

Modeling extreme values with a GEV mixture probability distributions

Standard Gumbel distribution

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```
# Load useful functions
```

```
path <- ".."
```

```
xfun::in_dir(dir = path, expr = source("./src/generate_gev_sample.R"))
xfun::in_dir(dir = path, expr = source("./src/calculate_gev_inverse_cdf.R"))
xfun::in_dir(dir = path, expr = source("./src/estimate_gev_mixture_model_parameters.R"))
xfun::in_dir(dir = path, expr = source("./src/plot_gev_mixture_model_pdf.R"))
xfun::in_dir(dir = path, expr = source("./src/plot_gev_mixture_model_cdf.R"))
xfun::in_dir(dir = path, expr = source("./src/estimate_gev_mixture_model_quantile.R"))
```

```
# Generate a random sample
```

```
n <- 20000
```

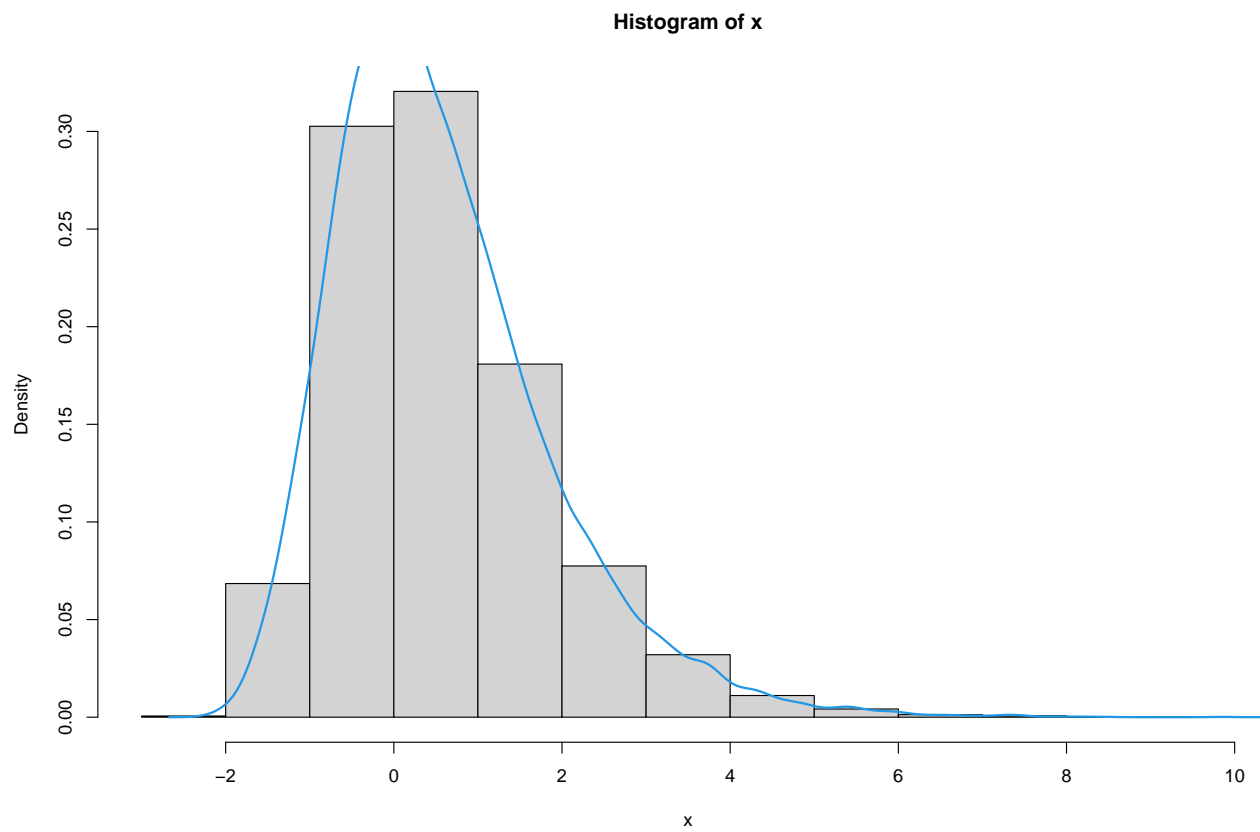
```
loc <- 0
scale <- 1
shape <- 0
```

```
set.seed(1122)
```

```
x <- generate_gev_sample(n = n, loc = loc, scale = scale, shape = shape)
```

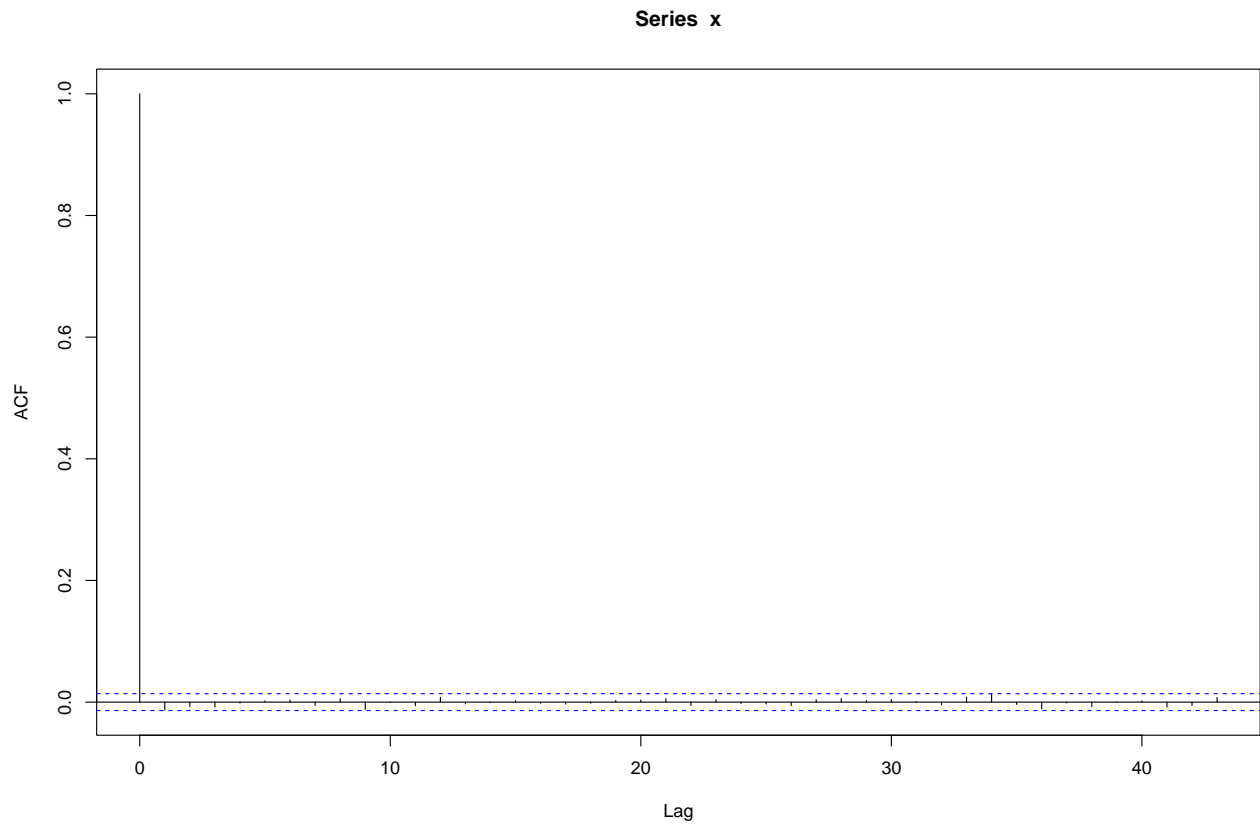
```
# Histogram of all data
```

```
hist(x, prob = TRUE)
lines(density(x),
      lwd = 2,
      col = 4)
```



```
# Autocorrelation function of all data
```

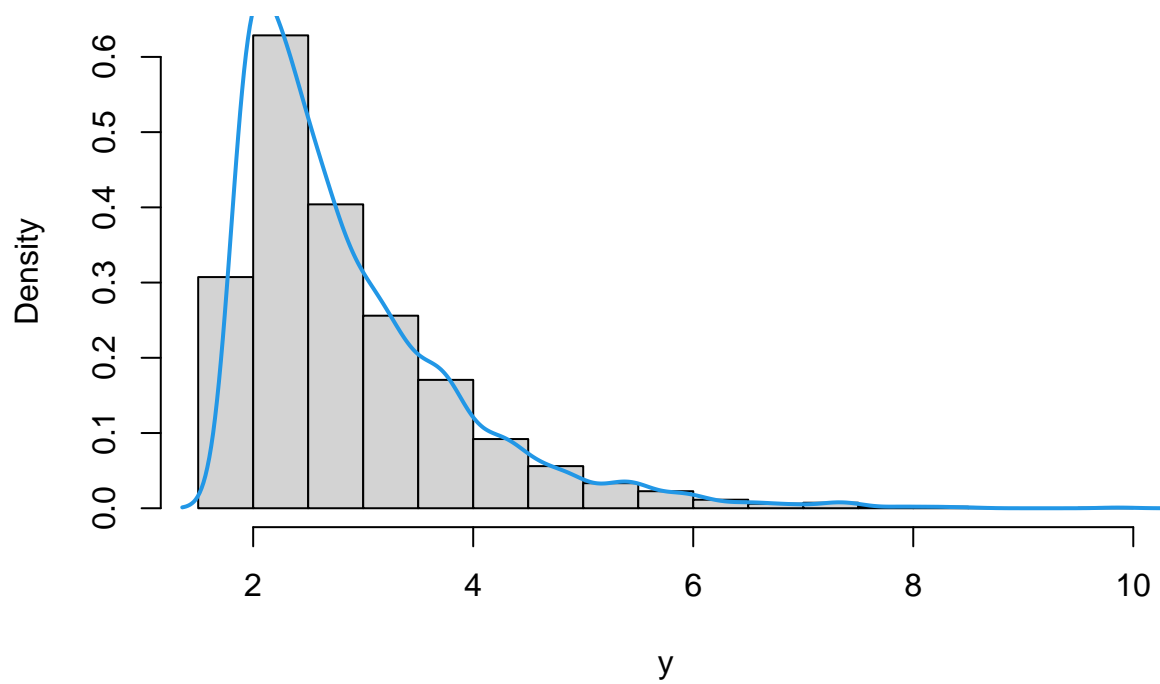
```
acf(x)
```



```
# Histogram of the largest data
```

```
nlargest <- 3000  
y <- extract_nlargest_sample(x, n = nlargest)  
hist(y, prob = TRUE)  
lines(density(y),  
      lwd = 2,  
      col = 4)
```

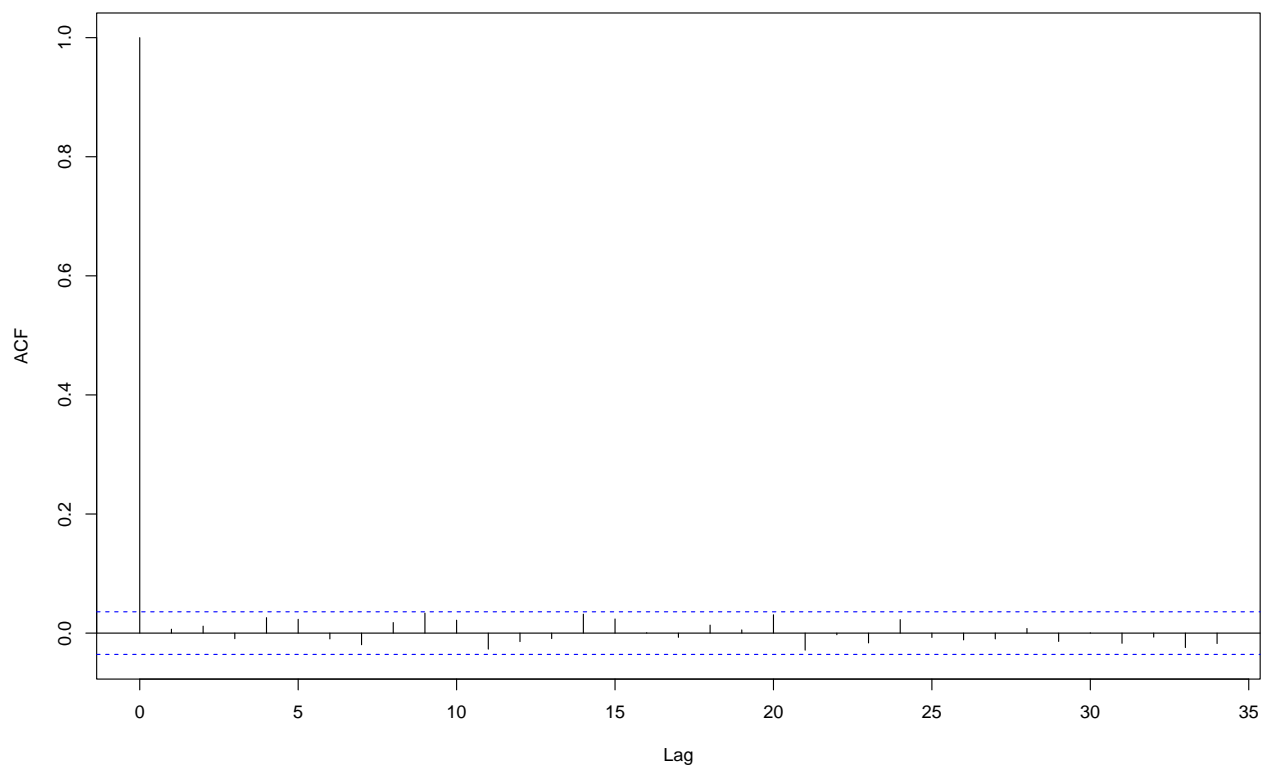
Histogram of y



Autocorrelation function of the largest data

`acf(y)`

Series y



```
# Estimation of gev mixture models
```

```
gev_mixture_model <- estimate_gev_mixture_model_parameters(x = x,
  block_sizes = NULL,
  minimum_nblocks = 50,
  threshold = NULL,
  nlargest = nlargest,
  confidence_level = 0.95,
  use_extremal_index = TRUE,
  use_lower_threshold = FALSE,
  maximum_iterations = 1500,
  log_mv = TRUE,
  log_pw = TRUE,
  trace = FALSE,
  method = "MLE")
```

```
## Successful convergence.
```

```
## Successful convergence.
```

```
gev_mixture_model$extremal_indexes
```

```
##          11          12          13          14          15          16
## 1.0000000000 0.9879441345 0.9541149980 0.9981560114 0.9867189207 0.9709389961
##          17          18          19          20          21          22
## 0.9788992889 0.9709389961 0.9709389961 1.0000000000 1.0000000000 0.9829783536
##          23          24          25          26          27          28
## 1.0000000000 1.0000000000 0.9737173241 1.0000000000 1.0000000000 0.9973014944
##          29          30          31          32          33          34
## 0.9677436233 0.9973014944 1.0000000000 0.9948576258 0.9737173241 1.0000000000
##          35          36          37          38          39          40
## 1.0000000000 1.0000000000 1.0000000000 1.0000000000 0.9471124881 1.0000000000
##          41          42          43          44          45          46
## 0.9471124881 1.0000000000 0.9948576258 0.9948576258 0.9737173241 0.9709850770
##          47          48          49          50          51          52
## 0.9189560440 0.9189560440 0.9189560440 0.9948576258 0.9189560440 0.9189560440
##          53          54          55          56          57          58
## 0.9189560440 0.9082517451 1.0000000000 0.9430905478 0.9189560440 0.9189560440
##          59          60
## 0.9364930765 0.9430905478
```

```
gev_mixture_model$normalized_gev_parameters_object
```

```
##          loc_star  scale_star  shape_star
## 11 2.02054328793 0.9344156768 0.005937382072
## 12 2.16753978551 0.8796595284 0.018151917288
## 13 2.24884421940 0.8245572959 0.033717663214
## 14 1.85683318186 1.0575703057 -0.028329854703
## 15 2.09428747745 0.9554711250 -0.009685426584
## 16 1.89778842003 1.0599597117 -0.035352984120
## 17 2.10111346064 0.9582364708 -0.011082316896
## 18 1.74318624546 1.1298938116 -0.047763372778
## 19 1.66157530093 1.2043828689 -0.068453143752
## 20 1.65620616947 1.1626833418 -0.051969671458
## 21 1.57847065962 1.2282035565 -0.068136141301
## 22 1.41327343958 1.3360713086 -0.089609648582
## 23 1.47508840367 1.2929542192 -0.082712728043
```

```

## 24 1.79324300242 1.1320566276 -0.051636208306
## 25 1.47025451512 1.2841324468 -0.076710805118
## 26 1.37774660752 1.3051428585 -0.079777364620
## 27 1.19700709911 1.3813332257 -0.087543464586
## 28 1.17232127028 1.4423652077 -0.104103127317
## 29 1.39934273156 1.3480785687 -0.090316075544
## 30 1.27737501017 1.3830641903 -0.093989128347
## 31 1.49562708015 1.3237377544 -0.093576449742
## 32 1.65493298923 1.1851937397 -0.060105209353
## 33 0.54430299771 1.8295902670 -0.153344924530
## 34 0.74771018609 1.6932898212 -0.136846023728
## 35 0.39866777056 1.8680425784 -0.154803049890
## 36 1.00453767211 1.5645190308 -0.123103049083
## 37 0.56235356389 1.7035326618 -0.131093178532
## 38 0.98802328822 1.4917135024 -0.105326366875
## 39 0.94791483565 1.5764155819 -0.122226343979
## 40 0.60961951002 1.7772453874 -0.147081258074
## 41 1.20385370968 1.4489174293 -0.106274579663
## 42 0.40188703818 1.8308467063 -0.148903099457
## 43 1.05805169874 1.5179667644 -0.113892669961
## 44 1.44154618374 1.3464155130 -0.094600805926
## 45 0.37692614568 1.8133418831 -0.145126488398
## 46 1.22513184692 1.4211660607 -0.102439677789
## 47 1.57640259224 1.2795360615 -0.085158054035
## 48 1.06259309424 1.5466360758 -0.120933882821
## 49 0.81159896597 1.6058549613 -0.122304724844
## 50 0.09996483251 1.9925184419 -0.162818154816
## 51 1.15455988685 1.4730215861 -0.109749402804
## 52 1.66800974392 1.2718443674 -0.087747906577
## 53 1.50434383503 1.3787335672 -0.105732384029
## 54 1.60158254216 1.3008029753 -0.090803525662
## 55 2.39437544354 0.9652821776 -0.034612707690
## 56 1.01673426372 1.5037746674 -0.111110551358
## 57 0.28300431933 1.8075164926 -0.137449124155
## 58 0.57316744045 1.7700547259 -0.144025052761
## 59 1.72478725589 1.2333498915 -0.078320729190
## 60 0.82765498700 1.6283619371 -0.128761306166

```

```

gev_mixture_model$full_normalized_gev_parameters_object

```

```

##          loc_star    scale_star    shape_star
## 11 2.02054328793 0.9344156768 0.005937382072
## 12 2.15687145797 0.8794658778 0.018151917288
## 13 2.21014453290 0.8232524329 0.033717663214
## 14 1.85488118296 1.0576256056 -0.028329854703
## 15 2.08151194250 0.9555948615 -0.009685426584
## 16 1.86651216964 1.0610654205 -0.035352984120
## 17 2.08067520280 0.9584629741 -0.011082316896
## 18 1.70984034528 1.1314865243 -0.047763372778
## 19 1.62602019958 1.2068167273 -0.068453143752
## 20 1.65620616947 1.1626833418 -0.051969671458
## 21 1.57847065962 1.2282035565 -0.068136141301
## 22 1.39031787376 1.3381283488 -0.089609648582
## 23 1.47508840367 1.2929542192 -0.082712728043
## 24 1.79324300242 1.1320566276 -0.051636208306

```

```

## 25 1.43601766115 1.2867587834 -0.076710805118
## 26 1.37774660752 1.3051428585 -0.079777364620
## 27 1.19700709911 1.3813332257 -0.087543464586
## 28 1.16842323036 1.4427710058 -0.104103127317
## 29 1.35507631476 1.3520765378 -0.090316075544
## 30 1.27363728431 1.3834154959 -0.093989128347
## 31 1.49562708015 1.3237377544 -0.093576449742
## 32 1.64882160815 1.1855610656 -0.060105209353
## 33 0.49547360582 1.8370780064 -0.153344924530
## 34 0.74771018609 1.6932898212 -0.136846023728
## 35 0.39866777056 1.8680425784 -0.154803049890
## 36 1.00453767211 1.5645190308 -0.123103049083
## 37 0.56235356389 1.7035326618 -0.131093178532
## 38 0.98802328822 1.4917135024 -0.105326366875
## 39 0.86197141801 1.5869201316 -0.122226343979
## 40 0.60961951002 1.7772453874 -0.147081258074
## 41 1.12489553053 1.4573086766 -0.106274579663
## 42 0.40188703818 1.8308467063 -0.148903099457
## 43 1.05022330789 1.5188583607 -0.113892669961
## 44 1.43460285473 1.3470723576 -0.094600805926
## 45 0.32853570202 1.8203646182 -0.145126488398
## 46 1.18322360712 1.4254591273 -0.102439677789
## 47 1.46786995561 1.2887784896 -0.085158054035
## 48 0.93120576246 1.5625252560 -0.120933882821
## 49 0.67517305059 1.6225404953 -0.122304724844
## 50 0.08968780858 1.9941917280 -0.162818154816
## 51 1.02948535959 1.4867484408 -0.109749402804
## 52 1.56011770935 1.2813116676 -0.087747906577
## 53 1.38729522160 1.3911093962 -0.105732384029
## 54 1.47585294136 1.3122196664 -0.090803525662
## 55 2.39437544354 0.9652821776 -0.034612707690
## 56 0.92833618803 1.5135966264 -0.111110551358
## 57 0.12934769944 1.8286364605 -0.137449124155
## 58 0.42265353473 1.7917324991 -0.144025052761
## 59 1.64365499879 1.2397042291 -0.078320729190
## 60 0.73188358882 1.6406935874 -0.128761306166

```

```
gev_mixture_model$automatic_weights_pw_shape
```

```

##          11          12          13          14          15
## 6.041530053e-02 6.589552390e-02 7.296633288e-02 4.515069667e-02 5.340569996e-02
##          16          17          18          19          20
## 4.205393053e-02 5.277893516e-02 3.658160064e-02 2.752626824e-02 3.472681520e-02
##          21          22          23          24          25
## 2.766111033e-02 1.859422840e-02 2.148598688e-02 3.487385723e-02 2.401378791e-02
##          26          27          28          29          30
## 2.271672228e-02 1.946054066e-02 1.251721725e-02 1.829803144e-02 1.675796656e-02
##          31          32          33          34          35
## 1.693099171e-02 3.113935450e-02 4.985194478e-07 8.468448214e-04 5.099055588e-07
##          36          37          38          39          40
## 4.674601769e-03 2.390010065e-03 1.200431063e-02 4.962086897e-03 4.496011795e-07
##          41          42          43          44          45
## 1.160672340e-02 4.638308384e-07 8.432748186e-03 1.650149257e-02 4.343363011e-07
##          46          47          48          49          50
## 1.321469500e-02 2.046070815e-02 5.500354041e-03 4.929446066e-03 5.724873367e-07

```

```
##          51          52          53          54          55
## 1.015823196e-02 1.937482671e-02 1.183406360e-02 1.809364926e-02 4.238034915e-02
##          56          57          58          59          60
## 9.591374512e-03 7.034852518e-04 4.257314968e-07 2.332898550e-02 3.056759202e-03
```

```
gev_mixture_model$automatic_weights_pw_scale
```

```
##          11          12          13          14
## 6.410507485e-02 7.055255376e-02 7.714707543e-02 4.964406506e-02
##          15          16          17          18
## 6.161972723e-02 4.924025156e-02 6.128314072e-02 4.097224332e-02
##          19          20          21          22
## 3.229470237e-02 3.730886535e-02 2.987584972e-02 1.753470157e-02
##          23          24          25          26
## 2.255144492e-02 4.090530033e-02 2.325232556e-02 2.119235254e-02
##          27          28          29          30
## 1.276412516e-02 6.086756212e-03 1.599465240e-02 1.253418254e-02
##          31          32          33          34
## 1.912351417e-02 3.469854436e-02 0.000000000e+00 0.000000000e+00
##          35          36          37          38
## 0.000000000e+00 0.000000000e+00 -1.355252716e-20 8.842484854e-04
##          39          40          41          42
## 0.000000000e+00 0.000000000e+00 4.517957851e-03 0.000000000e+00
##          43          44          45          46
## 0.000000000e+00 1.654718416e-02 0.000000000e+00 7.954816654e-03
##          47          48          49          50
## 2.302384188e-02 0.000000000e+00 0.000000000e+00 1.355252716e-20
##          51          52          53          54
## 1.370271056e-03 2.386852748e-02 1.168939428e-02 2.040439657e-02
##          55          56          57          58
## 6.048286820e-02 0.000000000e+00 -1.355252716e-20 -1.355252716e-20
##          59          60
## 2.857504422e-02 1.355252716e-20
```

```
gev_mixture_model$automatic_weights_pw_loc
```

```
##          11          12          13          14          15
## 5.411134024e-02 6.259509697e-02 6.591092379e-02 4.476910796e-02 5.790422437e-02
##          16          17          18          19          20
## 4.540491015e-02 5.785213747e-02 3.683970648e-02 3.226678257e-02 3.390713675e-02
##          21          22          23          24          25
## 2.971453200e-02 1.961372627e-02 2.416486413e-02 4.139952455e-02 2.206730743e-02
##          26          27          28          29          30
## 1.893876216e-02 9.343319511e-03 7.848840890e-03 1.772153741e-02 1.335178962e-02
##          31          32          33          34          35
## 2.526746530e-02 3.350334682e-02 1.007441814e-05 6.708674265e-06 1.136599674e-05
##          36          37          38          39          40
## 3.281055460e-06 9.182058386e-06 3.501474307e-06 5.183820028e-06 8.551377287e-06
##          41          42          43          44          45
## 5.583599818e-03 1.132304776e-05 1.718555139e-03 2.199135053e-02 1.230164074e-05
##          46          47          48          49          50
## 8.622273865e-03 2.377734435e-02 4.259808761e-06 7.676640489e-06 1.548780207e-05
##          51          52          53          54          55
## 6.805289228e-04 2.872937791e-02 1.945144088e-02 2.420590716e-02 7.737627302e-02
##          56          57          58          59          60
```



```
## 4.298104446e-06 1.495878898e-05 1.104598791e-05 3.322084500e-02 6.919876206e-06
```

```
gev_mixture_model$weighted_normalized_gev_parameters_object[3, ]
```

```
##                loc_star scale_star    shape_star
## automatic_weights 1.825534678 1.10085912 -0.04731721557
```

```
gev_mixture_model$automatic_weights_mw
```

```
##          11          12          13          14          15
## 0.11245230416 0.27837194084 0.33619448515 0.00000000000 0.00000000000
##          16          17          18          19          20
## 0.00000000000 0.00000000000 0.00000000000 0.00000000000 0.00000000000
##          21          22          23          24          25
## 0.00000000000 0.00000000000 0.00000000000 0.00000000000 0.00000000000
##          26          27          28          29          30
## 0.00000000000 0.00000000000 0.00000000000 0.00000000000 0.00000000000
##          31          32          33          34          35
## 0.00000000000 0.00000000000 0.03204707978 0.00000000000 0.02404106142
##          36          37          38          39          40
## 0.00000000000 0.00000000000 0.00000000000 0.00000000000 0.00000000000
##          41          42          43          44          45
## 0.00000000000 0.00000000000 0.00000000000 0.00000000000 0.00000000000
##          46          47          48          49          50
## 0.00000000000 0.00000000000 0.00000000000 0.00000000000 0.12726283326
##          51          52          53          54          55
## 0.00000000000 0.00000000000 0.00000000000 0.00000000000 0.08963029539
##          56          57          58          59          60
## 0.00000000000 0.00000000000 0.00000000000 0.00000000000 0.00000000000
```

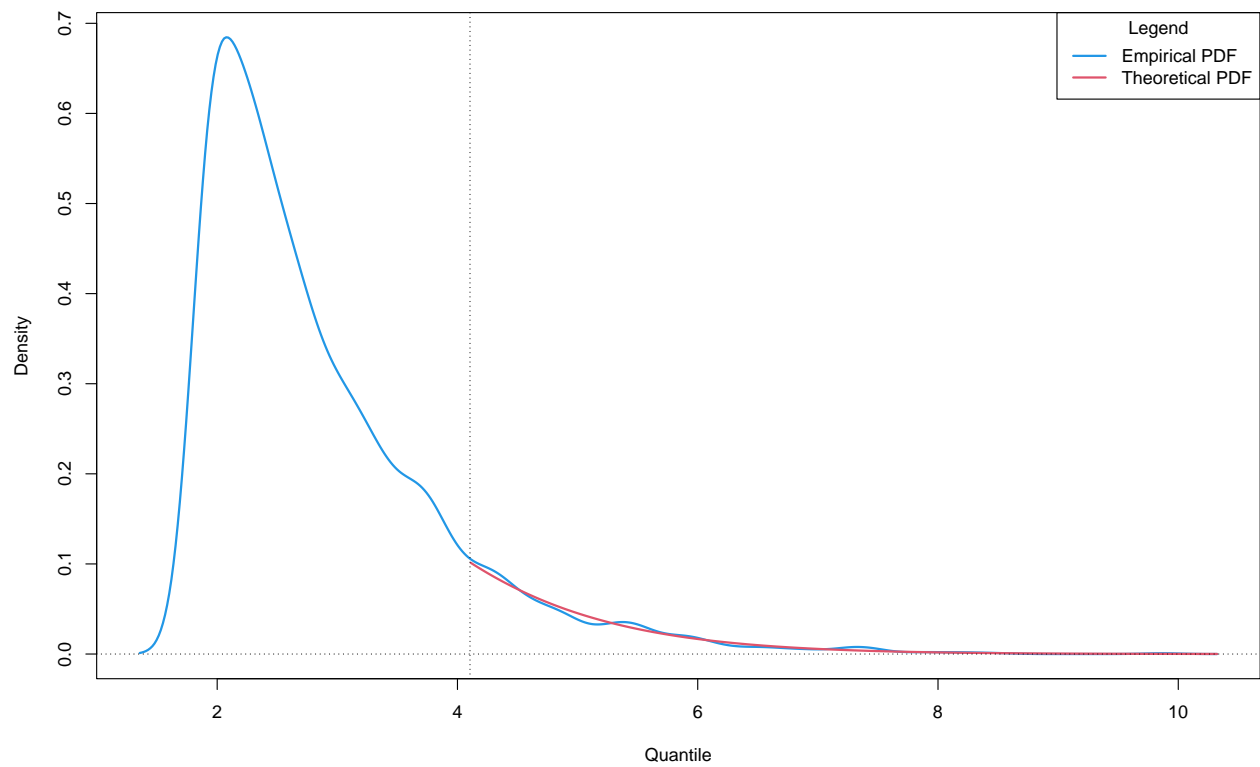
```
# Model diagnostics
```

```
## GEV mixture model with respect to parameters
```

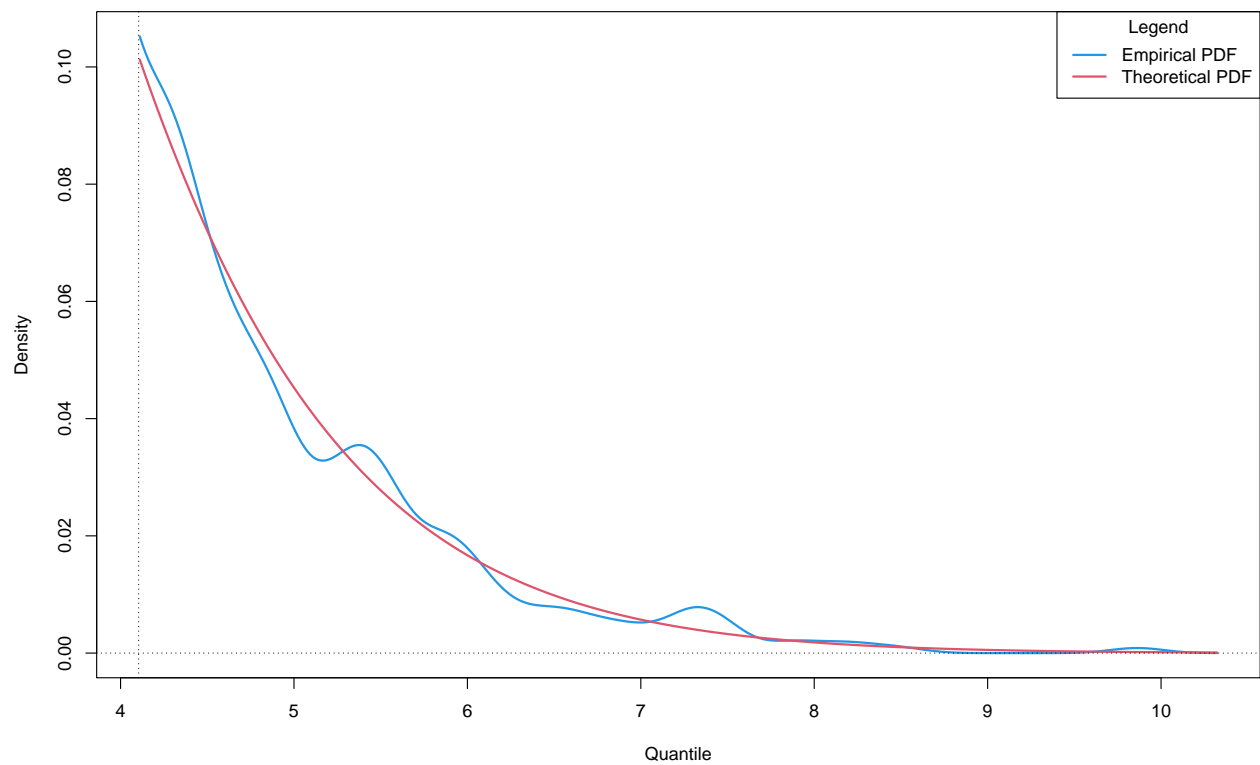
```
par(mfrow = c(2, 1))
plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = FALSE,
                           zoom = FALSE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")

plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = FALSE,
                           zoom = TRUE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")
```

Probability Density Function (PDF) Plot : automatic_weights – model_wise = FALSE : zoom = FALSE



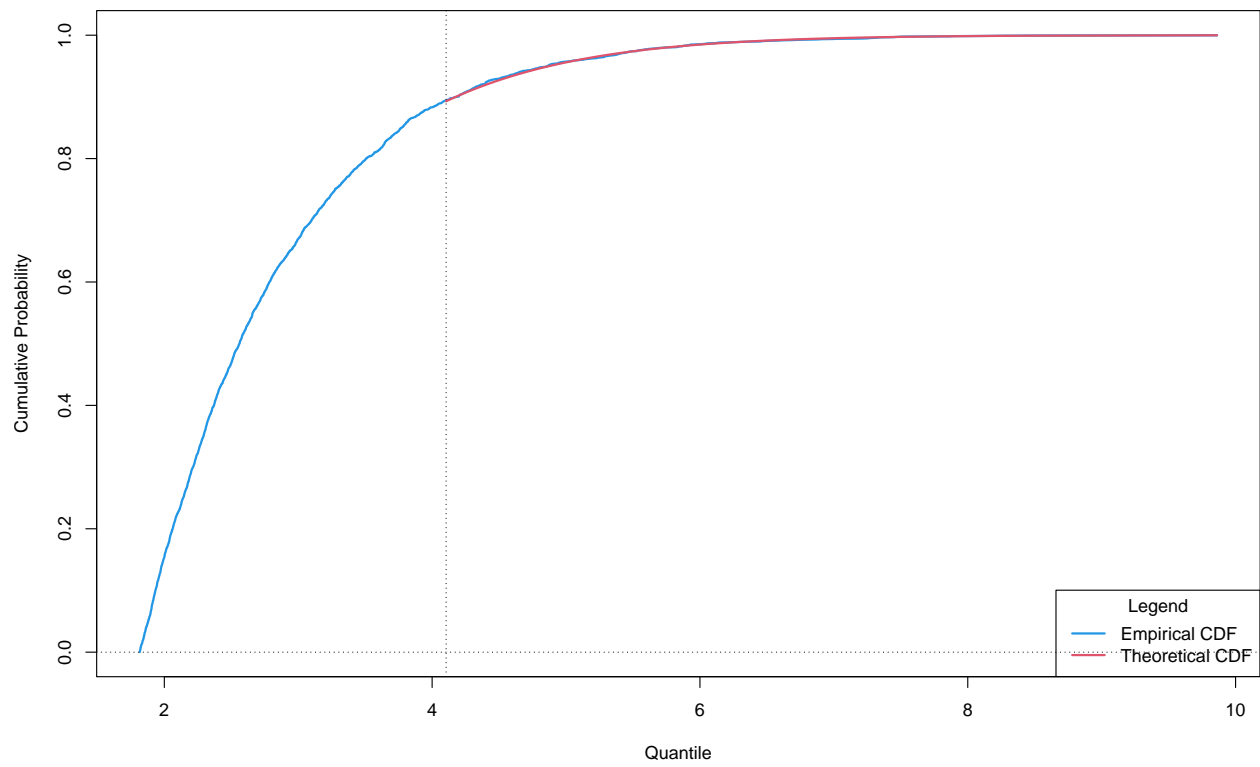
Probability Density Function (PDF) Plot : automatic_weights – model_wise = FALSE : zoom = TRUE



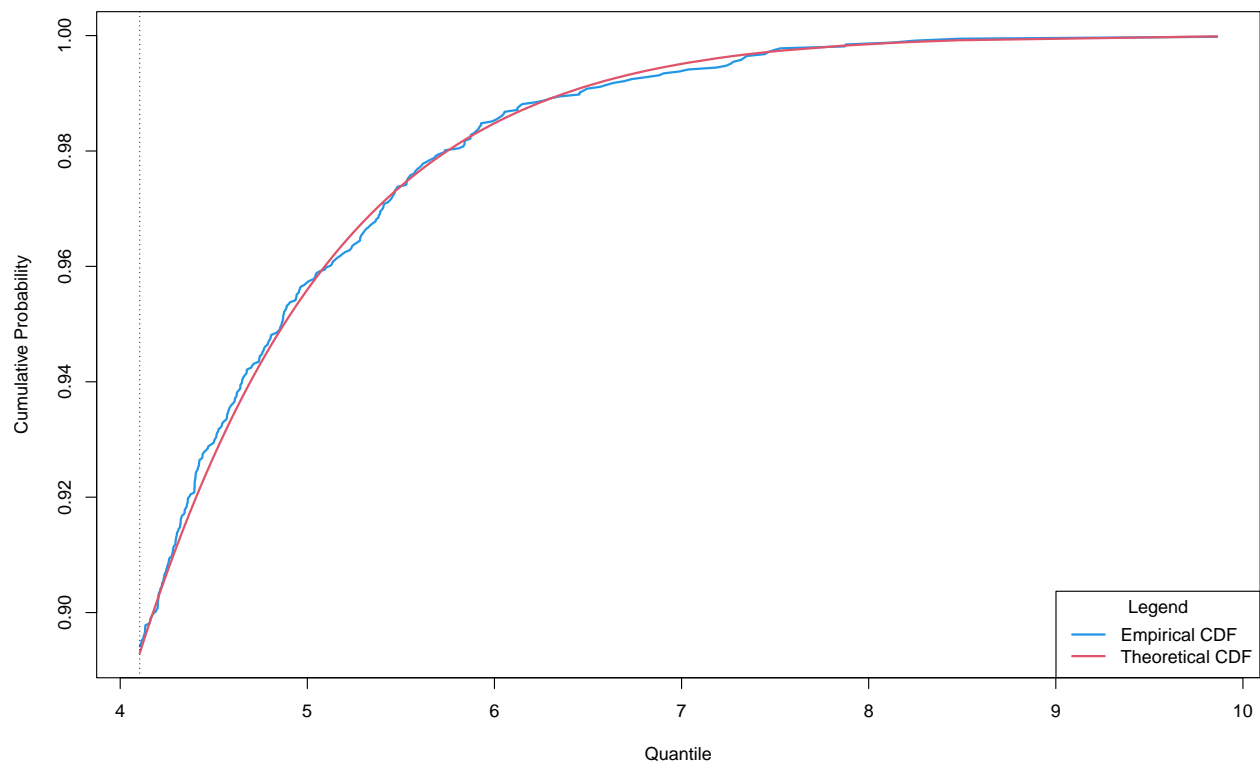
```
par(mfrow = c(2, 1))
plot_gev_mixture_model_cdf(gev_mixture_model,
                             type = "automatic_weights",
                             model_wise = FALSE,
                             zoom = FALSE,
                             xlab = "Quantile",
                             ylab = "Cumulative Probability",
                             main = "Cumulative Distribution Function (CDF) Plot")

plot_gev_mixture_model_cdf(gev_mixture_model,
                             type = "automatic_weights",
                             model_wise = FALSE,
                             zoom = TRUE,
                             xlab = "Quantile",
                             ylab = "Cumulative Probability",
                             main = "Cumulative Distribution Function (CDF) Plot")
```

Cumulative Distribution Function (CDF) Plot : automatic_weights – model_wise = FALSE : zoom = FALSE



Cumulative Distribution Function (CDF) Plot : automatic_weights – model_wise = FALSE : zoom = TRUE



```

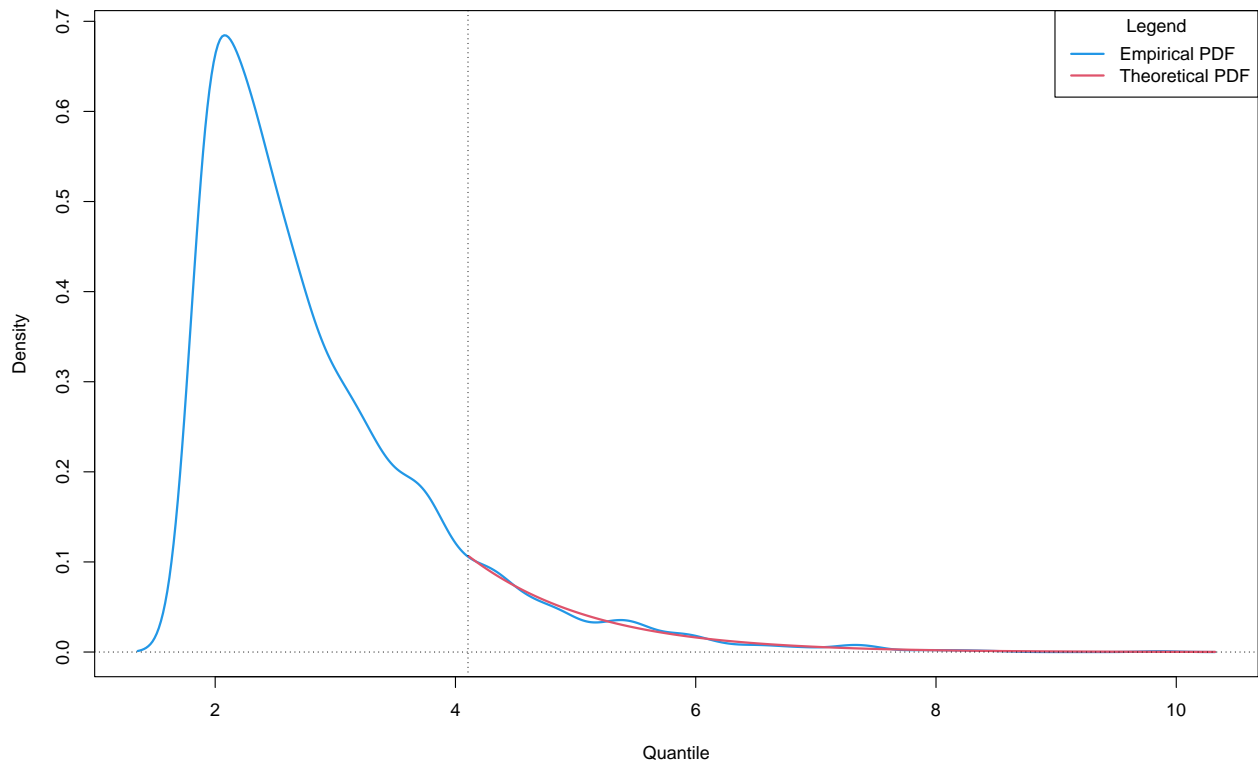
## GEV mixture model with respect to distribution functions

par(mfrow = c(2, 1))
plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = TRUE,
                           zoom = FALSE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")

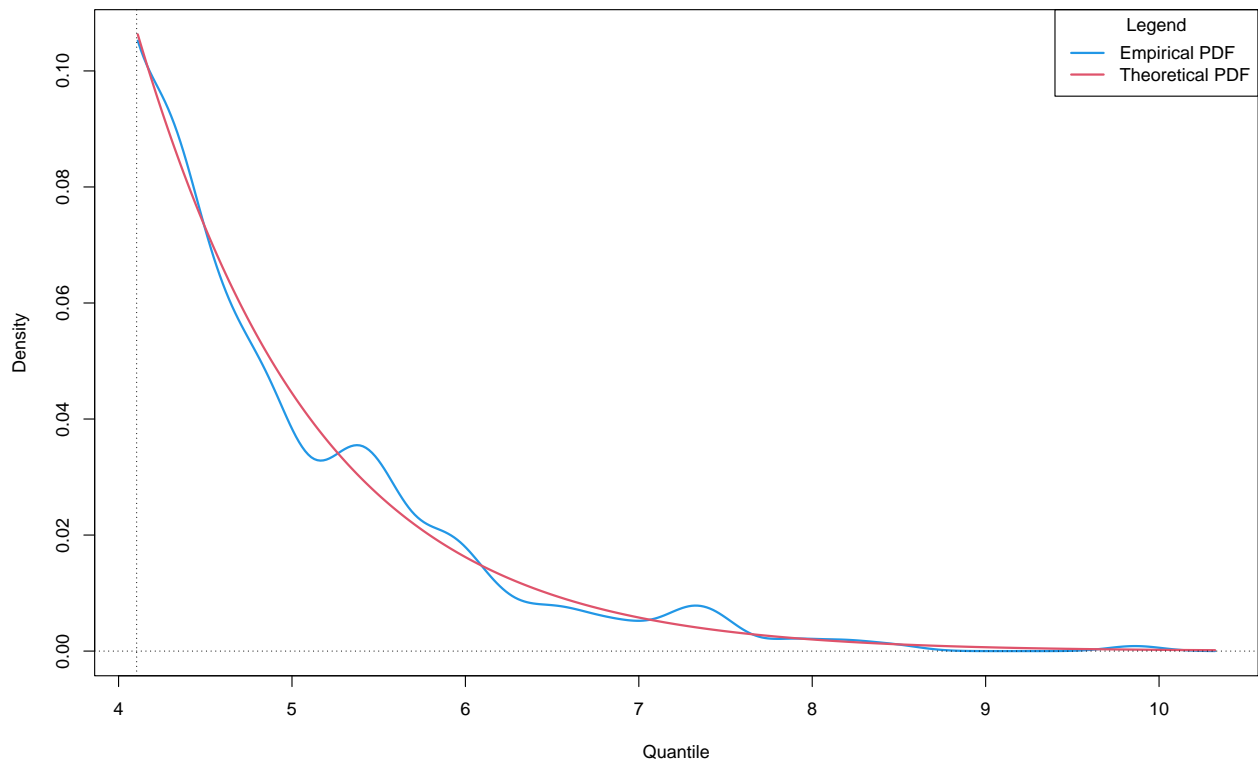
plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = TRUE,
                           zoom = TRUE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")

```

Probability Density Function (PDF) Plot : automatic_weights – model_wise = TRUE : zoom = FALSE



Probability Density Function (PDF) Plot : automatic_weights – model_wise = TRUE : zoom = TRUE



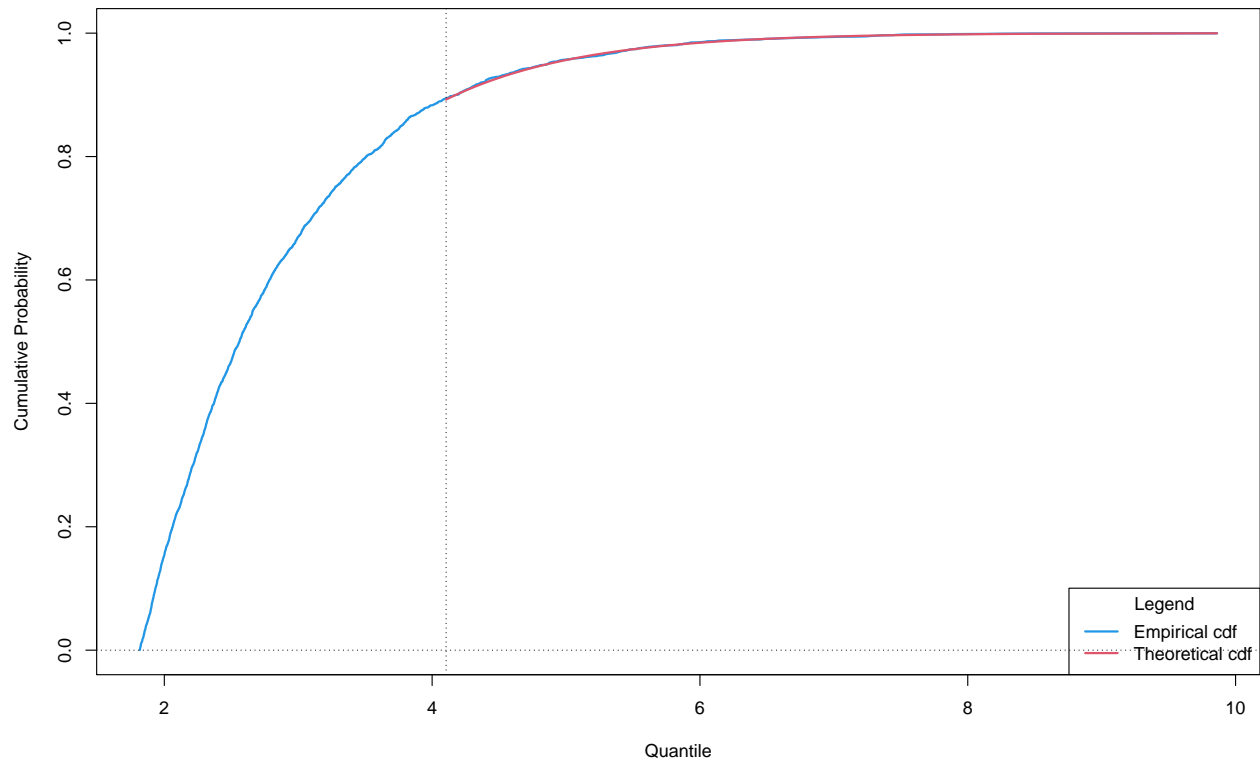
```

par(mfrow = c(2, 1))
plot_gev_mixture_model_cdf(gev_mixture_model,
    type = "automatic_weights",
    model_wise = TRUE,
    zoom = FALSE,
    xlab = "Quantile",
    ylab = "Cumulative Probability",
    main = "Cumulative Distribution Function (CDF) Plot")

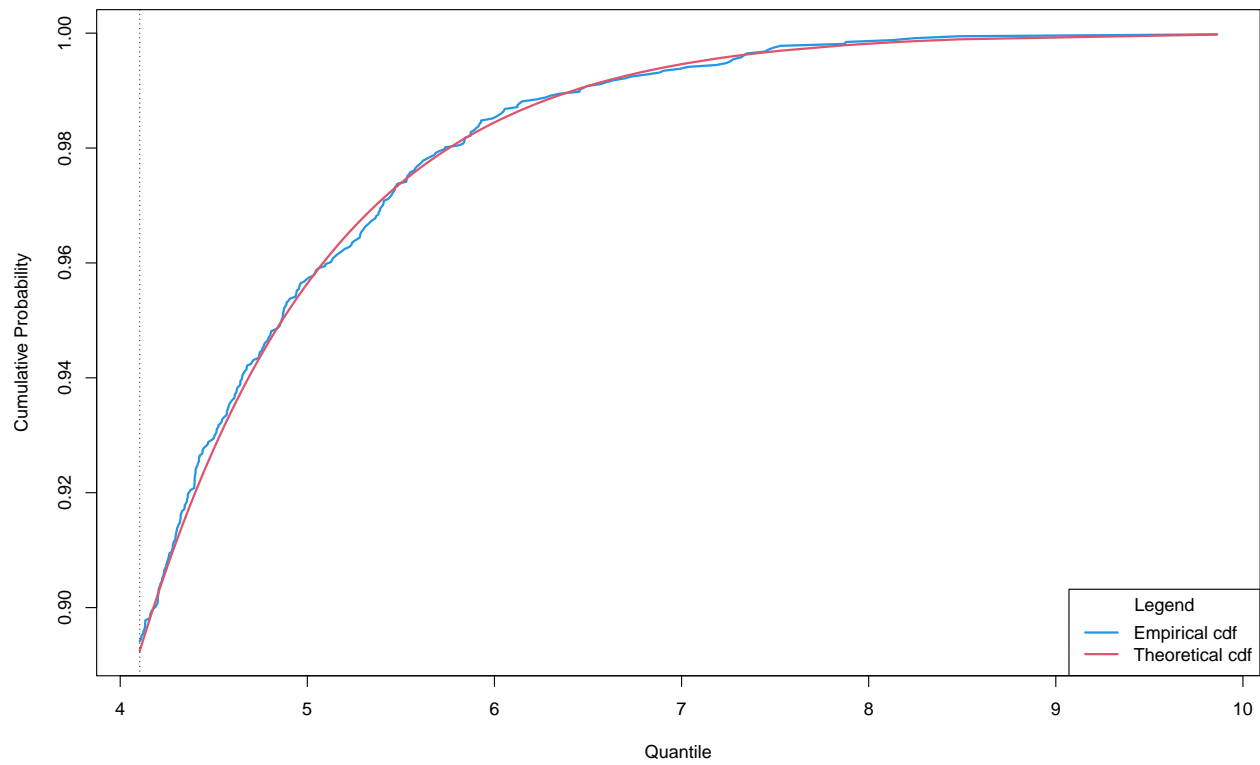
plot_gev_mixture_model_cdf(gev_mixture_model,
    type = "automatic_weights",
    model_wise = TRUE,
    zoom = TRUE,
    xlab = "Quantile",
    ylab = "Cumulative Probability",
    main = "Cumulative Distribution Function (CDF) Plot")

```

Cumulative Distribution Function (CDF) Plot : automatic_weights – model_wise = TRUE : zoom = FALSE



Cumulative Distribution Function (CDF) Plot : automatic_weights – model_wise = TRUE : zoom = TRUE




```

# Estimation of an extreme quantile

estimator_types <- c("automatic_weights_mw",
                    "pessimistic_weights_mw",
                    "identic_weights_mw",
                    "automatic_weights_pw",
                    "pessimistic_weights_pw",
                    "identic_weights_pw",
                    "empirical",
                    "confidence_interval_mw",
                    "confidence_interval_pw")

alpha <- 10^(-14)

## Quantile from the true distribution

true_rl <- calculate_gev_inverse_cdf(p = 1 - alpha,
                                   loc = loc,
                                   scale = scale,
                                   shape = shape)

true_rl

## [1] 32.2369909

## Quantile from GEV mixture model with respect to parameters

rl_pw <- estimate_gev_mixture_model_quantile(gev_mixture_model,
                                             alpha = alpha,
                                             confidence_level = 0.95,
                                             do.ci = TRUE,
                                             estimator_type = estimator_types[4])

rl_pw[2]

##      estimate
## 1 19.55448181

## Quantile from GEV mixture model with respect to distribution functions

rl_mw <- estimate_gev_mixture_model_quantile(gev_mixture_model,
                                             alpha = alpha,
                                             confidence_level = 0.95,
                                             do.ci = TRUE,
                                             estimator_type = estimator_types[1])

rl_mw[2]

##      estimate
## 1 43.37664867

## Quantiles from equivalent estimated GEV models

est_rl_pw <- estimate_gev_mixture_model_quantile(gev_mixture_model,
                                                  alpha = alpha,
                                                  confidence_level = 0.95,
                                                  do.ci = TRUE,

```

```
estimator_type = estimator_types[9])
```

```
est_rl_pw
```

##		lower	estimate	upper
## 11	-6.516411742	33.0837167	72.68384513	
## 12	-13.84734027	37.76149386	89.370328	
## 13	-28.16634973	45.81317766	119.792705	
## 14	-0.487219334	23.38264338	47.2525061	
## 15	-6.003543013	27.21161621	60.42677543	
## 16	0.7623373438	21.62228163	42.48222591	
## 17	-6.472641819	26.79037605	60.05339392	
## 18	1.608681945	19.84522168	38.08176142	
## 19	5.211411905	17.05075368	28.89009546	
## 20	0.7764479501	19.40521749	38.03398703	
## 21	3.616290933	17.32320228	31.03011362	
## 22	5.766118057	15.33968505	24.91325203	
## 23	5.275762243	15.8358521	26.39594196	
## 24	0.2984154504	19.14025434	37.98209322	
## 25	3.29981576	16.57710101	29.85438627	
## 26	4.037046936	16.28336301	28.52967909	
## 27	3.825087764	15.86769583	27.91030389	
## 28	6.156337811	14.43871904	22.72110026	
## 29	4.445843552	15.36188304	26.27792252	
## 30	4.90117977	15.14265186	25.38412395	
## 31	5.794118415	14.81438217	23.83464593	
## 32	-1.245837637	18.18993768	37.62571301	
## 33	7.951670666	12.36169682	16.77172298	
## 34	7.334808947	12.92668049	18.51855202	
## 35	7.92681784	12.35575774	16.78469764	
## 36	6.919829124	13.41010197	19.90037482	
## 37	6.730952376	13.31368991	19.89642744	
## 38	4.698532734	14.57089047	24.4432482	
## 39	6.304939818	13.52916788	20.75339595	
## 40	7.634160577	12.55365417	17.47314776	
## 41	5.125197366	14.29515514	23.46511292	
## 42	7.585725866	12.56323445	17.54074303	
## 43	5.307978487	13.96526761	22.62255673	
## 44	4.265390636	14.86725538	25.46912013	
## 45	7.347736507	12.71888793	18.09003935	
## 46	4.725112959	14.47828788	24.23146281	
## 47	2.979416616	15.46740046	27.9553843	
## 48	5.886841146	13.52555759	21.16427403	
## 49	5.497115358	13.62035653	21.74359771	
## 50	7.47551101	12.25007938	17.02464775	
## 51	4.806568439	14.09569249	23.38481653	
## 52	3.387825504	15.15067579	26.91352608	
## 53	5.348040574	14.01679538	22.68555018	
## 54	3.0114578	15.01572508	27.01999236	
## 55	-12.87102893	20.52451735	53.92006363	
## 56	4.764193635	14.08580452	23.4074154	
## 57	5.524070011	13.23025683	20.93644365	
## 58	6.828588528	12.70752896	18.58646939	
## 59	0.1029810382	16.00920027	31.9154195	

```
## 60 6.092752512 13.21968723 20.34662194
## Comparison of estimated quantiles

est_rl_pw_range <- range(as.matrix(est_rl_pw))

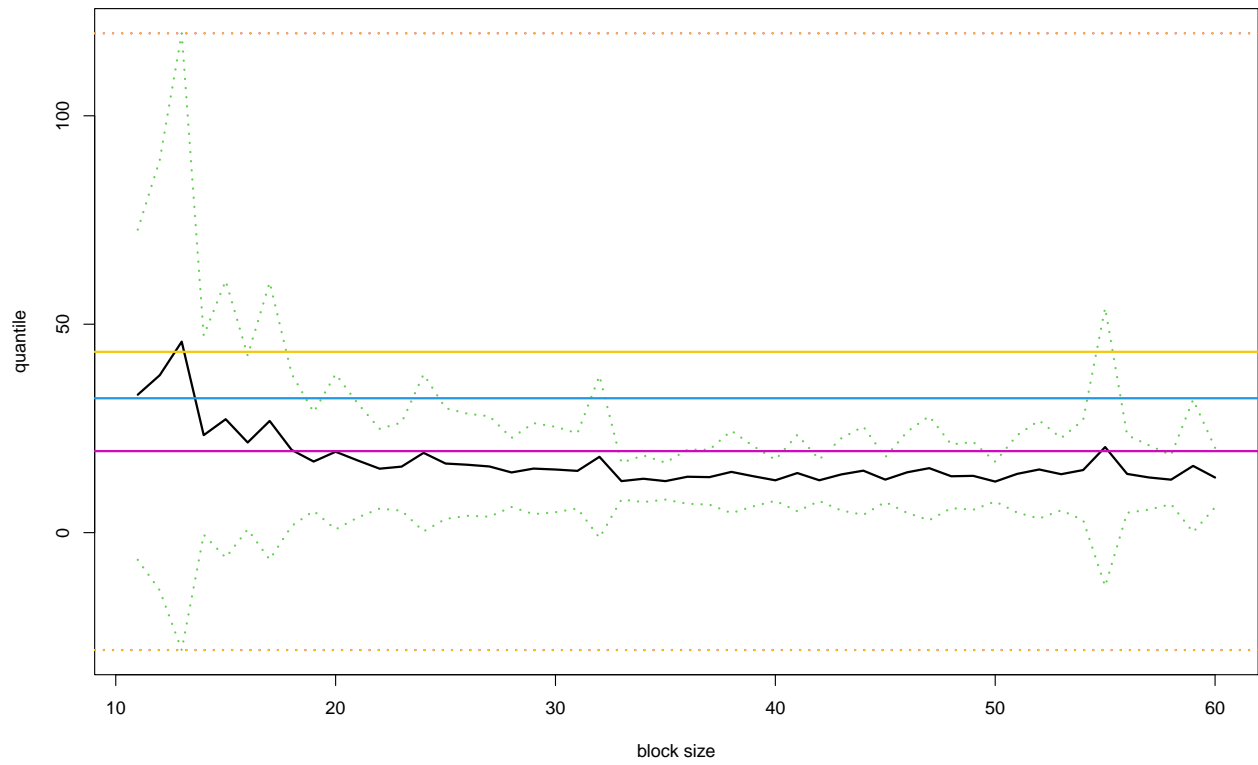
est_rl_mw <- estimate_gev_mixture_model_quantile(gev_mixture_model,
                                                  alpha = alpha,
                                                  confidence_level = 0.95,
                                                  do.ci = TRUE,
                                                  estimator_type = estimator_types[8])

est_rl_mw_range <- range(as.matrix(est_rl_mw))

matplot(x = rownames(est_rl_pw),
        y = est_rl_pw,
        xlab = "block size",
        ylab = "quantile",
        main = "Estimates of a quantile",
        ylim = range(c(est_rl_pw_range, true_rl)),
        cex = 1,
        cex.lab = 1,
        cex.axis = 1,
        type = "l",
        lty = c("dotted", "solid", "dotted"),
        lwd = c(2,2,2),
        col = c(3, 1, 3))

abline(h = true_rl, col = 4, lwd = 2)
abline(h = rl_mw[2], col = 7, lwd = 2)
abline(h = rl_pw[2], col = 6, lwd = 2)
abline(h = est_rl_pw_range, col = 6, lty = "dotted", lwd = 2)
abline(h = est_rl_mw_range, col = 7, lty = "dotted", lwd = 2)
```

Estimates of a quantile



Legend:
blue: Quantile from the true distribution
yellow: Quantile from GEV mixture model with respect to distribution functions
pink: Quantile from GEV mixture model with respect to parameters