[Coupling metrics to diagnose land-atmosphere interactions]

Associated Predictability Ratio

- Reference:
  - Schlosser, C. A., and P. C. D. Milly, 2002: A model-based investigation of soil moisture predictability and associated climate predictability. *J. Hydrometeor.*, **3**, 483-501, doi: 10.1175/1525-7541(2002)003<0483:AMBIOS>2.0.CO;2.
- Principle:
  - From a set of model simulations (e.g., subseasonal or seasonal) with M different initial soil moisture states and N different atmospheric initial states, an $M \times N$ matrix of simulations is compiled. They can be treated as M ensembles of N simulations, each ensemble member having identical soil moisture states and the same range of different atmospheric states.
    - Intra-ensemble variance of a state f is: $S_A^2 = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(f_{ij} - \overline{f}_i)^2, \quad \overline{f}_i = \frac{1}{N}\sum_{j=1}^{N} f_{ij}$
    - Total variance S_T^2 of a state f is: $S_T^2 = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(f_{ij} - \overline{f})^2, \quad \overline{f} = \frac{1}{M}\sum_{i=1}^{M}\overline{f}_i$
    - Relative inter-ensemble variance of f is: $R_f = \left(S_T^2 - S_A^2\right)\Big/ S_T^2$
      - As $t \to \infty$, $R_f \to \frac{M-1}{MN-1}$, $R_w(t=0)=1$, $R_a(t=0)=0$ where w and a indicate soil moisture and any associated variable respectively
    - $R_w(t) = (1 - R_\infty)e^{-t/t_w} + R_\infty$ gives an e-folding time or memory for soil moisture t_w that can be estimated by a least-squares fit through various forecast leads t.
    - The "associated predictability ratio" (APR) gives an indication of the predictability of atmospheric variable a at lead t derived from soil moisture:

$$A_a(t) = \frac{R_a(t) - R_\infty}{R_w(t) - R_\infty}$$

- Data needs:
  - Time series of the potentially affected variables from an ensemble of model simulations. Could use other land state variables.
- Observational data sources:
  - Cannot be determined observationally.
- Caveats:
  - APR is a characteristic of ensembles of model simulations. This makes it a very good metric to assess model behavior, compare models, explain predictability, etc., but it has no real-world analogue or equivalent.
  - By construct, it measures potential predictability in a model from soil moisture initialization, not realizable skill. It should be taken as an upper bound on what realistic soil moisture initialization can deliver, and as an indicator of the spatio-temporal distribution of atmospheric sensitivity to soil moisture states that is necessarily model dependent.

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