TRIGONOMETRY

By using

$$\begin{pmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{pmatrix}\begin{pmatrix} \cos B & -\sin B \\ \sin B & \cos B \end{pmatrix} = \begin{pmatrix} \cos(A+B) & -\sin(A+B) \\ \sin(A+B) & \cos(A+B) \end{pmatrix}$$

derive a formula for $\tan(A + B)$ in terms of $\tan A$ and $\tan B$.

Derive the following identities as consequences of Pythagoras' theorem:

(i) $\cos^2 A + \sin^2 A = 1$

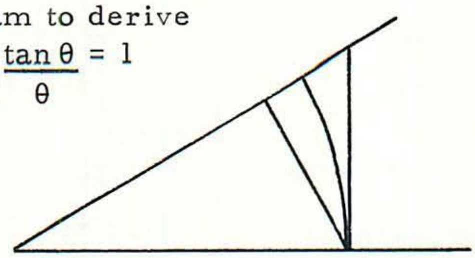
(ii) $1 + \tan^2 A = \sec^2 A$

(iii) $1 + \cot^2 A = \operatorname{cosec}^2 A$

By assuming that $5\cos u + 6\sin u$ can be written as $R\cos(u-\alpha)$, find the values of R and α.
Similarly find the maximum value of $p\cos x + q\sin x$ without using calculus.

Find formulae for $\sin A$, $\cos A$ and $\tan A$ in terms of $t = \tan(\frac{1}{2}A)$

Use this diagram to derive
$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = \lim_{\theta\to 0} \frac{\tan\theta}{\theta} = 1$



What about the value of $\lim_{\theta\to 0} \frac{\theta}{\cos\theta}$?

(What are these limits if θ is measured in degrees?)

Outline how to derive; $2\sin A\cos B = \sin(A + B) + \sin(A - B)$

$2\cos A\cos B = \cos(A + B) + \cos(A - B)$

$2\sin A\sin B = -\cos(A + B) + \cos(A - B)$

and consequently:

$\sin x + \sin y = 2\sin\frac{1}{2}(x + y)\cos\frac{1}{2}(x - y)$
$\sin x - \sin y = 2\cos\frac{1}{2}(x + y)\sin\frac{1}{2}(x - y)$
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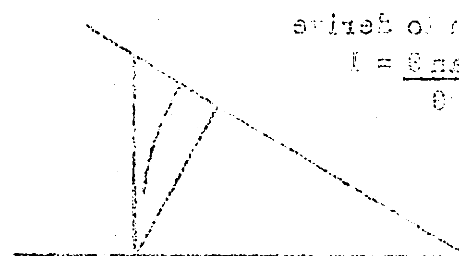

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Find the limit as $\theta \to 0$ of $\dfrac{\sin(x+\theta) - \sin(x-\theta)}{2\theta}$

(i) Prove that $\cos(A+B).\cos(A-B) = \cos^2 A - \sin^2 B$ and use this result to show that $\cos 15^\circ . \cos 105^\circ = -\frac{1}{4}$

(ii) By expressing $\sin\theta$ and $\cos\theta$ interms of t, where $t = \tan(\frac{1}{2}\theta)$ in the equation

$$5\sin\theta + 2\cos\theta = 5$$

form a quadratic in t and solve this to find, correct to the nearest 0.1°, the value of t in the range $40^\circ < t < 50^\circ$

Simplify $\dfrac{\sin 4\theta\,(1 - \cos 2\theta)}{\cos 2\theta\,(1 - \cos 4\theta)}$ where $\cos 2\theta\,(1 - \cos 4\theta) \neq 0$

(i) Given that $\cos\alpha = -\frac{1}{2}$, prove that

$$\sin\theta + \sin(\theta + \alpha) + \sin(\theta + 2\alpha) = 0$$

Use this result to find the value, in surd form, of $\sin 165^\circ + \sin 285^\circ$.

(ii) Express $4\cos x + \sin x$ in terms of t, where $t = \tan(\frac{1}{2}x)$ and hence solve the equation $4\cos x + \sin x = 1$ for values of x between 0° and 360°.

When $\cos\theta \neq 0$, simplify $(\cos 2\theta + \tan\theta \sin 2\theta)^{-1}$

Evaluate

(i) $\displaystyle\lim_{\theta \to 0} \frac{\sin 5\theta + \sin 7\theta}{6\theta}$

(ii) $\displaystyle\lim_{\theta \to 0} \frac{1 + \theta - \cos\theta}{\sin\theta}$

Find the limit as $\theta \to 0$ of $\dfrac{\sin 4\theta + \sin 2\theta}{3\sin 3\theta}$

(i) By expressing the numerator as a difference of sines, or otherwise, show that the greatest value, as θ varies, of

$$\frac{2\sin\theta\cos(\theta + \alpha)}{\cos^2\alpha}$$

where $0 < \alpha < \pi/2$, is $(1 + \sin\alpha)^{-1}$. Give in terms of α the smallest positive value of θ for which the expression has this greatest value.

(ii) By expressing cosec 2x and cot 2x in terms of tan x or otherwise, find the possible values of tan x for which

$$3\cot 2x + 7\tan x = 5\,\mathrm{cosec}\,2x$$

Given that X is the acute angle such that $\sin X = 4/5$ and Y is the obtuse angle such that $\sin Y = 12/13$, find the exact value of $\tan(X + Y)$.

Show that, provided x is not a multiple of $\pi/4$

$$\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$$

Find the limit, as $\theta \to 0$, of

$$\frac{\sin(A+\theta) - \sin A}{\sin 2\theta}$$
