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- We are going to explore how to combine different methods we have learnt to obtain a fast and precise approximation for the exponential e(x)
- We are going to start from the code present in the working directory Taylor0: exp.c and tester.c
- tester.c: it calculates the exponential by a number of methods for a number (to be provided) of random points x between -10.0 and 10.0
- exp.c contains the subroutines to calculate the exponential

- tester.c also calculates execution times for the different methods, and provides an average error with respect to the standard exponential function
- The code in Taylor0 performs a simple Taylor expansion: at what degree?
- The code in Pade0 performs a simple Pade expansion: what order of approximant is used?

- First task: check their performance (execution time and error)
- Second task: reduce the range of testing from [-10,10]
   to [-0.5,0.5]: what has changed? Has the execution
   time improved? And the error?
- Third task: exploit the insight from the second task and modify the Pade` code exploiting the other tricks: see following slide

- Our Pade` approximation works best in a neighbourhood of 0. The strategy is now to exploit this fact to use these approximation only in a range around 0, also when calculating e(x) for x in [-10,10]
- Moreover, we are going to work with integers when possible
- In particular, as an intermediate result we are going to calculate an exponential of 2<sup>y</sup>, exploiting the fact that e<sup>x</sup> = 2<sup>log(exp(x))</sup> = 2<sup>x log(e)</sup>, where log is the logarithm in base 2: log<sub>2</sub>(x)

#### Advanced exercise: workflow to calculate exercise

#### To calculate ex

- We calculate y = x log<sub>2</sub>(e)
- Then we calculate 2<sup>y</sup>:
  - Split y in integer part ipart and non-integer part fpart
  - We calculate 2<sup>ipart</sup> as integer and we use a Pade approximation for 2<sup>fpart</sup> in [-0.5,0.5]
  - $3 2^y = 2^{ipart} * 2^{ipart}$
- $e^{x} = 2^{y}$

<sup>a</sup> See next slide for details

### Advanced exercise: workflow to calculate exercise

Pade` approximation for 2<sup>fpart</sup>:
We are going to use the following Pade` approximant:

$$p(x) = a_0 x^5 + a_1 x^3 + a_2 x$$

$$q(x) = x^4 + b_1 x^2 + b_2$$

$$f(x) = 1 + 2 \frac{p(x)}{q(x) - p(x)}$$

#### where

$$a_0 = 2.30933477057345225087e-2$$

$$a_1 = 2.02020656693165307700e1$$

$$a_2 = 1.51390680115615096133e3$$

$$b_1 = 2.33184211722314911771e2$$

$$b_2 = 4.36821166879210612817e3$$

#### Advanced exercise:

Which is the fastest method to calculate exp(x)?

Which is the most accurate?