[Coupling metrics to diagnose land-atmosphere interactions]

Multivariate Mutual Information

- References:
  - Goodwell, A. E., and P. Kumar, 2017a: Temporal information partitioning: Characterizing synergy, uniqueness, and redundancy in interacting environmental variables. *Water Resour. Res.*, **53**, 5920–5942, doi: 10.1002/2016WR020216.
  - Hsu, H., and P. A. Dirmeyer, 2021: Nonlinearity and multivariate dependencies in land-atmosphere coupling. *Water Resour. Res.*, **57**, e2020WR028179, doi: 10.1029/2020WR028179.

- Principle:
  - Multivariate mutual information (MMI) measures the reduction in uncertainty of one target variable X_{tar} by knowledge of multiple source variables: $I(X_{s1}, \ldots X_{sn};\ X_{tar})$. The simplest case involves two source variables X_{s1} and X_{s2}.
  - This approach expands upon the standard definition of mutual information, using synchronized time series of each variable. For example, for two source variables:

$$I(I_{s1}, X_{s2}; X_{tar}) = \sum p(x_{s1}, x_{s2}, x_{tar}) \log_2 \left[\frac{p(x_{s1}, x_{s2}, x_{tar})}{p(x_{s1}, x_{s2}) p(x_{tar})}\right]$$

  - Other techniques that build upon mutual information can be applied equally well to MMI, and some such as temporal information partitioning require it.

- Data needs:
  - Although statistical, there is an underlying assumption that the variables X_{si} are physically related to X_{tar}. Thus, comparison of results among models or between models and observational data sets can be a powerful diagnostic tool.
  - As with mutual information, the joint probability distribution $p(x_{s1}, x_{s2}, x_{tar})$ must have exactly the same shape of data as the marginal probability distributions.

- Observational data sources:
  - Well suited to data where multiple components of the LoCo process chain are measured at the same location, such as flux tower data.

- Caveats:
  - Causality between variables is not assured, particularly if all variables are synchronous in time. Introduction of time lags in one or more variables can address causality.

Supported by NASA/MAP Grant: NNX13AQ21G