AANRDRSAENLGEECMTEINOTNSS

Some Problems:

1) A man living in Aberdeen wishes to visit Birmingham, Cardiff, Durham, and Edinburgh, before returning home. How many different ways can this be done?
2) How many ways are there of making 10p from 1p, 2p and 5p pieces? How much difference does the use of $\frac{1}{2}$p pieces make?
3) A certain computer has 8-bit bytes, which represent the letters A, B, ...Z, the digits 0 to 9 and ten other symbols such as ? or $. The rest of the codes are reserved for instruction codes. How many instruction codes are there?
4) How many edges and vertices does a dodecahedron have? What about an icosahedron? (Hint: in the dodecahedron 3 edges meet at a point, and in an icosahedron 5 edges meet at a point.
5) How many integers between 1 and 200 are divisible by 9 or 11 (or both)? How many integers between 1 and 500 are divisible by 9 or 12 (or both)?

Principle of Inclusion-Exclusion

How many of the integers 1, 2, 3, ... 600 are not divisible by at least one of the integers 2, 3, 5 and 7?

In general:

For any sets $A_1, A_2, A_3, \ldots\ldots A_n$

$$\left|A_1 \cup A_2 \cup A_3 \cup \ldots\ldots \cup A_n\right| = \left(|A_1| + |A_2| + |A_3| + \ldots. + |A_n|\right)$$
$$- \left(|A_1 \cap A_2| + |A_2 \cap A_3| + \ldots. + |A_{n-1} \cap A_n|\right)$$
$$+ \left(|A_1 \cap A_2 \cap A_3| + \ldots. + |A_{n-2} \cap A_{n-1} \cap A_n|\right)$$
$$- (\ldots\ldots\ldots$$

i.e. number of elements = sum taken one at a time
- sum taken two at a time
+ sum taken three at a time
- sum taken four at a time
etc.

Three Englishmen, four Frenchmen, and five Germans are available for selection to a European committee of four, on which each nation must be represented. In how many different ways can the committee be chosen?

The result (home-win, away-win, score-draw or no score-draw) is forecast for each of ten football matches. In how many ways can the results of these ten matches be predicted to give exactly seven correct results?

Given that a triangle can be formed by joining three non-collinear points, find the number of different triangles that can be formed by the points A, B, C, D, E, F, G, H, I, J, K, L if ABCDE and EFGHIJKL are two straight lines?

(i) A forecast is to be made of the results of four football matches, each of which can be a win, draw or loss for the home team. Find the number of different possible forecasts containing 0, 1, 2, 3, 4 wrong results respectively.
(ii) Find how many numbers less than 2000 can be constructed from some or all of the digits 7, 4, 2, 1
(a) if no digit can be used more than once,
(b) if each digit can be used any number of times.

Find the number of different sets of three pairs that can be selected to form a tennis team of six players where each pair is to consist of a boy and a girl and there are five boys and four girls from whom to choose.

A family of six, including two brothers, sit in six adjacent seats at the cinema. If the two brothers do not sit together, in how many different ways can the family be seated?

In how many ways can 8 children join hands to form a circle if they all face inwards? In how many of these ways will Jane hold hands with her special friends Susan and Louisa?

Naval signals are made by arranging coloured flags in a vertical line and the flags are then read from top to bottom. How many different signals can be made from one green, three identical red and two identical blue flags, if
(a) all six of the flags are used,
(b) at least five of them are used?

Find the number of permutations of the letters of the word TOPOLOGY. In how many of these permutations are no two O's together?

Rules:

Ordered selections with repetition: n^k

Ordered selections without repetition: $n(n-1)(n-2)\ldots(n-k+1)$

Unordered selections without repetition: $\frac{n!}{k!(n-k)!}$

Unordered selections with repetition: $\frac{(n+k-1)!}{k!(n-1)!}$

AANRDRSAENLGEECMTEINOTNSS

Some Problems:

1) A man living in Aberdeen wishes to visit Birmingham, Cardiff, Durham, and Edinburgh, before returning home. How many different ways can this be done?
2) How many ways are there of making 10p from 1p, 2p and 5p pieces? How much difference does the use of $\frac{1}{2}$p pieces make?
3) A certain computer has 8-bit bytes, which represent the letters A, B, ...Z, the digits 0 to 9 and ten other symbols such as ? or $. The rest of the codes are reserved for instruction codes. How many instruction codes are there?
4) How many edges and vertices does a dodecahedron have? What about an icosahedron? (Hint: in the dodecahedron 3 edges meet at a point, and in an icosahedron 5 edges meet at a point.
5) How many integers between 1 and 200 are divisible by 9 or 11 (or both)? How many integers between 1 and 500 are divisible by 9 or 12 (or both)?

Principle of Inclusion-Exclusion

How many of the integers 1, 2, 3, ...600 are not divisible by at least one of the integers 2, 3, 5 and 7?

In general:

For any sets $A_1, A_2, A_3, \ldots\ldots A_n$

$$\left|A_1 \cup A_2 \cup A_3 \cup \ldots\ldots \cup A_n\right| = \left(|A_1| + |A_2| + |A_3| + \ldots. + |A_n|\right)$$
$$- \left(|A_1 \cap A_2| + |A_2 \cap A_3| + \ldots. + |A_{n-1} \cap A_n|\right)$$
$$+ \left(|A_1 \cap A_2 \cap A_3| + \ldots. + |A_{n-2} \cap A_{n-1} \cap A_n|\right)$$
$$- (\ldots\ldots\ldots$$

i.e. number of elements = sum taken one at a time
− sum taken two at a time
+ sum taken three at a time
− sum taken four at a time
etc.