Physical Domain Ω^μ

$\mathrm{grad} := \frac{\partial}{\partial x_i^\mu} e_i$

$\mathrm{Grad} := \frac{\partial}{\partial x_i^p} e_i$

$$\int_{\Omega^\mu} \mathrm{grad}(v) \cdot\cdot \overset{<4>}{\mathbb{C}} \cdot\cdot \mathrm{grad}(w) \, dx^\mu$$

$$\mathbb{F}_\mu := \mathrm{Grad}(x^\mu) = \frac{\partial x^\mu}{\partial x^p}, \quad dx^\mu = |\det \mathbb{F}_\mu| \, dx^p$$

Reference Domain Ω^p

$$\int_{\Omega^p} \mathrm{Grad}(v) \cdot \mathbb{F}_\mu^{-1} \cdot\cdot \overset{<4>}{\mathbb{C}} \cdot\cdot \mathrm{Grad}(w) \cdot \mathbb{F}_\mu^{-1} \, |\det \mathbb{F}_\mu| \, dx^p$$

use of identity $\quad A \cdot B \cdot\cdot C = A \cdot\cdot C \cdot B^T$

$$\boxed{\int_{\Omega^p} \mathrm{Grad}(v) \cdot\cdot \overset{<4>}{\mathbb{C}} \cdot\cdot \mathrm{Grad}(w) \cdot \mathbb{F}_\mu^{-1} \cdot \mathbb{F}_\mu^{-T} \det(\mathbb{F}_\mu) \, dx^p}$$

EI of $\mathbb{F}_\mu^{-1} \mathbb{F}_\mu^{-T} \det(\mathbb{F}_\mu)$ to arrive at approx. affine decomposition

Given the transformation $\Phi_\mu : \Omega^p \to \Omega^\mu$

and $\quad x^\mu = \Phi(x^p) = x^p + d(x^p; \mu)$

with the transformation displacement $d(x^p; \mu)$

we have

$$\mathbb{F}_\mu = \frac{\partial x^\mu}{\partial x^p} = \mathbb{1} + \frac{\partial d(x^p; \mu)}{\partial x^p}$$

the Operator $\mathcal{L}(d; \mu) := \mathbb{F}_\mu^{-1} \cdot \mathbb{F}_\mu^{-T} \det(\mathbb{F}_\mu)$

will be available as some UFL expression.

With new dolfinx Expression.eval can be used and a mapping from „magic integration points" to cells (which is suboptimal but who cares).

if only $\mathcal{L}(d; \mu) = \mathbb{F}_\mu^{-1} \cdot \mathbb{F}_\mu^{-T} \det(\mathbb{F}_\mu)$

is interpolated then

$$\mathcal{L}_{EI}(d; \mu) = \sum_{i=1}^{M} \mathbb{B}_i(x^p) \, \lambda_i(\mu)$$

$$\sum_{i=1}^{M} \lambda_i(\mu) \underbrace{\int_{\Omega^p} \mathrm{Grad}(v) \cdot\cdot \overset{<4>}{\mathbb{C}} \cdot\cdot \mathrm{Grad}(w) \cdot \mathbb{B}_i(x^p) \, dx^p}_{\text{some matrix } A_i}$$

; can be projected onto subspace for displacement

$$\hat{A}_i := B^T A_i B$$

$$\sum_{i=1}^{M} \lambda_i(\mu) \, \hat{A}_i$$

- Determine coefficients $\lambda_i(\mu)$ for new parameter value μ.
- $\hat{A}_i$ can be saved and also stored as COO Matrix Operator.

$$\lambda(\mu) = P^{-1} \mathcal{L}_{EI}(d; \mu)$$

P: interpolation matrix

operator applied to "U"

"apply" $\hat{=}$ as_range_array(μ)

For each evaluation of $\mathcal{L}_{EI}(d; \mu)$

I first have to solve the auxiliary problem for $d(x; \mu)$.

$\mathcal{L}_{EI}$ maps from V_d to Q (Quadrature Space).