Taylor and Pade` approximations: warming-up exercise

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Taylor approximation

- First task: approximate the natural exponential e(x) with a Taylor approximation, for x in [-10; 10]
- We are just going to inspect the result graphically
- Write a program that prints the value of the built-in exponential function and its Taylor approximation on a grid of points between x=-10 and x=10
- Do it with the Taylor polynomial of degree 5 and of degree 7
- First we have to remember how the Taylor series looks like:

$$e(x) = \Sigma^{\infty}_{n=0}\, x^n/n!$$

Taylor approximation

- In this first exercise we just look qualitatively at the results: where does the approximation go REALLY wrong?
- Where does it look like to work best? Why? How is it connected to our choice of the Taylor expansion?
- Increasing further the degree of the polynomial, what do you expect to happen?

Pade` approximation

- Let's now turn to the Pade` approximation

$$f(x) \approx \frac{\sum_{j=0}^{m} a_j x^j}{\sum_{j=0}^{n} b_j x^j} = \frac{p(x)}{q(x)}$$

- This is the Pade` approximant of order [m/n]
- We are going to use the Pade` approximant of order [4/4]

Pade` approximation

- For a generic function one should first figure out the coefficients:
- f(0) = R(0)
- f'(0) = R'(0)
- ...
- But for the exponential there is a known closed formula which we can use: P(x)/Q(x) for order [p/q]

$$P_{pq}(x) = \sum_{j=0}^{p} \frac{(p+q-j)!\,p!}{(p+q)!\,j!\,(p-j)!} x^j$$

$$Q_{pq}(x) = \sum_{j=0}^{q} \frac{(p+q-j)!\,q!}{(p+q)!\,j!\,(q-j)!} (-x)^j$$

- Remember: we want p=q=4

Pade` approximation

- Pade` approximation of order [4/4]:
- Where is it completely wrong?
- Where does it work well?
- Compare it with the Taylor approximation: which works better? Where?