## Modeling extreme values with a GEV mixture probability distributions

## Pascal Alain Dkengne Sielenou

September 08th, 2023

```
source("./src/estimate gev mixture model parameters.R")
source("./src/plot_gev_mixture_model_pdf.R")
source("./src/generate_gev_sample.R")
source("./src/plot_normalized_gev_mixture_model_pdf.R")
source("./src/calculate_gev_inverse_cdf.R")
source("./src/calculate_gev_mixture_model_inverse_cdf.R")
source("./src/calculate_gev_mixture_model_cdf.R")
n <- 20000
nlargest <- 1000
x <- generate_gev_sample(n = n, loc = 1, scale = 0.5, shape = 0.1)
\#x \leftarrow rnorm(n = n)
gev_mixture_model <- estimate_gev_mixture_model_parameters(x,</pre>
                                                             nsloc = NULL,
                                                             std.err = FALSE,
                                                             block sizes = NULL,
                                                             minimum_nblocks = 50,
                                                             nlargest = nlargest,
                                                             confidence level = 0.95,
                                                             log_mv = TRUE,
                                                             log_pw = TRUE,
                                                             trace = TRUE)
```

```
## iter: 0 f-value: 0.0818045692229777 pgrad: 0.116557301520905
## iter: 10 f-value: 0.00645277788901075
                                          pgrad: 0.00490695616336123
## iter: 20 f-value: 0.00644207597742333
                                                  0.00490210395299373
                                          pgrad:
## iter: 30 f-value: 0.00643139528540268
                                          pgrad:
                                                  0.00489725654445849
## iter: 40 f-value: 0.00642073577085853
                                          pgrad:
                                                  0.00489241393299966
## iter: 50 f-value: 0.00641009739178396 pgrad: 0.00488757611386559
## iter: 60 f-value: 0.0063994801062556 pgrad: 0.00488274308230963
## iter: 70 f-value: 0.00638888387243312 pgrad: 0.00487791483359035
## iter: 80 f-value: 0.00637830864855929
                                          pgrad:
                                                  0.00487309136297054
                                          pgrad: 0.0048682726657171
## iter: 90 f-value: 0.00636775439295979
## iter: 100 f-value: 0.00635722106404289 pgrad: 0.00486345873710292
## iter: 110 f-value: 0.00634670862029943 pgrad: 0.00485864957240467
## iter: 120 f-value: 0.00633621702030267
                                           pgrad: 0.00485384516690379
## iter: 130 f-value: 0.00632574622270798 pgrad: 0.00484904551588683
## iter: 140 f-value: 0.00631529618625292 pgrad: 0.00484425061464422
```

```
0.00630486686975674 pgrad:
          150
               f-value:
                                                         0.00483946045847228
## iter:
                                                        0.00483467504267077
          160
               f-value:
                          0.0062944582321205
                                               pgrad:
   iter:
          170
                f-value:
                          0.00628407023232702
                                                 pgrad:
                                                         0.00482989436254466
          180
                          0.00627370282944006
   iter:
               f-value:
                                                 pgrad:
                                                         0.00482511841340383
                          0.00626335598260489
##
   iter:
          190
               f-value:
                                                 pgrad:
                                                         0.0048203471905619
          200
                f-value:
                          0.00625302965104794
                                                 pgrad:
                                                         0.00481558068933841
   iter:
  iter:
          210
                f-value:
                          0.00624272379407645
                                                 pgrad:
                                                         0.00481081890505652
          220
  iter:
               f-value:
                          0.00623243837107827
                                                 pgrad:
                                                         0.00480606183304455
   iter:
          230
                f-value:
                          0.00622217334152204
                                                 pgrad:
                                                         0.00480130946863488
   iter:
          240
                f-value:
                          0.00621192866495676
                                                 pgrad:
                                                         0.00479656180716537
  iter:
          250
                f-value:
                          0.00620170430101157
                                                         0.00479181884397784
                                                 pgrad:
          260
   iter:
               f-value:
                          0.006191500209396
                                              pgrad:
                                                       0.00478708057441901
          270
               f-value:
##
   iter:
                          0.00618131634989931
                                                 pgrad:
                                                         0.00478234699384028
   iter:
          280
                          0.00617115268239048
                f-value:
                                                 pgrad:
                                                         0.00477761809759725
          290
  iter:
                f-value:
                          0.00616100916681847
                                                 pgrad:
                                                         0.0047728938810504
          300
                f-value:
                          0.00615088576321122
   iter:
                                                 pgrad:
                                                         0.004768174339565
          310
                f-value:
                          0.00614078243167655
##
   iter:
                                                 pgrad:
                                                         0.00476345946851064
          320
                f-value:
                          0.00613069913240094
   iter:
                                                 pgrad:
                                                         0.00475874926326159
          330
               f-value:
                          0.00612063582565017
##
  iter:
                                                 pgrad:
                                                         0.00475404371919652
   iter:
          340
               f-value:
                          0.00611059247176879
                                                 pgrad:
                                                         0.00474934283169925
##
  iter:
          350
               f-value:
                          0.00610056903118001
                                                 pgrad:
                                                         0.00474464659615725
          360
  iter:
                f-value:
                          0.00609056546438562
                                                 pgrad:
                                                         0.00473995500796343
          370
               f-value:
                          0.00608058173196571
                                                 pgrad:
                                                         0.00473526806251501
## iter:
          380
  iter:
                f-value:
                          0.00607061779457868
                                                 pgrad:
                                                         0.00473058575521335
  iter:
          390
                f-value:
                          0.00606067361296091
                                                 pgrad:
                                                         0.004725908081465
  iter:
          400
                f-value:
                          0.00605074914792679
                                                 pgrad:
                                                         0.00472123503668076
          410
   iter:
               f-value:
                          0.00604084436036833
                                                 pgrad:
                                                         0.00471656661627606
##
   iter:
          420
               f-value:
                          0.00603095921125538
                                                         0.00471190281567024
                                                 pgrad:
          430
   iter:
                f-value:
                          0.00602109366163493
                                                 pgrad:
                                                         0.0047072436302884
          440
                f-value:
  iter:
                          0.00601124767263149
                                                 pgrad:
                                                         0.00470258905555954
   iter:
          450
                f-value:
                          0.00600142120544667
                                                 pgrad:
                                                         0.00469793908691674
##
          460
                f-value:
                          0.00599161422135897
                                                         0.0046932937197981
   iter:
                                                 pgrad:
          470
                f-value:
                          0.00598182668172387
                                                         0.00468865294964649
   iter:
                                                 pgrad:
          480
               f-value:
                          0.00597205854797332
   iter:
                                                 pgrad:
                                                         0.00468401677190888
                          0.0059623097816161
          490
   iter:
                f-value:
                                               pgrad:
                                                        0.0046793851820367
##
   iter:
          500
               f-value:
                          0.00595258034423708
                                                pgrad:
                                                         0.00467475817548624
  iter:
          510
                f-value:
                          0.0059428701974976
                                               pgrad:
                                                        0.00467013574771802
          520
                          0.00593317930313479
  iter:
                f-value:
                                                 pgrad:
                                                         0.00466551789419734
          530
                          0.00592350762296206
   iter:
                f-value:
                                                 pgrad:
                                                         0.00466090461039337
          540
                                               pgrad:
##
                f-value:
                          0.0059138551188682
                                                        0.00465629589178063
  iter:
  iter:
          550
                f-value:
                          0.00590422175281807
                                                pgrad:
                                                         0.00465169173383728
          560
                          0.0058946074868515
   iter:
               f-value:
                                               pgrad:
                                                        0.00464709213204706
   iter:
          570
               f-value:
                          0.00588501228308416
                                                pgrad:
                                                         0.00464249708189701
          580
                          0.0058754361037064
   iter:
                f-value:
                                               pgrad:
                                                        0.00463790657887908
## iter:
          590
                f-value:
                          0.00586587891098403
                                                pgrad:
                                                         0.00463332061849009
          600
## iter:
               f-value:
                          0.0058563406672575
                                                pgrad:
                                                        0.00462873919623052
##
  iter:
          610
                f-value:
                          0.00584682133494182
                                                 pgrad:
                                                         0.00462416230760632
          620
   iter:
                f-value:
                          0.00583732087652701
                                                 pgrad:
                                                         0.00461958994812728
   iter:
          630
                f-value:
                          0.00582783925457722
                                                         0.00461502211330722
                                                 pgrad:
          640
               f-value:
                          0.005818376431731
   iter:
                                              pgrad:
                                                       0.00461045879866551
          650
##
                          0.00580893237070086
   iter:
               f-value:
                                                 pgrad:
                                                         0.00460589999972505
  iter:
          660
                f-value:
                          0.00579950703427359
                                                 pgrad:
                                                         0.00460134571201354
          670
                          0.00579010038530959
## iter:
               f-value:
                                                 pgrad:
                                                         0.00459679593106277
          680
               f-value:
                          0.00578071238674303
                                                pgrad:
## iter:
                                                         0.00459225065240954
```

```
690
                f-value:
                          0.00577134300158158
                                                          0.00458770987159485
## iter:
                                                 pgrad:
          700
## iter:
                f-value:
                          0.00576199219290647
                                                 pgrad:
                                                          0.00458317358416382
   iter:
          710
                f-value:
                          0.00575265992387194
                                                 pgrad:
                                                          0.00457864178566647
          720
   iter:
                f-value:
                          0.00574334615770559
                                                 pgrad:
                                                          0.00457411447165679
##
   iter:
          730
                f-value:
                          0.00573405085770787
                                                 pgrad:
                                                          0.00456959163769355
          740
                f-value:
                          0.00572477398725196
                                                 pgrad:
##
   iter:
                                                          0.00456507327933942
  iter:
          750
                f-value:
                           0.00571551550978384
                                                 pgrad:
                                                          0.00456055939216193
## iter:
          760
                f-value:
                          0.00570627538882193
                                                 pgrad:
                                                          0.0045560499717332
   iter:
          770
                f-value:
                          0.00569705358795729
                                                 pgrad:
                                                          0.00455154501362898
          780
   iter:
                f-value:
                          0.00568785007085287
                                                 pgrad:
                                                          0.00454704451342969
  iter:
          790
                f-value:
                          0.00567866480124399
                                                          0.00454254846672075
                                                 pgrad:
          800
##
   iter:
                f-value:
                          0.0056694977429378
                                                pgrad:
                                                         0.00453805686909103
##
          810
                f-value:
   iter:
                          0.00566034885981314
                                                 pgrad:
                                                          0.0045335697161345
   iter:
          820
                                                          0.00452908700344934
##
                f-value:
                          0.00565121811582092
                                                 pgrad:
          830
##
  iter:
                f-value:
                          0.00564210547498315
                                                 pgrad:
                                                          0.00452460872663762
          840
                f-value:
                          0.0056330109013935
                                                         0.00452013488130631
   iter:
                                                pgrad:
          850
##
                f-value:
                          0.00562393435921681
   iter:
                                                 pgrad:
                                                          0.00451566546306637
          860
                f-value:
                           0.00561487581268884
   iter:
                                                 pgrad:
                                                          0.00451120046753328
          870
                f-value:
                          0.00560583522611678
##
   iter:
                                                 pgrad:
                                                          0.00450673989032724
   iter:
          880
                f-value:
                          0.00559681256387817
                                                 pgrad:
                                                          0.00450228372707207
##
   iter:
          890
                f-value:
                          0.0055878077904215
                                                pgrad:
                                                         0.00449783197339626
          900
  iter:
                f-value:
                           0.00557882087026555
                                                 pgrad:
                                                          0.00449338462493309
  iter:
          910
                f-value:
                          0.00556985176799984
                                                          0.00448894167731906
##
                                                 pgrad:
          920
   iter:
                f-value:
                          0.0055609004482839
                                                pgrad:
                                                         0.00448450312619608
                                                pgrad:
##
  iter:
          930
                f-value:
                          0.0055519668758476
                                                         0.00448006896720987
   iter:
          940
                f-value:
                          0.00554305101549046
                                                 pgrad:
                                                          0.00447563919601057
          950
                          0.00553415283208223
   iter:
                f-value:
                                                 pgrad:
                                                          0.00447121380825255
##
   iter:
          960
                f-value:
                          0.00552527229056205
                                                 pgrad:
                                                          0.00446679279959483
          970
                f-value:
##
   iter:
                          0.00551640935593881
                                                 pgrad:
                                                          0.00446237616570011
  iter:
          980
                f-value:
                          0.00550756399329078
                                                 pgrad:
                                                          0.00445796390223574
   iter:
          990
                f-value:
                          0.00549873616776556
                                                 pgrad:
                                                          0.00445355600487363
   iter:
          1000
                 f-value:
                            0.00548992584457976
                                                  pgrad:
                                                           0.0044491524692899
          1010
                 f-value:
                            0.00548113298901929
                                                           0.00444475329116412
   iter:
                                                  pgrad:
          1020
                 f-value:
                           0.00547235756643857
                                                           0.00444035846618152
   iter:
                                                  pgrad:
                            0.0054635995422611
          1030
                                                          0.00443596799003021
   iter:
                 f-value:
                                                 pgrad:
##
   iter:
          1040
                 f-value:
                           0.00545485888197876
                                                  pgrad:
                                                           0.00443158185840353
   iter:
          1050
                 f-value:
                            0.00544613555115211
                                                  pgrad:
                                                           0.00442720006699904
          1060
                            0.00543742951540975
  iter:
                 f-value:
                                                  pgrad:
                                                           0.00442282261151805
          1070
                            0.00542874074044874
   iter:
                 f-value:
                                                  pgrad:
                                                           0.00441844948766656
          1080
                                                 pgrad:
                 f-value:
                            0.0054200691920342
                                                          0.00441408069115434
   iter:
   iter:
          1090
                 f-value:
                            0.00541141483599901
                                                  pgrad:
                                                           0.00440971621769615
          1100
                            0.00540277763824387
   iter:
                 f-value:
                                                  pgrad:
                                                           0.00440535606301051
   iter:
          1110
                 f-value:
                            0.00539415756473733
                                                  pgrad:
                                                           0.00440100022281997
          1120
                 f-value:
   iter:
                            0.00538555458151521
                                                  pgrad:
                                                           0.00439664869285228
  iter:
          1130
                 f-value:
                            0.00537696865468089
                                                           0.00439230146883818
                                                  pgrad:
          1140
                 f-value:
   iter:
                            0.00536839975040499
                                                  pgrad:
                                                           0.00438795854651342
   iter:
          1150
                 f-value:
                            0.00535984783492521
                                                  pgrad:
                                                           0.00438361992161784
   iter:
          1160
                 f-value:
                            0.00535131287454635
                                                  pgrad:
                                                           0.00437928558989531
   iter:
          1170
                 f-value:
                            0.00534279483563974
                                                           0.00437495554709422
                                                  pgrad:
   iter:
          1180
                 f-value:
                            0.00533429368464393
                                                           0.00437062978896685
                                                  pgrad:
          1190
                            0.00532580938806365
   iter:
                 f-value:
                                                  pgrad:
                                                           0.00436630831126994
   iter:
          1200
                 f-value:
                            0.0053173419124703
                                                 pgrad:
                                                          0.00436199110976443
          1210
                 f-value:
                            0.0053088912245015
                                                          0.00435767818021515
## iter:
                                                 pgrad:
          1220
                 f-value:
                           0.00530045729086117
                                                  pgrad:
## iter:
                                                         0.0043533695183916
```

```
pgrad:
## iter: 1230
               f-value:
                           0.00529204007831924
                                                         0.00434906512006705
                f-value:
          1240
                           0.00528363955371155
## iter:
                                                 pgrad:
                                                          0.00434476498101921
  iter:
          1250
                f-value:
                           0.00527525568393974
                                                 pgrad:
                                                          0.00434046909703012
          1260
                f-value:
                           0.00526688843597127
                                                          0.00433617746388548
  iter:
                                                 pgrad:
   iter:
          1270
                f-value:
                           0.00525853777683894
                                                 pgrad:
                                                          0.00433189007737567
          1280
                f-value:
                           0.00525020367364101
                                                 pgrad:
   iter:
                                                          0.00432760693329481
          1290
  iter:
                f-value:
                           0.00524188609354119
                                                 pgrad:
                                                          0.00432332802744184
                           0.00523358500376812
## iter:
          1300
                f-value:
                                                 pgrad:
                                                          0.00431905335561922
                           0.00522530037161555
   iter:
          1310
                f-value:
                                                 pgrad:
                                                          0.00431478291363363
          1320
   iter:
                f-value:
                           0.00521703216444223
                                                 pgrad:
                                                          0.00431051669729621
  iter:
          1330
                f-value:
                           0.00520878034967143
                                                 pgrad:
                                                          0.00430625470242207
          1340
                           0.00520054489479136
                                                          0.00430199692483069
  iter:
                f-value:
                                                 pgrad:
   iter:
          1350
                f-value:
                           0.00519232576735442
                                                 pgrad:
                                                          0.00429774336034561
          1360
                                                 pgrad:
   iter:
                f-value:
                           0.00518412293497774
                                                          0.00429349400479406
          1370
                f-value:
                           0.00517593636534228
  iter:
                                                 pgrad:
                                                          0.00428924885400805
## iter:
          1380
                f-value:
                           0.00516776602619362
                                                 pgrad:
                                                          0.00428500790382313
          1390
   iter:
                f-value:
                           0.00515961188534097
                                                 pgrad:
                                                          0.00428077115007974
          1400
                f-value:
                           0.00515147391065747
                                                 pgrad:
                                                          0.00427653858862165
   iter:
  iter:
          1410
                           0.00514335207008011
                f-value:
                                                 pgrad:
                                                          0.00427231021529728
   iter:
          1420
                f-value:
                           0.00513524633160947
                                                 pgrad:
                                                          0.00426808602595896
  iter:
          1430
                f-value:
                           0.0051271566633095
                                                pgrad:
                                                        0.00426386601646289
          1440
                f-value:
                           0.00511908303330771
                                                 pgrad:
  iter:
                                                          0.00425965018266983
          1450
                           0.00511102540979471
## iter:
                f-value:
                                                 pgrad:
                                                          0.00425543852044452
          1460
                           0.00510298376102426
  iter:
                f-value:
                                                 pgrad:
                                                          0.00425123102565561
## iter:
                                                          0.00424702769417629
          1470
                f-value:
                           0.00509495805531309
                                                 pgrad:
  iter:
          1480
                f-value:
                           0.00508694826104092
                                                 pgrad:
                                                          0.00424282852188296
          1490
                f-value:
                           0.00507895434664998
                                                          0.00423863350465714
  iter:
                                                 pgrad:
                           0.00507097628064527
   iter:
          1500
                f-value:
                                                 pgrad:
                                                         0.00423444263838368
                       0.0818045692229777
                                             pgrad:
   iter:
             f-value:
                                                     0.116557301520905
## iter:
              f-value:
                        0.00565150664868701 pgrad:
                                                       0.00452923344402456
          10
## iter:
          20
              f-value:
                         0.0048173251476876
                                              pgrad:
                                                      0.00409896428094347
   iter:
          30
              f-value:
                         0.00428973234223551
                                              pgrad:
                                                       0.00208851171368815
   iter:
              f-value:
                         0.00399092398543415
                                               pgrad:
                                                       0.00179597829685192
   iter:
              f-value:
                         0.00376996021247007
                                              pgrad:
                                                       0.00154441941676664
          50
     Successful convergence.
## iter:
                        0.0808505295557911
          0
             f-value:
                                             pgrad:
                                                     0.298285027768035
## iter:
              f-value: 0.0221356618135646
                                             pgrad:
                                                      0.234220714924961
## iter:
          20
              f-value:
                         0.0163306973407655
                                             pgrad:
                                                      0.190501419463078
          30
                         0.0124882088894578
                                              pgrad:
   iter:
              f-value:
                                                      0.158815736163187
  iter:
          40
              f-value:
                         0.00988449826158982
                                              pgrad:
                                                       0.133649149798946
                         0.00808942464421206
  iter:
          50
              f-value:
                                              pgrad:
                                                       0.113010540495617
          60
                         0.00683521554817273
                                                       0.0958153237745902
  iter:
              f-value:
                                               pgrad:
                                              pgrad:
                         0.0059494522695678
   iter:
          70
              f-value:
                                                      0.0818905860696397
          80
                         0.00531825091912158
                                               pgrad:
   iter:
              f-value:
                                                       0.0703964119265424
## iter:
          90
              f-value:
                         0.0048648941017042
                                             pgrad:
                                                      0.0606954403924165
          100
               f-value:
                          0.00453689312699397
                                                pgrad:
## iter:
                                                        0.0524851990372676
  iter:
          110
               f-value:
                          0.00429788744934875
                                                pgrad:
                                                        0.0455207052300079
          120
   iter:
               f-value:
                          0.00412243453220038
                                                pgrad:
                                                        0.0396016542396601
  iter:
          130
               f-value:
                          0.00399258449686564
                                                pgrad:
                                                        0.0345629950449209
   iter:
          140
               f-value:
                          0.00389558738380447
                                                pgrad:
                                                        0.0302678489295685
          150
                          0.00382233545372471
   iter:
               f-value:
                                                pgrad:
                                                        0.0266020910484547
## iter:
          160
               f-value:
                          0.00376629166913751
                                                pgrad:
                                                        0.0234701370747463
## iter:
          170
               f-value:
                          0.00372274489276432
                                                pgrad:
                                                        0.0207916242757186
## iter:
          180
               f-value:
                          0.00368828756068676
                                                pgrad:
                                                        0.0184987634790533
```

```
190
                f-value:
                          0.00366044658901953
                                                 pgrad:
                                                          0.0165342065954048
          200
                                                          0.0148493110851241
  iter:
                f-value:
                          0.00363742077104073
                                                 pgrad:
   iter:
          210
                f-value:
                          0.00361789272201691
                                                 pgrad:
                                                          0.0134027164717722
          220
   iter:
                f-value:
                          0.00360089326425507
                                                 pgrad:
                                                          0.0121591666016866
##
   iter:
          230
                f-value:
                          0.00358570271097087
                                                 pgrad:
                                                          0.0110885253693718
          240
                f-value:
                          0.00357177793535135
##
   iter:
                                                 pgrad:
                                                          0.0101649463550474
  iter:
          250
                f-value:
                           0.00355869691071027
                                                 pgrad:
                                                          0.00961240986705104
  iter:
          260
                f-value:
                          0.00354611401726685
                                                 pgrad:
                                                          0.00935754512189675
   iter:
          270
                f-value:
                           0.00353371952748566
                                                 pgrad:
                                                          0.00913310492969202
          280
   iter:
                f-value:
                          0.00352119455004405
                                                 pgrad:
                                                          0.00893356010024722
   iter:
          290
                f-value:
                          0.00350814464643004
                                                          0.00875358666352545
                                                 pgrad:
          300
##
   iter:
                f-value:
                          0.00349396857067213
                                                 pgrad:
                                                          0.00858749626530403
##
          310
                f-value:
   iter:
                          0.00347751192665673
                                                 pgrad:
                                                          0.00842789335006082
          320
                           0.00345575412823342
                                                          0.00826104187227199
##
   iter:
                f-value:
                                                 pgrad:
          330
##
  iter:
                f-value:
                           0.0034131990024046
                                                pgrad:
                                                         0.00826018918590035
          340
                f-value:
                           0.00292864417948742
   iter:
                                                 pgrad:
                                                          0.00761390977254997
          350
##
                f-value:
                          0.00252683033632397
   iter:
                                                 pgrad:
                                                          0.00671738461178008
          360
                f-value:
                           0.00220459393006409
   iter:
                                                 pgrad:
                                                          0.005927204092121
          370
                f-value:
                          0.0019466869383843
##
   iter:
                                                pgrad:
                                                         0.00523043878627594
   iter:
          380
                f-value:
                          0.00174063934385388
                                                 pgrad:
                                                          0.00461575000344738
##
   iter:
          390
                f-value:
                          0.0015762941140159
                                                pgrad:
                                                         0.0040732320998191
          400
                           0.00144540619323016
  iter:
                f-value:
                                                 pgrad:
                                                          0.00359422118783862
          410
                f-value:
                          0.00134130370068582
                                                 pgrad:
                                                          0.00317113084982187
##
  iter:
          420
##
   iter:
                f-value:
                          0.00125860333682877
                                                 pgrad:
                                                          0.00279731114534569
##
  iter:
          430
                f-value:
                          0.00119297321077633
                                                 pgrad:
                                                          0.00249265860024324
   iter:
          440
                f-value:
                          0.00114093600658978
                                                 pgrad:
                                                          0.00222937851832185
          450
                          0.00109970671700223
##
   iter:
                f-value:
                                                 pgrad:
                                                          0.00199611840728461
##
   iter:
          460
                f-value:
                          0.00106705932643387
                                                          0.00178945686137494
                                                 pgrad:
          470
##
   iter:
                f-value:
                          0.0010412179898452
                                                pgrad:
                                                         0.0016063606294647
          480
                f-value:
                           0.00102076837792102
  iter:
                                                 pgrad:
                                                          0.00144414081289207
##
   iter:
          490
                f-value:
                          0.00100458581729854
                                                 pgrad:
                                                          0.00130041454384291
##
          500
                f-value:
                          0.000991777207072398
                                                           0.00117307016156581
   iter:
                                                  pgrad:
          510
                f-value:
                           0.000981634234613364
                                                           0.00106023713758893
   iter:
                                                  pgrad:
          520
                f-value:
                          0.00097359576398791
##
   iter:
                                                 pgrad:
                                                          0.000960257894546263
          530
                          0.000967217718211271
   iter:
                f-value:
                                                  pgrad:
                                                           0.000871664319894216
##
   iter:
          540
                f-value:
                          0.000962148963108319
                                                  pgrad:
                                                           0.00079315563734525
  iter:
          550
                f-value:
                           0.000958112097853491
                                                  pgrad:
                                                           0.000723579868489227
          560
##
  iter:
                f-value:
                           0.000954888116248917
                                                           0.000661916334756152
                                                  pgrad:
          570
   iter:
                f-value:
                          0.00095230423499862
                                                 pgrad:
                                                          0.000607261006493387
          580
##
                f-value:
                          0.000950224213991097
   iter:
                                                  pgrad:
                                                           0.000558813043581574
   iter:
          590
                f-value:
                           0.000948540675578742
                                                  pgrad:
                                                           0.000515862971224024
          600
##
   iter:
                f-value:
                          0.000947169033988041
                                                  pgrad:
                                                           0.000477782316846836
   iter:
          610
                f-value:
                          0.000946042674981681
                                                  pgrad:
                                                           0.000444014263882075
          620
                f-value:
##
   iter:
                          0.000945109156771679
                                                  pgrad:
                                                           0.000414065545779052
  iter:
          630
                f-value:
                          0.000944327197179668
                                                           0.000387498794790081
                                                  pgrad:
          640
                f-value:
##
  iter:
                          0.000943664304286144
                                                  pgrad:
                                                           0.000363926830688782
##
   iter:
          650
                f-value:
                          0.000943094897116522
                                                  pgrad:
                                                           0.000343006161274029
   iter:
          660
                f-value:
                           0.00094259882898013
                                                 pgrad:
                                                          0.000324432448531781
   iter:
          670
                f-value:
                          0.000942160216505754
                                                           0.000307935697241968
                                                  pgrad:
          680
                f-value:
                          0.000941766522224711
                                                           0.000293276500583167
   iter:
                                                  pgrad:
          690
##
                f-value:
                          0.000941407814139493
                                                           0.000280242310773005
   iter:
                                                  pgrad:
                f-value:
   iter:
          700
                          0.000941076193430645
                                                  pgrad:
                                                           0.000268644352344444
          710
                          0.000940765328668992
## iter:
                f-value:
                                                  pgrad:
                                                           0.000258314979156682
          720
                f-value:
                          0.000940470078829594
                                                  pgrad:
## iter:
                                                           0.000249104646711268
```

```
730
                f-value:
                           0.00094018618399507
                                                          0.000240880228878859
## iter:
                                                 pgrad:
          740
## iter:
                f-value:
                           0.000939910000006777
                                                  pgrad:
                                                           0.000233522528571284
   iter:
          750
                f-value:
                           0.000939638261876039
                                                  pgrad:
                                                           0.000226924569432824
          760
                                                           0.000220989349077855
##
   iter:
                f-value:
                           0.000939367834954649
                                                  pgrad:
##
   iter:
          770
                f-value:
                           0.000939095445047416
                                                  pgrad:
                                                           0.000215628352846803
          780
##
                f-value:
                           0.000938817332252797
   iter:
                                                  pgrad:
                                                           0.000210759378995007
  iter:
          790
                f-value:
                           0.000938528669871896
                                                  pgrad:
                                                           0.00020630328559354
## iter:
          800
                f-value:
                           0.000938222618070778
                                                  pgrad:
                                                           0.000202180684993752
##
   iter:
          810
                f-value:
                           0.000937888360326332
                                                           0.000198305035291252
                                                  pgrad:
##
   iter:
          820
                f-value:
                           0.000937506488340221
                                                  pgrad:
                                                           0.00019456783534777
  iter:
          830
                f-value:
                           0.000937035442629204
                                                           0.000190801001892005
                                                  pgrad:
          840
##
   iter:
                f-value:
                           0.000936397257838085
                                                  pgrad:
                                                           0.000187033302947198
##
          850
                f-value:
                           0.000935706462817269
                                                           0.000183523789402479
   iter:
                                                  pgrad:
   iter:
          860
                                                  pgrad:
##
                f-value:
                           0.000935223006430148
                                                           0.000183513129033946
          870
                f-value:
##
  iter:
                           0.000935219146702352
                                                  pgrad:
                                                           0.000183201340974048
##
   iter:
          880
                f-value:
                           0.000935215015275442
                                                           0.000182926262521338
                                                  pgrad:
          890
##
                f-value:
                           0.000935210423939326
   iter:
                                                  pgrad:
                                                           0.000182678254132734
          900
                f-value:
                           0.00093520500340034
                                                          0.000182447059354274
   iter:
                                                 pgrad:
##
          910
                f-value:
                           0.000935197760088436
                                                           0.000182216037949628
   iter:
                                                  pgrad:
##
   iter:
          920
                f-value:
                           0.000935184155737379
                                                  pgrad:
                                                           0.000181938531475356
##
   iter:
          930
                f-value:
                           0.000934318758398091
                                                  pgrad:
                                                           0.000178366635500304
          940
  iter:
                f-value:
                           0.000934314349951214
                                                  pgrad:
                                                           0.000178567084572145
          950
  iter:
                f-value:
                           0.000934309096820704
                                                           0.000178751573399472
##
                                                  pgrad:
          960
##
   iter:
                f-value:
                           0.000934302154344617
                                                  pgrad:
                                                           0.000178926831822301
##
  iter:
          970
                f-value:
                           0.000934289981239566
                                                  pgrad:
                                                           0.000179108874492351
  iter:
          980
                f-value:
                           0.000930753569932613
                                                  pgrad:
                                                           0.000183461456155498
          990
                           0.000930726268763319
##
   iter:
                f-value:
                                                  pgrad:
                                                           0.000169900661563638
##
   iter:
          1000
                 f-value:
                            0.00093072295021543
                                                  pgrad:
                                                           0.000170322732302969
          1010
##
   iter:
                 f-value:
                            0.000930719550275278
                                                   pgrad:
                                                            0.000170684314023145
  iter:
          1020
                 f-value:
                            0.000930716024126893
##
                                                   pgrad:
                                                            0.00017099606965551
##
   iter:
           1030
                 f-value:
                            0.00093071231049726
                                                  pgrad:
                                                           0.000171266749697169
##
   iter:
           1040
                 f-value:
                            0.000930708301400532
                                                            0.000171504593493327
                                                   pgrad:
##
           1050
                 f-value:
                            0.000930703788324123
                                                            0.000171717970788937
   iter:
                                                   pgrad:
          1060
##
                 f-value:
                            0.000930698316531485
                                                            0.000171916477463258
   iter:
                                                   pgrad:
                 f-value:
           1070
                            0.000930690590696089
   iter:
                                                   pgrad:
                                                            0.000172114594219064
          1080
##
   iter:
                 f-value:
                            0.000930672315827153
                                                   pgrad:
                                                            0.000172359272037126
   iter:
           1090
                 f-value:
                            0.000929552654633175
                                                   pgrad:
                                                            0.000177134678452956
          1100
                 f-value:
                            0.000929549533637528
##
  iter:
                                                   pgrad:
                                                            0.000176393707977451
          1110
                 f-value:
   iter:
                            0.000929546399217766
                                                   pgrad:
                                                            0.000175763890894326
          1120
##
                 f-value:
                            0.000929543241261897
                                                   pgrad:
                                                            0.000175227692106178
   iter:
                                                  pgrad:
   iter:
          1130
                 f-value:
                            0.00092954004816145
                                                           0.000174770523376971
          1140
                            0.000929536798010724
##
   iter:
                 f-value:
                                                   pgrad:
                                                            0.000174379259256785
   iter:
          1150
                 f-value:
                            0.000929533469612865
                                                   pgrad:
                                                            0.000174043447642325
          1160
                 f-value:
                            0.000929530017652257
##
   iter:
                                                   pgrad:
                                                            0.000173753319576309
  iter:
          1170
                 f-value:
                            0.000929526378872964
                                                            0.000173500553117212
                                                   pgrad:
          1180
                 f-value:
##
  iter:
                            0.000929522440221145
                                                   pgrad:
                                                            0.000173277165481489
##
   iter:
          1190
                 f-value:
                            0.000929517995659022
                                                   pgrad:
                                                            0.000173075732644024
          1200
##
   iter:
                 f-value:
                            0.000929512611801471
                                                   pgrad:
                                                            0.000172887768258856
   iter:
          1210
                 f-value:
                            0.000929504995397941
                                                            0.000172698449902198
                                                   pgrad:
   iter:
          1220
                 f-value:
                            0.00092948770820166
                                                           0.000172466126173637
                                                  pgrad:
##
          1230
                            0.000927033170227906
                                                            0.000243299761354954
   iter:
                 f-value:
                                                   pgrad:
   iter:
          1240
                 f-value:
                            0.000927027644571644
                                                   pgrad:
                                                            0.000221088601478625
  iter:
          1250
                 f-value:
                            0.000927022845171311
                                                            0.000202281227989404
                                                   pgrad:
          1260
                 f-value:
                            0.000927018566155787
                                                   pgrad:
## iter:
                                                            0.000186355718873238
```

```
## iter: 1270
                f-value:
                           0.000927014659951009 pgrad: 0.00017287043186609
          1280
                           0.00092701102073066
## iter:
                f-value:
                                                 pgrad:
                                                        0.00016145130913614
                                                  pgrad:
   iter:
          1290
                 f-value:
                           0.000927007572508781
                                                           0.000151781446470656
          1300
                           0.00092700426073075
                                                 pgrad:
   iter:
                 f-value:
                                                          0.000153378326749387
   iter:
          1310
                f-value:
                           0.000927001046167038
                                                  pgrad:
                                                           0.00015601953145622
          1320
                           0.000926997900522447
                 f-value:
                                                  pgrad:
                                                           0.000158255809321273
   iter:
  iter:
          1330
                 f-value:
                           0.000926994803171658
                                                  pgrad:
                                                           0.00016014956751445
  iter:
          1340
                f-value:
                           0.000926991738824516
                                                  pgrad:
                                                           0.000161753314439794
   iter:
          1350
                 f-value:
                           0.000926988696212463
                                                  pgrad:
                                                           0.000163111651907744
          1360
   iter:
                 f-value:
                           0.00092698566625103
                                                 pgrad:
                                                          0.000164262375107416
   iter:
          1370
                 f-value:
                           0.000926982641257248
                                                  pgrad:
                                                           0.000165237568242271
          1380
   iter:
                 f-value:
                           0.00092697961448026
                                                 pgrad:
                                                          0.000166064339816446
   iter:
          1390
                 f-value:
                           0.000926976578282681
                                                           0.000166765695417265
                                                  pgrad:
          1400
                                                  pgrad:
   iter:
                 f-value:
                           0.000926973524914943
                                                           0.00016736112949256
          1410
  iter:
                 f-value:
                           0.000926970443384563
                                                  pgrad:
                                                           0.000167867380941156
   iter:
          1420
                 f-value:
                           0.000926967319501393
                                                           0.000168298536900424
                                                  pgrad:
          1430
##
                 f-value:
                           0.000926964131443727
                                                           0.0001686670286353
   iter:
                                                   pgrad:
                f-value:
          1440
                           0.000926960848010878
                                                           0.000168983135368145
   iter:
                                                   pgrad:
          1450
##
                 f-value:
                           0.000926957421197675
                                                           0.000169255988840905
   iter:
                                                  pgrad:
   iter:
          1460
                 f-value:
                           0.000926953780870564
                                                  pgrad:
                                                           0.000169493181130076
##
   iter:
          1470
                 f-value:
                           0.000926949786904251
                                                   pgrad:
                                                           0.00016970270211572
          1480
                           0.000926945155729468
  iter:
                 f-value:
                                                  pgrad:
                                                           0.000169892711503361
  iter:
          1490
                 f-value:
                           0.000926939251411093
                                                  pgrad:
                                                           0.000170072110494274
                           0.000926929730026061
   iter:
          1500
                f-value:
                                                  pgrad:
                                                           0.000170258795915174
  iter:
             f-value:
                        0.0808505295557911
                                             pgrad:
                                                     0.298285027768035
   iter:
          10
              f-value:
                         0.00548798061356966
                                               pgrad:
                                                        0.0736229270327704
                         0.00375137544122695
   iter:
          20
              f-value:
                                               pgrad:
                                                        0.0220798236760894
##
   iter:
              f-value:
                         0.00352446703823117
                                               pgrad:
                                                        0.00872028563350478
          40
                         0.00245379051364538
   iter:
              f-value:
                                               pgrad:
                                                        0.006552974486036
  iter:
          50
              f-value:
                         0.00129724788647376
                                               pgrad:
                                                        0.00539672147484921
##
  iter:
          60
              f-value:
                         0.00116336349069024
                                               pgrad:
                                                        0.00333775280378497
##
   iter:
          70
              f-value:
                         0.00108903466397249
                                               pgrad:
                                                        0.00190921293452803
          80
              f-value:
                         0.00101864754852102
                                               pgrad:
                                                        0.00142504956225517
   iter:
          90
              f-value:
                         0.000976654378619111
                                                         0.000997552748068009
   iter:
                                                pgrad:
          100
               f-value:
                          0.000958449297472636
   iter:
                                                 pgrad:
                                                          0.000744402940192845
##
   iter:
          110
               f-value:
                          0.00095166681308669
                                                pgrad:
                                                         0.0006398708971532
   iter:
          120
                f-value:
                          0.000948031278701184
                                                 pgrad:
                                                          0.000493636979067735
  iter:
          130
                          0.000944572041868178
##
                f-value:
                                                 pgrad:
                                                          0.000389096222985164
          140
                f-value:
                          0.000943078043273489
   iter:
                                                 pgrad:
                                                          0.000351752915581288
          150
##
  iter:
                f-value:
                          0.000941792566572578
                                                 pgrad:
                                                          0.000260500752026244
                                                 pgrad:
   iter:
          160
                f-value:
                          0.000940741435894291
                                                          0.000276463268609362
          170
               f-value:
                          0.000940636432666963
   iter:
                                                 pgrad:
                                                          0.000240171951534279
   iter:
          180
                f-value:
                          0.000939234800584778
                                                 pgrad:
                                                          0.000238212914218244
          190
                f-value:
                          0.00093902315453756
   iter:
                                                pgrad:
                                                         0.000201735673817816
                                                 pgrad:
  iter:
          200
                f-value:
                          0.000938571292168387
                                                          0.000192644573155154
          210
                f-value:
                          0.000937952033971741
## iter:
                                                 pgrad:
                                                          0.000177217643574396
##
   iter:
          220
                f-value:
                          0.00093739079839151
                                                pgrad:
                                                         0.000244182425818085
          230
   iter:
                f-value:
                          0.000937207441240713
                                                 pgrad:
                                                          0.000185814173194215
   iter:
          240
               f-value:
                          0.000936779838412506
                                                 pgrad:
                                                          0.000181145989543663
   iter:
          250
               f-value:
                          0.000936199021137431
                                                 pgrad:
                                                          0.000175474987286844
          260
##
               f-value:
                          0.000933859784910175
                                                          0.000176623839547807
   iter:
                                                 pgrad:
  iter:
          270
                f-value:
                          0.000933323755560183
                                                 pgrad:
                                                          0.000177107718977415
## iter:
          280
                f-value:
                          0.000932422885391989
                                                          0.000176963697996163
                                                 pgrad:
## iter:
          290
               f-value:
                          0.000931944483337105
                                                 pgrad:
                                                          0.000211438375784329
```

```
300
                f-value:
                           0.000931416984018967
## iter:
                                                  pgrad:
                                                           0.000234455486610552
          310
  iter:
                f-value:
                           0.000930866248624855
                                                  pgrad:
                                                           0.000232578872019246
   iter:
          320
                f-value:
                           0.000930372144892009
                                                  pgrad:
                                                           0.000218021374214145
          330
##
   iter:
                f-value:
                           0.000929805626990325
                                                  pgrad:
                                                           0.000166046154887661
##
   iter:
           340
                f-value:
                           0.000928935132850013
                                                   pgrad:
                                                           0.000172391493145824
          350
##
                f-value:
                           0.000911299335602487
   iter:
                                                   pgrad:
                                                           0.000150133355183657
##
  iter:
           360
                f-value:
                           0.000910792702328878
                                                   pgrad:
                                                           0.000137121159005937
##
  iter:
          370
                f-value:
                           0.000910380532074656
                                                   pgrad:
                                                           0.000135984944698297
##
   iter:
           380
                f-value:
                           0.000910019026358382
                                                           0.000150158233286707
                                                   pgrad:
##
   iter:
           390
                f-value:
                           0.000909662998212293
                                                   pgrad:
                                                           0.000167893313677225
##
   iter:
           400
                f-value:
                           0.000909320259350745
                                                           0.000217217289964738
                                                   pgrad:
          410
##
   iter:
                f-value:
                           0.000909080264169477
                                                   pgrad:
                                                           0.000147726878078791
##
          420
                f-value:
                           0.000908564351074193
   iter:
                                                   pgrad:
                                                           0.000173908430467329
          430
##
   iter:
                f-value:
                           0.000908457529040839
                                                   pgrad:
                                                           0.000152011648711287
          440
##
   iter:
                f-value:
                           0.000908132796990118
                                                   pgrad:
                                                           0.000149683361802988
           450
                f-value:
                           0.000907777811892765
##
   iter:
                                                   pgrad:
                                                           0.000138964265146313
           460
##
                f-value:
                           0.000907330602978034
                                                           0.000142268590275829
   iter:
                                                   pgrad:
           470
                f-value:
                           0.000906992611653637
   iter:
                                                   pgrad:
                                                           0.000138196809484514
          480
                                                           0.000142670470115208
##
                f-value:
                           0.000906532298810811
   iter:
                                                   pgrad:
##
   iter:
          490
                f-value:
                           0.000905745636456882
                                                  pgrad:
                                                           0.000148490649227395
##
   iter:
          500
                f-value:
                           0.000904408525301197
                                                           0.00014785292495928
                                                   pgrad:
          510
##
  iter:
                f-value:
                           0.00090396535369911
                                                  pgrad:
                                                          0.000130137298605448
          520
##
                f-value:
                           0.000903655815251337
                                                           0.000152002905409288
  iter:
                                                   pgrad:
          530
##
   iter:
                f-value:
                           0.000901041716584621
                                                   pgrad:
                                                           0.00024087325351637
                           0.000900995864844732
##
   iter:
          540
                f-value:
                                                   pgrad:
                                                           0.000153050174727609
   iter:
          550
                f-value:
                           0.000900204100776305
                                                   pgrad:
                                                           0.000135934240603791
          560
##
   iter:
                f-value:
                           0.000885205148947248
                                                   pgrad:
                                                           0.000111407053386026
          570
##
   iter:
                f-value:
                           0.000884596670800475
                                                           0.000178650669660191
                                                   pgrad:
          580
##
   iter:
                f-value:
                           0.000884555542932767
                                                   pgrad:
                                                           0.000120847076920061
##
          590
                f-value:
  iter:
                           0.000884360728121233
                                                   pgrad:
                                                           0.000119138622282827
##
   iter:
          600
                f-value:
                           0.000884161146366584
                                                   pgrad:
                                                           0.000112818966605727
##
          610
                f-value:
                           0.000883938323542387
                                                           0.000115237668878582
   iter:
                                                   pgrad:
##
           620
                f-value:
                           0.000883740069550177
                                                           0.000136013136517454
   iter:
                                                   pgrad:
          630
                f-value:
                           0.000883252093375448
##
   iter:
                                                   pgrad:
                                                           0.000375274211366773
          640
                f-value:
##
   iter:
                           0.000882981640486224
                                                   pgrad:
                                                           0.000159285328574399
          650
##
   iter:
                f-value:
                           0.000882910483332704
                                                   pgrad:
                                                           0.000118839115670927
##
  iter:
           660
                f-value:
                           0.000882749149308992
                                                  pgrad:
                                                           0.000119455416399794
          670
                           0.000881677156954622
##
  iter:
                f-value:
                                                  pgrad:
                                                           0.000175297068429031
          680
##
   iter:
                f-value:
                           0.000881641854953563
                                                  pgrad:
                                                           0.000116008775739085
          690
##
                f-value:
                           0.000881456040322125
                                                           0.000110492970813725
   iter:
                                                   pgrad:
   iter:
           700
                f-value:
                           0.000880987318158473
                                                   pgrad:
                                                           0.000114432273138759
          710
                f-value:
                           0.000880683606117975
##
   iter:
                                                   pgrad:
                                                           0.000119135613326105
##
   iter:
          720
                f-value:
                           0.000880293184436292
                                                   pgrad:
                                                           0.000113563192447513
          730
                           0.000876705979090863
##
   iter:
                f-value:
                                                   pgrad:
                                                           0.000790468558171775
           740
##
  iter:
                f-value:
                           0.000876430521342055
                                                           0.000195832757759383
                                                  pgrad:
          750
##
  iter:
                f-value:
                           0.000876403272013374
                                                  pgrad:
                                                           0.000117113659757972
##
   iter:
           760
                f-value:
                           0.00087606310415655
                                                  pgrad:
                                                          0.000146040552894994
          770
##
   iter:
                f-value:
                           0.000876036817299847
                                                   pgrad:
                                                           0.000106351367205409
##
   iter:
          780
                f-value:
                           0.000875284973329705
                                                           0.00011862494375367
                                                   pgrad:
          790
                f-value:
                           0.000874715235623463
                                                           0.00197781697131819
##
   iter:
                                                   pgrad:
          800
                           0.000871503592612927
##
                f-value:
   iter:
                                                   pgrad:
                                                           0.000106341014158878
##
   iter:
          810
                f-value:
                           0.000870921476208527
                                                   pgrad:
                                                           0.000637461427453218
## iter:
          820
                           0.000870788360100893
                                                           0.000188148525523735
                f-value:
                                                  pgrad:
          830
                f-value:
                           0.000870769965051153
                                                  pgrad:
## iter:
                                                           9.24562728696621e-05
```

```
## iter:
          840
                f-value:
                          0.000869782707384213
                                                           0.000325517248099677
                                                  pgrad:
          850
## iter:
                f-value:
                          0.000869747206300914
                                                  pgrad:
                                                           0.000103503975828922
                                                           0.000257909816845953
  iter:
          860
                f-value:
                          0.000869570069737923
                                                  pgrad:
          870
                          0.000869540297747904
##
  iter:
                f-value:
                                                  pgrad:
                                                           9.11766312308437e-05
##
   iter:
          880
                f-value:
                          0.0008694246915281
                                                pgrad:
                                                         8.72849172989798e-05
          890
##
                f-value:
                          0.000868198110230017
                                                           0.000134129363432017
   iter:
                                                  pgrad:
  iter:
          900
                f-value:
                           0.000868176127136443
                                                  pgrad:
                                                           9.10889848200203e-05
## iter:
          910
                f-value:
                          0.000868072793085388
                                                  pgrad:
                                                           8.9365836384997e-05
##
   iter:
          920
                f-value:
                           0.000867813932850335
                                                  pgrad:
                                                           0.000106845205426198
##
   iter:
          930
                f-value:
                          0.000867717224645632
                                                  pgrad:
                                                           0.000103926104819463
   iter:
          940
                f-value:
                          0.000867682126906662
                                                           8.98477865260983e-05
                                                  pgrad:
          950
##
   iter:
                f-value:
                          0.000867341512251412
                                                  pgrad:
                                                           9.25783919814016e-05
##
          960
                f-value:
                          0.000867050873213932
   iter:
                                                  pgrad:
                                                           8.40392770761478e-05
   iter:
          970
                          0.000866938857328822
##
                f-value:
                                                  pgrad:
                                                           8.25200164079887e-05
          980
                f-value:
##
  iter:
                          0.000866679500668022
                                                  pgrad:
                                                           0.000133552854302749
##
   iter:
          990
                f-value:
                          0.000866661127223277
                                                           8.75565078733431e-05
                                                  pgrad:
           1000
##
                 f-value:
                            0.000866516710082035
   iter:
                                                   pgrad:
                                                            7.8479457386621e-05
                 f-value:
           1010
                            0.000866117637541737
                                                            0.000106777812799053
   iter:
                                                   pgrad:
          1020
##
                 f-value:
                            0.00086602629052526
                                                           7.64240393270332e-05
   iter:
                                                  pgrad:
                                                  pgrad:
##
   iter:
          1030
                 f-value:
                            0.00086599477481023
                                                           8.63397245310094e-05
##
   iter:
          1040
                 f-value:
                            0.000865895959497257
                                                   pgrad:
                                                            8.77184576159595e-05
           1050
  iter:
                 f-value:
                            0.000865795391732938
                                                   pgrad:
                                                            8.93039748645352e-05
          1060
##
  iter:
                 f-value:
                            0.000865672996838399
                                                            8.80494766625495e-05
                                                   pgrad:
          1070
##
   iter:
                 f-value:
                            0.000865498958359052
                                                   pgrad:
                                                            0.000102973811512891
##
          1080
  iter:
                 f-value:
                            0.000865318428573917
                                                   pgrad:
                                                            0.000247509317630151
   iter:
          1090
                 f-value:
                            0.000865184575917115
                                                   pgrad:
                                                            8.72462030020449e-05
          1100
                 f-value:
                            0.000865073110103693
##
   iter:
                                                   pgrad:
                                                            8.33993905146602e-05
##
   iter:
          1110
                 f-value:
                            0.000864957068275234
                                                            8.53151332724233e-05
                                                   pgrad:
          1120
                 f-value:
                            0.000864874306091054
##
   iter:
                                                   pgrad:
                                                            8.38849431639144e-05
##
  iter:
          1130
                 f-value:
                            0.000864792322682233
                                                   pgrad:
                                                            8.51009338558584e-05
##
   iter:
          1140
                 f-value:
                            0.000864700369003013
                                                   pgrad:
                                                            8.71433028523383e-05
##
   iter:
          1150
                 f-value:
                            0.000864610522984255
                                                            9.13145288353179e-05
                                                   pgrad:
##
   iter:
          1160
                 f-value:
                            0.00086449849776892
                                                           9.00534203512604e-05
                                                  pgrad:
          1170
##
   iter:
                 f-value:
                            0.000864108350910587
                                                            0.000148768819233355
                                                   pgrad:
          1180
                 f-value:
                            0.000864092889006071
   iter:
                                                   pgrad:
                                                            8.89389917064221e-05
          1190
##
   iter:
                 f-value:
                            0.000863800535660713
                                                   pgrad:
                                                            0.000124423856240209
   iter:
          1200
                 f-value:
                            0.000863787491205523
                                                   pgrad:
                                                            8.47025907825816e-05
          1210
                 f-value:
                            0.000863702727329401
##
  iter:
                                                   pgrad:
                                                            8.80376615201017e-05
          1220
                 f-value:
                            0.000862196264113388
   iter:
                                                   pgrad:
                                                            0.000120063345510867
          1230
##
   iter:
                 f-value:
                            0.000862184446974828
                                                            8.06323868154379e-05
                                                   pgrad:
           1240
   iter:
                 f-value:
                            0.000862102726797721
                                                   pgrad:
                                                            8.67939028335041e-05
          1250
                 f-value:
                            0.000862001793217786
##
   iter:
                                                   pgrad:
                                                            8.46278722012594e-05
          1260
##
   iter:
                 f-value:
                            0.000861927250550088
                                                   pgrad:
                                                            8.79723704710345e-05
##
          1270
                 f-value:
                            0.000861784158658091
   iter:
                                                   pgrad:
                                                            0.000109589034993918
          1280
##
  iter:
                 f-value:
                            0.000861773284966722
                                                            7.72994024770268e-05
                                                   pgrad:
          1290
                 f-value:
## iter:
                            0.000861699894980909
                                                   pgrad:
                                                            7.97618504348452e-05
##
   iter:
          1300
                 f-value:
                            0.000861549170700408
                                                   pgrad:
                                                            9.67140279438672e-05
          1310
##
   iter:
                 f-value:
                            0.00086152767315823
                                                  pgrad:
                                                           7.50012033618463e-05
##
   iter:
          1320
                 f-value:
                            0.000861428316846219
                                                            8.00046366158935e-05
                                                   pgrad:
   iter:
          1330
                 f-value:
                            0.000861345804007183
                                                            8.02023776463878e-05
##
                                                   pgrad:
          1340
##
                 f-value:
                            0.000861274417501135
                                                            8.42467795730828e-05
   iter:
                                                   pgrad:
##
  iter:
          1350
                 f-value:
                            0.000861178077337278
                                                   pgrad:
                                                            8.18326180374029e-05
## iter:
          1360
                 f-value:
                            0.000861100982858868
                                                            8.20784587116652e-05
                                                   pgrad:
## iter:
          1370
                 f-value:
                            0.000861026995136708
                                                   pgrad:
                                                            8.88299045077034e-05
```

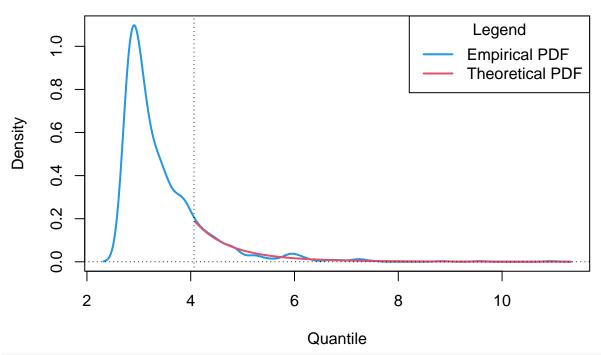
```
## iter: 1380 f-value: 0.000860831869927721
                                              pgrad: 0.000373105120426154
## iter: 1390 f-value: 0.000860770708730074 pgrad:
                                                      7.25359427864447e-05
## iter: 1400 f-value: 0.000860566696698535 pgrad:
                                                      7.94805589122705e-05
## iter: 1410 f-value: 0.000860372626439614 pgrad:
                                                      0.0004845223067656
## iter: 1420 f-value: 0.000860292452083116 pgrad:
                                                      7.33000123863681e-05
## iter: 1430 f-value: 0.000860215684247431 pgrad:
                                                      7.15001185034314e-05
## iter: 1440 f-value: 0.000860135281639201 pgrad:
                                                      6.7874554028069e-05
## iter: 1450 f-value: 0.000859878185426882 pgrad:
                                                      0.000203972963342069
## iter: 1460 f-value: 0.000859863859550065 pgrad:
                                                      6.48983722467134e-05
## iter: 1470 f-value: 0.000859798437004844
                                              pgrad:
                                                      7.1289266511787e-05
## iter: 1480 f-value: 0.000859718311564438 pgrad:
                                                      6.45226654421016e-05
## iter: 1490 f-value: 0.00085964745612598 pgrad: 9.08451055661175e-05
## iter: 1500 f-value: 0.000859624134684047 pgrad: 6.85731964621961e-05
## iter: 0 f-value: 0.0808505295557911 pgrad: 0.298285027768035
## iter: 10 f-value: 0.00107719875960105 pgrad: 0.00185150310625298
            f-value: 0.0192359365849244 pgrad: 0.189777432462782
## iter: 20
## iter: 30 f-value: 0.000939654216641966 pgrad: 0.000711360798332492
## iter: 40 f-value: 0.000920209469651887 pgrad:
                                                    0.00662663137460659
## iter: 50 f-value: 0.000896611743056737 pgrad: 0.00019918306567715
## iter: 60 f-value: 0.00088583552053203 pgrad: 0.000708953825057285
## iter: 70 f-value: 0.0464650323721035 pgrad: 0.228110269437323
## iter: 80 f-value: 0.000851978873905954 pgrad:
                                                    3.99448020233506e-05
## iter: 90 f-value: 0.000849216847718312 pgrad:
                                                    0.000320976213371849
## iter: 100 f-value: 0.000849144568390781 pgrad: 2.17019862071011e-05
         110 f-value: 0.000855877138303113 pgrad: 0.000482232001332183
    Successful convergence.
names(gev_mixture_model)
##
    [1] "data"
##
    [2] "data_largest"
##
   [3] "block_sizes"
   [4] "equivalent_block_sizes"
##
##
    [5] "rejected_block_sizes"
##
    [6] "block_maxima_indexes_object"
   [7] "gev_models_object"
   [8] "extremal_indexes"
##
##
   [9] "normalized_gev_parameters_object"
  [10] "weighted_normalized_gev_parameters_object"
  [11] "identic_weights_mw"
  [12] "pessimistic_weights_mw"
  [13] "pessimistic_weights_pw_shape"
  [14] "pessimistic_weights_pw_scale"
## [15] "pessimistic_weights_pw_loc"
## [16] "automatic weights mw"
## [17] "automatic_weights_mw_statistics"
  [18] "automatic_weights_pw_shape"
## [19] "automatic_weights_pw_scale"
## [20] "automatic_weights_pw_loc"
## [21] "automatic_weights_pw_statistics"
gev_mixture_model$block_sizes
```

## [1] 10 11 12 13 14 15 16 17 18 19 20

```
gev_mixture_model$normalized_gev_parameters_object
              loc star
                              scale star
                                                shape star
## 10 2.91331481647741 0.557778942787110 0.161970648893821
## 11 2.96206166835856 0.504669570501020 0.190946663105825
## 12 3.25805424025380 0.363819198216133 0.272634333744191
## 13 2.97569910760020 0.573679252734363 0.148186663152879
## 14 3.28103398642466 0.396035969252894 0.243229178127215
## 15 3.22872025550140 0.420675763009360 0.228491424009938
## 16 2.98489828985661 0.554891759147021 0.157253339653076
## 17 3.68872366009854 0.234473016638563 0.379269119513422
## 18 3.60179011004418 0.267525276338887 0.345794054989248
## 19 3.56487287280516 0.305417079312862 0.310152835762093
## 20 3.45934838387646 0.343904011149439 0.274174802601684
gev_mixture_model$weighted_normalized_gev_parameters_object
##
                               loc_star
                                               scale_star
                                                                  shape_star
## identic_weights
                       3.26531976284518 0.411169985371605 0.246554823959399
## pessimistic_weights 3.33813544954861 0.424583307490144 0.252164095173749
                       3.19056898022041 0.341195137135598 0.335591914464968
## automatic_weights
gev_mixture_model$automatic_weights_mw
##
                      10
                                            11
                                                                   12
##
   0.0000000000000e+00
                          1.0000000000000e+00
                                               -1.33226762955019e-15
##
                      13
                                            14
   0.0000000000000e+00
                          0.0000000000000e+00
##
                                                0.0000000000000e+00
##
                      16
                                            17
                                                                   18
   0.0000000000000e+00
                          0.0000000000000e+00
                                                0.0000000000000e+00
##
##
                      19
   0.0000000000000e+00
                          0.0000000000000e+00
gev_mixture_model$automatic_weights_mw_statistics
## $function_value
## [1] 0.0036225431298914
##
## $gradient_value
## [1] 1.42941214420489e-15
##
## $function_reduction
## [1] 0.0781820260930863
##
## $number iterations
## [1] 1560
##
## $convergence
## [1] 0
##
## $message
## [1] "Successful convergence"
gev_mixture_model$automatic_weights_pw_statistics
## $function_value
## [1] 0.000848757832158418
```

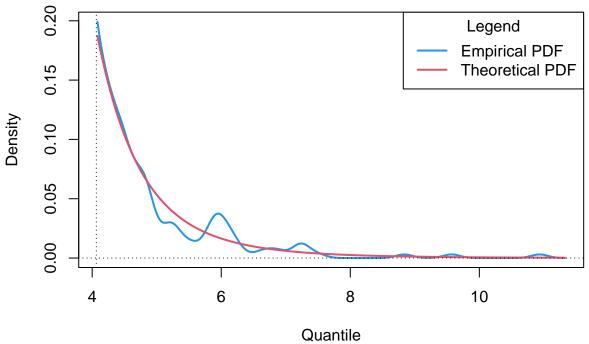
```
##
## $gradient_value
  [1] 1.36055174386751e-05
##
## $function_reduction
## [1] 0.0800017717236326
## $number_iterations
## [1] 3118
##
## $convergence
##
  [1] 0
##
## $message
## [1] "Successful convergence"
plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = TRUE,
                           zoom = FALSE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")
```

## bility Density Function (PDF) Plot : automatic\_weights - model\_wise = TRUE : zoo



```
main = "Probability Density Function (PDF) Plot")
```

## ability Density Function (PDF) Plot : automatic\_weights - model\_wise = TRUE : zoc



```
gev_mixture_model_parameters <- gev_mixture_model$normalized_gev_parameters_object</pre>
shapes <- gev_mixture_model_parameters$shape_star</pre>
scales <- gev_mixture_model_parameters$scale_star</pre>
locations <- gev_mixture_model_parameters$loc_star</pre>
weights <- gev_mixture_model$automatic_weights_mw</pre>
p <- 0.95
q_initial_guesses <- sapply(1:length(weights), function(j) calculate_gev_inverse_cdf(p = p,</pre>
                                                                                          loc = locations[j]
                                                                                          scale = scales[j],
                                                                                          shape = shapes[j])
q_initial_guesses
  [1] 5.04093114394992 4.97927579575552 4.92266737439822 5.11627189727924
    [5] 5.00607229061463 5.01687599937565 5.08556643559924 4.97762713999913
   [9] 4.98886604924849 5.05412052001466 5.03692884375668
range(q_initial_guesses)
## [1] 4.92266737439822 5.11627189727924
block_size <- max(gev_mixture_model$block_sizes)</pre>
```

threshold <- find\_threshold\_associated\_with\_given\_block\_size(x = y, block\_size = block\_size)</pre>

y <- gev\_mixture\_model\$data\_largest

```
library(evd)
data <- y[y > threshold]
M3 <- fgev(data, prob = 0.95)
МЗ
##
## Call: fgev(x = data, prob = 0.95)
## Deviance: 278.014037403653
## Estimates
         quantile scale
                                            shape
## 4.120534480188  0.373555979542  0.680349618047
##
## Standard Errors
         quantile
                             scale
                                              shape
## 0.0177001900922 0.0404264382965 0.1156345016665
## Optimization Information
##
    Convergence: successful
##
    Function Evaluations: 65
    Gradient Evaluations: 14
M4 <- fgev(data)
M4
##
## Call: fgev(x = data)
## Deviance: 278.01403743192
##
## Estimates
      loc
                          scale
                                           shape
## 4.409323933609 0.373559795532 0.680363739474
## Standard Errors
##
              loc
                              scale
                                               shape
## 0.0381669578725 0.0404251481620 0.1156445110604
## Optimization Information
##
    Convergence: successful
##
     Function Evaluations: 61
    Gradient Evaluations: 14
##
Fn <- ecdf(y)
p \leftarrow seq(from = Fn(threshold), to = 0.999, length.out = 20)
## [1] 0.85600000000000 0.863526315789474 0.871052631578947 0.878578947368421
## [5] 0.886105263157895 0.893631578947368 0.901157894736842 0.908684210526316
## [9] 0.916210526315789 0.923736842105263 0.931263157894737 0.938789473684211
## [13] 0.946315789473684 0.953842105263158 0.961368421052632 0.968894736842105
## [17] 0.976421052631579 0.983947368421053 0.991473684210526 0.9990000000000000
```

```
quantiles <- calculate_gev_mixture_model_inverse_cdf(p = p*0.1, locations, scales, shapes, weights, ite
quantiles
## [1] 2.54500903948768 2.54652594666024 2.54803607637323 2.54953953953945
   [5] 2.55103644445846 2.55252689690042 2.55401100018660 2.55548885526666
## [9] 2.55696056079287 2.55842621319139 2.55988590673084 2.56133973358820
## [13] 2.56278778391210 2.56423014588387 2.56566690577614 2.56709814800931
## [17] 2.56852395520596 2.56994440824319 2.57135958630309 2.57276956692134
probaility <- calculate_gev_mixture_model_cdf(q = quantiles, locations, scales, shapes, weights)</pre>
probaility
## [1] 0.085600000000000 0.0863526315789475 0.0871052631578949 0.0878578947368423
## [5] 0.0886105263157893 0.0893631578947368 0.0901157894736844 0.0908684210526317
## [9] 0.0916210526315789 0.0923736842105263 0.0931263157894738 0.0938789473684212
## [13] 0.0946315789473683 0.0953842105263156 0.0961368421052631 0.0968894736842103
## [17] 0.0976421052631580 0.0983947368421055 0.0991473684210526 0.09990000000000000
qnorm(p = p)
## [1] 1.06251930227087 1.09630032226387 1.13138106688499 1.16791239148275
## [5] 1.20607266469006 1.24607505418858 1.28817741907656 1.33269602289698
## [9] 1.38002500058479 1.43066476621983 1.48526483109878 1.54469087695126
## [13] 1.61013484724568 1.68330641362096 1.76679146935076 1.86479227652848
## [17] 1.98487936474799 2.14309758739059 2.38557097390240 3.09023230616781
calculate\_gev\_inverse\_cdf(p = p*0.1, loc = 2.52214, scale = 0.5222, shape = 0.1487)
## [1] 2.08253632084543 2.08416658253954 2.08578931634005 2.08740464488087
   [5] 2.08901268789773 2.09061356232131 2.09220738236667 2.09379425961898
##
   [9] 2.09537430311604 2.09694761942739 2.09851431273046 2.10007448488366
## [13] 2.10162823549681 2.10317566199874 2.10471685970246 2.10625192186782
## [17] 2.10778093976187 2.10930400271701 2.11082119818693 2.11233261180067
calculate_gev_inverse_cdf(p = p, loc = 1, scale = 0.5, shape = 0.1)
## [1] 2.02283799771981 2.05784063947258 2.09488419187596 2.13422869874810
##
   [5] 2.17618590500976 2.22113381047765 2.26953669811663 2.32197329748089
## [9] 2.37917738058723 2.44209798210797 2.51199179015200 2.59057069417566
## [13] 2.68024915905050 2.78458473159841 2.90912515505341 3.06321218520797
## [17] 3.26441171406689 3.55204289090937 4.04837225469639 5.97581256378162
```