# Introduction

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Duke STA 610

Multilevel data

Subpopulation inferences

Population inferences

Cross-level inferences

## Multilevel data

**Multilevel data:** Data for which there are

- multiple nested levels of sampling, and/or
- multiple nested sources of variability.

Such data are also often called *hierarchical data* or *clustered data*.

**Examples:**

Educational testing: students nested within classes;

Small area estimation: households nested within counties;

Agricultural experiments: subplots nested within whole plots;

Clinical trials: measurements nested within patients, patients within hospitals.

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## Terminology

*observational unit:* an object or condition for which data are measured.

*macro-level unit:* a unit within which other units are nested.

*micro-level unit:* a unit nested within another unit.

**Synonyms:**

- macro-level unit, top-level unit, clusters, groups;
- micro-level unit, bottom-level unit, units.

If there are only two levels, we will say *units* are nested within *groups*.

**Notation:** $y_{i,j}$ = measurement of $i$th unit in $j$th group.

**Populations:**

- The population: all possible units from all possible groups;
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## Types of multilevel inference

Subpopulation inferences: Group-specific features are of primary interest.

- What is the mean within each group, based on a sample from each group?
- What is the treatment effect for each group?
- Do the groups differ? If so, how do they differ?

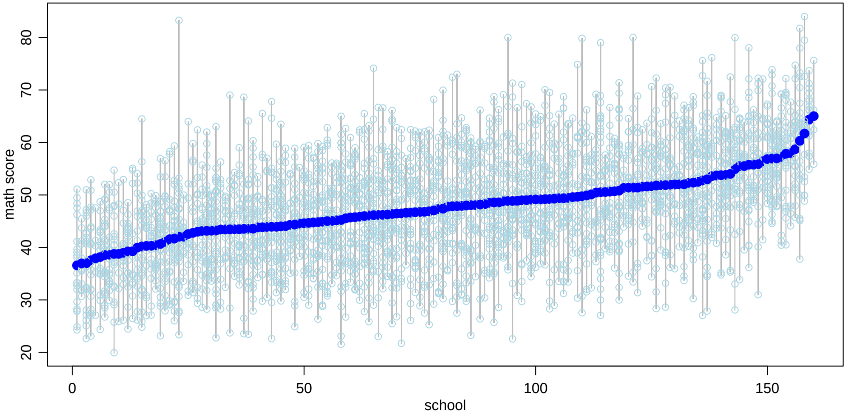
Population inferences: Across-group averages are of primary interest.

- What is the population mean, based on cluster sample?
- What is the population treatment effect?

Cross-level inferences: Both types of features are important.

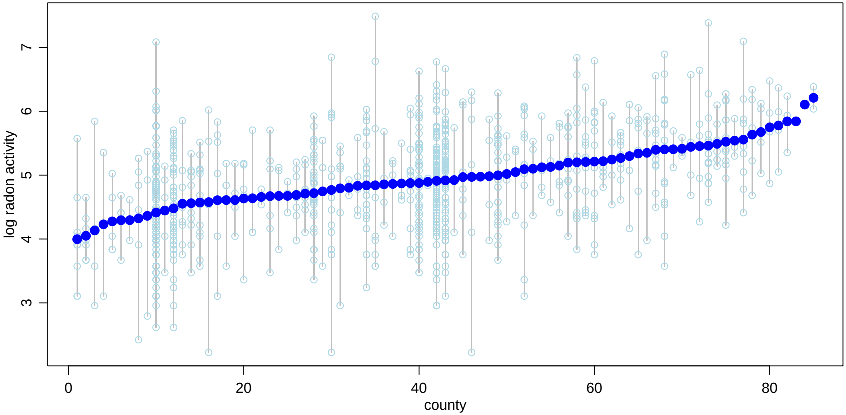
- What is the average treatment effect, adjusting for group differences?

# Example: Educational testing data




Exercise: Identify the populations and subpopulations.

# Example: Environmental monitoring data

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## Group-specific inferences

Targets of inference: Subpopulation means $\theta_1, \ldots, \theta_p$.

Data: Subpopulation samples $\{y_{1,1}, \ldots, y_{1,n_1}\}, \ldots, \{y_{1,p}, \ldots, y_{1,n_p}\}$.

Statistical methods:

- Variance tests and estimation: What is $\text{Var}[\theta_1, \ldots, \theta_p]$? Is it zero?
- Estimates of $\theta_j$: $\hat{\theta}_j = \bar{y}_{\cdot j}$ or $\hat{\theta}_j = w\bar{y}_{\cdot j} + (1-w)\bar{y}_{\cdot\cdot}$;
- Confidence intervals: $\Pr(\theta_j \in C(\mathbf{y})|\theta_j) = 1-\alpha$, or $\Pr(\theta^* \in C(\mathbf{y})) = 1-\alpha$;

## Cluster sampling

Survey design: Consider the costs of obtaining soil samples from

- 100 randomly sampled locations in a city, versus
- 10 randomly sampled locations from 10 randomly sampled neighborhoods.

Cluster sampling:
The second sampling scheme is called *cluster sampling* or *two-stage sampling*.

Cluster sampling

- is often cheaper per sampled unit;
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Task: Estimate the population mean $\mu$ from sample data.

Questions:

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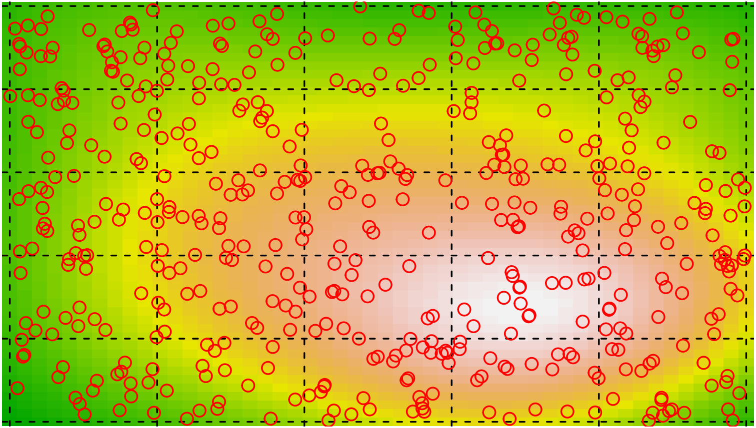

$\mu$=2.0494009

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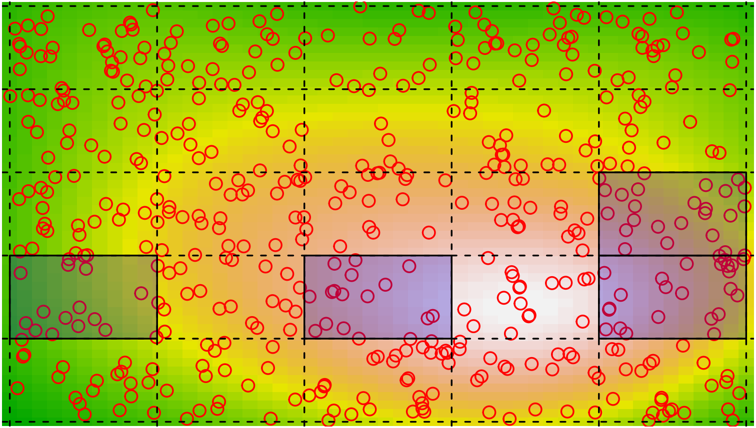

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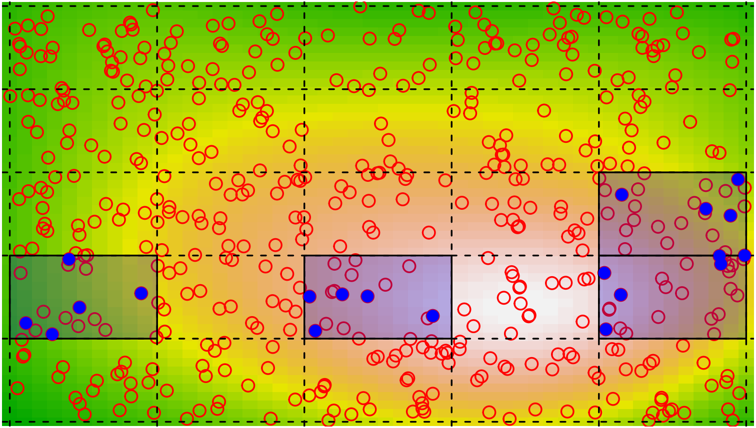

$\mu$=2.0494009 , $\bar{y}$=2.3547727

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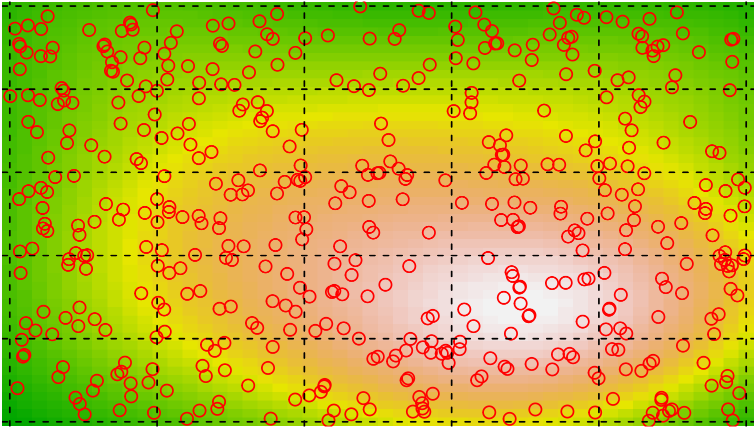

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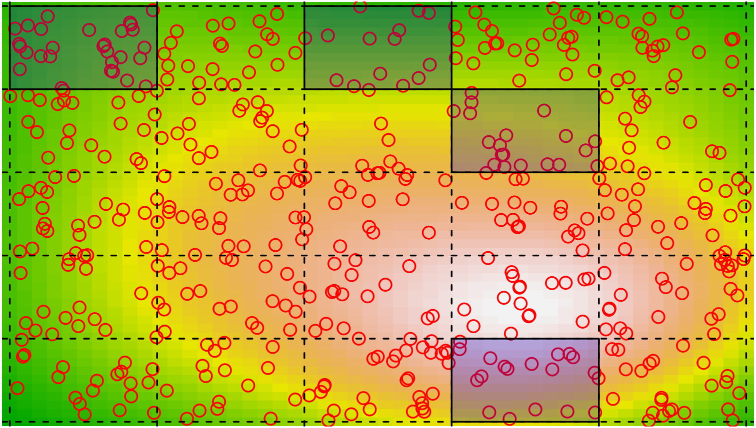

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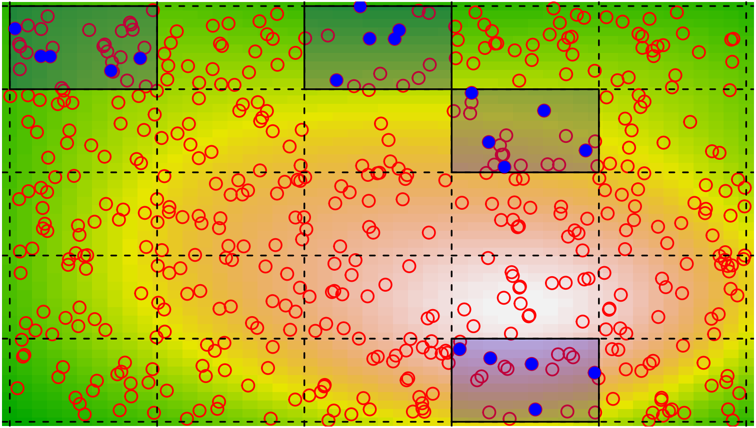
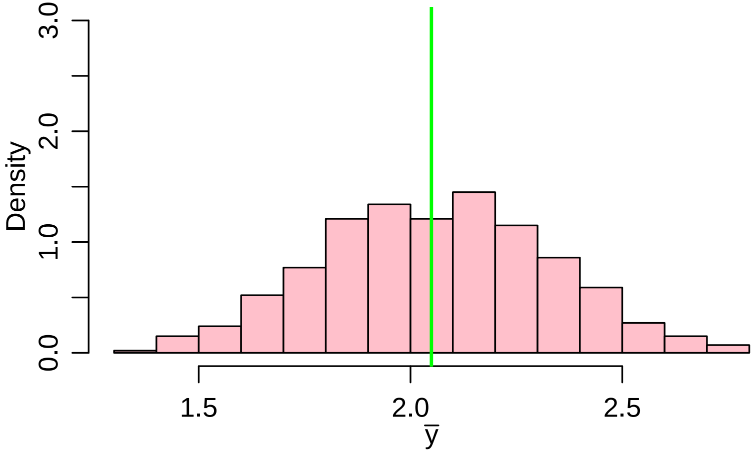


$\mu$=2.0494009 , $\bar{y}$=1.896463

# Variability of sample mean



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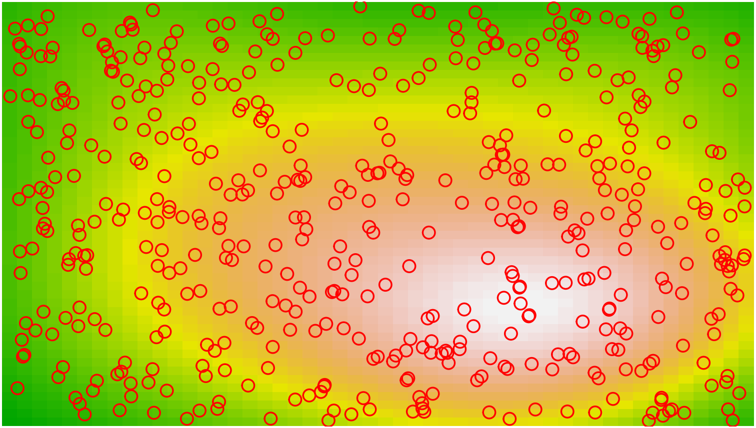

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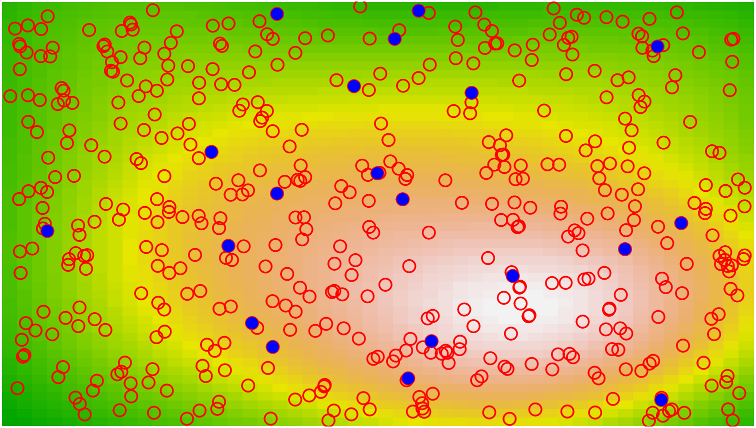

$\mu$=2.0494009 , $\bar{y}$=2.1696295

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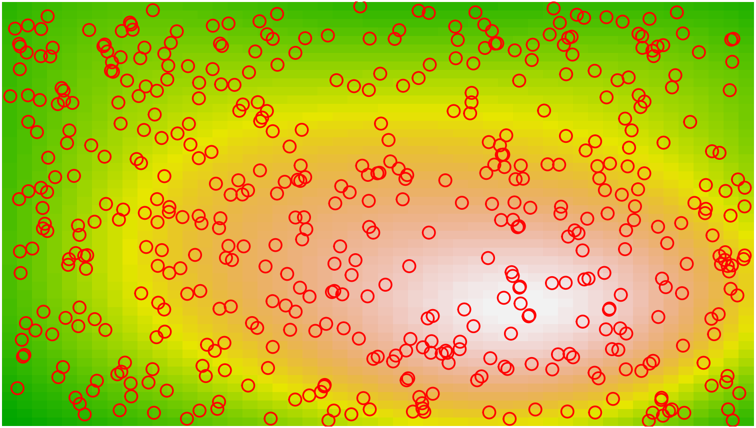

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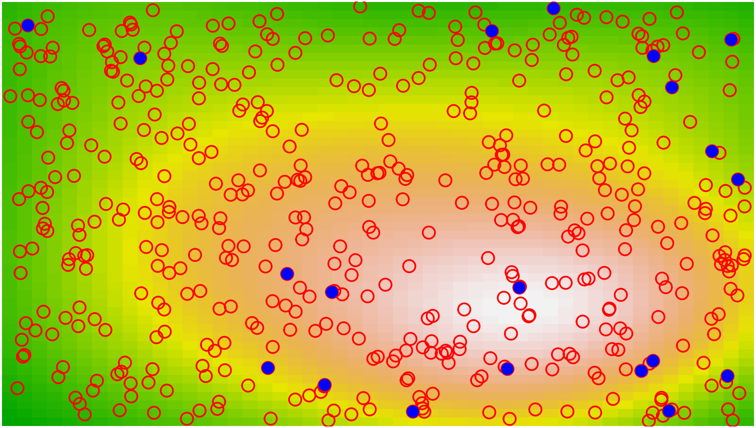

$\mu$=2.0494009 , $\bar{y}$=1.9804926

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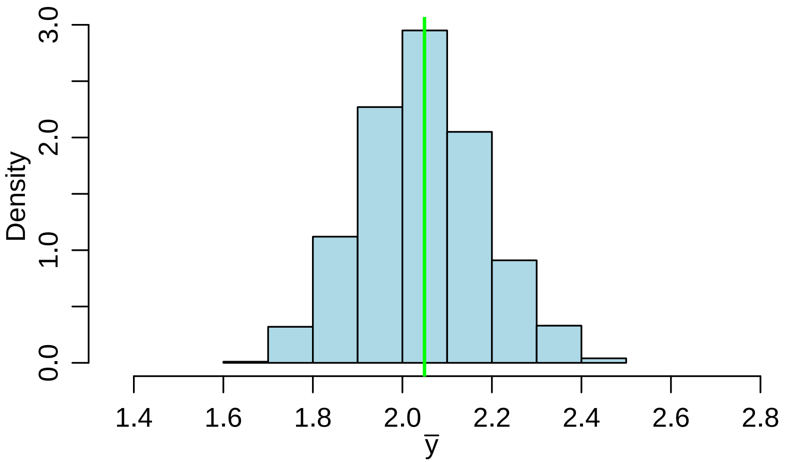
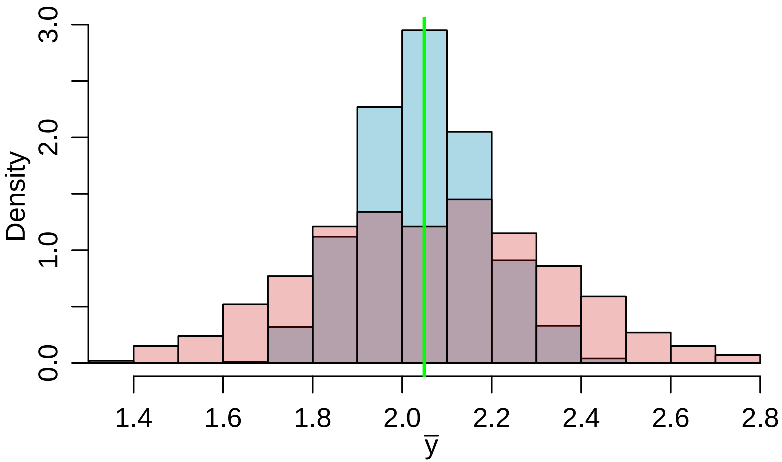


## Comparison of sampling variability



## Heterogeneity, homogeneity and dependence

As we will show mathematically,

$$\text{across-group heterogeneity} \Leftrightarrow \text{within-group homogeneity}$$
$$\Leftrightarrow \text{within-group correlation or dependence}$$

Across-group heterogeneity increases the variance of the sample mean, and so

$$\text{Var}[\bar{y}_{tss}] \geq \text{Var}[\bar{y}_{srs}]$$

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# Ignoring across-group heterogeneity

Task: Construct a 95% CI for the population mean.

$t$-interval for SRS:
If $y_1, \ldots, y_n$ is an iid sample with $\mathrm{E}[y_i] = \mu$ and $\mathrm{Var}[y_i] = \sigma^2$,

$$\mathrm{E}[\bar{y}] = \mu \ , \ \mathrm{Var}[\bar{y}] = \sigma^2/n.$$

By the central limit theorem,

$$\bar{y} \dot{\sim} N(\mu, \sigma^2/n) \ , \ \frac{\bar{y} - \mu}{\sigma/\sqrt{n}} \dot{\sim} N(0,1).$$

As $\sigma^2$ is generally unknown, we use

$$\frac{\bar{y} - \mu}{s/\sqrt{n}} \dot{\sim} t_{n-1}, \ \text{, where } s^2 = \frac{1}{n-1} \sum (y_i - \bar{y})^2.$$

From this, we have

$$\bar{y} \pm t_{n-1,.975} \times s/\sqrt{n} \ \text{ is a 95\% CI for } \mu.$$

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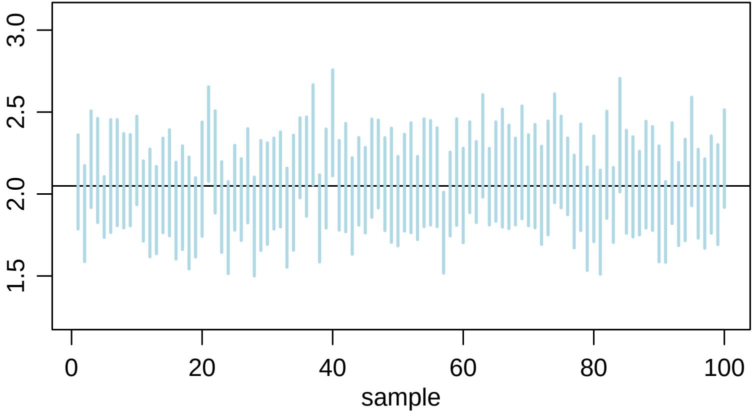

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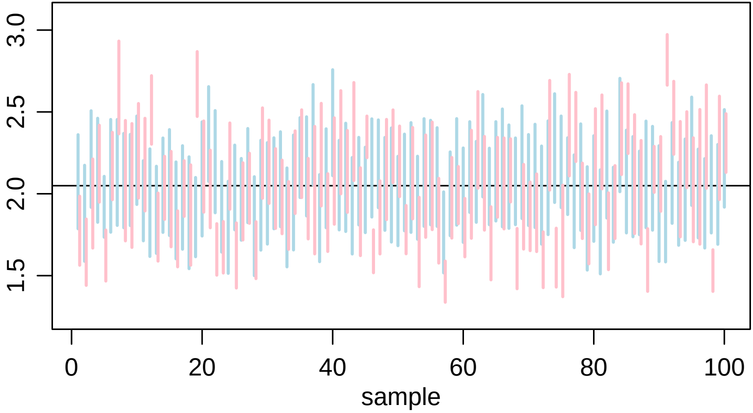

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Summary:

- Across-group heterogeneity = within-group similarity.
- Within-group similarity leads to positively correlated cluster sample data.
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- Statistical inference ignoring such correlation will be inaccurate.

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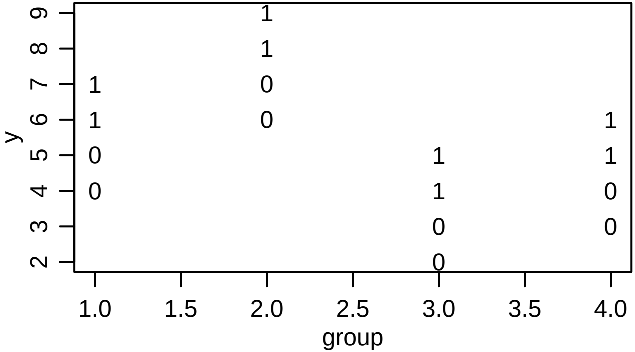
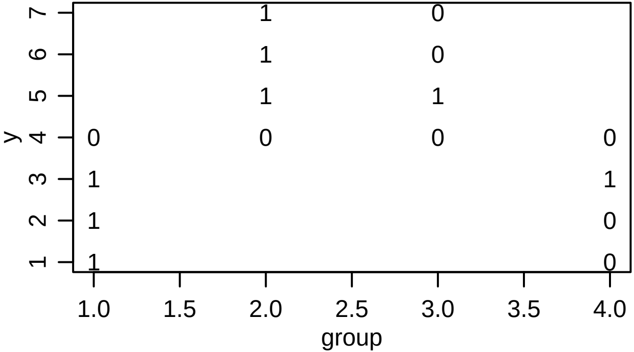
## Overconservative analysis



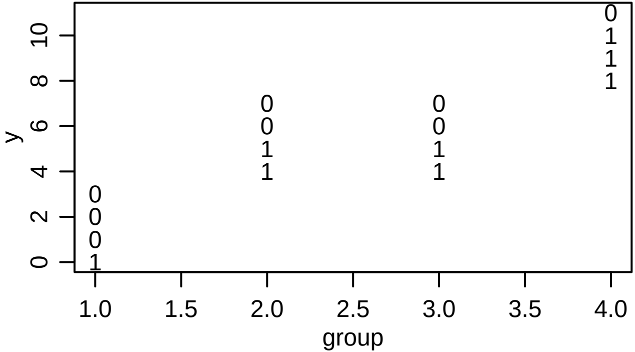

- Overlap across groups, no overlap within groups.
- Across-group variation is *large* compared to the treatment effect.
- Ignoring group differences can lead to *overconservative analysis.*

## Underconservative analysis




- The population mean difference is zero.
- The sample mean difference based on pairs of two groups is not zero.
- Ignoring group differences can lead to *underconservative analysis.*

# Effect reversal




- $\mu_1 - \mu_0 > 0$ in population, $\mu_{1,j} - \mu_{0,j} < 0$ in every group.
- Within-group effects may be different from population effects.
- This is sometimes called *Simpson's paradox*.

# Consequences of across-group heterogeneity

Summary:

- Across-group heterogeneity can lead to *over or under* conservative analysis.
- Population-level effects may be different from group-level effects.
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# Macro and micro effects

$X$, $x$ are macro and micro level explanatory variables

$Y$, $y$ are macro and micro level outcome variables

$$\begin{array}{ccc} X & & Y \\ \hdashline x & \longrightarrow & y \end{array}$$

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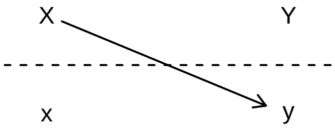


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$$X \longrightarrow Y$$

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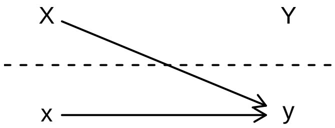



What are the effects of State GDP ($X$) and SES ($x$) on political opinion ($y$)?

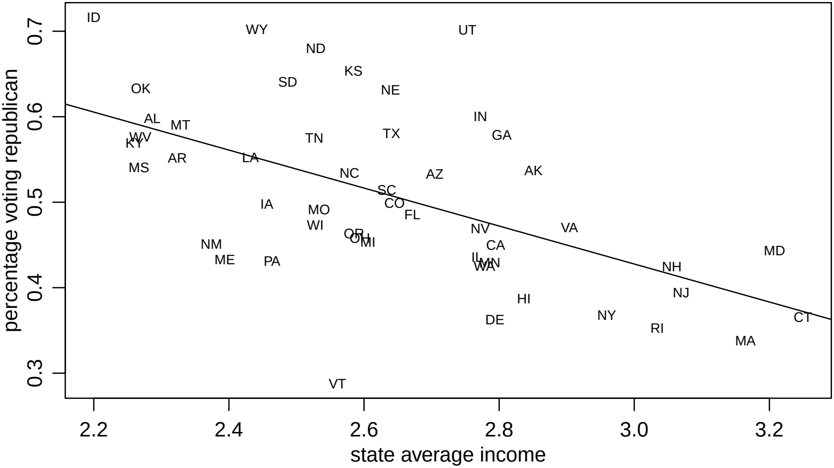
(*multilevel effects*)

## Example: Income and voting patterns

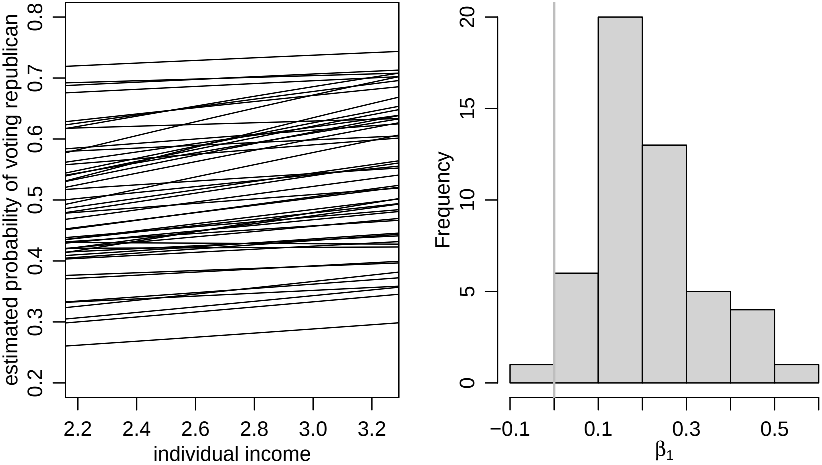
Exit poll data from 2004 presidential election

- $j \in \{1, \ldots, 50\}$ indexes the states,
- $y_{i,j}$ is the voting variable for person $i$ in state $j$,
- $x_{i,j}$ is a measure of income for person $(i, j)$.

# Macro effects



# Micro effects



## Joint estimation of effects

In general we may be interested in understanding all of the following:

- macro level effects,
- micro level effects,
- macro effects on micro variables,
- heterogeneity of micro effects across groups.

Inference for these items can be made with LME and GLME models:

$$
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