Modeling extreme values with a GEV mixture probability distributions

Standard exponential distribution

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```r
# Load useful functions
```

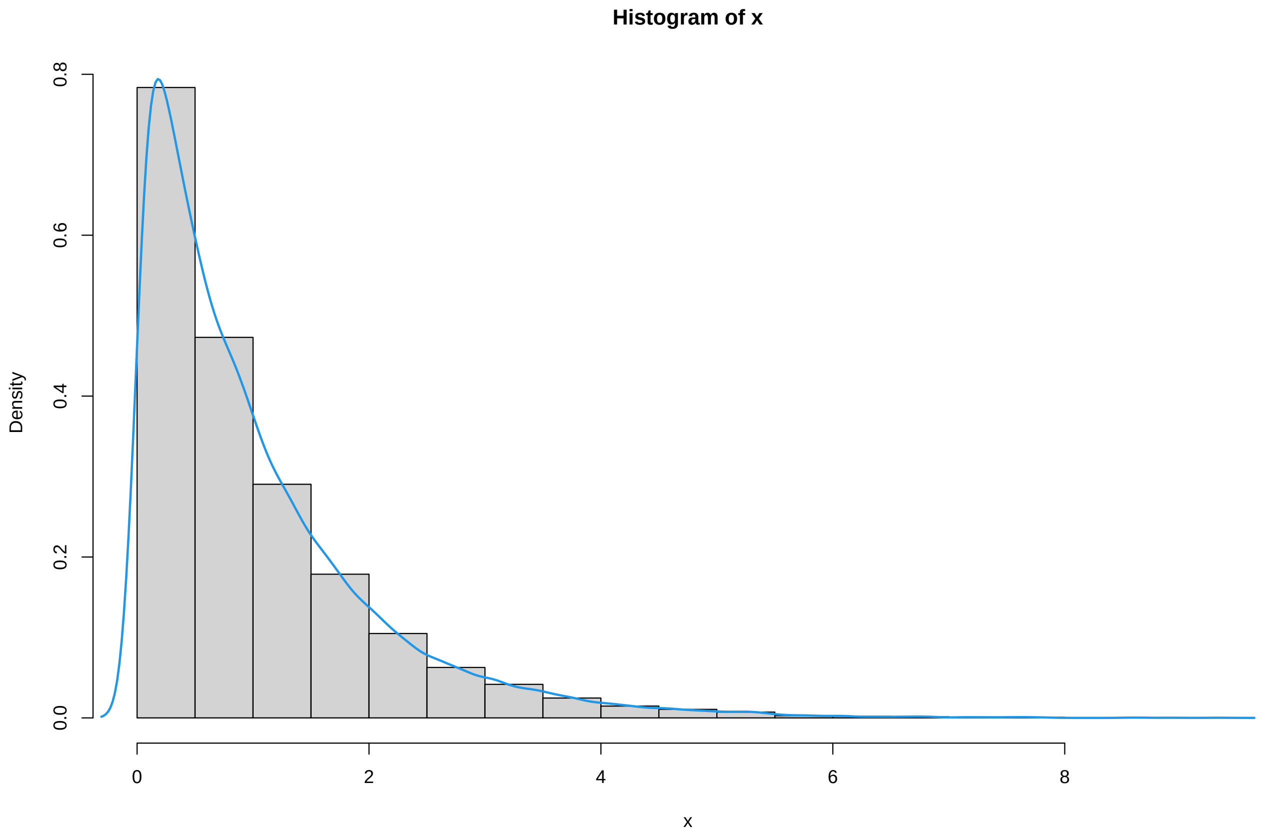
```r
path <- ".."
```

```r
xfun::in_dir(dir = path, expr = source("./src/generate_gev_sample.R"))
xfun::in_dir(dir = path, expr = source("./src/calculate_gev_inverse_cdf.R"))
xfun::in_dir(dir = path, expr = source("./src/estimate_gev_mixture_model_parameters.R"))
xfun::in_dir(dir = path, expr = source("./src/plot_gev_mixture_model_pdf.R"))
xfun::in_dir(dir = path, expr = source("./src/plot_gev_mixture_model_cdf.R"))
xfun::in_dir(dir = path, expr = source("./src/estimate_gev_mixture_model_quantile.R"))


# Generate a random sample
```

```r
n <- 20000
```

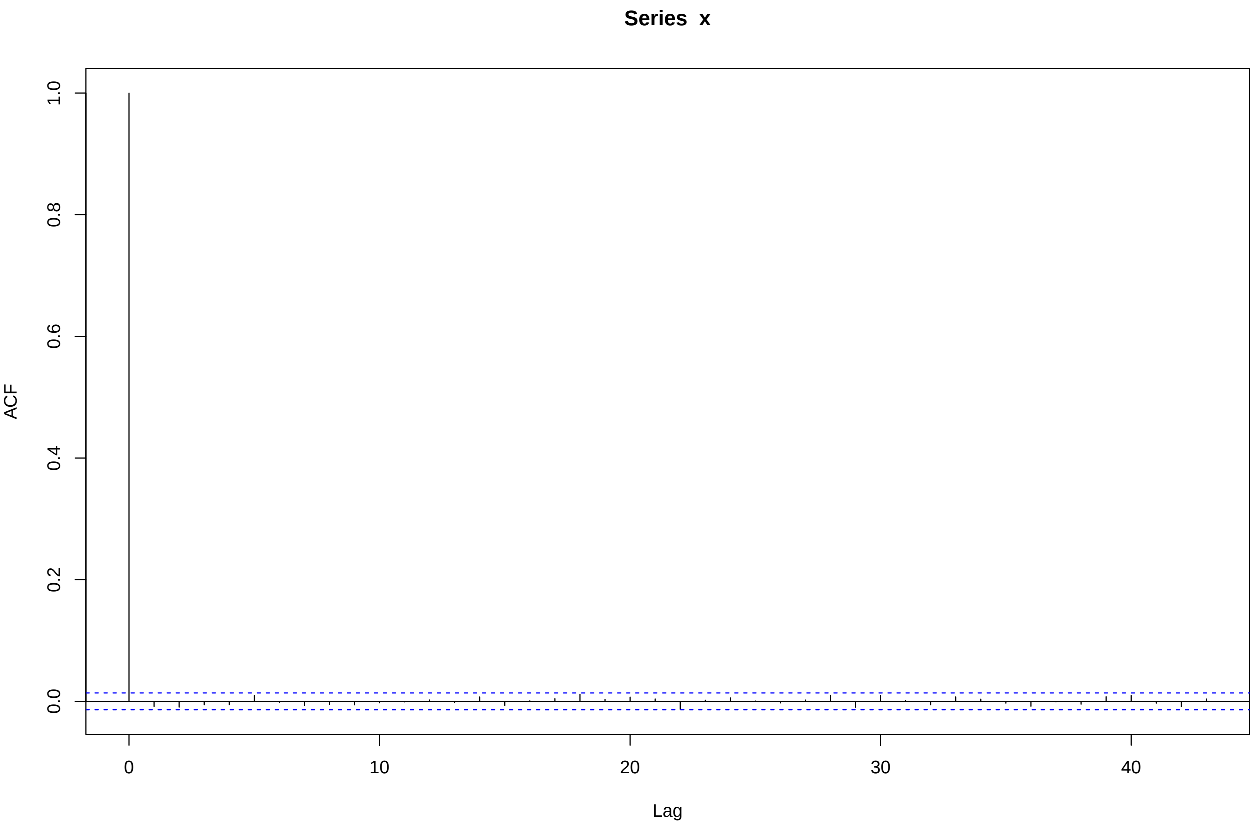
```r
set.seed(1122)
x <- rexp(n = n)

# Histogram of all data
```

```r
hist(x, prob = TRUE)
lines(density(x),
      lwd = 2,
      col = 4)
```



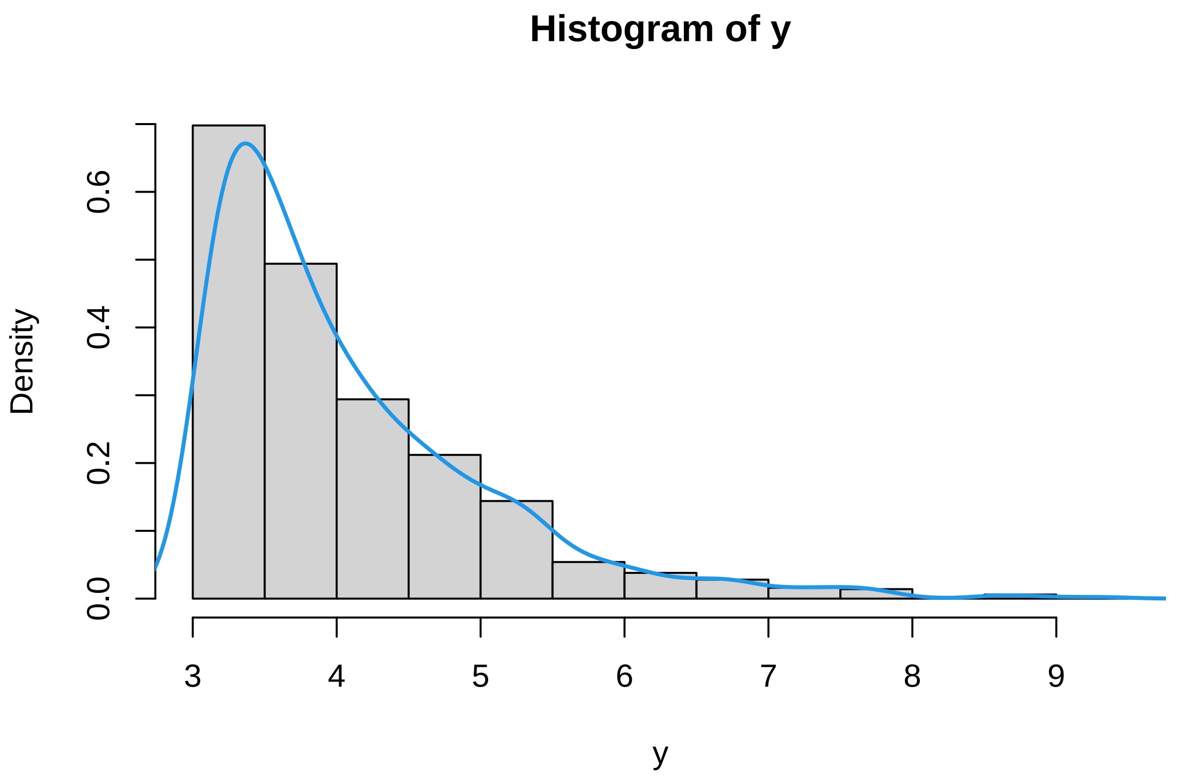
```
# Autocorrelation function of all data
```

```
acf(x)
```



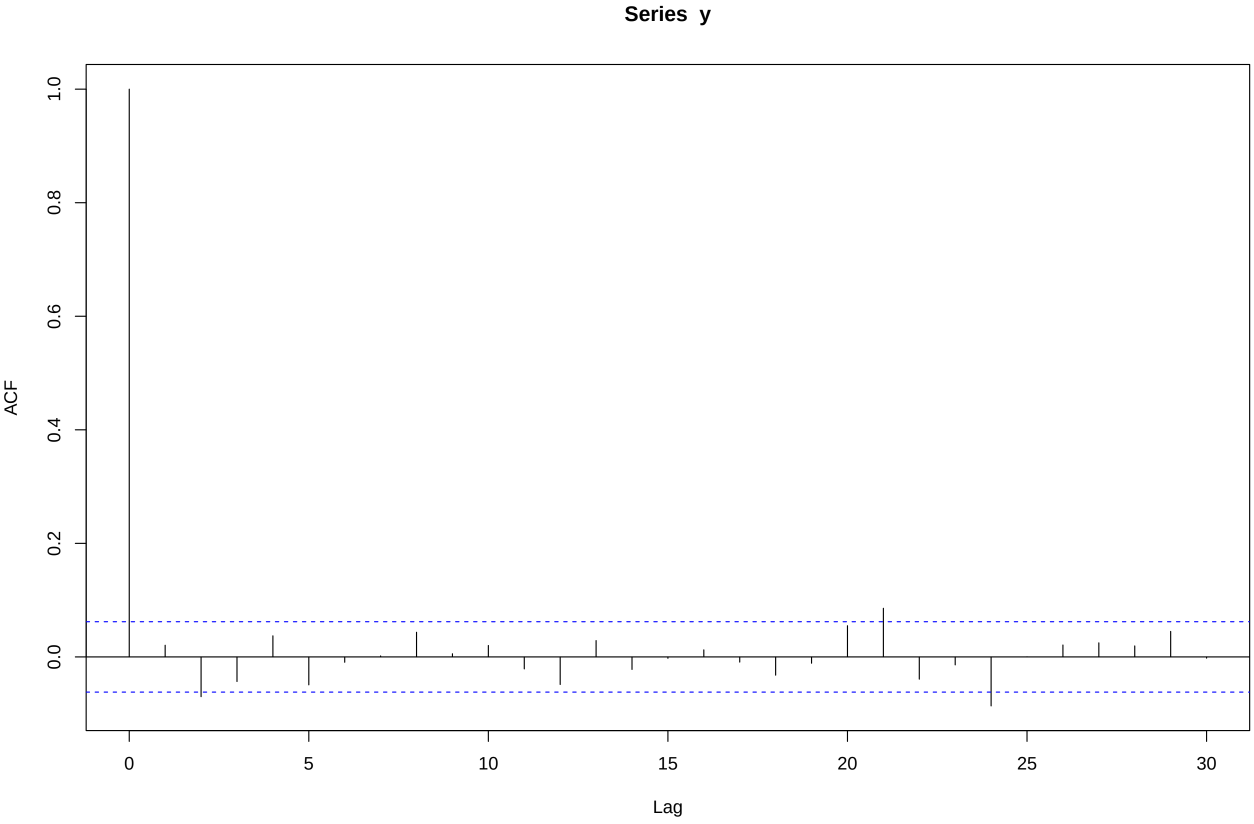
```
# Histogram of the largest data
```

```
nlargest <- 1000
y <- extract_nlargest_sample(x, n = nlargest)
hist(y, prob = TRUE)
lines(density(y),
      lwd = 2,
      col = 4)
```



```
# Autocorrelation function of the largest data
```

```
acf(y)
```



```r
# Estimation of gev mixture models
```

```r
gev_mixture_model <- estimate_gev_mixture_model_parameters(x = x,
                                                           block_sizes = NULL,
                                                           minimum_nblocks = 50,
                                                           threshold = NULL,
                                                           nlargest = nlargest,
                                                           confidence_level = 0.95,
                                                           use_extremal_index = TRUE,
                                                           use_lower_threshold = FALSE,
                                                           maximum_iterations = 1500,
                                                           log_mv = TRUE,
                                                           log_pw = TRUE,
                                                           trace = FALSE,
                                                           method = "MLE")
```

```
##   Successful convergence.
##   Successful convergence.
```

```r
gev_mixture_model$extremal_indexes
```

```
##           11           12           13           14           15           16 
## 0.9900009242 1.0000000000 0.9609670108 1.0000000000 0.9609670108 0.9609670108 
##           17           18           19           20 
## 0.9609670108 1.0000000000 1.0000000000 1.0000000000
```

```r
gev_mixture_model$normalized_gev_parameters_object
```

```
##       loc_star   scale_star      shape_star
## 11 3.295748201 0.9764477537 -0.036678131753
## 12 3.522773537 0.8042194762  0.023268099931
## 13 3.347347009 0.9754901572 -0.039148696475
## 14 3.377601536 0.9349268788 -0.023112178277
## 15 3.122292373 1.1696169467 -0.094710563158
## 16 2.931105537 1.1775333422 -0.080313159733
## 17 2.657091744 1.3690784947 -0.119321409565
## 18 3.752558379 0.7760365143  0.008517987581
## 19 3.878534390 0.6894152458  0.045461735709
## 20 3.578059712 0.9702756582 -0.063034629214
```

```r
gev_mixture_model$full_normalized_gev_parameters_object
```

```
##       loc_star   scale_star      shape_star
## 11 3.285933676 0.9768077321 -0.036678131753
## 12 3.522773537 0.8042194762  0.023268099931
## 13 3.308477389 0.9770118521 -0.039148696475
## 14 3.377601536 0.9349268788 -0.023112178277
## 15 3.075635928 1.1740358048 -0.094710563158
## 16 2.884146774 1.1813047488 -0.080313159733
## 17 2.602451923 1.3755981952 -0.119321409565
## 18 3.752558379 0.7760365143  0.008517987581
## 19 3.878534390 0.6894152458  0.045461735709
## 20 3.578059712 0.9702756582 -0.063034629214
```

```r
gev_mixture_model$automatic_weights_pw_shape
```

```
##                 11                 12                 13                 14
##  0.000000000e+00  5.003931839e-17  0.000000000e+00 -3.388131789e-21
##               15               16               17               18
## -1.355252716e-20  0.000000000e+00  0.000000000e+00  0.000000000e+00
##               19               20
##  1.000000000e+00 -6.776263578e-21
```

```
gev_mixture_model$automatic_weights_pw_scale
```

```
##              11              12              13              14              15
## 1.979257069e-05 2.106932985e-01 1.937756174e-05 1.055065246e-04 2.831812967e-04
##              16              17              18              19              20
## 2.955207653e-04 6.243595086e-04 2.829522561e-01 5.049735388e-01 3.316840595e-05
```

```
gev_mixture_model$automatic_weights_pw_loc
```

```
##            11            12            13            14            15
## 0.03820822164 0.11818422716 0.04523113754 0.06725519263 0.01222334111
##            16            17            18            19            20
## 0.02136803196 0.03469324583 0.22884552886 0.29019149249 0.14379958077
```

```
gev_mixture_model$weighted_normalized_gev_parameters_object[3, ]
```

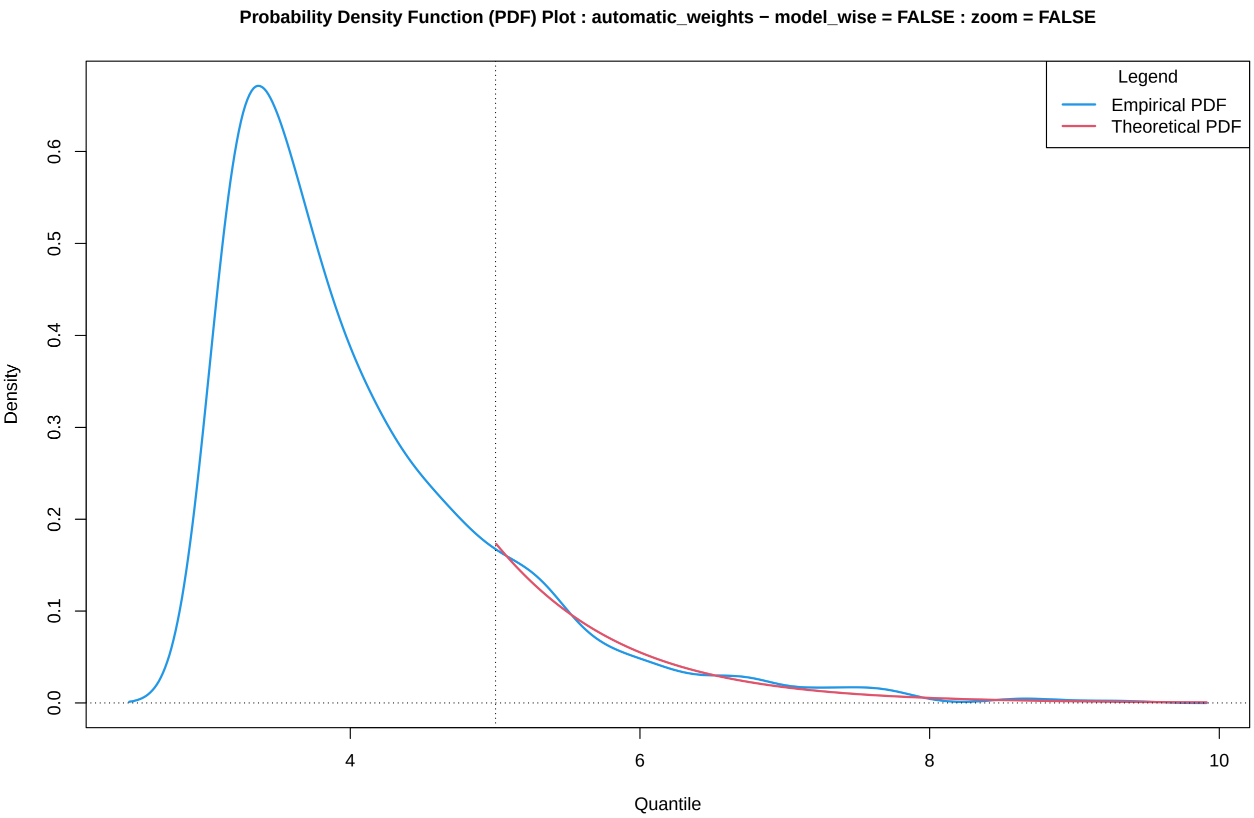
```
##                      loc_star   scale_star    shape_star
## automatic_weights 3.612603816 0.7388644418 0.04546173571
```

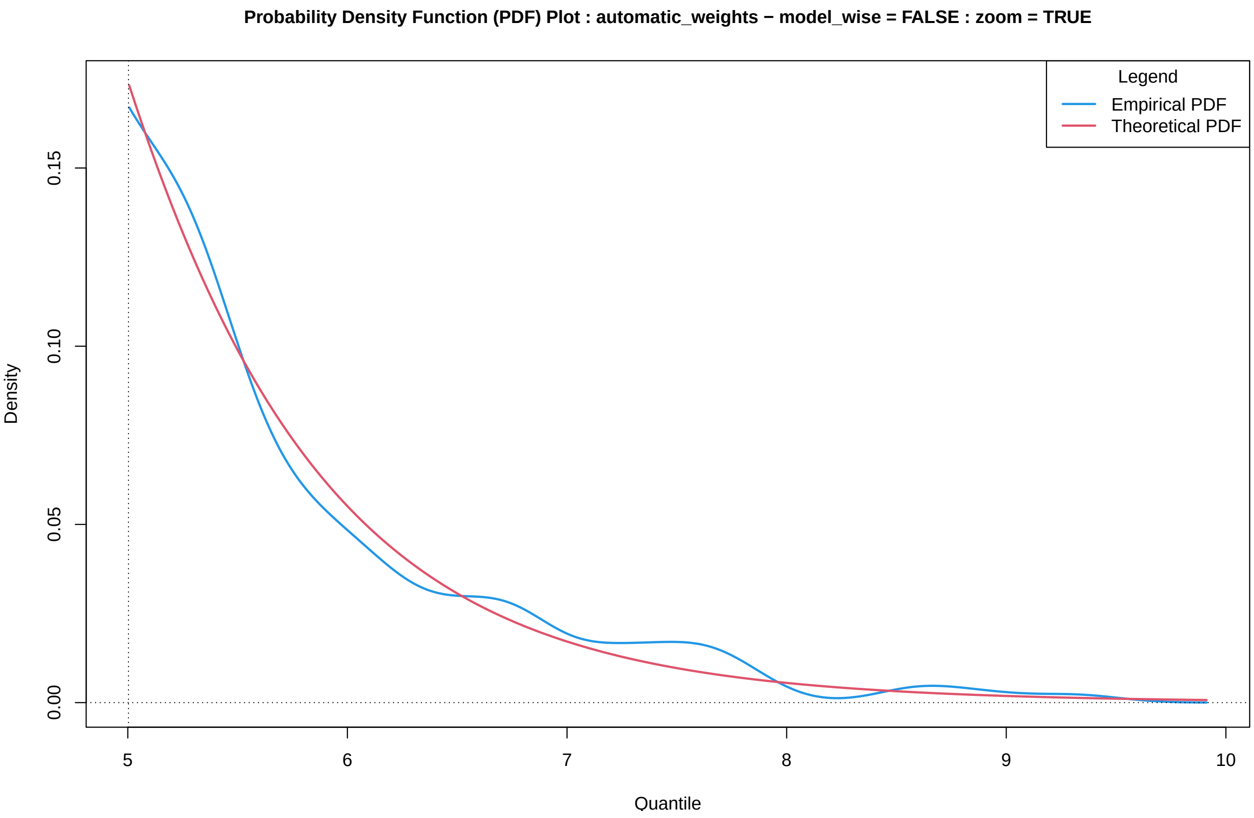
```
gev_mixture_model$automatic_weights_mw
```

```
##           11           12           13           14           15           16
## 0.0000000000 0.8210939617 0.0000000000 0.0000000000 0.0000000000 0.1789060383
##           17           18           19           20
## 0.0000000000 0.0000000000 0.0000000000 0.0000000000
```

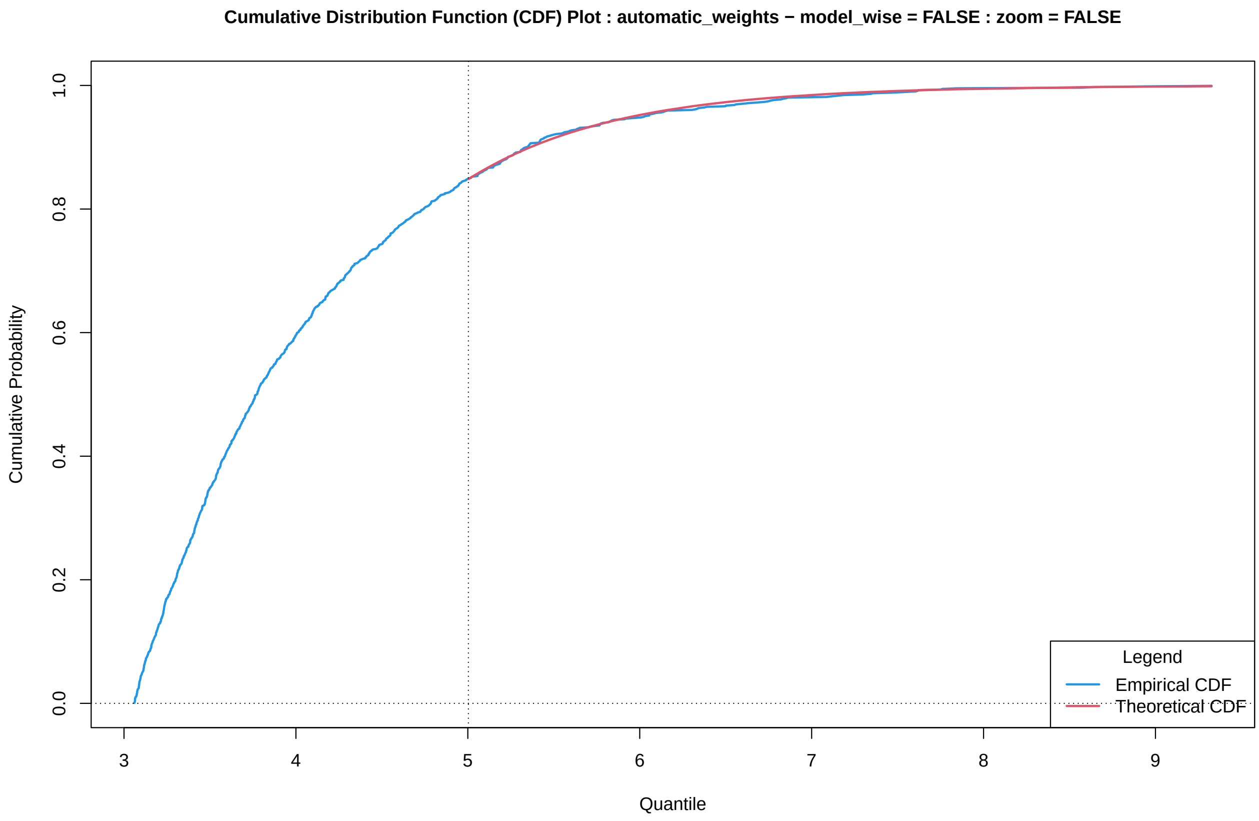
```
# Model diagnostics

## GEV mixture model with respect to parameters
```

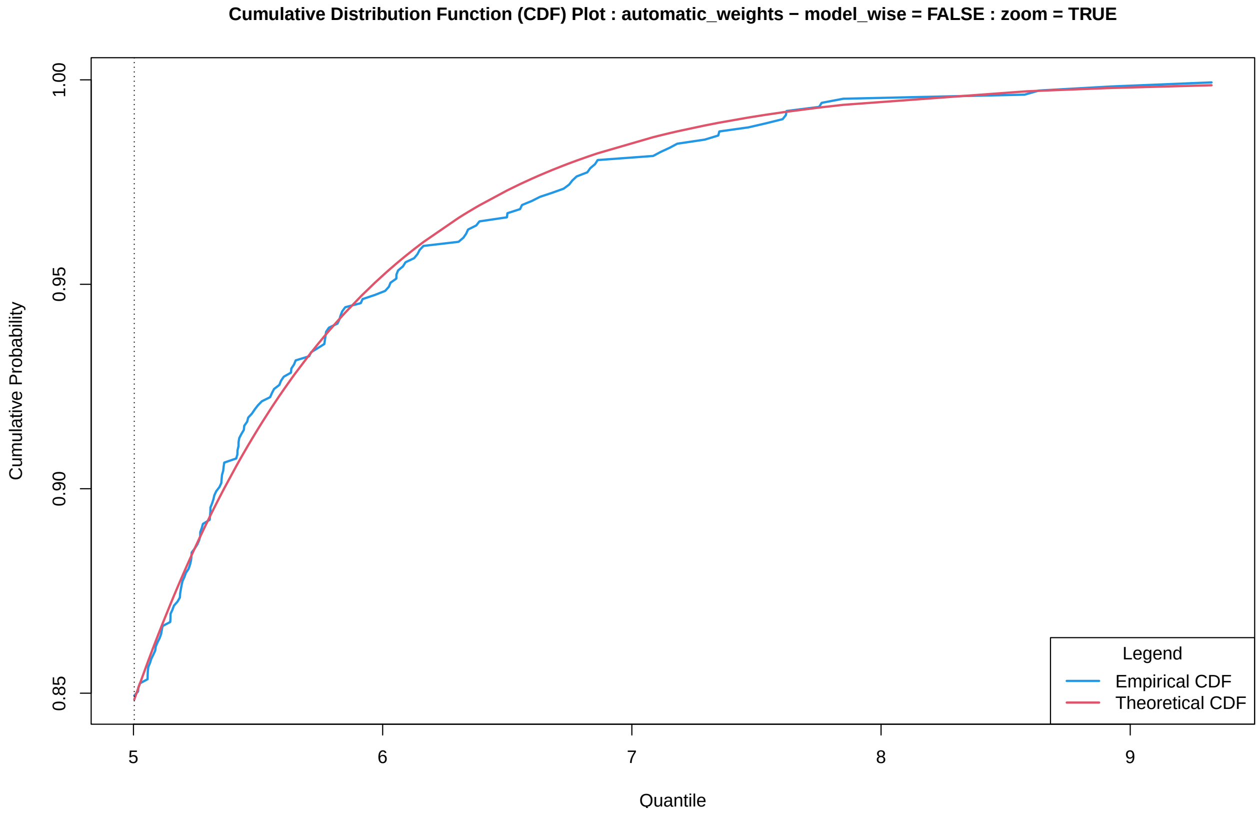
```
par(mfrow = c(2, 1))
plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = FALSE,
                           zoom = FALSE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")

plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = FALSE,
                           zoom = TRUE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")
```


Probability Density Function (PDF) Plot : automatic_weights − model_wise = FALSE : zoom = FALSE
Legend
Empirical PDF
Theoretical PDF
Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
4
6
8
10
Quantile


Probability Density Function (PDF) Plot : automatic_weights − model_wise = FALSE : zoom = TRUE
Legend
Empirical PDF
Theoretical PDF
Density
0.00
0.05
0.10
0.15
5
6
7
8
9
10
Quantile

```r
par(mfrow = c(2, 1))
plot_gev_mixture_model_cdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = FALSE,
                           zoom = FALSE,
                           xlab = "Quantile",
                           ylab = "Cumulative Probability",
                           main = "Cumulative Distribution Function (CDF) Plot")

plot_gev_mixture_model_cdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = FALSE,
                           zoom = TRUE,
                           xlab = "Quantile",
                           ylab = "Cumulative Probability",
                           main = "Cumulative Distribution Function (CDF) Plot")
```

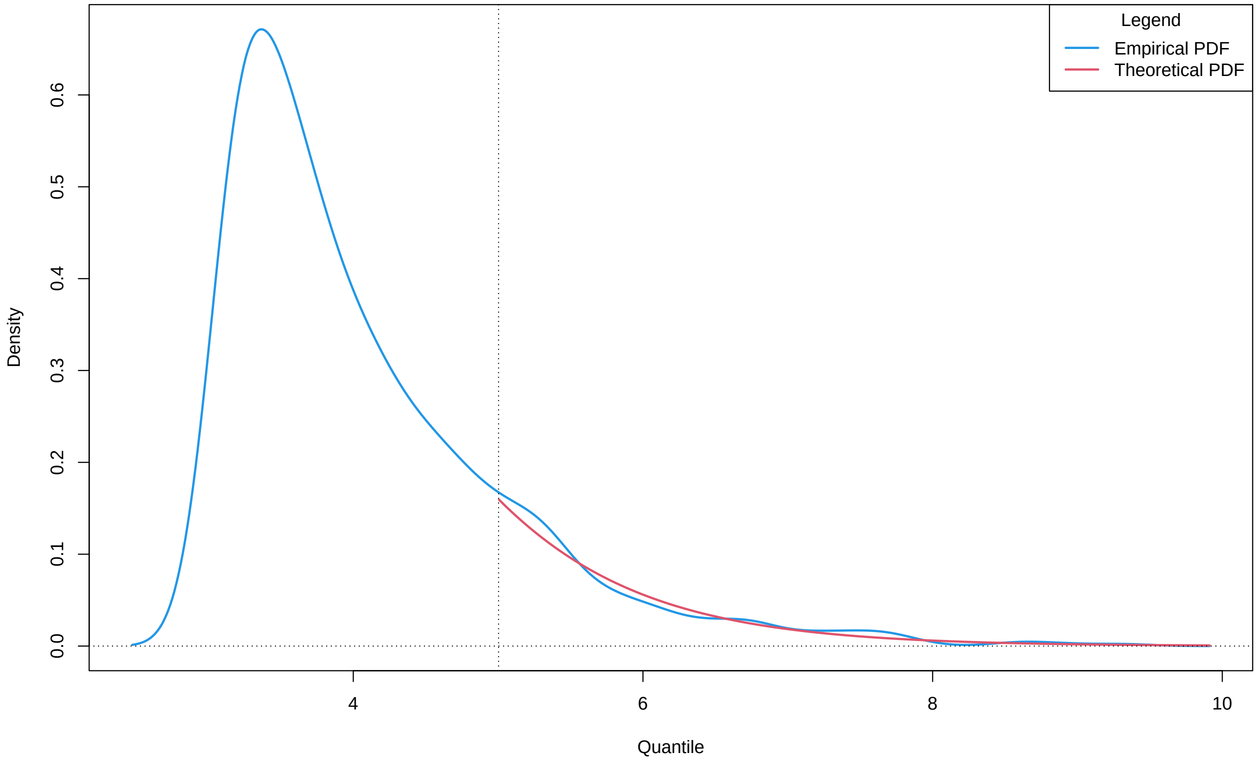

Cumulative Distribution Function (CDF) Plot : automatic_weights − model_wise = FALSE : zoom = FALSE
Cumulative Probability
0.0
0.2
0.4
0.6
0.8
1.0
3
4
5
6
7
8
9
Quantile
Legend
Empirical CDF
Theoretical CDF


Cumulative Distribution Function (CDF) Plot : automatic_weights − model_wise = FALSE : zoom = TRUE
Cumulative Probability
0.85
0.90
0.95
1.00
5
6
7
8
9
Quantile
Legend
Empirical CDF
Theoretical CDF

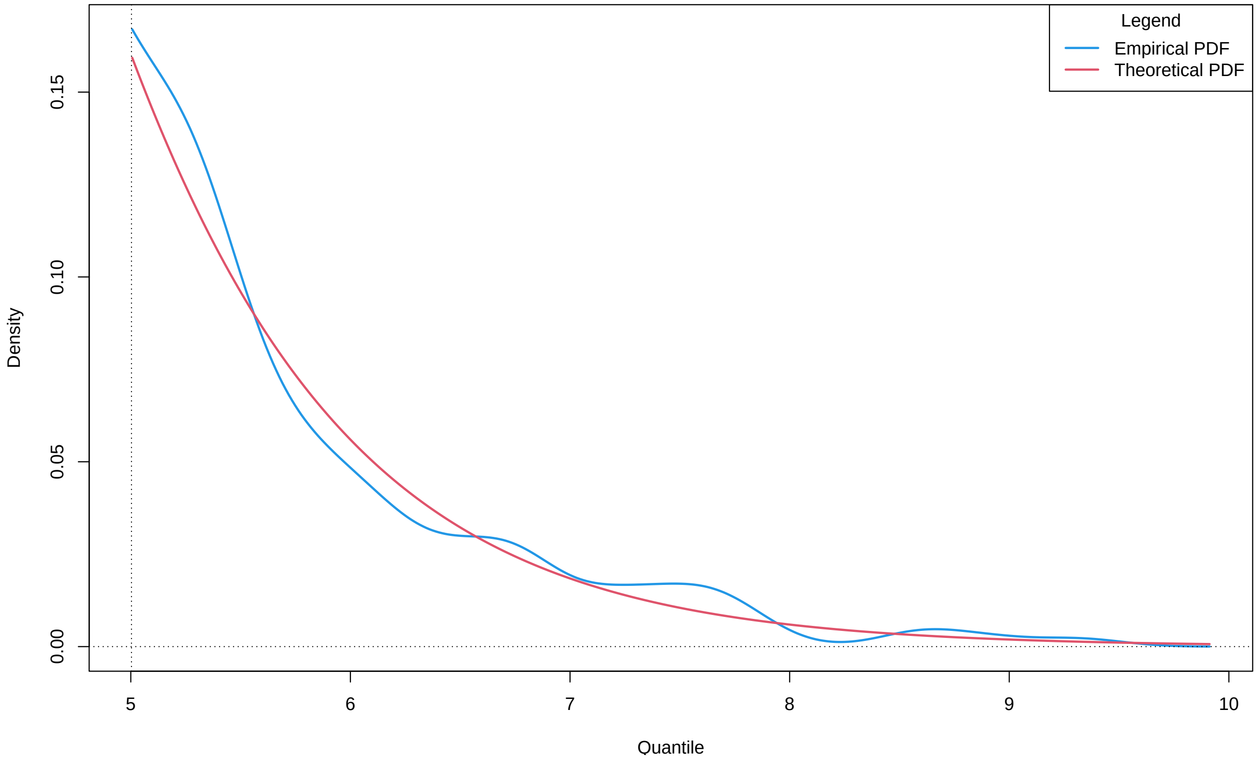
```
## GEV mixture model with respect to distribution functions
```

```
par(mfrow = c(2, 1))
plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = TRUE,
                           zoom = FALSE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")

plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = TRUE,
                           zoom = TRUE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")
```

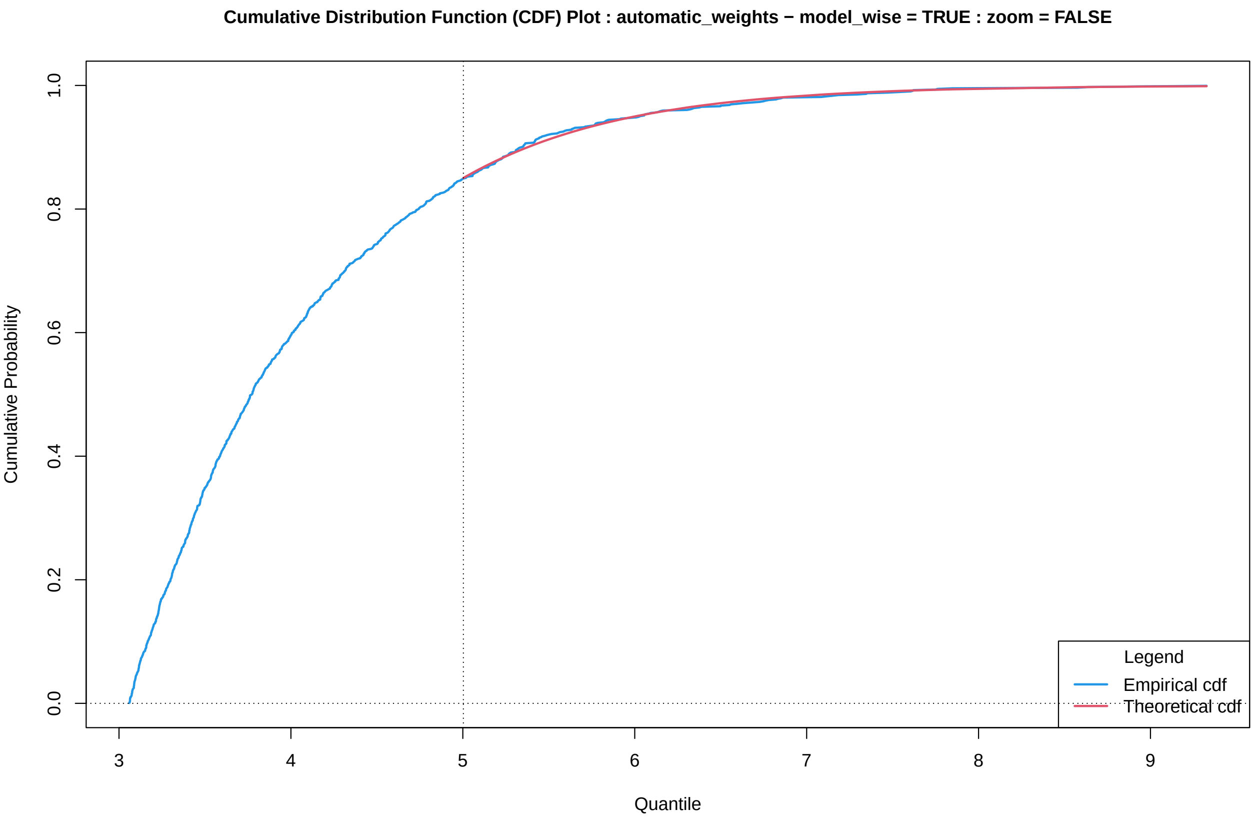
Probability Density Function (PDF) Plot : automatic_weights − model_wise = TRUE : zoom = FALSE

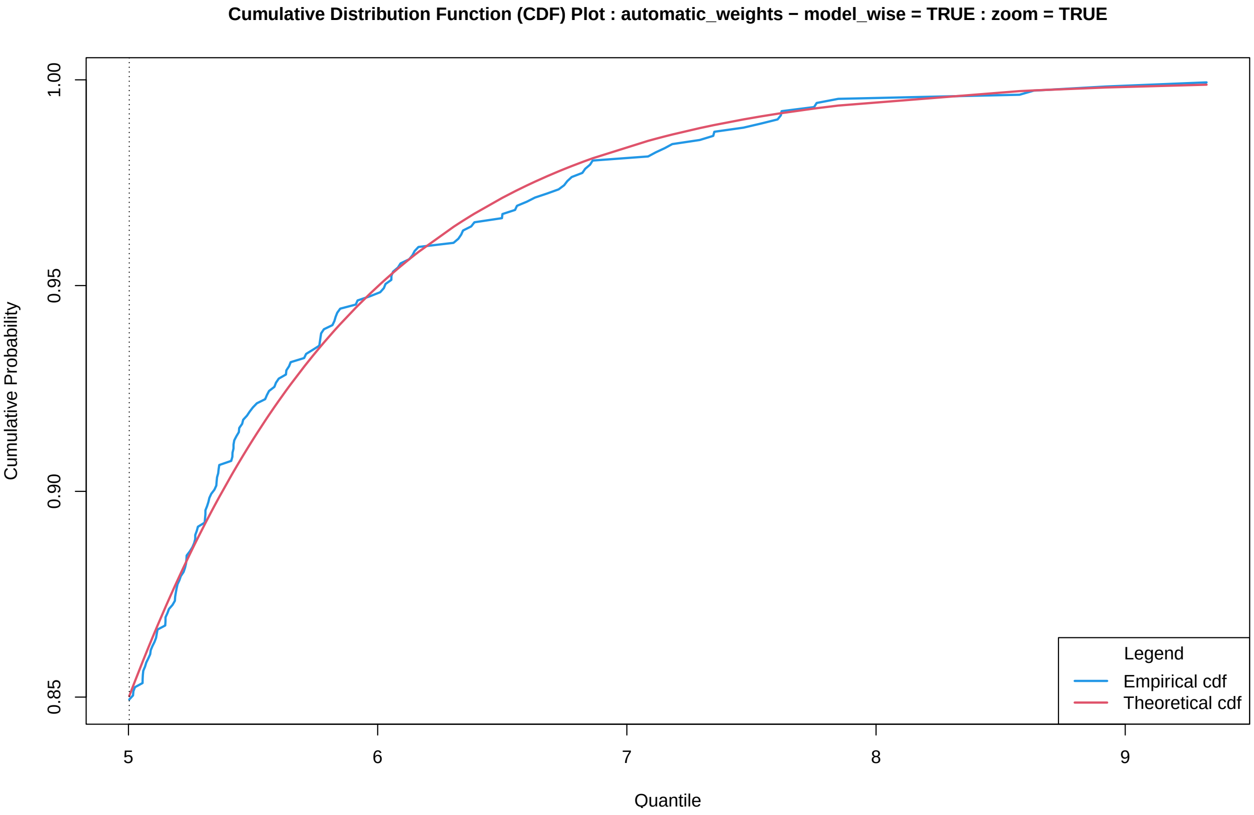


Probability Density Function (PDF) Plot : automatic_weights − model_wise = TRUE : zoom = TRUE



```r
par(mfrow = c(2, 1))
plot_gev_mixture_model_cdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = TRUE,
                           zoom = FALSE,
                           xlab = "Quantile",
                           ylab = "Cumulative Probability",
                           main = "Cumulative Distribution Function (CDF) Plot")

plot_gev_mixture_model_cdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = TRUE,
                           zoom = TRUE,
                           xlab = "Quantile",
                           ylab = "Cumulative Probability",
                           main = "Cumulative Distribution Function (CDF) Plot")
```


Cumulative Distribution Function (CDF) Plot : automatic_weights − model_wise = TRUE : zoom = FALSE
Cumulative Probability
0.0
0.2
0.4
0.6
0.8
1.0
3
4
5
6
7
8
9
Quantile
Legend
Empirical cdf
Theoretical cdf


Cumulative Distribution Function (CDF) Plot : automatic_weights − model_wise = TRUE : zoom = TRUE
Cumulative Probability
0.85
0.90
0.95
1.00
5
6
7
8
9
Quantile
Legend
Empirical cdf
Theoretical cdf

```r
# Estimation of an extreme quantile
```

```r
estimator_types <- c("automatic_weights_mw",
                     "pessimistic_weights_mw",
                     "identic_weights_mw",
                     "automatic_weights_pw",
                     "pessimistic_weights_pw",
                     "identic_weights_pw",
                     "empirical",
                     "confidence_interval_mw",
                     "confidence_interval_pw")
```

```r
alpha <- 10^(-14)


## Quantile from the true distribution
```

```r
true_rl <- qexp(p = 1 - alpha)
true_rl
```

```
## [1] 32.2369909
```

```r
## Quantile from GEV mixture model with respect to parameters
```

```r
rl_pw <- estimate_gev_mixture_model_quantile(gev_mixture_model,
                                             alpha = alpha,
                                             confidence_level = 0.95,
                                             do.ci = TRUE,
                                             estimator_type = estimator_types[4])

rl_pw[2]
```

```
##      estimate
## 1 48.77039677
```

```r
## Quantile from GEV mixture model with respect to distribution functions
```

```r
rl_mw <- estimate_gev_mixture_model_quantile(gev_mixture_model,
                                             alpha = alpha,
                                             confidence_level = 0.95,
                                             do.ci = TRUE,
                                             estimator_type = estimator_types[1])

rl_mw[2]
```

```
##      estimate
## 1 36.89561953
```

```r
## Quantiles from equivalent estimated GEV models
```

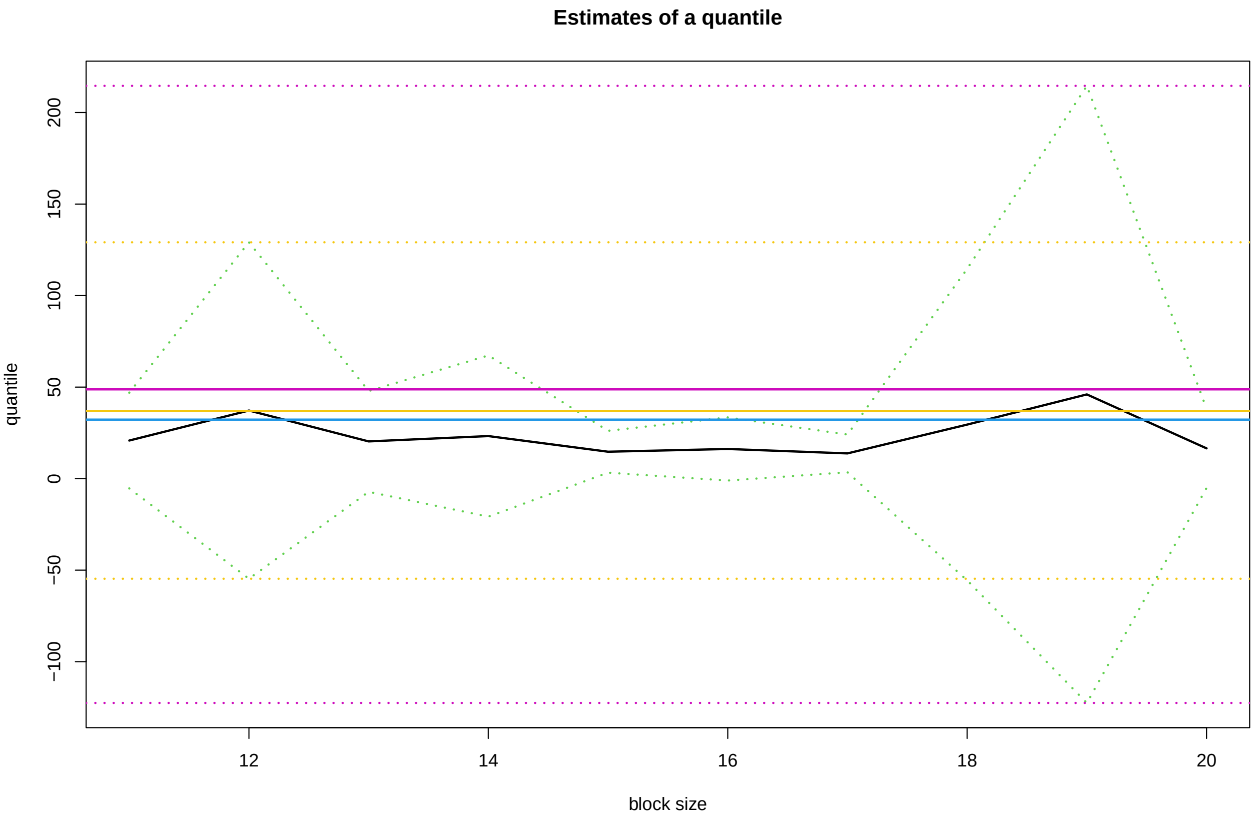
```r
est_rl_pw <- estimate_gev_mixture_model_quantile(gev_mixture_model,
                                                 alpha = alpha,
                                                 confidence_level = 0.95,
                                                 do.ci = TRUE,
                                                 estimator_type = estimator_types[9])

est_rl_pw
```

```
##           lower    estimate       upper
## 11 -5.337255338 20.80888561 46.95502657
## 12 -54.68513765 37.20834533 129.1018283
## 13 -7.200515728 20.33336847 47.86725266
## 14 -20.79259828 23.24975189 67.29210206
## 15  3.259477333 14.69735681 26.13523629
## 16 -1.048923468 16.19235004 33.43362354
## 17  3.475653089 13.78065494 24.08565679
## 18 -55.57779421  29.5201085 114.6180112
## 19 -122.5626977 46.01349074 214.5896792
## 20 -5.059712706 16.53387999 38.12747268
```

```r
## Comparison of estimated quantiles
```

```r
est_rl_pw_range <- range(as.matrix(est_rl_pw))

est_rl_mw <- estimate_gev_mixture_model_quantile(gev_mixture_model,
                                                 alpha = alpha,
                                                 confidence_level = 0.95,
                                                 do.ci = TRUE,
                                                 estimator_type = estimator_types[8])

est_rl_mw_range <- range(as.matrix(est_rl_mw))
```

```r
matplot(x = rownames(est_rl_pw),
        y = est_rl_pw,
        xlab = "block size",
        ylab = "quantile",
        main = "Estimates of a quantile",
        ylim = range(c(est_rl_pw_range, true_rl)),
        cex = 1,
        cex.lab = 1,
        cex.axis = 1,
        type = "l",
        lty = c("dotted", "solid", "dotted"),
        lwd = c(2,2,2),
        col = c(3, 1, 3))

abline(h = true_rl, col = 4, lwd = 2)
abline(h = rl_mw[2], col = 7, lwd = 2)
abline(h = rl_pw[2], col = 6, lwd = 2)
abline(h = est_rl_pw_range, col = 6, lty = "dotted", lwd = 2)
abline(h = est_rl_mw_range, col = 7, lty = "dotted", lwd = 2)
```



```
# Legend:
# blue: Quantile from the true distribution
# yellow: Quantile from GEV mixture model with respect to distribution functions
# pink: Quantile from GEV mixture model with respect to parameters
```