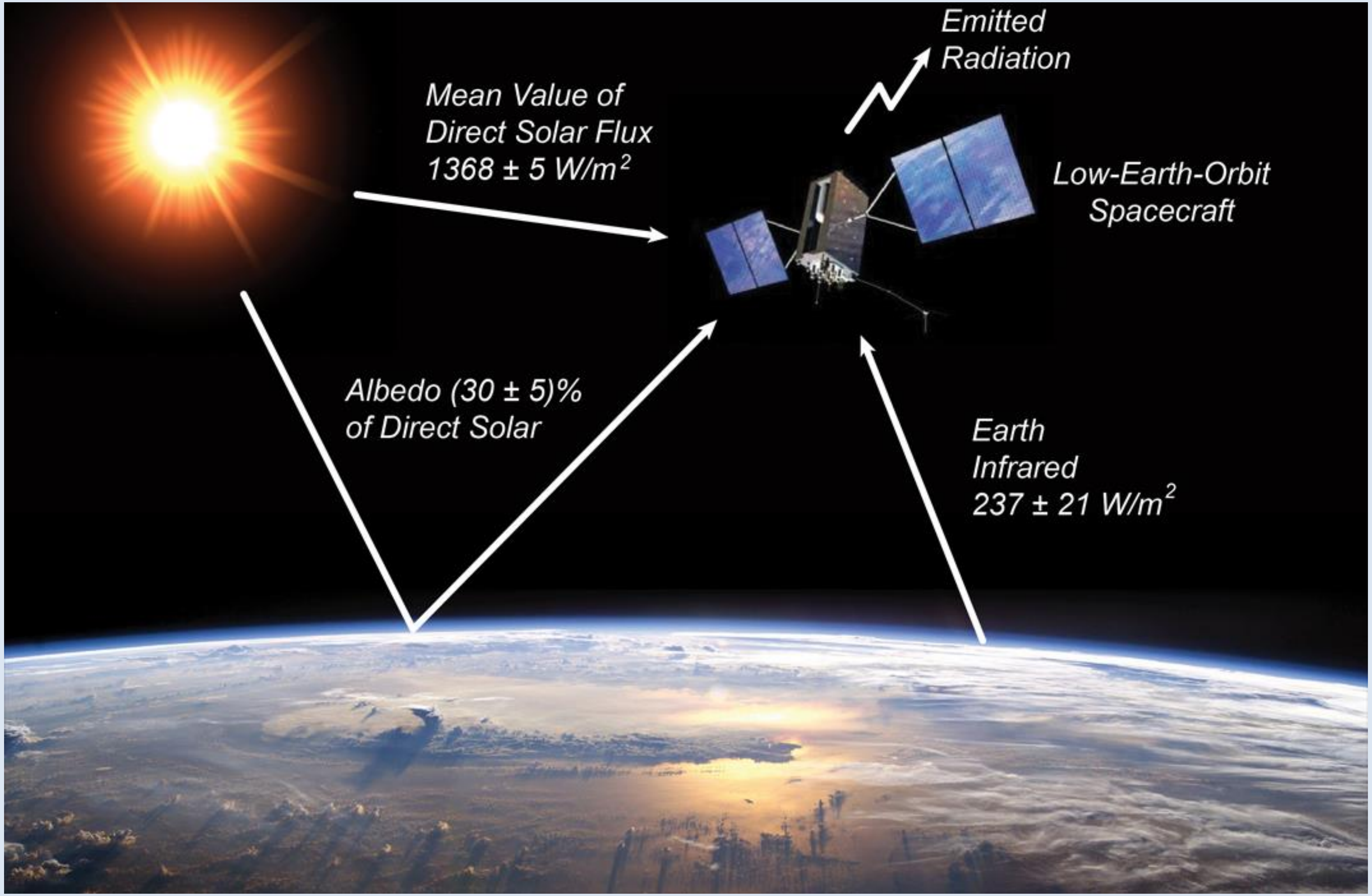
Credits: NASA

Solar flux and albedo of the planets

Planet	Mean Distance From Sun (10^6 km)	Mean Solar Flux $\left(\frac{W}{m^2}\right)$	Relative Mass (Earth = 1)	Planetary Albedo*
Mercury	58	9114	0.055	0.12
Venus	108	2619	0.815	0.59
Earth	150	1368	1.000	0.30
Mars	228	589	0.107	0.29
Jupiter	778	50	318.0	0.34
Saturn	1430	15	95.1	0.34
Uranus	2870	3.7	14.5	0.34
Neptune	4500	1.5	17.2	0.28

* Albedo is the diffuse reflectivity or reflecting power of a surface measured from zero for no reflecting power of a perfectly black surface, to 1 for perfect reflection.

Radiation balance for a spacecraft close to Earth



Credits: NASA

Stefan-Boltzmann law

The law relates the amount of radiation emitted by a black object to its temperature:

Per unit surface

$$P_e = \sigma T^4$$

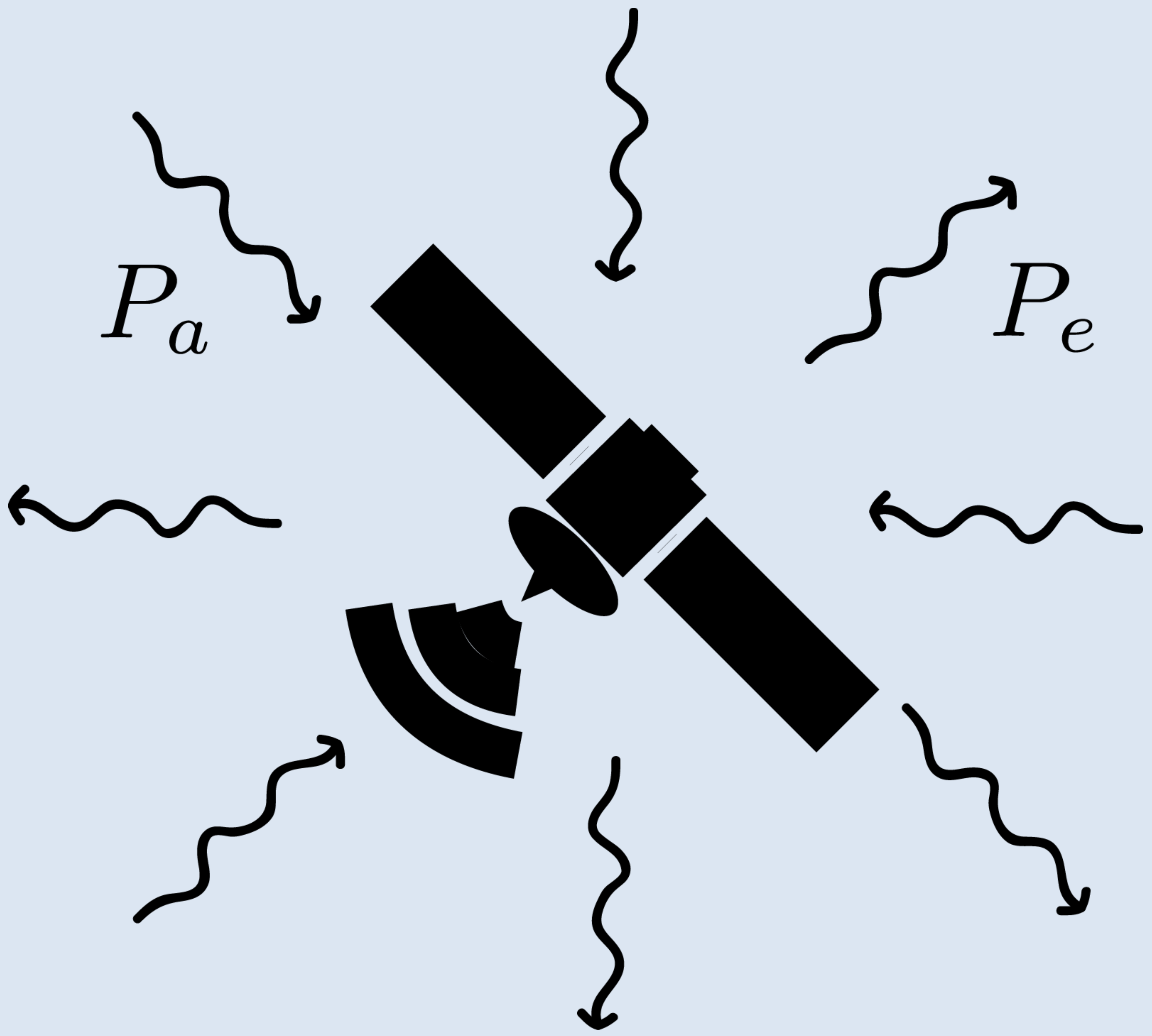
P

P_e: Total amount of radiation emitted by an object per square meter (W/m^2)

σ: Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8} \frac{\mathrm{W}}{\mathrm{m^2K^4}}$

T: The temperature of the object (K)

Radiation balance with the Sun as the only incident radiation source



$$P_a = \alpha \cdot S \cdot A_n$$

$$P_e = \epsilon \cdot \sigma T^4 \cdot A_{\mathrm{tot}}$$

α: solar absorptivity
ε: IR emissivity
S: Solar constant
A_n: Surface perpendicular to the Sun
A_{tot}: Total surface

Radiation balance and estimation of the temperature

- We have then:

$$T = \left(\frac{\alpha \times S \times A_n}{\epsilon \times \sigma \times A_{\text{tot}}} \right)^{\frac{1}{4}}$$

$$= \left(\frac{\alpha}{\epsilon} \right)^{\frac{1}{4}} \times \left(\frac{S \times A_n}{\sigma \times A_{\text{tot}}} \right)^{\frac{1}{4}}$$

- The importance of the factor $\left(\frac{\alpha}{\epsilon} \right)$ is clear, and its influence on the external temperature of a spacecraft has to be considered in the design.

Properties of materials

No.	Material	Measurement Temperature (K)	Surface Condition	Solar Absorption α	Infrared Emissivity ϵ	Absorptivity/ Emissivity ratio	Equilibrium Temperature ($°C$)
1	Aluminium (6061-T6)	294	As Received	0.379	0.0346	10.95	450
2	Aluminium (6061-T6)	422	As Received	0.379	0.0393	9.64	428
3	Aluminium (6061-T6)	294	Polished	0.2	0.031	6.45	361
4	Aluminium (6061-T6)	422	Polished	0.2	0.034	5.88	346
5	Gold	294	As Rolled	0.299	0.023	13.00	482
6	Steel (AM 350)	294	As Received	0.567	0.267	2.12	207
7	Steel (AM 350)	422	As Received	0.567	0.317	1.79	187
8	Steel (AM 350)	611	As Received	0.567	0.353	1.61	175
9	Steel (AM 350)	811	As Received	0.567	0.375	1.51	168
10	Steel (AM 350)	294	Polished	0.357	0.095	3.76	281
11	Steel (AM 350)	422	Polished	0.357	0.111	3.22	259
12	Steel (AM 350)	611	Polished	0.357	0.135	2.64	234
13	Steel (AM 350)	811	Polished	0.357	0.155	2.30	217
14	Titanium (6AL-4V)	294	As Received	0.766	0.472	1.62	176
15	Titanium (6AL-4V)	422	As Received	0.766	0.513	1.49	166
16	Titanium (6AL-4V)	294	Polished	0.448	0.129	3.47	270
17	Titanium (6AL-4V)	422	Polished	0.448	0.148	3.03	251
18	White Enamel	294	Al. Substrate	0.252	0.853	0.30	20
19	White Epoxy	294	Al. Substrate	0.248	0.924	0.27	13
20	White Epoxy	422	Al. Substrate	0.248	0.888	0.28	16

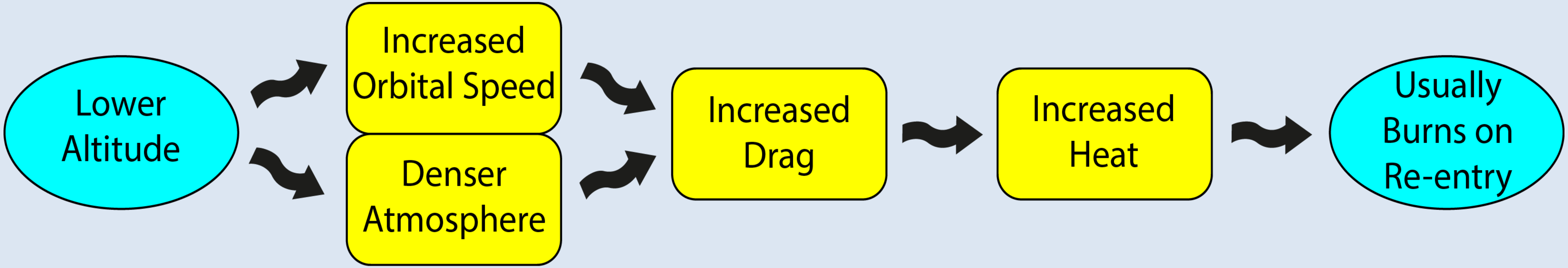
Spacecraft design strategies

Design strategies to take into account the characteristics of the space environment:

- Conducting surface on the spacecraft to avoid local voltage deltas and possible resulting electrical discharges;
- Filtering and hardening of electronic components;
- Proper choice of the $\left(\frac{\alpha}{\epsilon}\right)$ ratio for exposed surfaces.

Orbit decay and limitation of lifetime on LEO

- **Orbital decay**: Reduction in the altitude of a satellite's orbit.
- **Major cause:** Drag due to the Earth's upper atmosphere.



Drag equations and deceleration

EPFL

$$F_{\text{Drag}} = \frac{1}{2}\rho V^2 C_D A_n$$

Aerodynamics

$$a_{\text{Drag}} = \frac{1}{2}\rho V^2 \frac{C_D A_n}{m} = \frac{1}{2}\rho V^2 \times \frac{1}{BC}$$

ρ: Density (kg/m^3)

V: Velocity (m/s)

C_D: Drag coefficient (-)

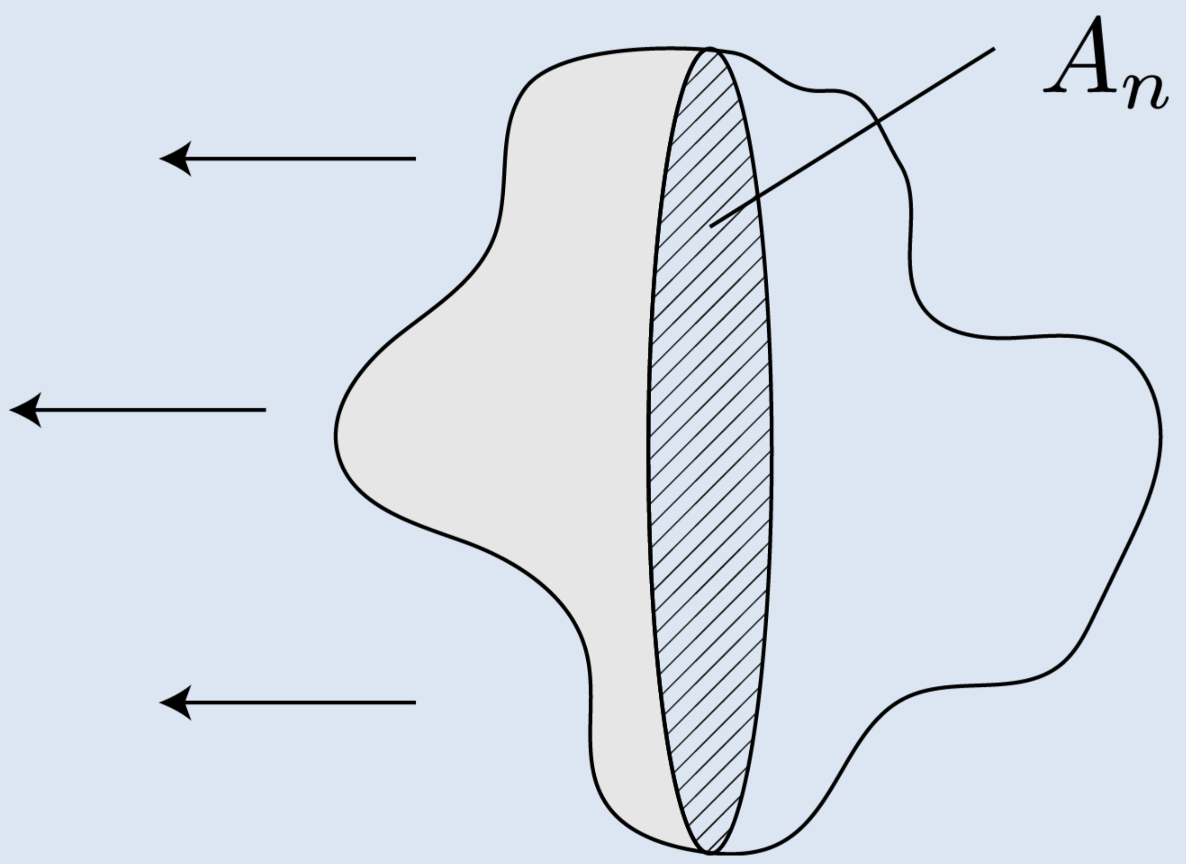
A_n: Reference area (m^2)

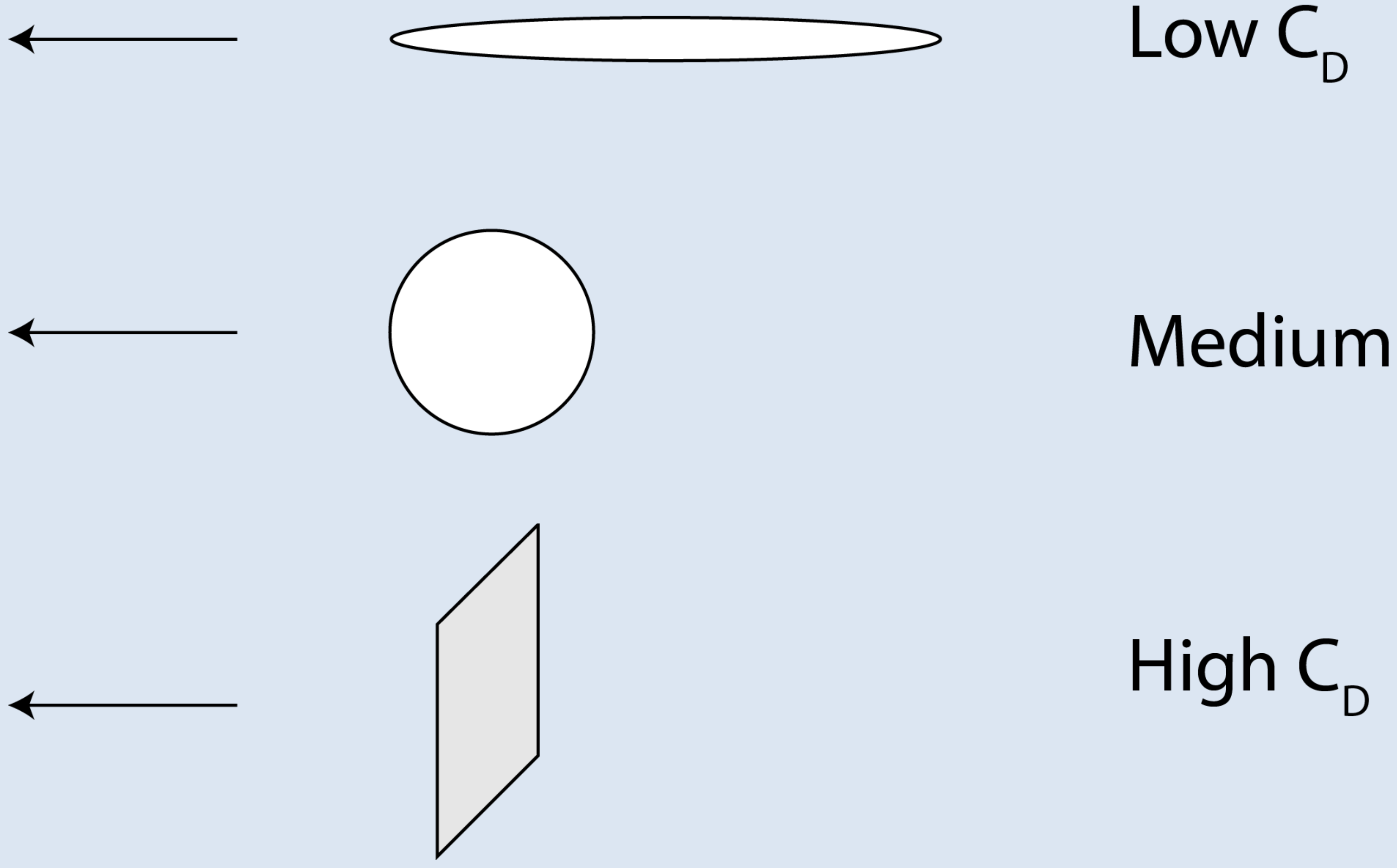
BC: Ballistic coefficient (kg/m^2)

Ballistic coefficient

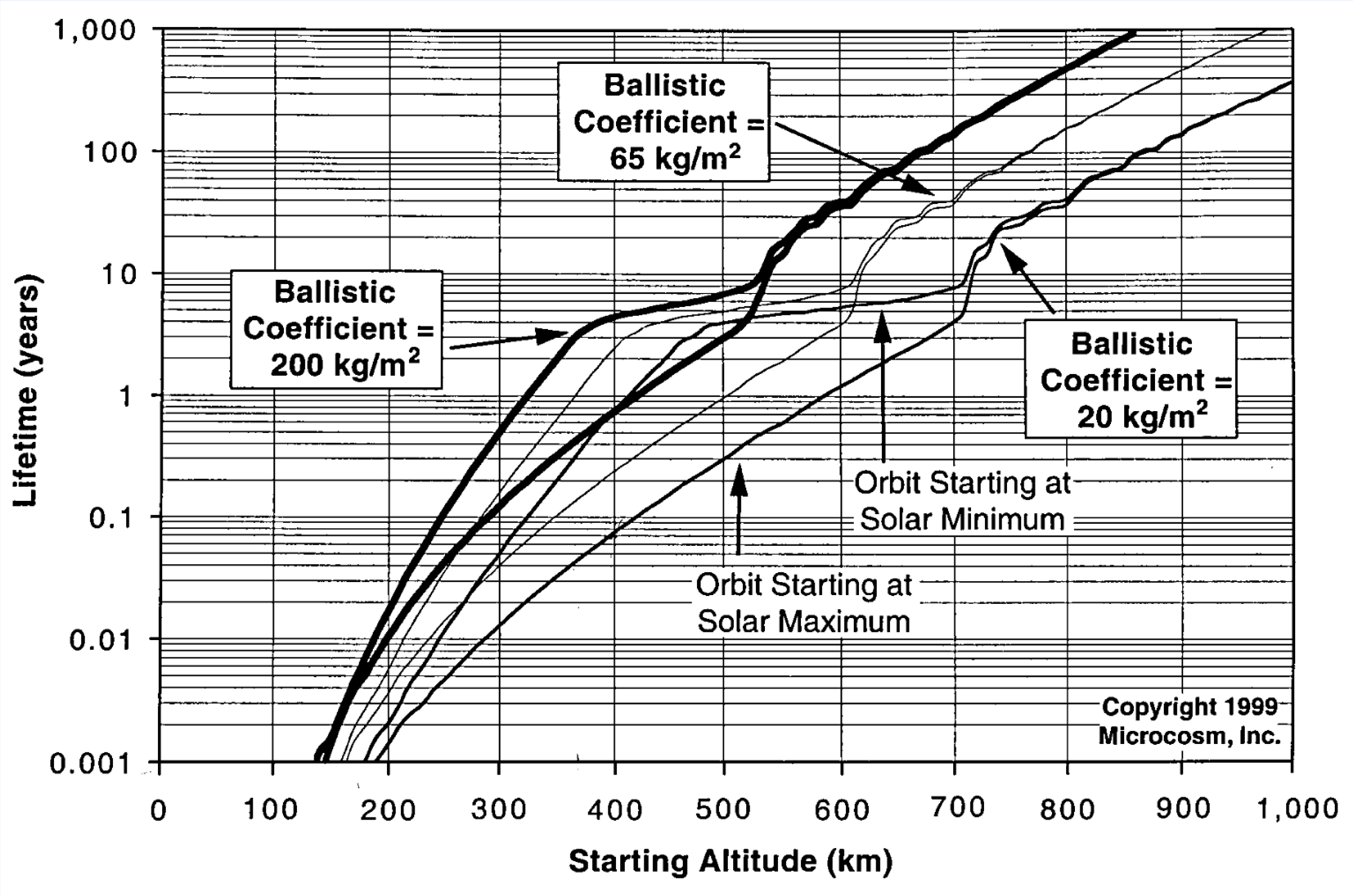
- **Ballistic coefficient BC:**
  - Measure of the resistance to orbit decay caused by atmospheric drag.

$$BC = \frac{m}{C_D A_n} \quad \left(\frac{\text{kg}}{\text{m}^2}\right)$$





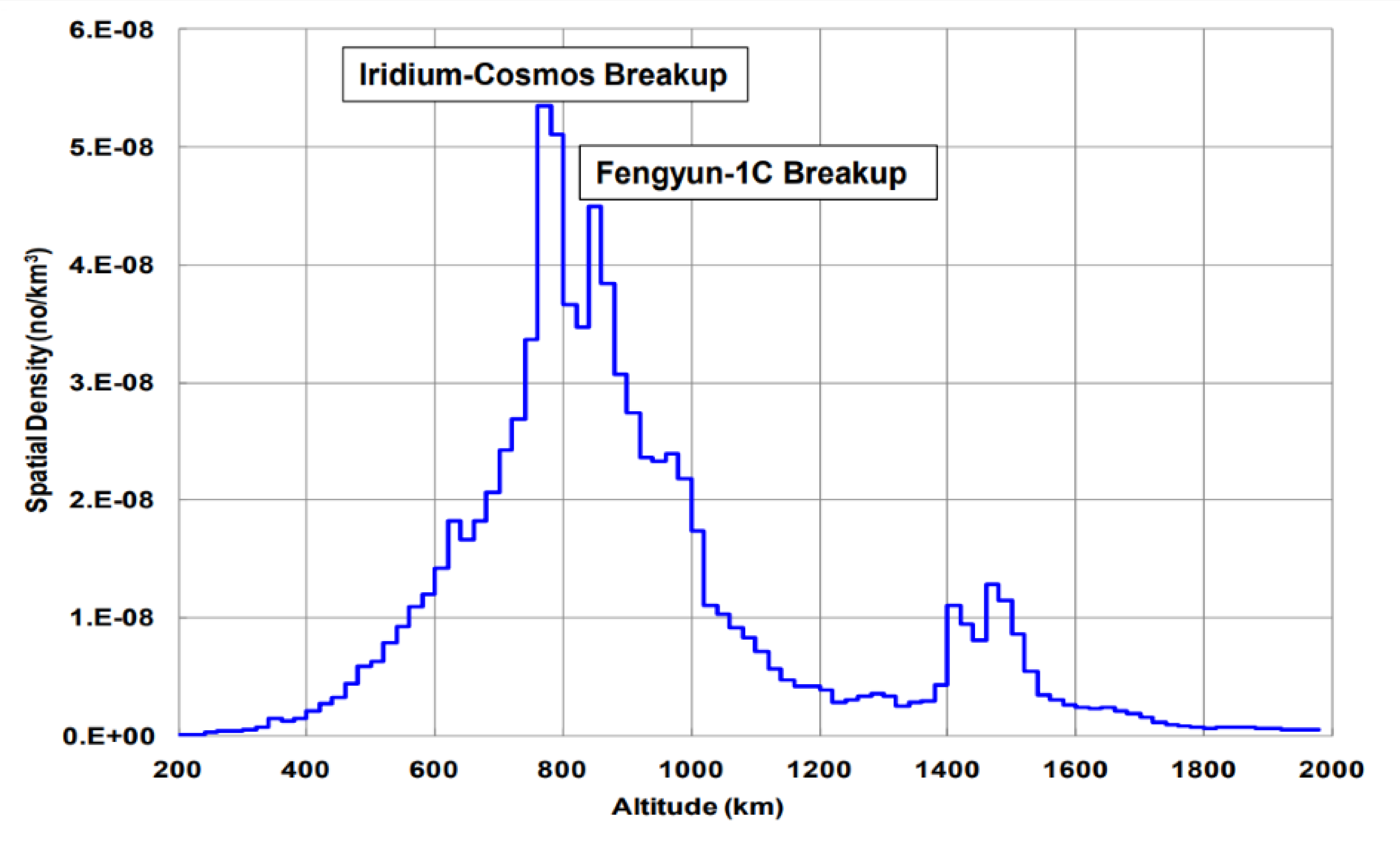
Lifetime and ballistic coefficient



- BC is inversely proportional to the drag deceleration.
- A high number for BC indicates a small value of the deceleration and consequently a long lifetime

Credits: J. R. Wertz & W. J. Larson, *Space Mission Analysis and Design,* 1992

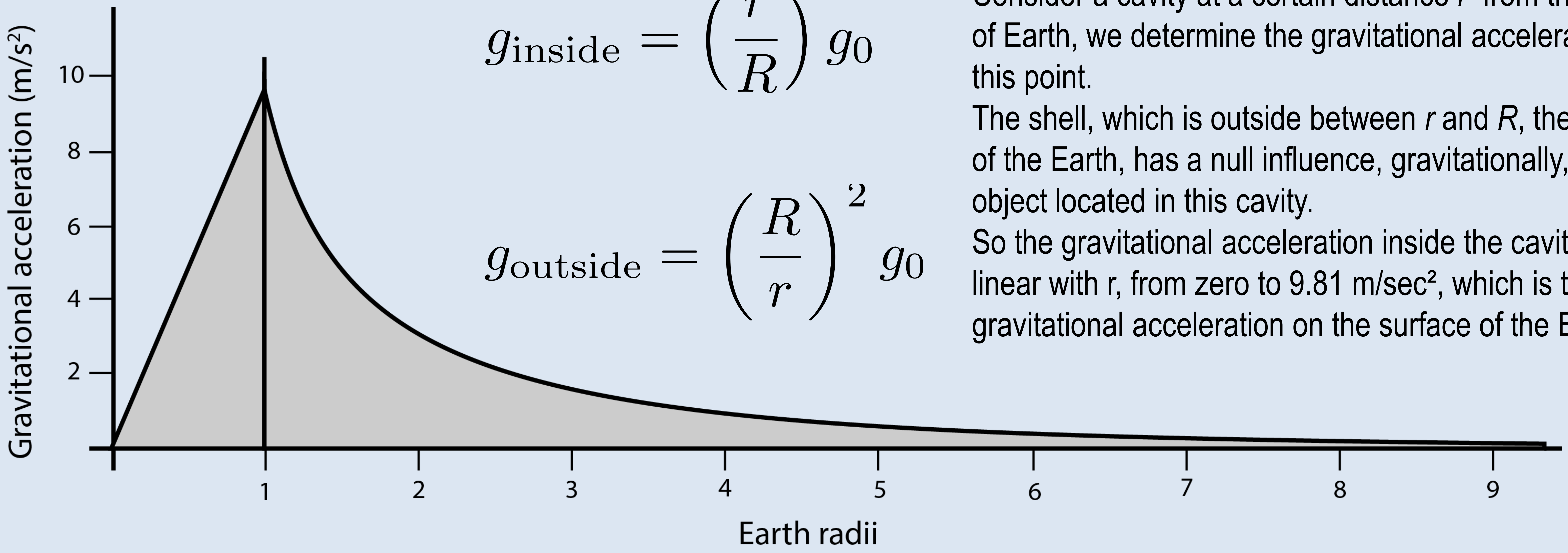
Spatial density of LEO space debris / altitude – NASA report 2011



Credits: NASA

Gravitational acceleration profile inside and outside Earth

EPFL



$$g_{\text{inside}} = \left(\frac{r}{R}\right) g_0$$

$$g_{\text{outside}} = \left(\frac{R}{r}\right)^2 g_0$$

Consider a cavity at a certain distance *r* from the center of Earth, we determine the gravitational acceleration at this point.
The shell, which is outside between *r* and *R*, the radius of the Earth, has a null influence, gravitationally, on any object located in this cavity.
So the gravitational acceleration inside the cavity is linear with r, from zero to 9.81 m/sec², which is the gravitational acceleration on the surface of the Earth.

The linear gravitational acceleration profile « Inside Earth » is valid only for an homogeneous Earth.

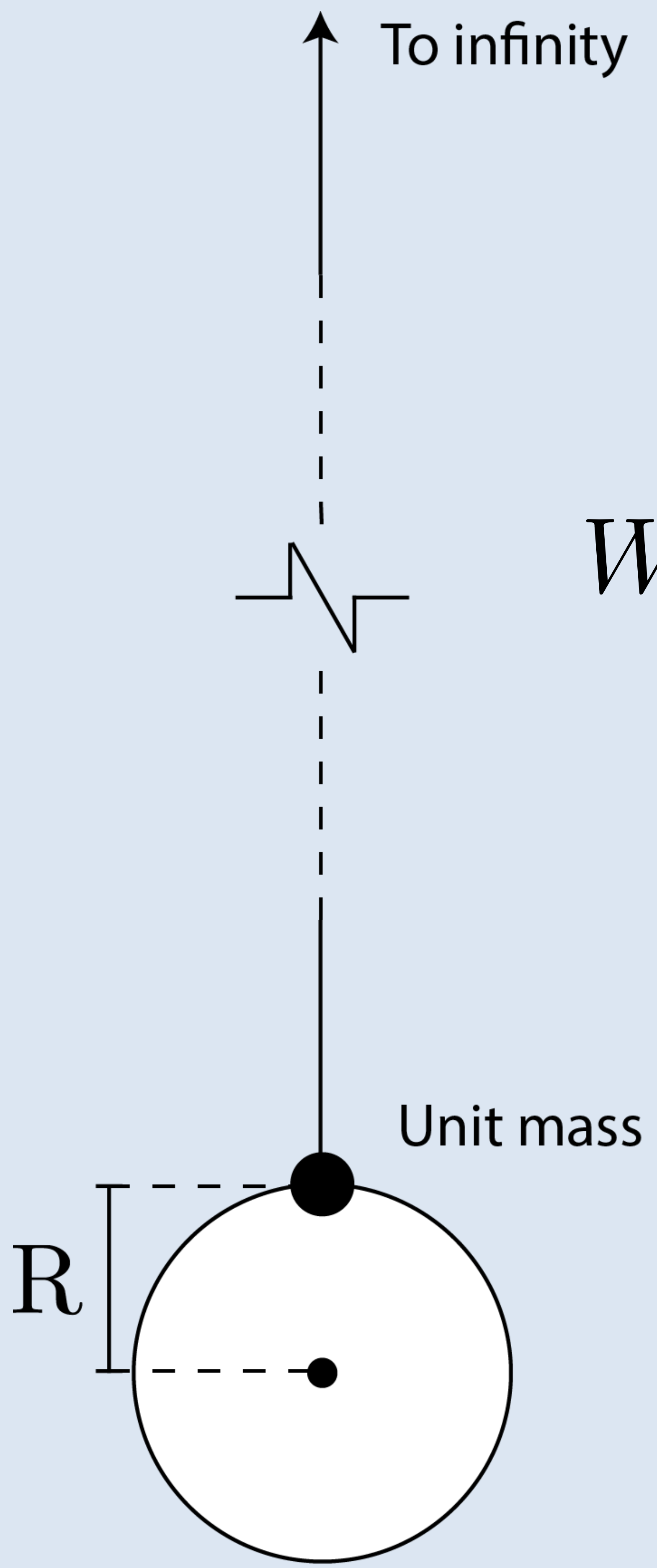
Gravitational figures for bodies in the solar system

Body	Gravity E=1	Time to fall 4.9 m (sec)	Escape velocity $\left(\frac{km}{s}\right)$	Circ. velocity on surface $\left(\frac{km}{s}\right)$
Sun	28	0.2	618	437
Mercury	0.26	2.0	3.5	2.5
Venus	0.90	1.1	10.4	7.3
Earth	1	1	11.2	7.9
Moon	0.16	2.5	2.3	1.6
Mars	0.38	1.6	5.0	3.6
Phobos	0.001∗	30.0∗	0.01∗	0.01∗
Jupiter	2.65	0.6	60	42.5
Ganymede	0.2∗	2.0∗	3.0∗	2.0∗
Saturn	1.14	0.9	36	25
Titan	0.2∗	2.0∗	3.0∗	2.0∗
Uranus	0.96	1.0	22	15.5
Neptune	1.0	1.0	23	16

∗Approximative figures

The escape velocity is equal to the square root of two times the circular velocity for a given distance to the center of the attracting object.

The concept of gravitational well: work



- Work performed to bring a unit mass to infinity, from the Earth's surface

$$W_R = \int_R^{\infty} \frac{\mu}{r^2} dr = \frac{\mu}{R^2} R = g_0 R$$

$$\boxed{W_R = g_0 R}$$

- ***R***: Earth's radius
- ***g_0***: Gravitational acceleration on Earth's surface.

The work necessary to lift a unit mass from the surface of Earth to infinity is the constant gravitational acceleration g_0 times the Earth's radius.

We say: The depth of the Earth's gravitational well is equal to the radius of the Earth R

The concept of gravitational well: work

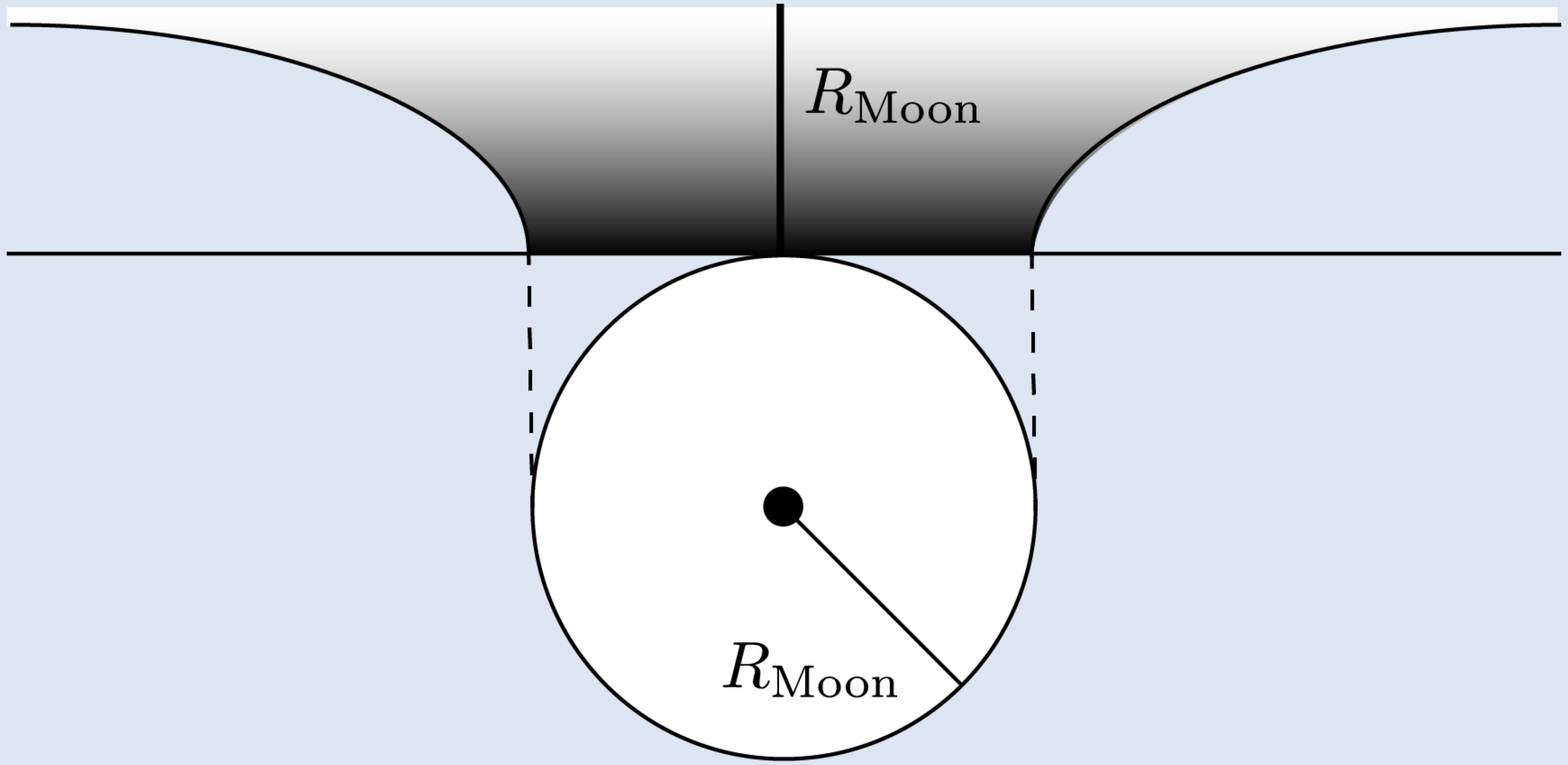
- Starting at a distance ***r*** from the Earth's center, the work performed is:

$$W_r = g_r r = \frac{g_r}{g_0}\frac{r}{R}W_R = \frac{R}{r}W_R$$

The work necessary to lift a unit mass from the distance *r*, larger than the radius of the Earth,to infinity, is equal to the work from the surface of the Earth multiplied by the factor *R/r*

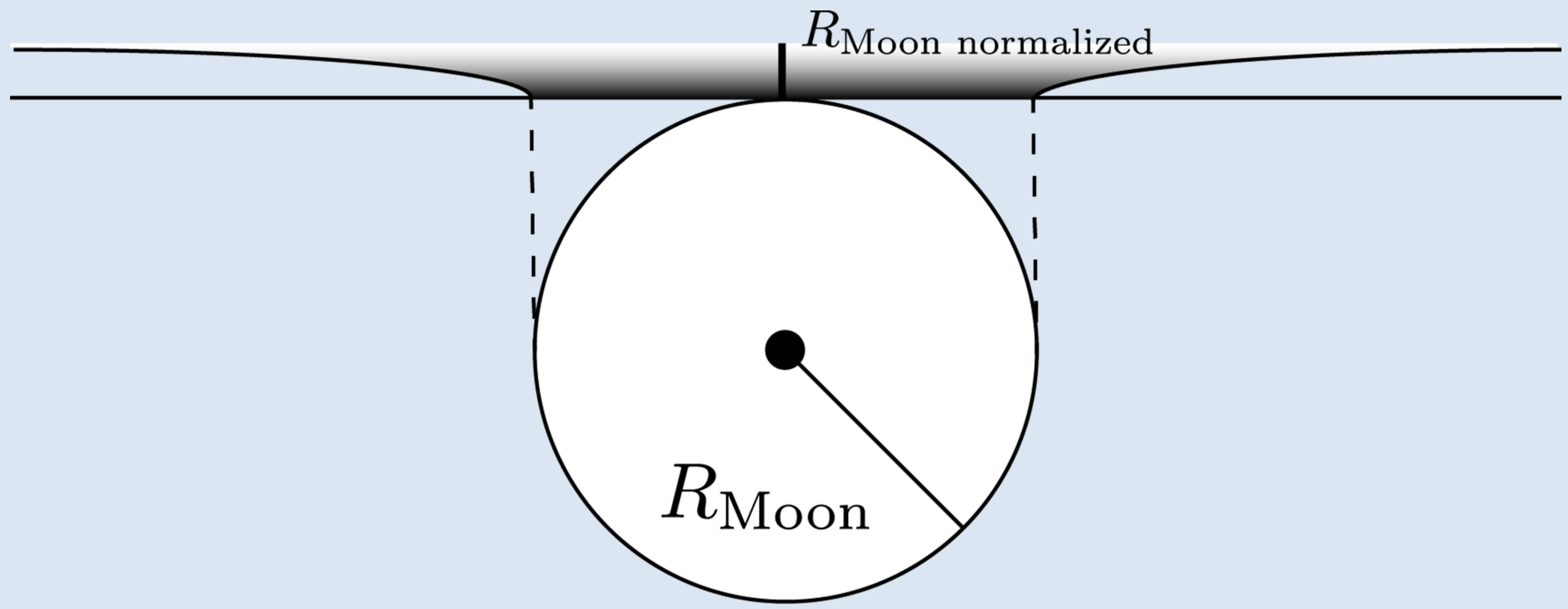
The profile of the gravitational well is in *1/r*

Moon's gravitational well: depth = R_{Moon}



The work needed to bring a unit mass from the surface of the Moon to infinity is equal to the work done to take that unit mass from the surface of the Moon to the radius of the Moon away from the Moon's surface, with a constant force equal to the gravitational force on the surface.

Moon's gravitational well normalized



Considering that the gravitational acceleration on the Moon is only 1/6 of the value on Earth, the normalized depth of the gravitational well of the Moon is the radius of the Moon divided by six.

The depth of gravitational well of any spherical objects in the solar system or elsewhere, is always normalized to the gravitational acceleration of the Earth for comparison purposes. It is the radius of that object, multiplied by the ratio between the gravitational acceleration on the surface of that object and the one on the surface of the Earth

Concept of Escape Velocity

- How to escape from the Earth's gravitational influence, starting from the Earth surface?
- Slow method with a low thrust rocket: inefficient.
- Rapid, with a single impulse: **Escape Velocity concept.**
- Work performed to bring a unit mass from the Earth's surface to infinity = $\boldsymbol{g_0 R}$.

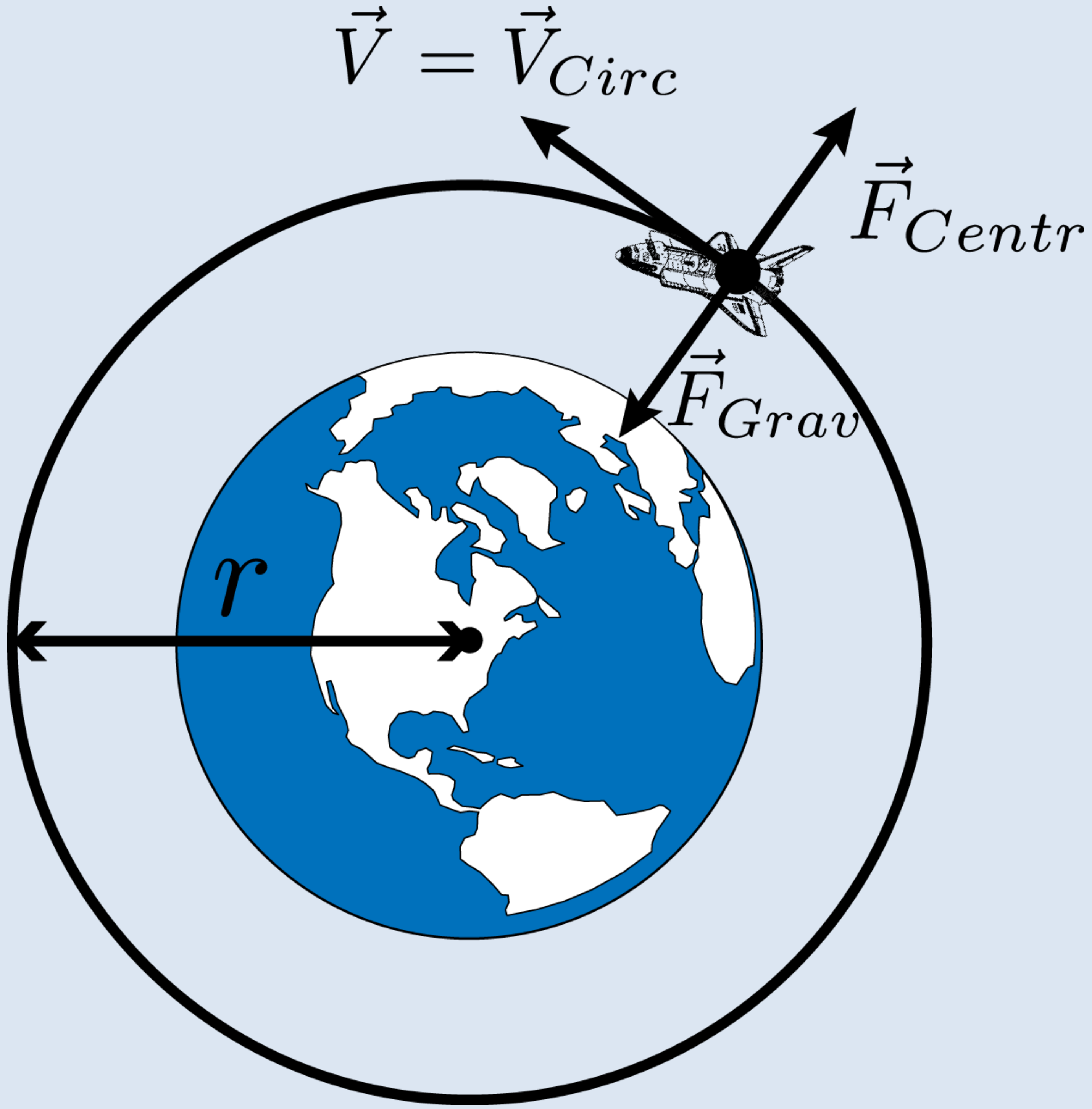
$$\frac{1}{2}V_{Esc}^2 = g_0 R \Rightarrow V_{Esc} = \sqrt{2g_0 R} = \sqrt{\frac{2\mu}{R}}$$

Escape velocity is the velocity at which a spacecraft has to leave the surface of the Earth in order to reach infinity with a zero velocity (in the absence of an atmosphere).

If velocity at infinity is not zero, you have done more than what is needed to just escape the gravitational influence of the Earth.

The work needed to bring a unit mass (the spacecraft) from the surface of the Earth to infinity is equal to the initial kinetic energy.

Circular velocity



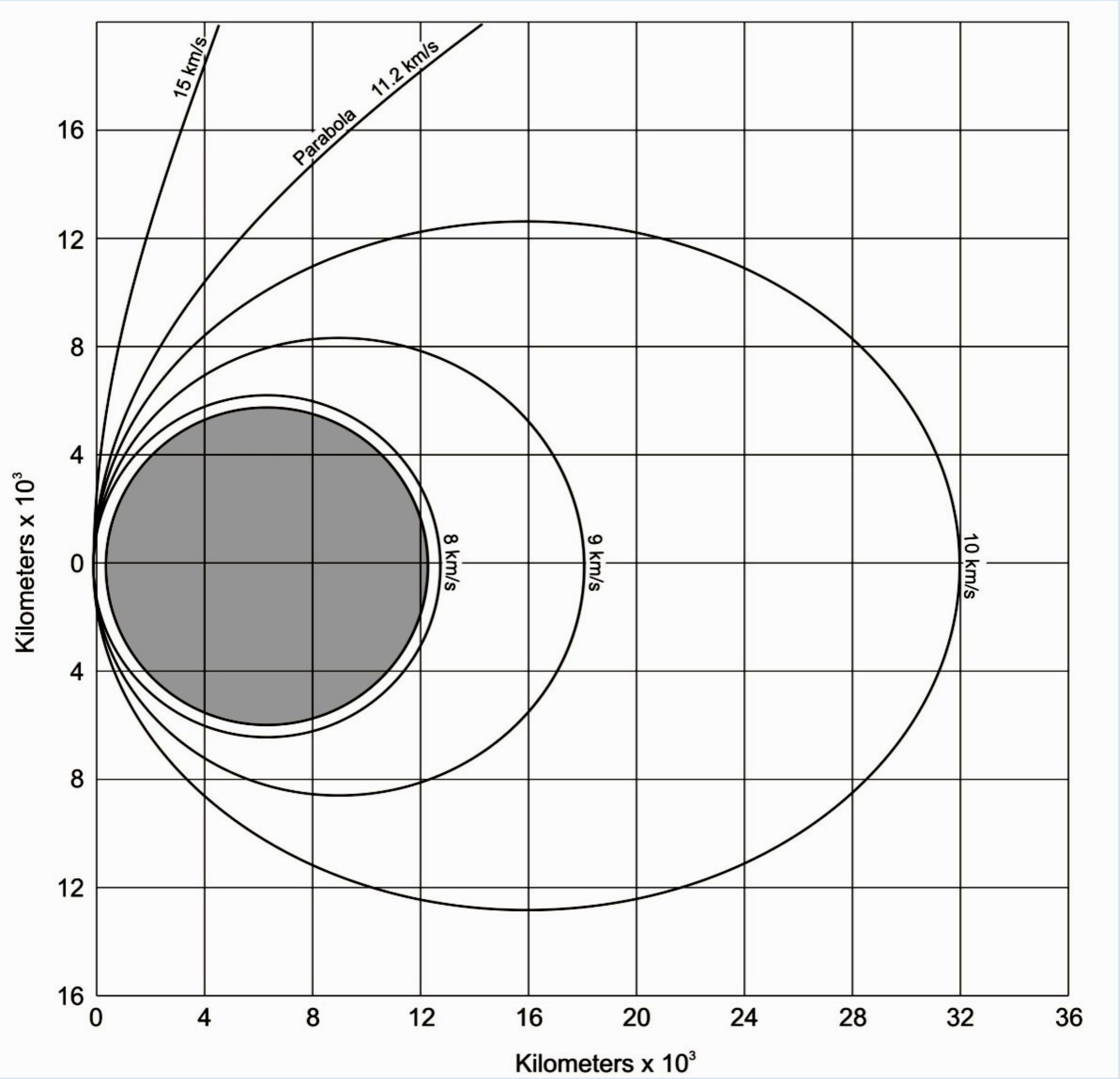
The Space Shuttle on a circular Low Earth Orbit has a velocity of the order of 7.7—7.8 km/s, or going around the Earth in about an hour and a half.

The centrifugal force resulting from the curved orbital trajectory is equal in magnitude to the gravitational force on the orbiting Shuttle:

$$F_{\text{Centr}} = F_{\text{Grav}}$$

$$\Rightarrow \frac{V_{\text{Circ}}^2}{r} = \frac{\mu}{r^2}$$

$$\Rightarrow V_{\text{Circ}} = \sqrt{\frac{\mu}{r}}$$



2.3.1 Orbital motion and Kepler's laws

Space Mission Design and Operations

Prof. Claude Nicollier

Credits: Adapted from « Ascent to Orbit », Arthur C. Clarke

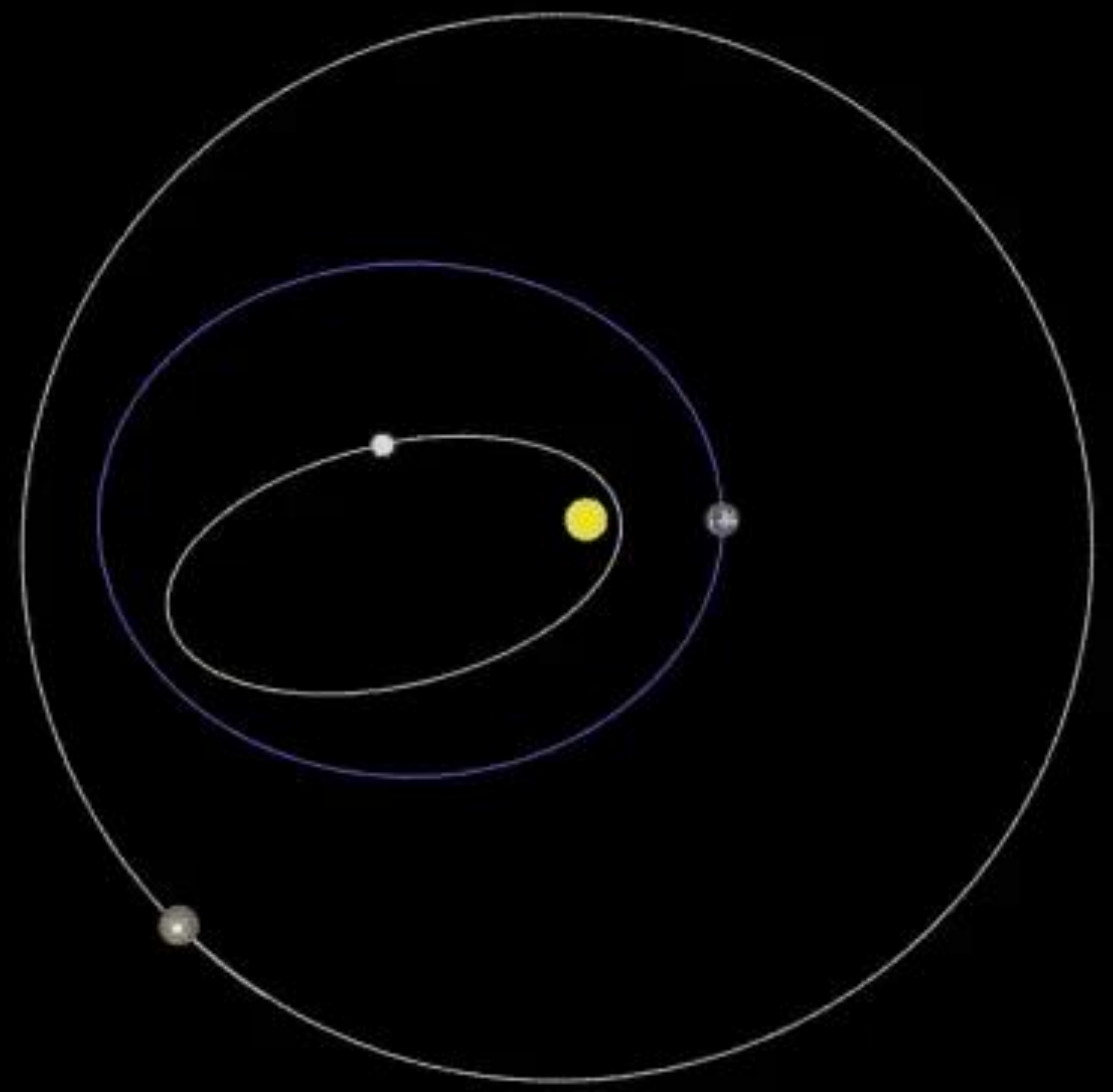
Newton's law and hypotheses for the rest of the course

$$F_{Grav} = G\frac{Mm}{r^2} = m\frac{\mu}{r^2}$$

- **Hypotheses:** Unless otherwise noted, we will consider:
  - Central body of mass ***M*** + spacecraft only.
  - Mass of spacecraft ***m*** << mass of the central body ***M***.
  - Bodies spherical and homogeneous.
  - No perturbations.

Kepler's laws (1609-1619) – First law

The orbit of every planet is an ellipse with the Sun at one of the two foci.



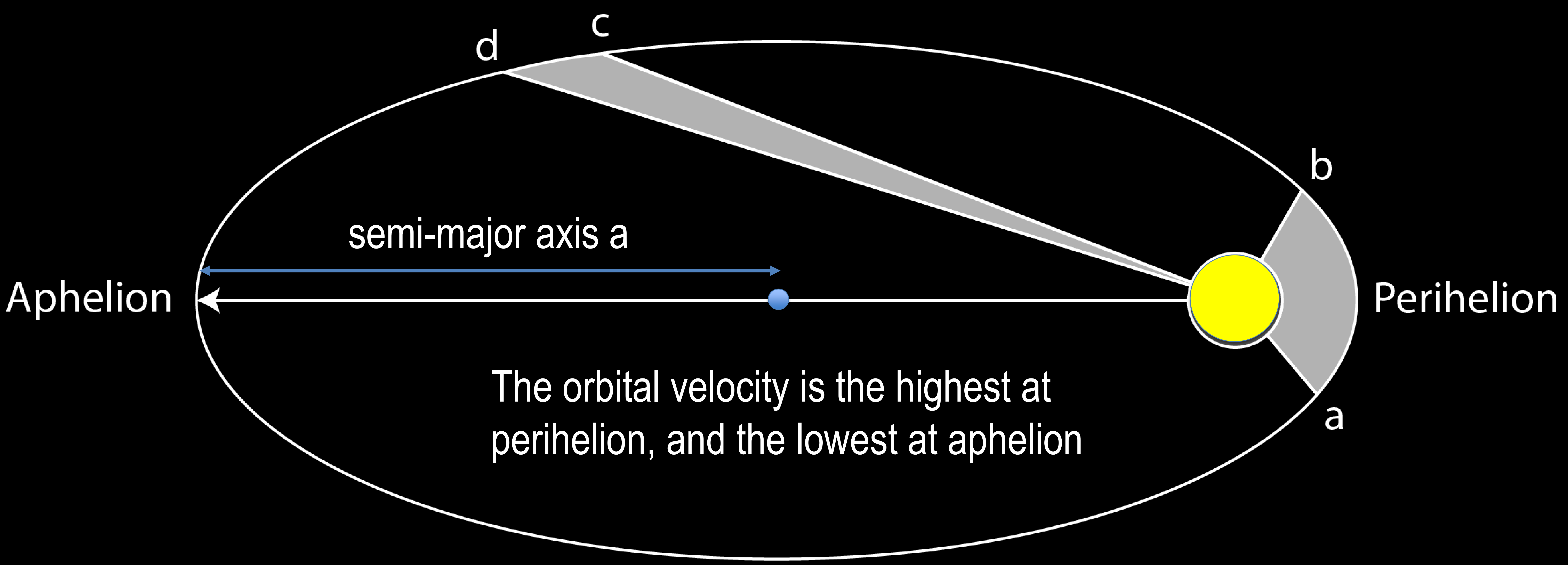
Kepler's laws were established at the beginning of the 17th century from observations of the motion of Mars in the sky made by Tycho Brahe.

The first Kepler's law can be generalized in the case of a two-body problem: the orbit of the small body versus the large body is, generally speaking, a conic, i.e. an ellipse, a parabola, or hyperbola.

Credits: Animations for Physics and Astronomy Education

Kepler’s laws (1609-1619) – Second law

A line joining a planet and the Sun sweeps equal areas during equal intervals of time.



Kepler's laws (1609-1619) – Third law

The square of the orbital period of a planet is proportional to the cube of the semi-major axis of the orbit.

$$T^2 \sim a^3 \longrightarrow T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

Orbital period

Velocity on a circular orbit:

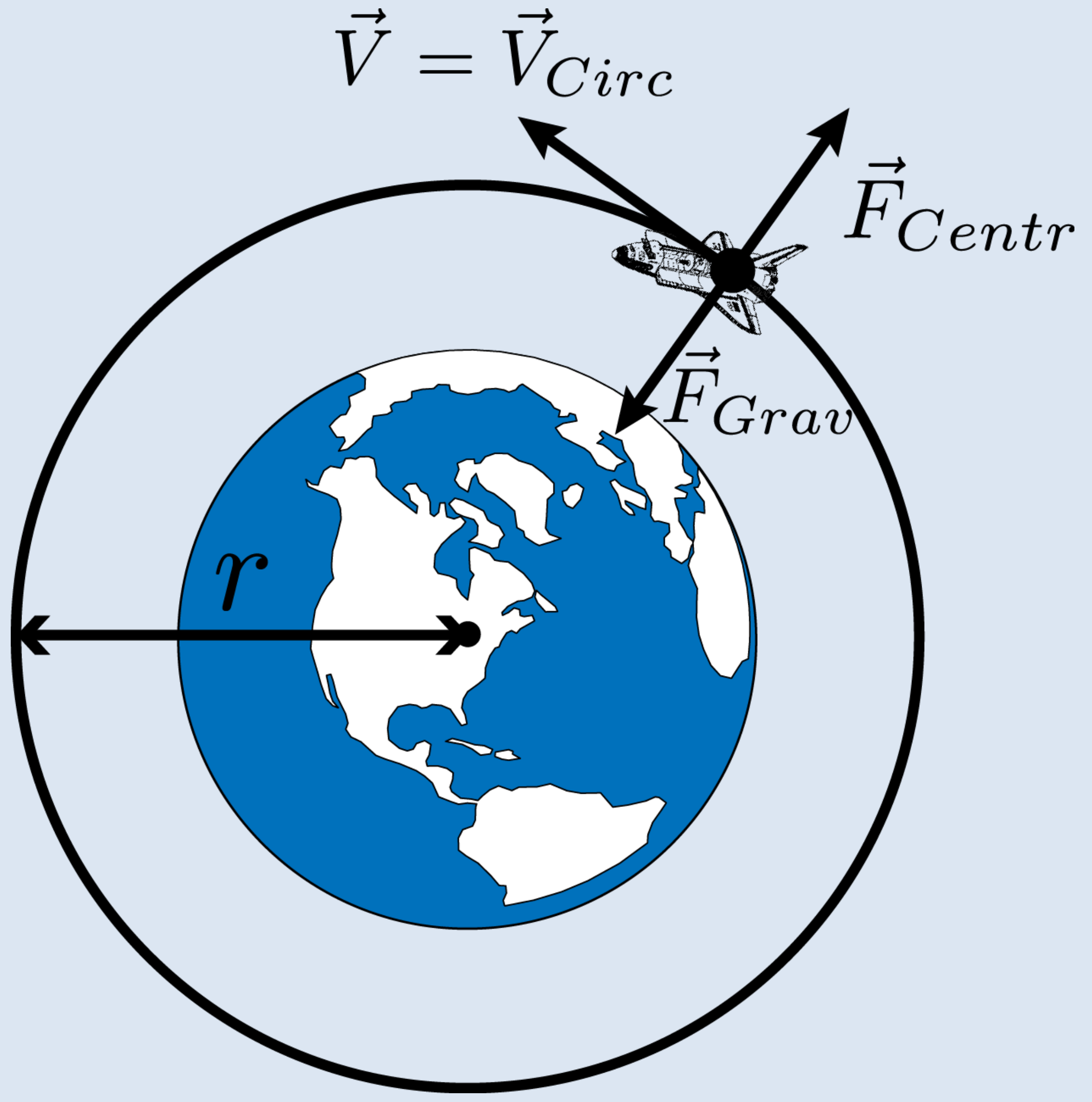
$$F_C = F_{\text{Grav}} \Rightarrow \frac{V^2}{r} = \frac{\mu}{r^2} \Rightarrow V = \sqrt{\frac{\mu}{r}}$$

Period (circular orbit):

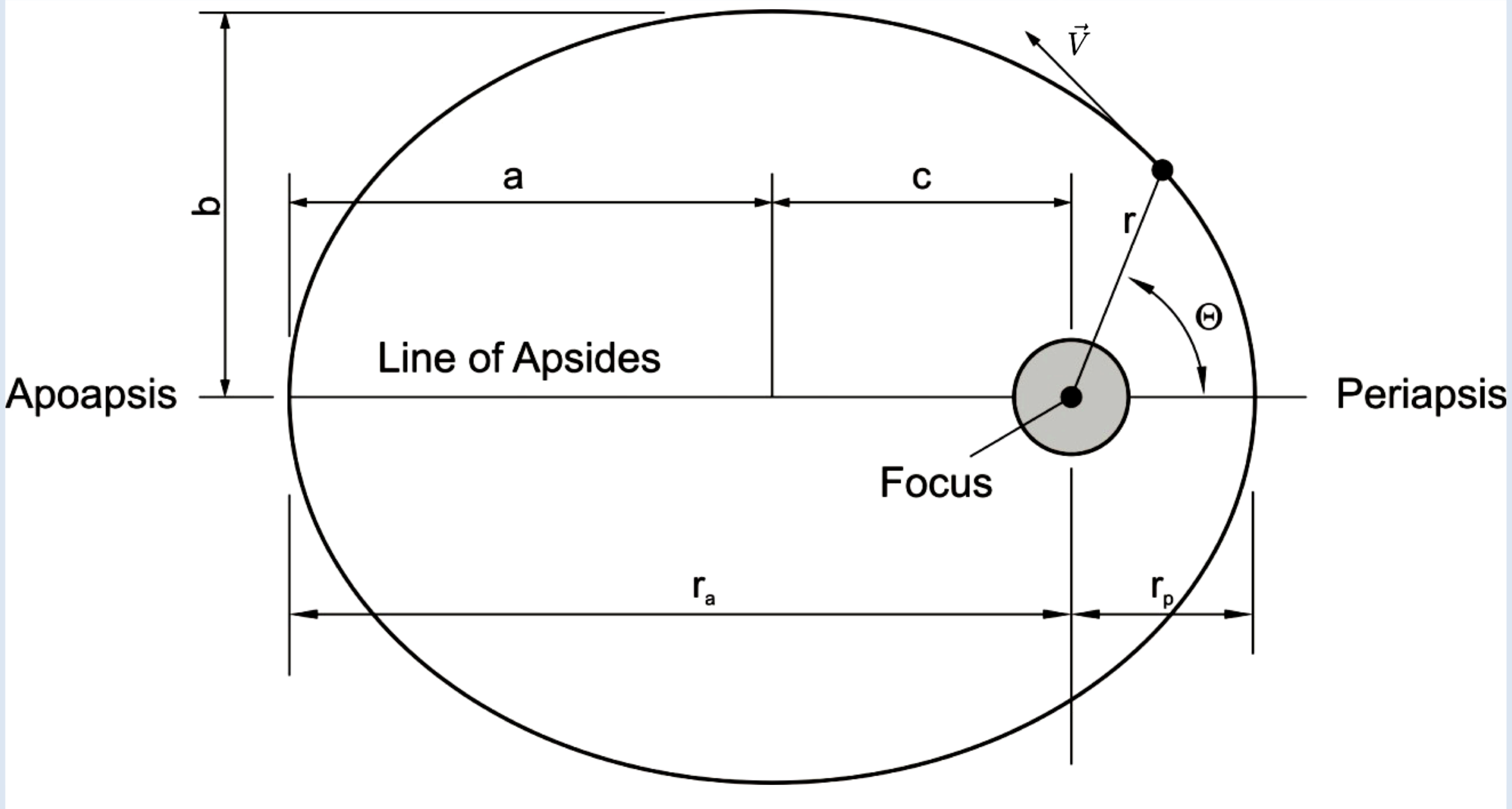
$$T = \frac{2\pi r}{V} = 2\pi\sqrt{\frac{r^3}{\mu}}$$

and for an elliptical orbit:

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$



Elliptical orbits



- ***a***: Semi-major axis
- ***b***: Semi-minor axis
- ***c*** = ***ae***: Eccentricity ***e*** < 1

- r_a: Distance to the apoastris
- r_p: Distance to the periastris

- $\vec{V}$: Velocity
- Θ: True anomaly

Periapsis and apoapsis are general terms. Periastris and apoastris sometimes used for a star as central body. If the Earth is the central body, we talk about perigee and apogee; if it is the Sun, perihelion and aphelion.
The *True anomaly* is the angle between the direction of the periapsis from the central body and the radius vector to the spacecraft or the planet.
if *e* = 0 the orbit is circular. If e = 1, the orbit is parabolic (a to infinity)

Energy of the orbital motion and orbital velocity

- Energy of the orbital motion, per unit mass:

$$\epsilon = \frac{V^2}{2} - \frac{\mu}{r}$$

$$\epsilon = -\frac{\mu}{2a}$$

depends on a only

- Orbital velocity at any location on an elliptical or circular orbit:

elliptical (Vis Viva equation):

$$V = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$$

circular:

$$V = \sqrt{\frac{\mu}{r}}$$

The total energy in a gravitational field is the sum of the kinetic energy and the potential energy.

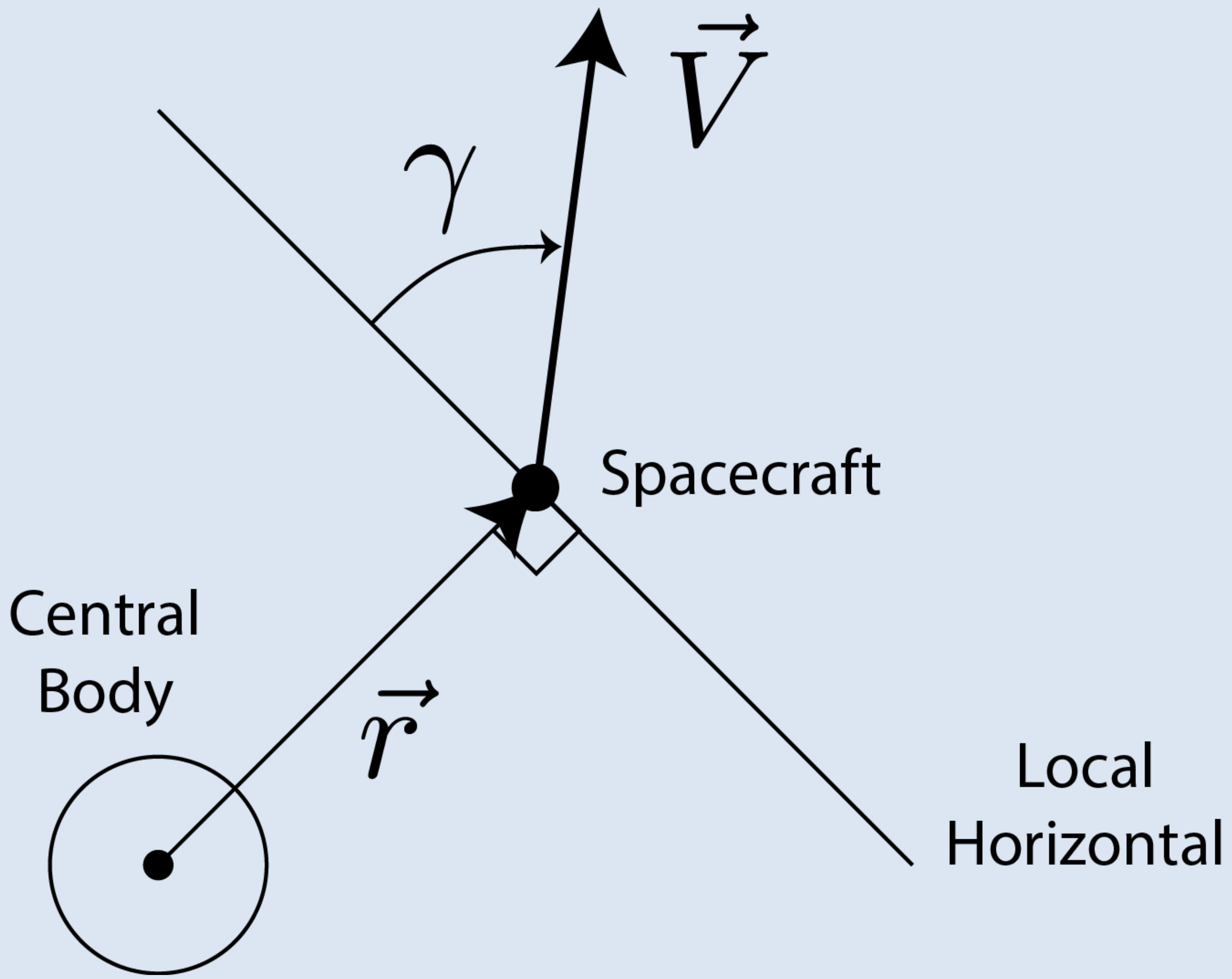
In case of a very elongated ellipse, the total energy is close to zero.

In the limit case of a parabolic orbit, the total energy is equal to zero.

If V < escape velocity, which is the case for a closed orbit, elliptical or circular, the total energy is negative.

If the orbit is hyperbolic, the total energy is positive

Angular momentum



- Angular momentum of a spacecraft $\vec{j}$, per unit mass:

$$\vec{j} = \vec{r} \times \vec{V}$$

$$|\vec{j}| = r \cdot V \cos\gamma$$

- γ is the flight path angle.

The flight path angle is the angle between the direction of the velocity vector and the perpendicular to the radius vector at the point where the spacecraft is.

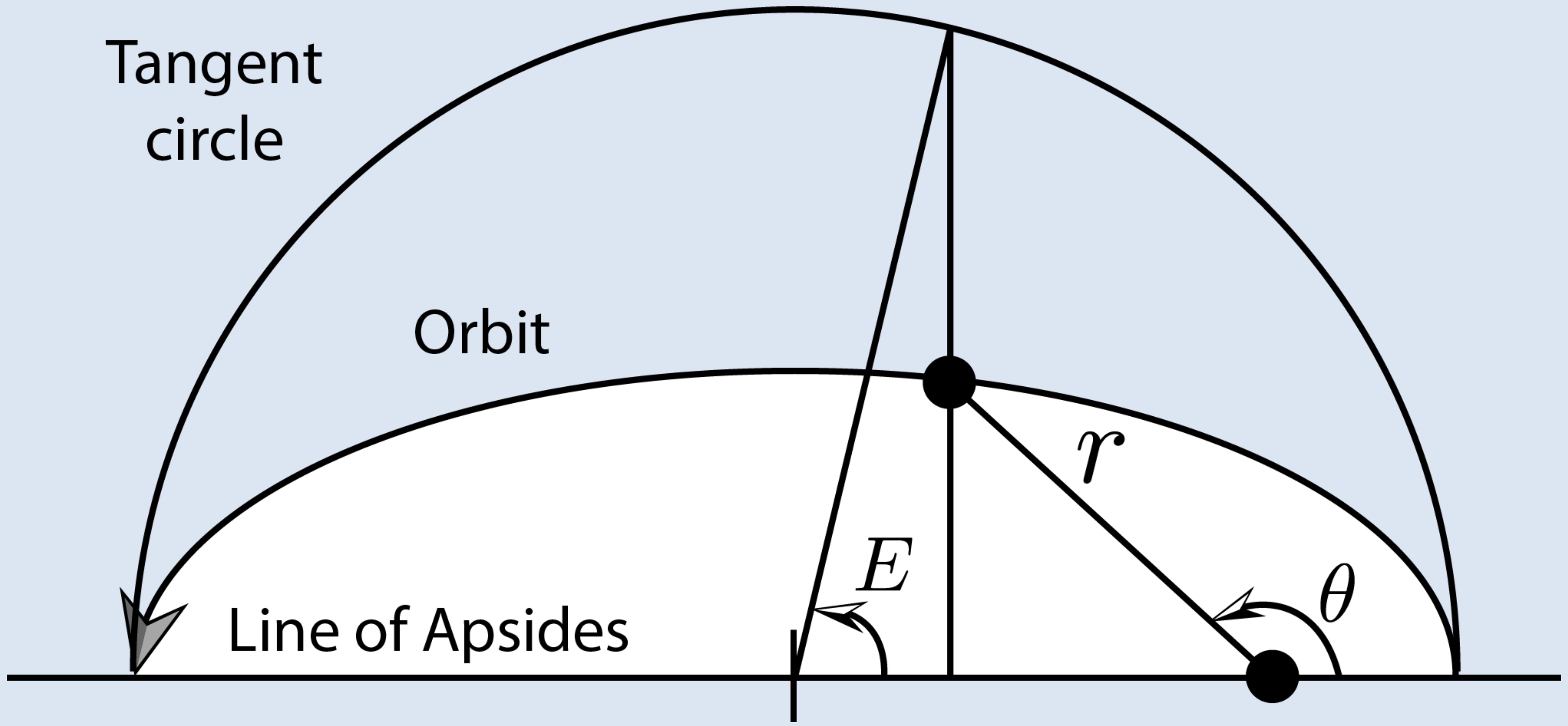
Elliptical orbits – Useful formulas

- Eccentricity $$e = \frac{c}{a} = \frac{(r_a - r_p)}{(r_a + r_p)} = \frac{r_a}{a} - 1 = 1 - \frac{r_p}{a} = \frac{r_2 - r_1}{r_1 \cos\theta_1 - r_2 \cos\theta_2}$$
- Flight path angle $$\tan\gamma = \frac{e \sin\theta}{1 + e\cos\theta}$$
- Mean motion (rad/sec) $$n = \sqrt{\frac{\mu}{a^3}}$$
- Period $$T = \frac{2\pi}{n} = 2\pi\sqrt{\frac{a^3}{\mu}}$$
- Radius $$r = \frac{a(1-e^2)}{(1 + e\cos\theta)} = \frac{r_p(1+e)}{(1+e\cos\theta)}$$
- Apoapsis radius $$r_a = a(1+e) = 2a - r_p = r_p\frac{(1+e)}{(1-e)}$$

Elliptical orbits – Useful formulas

- Periapsis radius $$r_p = a(1-e) = r_a\frac{(1-e)}{(1+e)} = 2a - r_a = \frac{r_1(1+e\cos\theta_1)}{1+e}$$
- True anomaly $$\cos\theta = \frac{r_p(1+e)}{re} - \frac{1}{e} = \frac{a(1-e^2)}{re} - \frac{1}{e}$$
- Semi-major axis $$a = \frac{(r_a+r_p)}{2} = \frac{r_p}{(1-e)} = \frac{r_a}{(1+e)}$$
- Time since periapsis $$t = \frac{(E - e\sin E)}{n}$$
- Eccentric anomaly $$cos E = \frac{e+\cos\theta}{1+e\cos\theta}$$
- Velocity $$V = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} \qquad r_pV_p = r_aV_a$$

Elliptical orbits – Kepler's equation

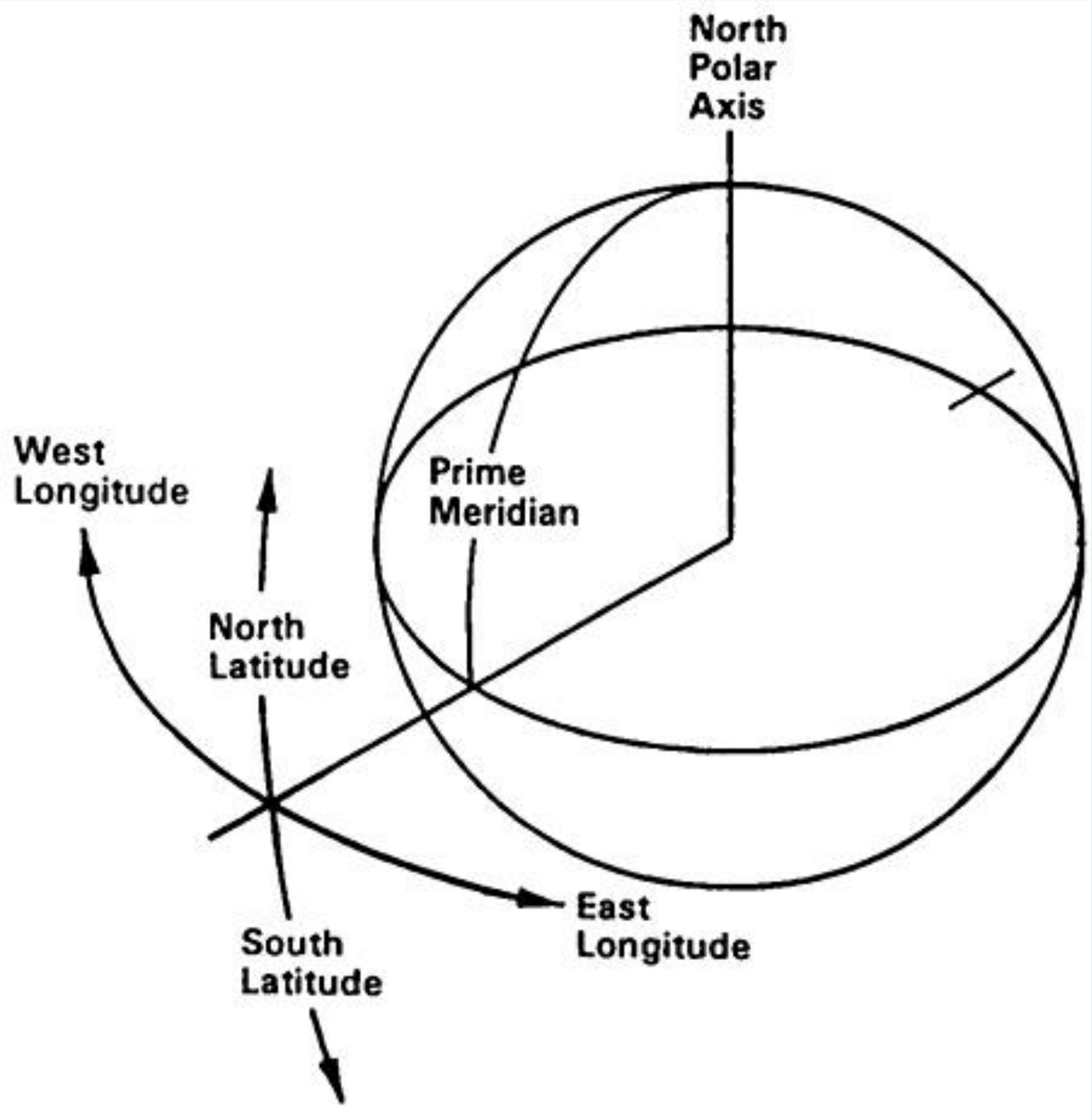


E, the eccentric anomaly, is an angular parameter that defines the position of a body that is moving along an elliptical Keplerian orbit.

The Kepler's equation is a transcendental equation that cannot be solved for *E* but expresses the time evolution of *E*, the eccentric anomaly since passing the periapsis.

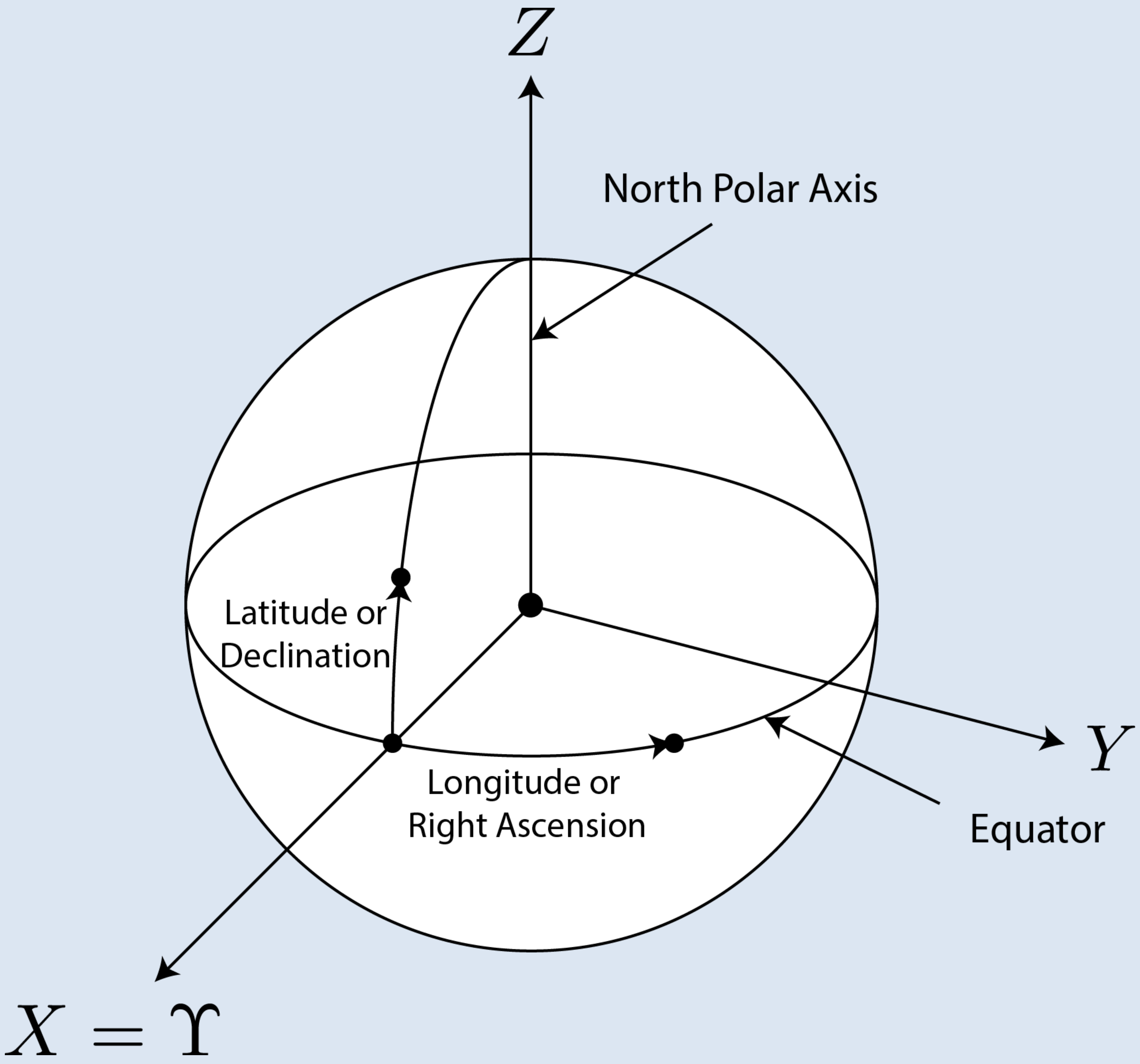
$$t = \frac{(E - e\sin E)}{n}$$

Geographic coordinate system



The geographic coordinate system (longitude, latitude) is used to specify a location on the surface of the Earth.

Geocentric-inertial coordinate system



An inertial frame is an orthogonal frame of reference XYZ, with respect to which the laws of motion, the laws of Newton, are valid.

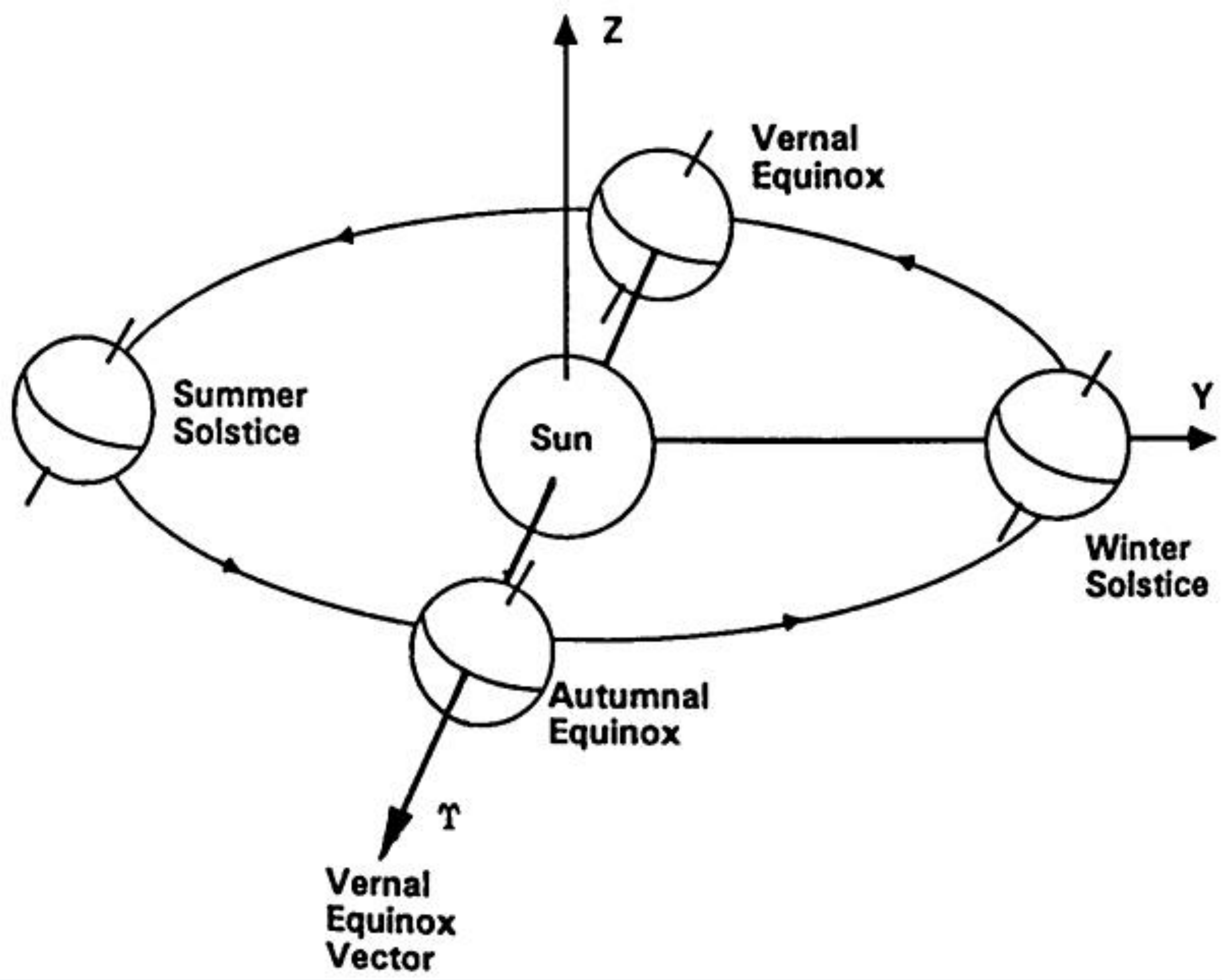
The center of the geocentric inertial coordinate system is the center of the Earth.

The plane of reference is the plane of the Equator, where the direction of X is the direction of the vernal equinox.

The vernal equinox ♈ is a point on the Equator that the Sun crosses when it goes from the southern celestial hemisphere to the northern celestial hemisphere around the 21st of March.

This point is very slowly migrating to the west (precession of equinoxes, about 0.014 degrees per year), so, when using the geocentric-inertial coordinate system, the year shall be specified. Currently the year 2000 is used.

Heliocentric-inertial coordinate system

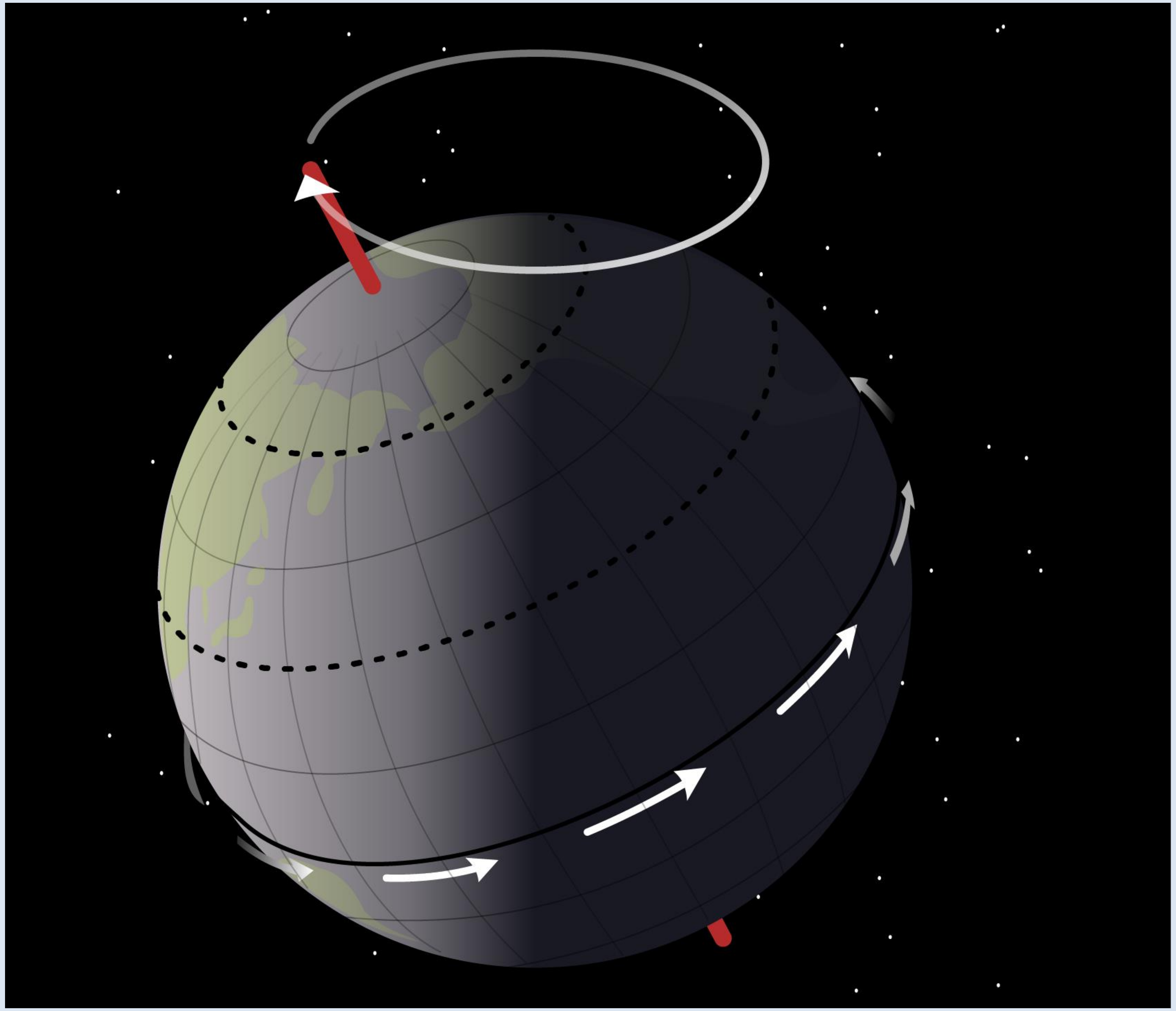


The heliocentric-inertial coordinate system has the same directions of axes as the geocentric-inertial coordinate system.

The center of this coordinate system is in the center of the Sun.

The plane of reference is the plane of the ecliptic, or plane of the Earth's orbit around the Sun

Precession of the equinoxes

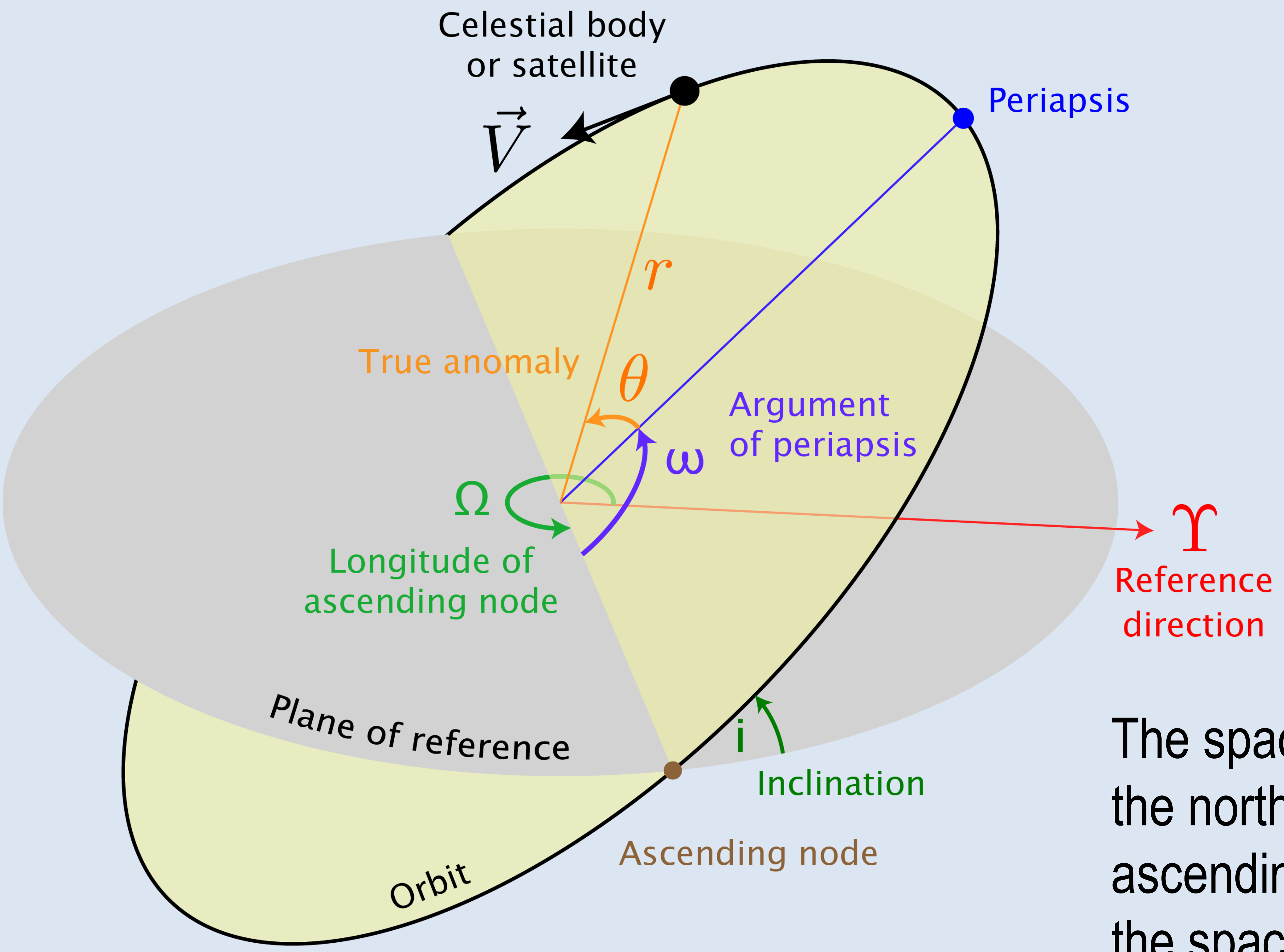


Credits: NASA GSFC, Robert Simmon

- Earth's rotational axis has a tilt of 23.5 degrees vs. a perpendicular to the ecliptic plane
- Axial precession is the displacement of the rotational axis of an astronomical body.
- Earth goes through one such complete precessional cycle in about 26'000 years.

The Earth is not a perfect sphere, but has an equatorial bulge, and the gravitational force, from the Sun and the Moon, on a non-spherical body, causes the precession.

Classical orbital parameters



- ***e***, ***a***, then:
- ***i*** = inclination of the orbital plane.
- ***Ω*** = longitude or Right Ascension of the Ascending Node (RAAN) in the plane of reference).
- ***ω*** = argument of periapsis (in the orbital plane).
- T_p = time of periapsis transit.
- Current time ***t*** allowing a determination of the exact position of the celestial body or satellite.

The spacecraft is passing from the southern celestial hemisphere to the northern on a point on the plane of reference which is called the ascending node. The descending node is on the other side, where the spacecraft goes from the north to the south.

Credits: Adapted from Wikipedia, Lasunnkty

Spacecraft's state vector

$$(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}, t)$$

$$\Updownarrow$$

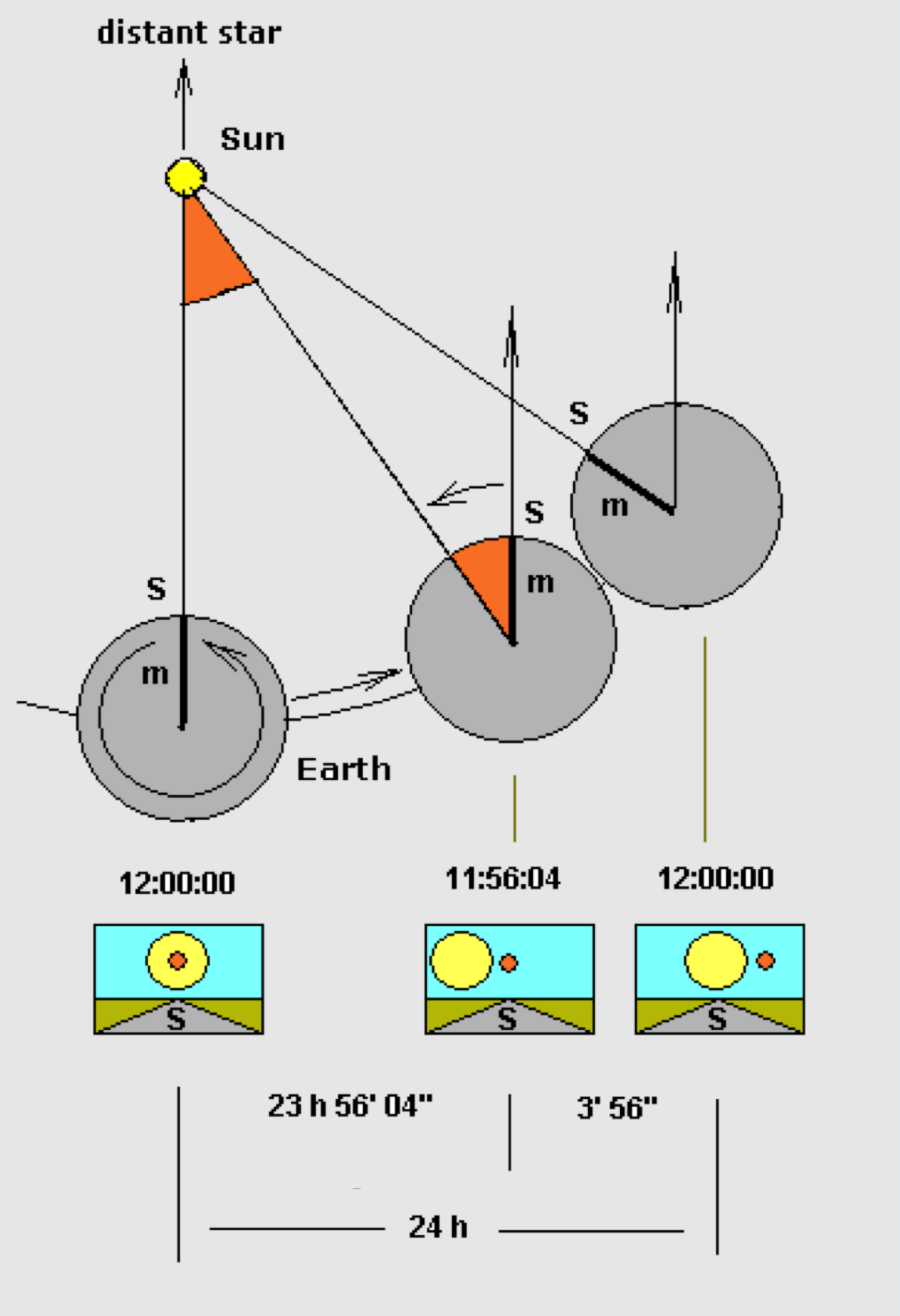
$$(a, e, i, \Omega, \omega, T_p, t)$$

The spacecraft's state vector is functionally equivalent to the six orbital parameters plus the time ***t***.

X, Y, Z, $\dot{X}$, $\dot{Y}$, $\dot{Z}$ are in the geocentric-inertial (or heliocentric-inertial) coordinate system.

On board the Space Shuttle, the current state vector was constantly propagated for a knowledge of the estimated current position and velocity vector. When GPS became available, there was a constant updating of this current position and velocity estimation, with a substantially increased accuracy!

Mean solar day – sidereal day



Credits: Wikipedia, Francisco Javier Blanco González

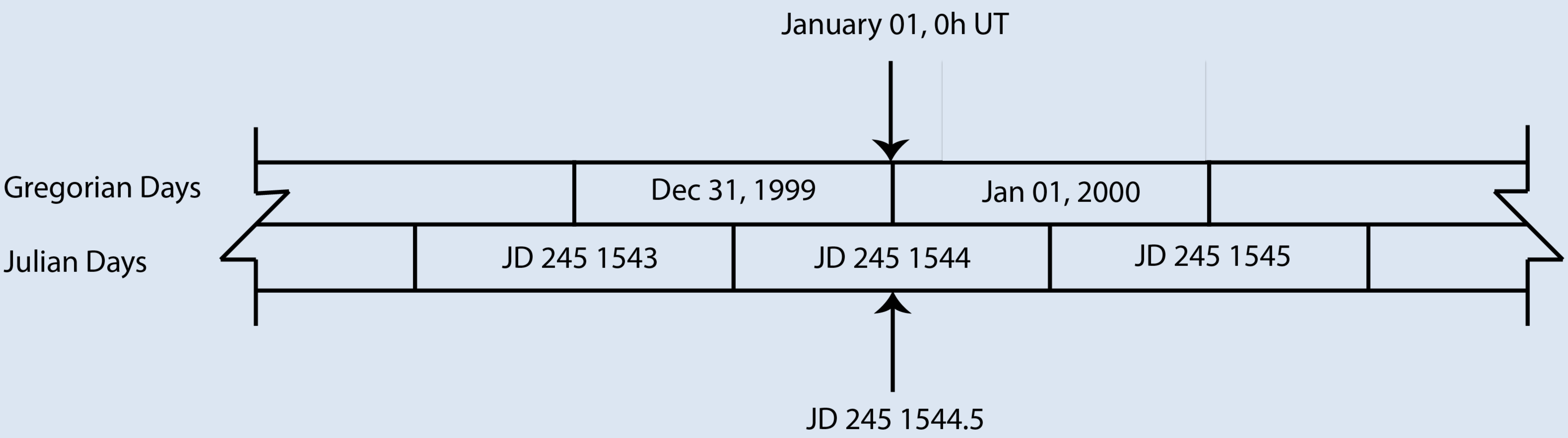
- Picture left: a distant star (the small red circle) and the Sun are at culmination, on the local astronomical meridian.
- Picture center: the distant star is again at culmination, after one sidereal day.
- Picture right: few minutes later the Sun is on the local astronomical meridian again at culmination. A solar day is complete.

The sidereal day is the time it takes for the Earth to make one full rotation with respect to the stars.

The mean solar day is the time it takes for the Earth to make one full rotation with respect to the mean Sun.

The duration of the mean solar day is 24 hours, but the duration of the sidereal day is about four minutes less.

Gregorian days vs. Julian days



Julian day is used in the Julian date (JD) system of time measurement for scientific use by the astronomy community, presenting the interval of time in days and fractions of a day since January 1st, 4713 BC Greenwich noon. Julian date is recomended for astronomical use by the International Astronomical Union.

Conversion of Gregorian days to Julian days

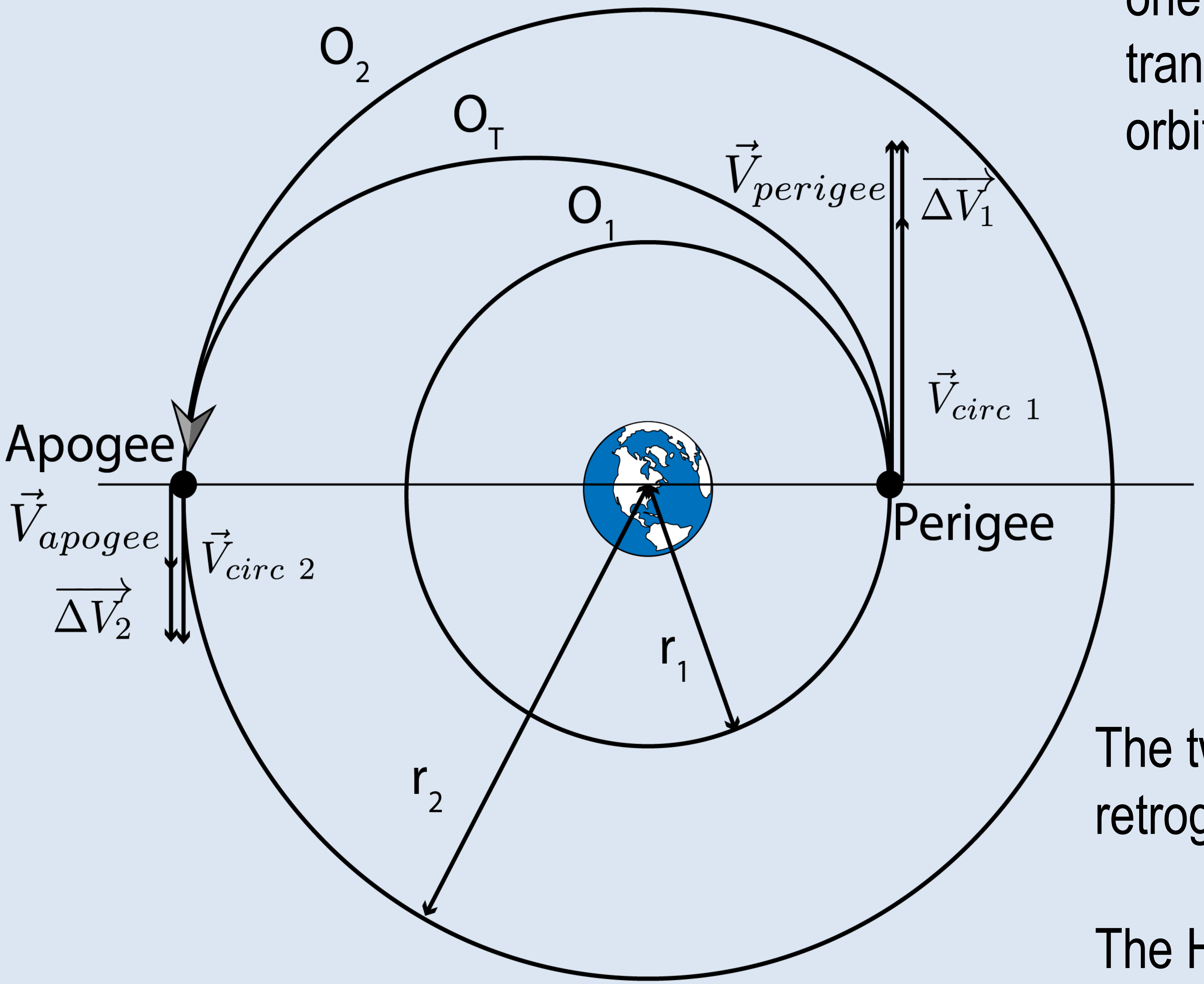
$$J = 367Y - 7\left[\frac{Y + \frac{M+9}{12}}{4}\right] + \frac{275M}{9} + D + 1'721'013.5$$

- ***J*** = Julian day number
- ***Y*** = calendar year
- ***M*** = calendar month number (e.g., July = 7)
- ***D*** = calendar day and fraction

- All divisions must be integer divisions. Only the integer is kept; the fraction is discarded.
- Good from 1901 to 2099.

Hohmann Transfer

EPFL



The Hohmann transfer is a very common method of transfer from one circular orbit to another, around the same central body. The transfer orbit is tangent to both the initial orbit and the destination orbit.

$$\Delta V_1 = \sqrt{\frac{2\mu r_2}{r_1(r_1+r_2)}} - \sqrt{\frac{\mu}{r_1}}$$

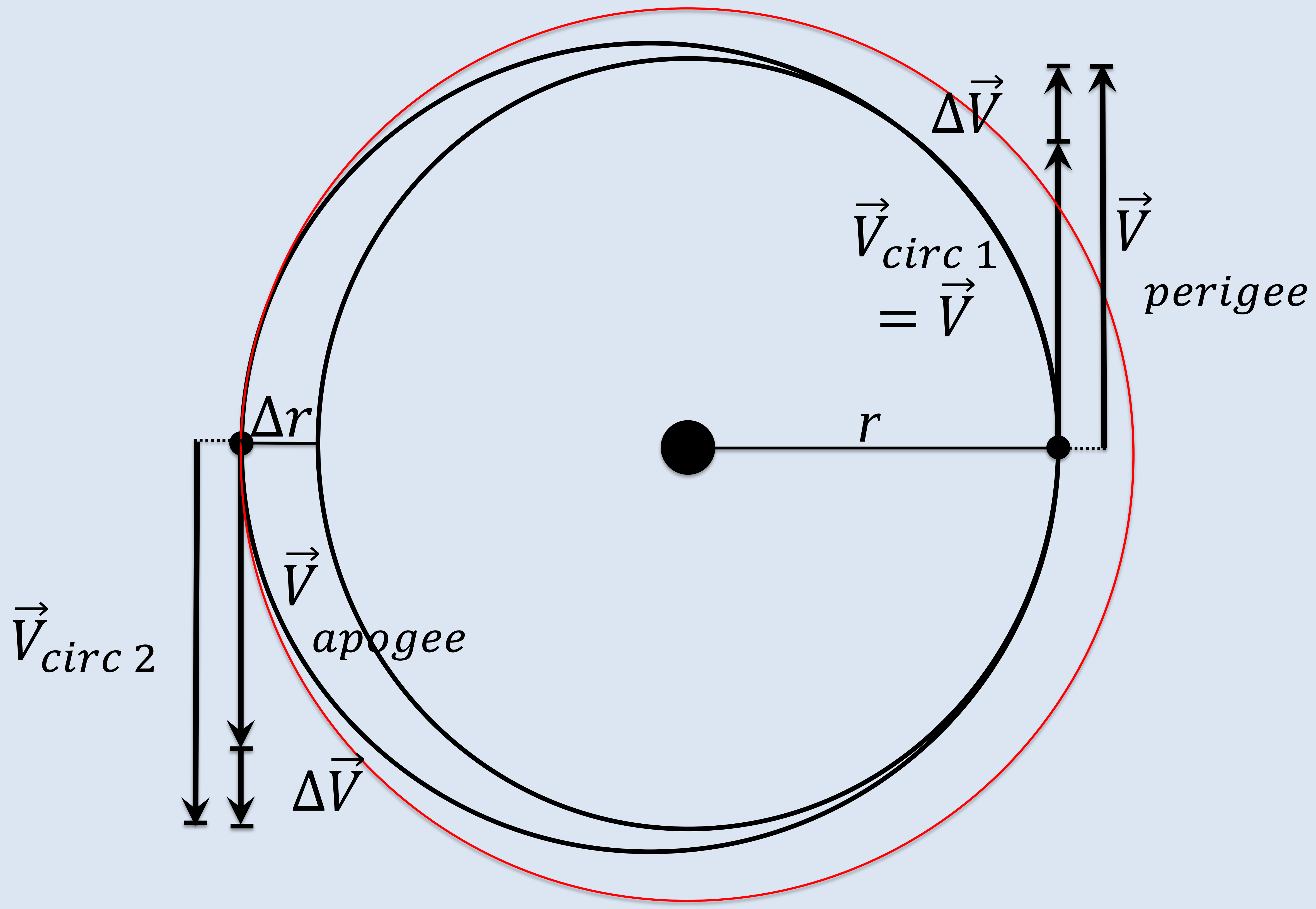
$$\Delta V_2 = -\sqrt{\frac{2\mu r_1}{r_2(r_1+r_2)}} + \sqrt{\frac{\mu}{r_2}}$$

The two ΔVs are posigrade for a transfer to a higher orbit, and retrograde for a transfer to a smaller orbit.

The Hohmann transfer is the most efficient transfer because the changes in velocity are used entirely for changes in kinetic energy.

Hohmann transfer: $\Delta V_{total} \cong 2\Delta V$ for the full transfer (small ΔV)

EPFL



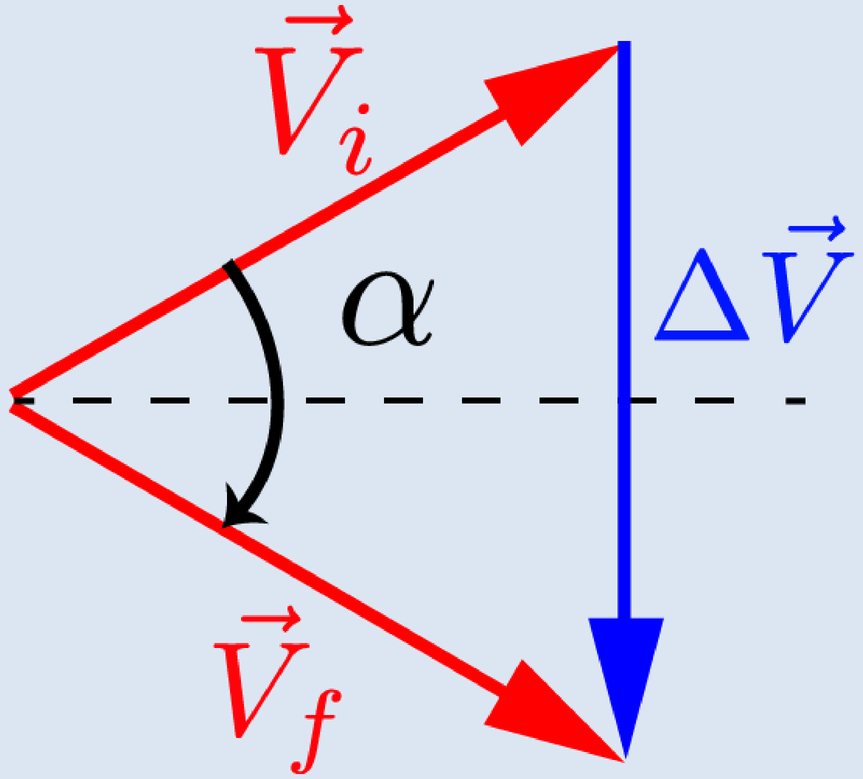
Resulting orbit in red following the 2 ΔVs

$$\Delta V_{total} \cong 2\Delta V \cong \frac{1}{2} V \frac{\Delta r}{r}$$

For LEO $\Delta V_{total} \cong 0.57\Delta r$

Δr in km and ΔV in $\frac{m}{s}$

ΔV needed for a change of orbital plane



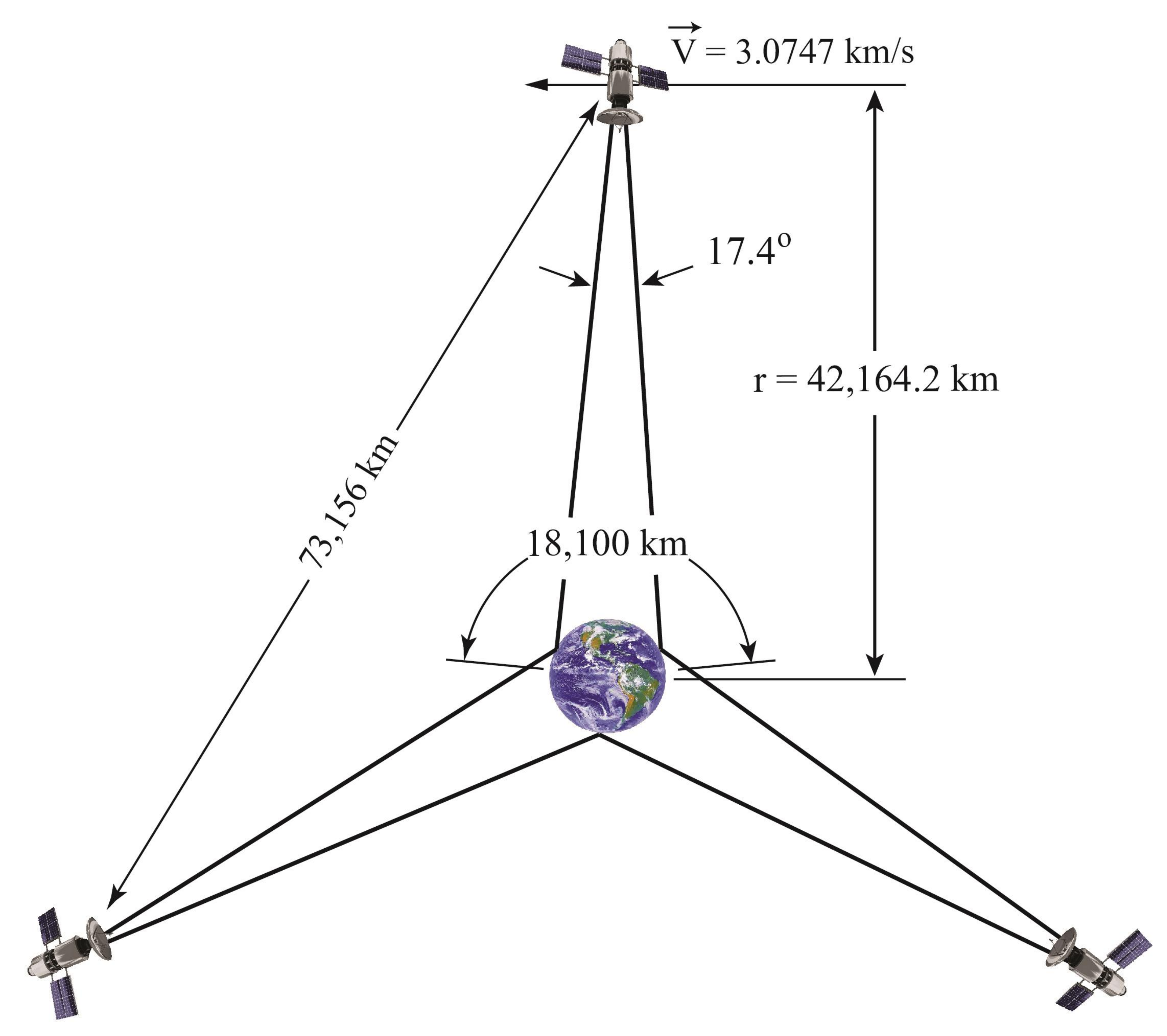
$$|\vec{V}_i| = |\vec{V}_f|$$

Because orbital velocities in LEO are large, of the order of 7.7 km/sec, it is clear that any orbital plane change with an out-of-plane ΔV will be expensive in propellant!

Change of orbital plane best done at equator crossing.

$$\Delta V = 2V_i \sin\frac{\alpha}{2}$$

Geosynchronous and geostationary orbits

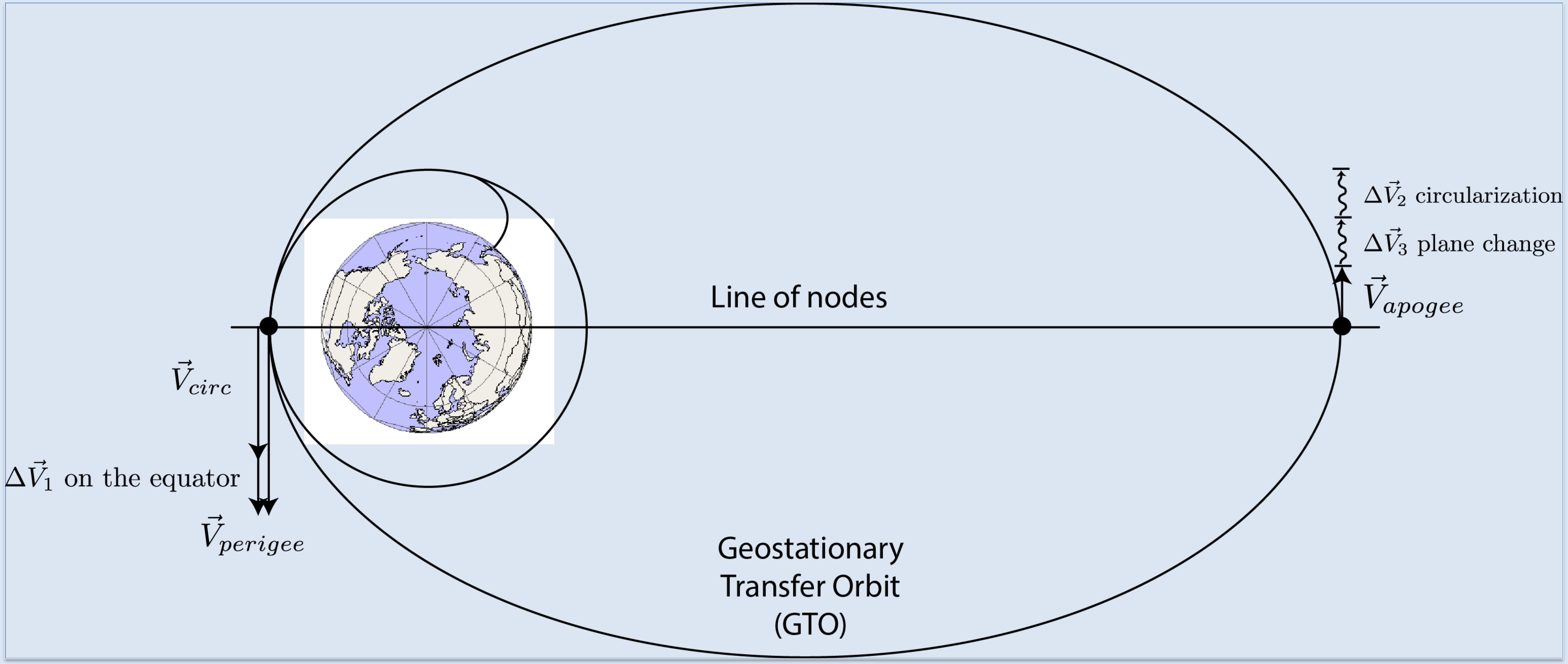


$$T = \frac{2\pi r}{V} = 2\pi\sqrt{\frac{r^3}{\mu}}$$

- Look for **r** satisfying ***T*** = 23h 56min 4.09s. this is the condition for a geosynchronous orbit.
- Geostationary = geosynchronous circular on the equatorial plane (**e** = 0 and ***i*** = 0).

Strategy to reach the geostationary orbit

The line of nodes is the intersection between the plane of the satellite orbit and the equatorial plane.
To reach the geostationary orbit, a Hohmann transfer is necessary followed by a plane change of the orbit from the initial plane to the equatorial condition. The apogee shall be at the geostationary altitude, about 36,000 km above the Earth's surface.
The geostationary transfer orbit is not on the Equator, it has a certain inclination versus the Equator which is usually equal to the latitude of the launch site. Typically 7 degrees for a launch from French Guiana, in Kourou, and 28.5 degrees if the launch takes place from Kennedy Space Center, Florida.

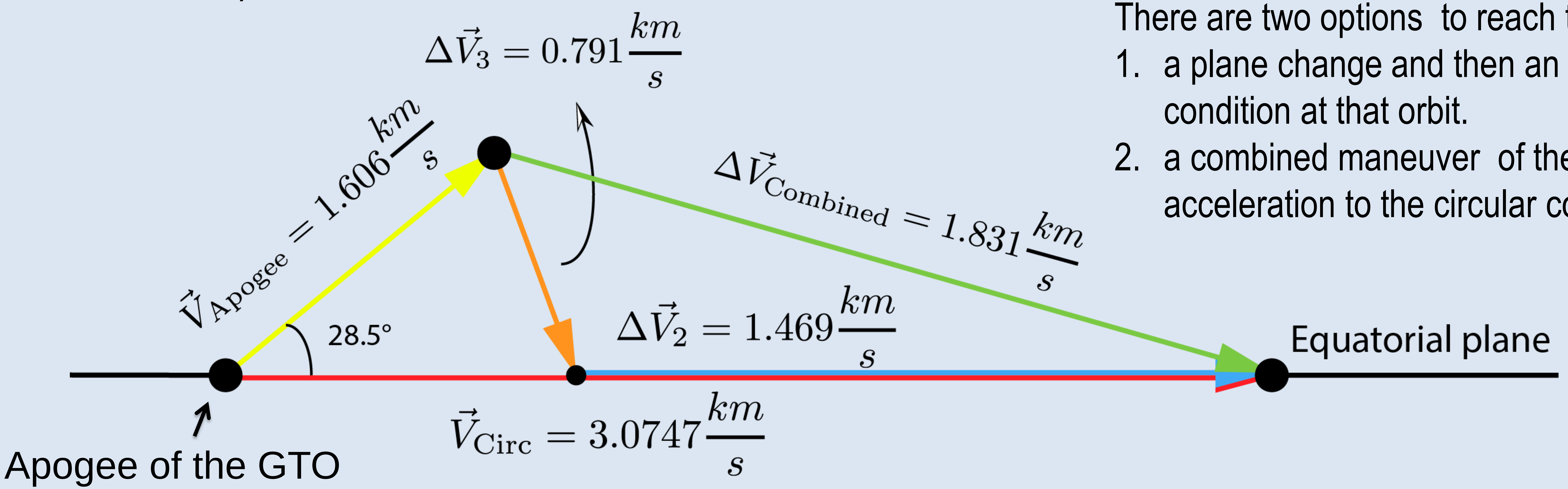


Combined maneuver

Exemple of a combined maneuver at the apogee of the transfer orbit for insertion into a geostationary equatorial orbit for a launch from Kennedy Space Center, Florida (Lat. 28.5°).

There are two options to reach the geostationary orbit:

1. a plane change and then an acceleration to the circular condition at that orbit.
2. a combined maneuver of the plane change and acceleration to the circular conditions



$$\text{Separate maneuvers: } |\Delta \vec{V}_{\text{Total}}| = |\Delta \vec{V}_2| + |\Delta \vec{V}_3| = 2.260 \frac{km}{s}$$

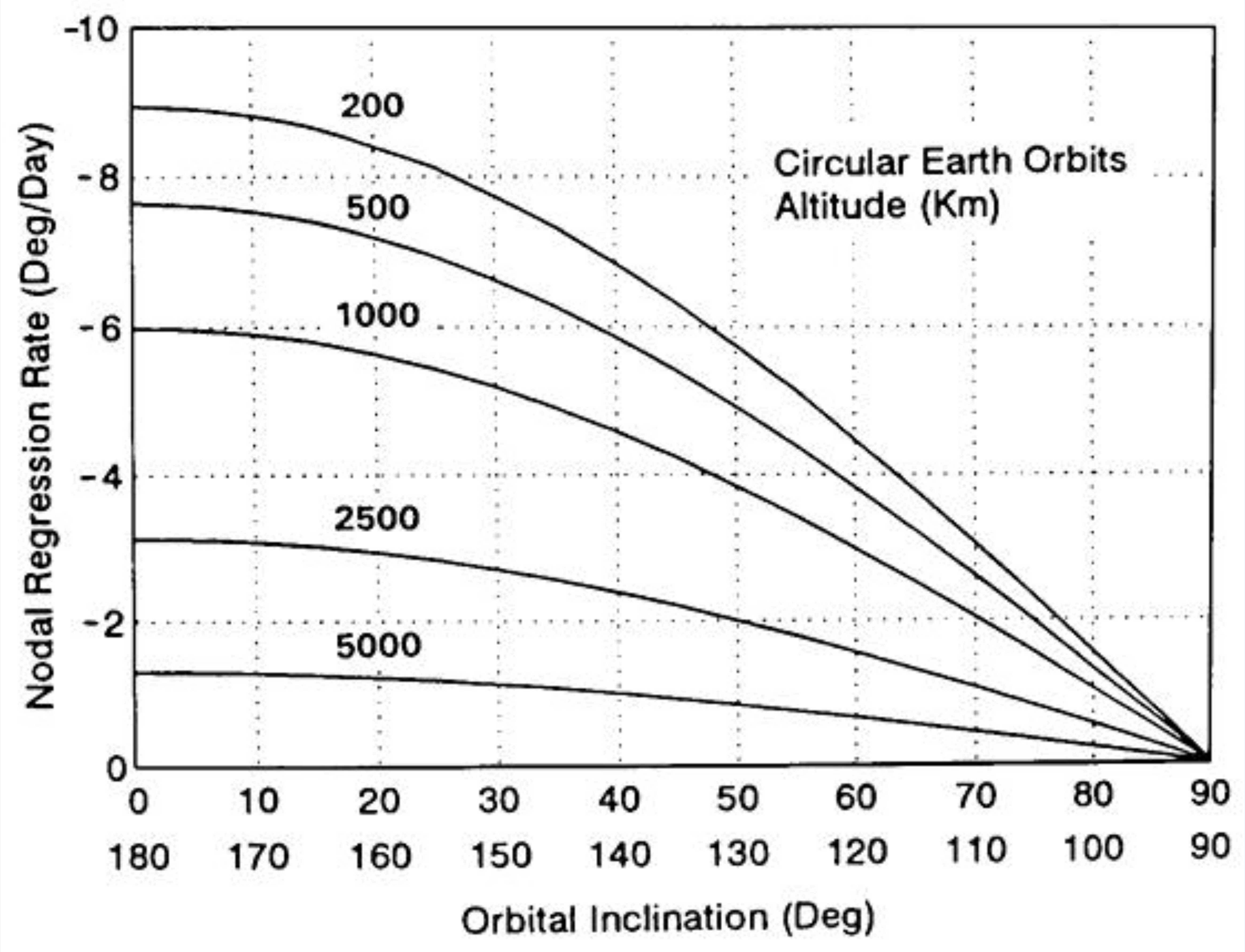
$$\text{Combined maneuvers: } |\Delta \vec{V}_{\text{Total}}| = |\Delta \vec{V}_{\text{Combined}}| = 1.831 \frac{km}{s}$$

Nodal regression rate for LEO

$$\frac{d\Omega}{dt} = -2.06474 \times 10^{14} \frac{\cos i}{a^{3.5}(1-e^2)^2}$$

in degrees per mean solar day with the semi-major axis ***a*** in kilometers.

Nodal regression rate vs. orbital inclination and altitude



The line of nodes drifts:

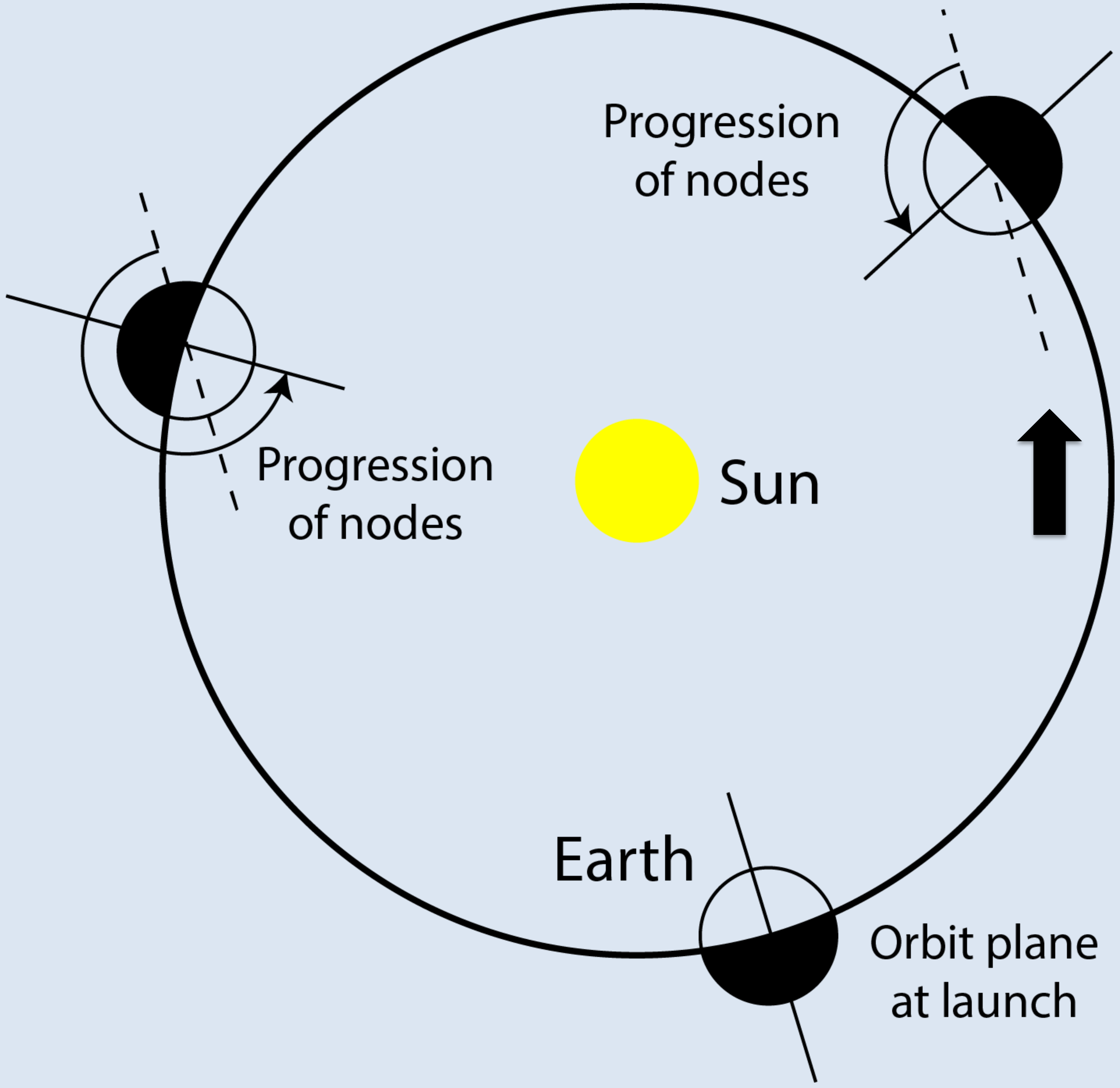
- To the west for i = 0 – 90°
  (prograde or direct orbit)
  Nodal regression

- To the east for i = 90 – 180°
  (retrograde orbit)
  Nodal progression

Sun-synchronous orbit

EPFL

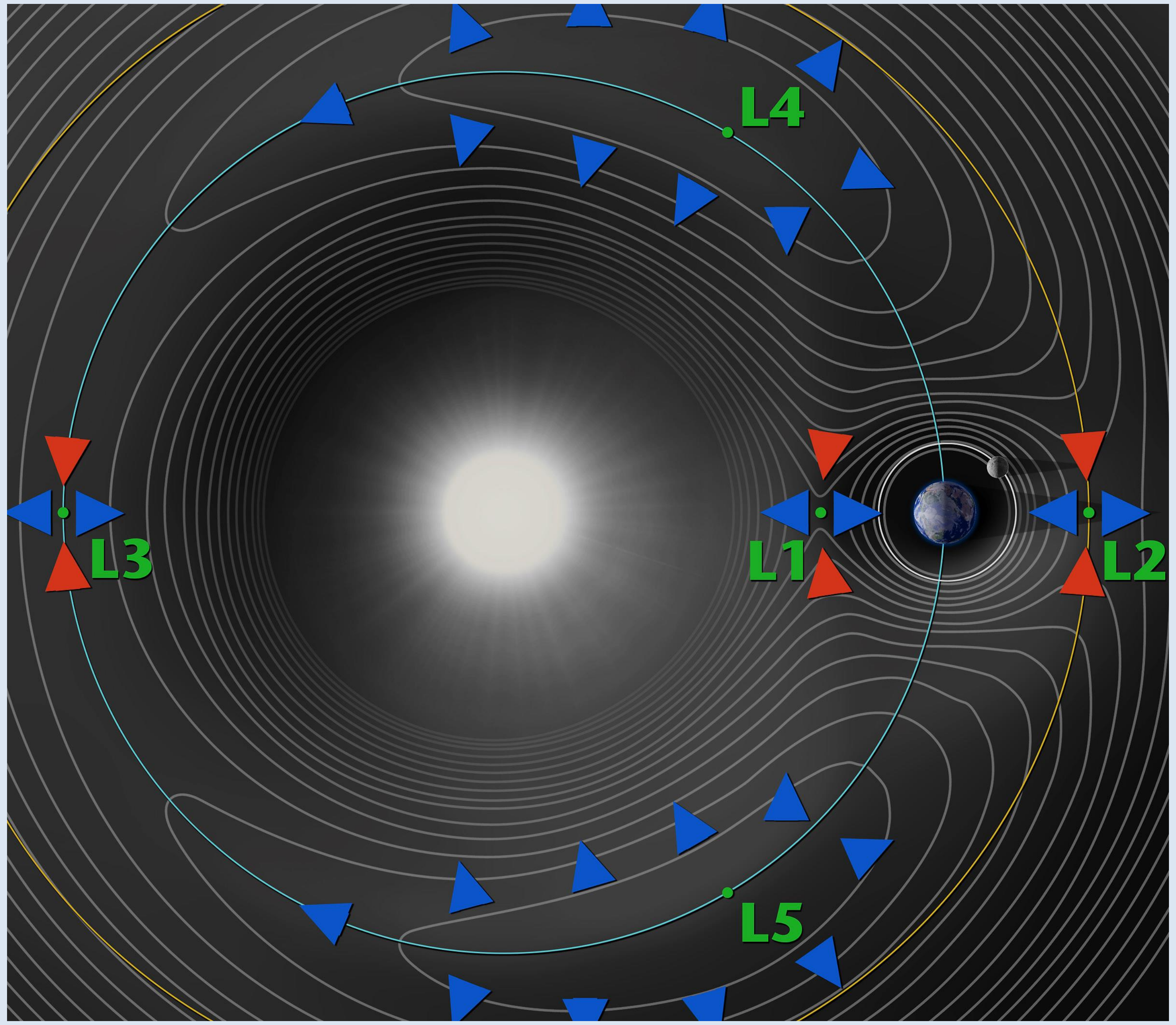
A Sun-synchronous orbit is an Earth centered orbit which maintains a constant orientation with respect to the Sun during the whole year. It is easy to understand that it means a drift of the line of nodes to the east, of about one degree per day.



Requirement on $\frac{d\Omega}{dt}$ for a Sun-synchronous orbit:

$$\frac{360 \text{ deg}}{365.242 \text{ days}} = 0.9856 \frac{\text{deg}}{\text{day}}$$

3.3.3 Lagrange points

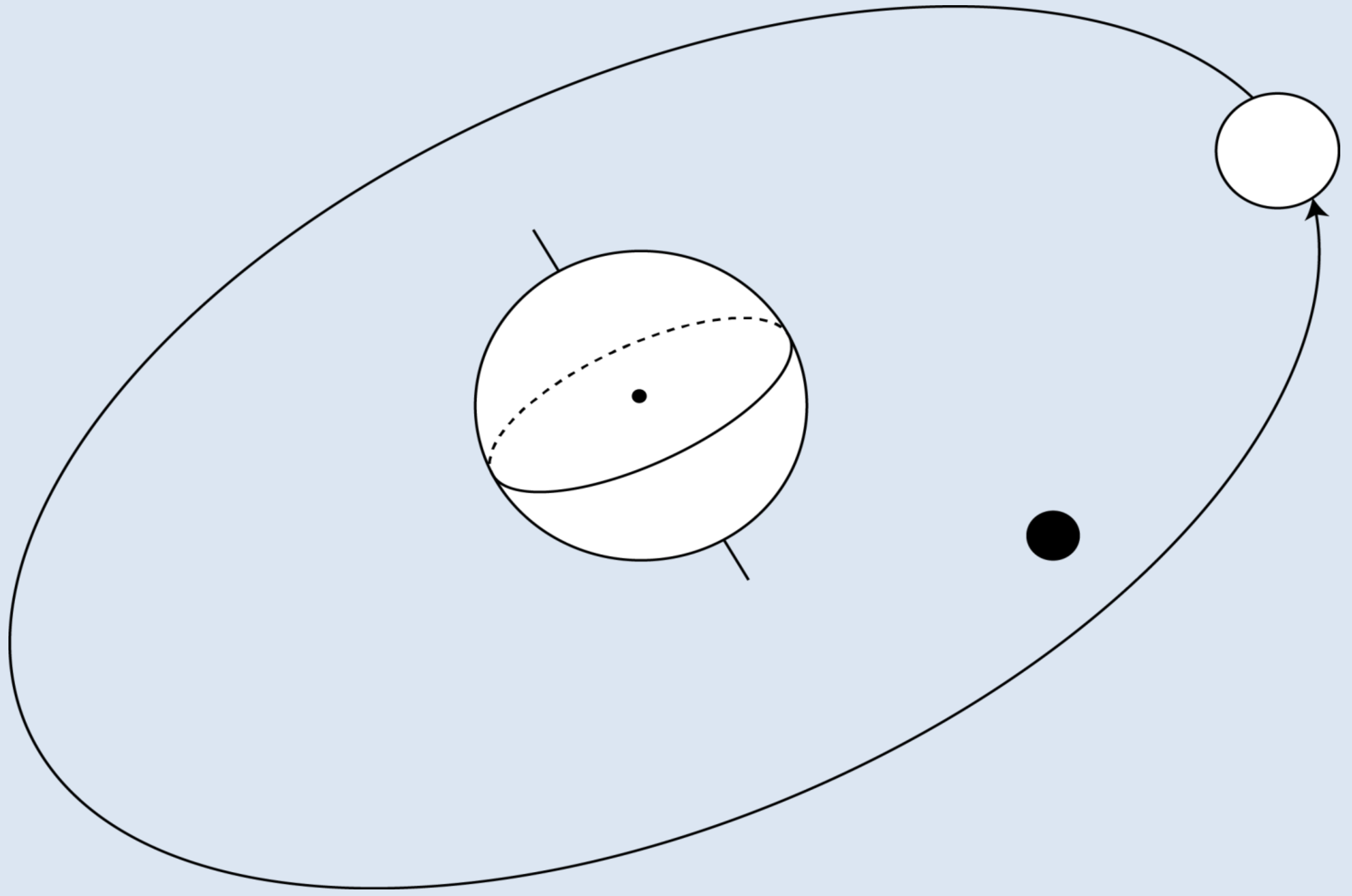


Credits: NASA, WMAP Science Team

Space Mission Design and Operations

Prof. Claude Nicollier

Restricted three-body problem



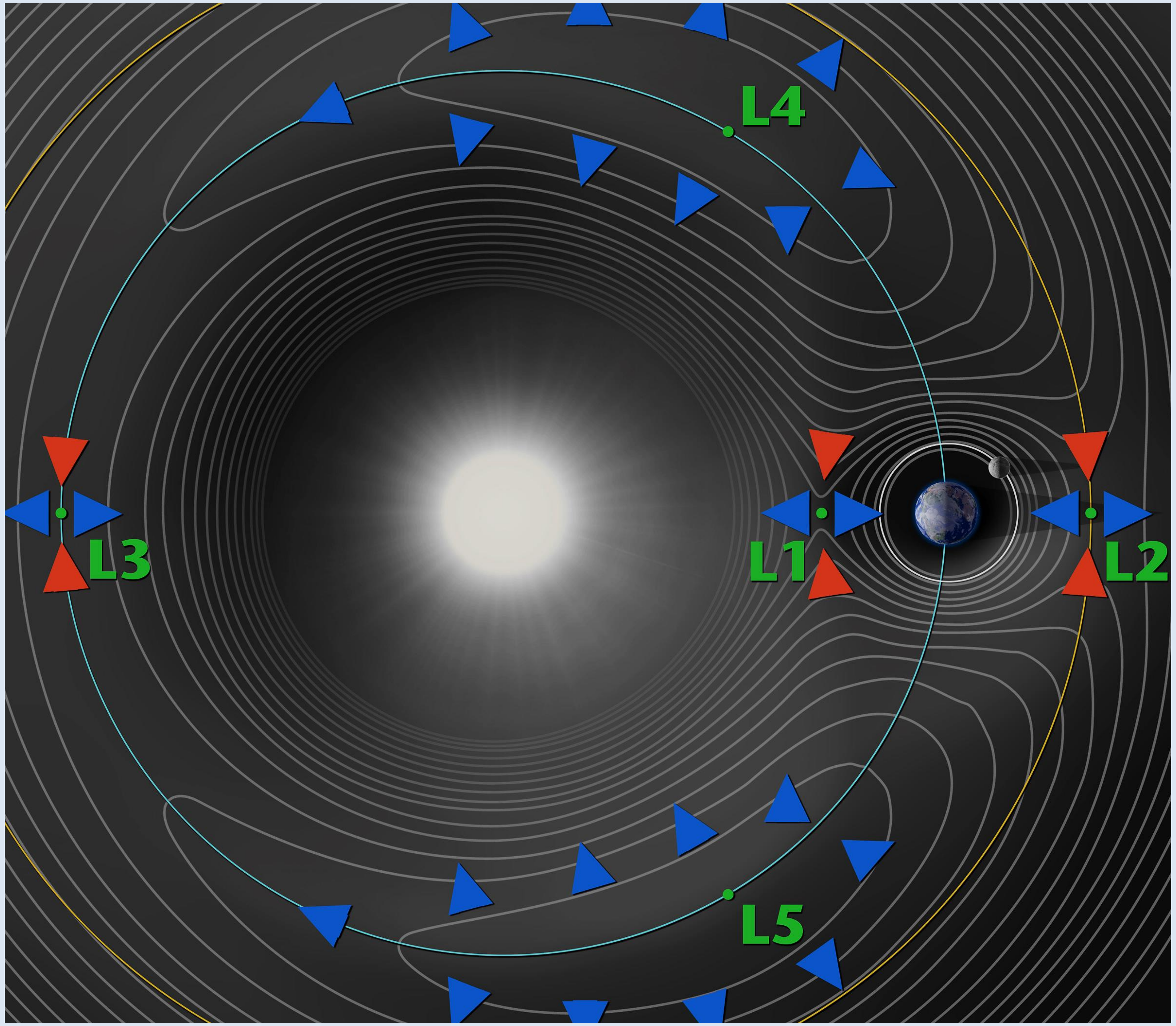
The general three-body problem is complex and not covered in this course. We will consider only the restricted three-body problem: two relatively large bodies and a smaller body : the spacecraft.

Assumptions:

- The two main bodies are on circular orbits around the center of mass of the system.
- The mass of the third body (satellite) is very small compared to the mass of the two main bodies.
- The third body is in an orbit contained in the plane of the orbits of the two main bodies.

Lagrange points

EPFL

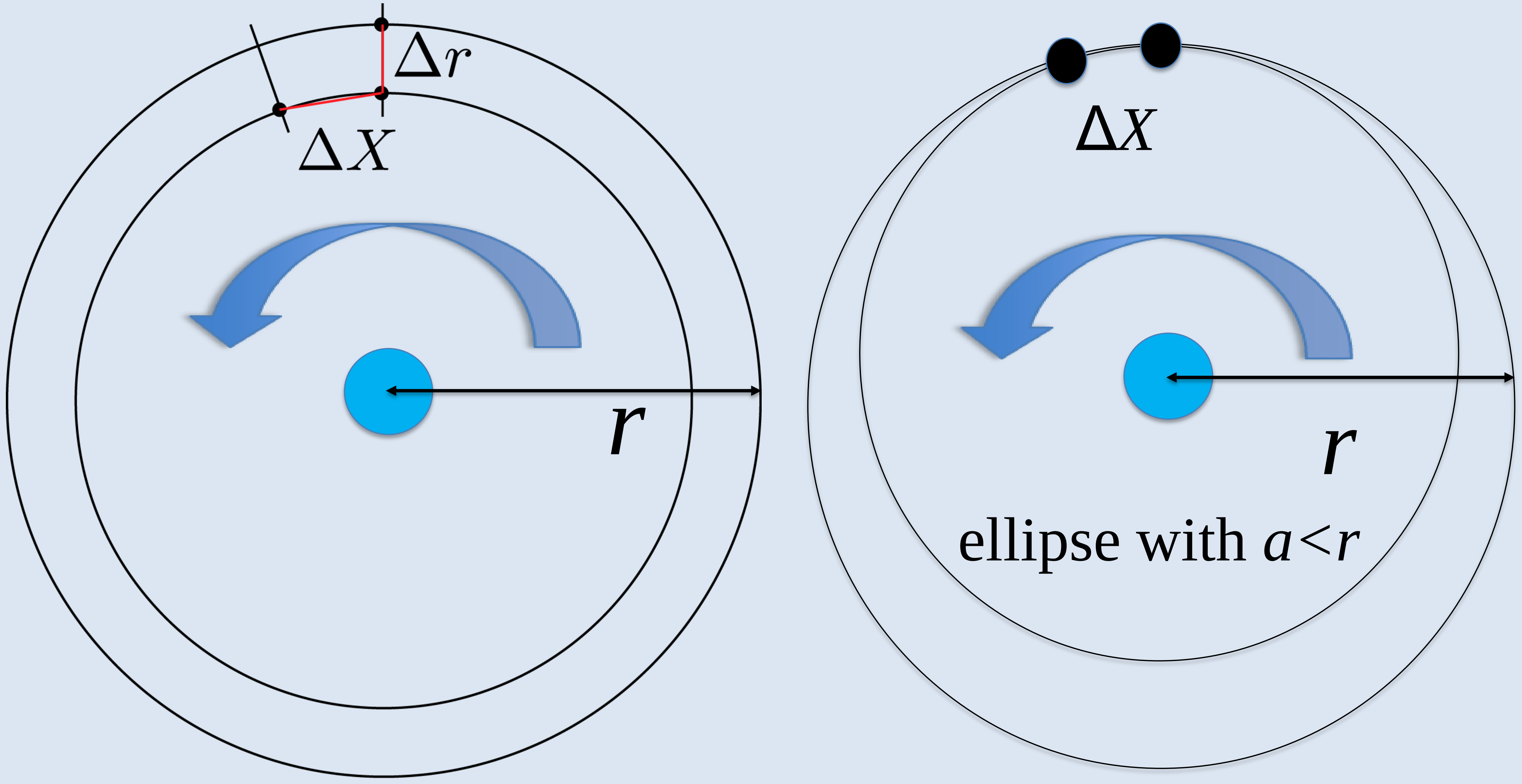


Credits: NASA, WMAP Science Team

The gravitational fields of two massive bodies with the addition of the inertial force due to the satellite's circular motion are in balance at the Lagrange points, allowing the third body or satellite to be stationary with respect to the first two bodies.

- In the Sun-Earth system, the distance to the Earth of the L1 Lagrange point is about 1.5 million km. The Lagrange point L2 is 1.5 million km away from the Earth in the anti-Sun direction.
- L4 and L5 are locally stable points.
- For L1, L2, and L3 there is stability across the L2 to L3 line
- Red arrows indicate stability, blue arrows instability

Catch up/overtake rate for nearby orbits (summary)



- For two circular orbits:

$$\boxed{\Delta X \cong 3\pi \Delta r}$$

$$\text{with } \Delta r \ll r$$

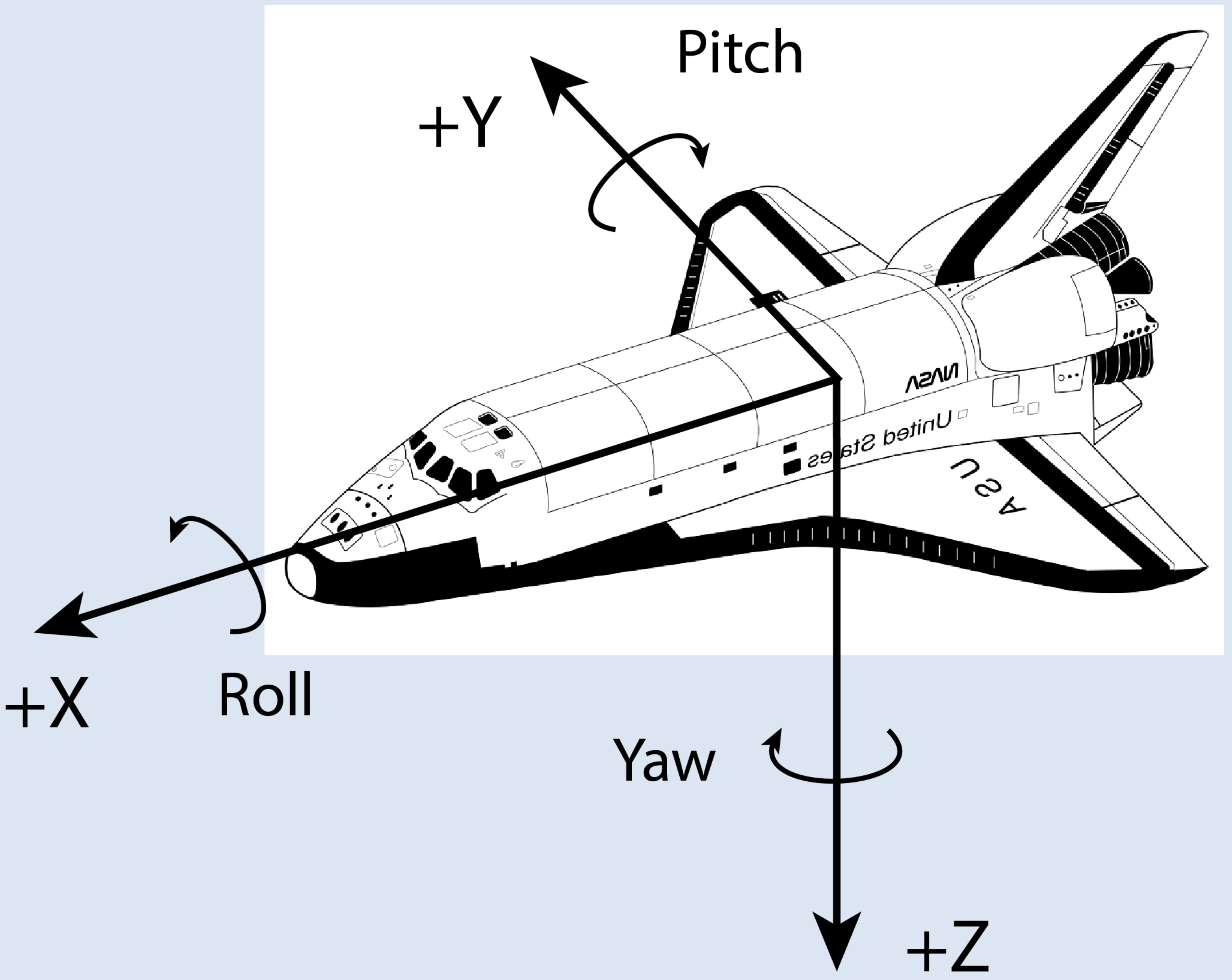
- For circular/close elliptical orbits:

$$\boxed{\Delta X \cong 3\pi (r-a)}$$

$$\text{with } |r-a| \ll r$$

Shuttle coordinate system

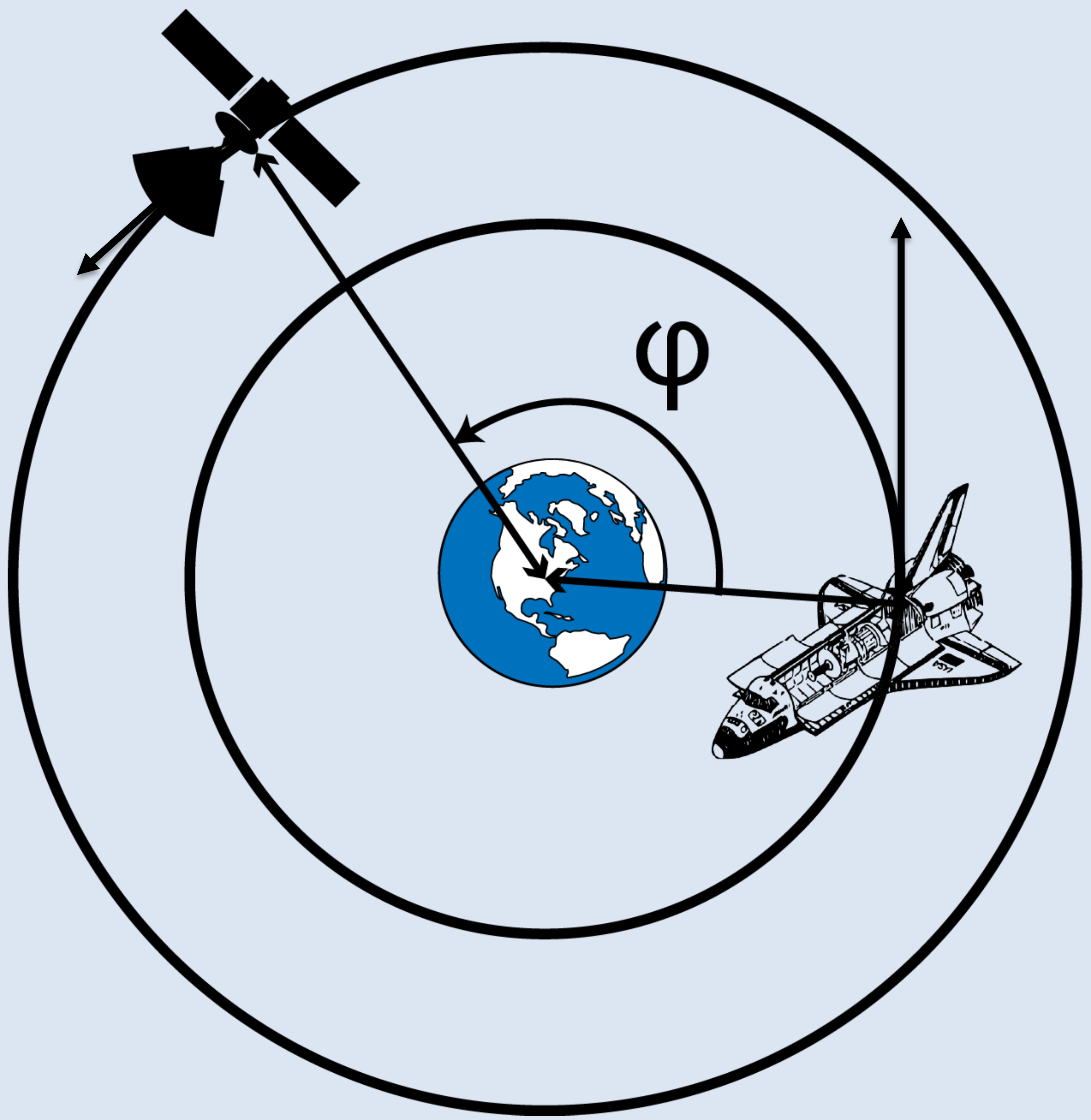
Body attached coordinate system for the Space Shuttle



Credits: SSM2007 Commander's Reference Manual

Phasing

The orbit of the chaser versus the target is represented in a two-dimensional plane which is the plane of the orbit of the target, which is also the plane of the orbit of the chaser at the end of the rendezvous (remember the nodal regression!). Very generally, the chaser approaches the target from behind and below, the two orbits being essentially coplanar at the end of the rendezvous.



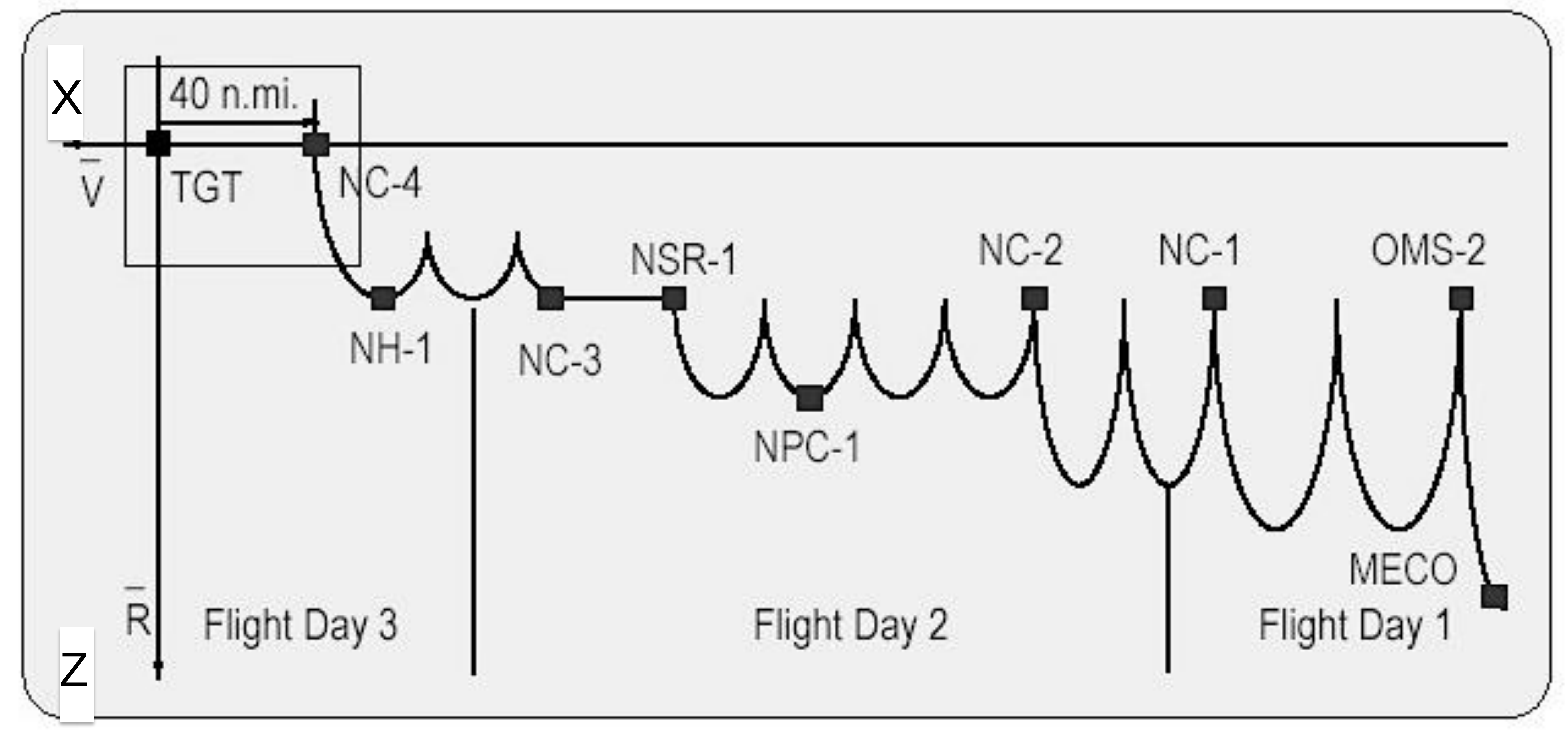
- The **Phase angle** is the angle between the chaser and target, measured from the center of the Earth.
- **The Phasing rate or catch-up rate** is the rate at which phase angle changes.
- The Phasing rate is a function of the differential altitude (third law of Kepler)

Shuttle RNDZ profile in TGT centered coordinate system

MECO: Main Engine Cutoff.

It happened 8.5 minutes after lift-off of the Shuttle, when it had reached orbital velocity and 18 seconds before the separation of the External Tank.

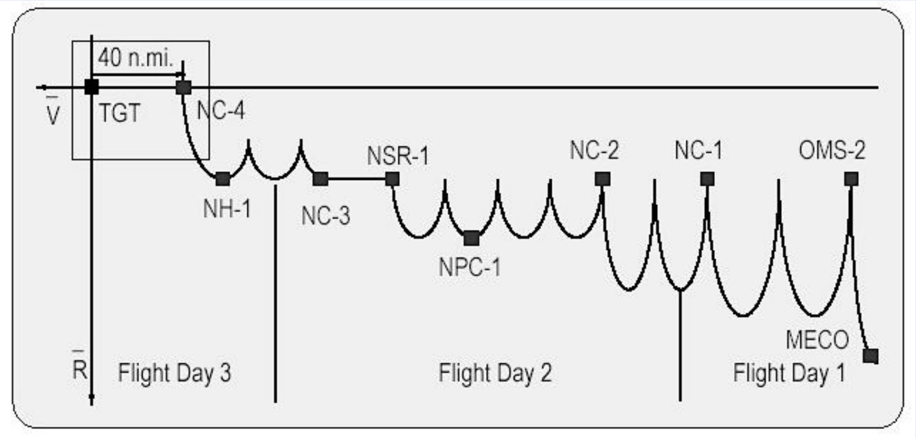
At MECO, the orbit of the Space Shuttle was elliptical, with apogee at the location of “OMS-2”.

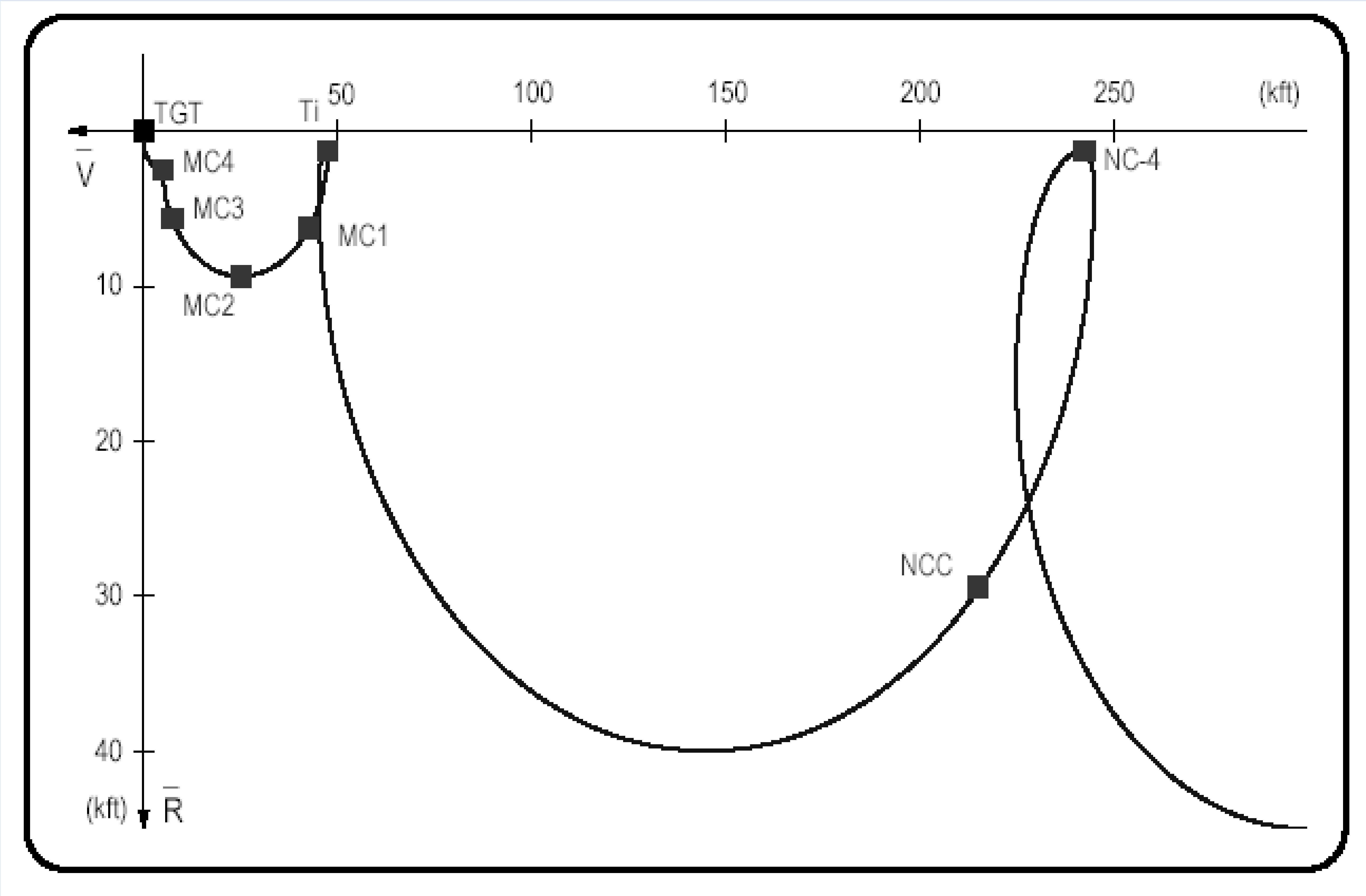


Typical relative orbits of the chaser are represented in the coordinate frame centered on the target with Z to the CoE (Center of Earth). Each loop represents one full orbit of the chaser with duration around 1:30 hour. Note that the X and Z are often designated Vbar and Rbar respectively.

OMS-2 maneuver was a ΔV posigrade. Several maneuvers called NC-1, NC-2, NPC-1, etc. were done in order to gradually increase the energy of the orbit of the chaser.

Detail of RNDZ profile





Loop at NC4: the motion of the chaser versus the target is toward the right with a lower velocity of the chaser when it reaches the altitude of the circular orbit of the target.

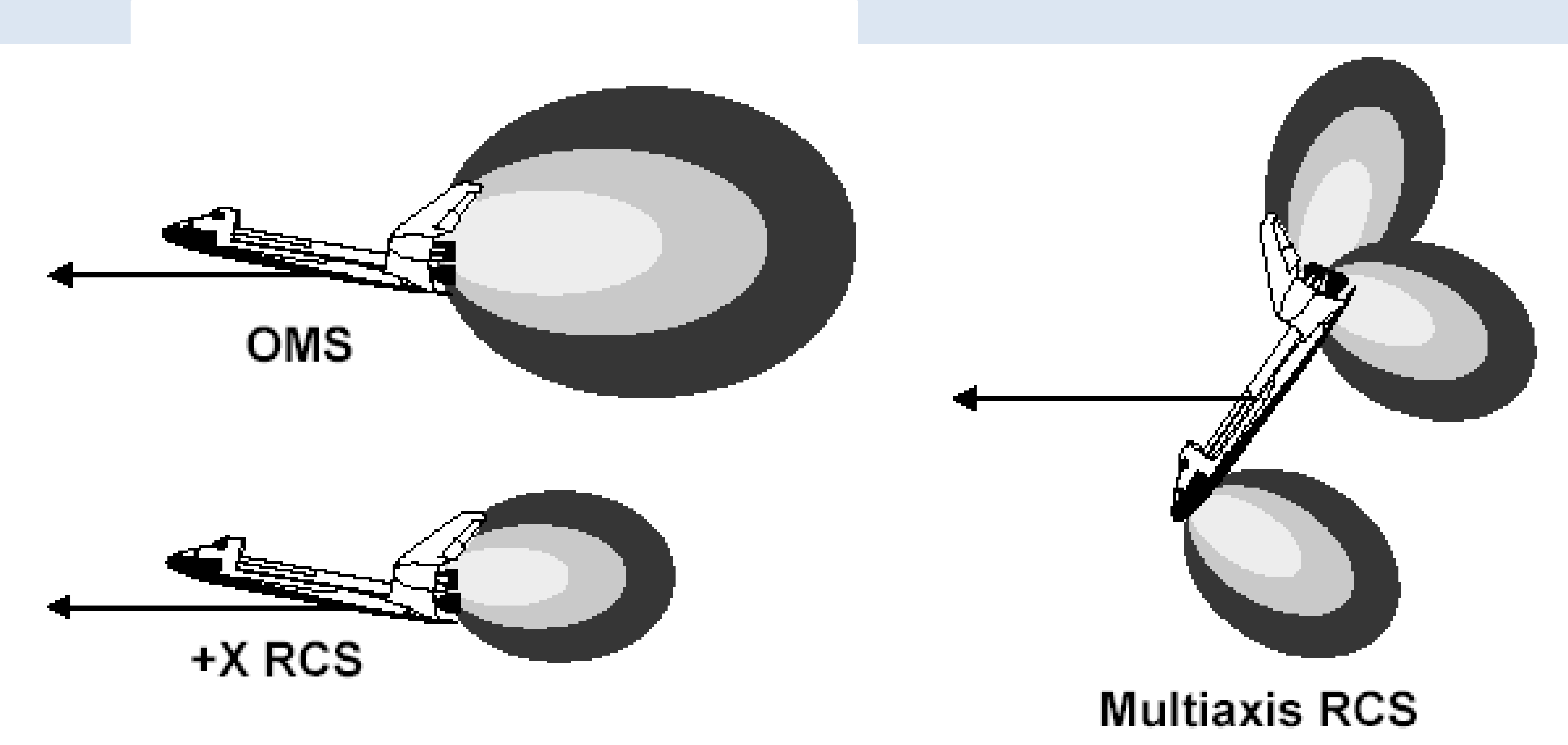
NC4 was a maneuver performed to increase the energy of the orbit of the chaser. At Ti (Terminal insertion), energy was again increased such that the *ΔX* performed in the last orbit would be exactly equal to the distance between Ti and the target.

Rendezvous burn execution

The Shuttle was equipped with different kinds of thrusters to perform maneuvers.

OMS (Orbital Maneuvering System) - Thrusters were the most powerful ones, located in the aft fuselage, for large translations only, no rotations.

RCS (Reaction Control System) - There were 38 total small thrusters, in the nose and in the aft portion of the fuselage of the Shuttle, for translations of smaller amplitude than in case of OMS, and attitude control.

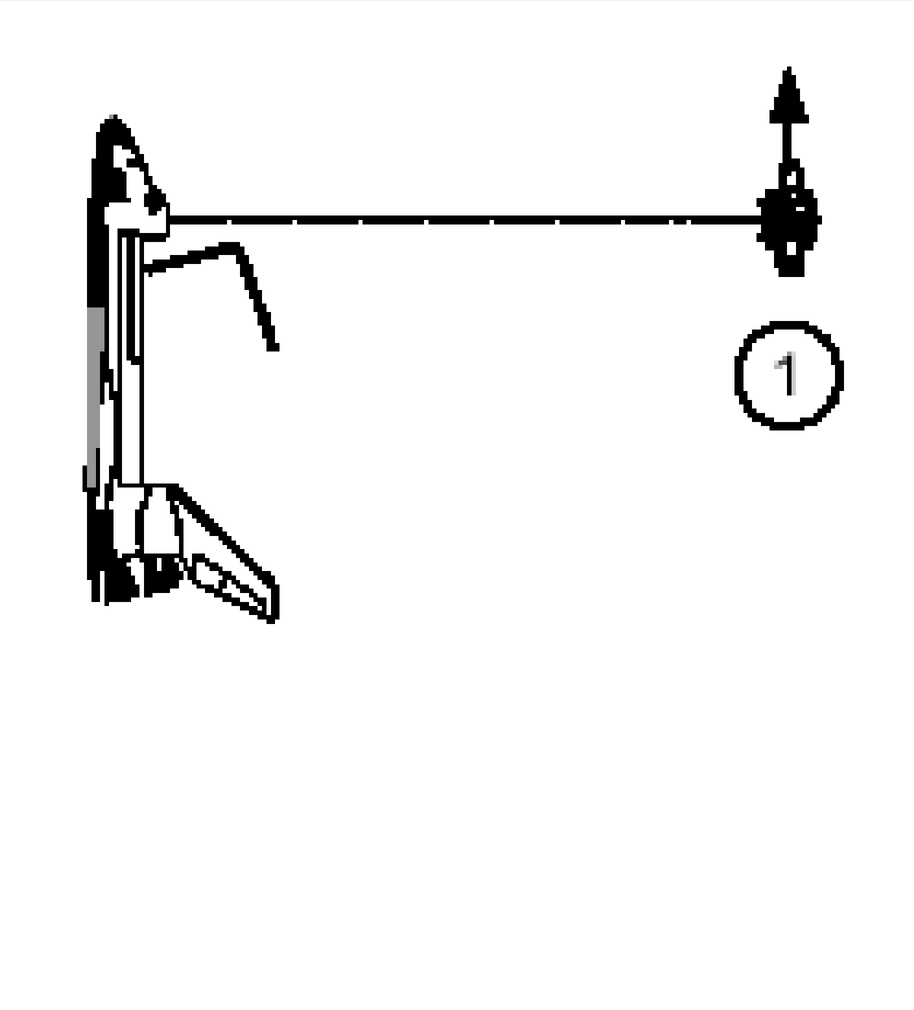


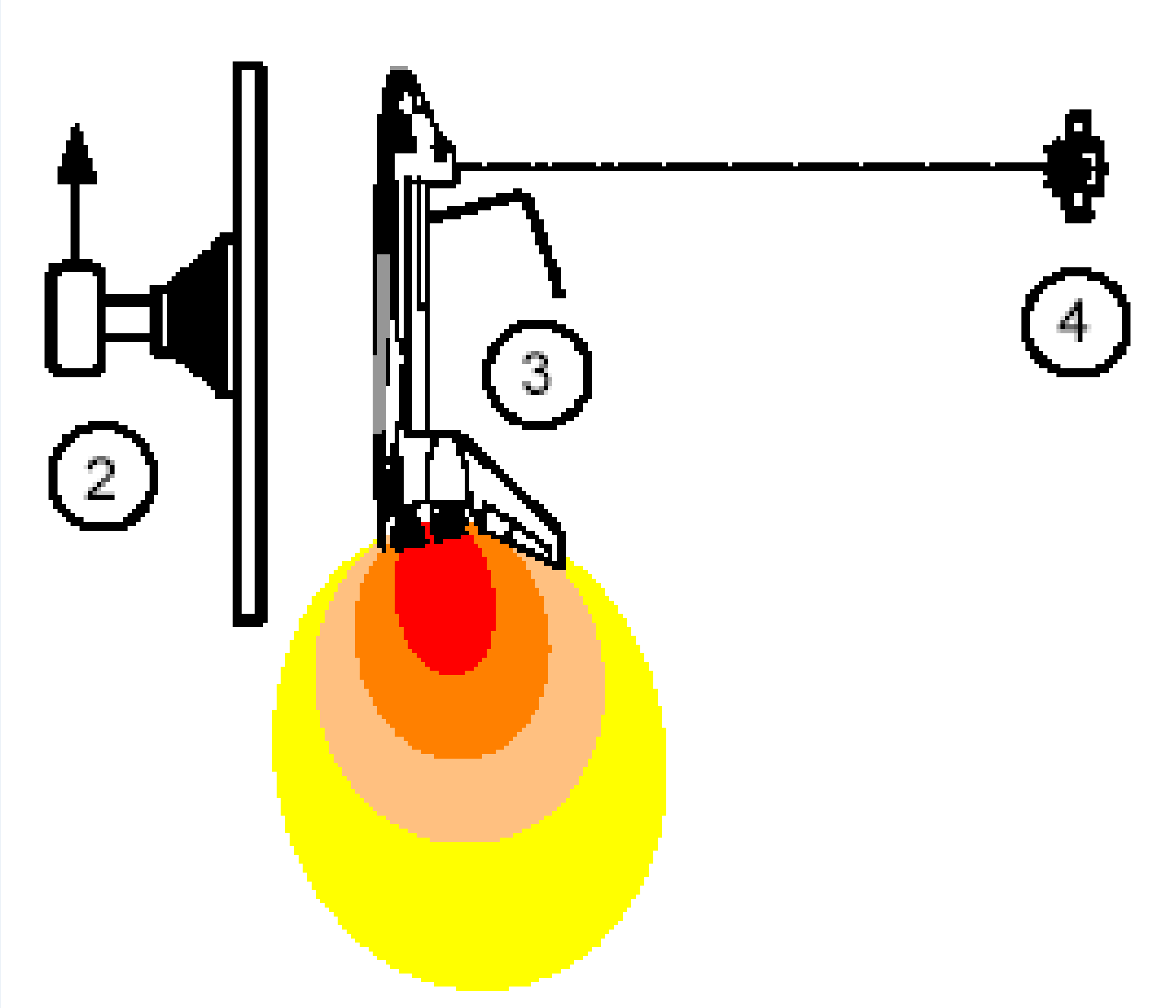
THC input and consequence

Translation hand controller set in + X, - X mode:

If the free flying spacecraft on the right of the orbiter was moving forwards (towards the nose of the orbiter) seen in the Crew Optical Alignment Sight (COAS) aligned along the –Z body axis of the orbiter from the crew cabin, and this motion had to be stopped, then following action from the crew was required:

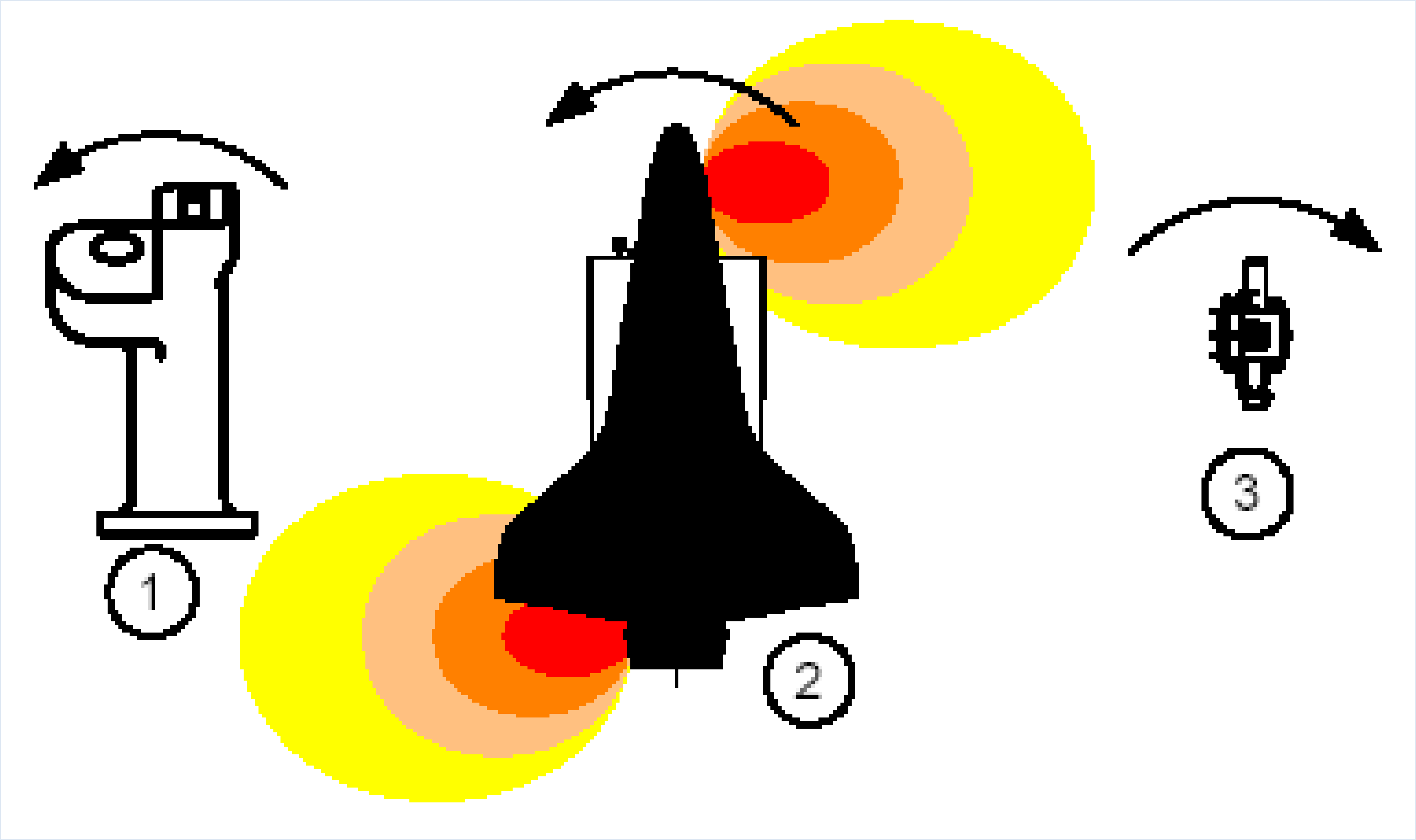
THC moved upwards, firing thrusters in the rear of the orbiter and making the orbiter move towards its nose.



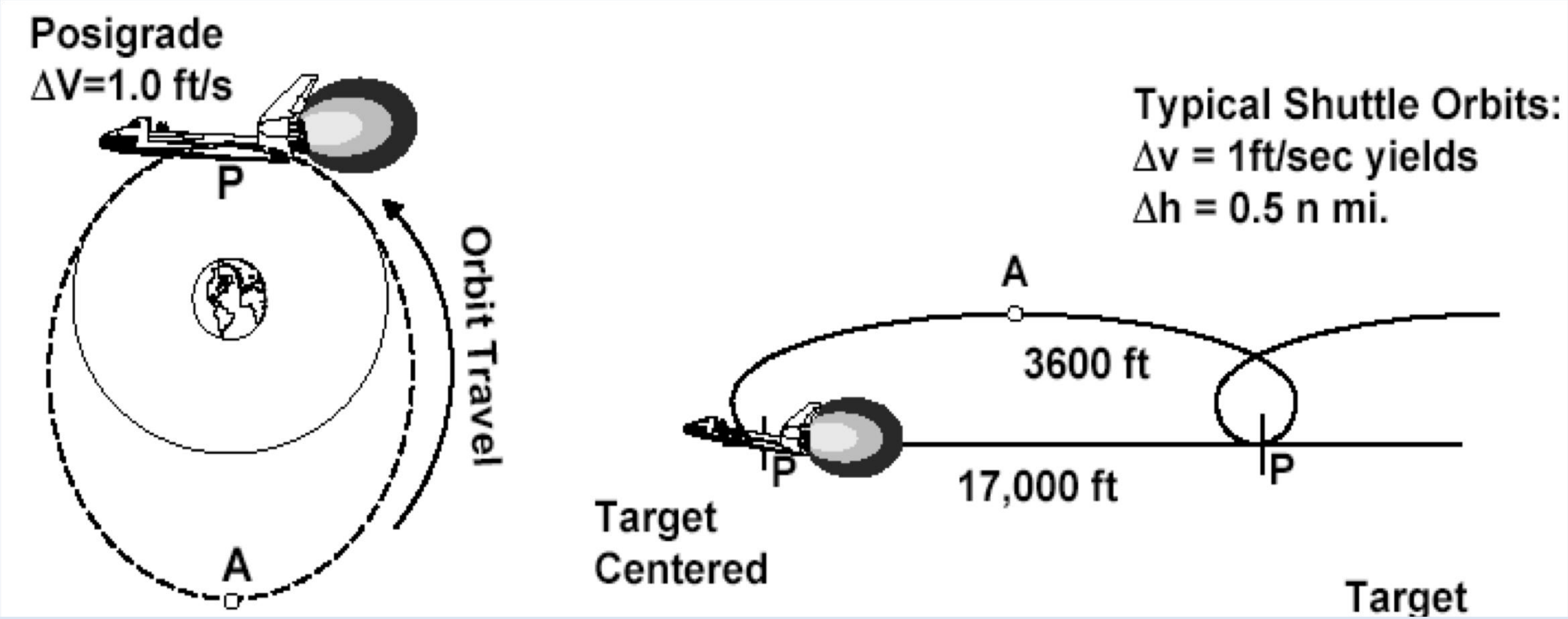


RHC input and consequence

RHC roll to the left in the drawing, or a negative roll. This fired thrusters in one direction in the forward RCS thruster pod, and in the other direction in the aft RCS thruster pod, causing the orbiter rotation around the +Z body axis, and, consequently, an opposite rotation of the other spacecraft located along the orbiter's –Z axis.



Effects of burns on relative motion

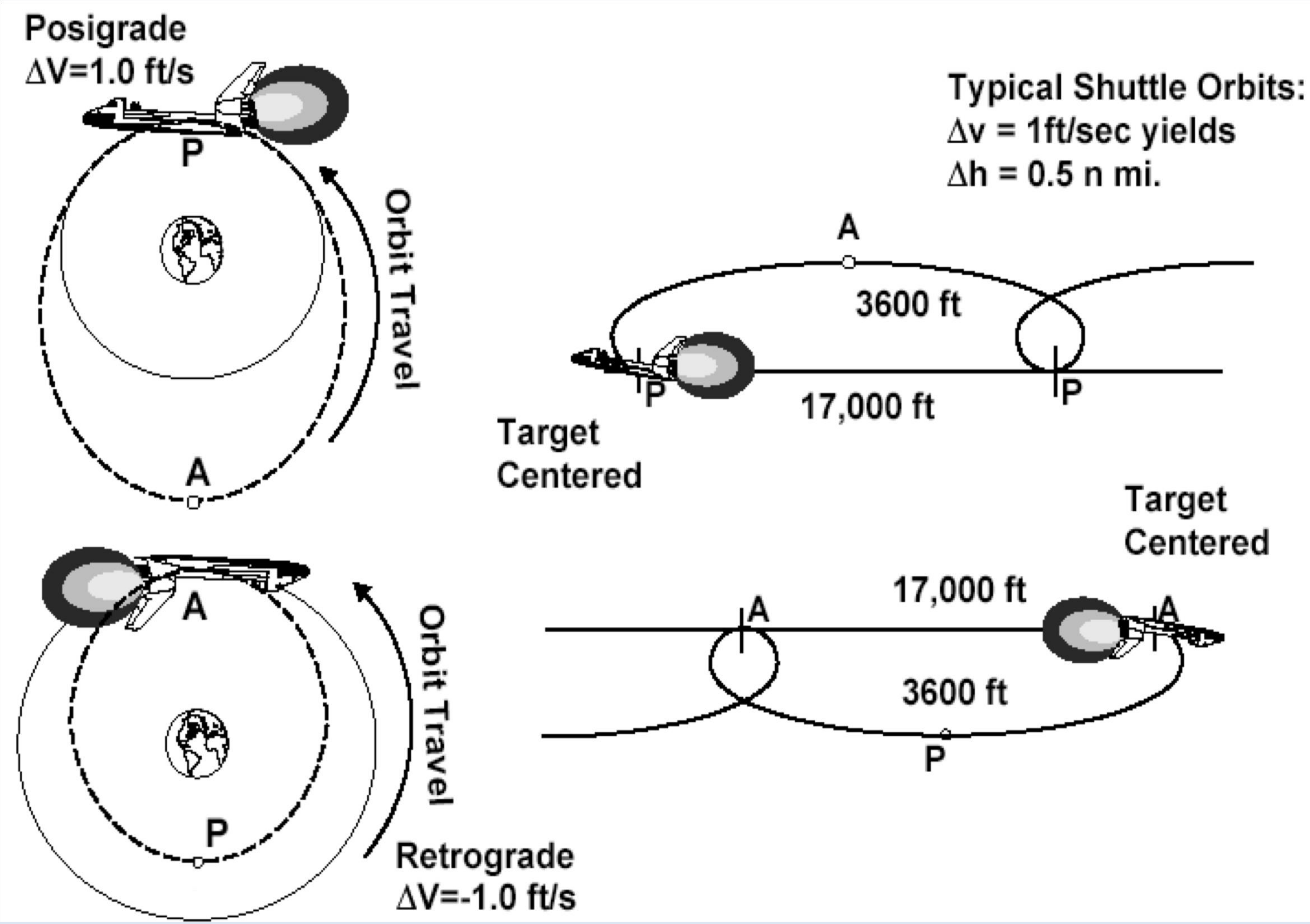


Initial condition: Shuttle and ISS are colocated. On the left: the Shuttle is moving away from the ISS, with a locally increased orbital velocity and transition to a higher energy elliptical orbit. The Space Shuttle reaches the apogee (A) after half an orbit or 45 minutes, and then comes back to the same altitude as it had originally
On the right: resulting motion of the Shuttle versus ISS, going over and behind.

Effects of burns on relative motion

A posigrade burn of the Shuttle with respect to ISS results in a higher and slower orbit with a longer period, so that the Shuttle trails behind ISS.

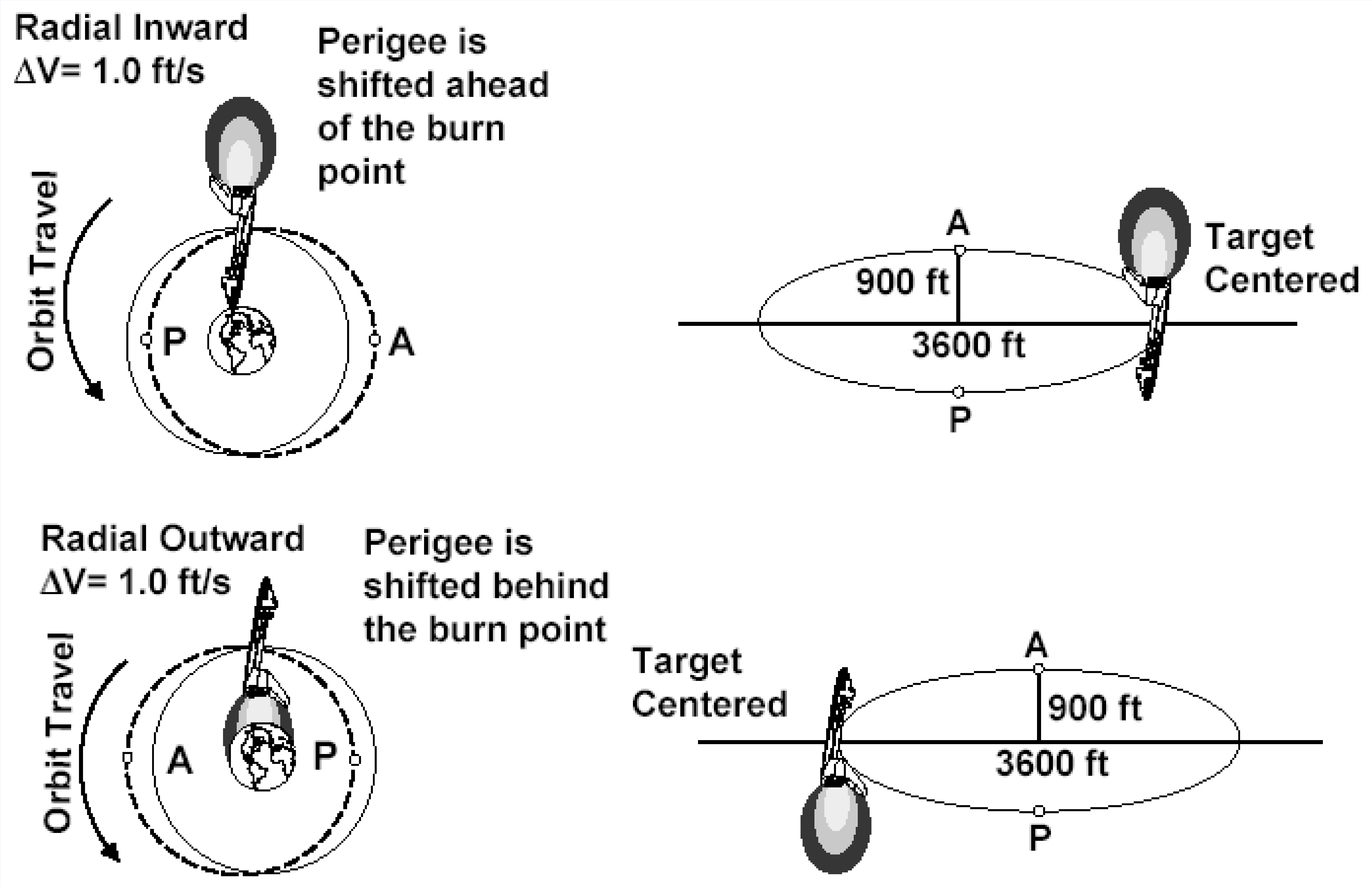
A retrograde burn brings the Shuttle on an orbit with lower altitude, shorter period, and the Shuttle comes in front of ISS.



Effects of burns on relative motion

The case of a radial inward burn

The case of a radial outward burn



The Astronomical Unit (AU)

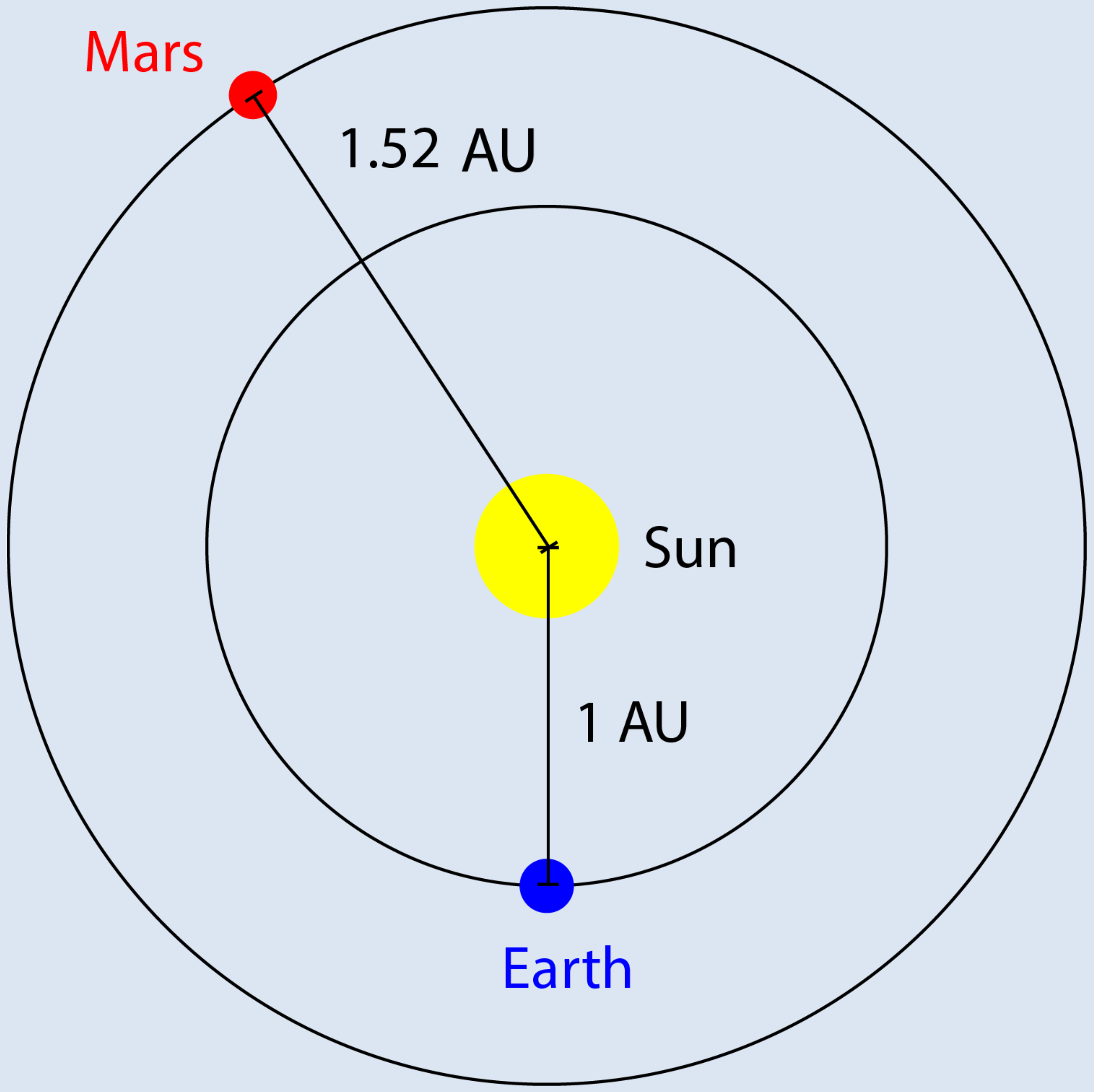
Astronomical unit = average distance Sun - Earth.

The orbit of the Earth around the Sun is slightly elliptical, eccentricity 0.017 at this time.

At perihelion on January 3rd, the Earth is about 147 million km to the Sun, at aphelion, on July 3rd, the distance is 152 million km to the Sun.

Mars has a 1.52 AU average distance to the Sun, which means that its distance to Earth varies from 0.52 AU to 2.52 AU on the average. Its orbit has a large eccentricity of 0.094 at this time.

The boundary of the planetary component of the Solar System is about 30 AU from the Sun (orbit of Neptune)



1 AU = 149.5978707 x 10^6 km

Orbital characteristics of planets

Planets	Semi-major axis a (AU)	Perihelion r_p (10^6 km)	Orbital eccentricity e	Orbital inclination i (deg)	Orbital velocity V $\left(\frac{km}{s}\right)$
Mercury	0.39	46.0	0.205	7.0	47.4
Venus	0.72	107.5	0.007	3.4	35.0
Earth	1.00	147.1	0.017	0.0	29.8
Mars	1.52	206.6	0.094	1.9	24.1
Jupiter	5.20	740.5	0.049	1.3	13.1
Saturn	9.65	1353.6	0.057	2.5	9.7
Uranus	19.20	2741.3	0.046	0.8	6.8
Neptune	30.04	4444.5	0.011	1.8	5.4

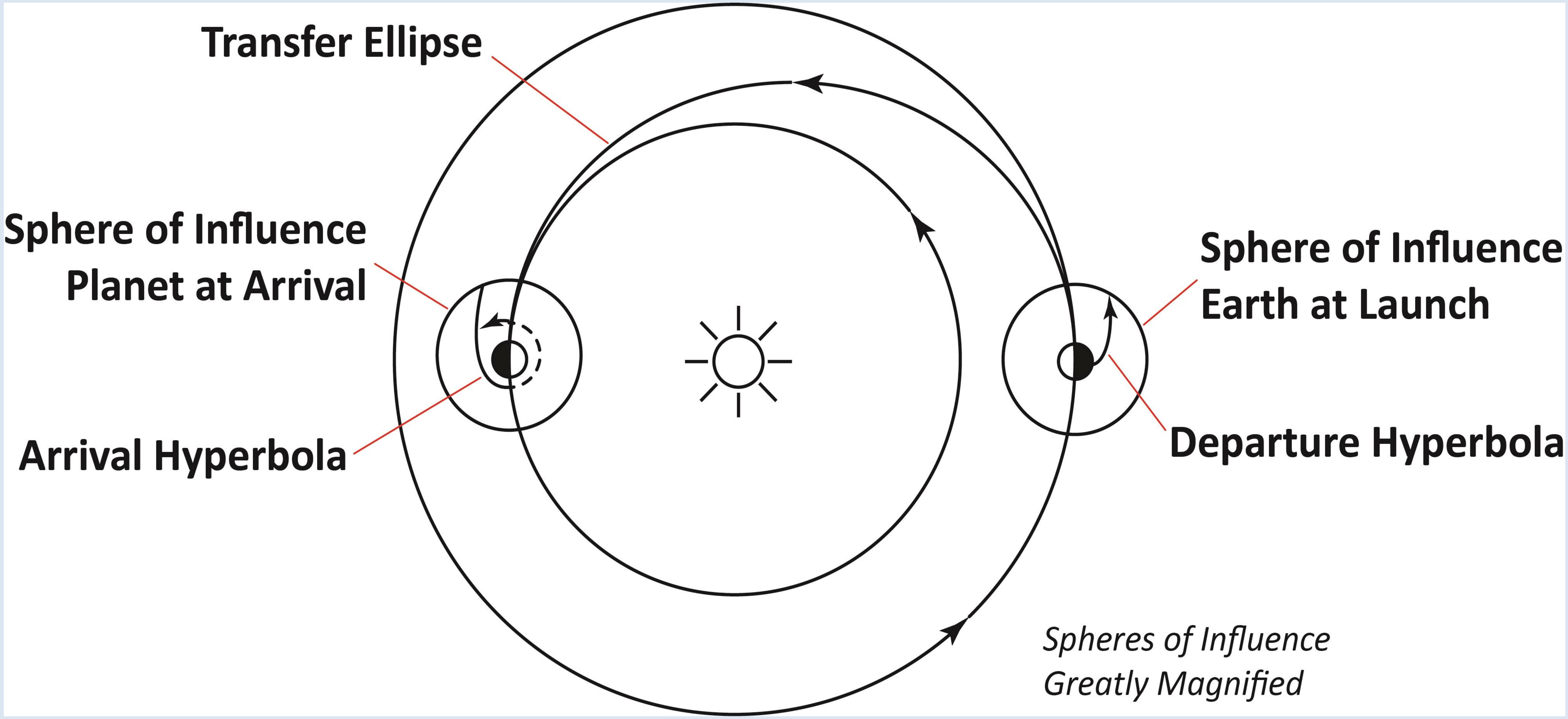
http://nssdc.gsfc.nasa.gov/planetary/factsheet/

Credits: NASA

Interplanetary trajectories – Patched conics approximation

EPFL

In order to plan for and execute a mission to another planet, we consider the Sun, the planet of departure (the Earth), the planet of destination, and the spacecraft. It is a four-body problem that we divide into three segments, each of them a two-body problem.



Credits: Charles D. Brown, *Elements of Spacecraft Design*, AIAA

- Departure phase (planetocentric 1)
- Cruise phase (heliocentric)
- Arrival phase (planetocentric 2)

The concept of sphere of influence

EPFL

- Sphere around each planet inside which the motion of a spacecraft is considered to be two-body Keplerian.
- The radius of the sphere of influence R_S has been determined by Laplace as:

$$R_S = R\left(\frac{\mu_{\text{Planet}}}{\mu_{\text{Sun}}}\right)^{\frac{2}{5}}$$

- R is the average distance between Sun and the planet.

As long as the spacecraft is within the sphere of influence of the Earth, its motion with respect to the Earth is a two-body problem with the Earth as a central body. We can ignore the Sun's gravitational influence.

When the spacecraft leaves the sphere of influence of the Earth, which happens to be about 1,000,000 km in radius, it comes on a heliocentric elliptical trajectory (Hohmann transfer) towards the destination planet, either larger than the Earth's orbit (outer planets) , or smaller (inner planets). The gravitational influence of both the departure and the destination planets is negligible on this heliocentric arc.

At the end its elliptic heliocentric arc, in the vicinity of the destination planet, the spacecraft enters the sphere of influence of the destination planet, then we can ignore the Sun and determine the spacecraft's trajectory as a two-body problem with the destination planet as the main and only attracting body.

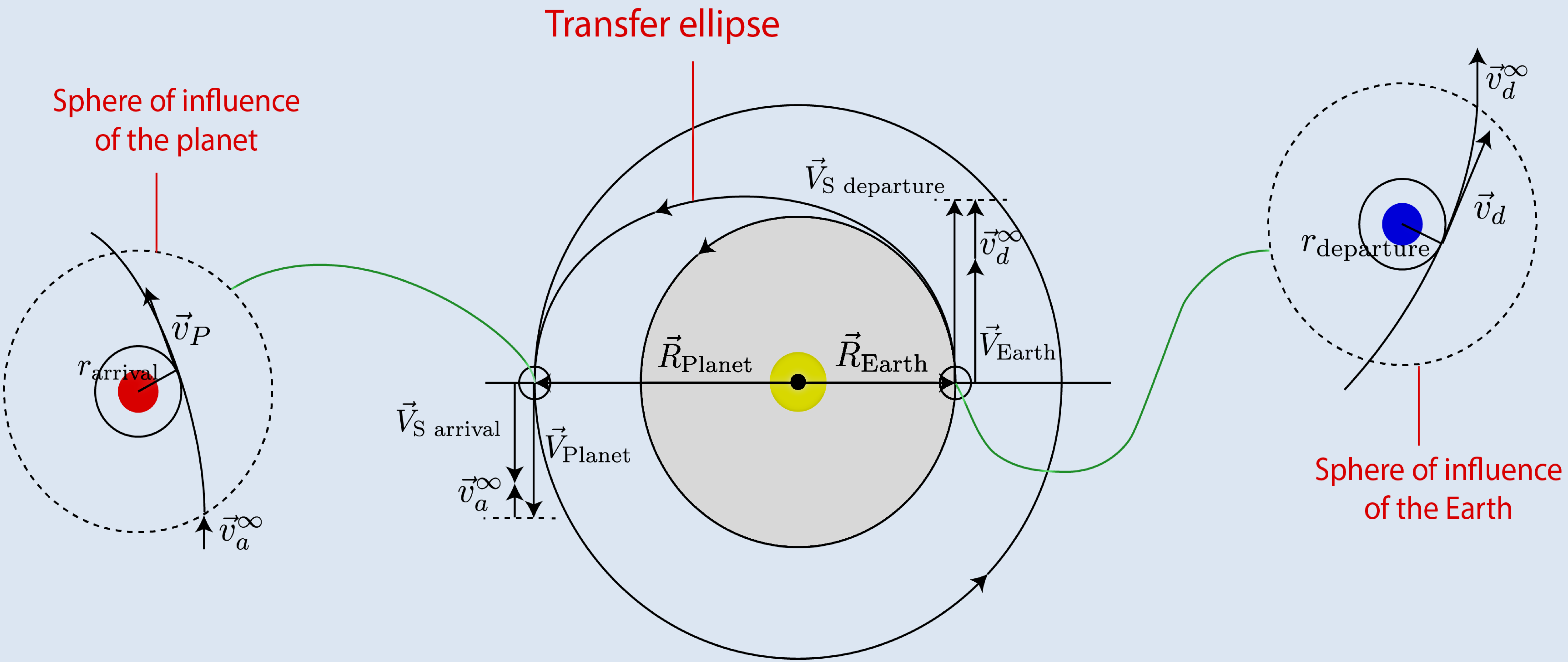
Spheres of influence in the solar system

Planets	R_S (10^6 km)
Mercury	0.111
Venus	0.616
Earth	0.924
Mars	0.577
Jupiter	48.157
Saturn	54.796
Uranus	51.954
Neptune	80.196
Moon	0.0662

- The concept of the sphere of influence is usable for the motion of a spacecraft from the Earth to another planet
- For the Moon, the sphere of influence is calculated with the Laplace equation with R = distance Earth-Moon. It means is that within about 66’200 km from the Moon center, motion of a spacecraft is essentially dominated by the Moon’s gravitational attraction.
- For all planets beyond Mercury, R_S is really large and will often be considered as being a location of zero potential energy with respect to the central body.

Credits: Charles D. Brown, *Elements of Spacecraft Design*, AIAA

Strategy for interplanetary transfer



Symbol convention for position and velocity

$$\begin{aligned} \vec{R}_S &= \vec{r}_S + \vec{R}_P \\ \vec{V}_S &= \vec{v}_S + \vec{V}_P \end{aligned}$$

$$\begin{aligned} \vec{r}_S &= \vec{R}_S - \vec{R}_P \\ \vec{v}_S &= \vec{V}_S - \vec{V}_P \end{aligned}$$

Heliocentric movement of the spacecraft

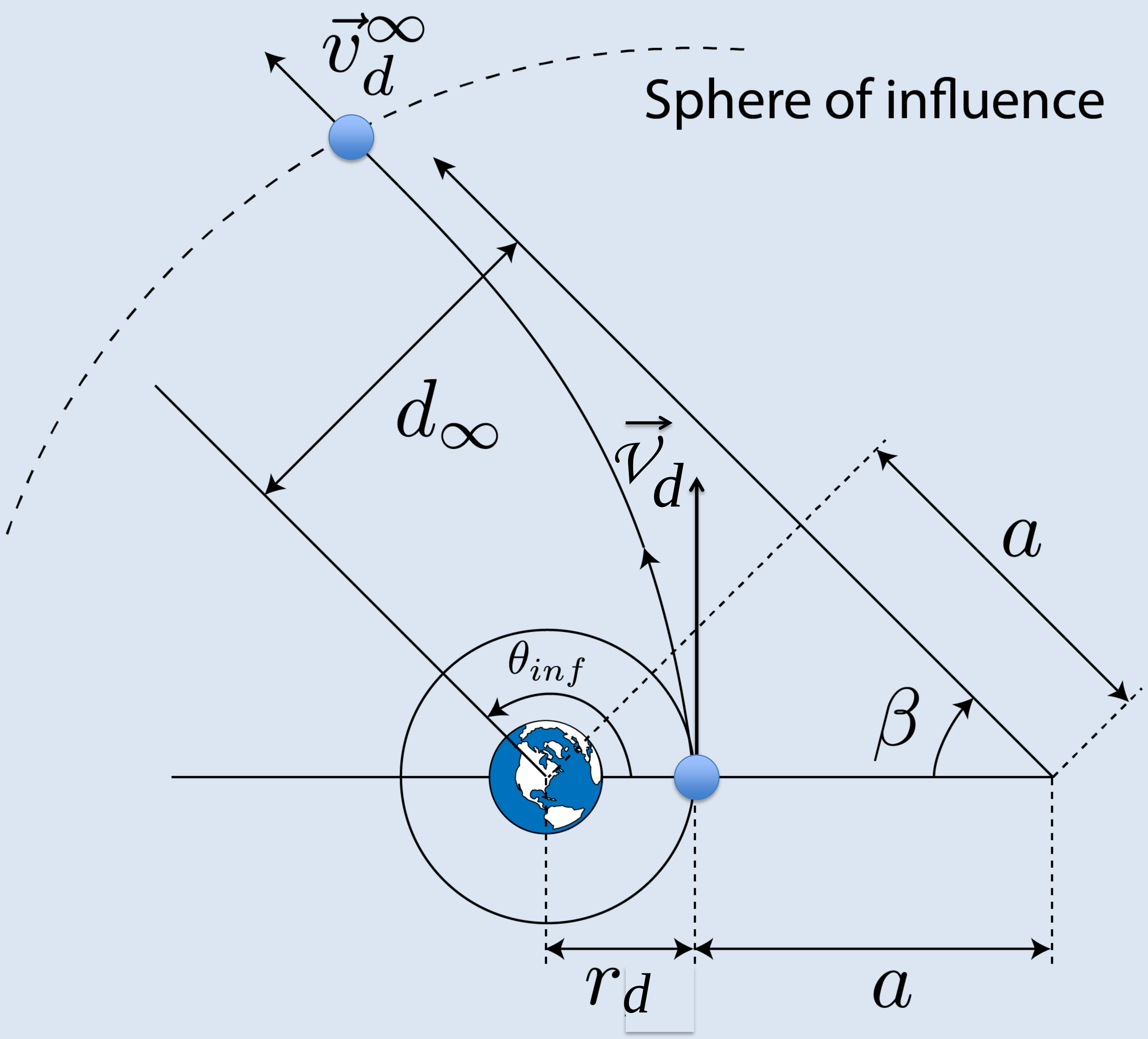
Planetocentric movement of the spacecraft

Heliocentric movement of the planet

Planetocentric movement of the planet

Difference between the heliocentric movement of the spacecraft and the planet

Reaching the sphere of influence



$$r_d = r_{\text{departure}} = r_{\text{perigee}}$$

Conservation of total mechanical energy →

$$\frac{(v_d^\infty)^2}{2} - \left(\frac{\mu}{r}\right)_\infty = \frac{v_d^2}{2} - \frac{\mu}{r_d}$$

$$\frac{(v_d^\infty)^2}{2} \cong \frac{v_d^2}{2} - \frac{v_{Erd}^2}{2}$$

$$v_d^2 = (v_d^\infty)^2 + v_{Erd}^2$$

- Reminder
  - $v_{E\ \text{Surface}}$ is 11.2 km/s for Earth
  - v_{Erd} is escape velocity at distance r_d

Vis-Viva equation – Velocities on elliptical and hyperbolic orbits

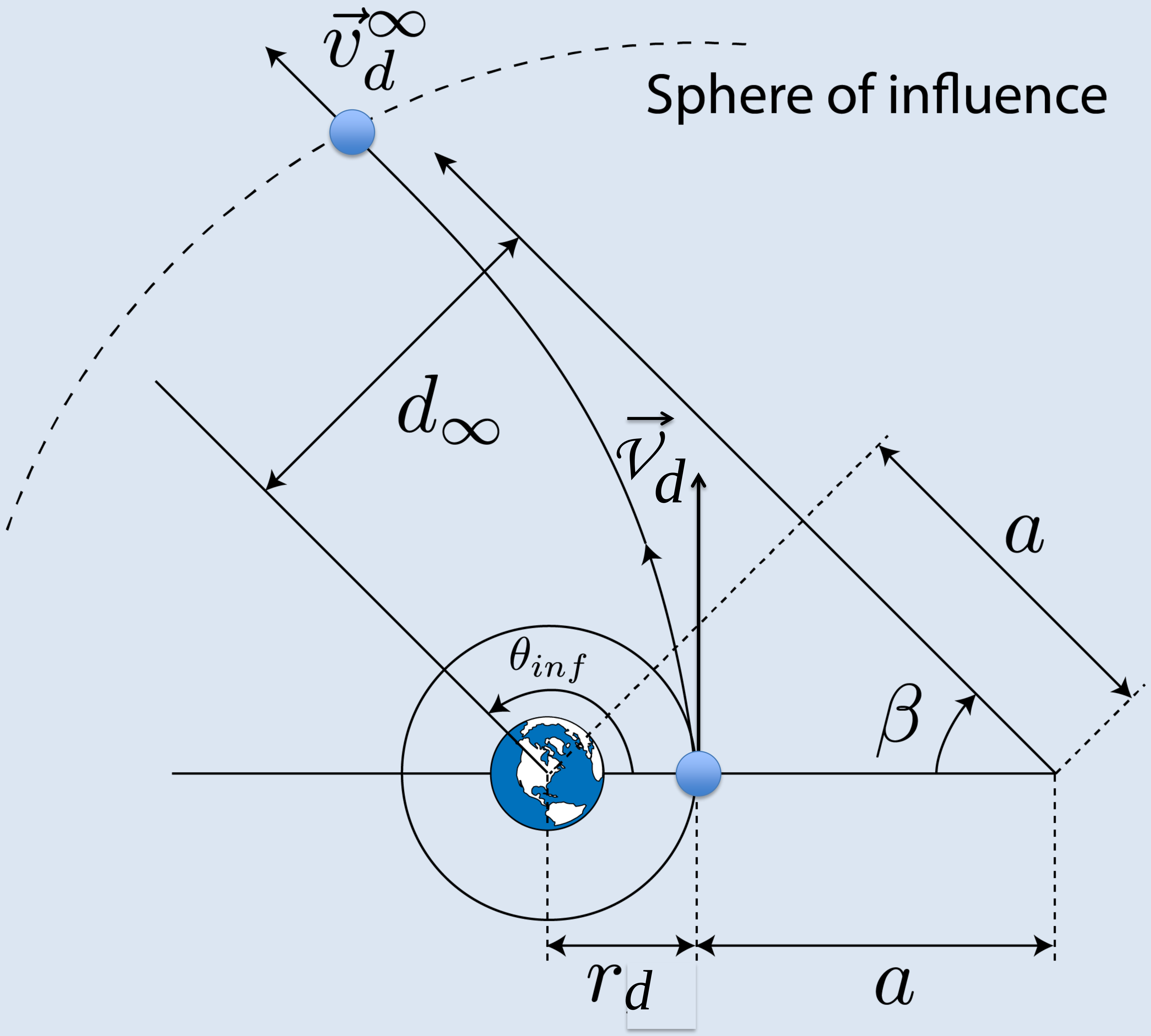
EPFL

Ellipse: $$V = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$$ $$\epsilon = -\frac{\mu}{2a}$$

Hyperbola: $$V = \sqrt{\frac{2\mu}{r} + \frac{\mu}{a}}$$ $$\epsilon = +\frac{\mu}{2a}$$

The energy per unit mass on an elliptical or hyperbolic trajectory is only dependent on the mass of the central object μ, and on the value of the semi-major axis a, and ***not*** on the eccentricity.

Departure from a planet



$r_d = r_{\text{departure}} = r_{\text{perigee}}$

$$v = \sqrt{\frac{2\mu}{r} + \frac{\mu}{a}}$$

$$a \cong \frac{\mu}{\left(v_d^\infty\right)^2}$$

$$e = \frac{a + r_d}{a} = \frac{c}{a} > 1$$

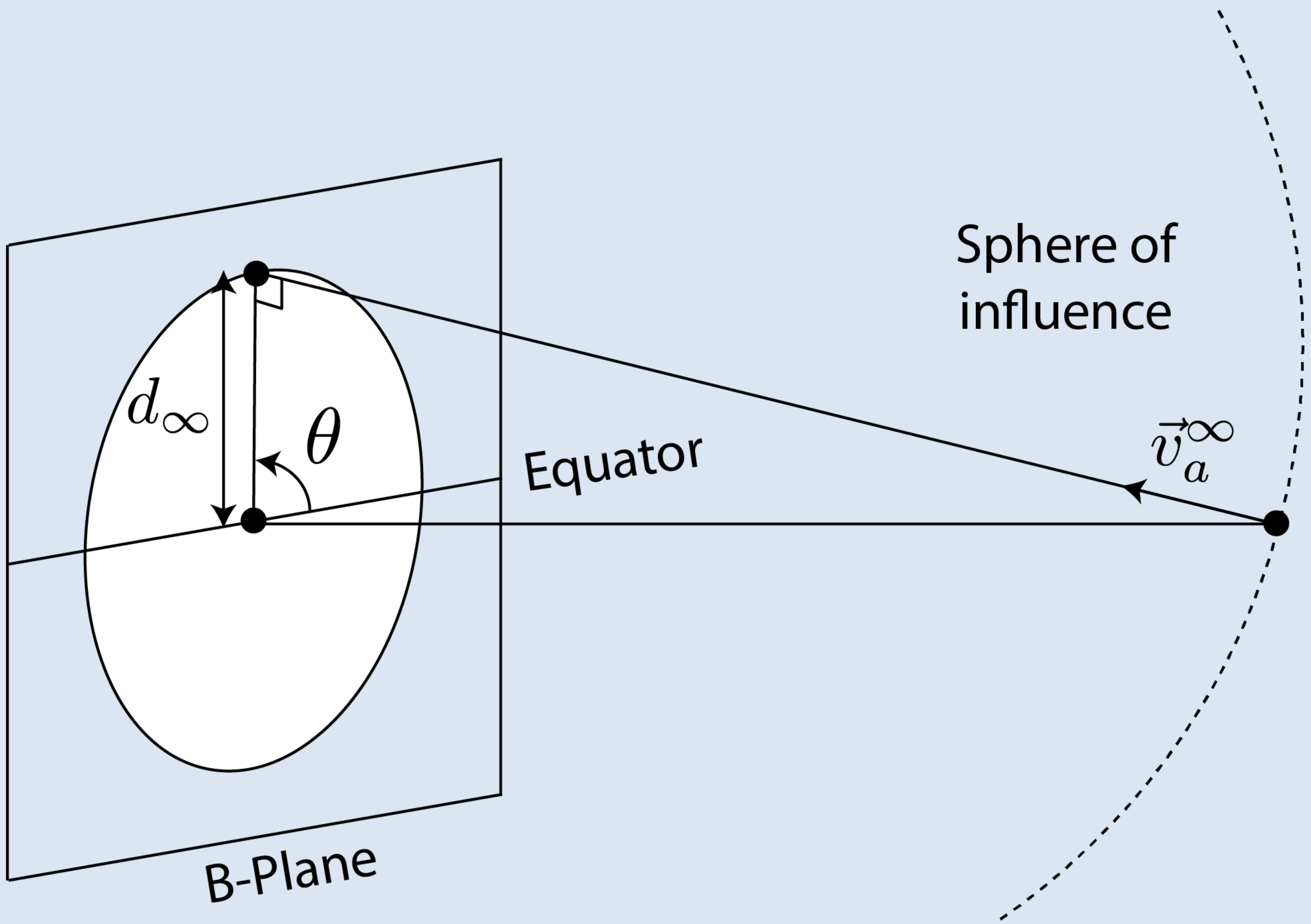
$$\theta_{\text{inf}} = \arccos\left(-\frac{1}{e}\right)$$

Definitions

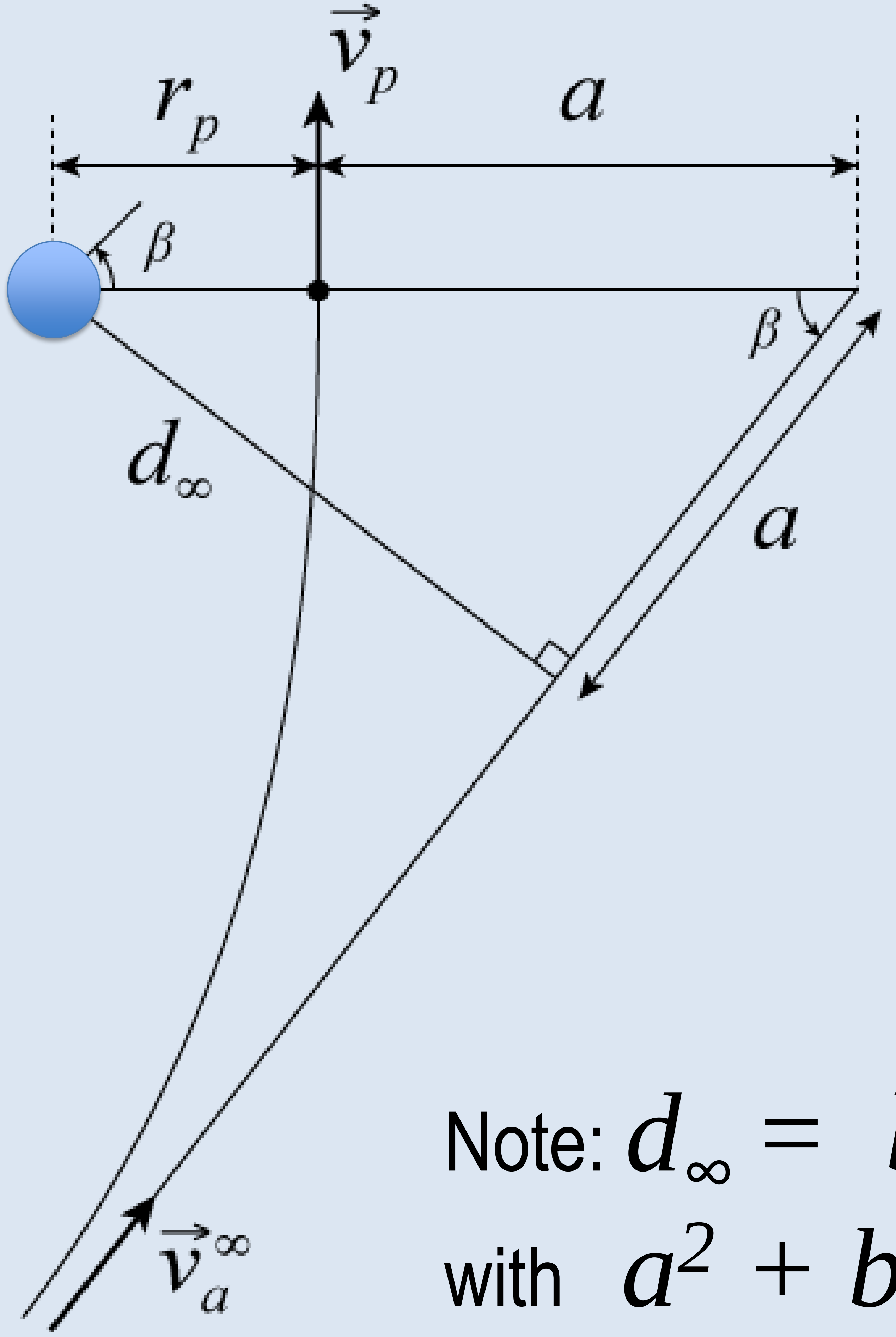
On its journey to the planet of destination, the spacecraft is on an elliptical heliocentric trajectory, until it gets close to that planet. Then only the motion of the spacecraft with respect to the destination planet is considered as hyperbolic trajectory inside the sphere of influence of this planet.

Flight controllers steer the motion of the spacecraft and select the value of d_∞, the impact parameter, and θ in order to accomplish the mission objective, either a flyby (and possible orbit insertion) or a direct landing on the surface of the planet.

Arrival at velocity $\vec{v}_a^{\infty} = \vec{V}_S - \vec{V}_P$ on the sphere of influence of the destination planet.



Determination of important parameters



Conservation of total energy → $$v_p^2 = (v_a^\infty)^2 + v_{Erp}^2$$

also $$a \cong \frac{\mu}{(v_a^\infty)^2}$$

$$r_p = -\frac{\mu}{(v_a^\infty)^2} + \sqrt{\frac{\mu^2}{(v_a^\infty)^4} + d_\infty^2}$$

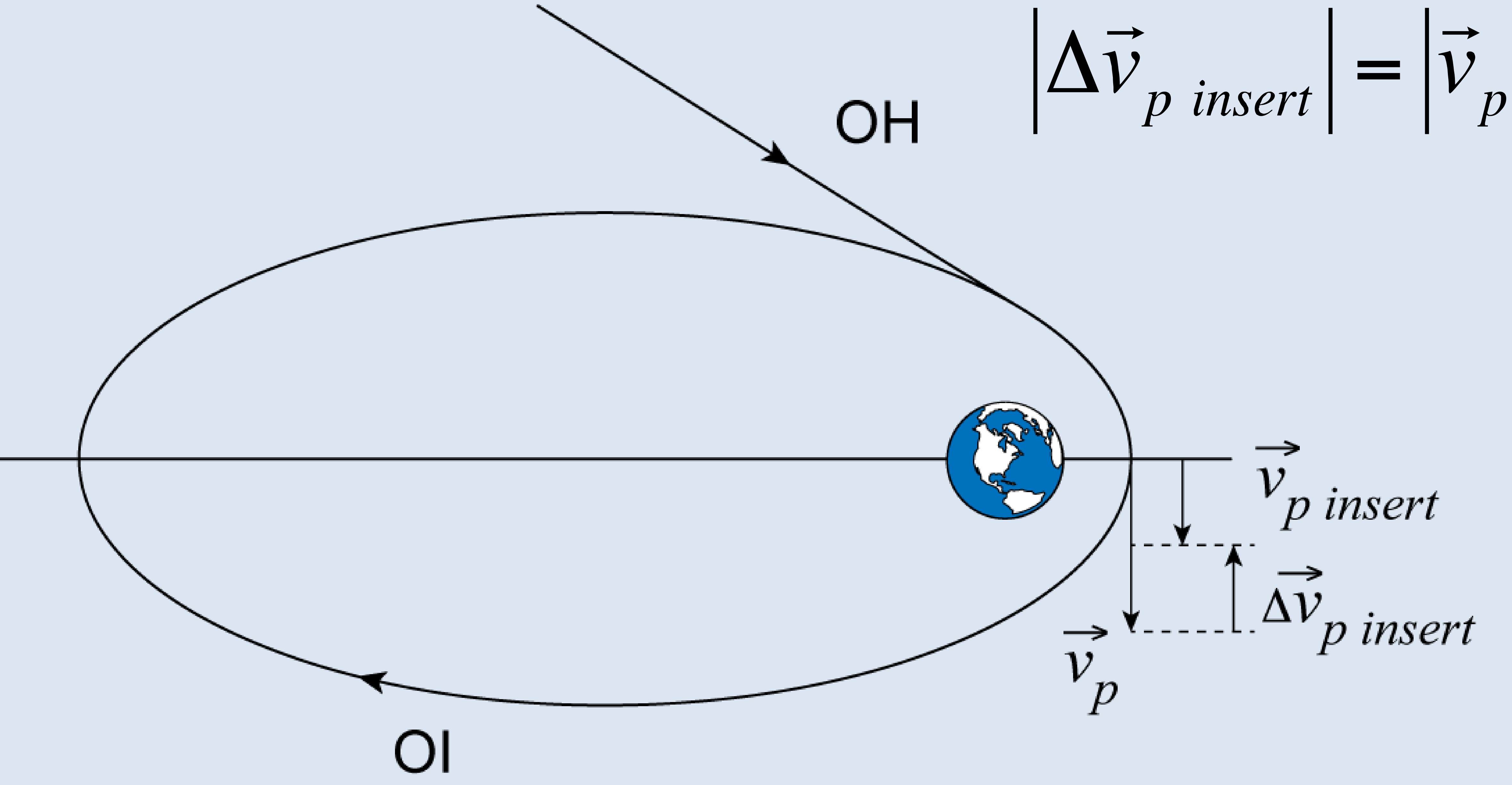
$$\cos\beta = \frac{a}{a + r_p} = \frac{a}{c}$$

Note: $d_\infty = b$

with $a^2 + b^2 = c^2 = (a + r_p)^2$

Orbit insertion around the destination planet

EPFL



$$\left|\Delta\vec{v}_{p\ insert}\right| = \left|\vec{v}_p\right| - \left|\vec{v}_{p\ insert}\right| = \sqrt{\frac{2\mu}{r_p} + \left(v_a^{\infty}\right)^2} - \sqrt{\frac{2\mu}{r_p} - \frac{\mu}{a_i}}$$

p for "periapsis"

a_i is the semi major axis of the elliptical orbit at insertion

$\Delta v_{p\ insert}$ is how much the spacecraft needs to brake in km/sec in order to be inserted in an elliptical orbit around the destination planet.

Aerodynamic braking maneuvers

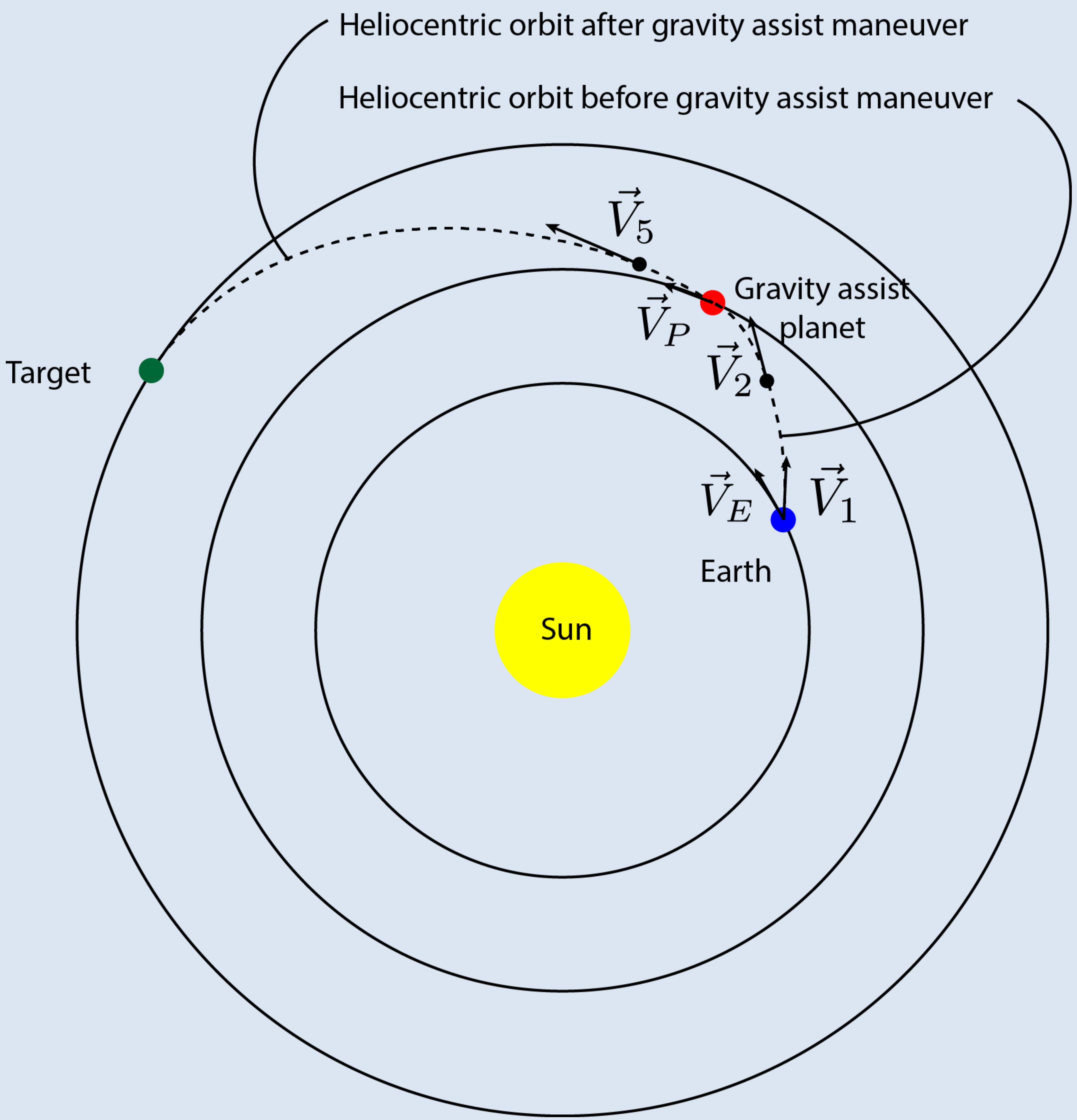
The three techniques use braking of a spacecraft through the atmosphere of a planet (Earth, Mars, Venus, or other) for capture by the planet, or to cause a change of trajectory, or else a full entry in the atmosphere of the planet.

In this last case, the entry is followed by the deployment of a parachute for a capsule (Apollo CM, Soyuz, Orion), or transition to atmospheric flight for a winged spacecraft. (Shuttle).

A thermal shield on the spacecraft is needed to avoid overheating during the braking maneuver.

- **Aerocapture**
  - Transfers the spacecraft from a hyperbolic approach trajectory to an elliptical orbit around the target planet.
  - Further loss of energy will occur at every subsequent crossing of the periapsis (through aerobraking).
- **Aerobraking**
  - Transfers the spacecraft from an initial elliptical orbit to a less energetic (i.e. lower apoapsis) elliptical orbit.
  - Involves relatively small ΔV.
- **Aeroentry**
  - Transfers the spacecraft from either a hyperbolic, parabolic or elliptical approach orbit to the planet surface.

Definitions



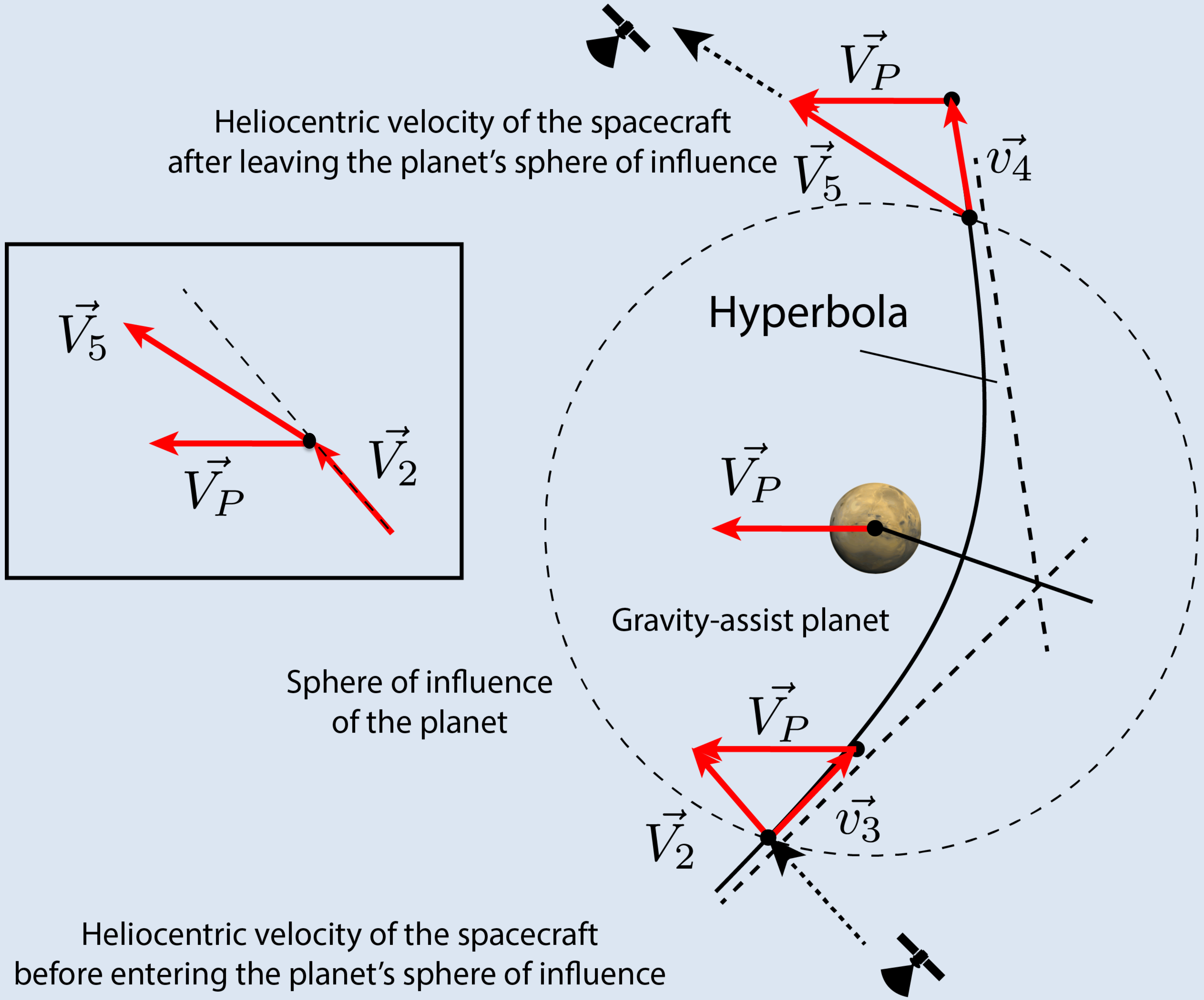
- $\vec{V}_P$: Heliocentric velocity of the gravity-assist-planet.
- $\vec{V}_E$: Heliocentric velocity of the Earth.
- $\vec{V}_1$: Heliocentric velocity of the spacecraft after leaving Earth.
- $\vec{V}_2$: Heliocentric velocity of the spacecraft entering the gravity-assist planet's sphere of influence.
- $\vec{V}_5$: Heliocentric velocity of the spacecraft leaving the gravity-assist planet's sphere of influence.

Slingshot maneuver profile

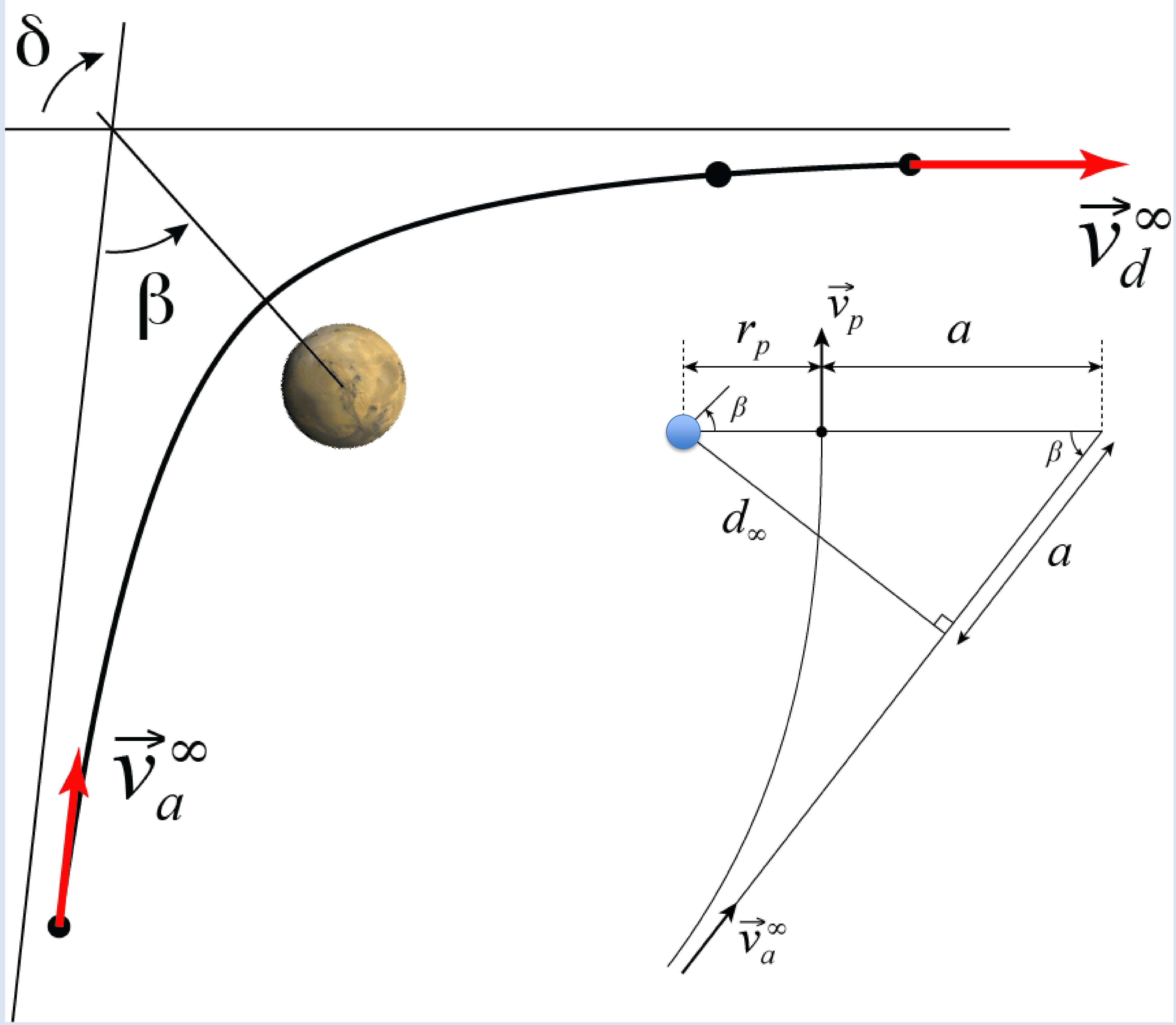
- $\vec{V}_2$: Heliocentric velocity of the spacecraft entering the planet's sphere of influence.
- $\vec{V}_5$: Heliocentric velocity of the spacecraft leaving the planet's sphere of influence.
- $\vec{V}_P$: Heliocentric velocity of the planet.
- $\vec{v}_3$: Planetocentric velocity of the spacecraft entering the planet's sphere of influence.
- $\vec{v}_4$: Planetocentric velocity of the spacecraft leaving the planet's sphere of influence.

$$|\vec{v}_3| = |\vec{v}_4|$$

$$|\vec{V}_P + \vec{v}_4| > |\vec{V}_P + \vec{v}_3|$$



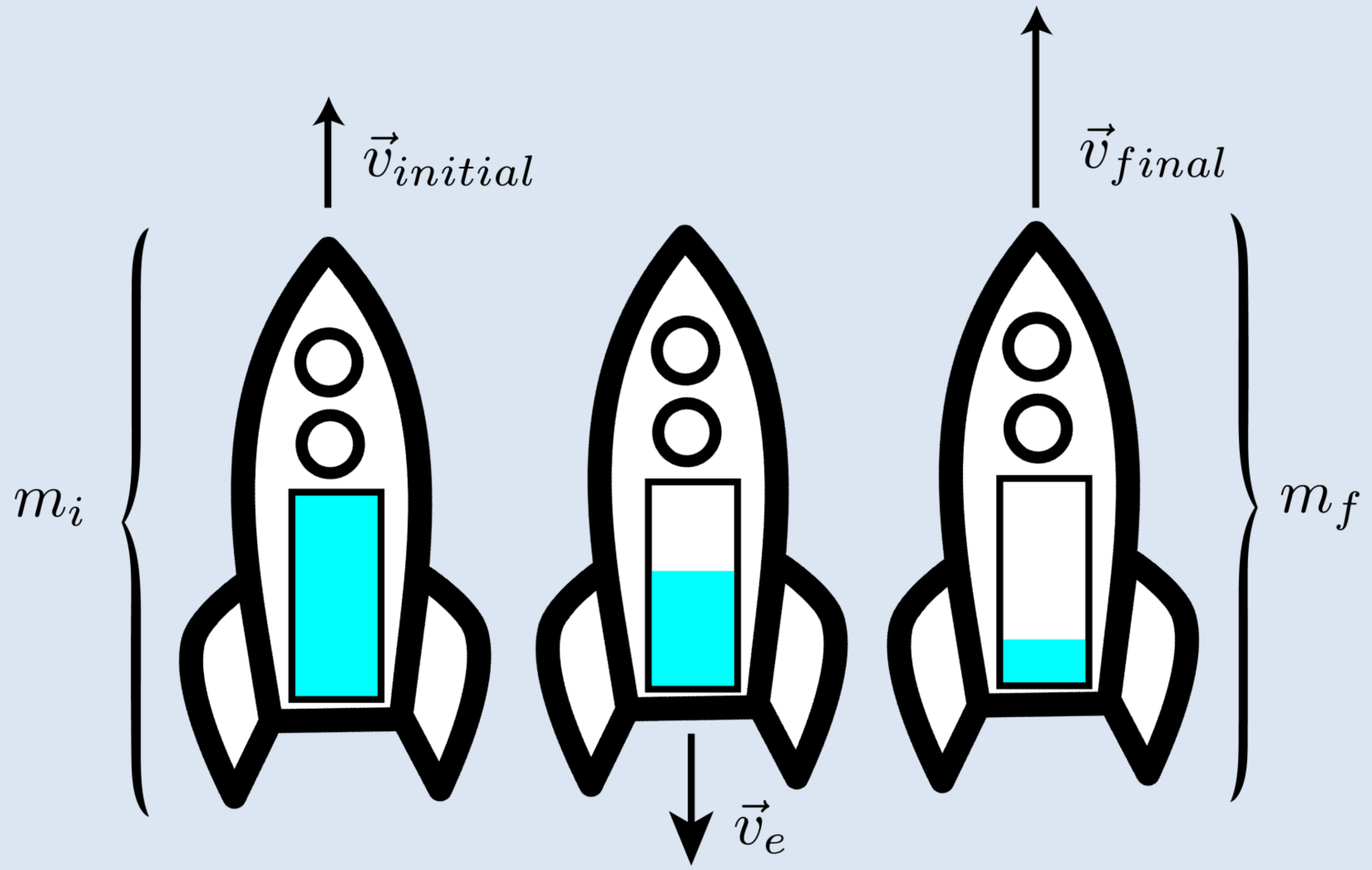
Slingshot maneuver parameters



- In the vicinity of a planet, the trajectory of a spacecraft is hyperbolic with a periapsis at distance r_p to the center of the planet.
- δ is the angle between the directions of $\vec{v}_a^\infty$ and $\vec{v}_d^\infty$.

$$\cos\left(\frac{\delta}{2}\right) = \cos\beta = \frac{a}{a + r_p}$$

Tsiolkovsky equation or rocket equation



$$\Delta V = v_e \log_e \left(\frac{m_i}{m_f} \right)$$

- ΔV = change of velocity induced by the propulsion system
- v_e = exhaust velocity of the gas in the propulsion system
- m_i, m_f = initial and final mass
- Valid in free space – gravitational field-induced and drag-induced ΔVs will be added to the propulsion-induced ΔVs.

Thrust and acceleration

- Thrust of the propulsion system (static case):

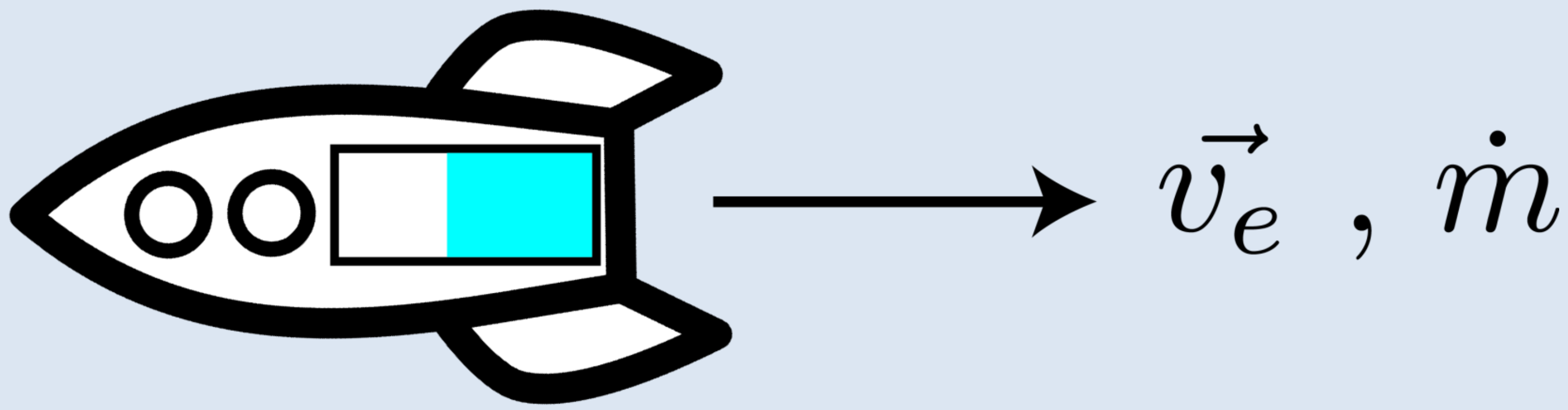
$$F = v_e \dot{m} \text{ with } \dot{m} = \text{mass flow and } v_e = \text{ejection velocity}$$

- Resulting acceleration of the spacecraft:

$$\frac{dV}{dt} = \frac{F}{m} \rightarrow \frac{dV}{dt} = v_e \frac{\dot{m}}{m}$$

- Integrating between the initial and final conditions:

$$\boxed{\Delta V = v_e \log_e \left(\frac{m_i}{m_f} \right)}$$



Propulsion system efficiency

- Other form of the Tsiolkovsky equation:

$$\Delta V = g \cdot I_{sp} \log_e \left(\frac{m_i}{m_f} \right)$$

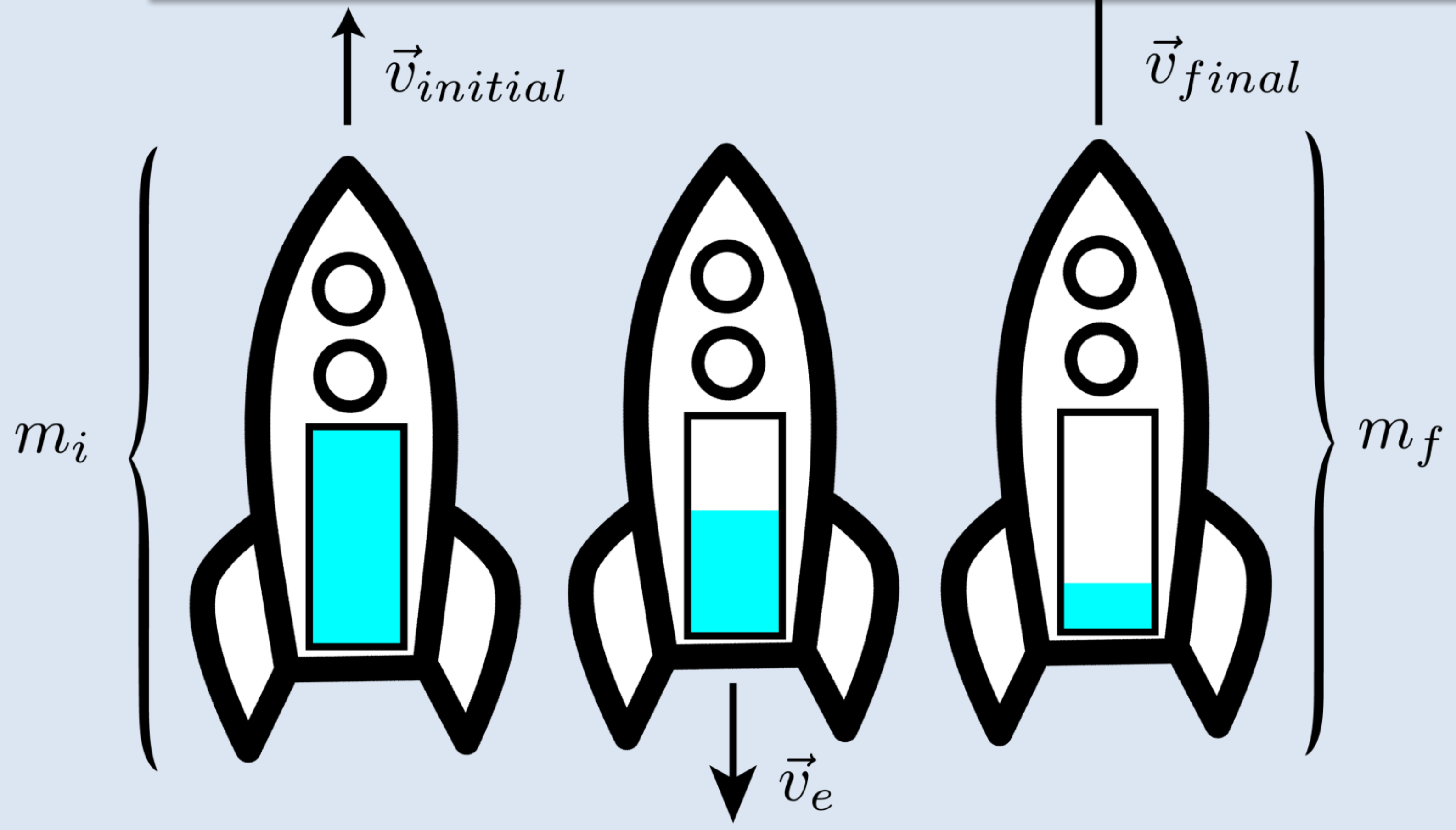
- I_{sp} = specific impulse (s).
- g = Earth's gravity acceleration, 9.81 (m/s^2).
- I_{sp} is a measure of the propulsion system efficiency, it is its thrust (kg-force) divided by the mass flow of propellants (fuel and oxidizer, kg/s), in seconds $I_{sp} = F/\dot{m}g$

$$F = v_e \dot{m} \text{ with } \dot{m} = \text{mass flow and } v_e = \text{ejection velocity}$$

There is a factor of about 10 between the value of I_{sp} in seconds and the exhaust velocity in m/s.

Mass of propellant needed

$$\Delta V = g I_{sp} \log_e\left(\frac{m_i}{m_f}\right) \Rightarrow \begin{cases} m_p = m_i\left[1 - \exp\left(-\frac{\Delta V}{g_0 I_{sp}}\right)\right] \\ m_p = m_f\left[\exp\left(\frac{\Delta V}{g_0 I_{sp}}\right) - 1\right] \end{cases}$$



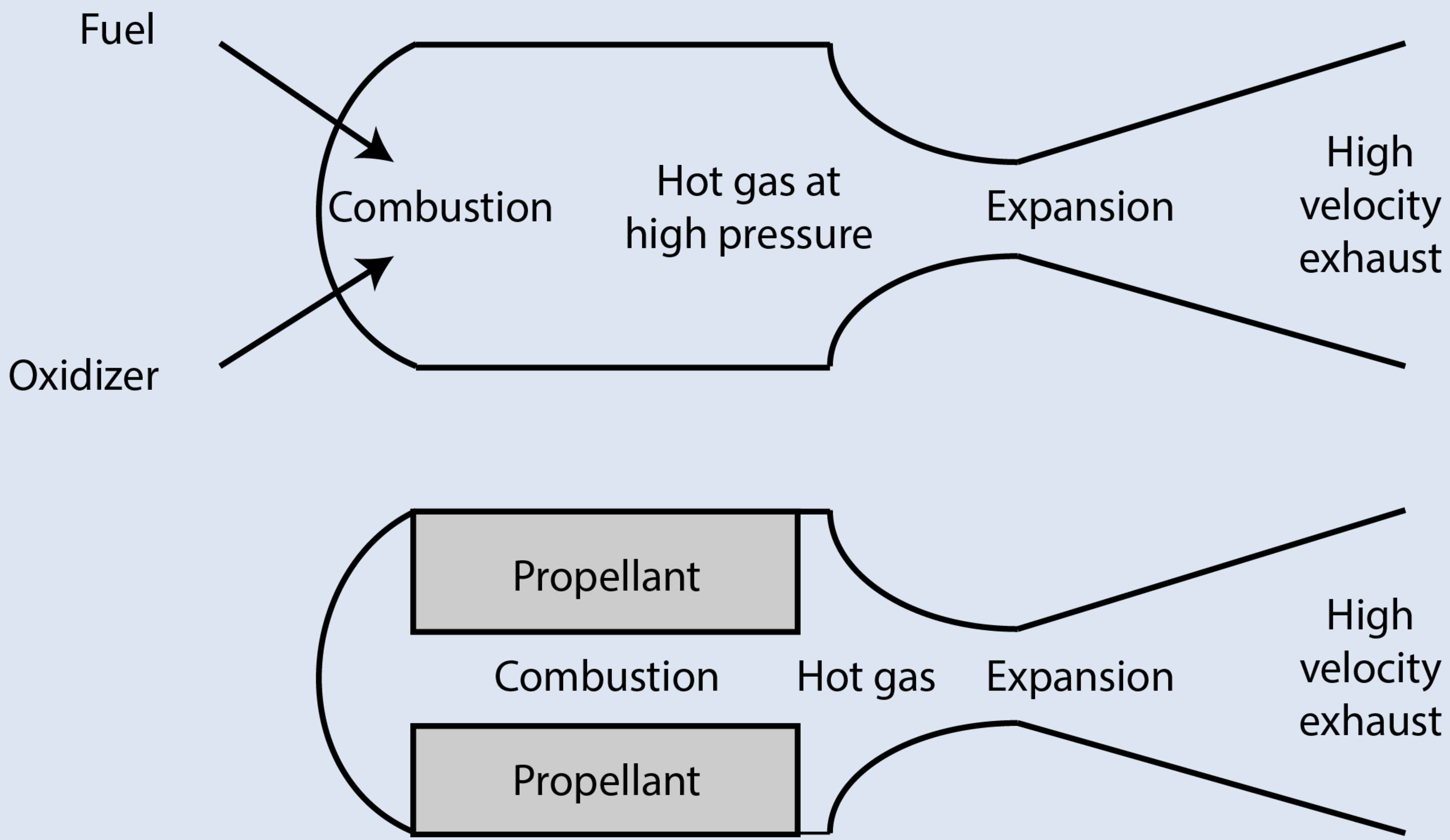
$$m_p = m_i - m_f$$

- m_i : initial vehicle mass (kg).
- m_f : final vehicle mass (kg).
- m_p : propellant mass consumed to produce the given ΔV (kg).
- ΔV : velocity increase of the vehicle (m/s).
- g : gravitational acceleration 9.81 (m/s^2).
- I_{sp} : specific impulse.

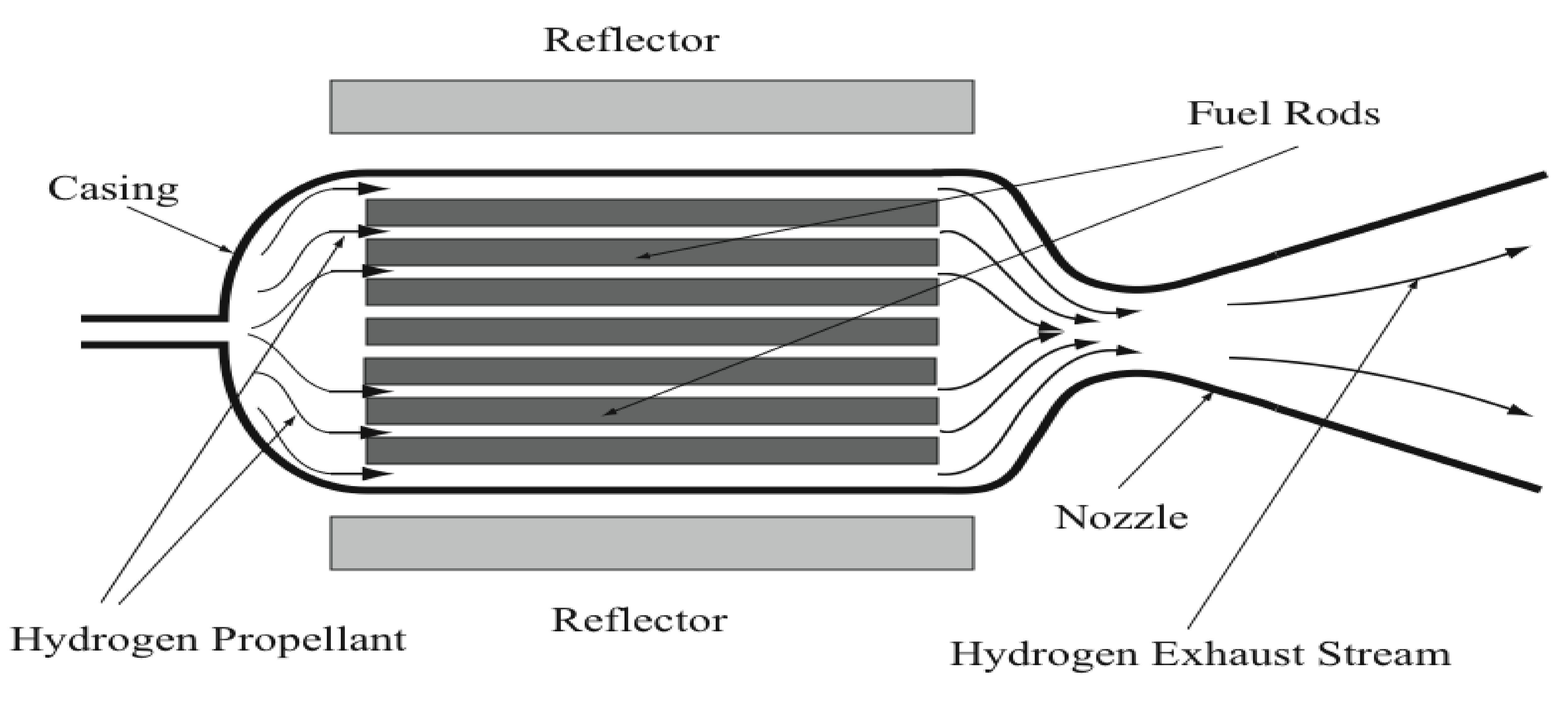
Liquid and solid propellant rocket motors

In addition to the 3 orbiter-attached liquid propellant main engines (SSMEs with LH2 and LOX), the Space Shuttle had 2 solid rocket boosters (SRBs) with a central cavity in order to increase the surface of the burning propellant generating thrust.

The same for Ariane 5 solid propellant boosters.



Nuclear rocket principle

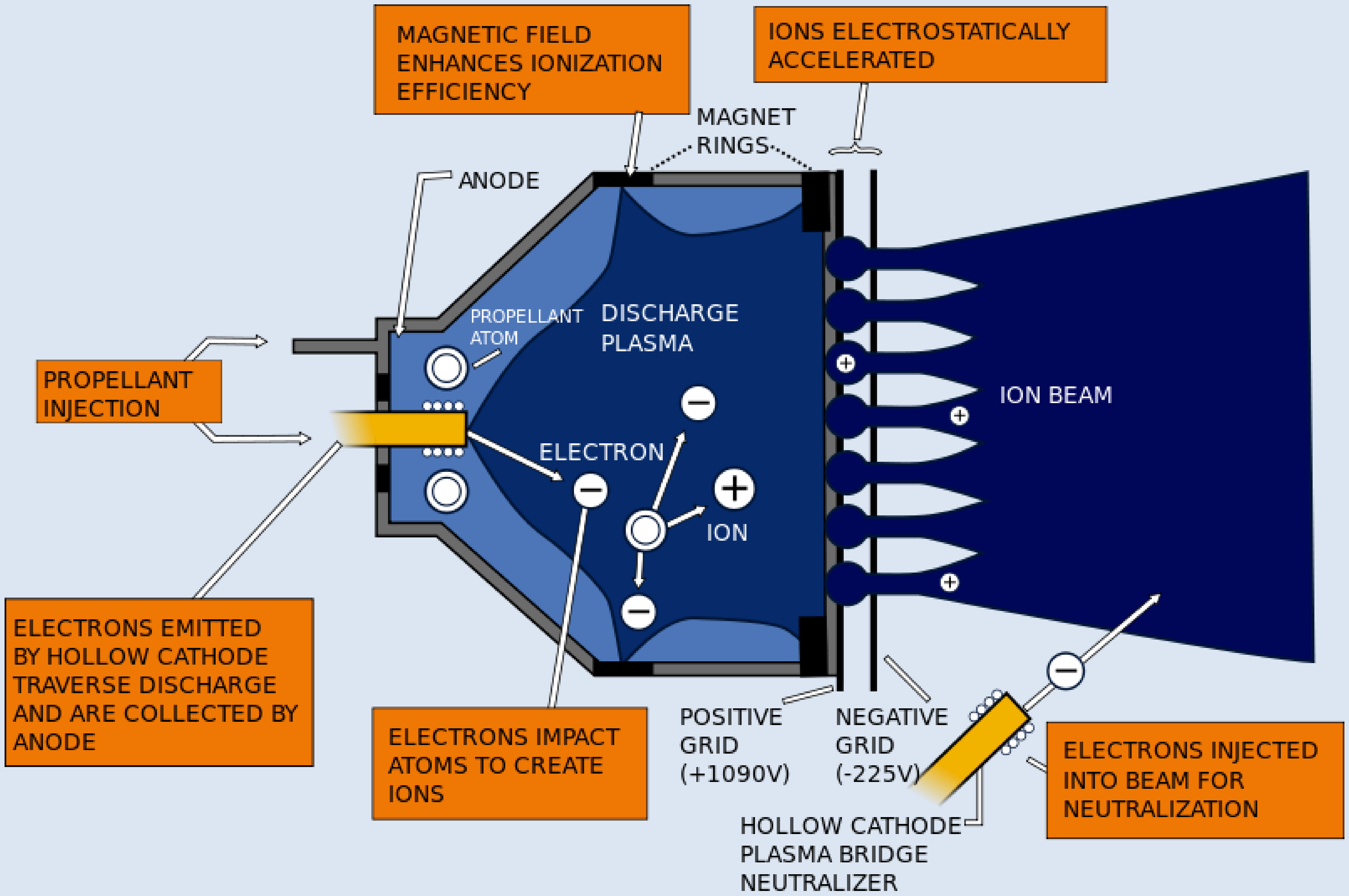


- Hot fuel rods heat hydrogen propellant.
- The hot hydrogen expands in the nozzle as in a chemical rocket motor.

Electric or ion propulsion

Ionization of the propellant material, and acceleration with an electric field. Higher exhaust velocities than with a liquid-fueled or solid propellant rocket engine.

There is very high ejection velocity and high efficiency, but relatively low thrust, of the order of a fraction of a Newton.
Such a system can be used for propulsion in space, but not for leaving the Earth's surface and bring a spacecraft to orbit.



Credits: Wikipedia, Chabacano, retrieved from NASA

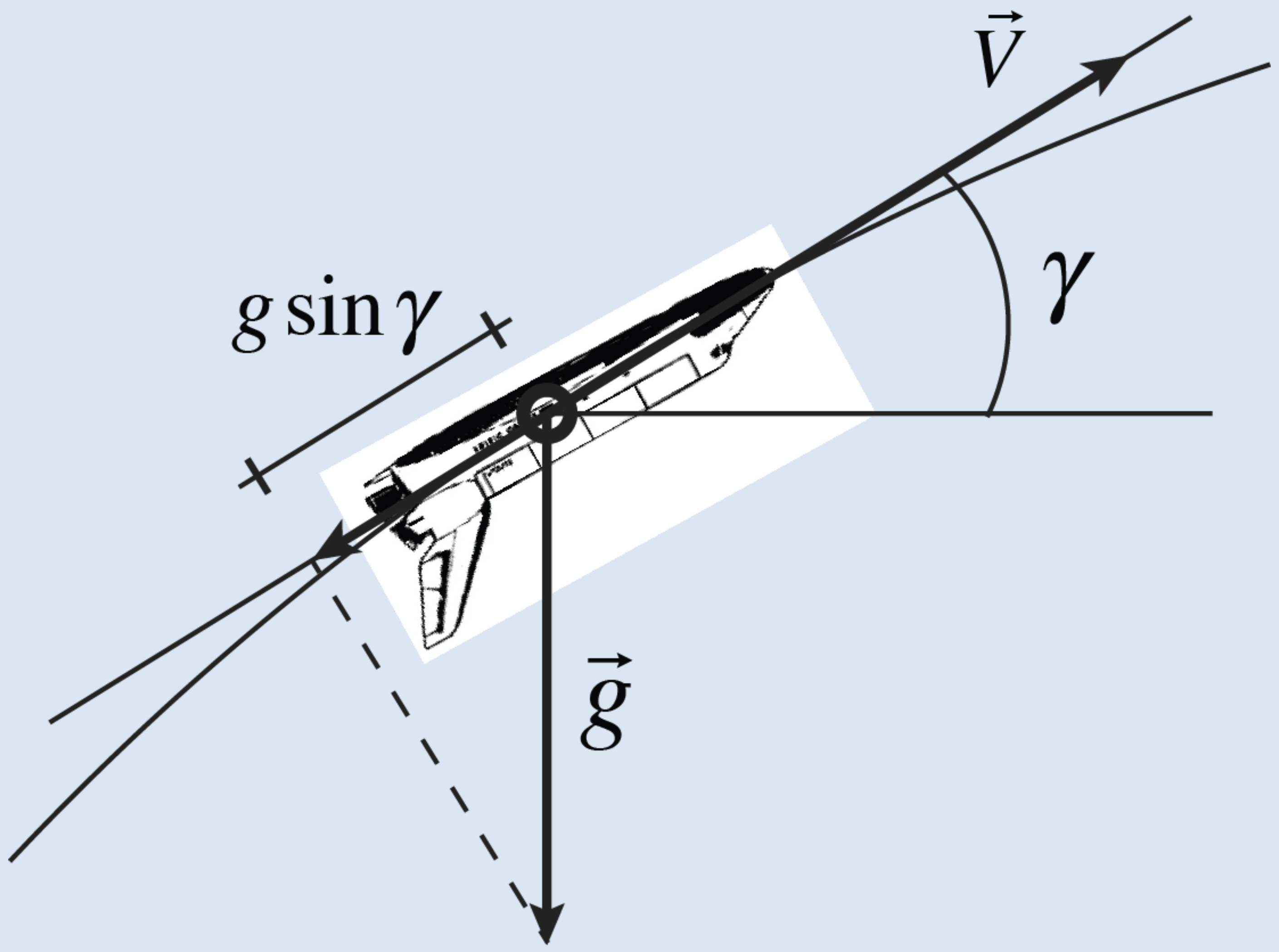
Losses during ascent to orbit

ΔV achieved by a rocket **in reality**, in case of an ascent to orbit from the Earth surface:

$$\Delta V = g I_{sp} \log_e\left(\frac{m_i}{m_f}\right) - \left(\int_{t_0}^{t_f} g \sin\gamma dt + \int_{t_0}^{t_f} \frac{D}{m} dt\right)$$

Losses during ascent to orbit: **gravity loss** and **drag loss**

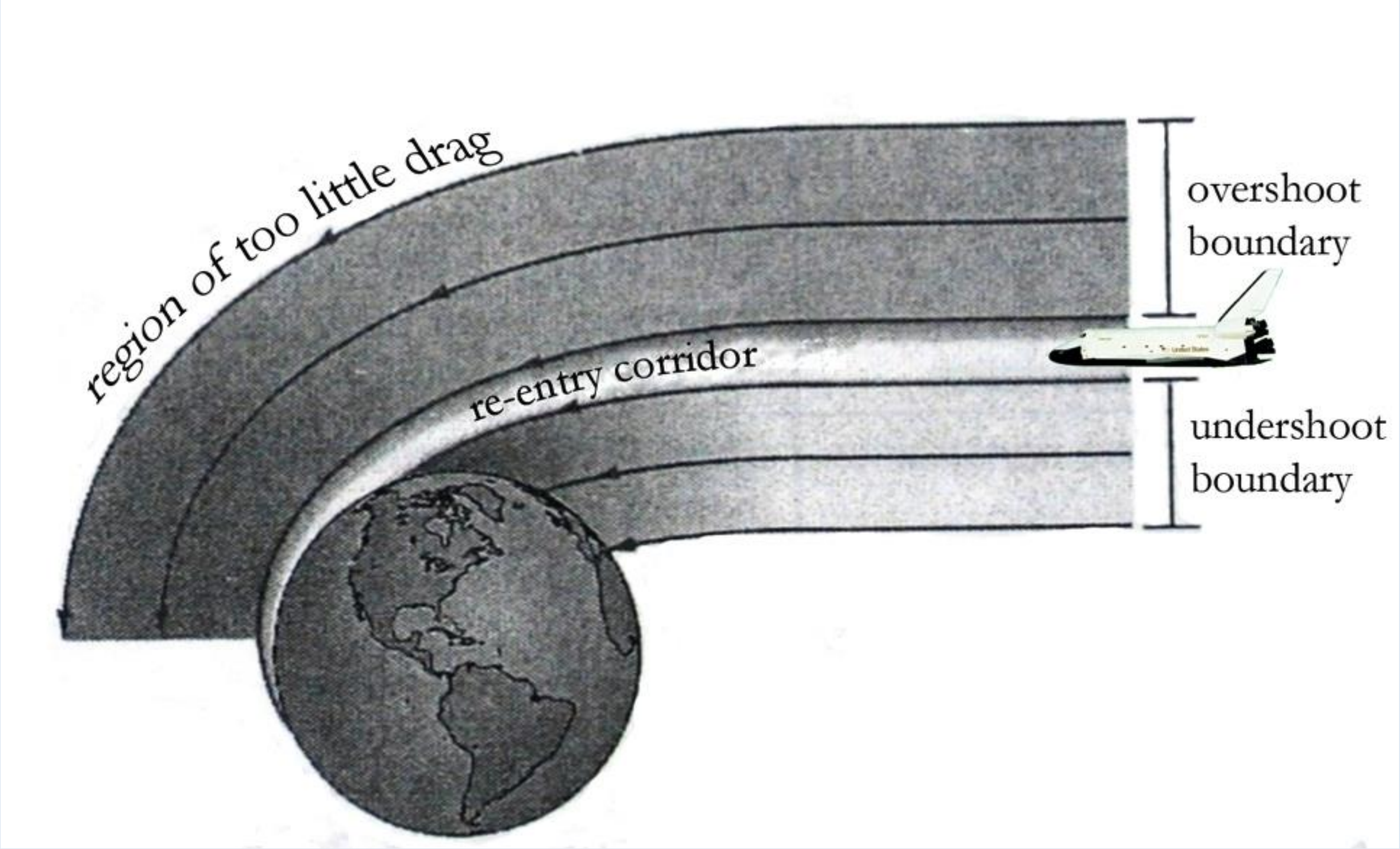
The planned and actual ascent trajectory is shaped to minimize these losses.



- D: drag force in Newton
- Y: flight path angle

Re-entry through the atmosphere

- Entry requirements and constraints:
  - Deceleration: Human limit is about 12g's for short duration.
  - Heating: Must withstand both total heat load and peak heating rate.
  - Accuracy of landing or impact: Function primarily of trajectory and vehicle design.
  - Size of the entry corridor: The size of the corridor depends on three constraints (deceleration, heating and accuracy).



Credits: Documention of the training division for NASA astronauts in the 90's.

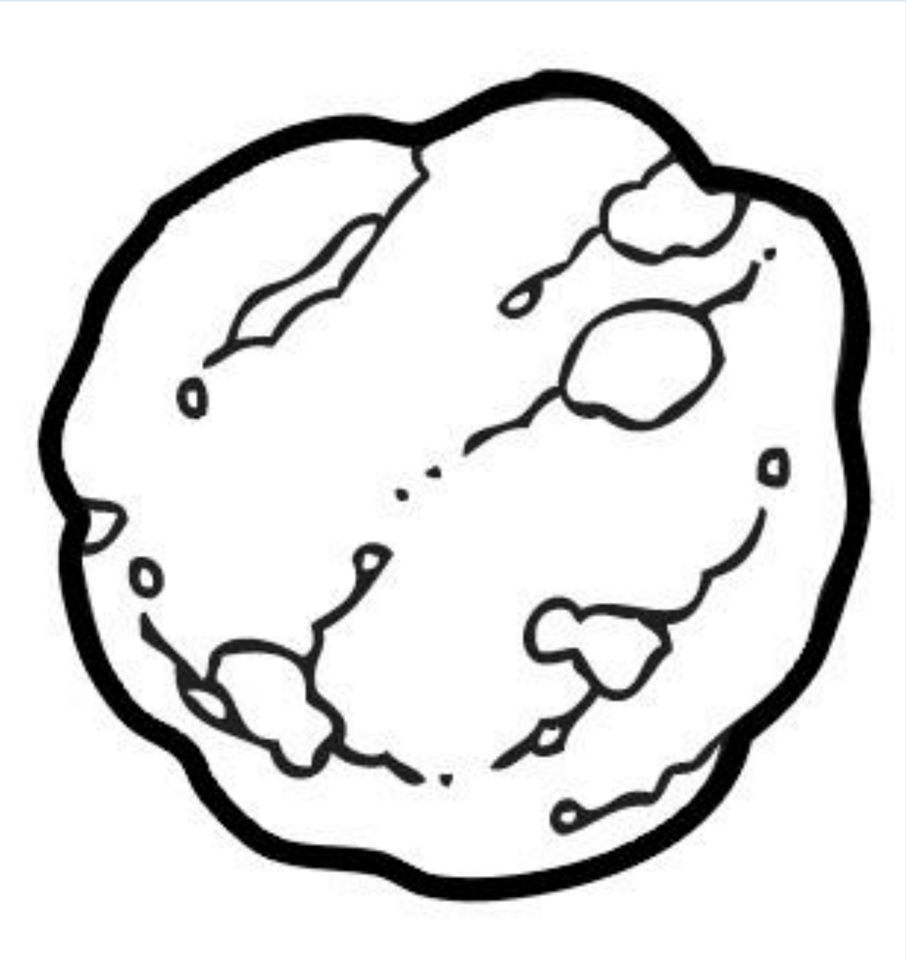
Attitude Measurement and Control System – AMCS

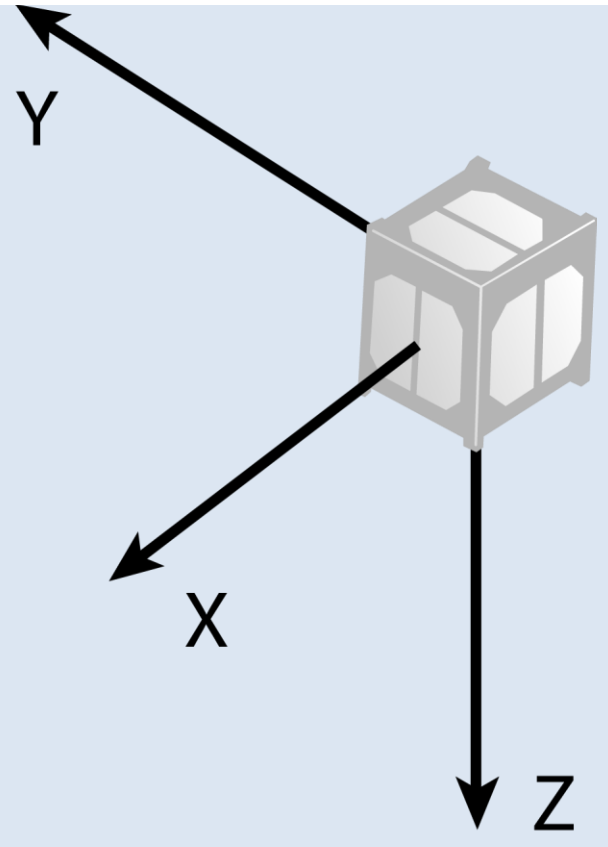
A man-made spacecraft or natural object, like an asteroid or a comet nucleus, is usually slowly rotating (with respect to an inertial reference frame), subject to gravity gradient forces if located in the vicinity of a large body, to solar radiation flux, solar wind, atmospheric effect or magnetic torques, if such conditions exist.

Orientation influenced by

- Gravity gradient
- Solar radiation and solar wind
- Atmosphere
- Magnetic field

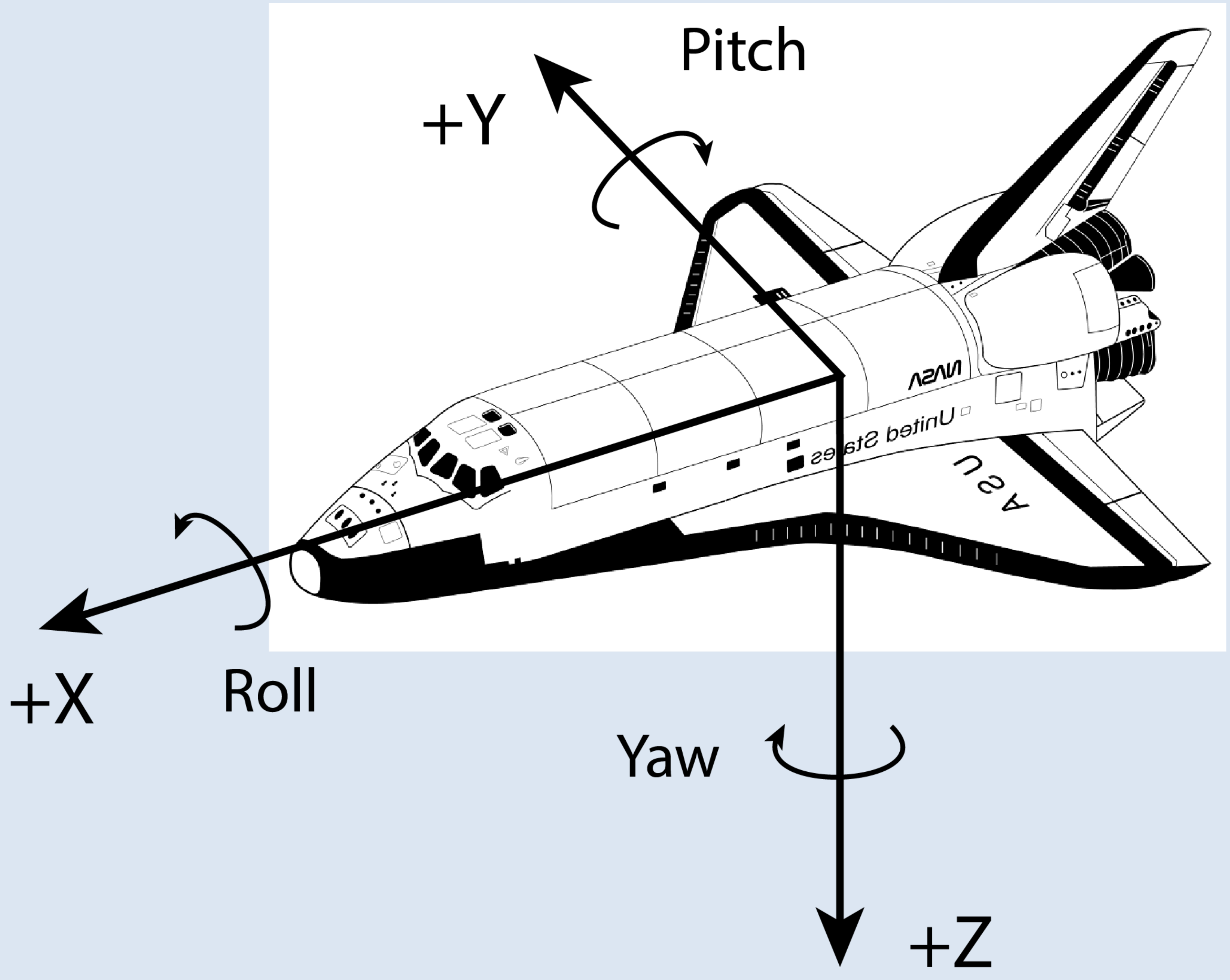
➢ Need for attitude control, or control of the orientation of a coordinate frame attached to the spacecraft, with respect to a standard reference frame, inertial, or LVLH, or other.

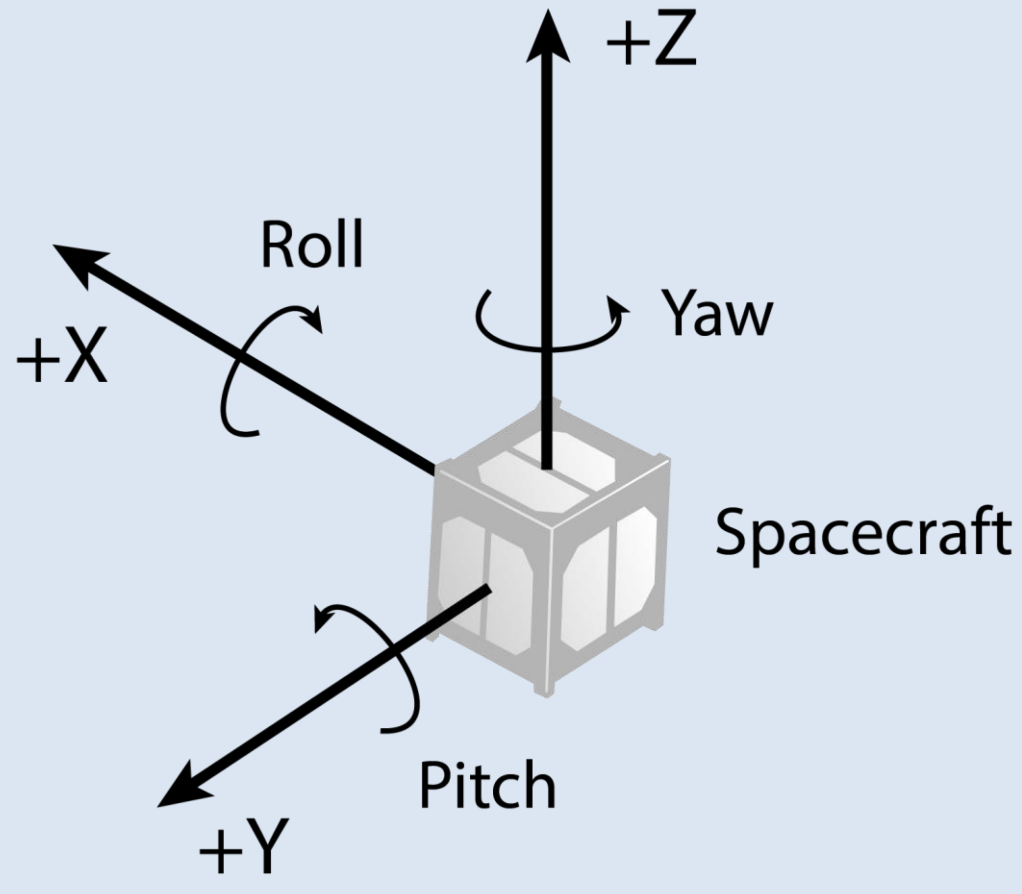




Attitude Measurement and Control System – AMCS

Body attached coordinate system for the Space Shuttle





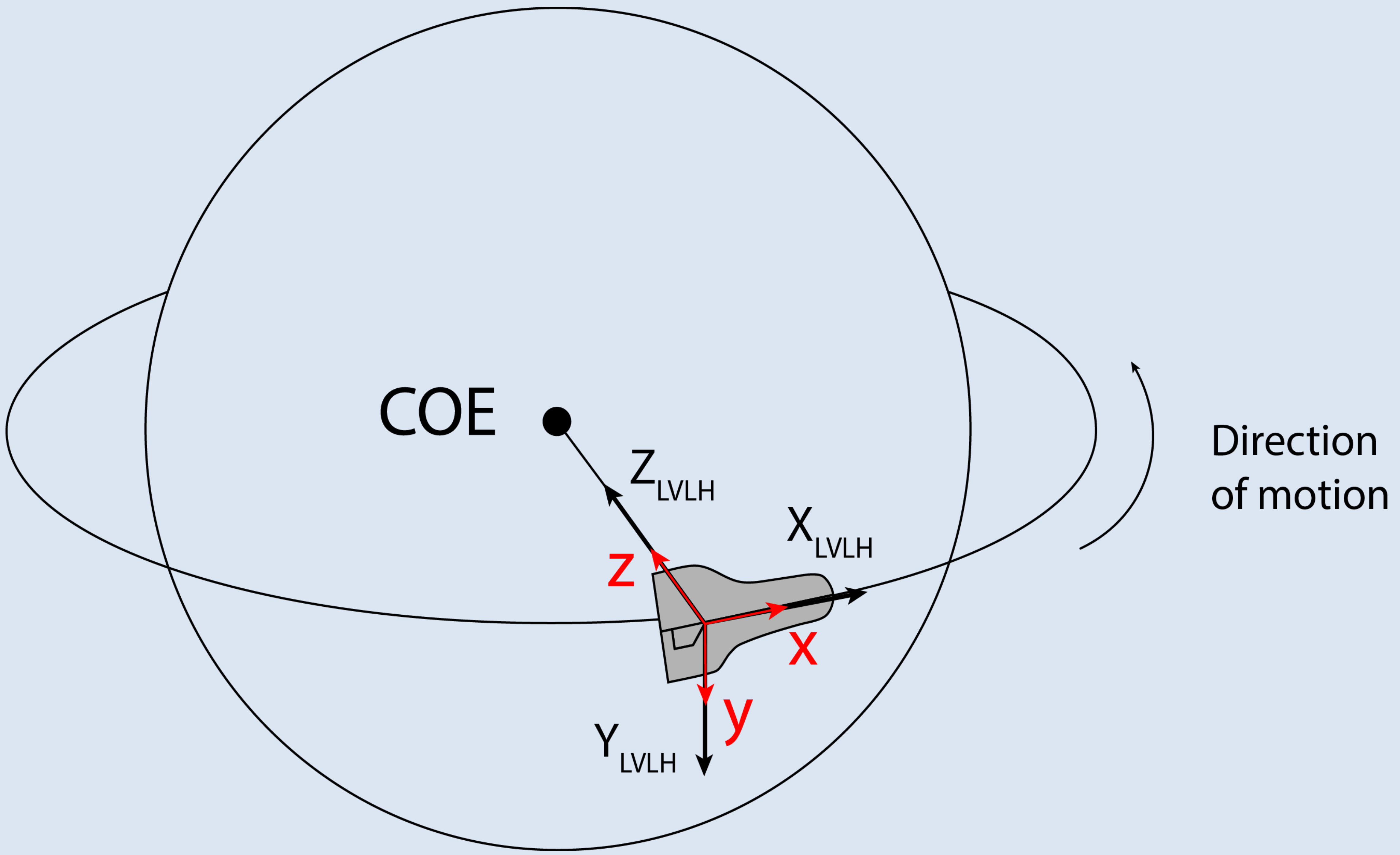
The AMCS consists in measuring and maintaining, or changing in a controlled manner, the orientation of a coordinate system attached to the spacecraft with respect to an inertial or any other reference system. The attitude is maintained or controlled within a specified deadband.

Credits: SSM2007 Commander's Reference Manual

LVLH coordinate system

Frame centered at the Center of Mass of a spacecraft orbiting the Earth, +Z to the Center of Earth (COE), +X in the direction of the velocity vector for a circular orbit

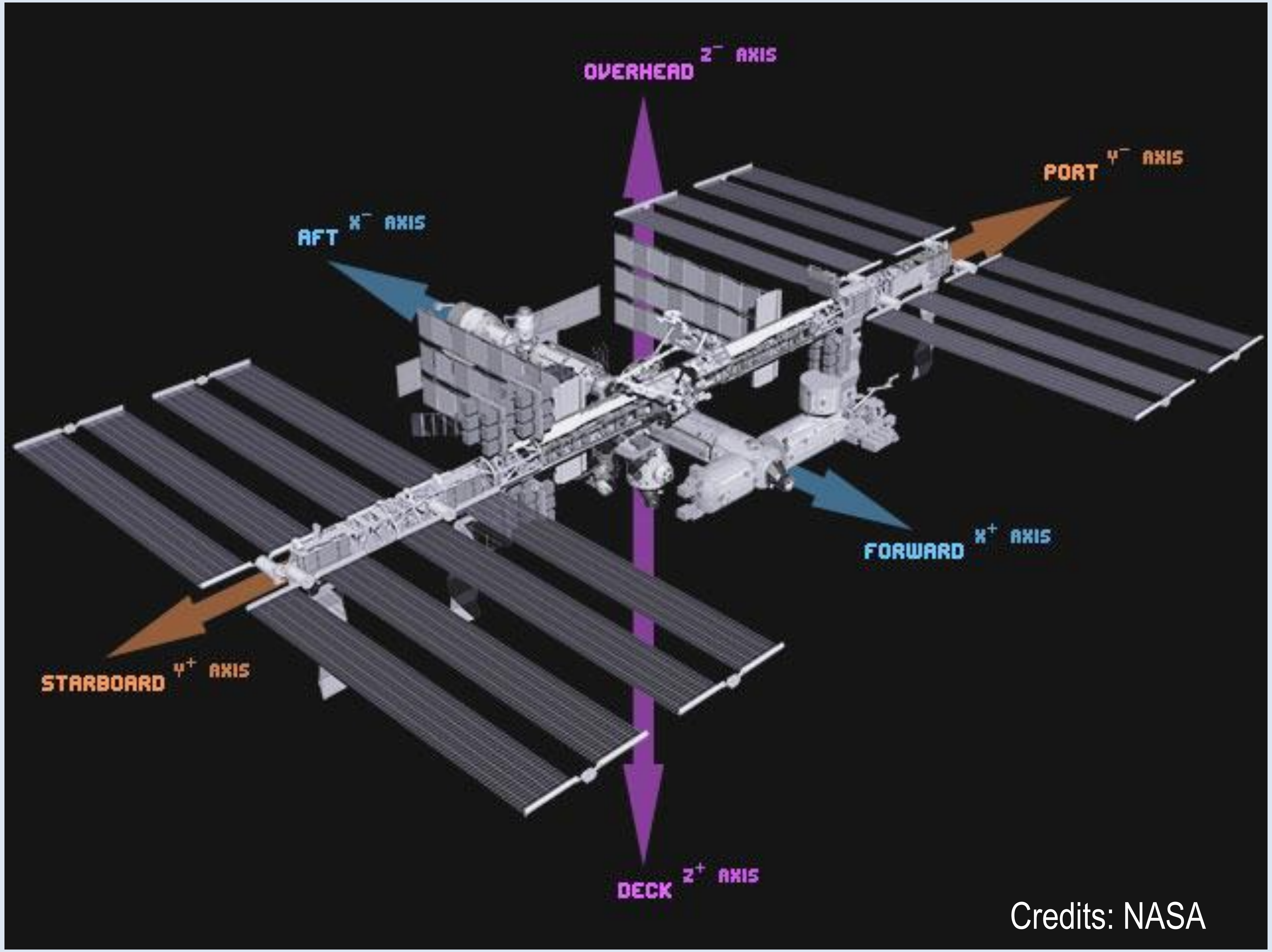
Example: Space Shuttle at (P, Y, R) = $(0, 0, 0)_{LVLH}$



ISS coordinate system

Usually the LVLH attitude of ISS is close to LVLH (0,0,0) with the X axis toward the velocity vector and Z down to the Center of the Earth.

The bias to LVLH (0,0,0) is to orient ISS close to the TEA or Torque Equilibrium Attitude. TEA is the attitude at which the gravitational torques and atmospheric torques best cancel each other along a circular orbit at around 400 km altitude.



Spacecraft attitude control methods

Passive or active?

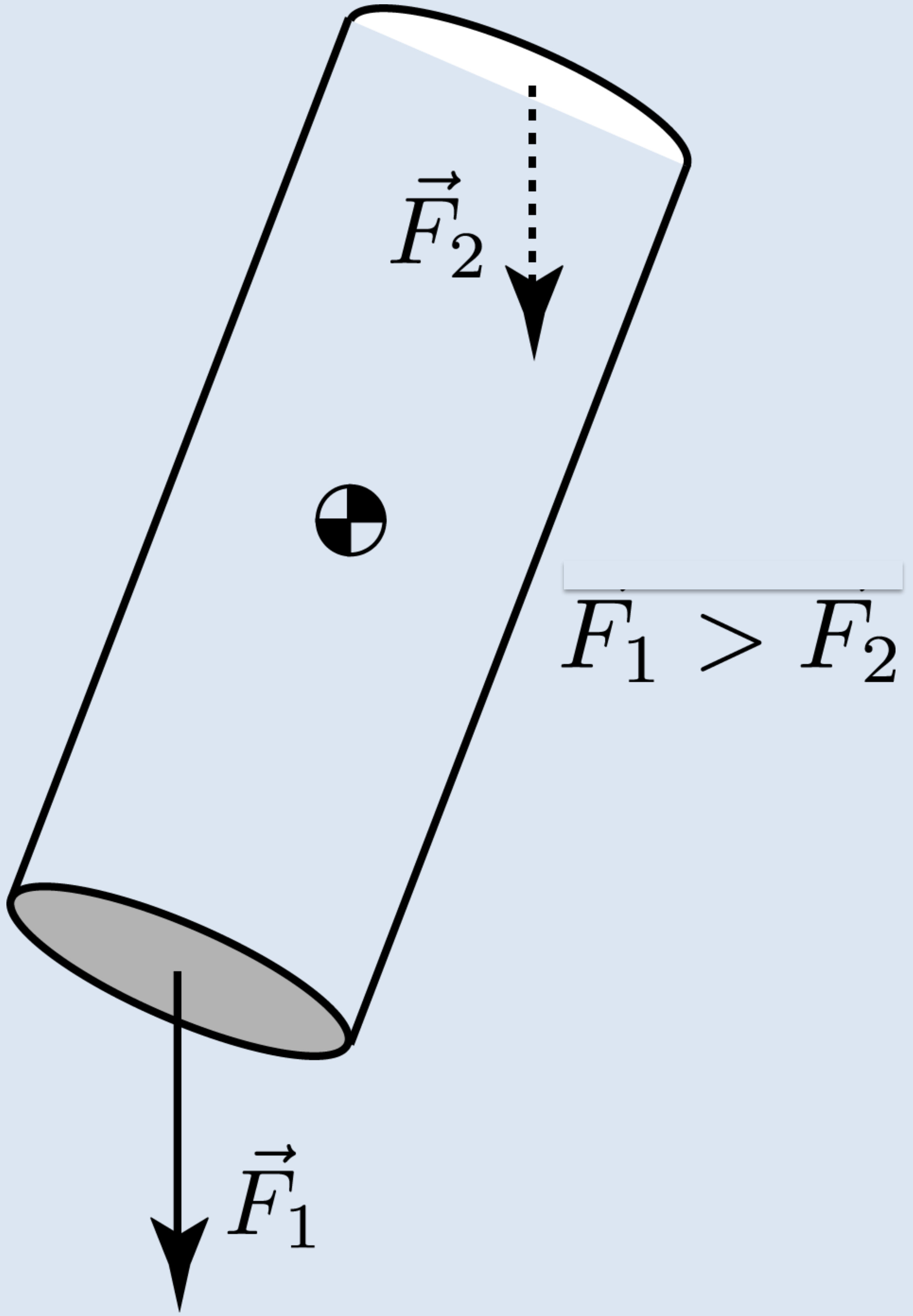
- Passive: gravity gradient
- Active: thrusters, spinning spacecraft, spinning wheels, or magnetic torquers

Magnetic torquers can only be used on orbit around a planet that has a magnetic field.

1. Gravity gradient
2. Thrusters
3. Spinning spacecraft
4. Momentum devices: Reaction Wheels or Control Moment Gyros (CMG)
5. Magnetic torquers

Gravity gradient

An elongated object in orbit around the Earth will take an orientation such that its long axis will be along the local vertical, with possible oscillations (in plane and out of plane) around the Center of Gravity (CG), normally close to the Center of Mass (CM)



Magnetic torquers

- A magnetic torquer is an elongated bar with a wire coil wrapped around it and an external protection.
- A current through the coil will produce a magnetic field which will try to align itself along the geomagnetic field (or planetary mag field) with a torque expressed by:

$$T = NBAI sin\theta$$

Example:
Magnetic Torquer VMT-35
Vectronic Aerospace GmbH

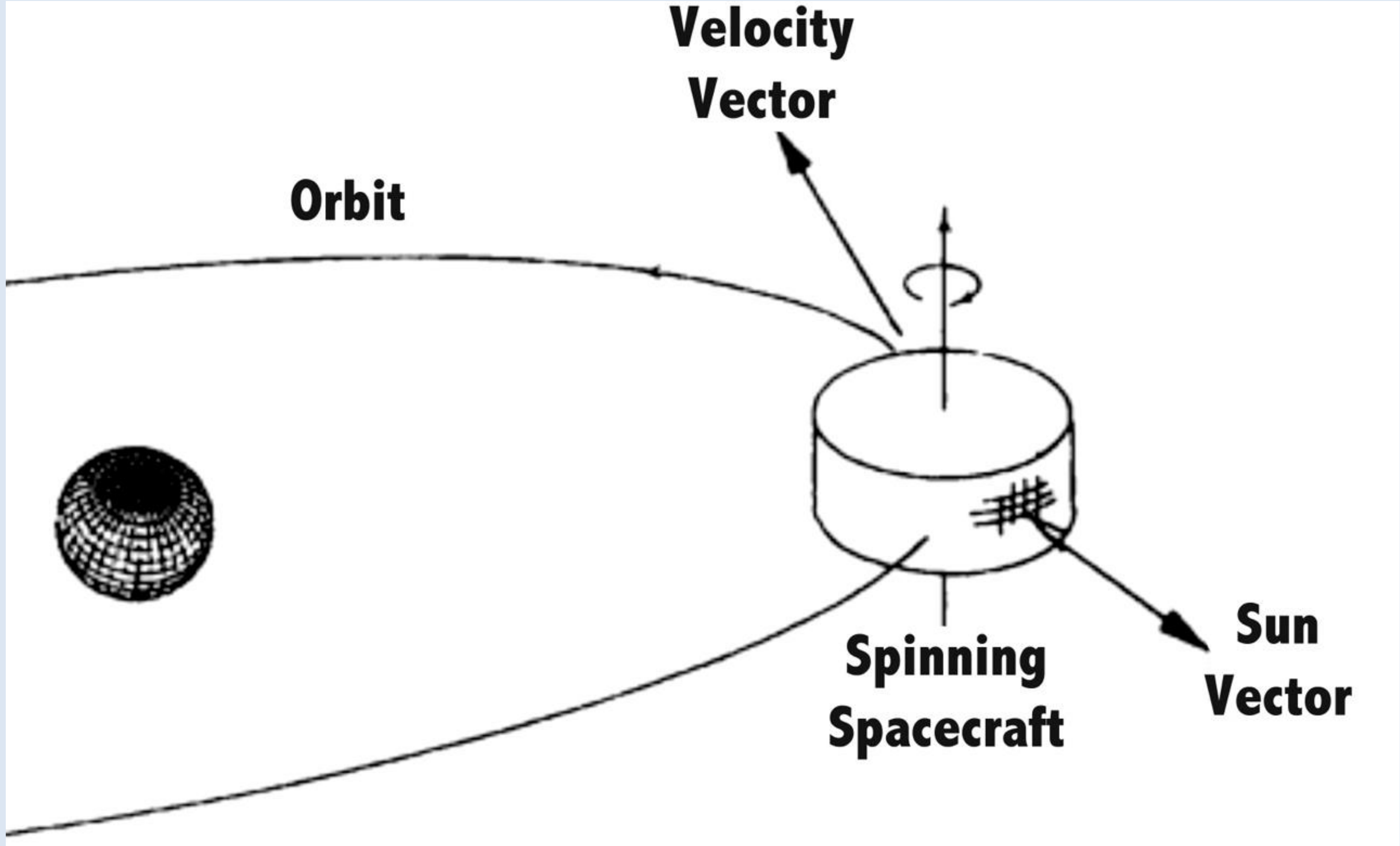
T = torque (Nm)
N = number of loops in the coil
B = Ambient magnetic field (Tesla), for Earth 3.1×10^{-5} Tesla on the surface

A = area of the coil (m^2)
I = current (A)
Θ = angle between the Earth's magnetic field and the bar

Spinning spacecraft

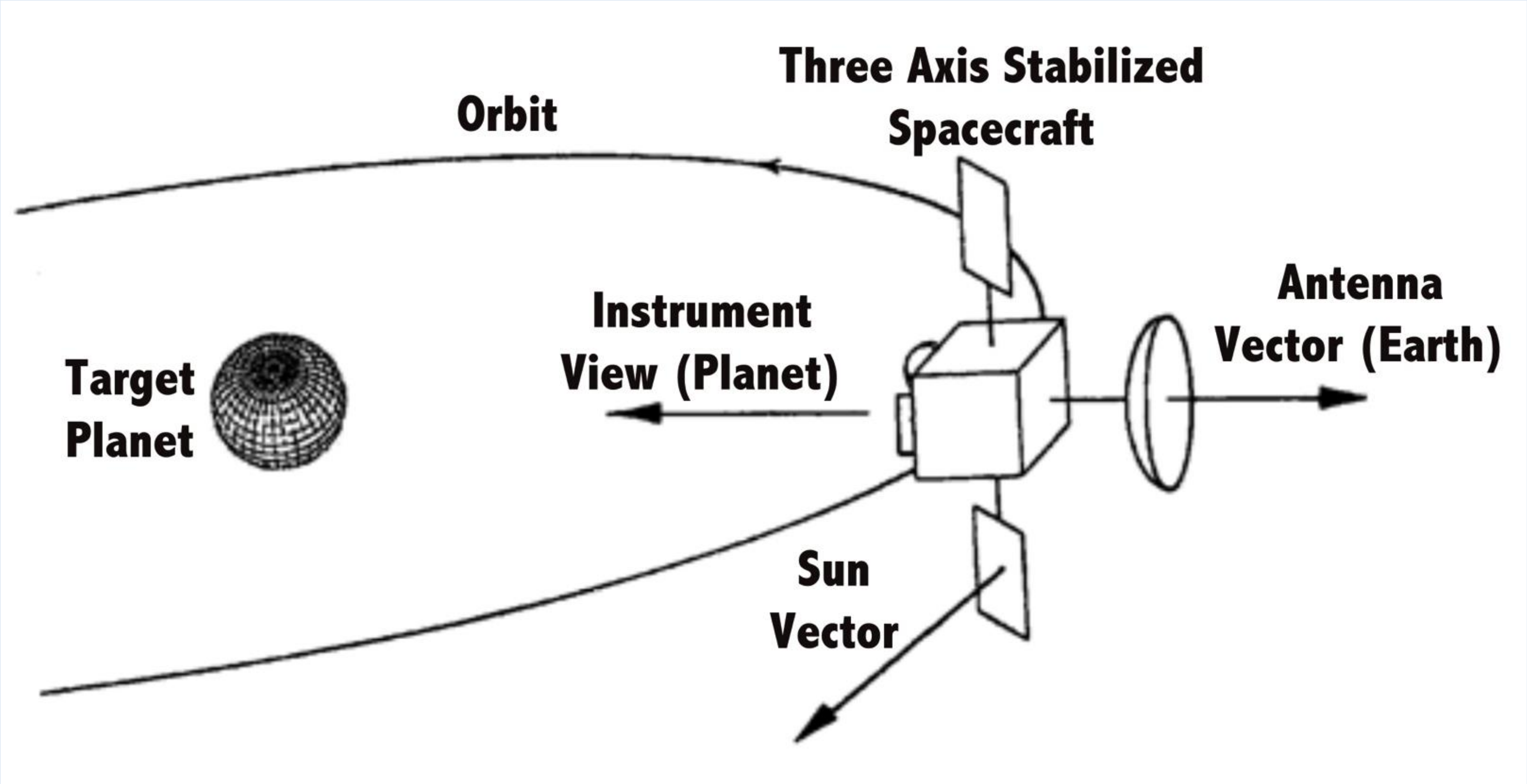
- **Advantage:** cheap, propellant flow from tanks provided by inertial forces

- **Disadvantages:** low accuracy in controlled attitude (0.3 - 1°), translations only possible along rotation axis, pointing of antennas and other devices impossible except in the direction of spin



Credits: Charles D. Brown, *Elements of Spacecraft Design*, AIAA

Three axis stabilized spacecraft

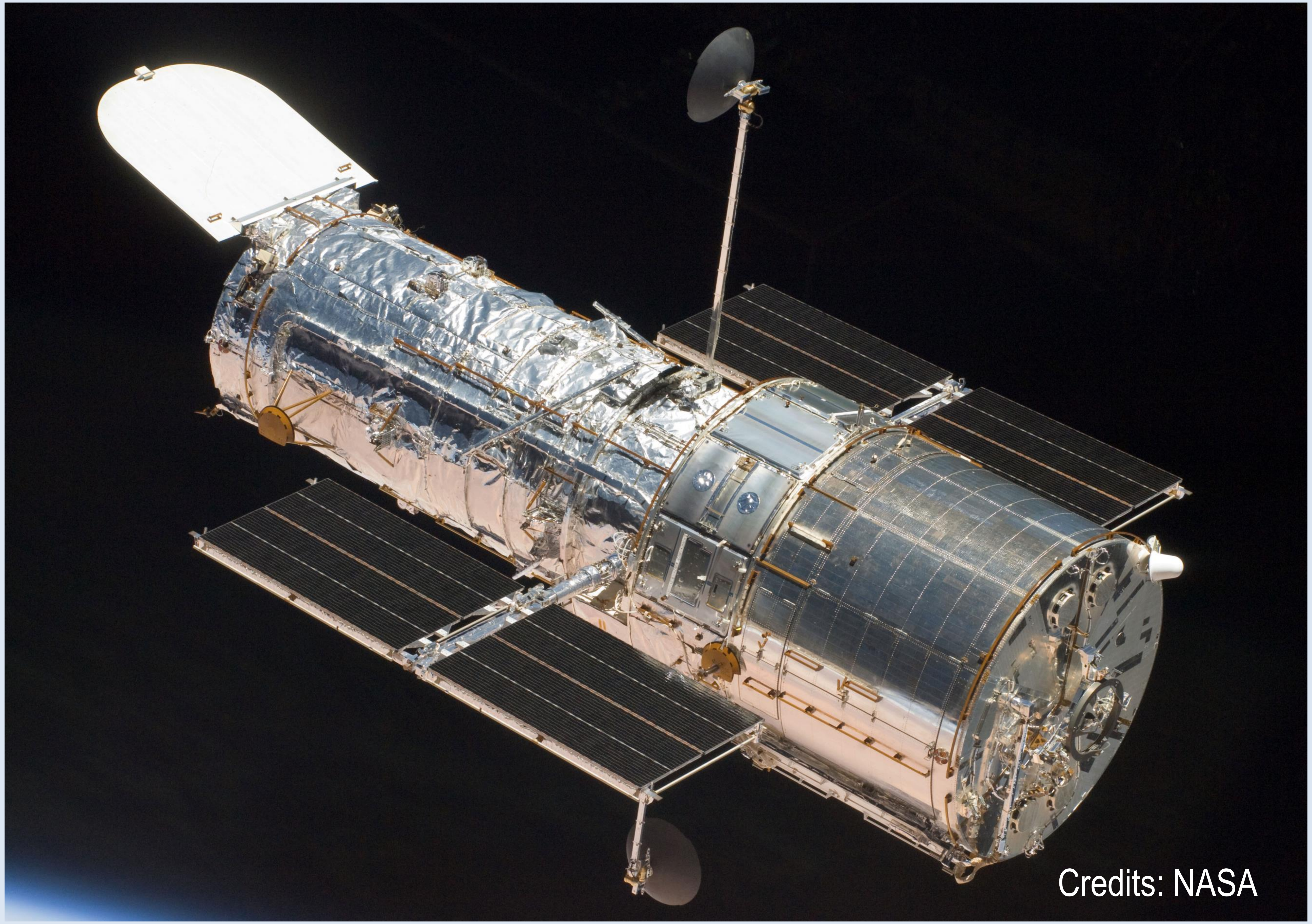


Attitude controlled by thrusters, Reaction Wheels, or Control Moment Gyros (CMGs)

Credits: Charles D. Brown, *Elements of Spacecraft Design*, AIAA

Required accuracy of attitude control systems

- Better than 10° for solar arrays
- 0.1-0.5° for high gain antennas
- 0.0001 to 0.1° for optical systems
- Hubble pointing accuracy 0.007 arcsec!



Main techniques for three axis stabilization

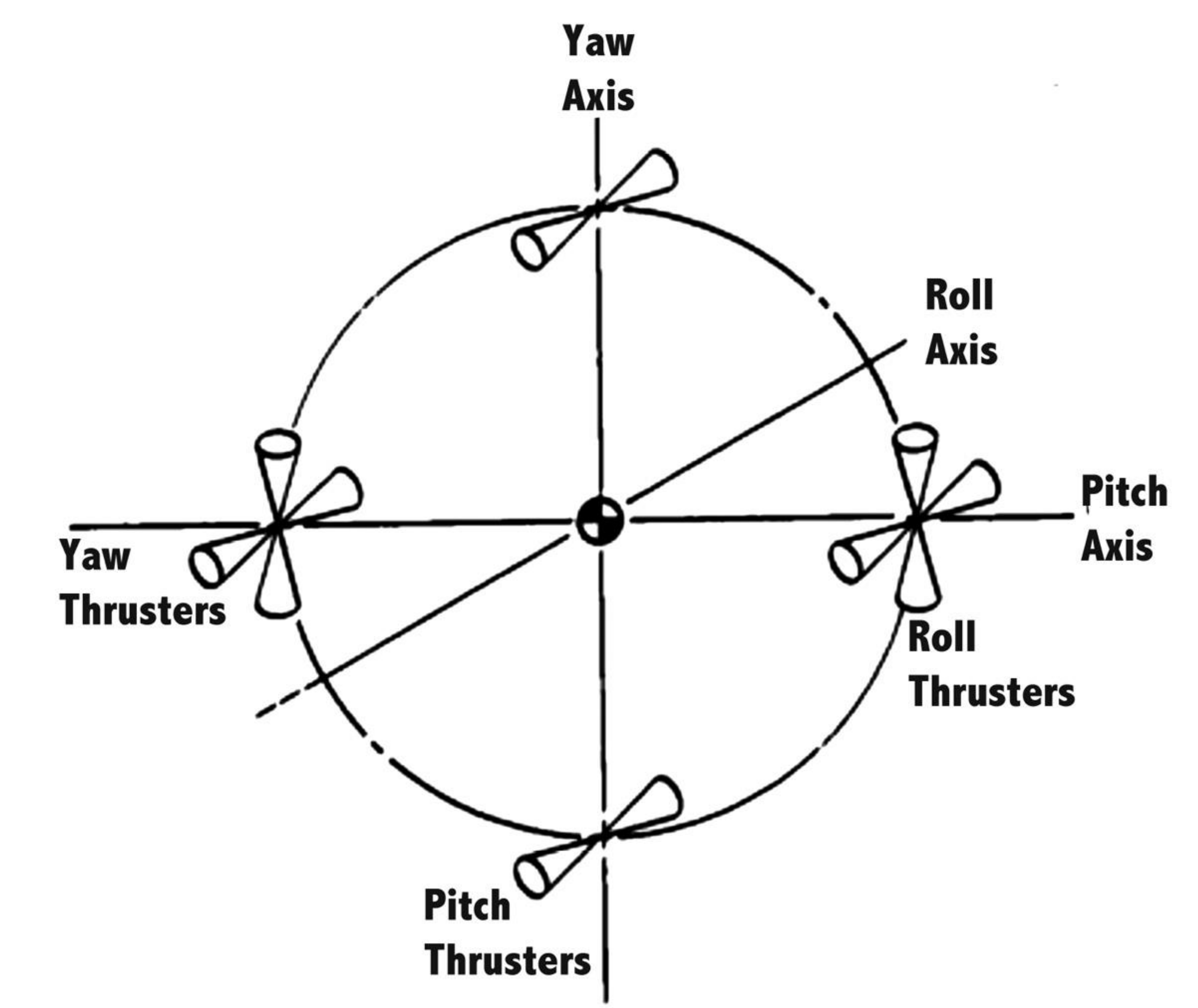
- **Thrusters:** Examples are the Space Shuttle, Soyuz capsule, Crew Dragon, ISS Russian Orbital Segment (ROS)

- **Reaction (or Momentum) Wheels:** Torques on the spacecraft induced by variations of the wheel's rotational speed. Requires thrusters or magnetic torquers to get out of wheels saturation. Hubble is an example.

- **Control Moment Gyros or CMG's:** Constant angular velocity. Mounted on gimbals. A torque generated along the input axis produces a corresponding torque reaction along the output axis. ISS in the US Orbital Segment (USOS) is an example.

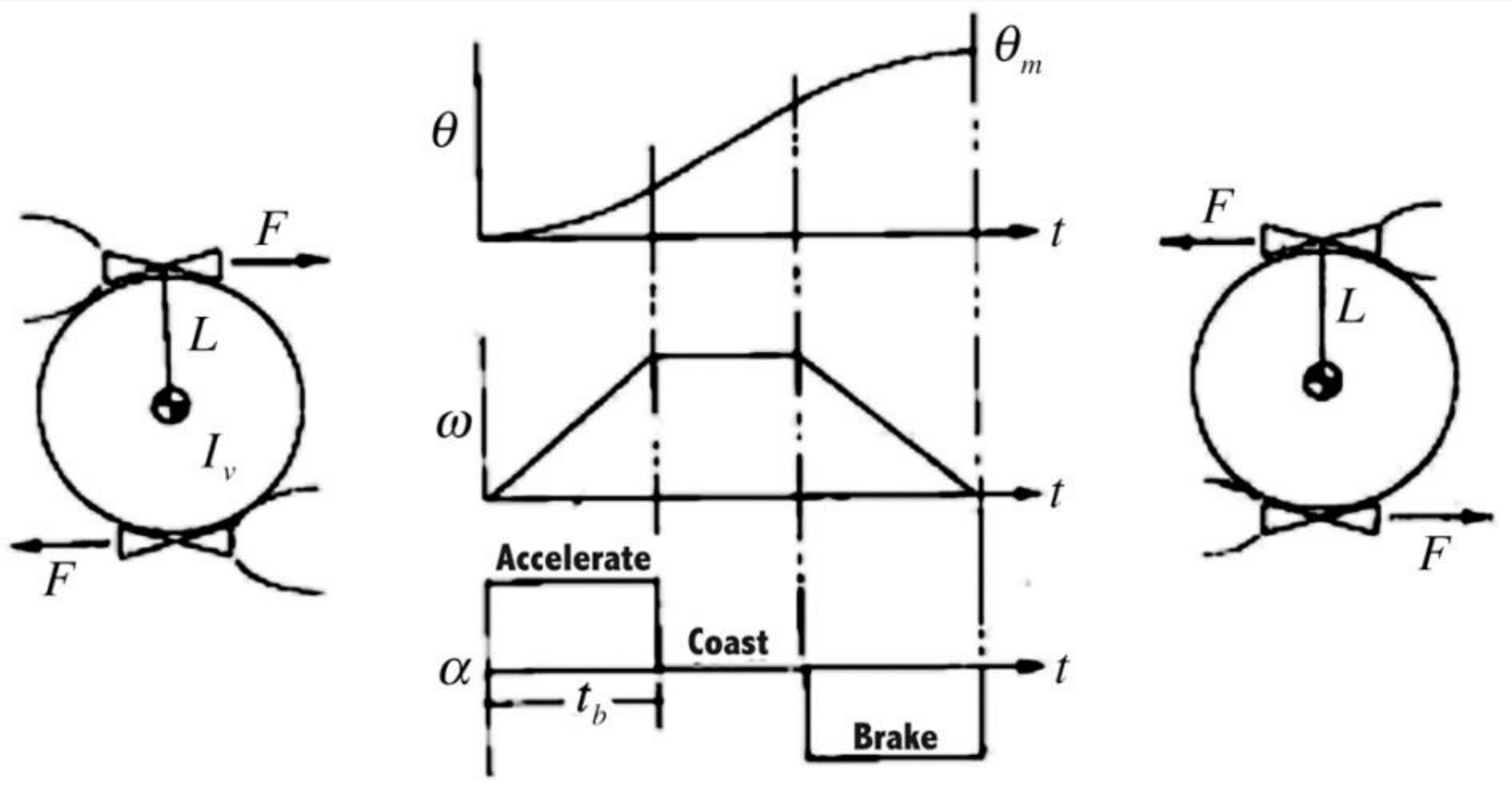
Geometry of 3 axis attitude control thrusters (min. 12)

Minimum 12 thrusters are needed for pure rotation maneuvers (without induced translation) - 4 on each axis: 2 thrusters for acceleration pulse to initiate the rotation and 2 opposite thrusters for the braking pulse



Credits: Charles D. Brown, *Elements of Spacecraft Design*, AIAA

Attitude maneuver around one axis using thrusters



- F = thrust for each thruster (N)
- T = torque (Nm)
- n = number of thrusters used to initiate or to stop the maneuver
- L = perpendicular distance from the center of mass to the thrust vector (m)
- θ = angle of rotation (rad)
- ω = angular velocity (rad/s)
- α = angular acceleration (rad/s^2)
- I_v = moment of inertia of the vehicle around the rotational axis (kg·m^2)
- t_b = duration of the burn (s)

During the initial rise of angular velocity, we have:

$$\alpha = \frac{T}{I_v} \qquad \omega = \alpha \times t \qquad \theta = \frac{1}{2}\alpha \times t^2$$

$$\omega_{\max} = \alpha \times t_b$$

Credits: Charles D. Brown, *Elements of Spacecraft Design*, AIAA

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$$\alpha = \frac{nFL}{I_v} \qquad \omega_{\max} = \frac{nFL}{I_v} t_b$$

$$\theta(t) = \frac{nFL \cdot t^2}{2I_v}$$

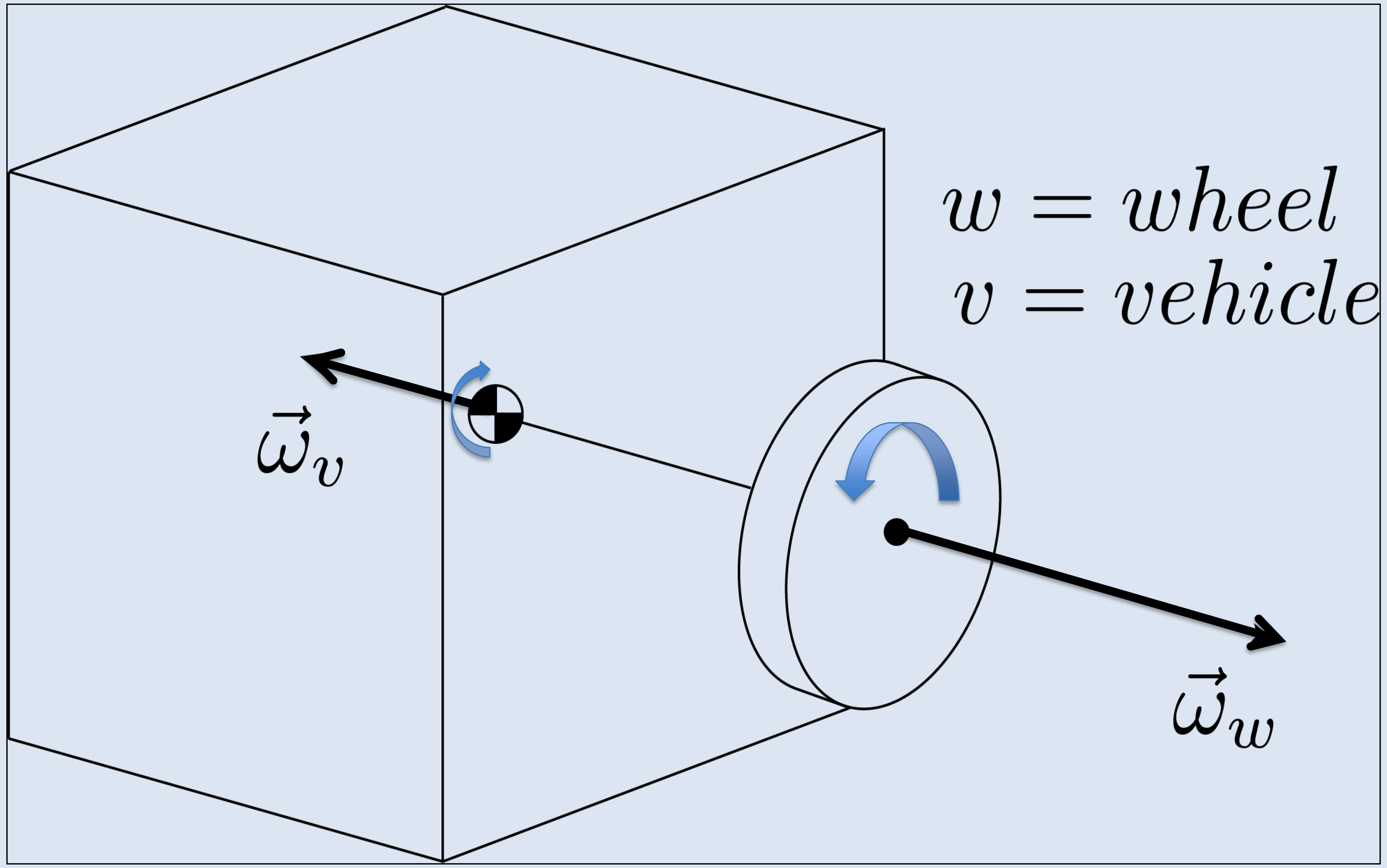
$$\theta_m = \frac{nFL}{I_v} t_b^2 + \frac{nFL}{I_v} t_b \times t_c$$

Determination of the propellant used in the maneuver:

$$I_{sp} = \frac{nF(kg)}{\dot{m}_p} \rightarrow m_p = 2t_b \times \dot{m}_p = \frac{2nF \times t_b}{g \times I_{sp}}$$

Reaction Wheel

Principle: If the angular rotation speed of the reaction wheel is increased, the angular rotation speed of the spacecraft will increase in the opposite direction.

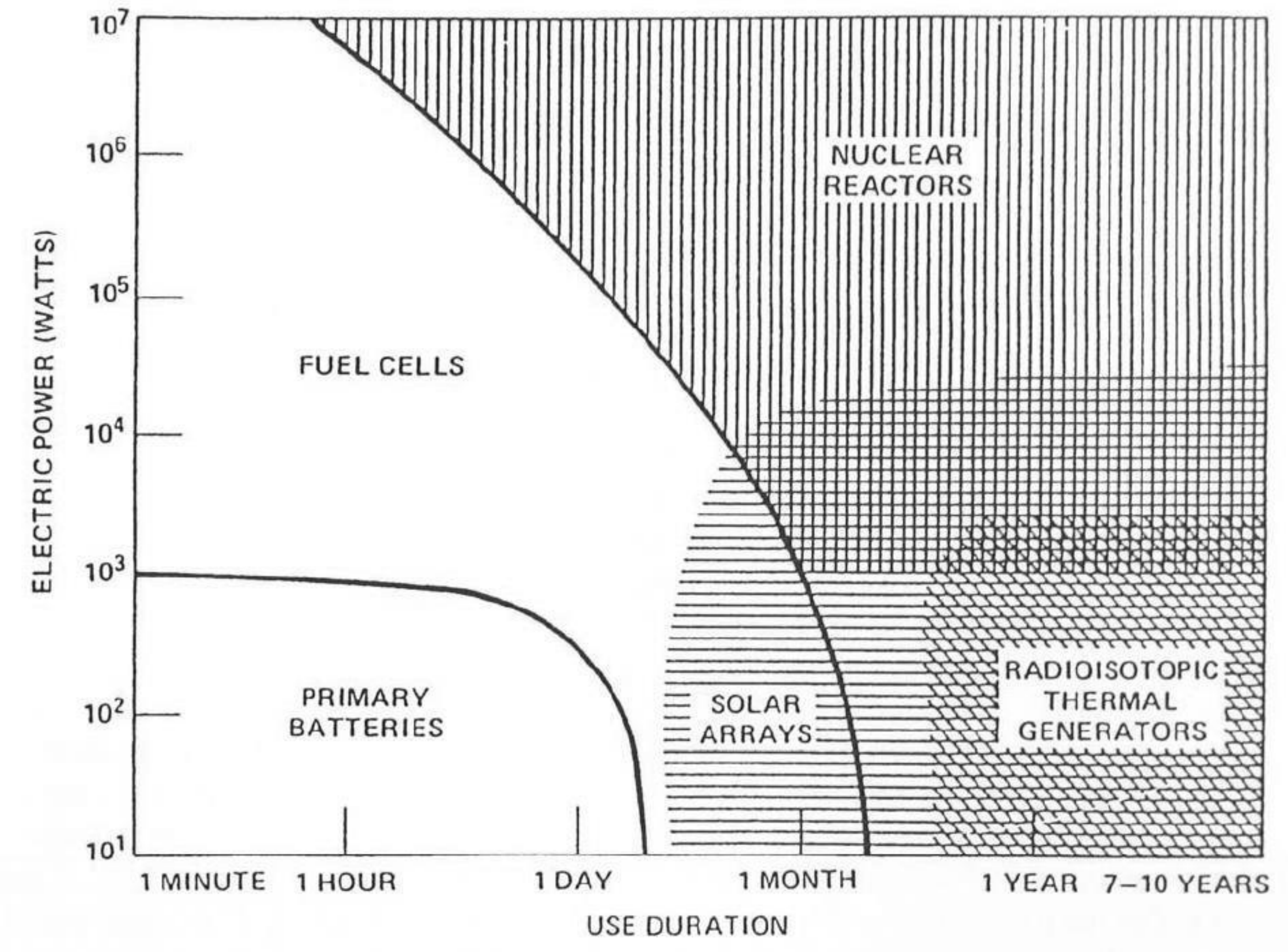


Angular drift of the spacecraft, around the reaction wheel axis, as a consequence of a maneuver of total time t_m duration:

$$\Delta\theta_v = \frac{\alpha_w I_w t_m^2}{4 I_v}$$

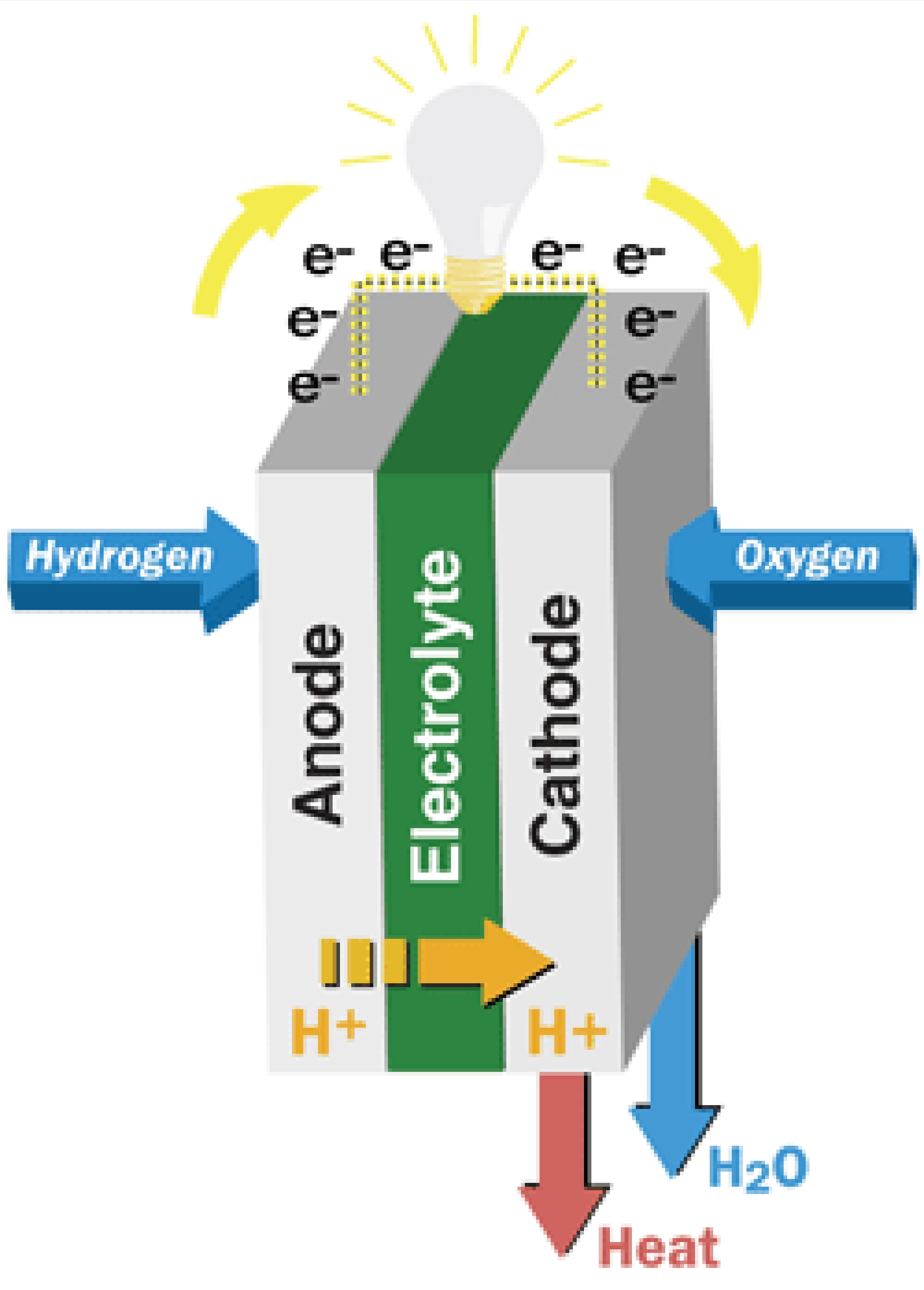
t_m = total maneuver time on a triangular profile of ω_w rising linarly until time $t_m/2$, then decreasing linearly until time t_m

Operating regimes of spacecraft power sources



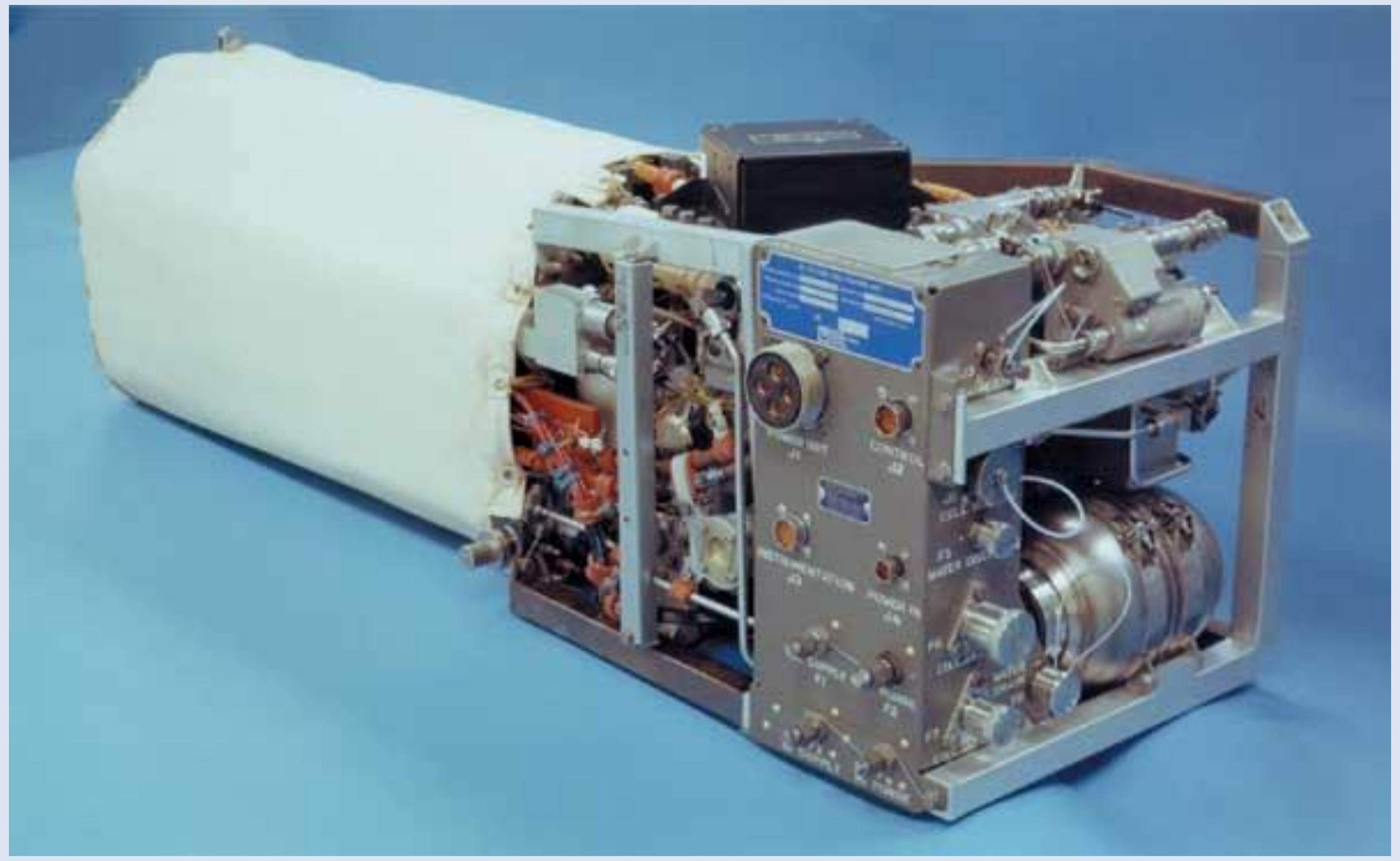
Credits: Charles D. Brown, *Elements of Spacecraft Design*, AIAA

Fuel Cells (FC)



Fuel cells convert chemical energy from reactants into electricity through a chemical reaction of positively charged hydrogen ions with oxygen (or other oxidizing agent). They require a continuous source of reactants to sustain the chemical reaction. The by-products of this reaction are *water* and *heat*.

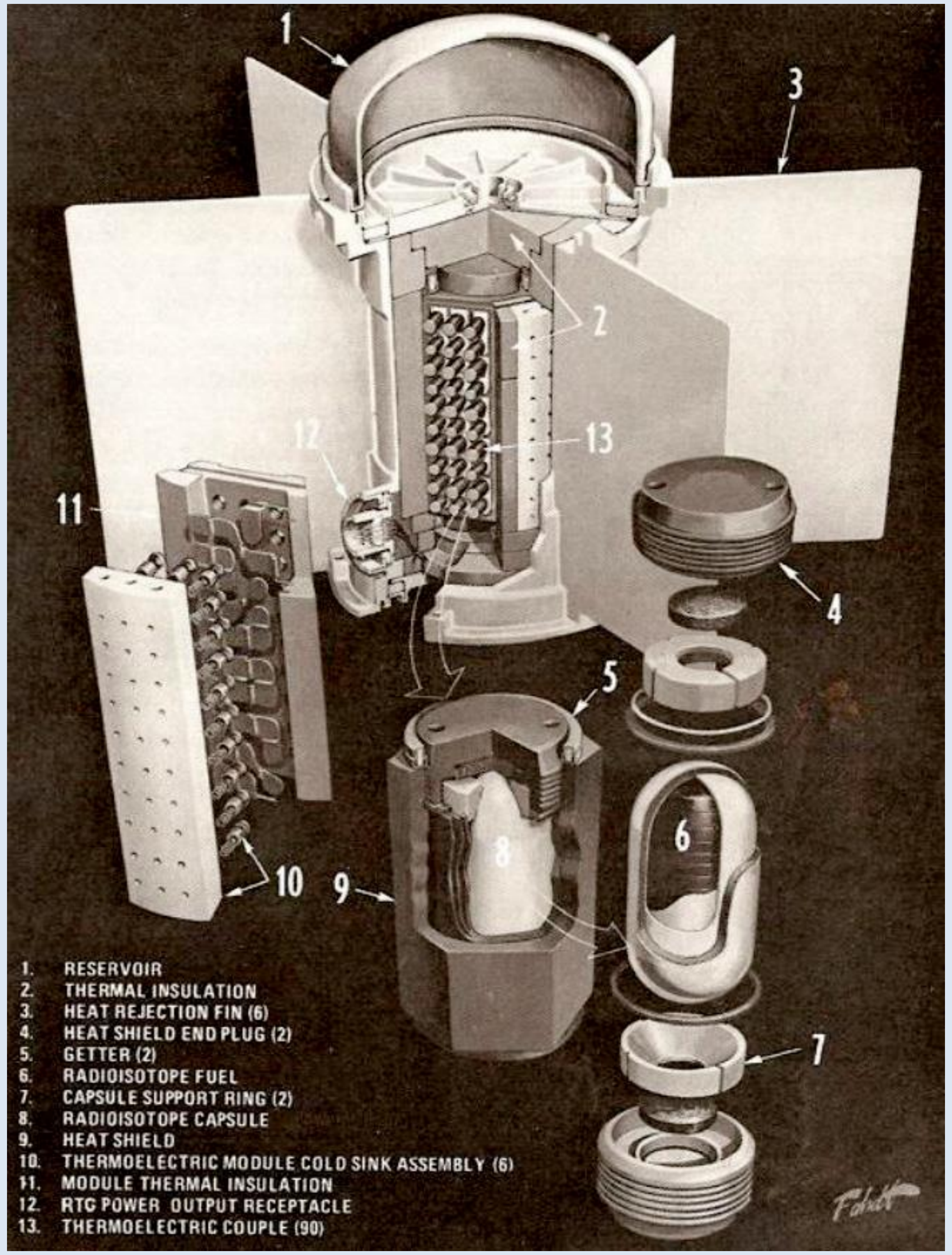
Fuel cells have been successfully used in the Gemini, Apollo and Shuttle programs. The three Shuttle fuel cells generated each around 7 kW.



One of three Shuttle Fuel Cell

Credits: 1. Wikipedia, 2. NASA

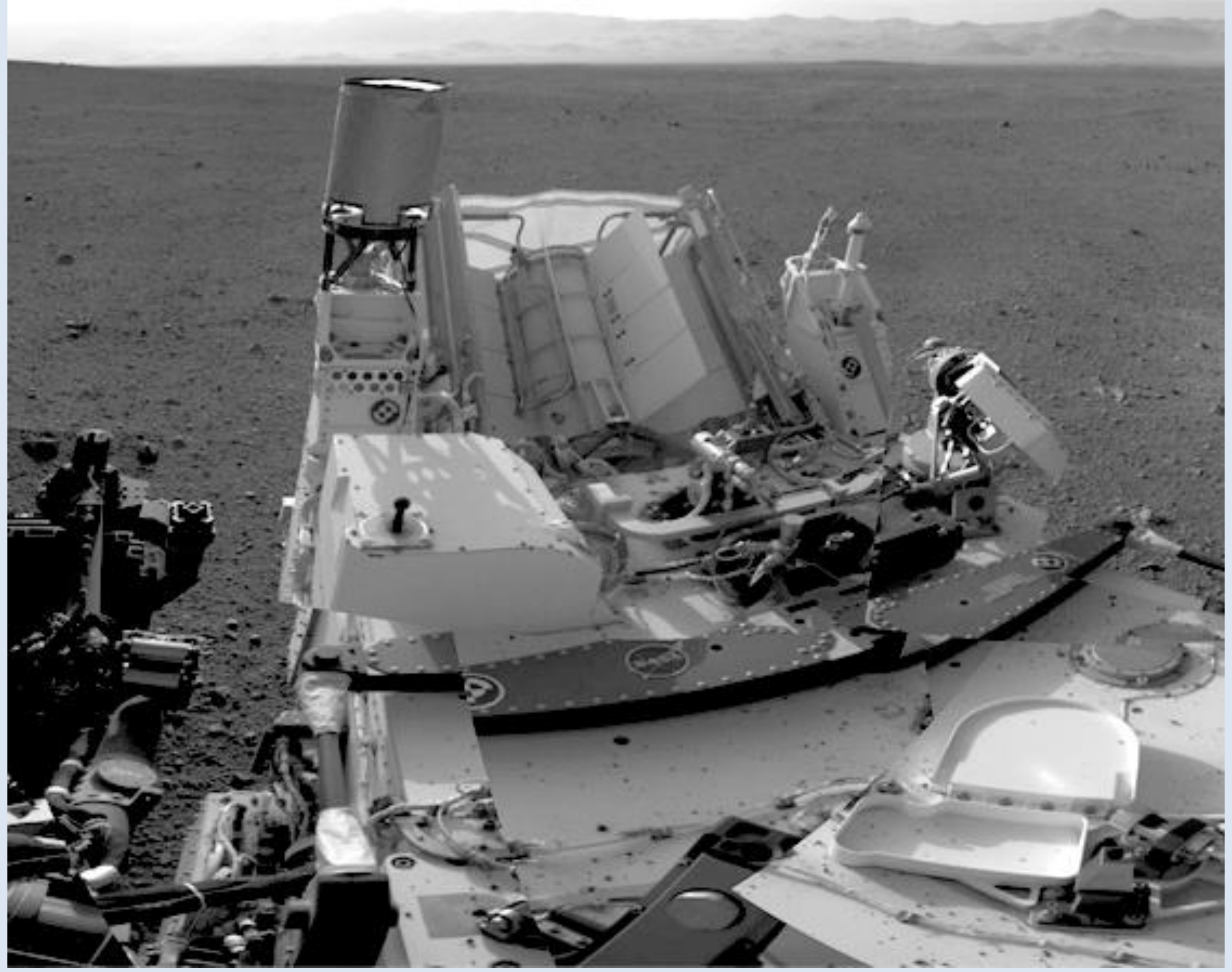
Radioisotope Thermoelectric Generators (RTG)



Viking spacecraft RTG

A Radioisotope Thermoelectric Generator (RTG), uses the fact that radioactive materials (such as plutonium 238) generate heat as they decay into non-radioactive materials. The heat is converted into electricity by an array of *thermocouples*.

RTGs are very reliable and long lasting, but of low efficiency (less than 10%). Used in the Apollo ALSEP program, Viking, Pioneer 10 and 11, Voyager 1 and 2, Cassini, New Horizons, Curiosity.



RTG installed in the Curiosity rover on the surface of Mars

Credits: NASA

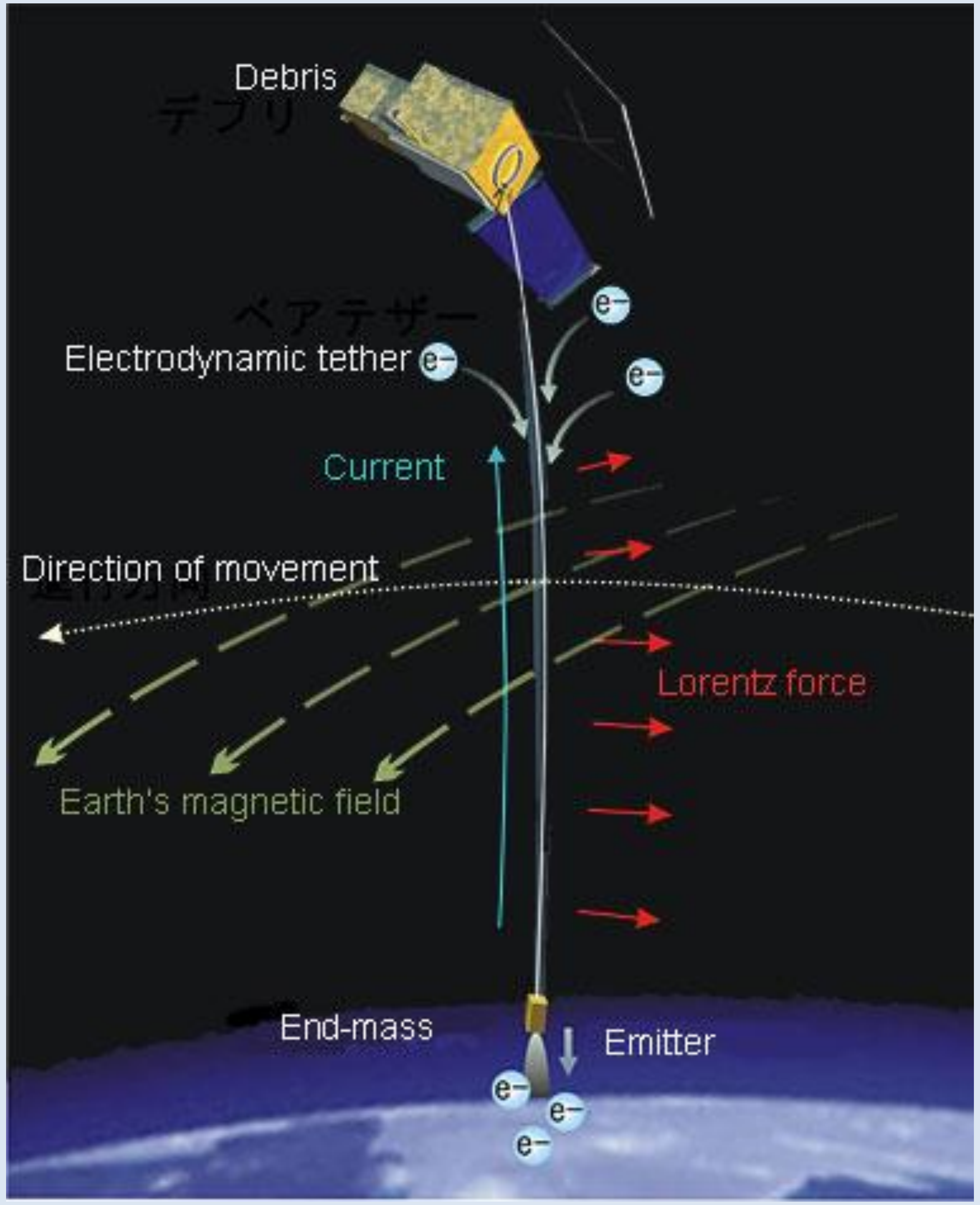
Tether in space, concept and applications

A space tether is a long cable which is used to couple spacecraft to each other or to other objects in space, like an asteroid or a spent rocket upper stage.

Tethers are usually made of a strong material like high-strength fibers or Kevlar, with or without an electrically conducting material in the core.

Space tethers have several useful applications listed here.

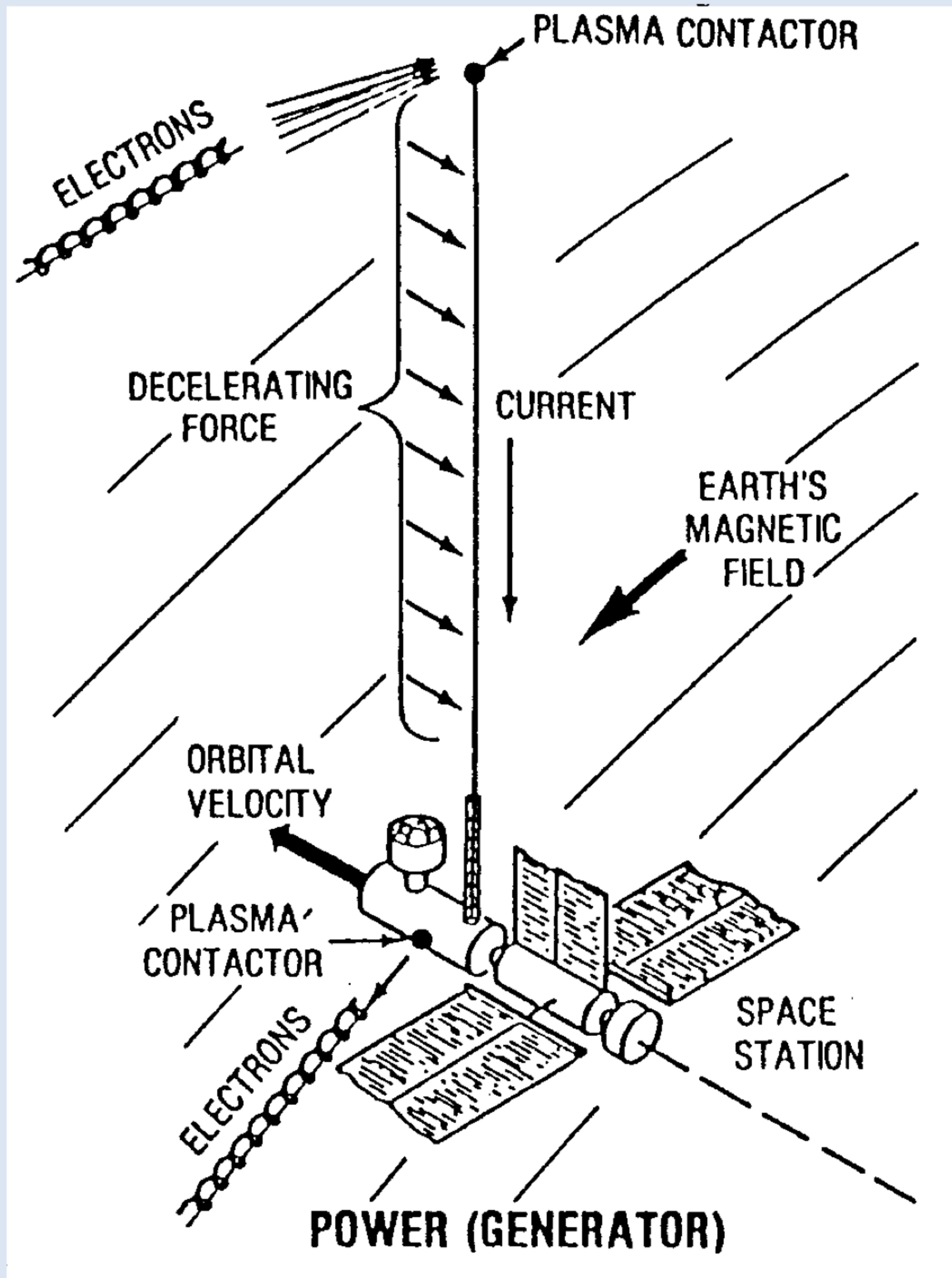
- Applications of electrodynamic tethers
  - Electrical power generation
  - Orbit transfers
  - Ionospheric studies
- Applications of non-electrodynamic tethers
  - Angular momentum transfer
  - Space debris removal
  - Provision of artificial gravity for long journeys in the Solar System
  - Space Elevator

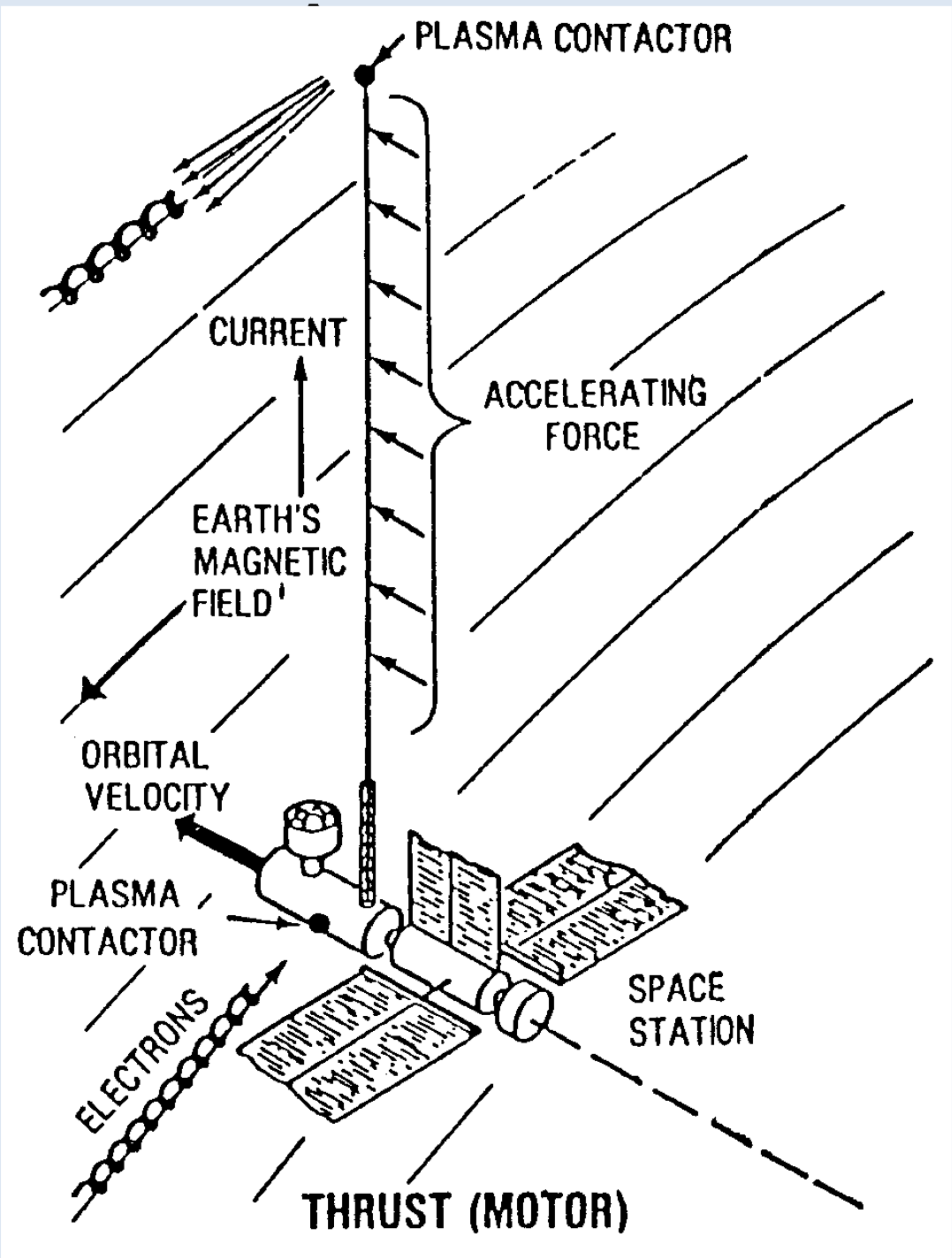


Credits: JAXA

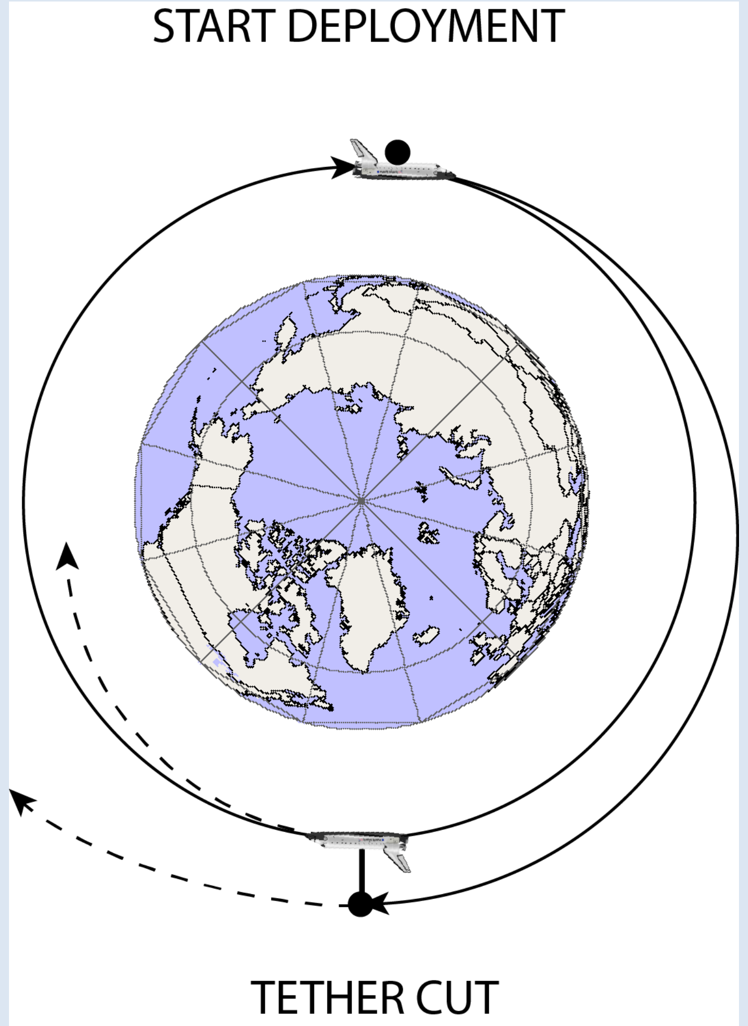
Summary of the electrodynamic applications of a tether

$$U_i = (\vec{V} \times \vec{B}) \cdot \vec{L} \quad \text{(Faraday)} \qquad \vec{F} = \int (I d\vec{L}) \times \vec{B} = I \int d\vec{L} \times \vec{B} \quad \text{(Lorentz)}$$





Credits: NASA, MSFC



5.3.3 Application of a non-electrodynamic tether

Space Mission Design and Operations

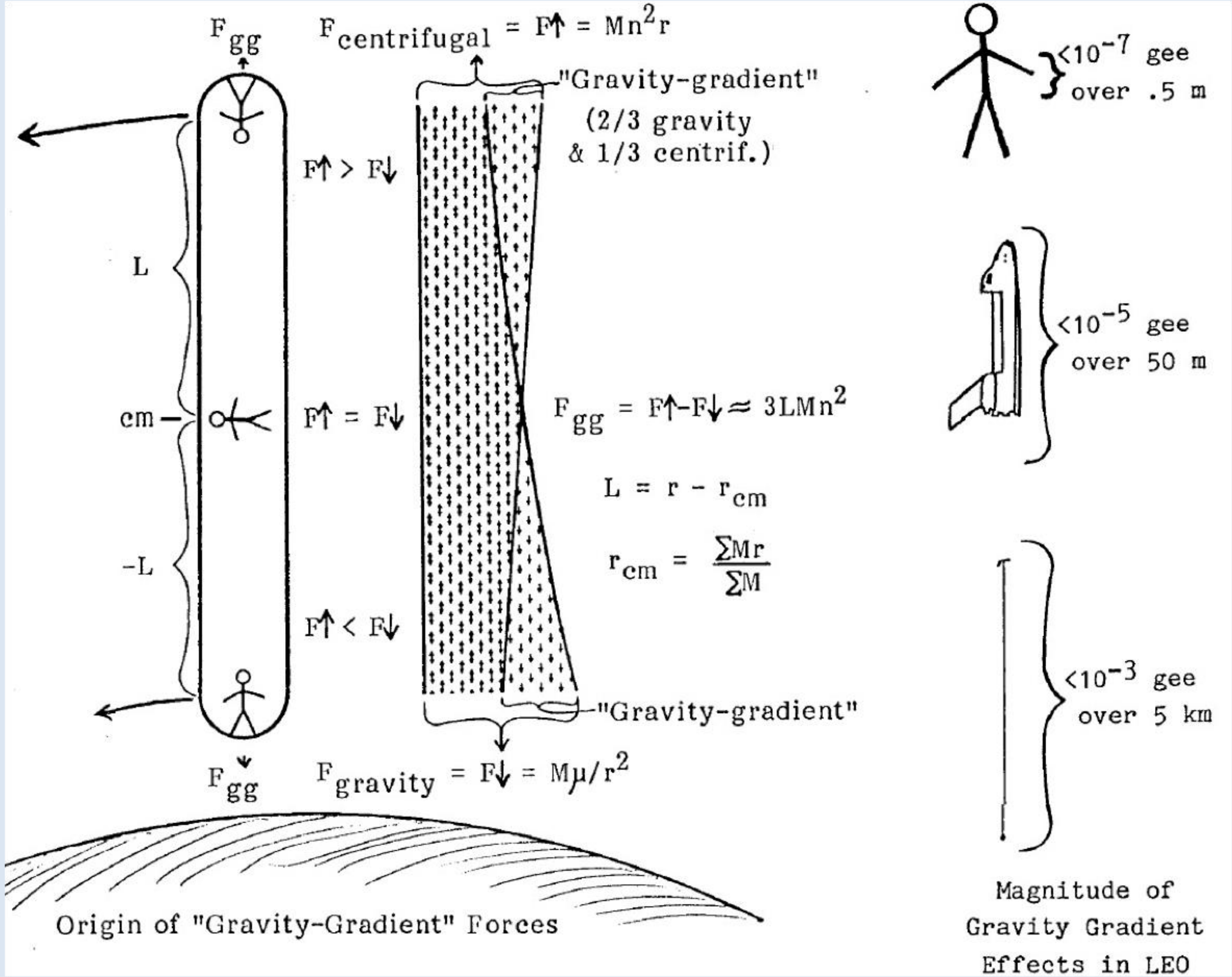
Prof. Claude Nicollier

Gravity gradient effects

Forces inside a large orbiting cylinder oriented along the local vertical, without oscillations

M = element of mass in the cylinder

n = mean motion in Rad/sec



Credits: NASA, MSFC

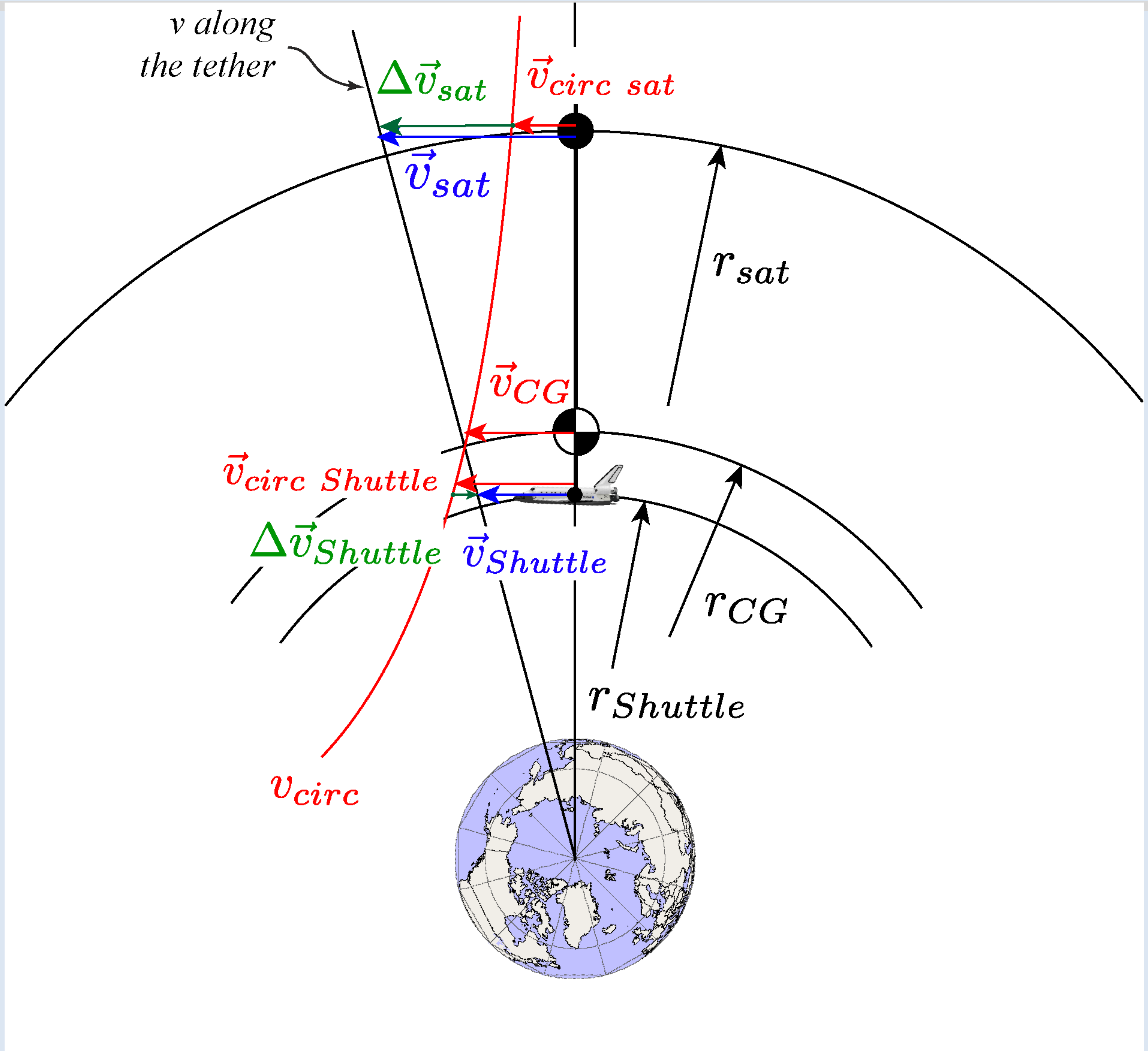
Space tether – velocity profile

As the space tether remains oriented along the local vertical, all velocities along the tether are proportional to the distance to the center of the Earth

As the circular velocity along the distance covered by the tether is

$$V_{circ} = \sqrt{\frac{\mu}{r}}$$

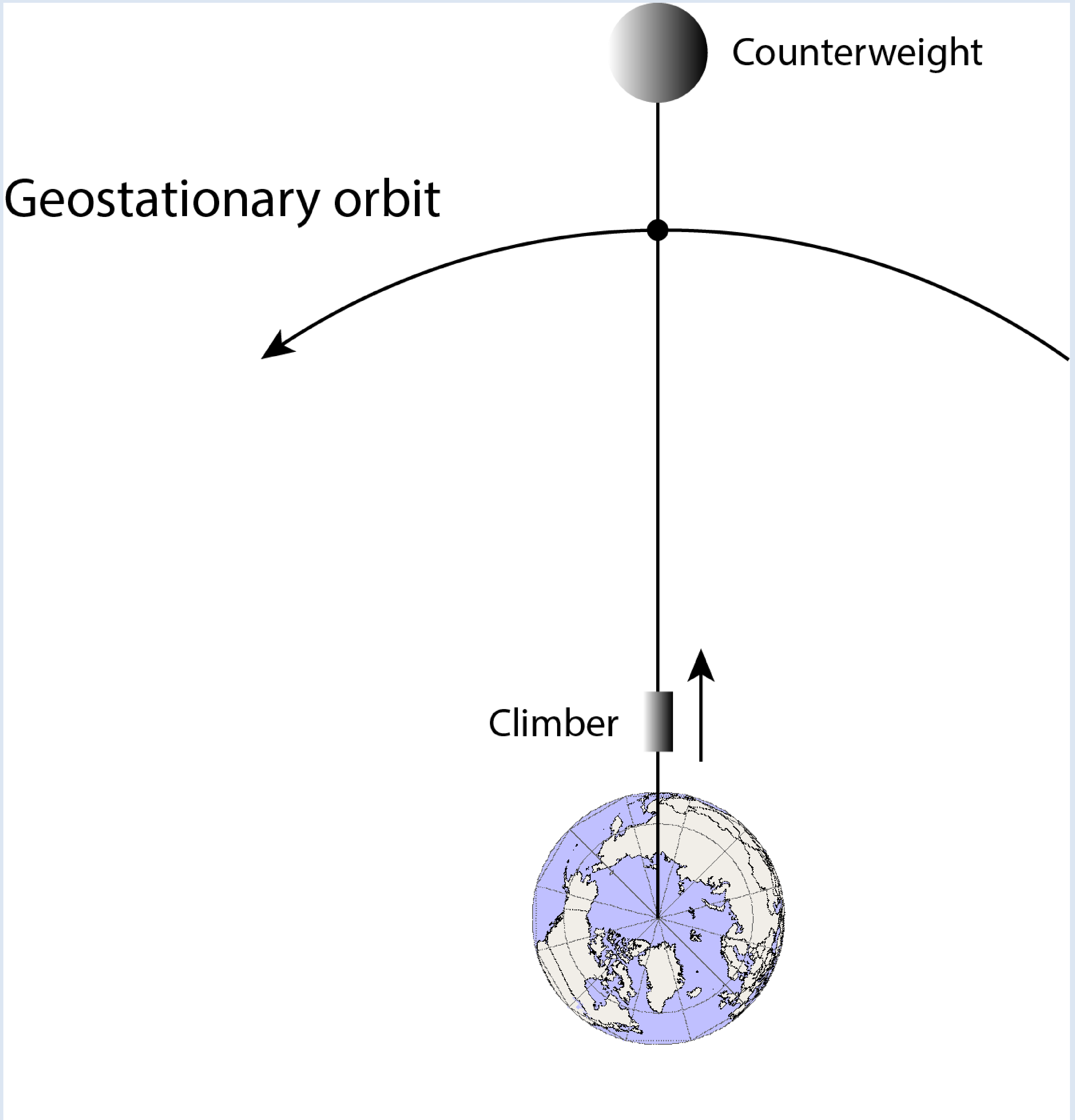
We see that in the upper portion of the tether, any part of this tether is forced to move faster, in the orbital direction, than a free satellite at the same altitude. The reverse is true for the low portions of the tether, where the tether is forced to move slower than would a free satellite at the same altitude



Space elevator concept

Originally proposed by Tsiolkovsky, a space elevator consists in a cable anchored at a location on the equator, and longer than the geostationary distance, with a counterweight at the end, and a climber able to move upwards and downwards along this cable

It would allow access to nearby space without using a rocket!



Reliability and redundancy – Definitions

- Reliability R(t) = probability that the system will not fail in the interval (0, t).
- MTTF = Mean Time To Failure, average time duration until first failure.
- MTBF = Mean Time Between Failures, average time duration between two consecutive failures.
- Failure rate $\lambda(t) = MTBF^{-1}$, in $hours^{-1}$ or $months^{-1}$.
- Probability of failure $\lambda(t)dt$ = probability that the system will fail between t and t+dt, knowing that it still works at time t.

Reliability and redundancy – Equations

- If λ is constant and expressed in hours^{-1} or months^{-1}.

$$R(t+dt) = R(t)\left[1 - \lambda(t)\,dt\right]$$

$$\frac{\left[R(t+dt) - R(t)\right]}{dt} = -R(t)\,\lambda(t)$$

$$R(t) = \exp(-\lambda t)$$

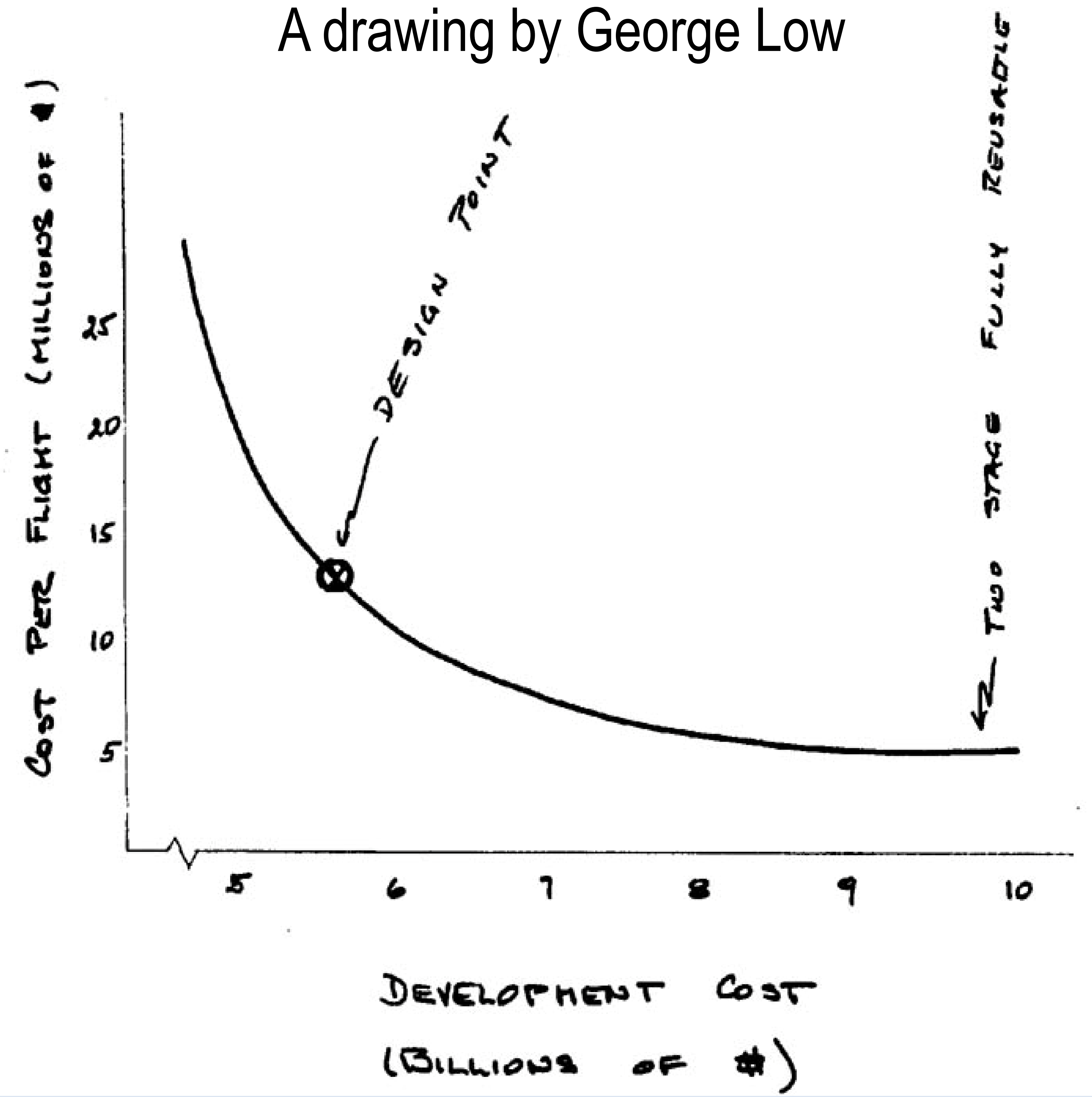
- For $\lambda t < 0.1$, the approximation $\exp(-\lambda t) = 1 - \lambda t$ can be used.

Costs trade-off

A fully reusable spacecraft would have induced quite high development cost for a relatively low operational cost.

Due to budget limitations, NASA and the US government had to choose a configuration leading to a lower development costs but a higher cost per flights as the chosen design was only partially reusable.

A drawing by George Low



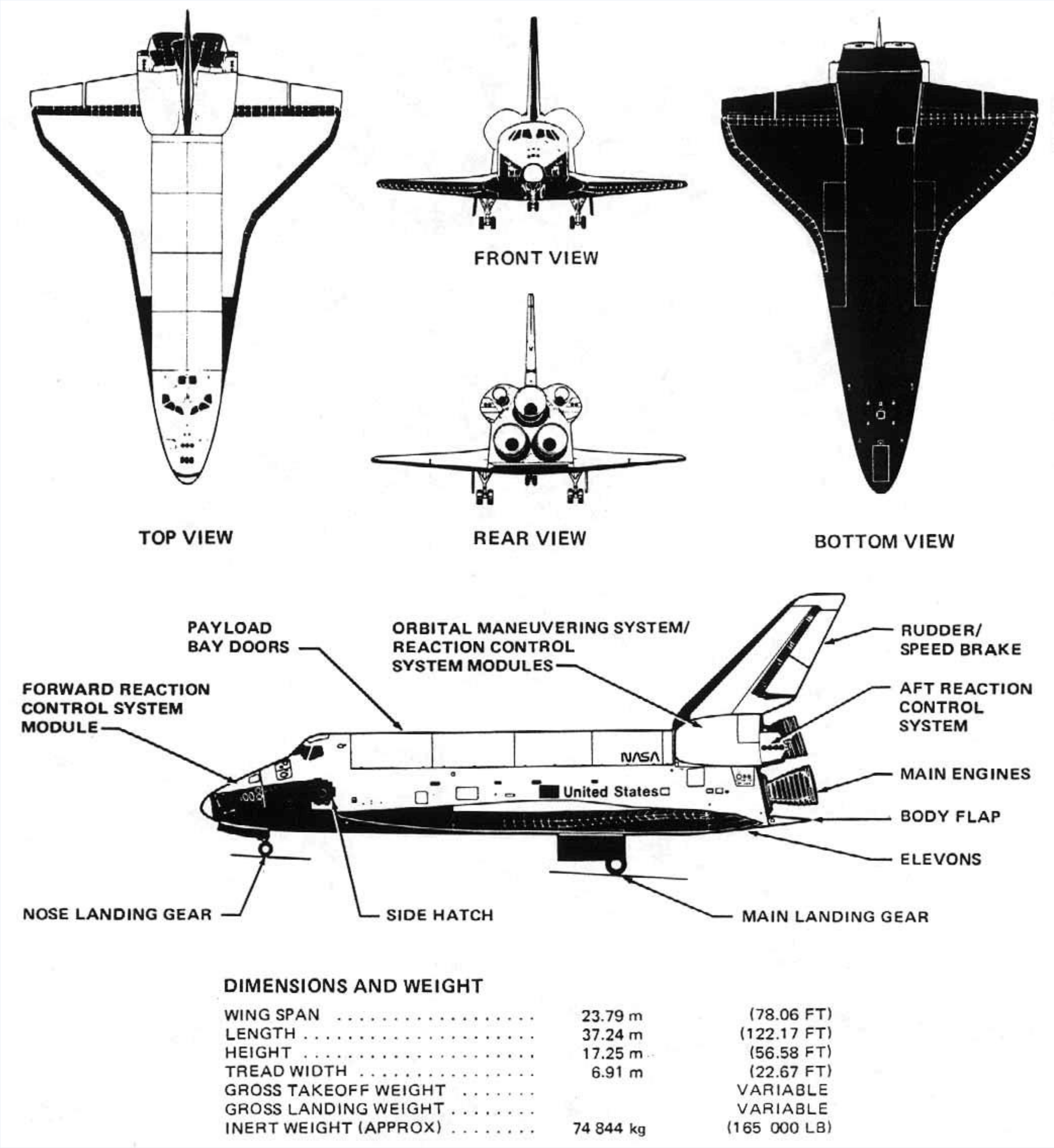
George Low was NASA Deputy Administrator from December 1969 to the end of 1976. As such, he became one of the leading figures in the early development of the Space Shuttle

Credits: NASA, George Low, John Logsdon

Shuttle – Chosen configuration

SRBs, Solid Rocket Boosters, provided the highest fraction of thrust until 2 min after lift off when they were fully burned out, jettisoned and were parachuted down in the Atlantic Ocean.

They were recovered by boats and reused on later flights after refurbishment.



Credits: NASA

Shuttle – Political go-ahead and industrial team

President Nixon and NASA Administrator James Fletcher discuss the Space Shuttle in January 1972, three months before Congress approved funding for the program

The prime contractor for the program was North American Rockwell (later Rockwell International, now Boeing), the same company responsible for building the Apollo Command/Service Module. The contractor for the Space Shuttle Solid Rocket Boosters was Morton Thiokol, for the External Tank, Martin Marietta (now Lockheed Martin), and for the Space Shuttle Main Engines, Rocketdyne (now Pratt & Whitney Rocketdyne, part of United Technologies).

Credits: Wikipedia/ NASA

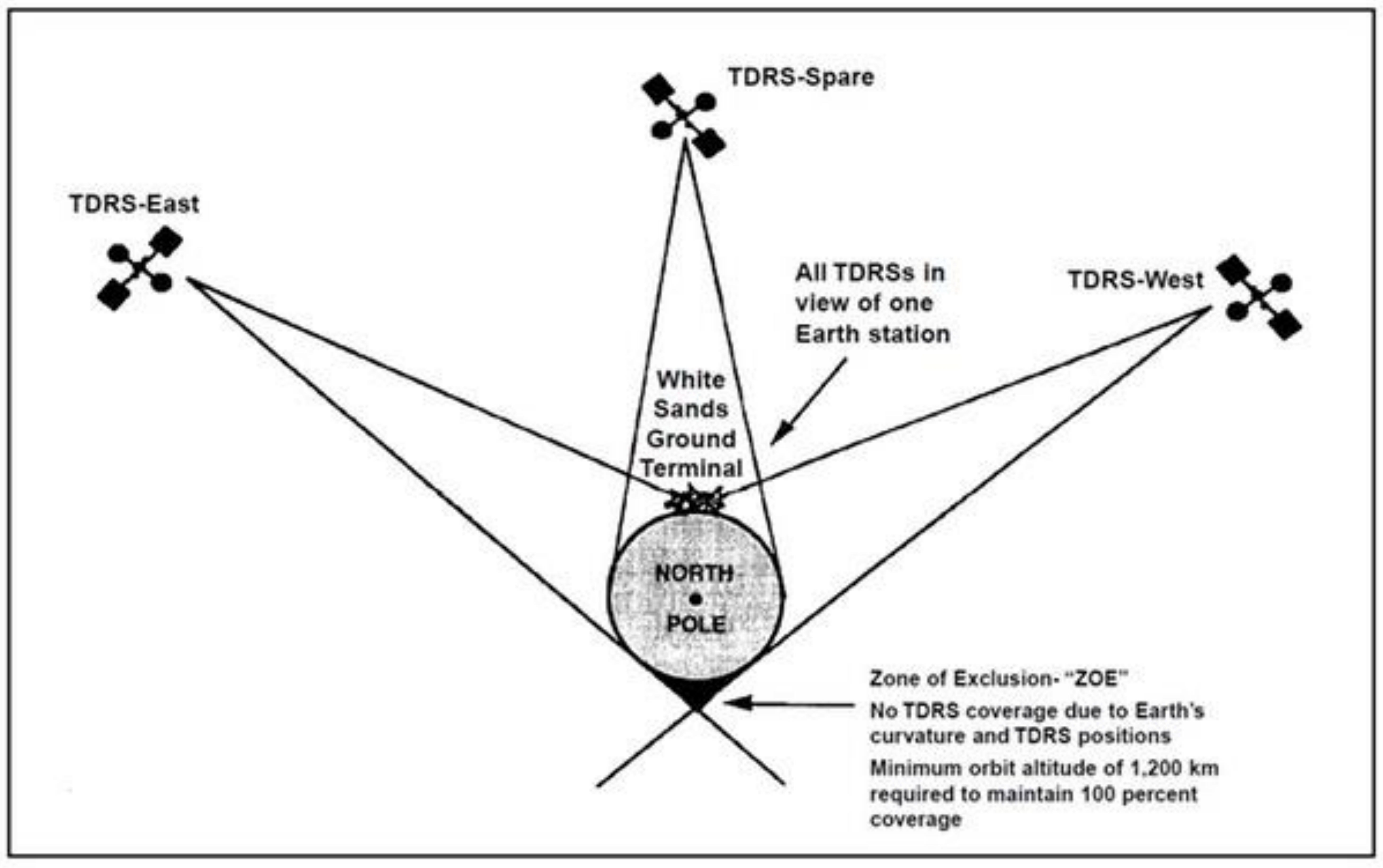
Space Shuttle – Summary of the program

- ALT in 1977
- 135 orbital flights total in 30 years, including Challenger and Columbia accidents
- OFT (orbital Flight Test): STS-1 in April 1981 to STS-4 in 1982
- Operational flights: STS-5 in 1982 to STS-135 in 2011

Credits: NASA

Communication and telemetry data/command flow while on-orbit

Via the TDRS satellites – The latest generation of these satellites provides ground reception rates of 6 Mbit/s in S-band and 800 Mbit/s in the K-band. At the time of Shuttle, we just had TDRS east and TDRS west – the system has been expanded now and is in use by ISS, Hubble, other science spacecraft, and the military.



Credits: NASA

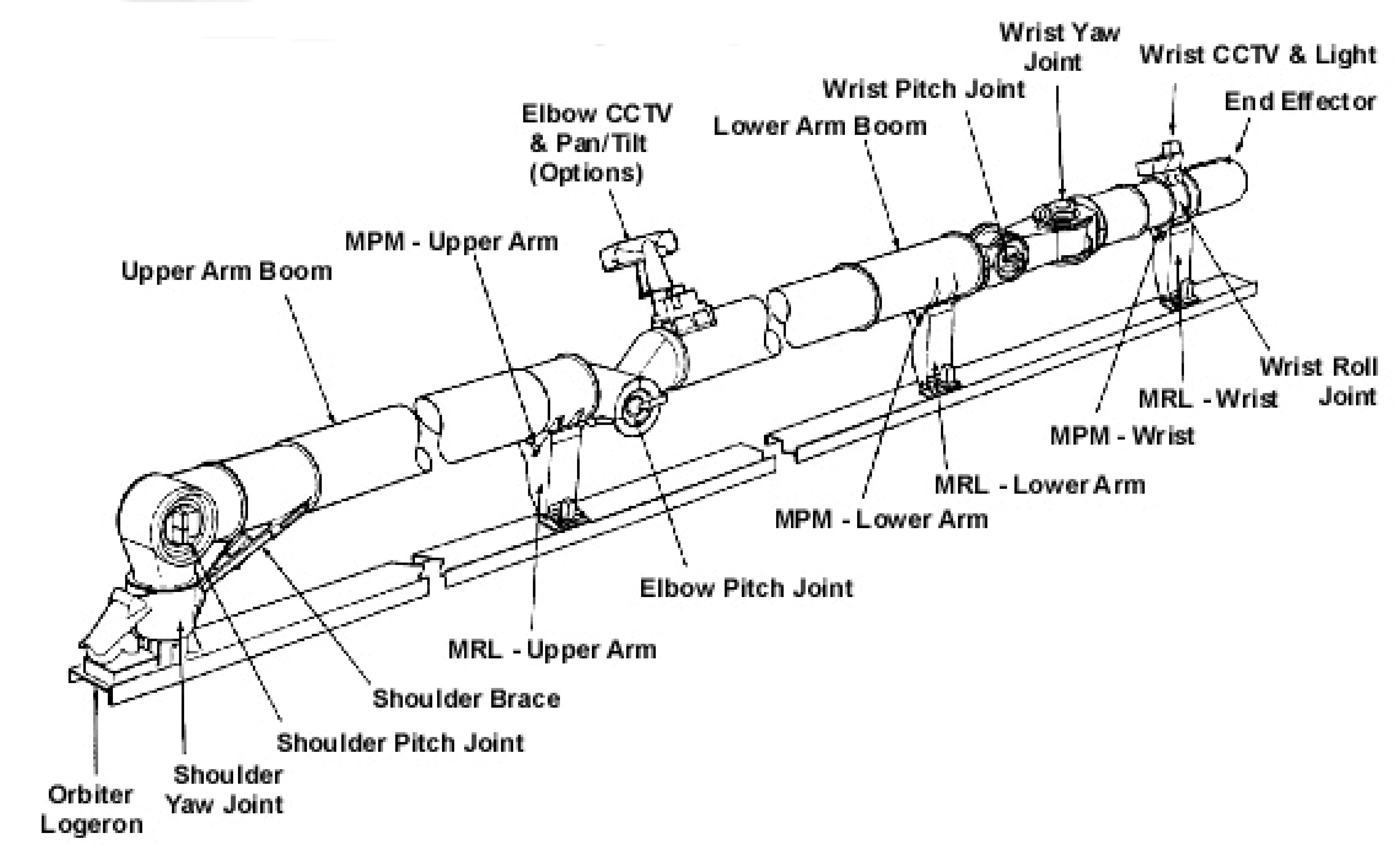
NASA's Commercial Crew and Cargo to ISS program

- General concept: Commercial Orbital Transportation Services (COTS), 2 parts:

- Commercial Resupply Services (CRS)
  - Phase 1: Dragon from Space X and Cygnus from Northrup Grumann (was Orbital ATK before)
  - Phase 2: Dream Chaser from Sierra Nevada Corporation
- Commercial Crew Development (CCDev)
  - Crew Dragon from Space X and CST-100 Starliner from Boeing

Credits: NASA

Shuttle Remote Manipulator System (SRMS) or Canadarm 1



Credits: Spar

End effector and Grapple Fixture (SRMS)

The Artemis program

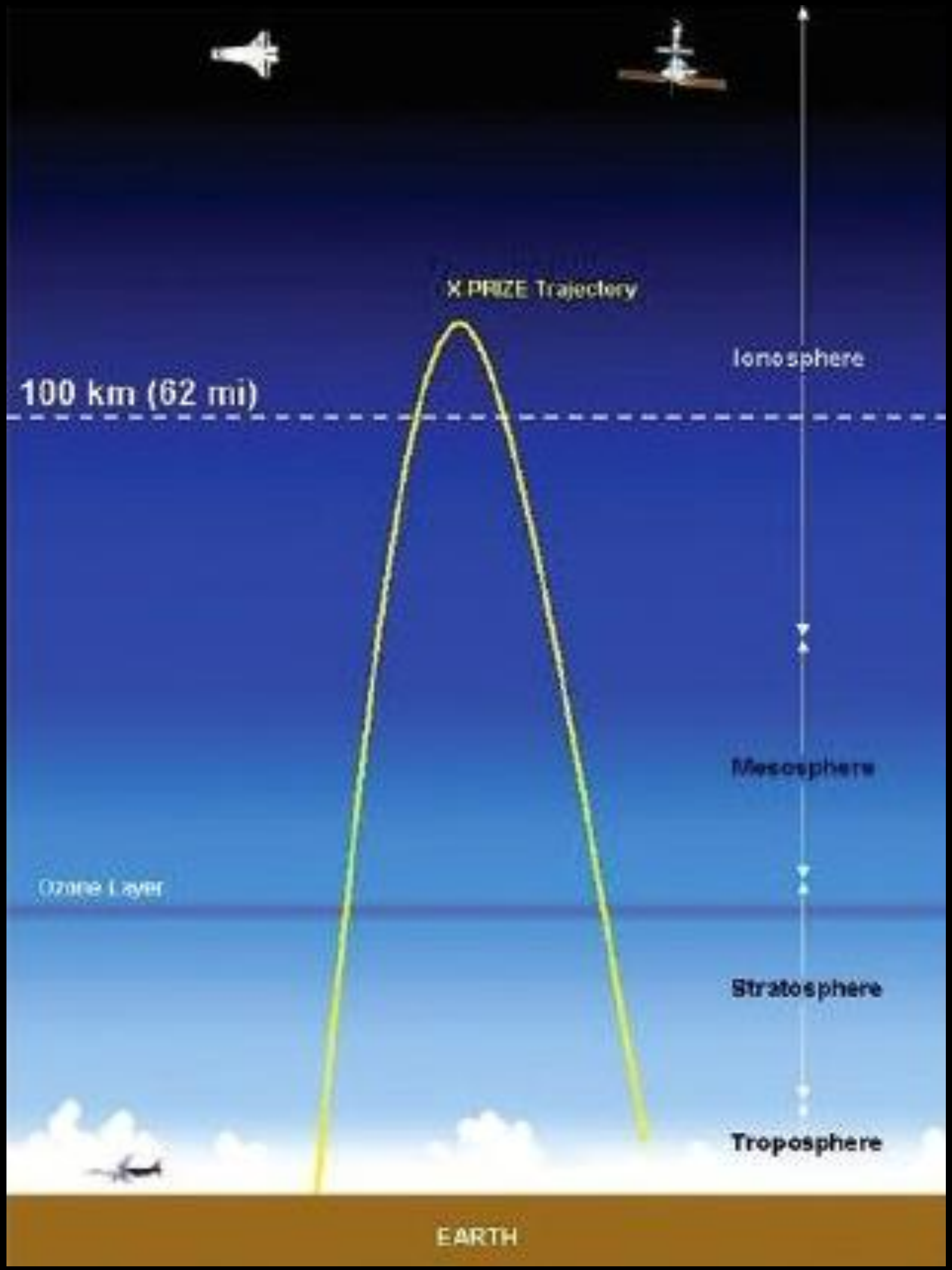
- The Artemis program has several international partners involved, mainly in the ORION vehicle (ESA) and the Gateway (ESA, CSA, JAXA, UAE), also commercial companies in the US.

- The project will be dynamic in its mission design and architecture, as well as in its timing, mainly because of budget constraints.

- The main goal of the Artemis program is to gain experience in long term sustainable human presence on the Moon, with the clear objective of sending people to Mars in the future.

Credits: NASA

Definition a of suborbital spaceflight



←Karman line" at 100 km

The boundary of space according to the FAI

Credits: quora.com

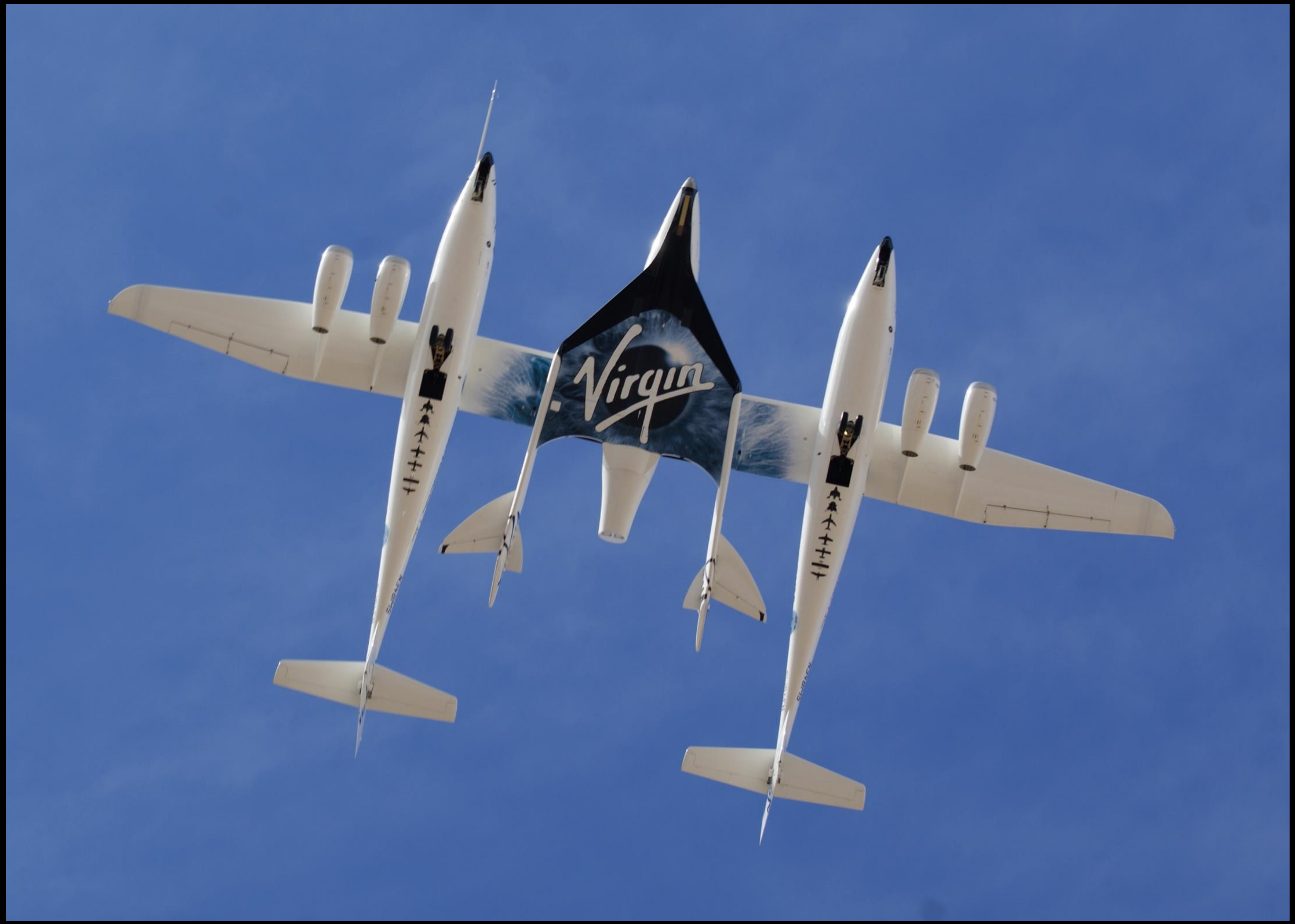
Scaled Composites, SpaceShip One – 2004

First privately funded human spaceflight in June 2004, then winner of the Ansari X prize with two spaceflights within 2 weeks in September and October 2004

SCALED

Credits: Scaled Composites

Virgin Galactic, SpaceShip Two



↑ VSS Enterprise carried by White Knight Two Carrier aircraft. The spaceplane was lost in an accident in October 2014. VSS Unity from 2016 →

Credits Virgin Galactic

Blue Origin – New Shepard rocket for suborbital flights

EPFL

BLUE ORIGIN