Modeling extreme values with a GEV mixture probability distributions

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```
source("./src/estimate gev mixture model parameters.R")
source("./src/plot_gev_mixture_model_pdf.R")
source("./src/generate_gev_sample.R")
source("./src/plot_normalized_gev_mixture_model_pdf.R")
source("./src/calculate_gev_inverse_cdf.R")
source("./src/calculate_gev_mixture_model_inverse_cdf.R")
source("./src/calculate_gev_mixture_model_cdf.R")
n <- 10000
nlargest <- 1000
\#x \leftarrow generate\_gev\_sample(n = n, loc = 1, scale = 0.5, shape = 0.1)
x \leftarrow rnorm(n = n)
gev_mixture_model <- estimate_gev_mixture_model_parameters(x,</pre>
                                                              nsloc = NULL,
                                                              std.err = FALSE,
                                                              block sizes = NULL,
                                                              minimum_nblocks = 50,
                                                              nlargest = nlargest,
                                                              confidence level = 0.95,
                                                              log_mv = TRUE,
                                                              log_pw = TRUE,
                                                              trace = TRUE)
```

```
## iter: 0 f-value: 0.177980208584727 pgrad: 0.853356525539112
## iter: 10 f-value: 0.016516029746572 pgrad: 0.0120712977513535
## iter: 20 f-value: 0.0164756351361383 pgrad: 0.0120314851809251
## iter: 30 f-value: 0.0164094260863119 pgrad:
                                                0.0119776028165485
## iter: 40 f-value: 0.0162513521251769 pgrad:
                                               0.0118732801445815
## iter: 50 f-value: 0.015916994740855 pgrad: 0.0116732311841027
## iter: 60 f-value: 0.0155576853629453 pgrad: 0.011456634172248
## iter: 70 f-value: 0.0152110369487963 pgrad: 0.0112438054758365
## iter: 80 f-value: 0.014876946934557 pgrad: 0.0110348760770005
## iter: 90 f-value: 0.0145549610062173 pgrad: 0.0107672215733081
## iter: 100 f-value: 0.0142446397905044 pgrad: 0.00968676975184177
## iter: 110 f-value: 0.0142008753132774 pgrad: 0.00958803745366726
## iter: 120 f-value: 0.0141763978362612 pgrad: 0.00956134082254254
## iter: 130 f-value: 0.0141315053956398 pgrad: 0.00952304364897447
## iter: 140 f-value: 0.0140121627960945 pgrad: 0.00944387784966538
```

```
150
               f-value:
                          0.0138134938763977
                                                         0.00932130225686695
## iter:
                                                pgrad:
          160
## iter:
                f-value:
                          0.0136141953627531
                                                pgrad:
                                                         0.00919712671138523
                          0.0134200902720165
   iter:
          170
                f-value:
                                                pgrad:
                                                         0.00895368915187497
          180
##
  iter:
                f-value:
                          0.0132073205643863
                                                pgrad:
                                                         0.00761635887316917
##
   iter:
          190
                f-value:
                          0.0131748907198041
                                                pgrad:
                                                         0.00722294679600685
          200
                f-value:
                          0.0131693125416204
                                                         0.00713756058202067
##
   iter:
                                                pgrad:
  iter:
          210
                f-value:
                          0.0131638000829557
                                                pgrad:
                                                         0.00705836199907511
          220
## iter:
                f-value:
                          0.013158300169665
                                               pgrad:
                                                        0.00698383410916281
##
   iter:
          230
                f-value:
                          0.0131527487587821
                                                pgrad:
                                                         0.00691250776299573
##
   iter:
          240
                f-value:
                          0.0131470575480164
                                                pgrad:
                                                         0.00684281486371627
  iter:
          250
                f-value:
                          0.0131410927324594
                                                         0.00677286541731575
                                                pgrad:
          260
##
   iter:
                f-value:
                          0.0131346347833411
                                                pgrad:
                                                         0.00670004332624699
                                                         0.00662013744607978
##
          270
                f-value:
                          0.0131272894091107
   iter:
                                                pgrad:
   iter:
          280
##
                f-value:
                          0.0131182545099014
                                                pgrad:
                                                         0.00652510599346015
          290
                f-value:
##
  iter:
                          0.0131055646160389
                                                pgrad:
                                                         0.00639590572922713
   iter:
          300
                f-value:
                          0.0130931916219669
                                                         0.00629211057363543
##
                                                pgrad:
##
          310
                f-value:
                          0.0130888913730386
   iter:
                                                pgrad:
                                                         0.00628509981096401
          320
                f-value:
                           0.0130830812802344
   iter:
                                                pgrad:
                                                         0.00627770970120481
          330
##
                f-value:
                          0.0130734017216692
                                                         0.00626873461470742
  iter:
                                                pgrad:
##
   iter:
          340
                f-value:
                          0.0130505499860338
                                                pgrad:
                                                         0.00625471514134543
##
  iter:
          350
                f-value:
                          0.0130025970107464
                                                pgrad:
                                                         0.00623198676885567
          360
##
  iter:
                f-value:
                           0.0129493241346159
                                                pgrad:
                                                         0.0062073615830614
## iter:
          370
                f-value:
                          0.012896384838806
                                                        0.00618280371152014
                                               pgrad:
          380
##
  iter:
                f-value:
                          0.0128438556280118
                                                pgrad:
                                                         0.00615834134746943
          390
## iter:
                f-value:
                          0.0127917338346923
                                                pgrad:
                                                         0.00613397430696333
  iter:
          400
                f-value:
                          0.0127400162949079
                                                pgrad:
                                                         0.006109702219848
          410
                          0.0126886998659585
##
   iter:
                f-value:
                                                pgrad:
                                                         0.00608552471619195
##
   iter:
          420
                f-value:
                          0.0126377814295599
                                                         0.00606144142749757
                                                pgrad:
          430
                f-value:
##
   iter:
                          0.0125872578916737
                                                pgrad:
                                                         0.00603745198670277
## iter:
          440
                f-value:
                          0.012537126182317
                                               pgrad:
                                                        0.00601355602817411
##
   iter:
          450
                f-value:
                          0.0124873832553802
                                                pgrad:
                                                         0.00598975318770456
##
   iter:
          460
                f-value:
                          0.0124380260884353
                                                         0.00596604310250343
                                                pgrad:
##
          470
                f-value:
                          0.0123890516825584
                                                         0.00594242541119611
   iter:
                                                pgrad:
          480
                f-value:
                          0.0123404570621411
                                                         0.00591889975381371
##
   iter:
                                                pgrad:
          490
                f-value:
                          0.0122922392747112
##
   iter:
                                                pgrad:
                                                         0.0058954657717899
          500
##
   iter:
                f-value:
                          0.0122443953907578
                                                pgrad:
                                                         0.00587212310795732
  iter:
          510
                f-value:
                           0.0121969225035401
                                                pgrad:
                                                         0.00584887140653617
          520
                           0.0121498177289276
## iter:
                f-value:
                                                pgrad:
                                                         0.00582571031313711
          530
                f-value:
                           0.0121030782052067
##
   iter:
                                                pgrad:
                                                         0.00580263947474707
          540
##
                f-value:
                          0.0120567010929208
  iter:
                                                pgrad:
                                                         0.00577965853973145
  iter:
          550
                f-value:
                           0.0120106835746888
                                                pgrad:
                                                         0.00575676715782431
          560
                f-value:
                          0.0119650228550357
##
  iter:
                                                pgrad:
                                                         0.00559366517379045
##
   iter:
          570
                f-value:
                          0.0119302796005655
                                                pgrad:
                                                         0.00527169499254976
          580
                f-value:
                          0.0119249407523897
##
   iter:
                                                pgrad:
                                                         0.00526534119797634
          590
## iter:
                f-value:
                          0.0119138804134625
                                                         0.00525665443801282
                                                pgrad:
          600
                f-value:
## iter:
                          0.0118861518612309
                                                pgrad:
                                                         0.0052421760191988
##
  iter:
          610
                f-value:
                          0.0118493064344785
                                                pgrad:
                                                         0.00522461362207732
          620
##
   iter:
                f-value:
                          0.0118123949625558
                                                pgrad:
                                                         0.00520700047429212
##
   iter:
          630
                f-value:
                          0.0117757263332758
                                                         0.00518944531068594
                                                pgrad:
   iter:
          640
                f-value:
                          0.0117393009559286
                                                         0.00517194865703946
##
                                                pgrad:
          650
##
                f-value:
                          0.0117031172247939
                                                         0.00515451032275099
   iter:
                                                pgrad:
  iter:
          660
                f-value:
                          0.0116671735325456
                                                pgrad:
                                                         0.005137130113474
## iter:
          670
                f-value:
                          0.0116314682824786
                                                         0.00511980783548427
                                                pgrad:
          680
                f-value:
                          0.0115959998885111
                                                pgrad:
                                                         0.00510254329570298
## iter:
```

```
f-value:
                          0.0115607667751129
                                                pgrad:
                                                        0.0048462725498345
          700
## iter:
                f-value:
                          0.0115398391793801
                                                pgrad:
                                                        0.00456093494936505
   iter:
          710
                f-value:
                          0.0115384135635914
                                                pgrad:
                                                         0.00455237595057084
          720
  iter:
                f-value:
                          0.011536929264036
                                               pgrad:
                                                       0.00454496746000116
##
   iter:
          730
                f-value:
                          0.011535343539546
                                               pgrad:
                                                       0.00453843277343075
          740
                f-value:
                          0.0115335828135115
                                                pgrad:
##
   iter:
                                                        0.00453251530344834
  iter:
          750
                f-value:
                          0.0115315077080542
                                                pgrad:
                                                         0.00452695033661304
## iter:
          760
                f-value:
                          0.0115288131431757
                                                pgrad:
                                                         0.00452141023838176
   iter:
          770
                f-value:
                          0.0115246644492003
                                                pgrad:
                                                         0.00451536785170076
          780
   iter:
                f-value:
                          0.0115159595427127
                                                pgrad:
                                                         0.00450771230288816
  iter:
          790
                f-value:
                          0.0114945076250979
                                                         0.00449668764719469
                                                pgrad:
          800
   iter:
                f-value:
                          0.0114665813333646
                                                pgrad:
                                                         0.00448404695626453
##
          810
                f-value:
                          0.011438601388698
   iter:
                                               pgrad:
                                                       0.00447138503241257
          820
##
   iter:
                f-value:
                          0.0114107764123215
                                                pgrad:
                                                         0.00445875825554487
          830
##
  iter:
                f-value:
                          0.011383107001005
                                               pgrad:
                                                        0.00444616692794425
          840
                f-value:
                          0.0113555922925043
                                                         0.00443361095273709
   iter:
                                                pgrad:
          850
##
                f-value:
                          0.0113282314197989
                                                         0.00442109023068399
   iter:
                                                pgrad:
          860
                f-value:
                          0.0113010235206653
   iter:
                                                pgrad:
                                                         0.00440860466280768
          870
                f-value:
                          0.0112739677377127
                                                         0.00439615415040695
##
  iter:
                                                pgrad:
   iter:
          880
                f-value:
                          0.0112470632183562
                                                pgrad:
                                                         0.00438373859505858
##
  iter:
          890
                f-value:
                          0.0112203091147931
                                                pgrad:
                                                         0.00437135789861648
          900
  iter:
                f-value:
                          0.0111937045839724
                                                pgrad:
                                                         0.00435901196320904
## iter:
          910
                f-value:
                          0.0111672487875656
                                                         0.00434670069123849
                                                pgrad:
          920
   iter:
                f-value:
                          0.0111409408919517
                                                pgrad:
                                                         0.0040826513446211
  iter:
          930
                f-value:
                          0.0111253093175706
                                                pgrad:
                                                         0.00378224701938285
  iter:
          940
                f-value:
                          0.0111240967261606
                                                pgrad:
                                                         0.0037769248698441
          950
                f-value:
                          0.0111226996327526
   iter:
                                                pgrad:
                                                         0.00377202094301388
##
   iter:
          960
                f-value:
                          0.0111209533455099
                                                         0.00376729178840451
                                                pgrad:
          970
                f-value:
   iter:
                          0.0111184437401746
                                                pgrad:
                                                         0.00376240175089304
  iter:
          980
                f-value:
                          0.0111137726941247
                                                pgrad:
                                                         0.00375676846042555
   iter:
          990
                f-value:
                          0.0111016776586541
                                                pgrad:
                                                         0.00374946320816072
   iter:
          1000
                 f-value:
                           0.0110815741022807
                                                          0.00374074305151428
                                                 pgrad:
          1010
                 f-value:
                           0.0110608931097228
                                                          0.00373191089021419
   iter:
                                                 pgrad:
          1020
                 f-value:
                           0.0110403046643594
                                                          0.00372309854187475
   iter:
                                                 pgrad:
          1030
                 f-value:
                            0.0110198133773764
   iter:
                                                 pgrad:
                                                          0.00371430701287206
          1040
##
   iter:
                 f-value:
                           0.0109994188193799
                                                 pgrad:
                                                         0.00370553626013037
   iter:
          1050
                 f-value:
                            0.0109791205339025
                                                 pgrad:
                                                          0.00369678623458669
          1060
                 f-value:
                            0.0109589180664633
  iter:
                                                 pgrad:
                                                          0.00368805688725832
          1070
   iter:
                 f-value:
                            0.0109388109647245
                                                 pgrad:
                                                          0.00367934816927751
          1080
   iter:
                 f-value:
                            0.0109187987784861
                                                 pgrad:
                                                          0.00367066003189293
   iter:
          1090
                 f-value:
                            0.010898881059672
                                                pgrad:
                                                        0.00366199242646864
          1100
                           0.0108790573623207
   iter:
                 f-value:
                                                 pgrad:
                                                         0.0036533453044823
   iter:
          1110
                 f-value:
                           0.0108593272425768
                                                 pgrad:
                                                         0.0036447186175273
          1120
                 f-value:
                            0.0108396902586806
   iter:
                                                 pgrad:
                                                          0.00363611231731109
  iter:
          1130
                 f-value:
                            0.0108201459709576
                                                          0.00362752635565555
                                                 pgrad:
          1140
                 f-value:
  iter:
                            0.0108006939418096
                                                 pgrad:
                                                          0.00361896068449641
   iter:
          1150
                 f-value:
                            0.0107813337357022
                                                 pgrad:
                                                          0.00361041525588235
   iter:
          1160
                 f-value:
                            0.01076206491916
                                              pgrad:
                                                       0.00360189002197686
   iter:
          1170
                 f-value:
                            0.0107428870607509
                                                 pgrad:
                                                          0.00359338493505357
   iter:
          1180
                 f-value:
                           0.0107237997310823
                                                         0.00358489994750255
                                                 pgrad:
          1190
                           0.0107048025027866
   iter:
                 f-value:
                                                 pgrad:
                                                          0.00357643501182381
                                                 pgrad:
   iter:
          1200
                 f-value:
                            0.0106858949505135
                                                          0.00356799008063002
## iter:
          1210
                 f-value:
                            0.0106670766509232
                                                          0.00355956510664657
                                                 pgrad:
          1220
                f-value:
                           0.0106483471826715
                                                 pgrad:
## iter:
                                                         0.00355116004271006
```

```
## iter: 1230
               f-value: 0.0106297061264068
                                                pgrad:
                                                        0.00354277484176979
          1240
                           0.0106111530647536
## iter:
                f-value:
                                                pgrad:
                                                        0.00353440945688377
                                                       0.00352606384122234
   iter:
          1250
                f-value:
                           0.010592687582308
                                               pgrad:
          1260
                           0.0105743092656282
  iter:
                f-value:
                                                pgrad:
                                                         0.0035177379480672
   iter:
          1270
                f-value:
                           0.0105560177032229
                                                pgrad:
                                                        0.00350943173080953
          1280
                f-value:
                           0.0105378124855425
                                                pgrad:
                                                        0.0035011451429503
   iter:
  iter:
          1290
                f-value:
                           0.0105196932049756
                                                pgrad:
                                                         0.00349287813810251
  iter:
          1300
                f-value:
                           0.0105016594558279
                                                pgrad:
                                                         0.0034846306699855
   iter:
          1310
                f-value:
                           0.0104837108343269
                                                pgrad:
                                                         0.0034764026924311
          1320
   iter:
                f-value:
                           0.010465846938601
                                               pgrad:
                                                       0.00346819415937738
  iter:
          1330
                f-value:
                           0.0104480673686803
                                                pgrad:
                                                         0.00346000502487387
          1340
   iter:
                f-value:
                           0.0104303717264784
                                                pgrad:
                                                         0.00300971737171973
   iter:
          1350
                f-value:
                           0.0104249767483478
                                                pgrad:
                                                        0.00276784130332364
   iter:
          1360
                           0.0104243858759738
                                                         0.00273660244990526
                f-value:
                                                pgrad:
          1370
                f-value:
  iter:
                           0.0104238023216592
                                                pgrad:
                                                         0.00270983277042691
   iter:
          1380
                f-value:
                           0.0104232243920631
                                                pgrad:
                                                         0.00268686968160414
          1390
                f-value:
                           0.0104226505893126
                                                pgrad:
                                                         0.00266714619214603
   iter:
                f-value:
          1400
                           0.0104220794267309
                                                         0.00265017678818535
   iter:
                                                pgrad:
          1410
                f-value:
                           0.0104215092458276
                                                pgrad:
                                                         0.00263554534273347
  iter:
   iter:
          1420
                f-value:
                           0.0104209380149787
                                                pgrad:
                                                         0.00262289473057065
  iter:
          1430
                f-value:
                           0.0104203630847052
                                                pgrad:
                                                         0.00261191786338977
          1440
                f-value:
                                                pgrad:
  iter:
                           0.0104197808619877
                                                         0.00260234988118047
          1450
                f-value:
                           0.0104191863404785
## iter:
                                                pgrad:
                                                         0.00259396124136502
          1460
  iter:
                f-value:
                           0.0104185723711639
                                                pgrad:
                                                         0.00258655142940856
## iter:
          1470
                f-value:
                           0.0104179284466667
                                                pgrad:
                                                         0.0025799429570087
  iter:
          1480
                f-value:
                           0.0104172385194908
                                                pgrad:
                                                         0.00257397518272751
          1490
                           0.0104164767479779
  iter:
                f-value:
                                                pgrad:
                                                         0.00256849721720587
   iter:
          1500
                f-value:
                           0.0104155983292915
                                                pgrad:
                                                         0.0025633586488758
                        0.177980208584727
                                            pgrad:
   iter:
             f-value:
                                                    0.853356525539112
## iter:
              f-value:
                         0.0148904808745826
          10
                                              pgrad:
                                                      0.0111208340349231
## iter:
          20
              f-value:
                         0.0137847829071712
                                              pgrad:
                                                      0.0918508163662956
   iter:
          30
              f-value:
                         0.0124966546288499
                                              pgrad:
                                                      0.0193601209160939
   iter:
          40
              f-value:
                         0.0124096169583755
                                              pgrad:
                                                       0.005720676164024
   iter:
          50
              f-value:
                         0.0118256688202246
                                              pgrad:
                                                      0.00874019123275627
          60
              f-value:
                         0.0117862397163269
                                              pgrad:
   iter:
                                                      0.00517164334130216
##
   iter:
          70
              f-value:
                         0.0115779743653704
                                              pgrad:
                                                      0.00508893389437498
  iter:
          80
              f-value:
                         0.0111381591513354
                                              pgrad:
                                                      0.00423642218333864
                         0.0110484600399126
                                              pgrad:
## iter:
          90
              f-value:
                                                       0.00372291584987905
          100
               f-value:
                          0.0102931321421189
                                               pgrad:
  iter:
                                                        0.00522936799892953
          110
  iter:
               f-value:
                          0.0102798591808577
                                               pgrad:
                                                       0.00251174468015652
                          0.0104294668892665
  iter:
          120
               f-value:
                                               pgrad:
                                                       0.025995203035836
          130
               f-value:
                          0.0101235547099642
                                                       0.00911427177924784
   iter:
                                               pgrad:
   iter:
          140
               f-value:
                          0.0100859489569268
                                               pgrad:
                                                       0.00319558766098105
          150
               f-value:
                          0.010081326085537
   iter:
                                              pgrad:
                                                      0.001120416501322
                                              pgrad:
  iter:
          160
               f-value:
                          0.010080757796428
                                                       0.000392833265619008
          170
                          0.0100806879367057
## iter:
               f-value:
                                               pgrad:
                                                       0.000137732686366965
   iter:
          180
               f-value:
                          0.0100806793488571
                                               pgrad:
                                                       4.82909533238152e-05
     Successful convergence.
   iter:
             f-value: 0.221052805326838 pgrad:
                                                    0.909090909090909
   iter:
              f-value: 0.0990828571759875
                                              pgrad:
                                                      0.145699618434884
          10
   iter:
          20
                         0.0713330372432604
                                              pgrad:
                                                      0.123270702304469
              f-value:
  iter:
          30
              f-value:
                         0.0518234140388592
                                              pgrad:
                                                      0.099463155353744
## iter:
          40
                         0.0375032104882873
                                              pgrad:
                                                      0.15535209595742
              f-value:
## iter:
          50
              f-value:
                         0.0373245410300004
                                              pgrad:
                                                     0.144031544668052
```

```
60
             f-value:
                         0.0371580116500296
                                              pgrad:
                                                       0.135182467026881
## iter:
          70
              f-value:
                         0.0369954444075927
                                              pgrad:
                                                       0.128150742712242
  iter:
          80
              f-value:
                         0.0368288525130721
                                              pgrad:
                                                       0.122183946810691
          90
                         0.0366464121773918
                                                       0.117230689790636
  iter:
              f-value:
                                              pgrad:
##
   iter:
          100
               f-value:
                          0.0364210978269835
                                               pgrad:
                                                        0.112844295310678
                                              pgrad:
          110
                f-value:
                          0.036043254396454
   iter:
                                                       0.108278102030623
  iter:
          120
                f-value:
                          0.0320926317493761
                                               pgrad:
                                                        0.0969824647885573
## iter:
          130
               f-value:
                          0.0238165226268847
                                               pgrad:
                                                        0.0763573812095516
   iter:
          140
                f-value:
                          0.0193790238819223
                                               pgrad:
                                                        0.0623155775162049
          150
   iter:
                f-value:
                          0.0179930993943306
                                                pgrad:
                                                        0.0505764328254205
  iter:
          160
                f-value:
                          0.0179520672184054
                                                        0.0483818357656113
                                               pgrad:
          170
               f-value:
                          0.0179019744405778
   iter:
                                               pgrad:
                                                        0.0463264759941295
##
          180
               f-value:
                          0.0178272179300348
   iter:
                                               pgrad:
                                                        0.0441873125232421
   iter:
          190
##
                f-value:
                          0.0175818021895508
                                                pgrad:
                                                        0.0425674586759794
          200
                f-value:
  iter:
                          0.0155509075425813
                                               pgrad:
                                                        0.0358884318073929
   iter:
          210
                f-value:
                          0.0143988590181833
                                                        0.0311998712855412
                                                pgrad:
          220
##
                f-value:
                          0.0135177590939083
   iter:
                                               pgrad:
                                                        0.0258221323094635
          230
                f-value:
                          0.0128736193372711
   iter:
                                               pgrad:
                                                        0.0211721241374879
          240
               f-value:
                          0.0124718216744791
                                                        0.0174284270870084
##
  iter:
                                               pgrad:
   iter:
          250
               f-value:
                          0.0121678077007178
                                               pgrad:
                                                        0.0151842477499732
##
  iter:
          260
               f-value:
                          0.0121435990096132
                                               pgrad:
                                                        0.0154455077489475
          270
  iter:
                f-value:
                          0.0120047630179772
                                               pgrad:
                                                        0.0135233795284702
          280
                          0.0119903017245452
## iter:
               f-value:
                                               pgrad:
                                                        0.0129627060705776
          290
  iter:
                f-value:
                          0.011875729167634
                                              pgrad:
                                                       0.011595235200242
## iter:
          300
                f-value:
                          0.0117666697433981
                                               pgrad:
                                                        0.0103192666545281
  iter:
          310
                f-value:
                          0.0116825758369284
                                               pgrad:
                                                        0.00928938318628392
          320
                          0.011614872159814
##
   iter:
               f-value:
                                              pgrad:
                                                       0.00831991635058044
##
   iter:
          330
               f-value:
                          0.0115587866603611
                                               pgrad:
                                                        0.00745057930133952
          340
               f-value:
                          0.0115109306486427
##
   iter:
                                                pgrad:
                                                        0.00670105267562149
## iter:
          350
                f-value:
                          0.0114688950747482
                                                        0.00605222255162668
                                               pgrad:
## iter:
          360
               f-value:
                          0.0114565466799454
                                                pgrad:
                                                        0.0061102801139114
##
   iter:
          370
                f-value:
                          0.0114560782635908
                                                        0.00617749272525597
                                               pgrad:
          380
                f-value:
                          0.0114555626353106
                                                        0.00623907936599111
   iter:
                                                pgrad:
          390
               f-value:
                          0.0114549708363114
                                                        0.00629631525953311
##
   iter:
                                               pgrad:
          400
               f-value:
                          0.0114542430637935
   iter:
                                                pgrad:
                                                        0.00635067733210232
##
   iter:
          410
               f-value:
                          0.0114532226505087
                                               pgrad:
                                                        0.006404089630558
  iter:
          420
                f-value:
                          0.0114512216140231
                                               pgrad:
                                                        0.00645813597044409
          430
               f-value:
                          0.0114356431789195
## iter:
                                               pgrad:
                                                        0.00635807538779465
          440
                f-value:
                          0.0114038764076723
  iter:
                                               pgrad:
                                                        0.00603417158687863
          450
                          0.0113731736243587
##
                f-value:
                                                pgrad:
                                                        0.00575301526035965
  iter:
  iter:
          460
                f-value:
                          0.0113429807192508
                                               pgrad:
                                                        0.00550779189400707
          470
               f-value:
                          0.0113127517343665
  iter:
                                                pgrad:
                                                        0.00529251482049575
   iter:
          480
               f-value:
                          0.0112818661738723
                                               pgrad:
                                                        0.00510182450973173
          490
                          0.011249502728779
   iter:
                f-value:
                                              pgrad:
                                                       0.00493075480185318
          500
## iter:
                f-value:
                          0.0112144012289523
                                               pgrad:
                                                        0.0047744041315953
                          0.0111743103074669
          510
               f-value:
## iter:
                                               pgrad:
                                                        0.00462733583221631
##
  iter:
          520
                f-value:
                          0.0111243774633905
                                               pgrad:
                                                        0.00452751719953231
          530
   iter:
                f-value:
                          0.011059734063939
                                              pgrad:
                                                       0.00459744660586355
  iter:
          540
                f-value:
                          0.011017854748143
                                              pgrad:
                                                       0.00462478855716758
   iter:
          550
               f-value:
                          0.0109814199091766
                                               pgrad:
                                                        0.00464315344537625
          560
                          0.010933886271514
##
                                              pgrad:
                                                       0.00465259484978957
   iter:
               f-value:
## iter:
          570
                f-value:
                          0.0109308804866703
                                               pgrad:
                                                        0.00465762124890971
## iter:
          580
                f-value:
                          0.0108789639529254
                                               pgrad:
                                                        0.00464751814526561
## iter:
          590
               f-value:
                          0.0108785810072144
                                               pgrad:
                                                        0.00465053922770288
```

```
600
               f-value:
                          0.0108781318125949
                                                pgrad:
                                                        0.00465343578538092
## iter:
          610
               f-value:
                          0.0108775580614084
                                                pgrad:
                                                        0.00465632559971572
   iter:
          620
                f-value:
                          0.0108766775606083
                                                pgrad:
                                                        0.00465946101462325
          630
##
  iter:
               f-value:
                          0.0108739974516835
                                                pgrad:
                                                        0.00466381923768597
##
   iter:
          640
               f-value:
                          0.0107445383515642
                                                pgrad:
                                                        0.00458625187069939
          650
                f-value:
                          0.0107442873816401
                                                        0.00459443905900046
##
   iter:
                                                pgrad:
  iter:
          660
                f-value:
                          0.0107440345113753
                                                pgrad:
                                                        0.00460171590226985
## iter:
          670
               f-value:
                          0.0107437790158301
                                                pgrad:
                                                        0.00460819098946605
   iter:
          680
                f-value:
                          0.0107435199636922
                                                pgrad:
                                                        0.00461396187986039
          690
   iter:
                f-value:
                          0.0107432562080345
                                                pgrad:
                                                        0.00461911394708021
  iter:
          700
                f-value:
                          0.0107429861037732
                                                        0.00462372558801045
                                                pgrad:
          710
##
   iter:
               f-value:
                          0.0107427072974923
                                                pgrad:
                                                        0.00462786915481327
                                                        0.00463160940471597
##
          720
               f-value:
                          0.0107424165924229
   iter:
                                                pgrad:
   iter:
          730
                          0.0107421093224466
##
                f-value:
                                                pgrad:
                                                        0.00463500542713129
          740
                f-value:
##
  iter:
                          0.0107417776442926
                                                pgrad:
                                                        0.00463812112296391
          750
                f-value:
                          0.0107414090074502
                                                        0.00464101665557812
   iter:
                                                pgrad:
          760
##
                f-value:
                          0.0107409798635358
   iter:
                                                pgrad:
                                                        0.00464376463261342
          770
                f-value:
                          0.0107404388353668
   iter:
                                                pgrad:
                                                        0.00464646381976248
          780
##
               f-value:
                          0.0107396325371596
                                                        0.00464930661678298
  iter:
                                                pgrad:
   iter:
          790
               f-value:
                          0.0107375578608406
                                                pgrad:
                                                        0.00465284231282449
##
  iter:
          800
               f-value:
                          0.0105413042653027
                                                pgrad:
                                                        0.00466545287093988
          810
  iter:
                f-value:
                          0.0105410587916714
                                                pgrad:
                                                        0.00465855562774665
          820
## iter:
                f-value:
                          0.0105408103450169
                                                        0.00465239802490995
                                                pgrad:
          830
  iter:
                f-value:
                          0.0105405579446167
                                                pgrad:
                                                        0.00464688933166092
##
  iter:
          840
                f-value:
                          0.0105403001275799
                                                pgrad:
                                                        0.00464194631912123
  iter:
          850
                f-value:
                          0.0105400351796826
                                                pgrad:
                                                        0.0046374971870862
          860
               f-value:
                          0.0105397602958604
##
   iter:
                                                pgrad:
                                                        0.00463347048229589
          870
##
   iter:
               f-value:
                          0.0105394717085622
                                                        0.0046298034046064
                                                pgrad:
          880
                f-value:
##
   iter:
                          0.0105391634638854
                                                pgrad:
                                                        0.00462643164151041
## iter:
          890
                f-value:
                          0.0105388262806417
                                                pgrad:
                                                        0.00462329339900365
##
   iter:
          900
                f-value:
                          0.0105384429956797
                                                pgrad:
                                                        0.0046203095658105
##
   iter:
          910
                f-value:
                          0.0105379799375065
                                                pgrad:
                                                        0.00461738056091832
          920
                f-value:
                          0.01053735180958
                                             pgrad:
   iter:
                                                      0.00461432423363111
          930
                          0.0105362152236143
##
   iter:
                f-value:
                                               pgrad:
                                                        0.00461063401315343
                                               pgrad:
          940
                          0.010472414285985
   iter:
                f-value:
                                                       0.0045448754907887
          950
##
   iter:
               f-value:
                          0.0104714816263053
                                               pgrad:
                                                        0.00456232028008869
  iter:
          960
                f-value:
                          0.0104711852248868
                                               pgrad:
                                                        0.00456554402869258
          970
                          0.0104708640440809
##
  iter:
                f-value:
                                               pgrad:
                                                        0.00456849971205481
          980
                          0.010470504715041
   iter:
                f-value:
                                               pgrad:
                                                       0.00457124806757525
          990
##
                f-value:
                          0.0104700817167277
                                                pgrad:
                                                        0.00457385915600405
  iter:
  iter:
          1000
                 f-value:
                           0.010469536898102
                                                pgrad:
                                                        0.00457643400763648
          1010
                           0.0104686858549049
   iter:
                 f-value:
                                                 pgrad:
                                                         0.00457915898696955
   iter:
          1020
                 f-value:
                           0.010465752053106
                                                pgrad:
                                                        0.00458247984066262
          1030
                           0.010340981452539
   iter:
                 f-value:
                                                pgrad:
                                                        0.00462855364042539
  iter:
          1040
                 f-value:
                           0.0103407498473996
                                                         0.00461845405038484
                                                 pgrad:
          1050
## iter:
                 f-value:
                           0.0103405173859365
                                                 pgrad:
                                                         0.00460947541973022
   iter:
          1060
                 f-value:
                           0.0103402836269936
                                                 pgrad:
                                                         0.00460148583851137
          1070
   iter:
                 f-value:
                           0.0103400480266351
                                                 pgrad:
                                                         0.00459436811354136
   iter:
          1080
                 f-value:
                           0.0103398098952646
                                                         0.00458801758106771
                                                 pgrad:
   iter:
          1090
                 f-value:
                           0.0103395684015008
                                                         0.00458234158950965
                                                 pgrad:
          1100
                           0.0103393223677819
   iter:
                 f-value:
                                                 pgrad:
                                                         0.00457725545755214
   iter:
          1110
                 f-value:
                           0.0103390701483547
                                                 pgrad:
                                                         0.00457268210512304
          1120
                 f-value:
                           0.0103388097512725
                                                         0.00456855519955073
## iter:
                                                 pgrad:
          1130
                f-value:
                           0.010338537820798 pgrad: 0.0045648063107251
## iter:
```

```
## iter: 1140
                f-value: 0.0103382497678857
                                                         0.00456137543771515
                                                pgrad:
          1150
                           0.0103379381420235
## iter:
                f-value:
                                                pgrad:
                                                         0.00455819861180423
                                                         0.00455520876700571
   iter:
          1160
                 f-value:
                           0.0103375904193232
                                                pgrad:
          1170
                 f-value:
                           0.0103371830918969
  iter:
                                                pgrad:
                                                         0.00455232070316461
   iter:
          1180
                f-value:
                           0.0103366623013498
                                                pgrad:
                                                         0.00454939772091699
          1190
                 f-value:
                           0.0103358628062952
                                                pgrad:
                                                         0.00454614592498849
   iter:
  iter:
          1200
                 f-value:
                           0.0103333743743267
                                                pgrad:
                                                         0.0045411105904654
  iter:
          1210
                f-value:
                           0.010033699768066
                                               pgrad:
                                                        0.00449322900934745
   iter:
          1220
                 f-value:
                           0.0100334794709672
                                                pgrad:
                                                         0.00440280355359574
          1230
   iter:
                 f-value:
                           0.0100332604527638
                                                pgrad:
                                                         0.00432264908980029
   iter:
          1240
                 f-value:
                           0.0100330423060151
                                                pgrad:
                                                         0.00431917839877566
          1250
   iter:
                 f-value:
                           0.0100328246714181
                                                pgrad:
                                                         0.00433278676352634
          1260
                           0.010032607229019
                                               pgrad:
                                                        0.00434484619053135
   iter:
                 f-value:
          1270
                                                         0.00435553721403974
   iter:
                 f-value:
                           0.0100323896531002
                                                pgrad:
          1280
                 f-value:
  iter:
                           0.0100321716188555
                                                pgrad:
                                                         0.00436501746521979
   iter:
          1290
                 f-value:
                           0.0100319527572889
                                                pgrad:
                                                         0.00437342847227476
          1300
                 f-value:
                           0.0100317326387031
                                                pgrad:
                                                         0.00438089542704392
   iter:
          1310
                 f-value:
                           0.0100315107535798
                                                         0.00438752955053438
   iter:
                                                pgrad:
                           0.0100312864625905
          1320
                 f-value:
                                                pgrad:
                                                         0.0043934301162396
  iter:
   iter:
          1330
                 f-value:
                           0.0100310589156517
                                                pgrad:
                                                         0.00439868587103562
  iter:
          1340
                f-value:
                           0.010030826975778
                                               pgrad:
                                                        0.00440337694337681
          1350
                           0.0100305891902665
                                                pgrad:
  iter:
                 f-value:
                                                         0.0044075736384516
          1360
                 f-value:
                           0.0100303433404639
## iter:
                                                pgrad:
                                                         0.00441134282827296
          1370
                 f-value:
   iter:
                           0.0100300865324167
                                                pgrad:
                                                         0.00441474143542625
          1380
  iter:
                 f-value:
                           0.0100298140859327
                                                pgrad:
                                                         0.00441782842473942
  iter:
          1390
                 f-value:
                           0.0100295188508952
                                                pgrad:
                                                         0.00442065750640533
          1400
                           0.0100291885346481
                                                         0.00442328670850126
   iter:
                 f-value:
                                                pgrad:
   iter:
          1410
                 f-value:
                           0.0100287999305713
                                                pgrad:
                                                         0.0044257816098827
          1420
                           0.0100282997626373
   iter:
                 f-value:
                                                pgrad:
                                                         0.00442823698795526
  iter:
          1430
                 f-value:
                           0.010027518270187
                                               pgrad:
                                                        0.00443083092469997
   iter:
          1440
                 f-value:
                           0.0100248422140945
                                                pgrad:
                                                         0.00443393935992636
   iter:
          1450
                 f-value:
                           0.00991526233578761
                                                 pgrad:
                                                          0.00914231346381211
          1460
                 f-value:
                           0.00991468458356152
                                                 pgrad:
                                                          0.00816337090435632
   iter:
          1470
                 f-value:
                           0.00991421122311296
                                                          0.00737132642623728
   iter:
                                                 pgrad:
          1480
                           0.00991380615622386
   iter:
                 f-value:
                                                 pgrad:
                                                          0.00673028582270119
   iter:
          1490
                f-value:
                           0.00991344578257344
                                                 pgrad:
                                                          0.00621123403702195
  iter:
                f-value:
                           0.00991311464628044 pgrad:
                                                          0.00579071322638824
                        0.221052805326838 pgrad:
                                                    0.909090909090909
## iter:
             f-value:
                        0.0598330477881705
   iter:
          10
              f-value:
                                              pgrad:
                                                       0.276601939183102
                         0.0517838300351476
  iter:
          20
              f-value:
                                              pgrad:
                                                       0.0995962734271535
                         0.0204532964546295
  iter:
              f-value:
                                              pgrad:
                                                       0.0728362168906653
          40
                         0.0166007446012626
   iter:
              f-value:
                                              pgrad:
                                                       0.0392081474802859
   iter:
          50
              f-value:
                         0.0139448934804651
                                              pgrad:
                                                       0.0435650194891775
          60
              f-value:
                         0.0129694538061667
   iter:
                                              pgrad:
                                                       0.0373296985754654
## iter:
          70
              f-value:
                         0.0123771790679941
                                              pgrad:
                                                       0.0231575950977154
## iter:
          80
              f-value:
                         0.012038976321714
                                             pgrad:
                                                      0.0130733866111451
   iter:
          90
              f-value:
                         0.0118057866304535
                                              pgrad:
                                                       0.0123411034104632
   iter:
          100
               f-value:
                          0.0116174461625885
                                               pgrad:
                                                        0.0112271883680012
   iter:
          110
               f-value:
                          0.0115214583356063
                                               pgrad:
                                                        0.00773958701972899
   iter:
          120
               f-value:
                          0.0114533011839936
                                                        0.00550205705865736
                                               pgrad:
          130
               f-value:
                          0.0113707039080242
                                                        0.00513826652450217
   iter:
                                               pgrad:
  iter:
          140
               f-value:
                          0.0112520567937681
                                               pgrad:
                                                        0.0101710279318393
## iter:
          150
               f-value:
                          0.0112173262112534
                                               pgrad:
                                                        0.00531914464080646
## iter:
          160
               f-value:
                          0.0111116387883174
                                               pgrad:
                                                       0.00467488840883384
```

```
170
                f-value:
                          0.0110799794079837
                                                pgrad:
                                                         0.00464752715478857
          180
  iter:
                f-value:
                          0.0109106819332039
                                                pgrad:
                                                         0.00950822151265035
                          0.0109044931313208
   iter:
           190
                f-value:
                                                pgrad:
                                                         0.00483650441527805
          200
   iter:
                f-value:
                          0.0108768860509247
                                                pgrad:
                                                         0.00477631857459176
##
   iter:
          210
                f-value:
                          0.0107262245747569
                                                pgrad:
                                                         0.0291393490234714
          220
                f-value:
                          0.0107028089366294
##
   iter:
                                                pgrad:
                                                         0.00582867402563675
  iter:
          230
                f-value:
                          0.0106625039432071
                                                pgrad:
                                                         0.0118012350109123
  iter:
          240
                f-value:
                          0.0106572831503815
                                                pgrad:
                                                         0.00517154759988037
##
   iter:
          250
                f-value:
                          0.0106090561981512
                                                pgrad:
                                                         0.00451869231111129
##
   iter:
          260
                f-value:
                          0.0105627997704457
                                                pgrad:
                                                         0.00472118993205539
  iter:
          270
                f-value:
                          0.0104614490146434
                                                         0.00882263869166761
                                                pgrad:
          280
##
   iter:
                f-value:
                          0.0104551608471217
                                                pgrad:
                                                         0.00453806068461193
##
          290
                f-value:
                          0.0104044644029554
   iter:
                                                pgrad:
                                                         0.00459911758771389
   iter:
          300
                                                pgrad:
##
                f-value:
                           0.0103424775000279
                                                         0.00755155444469646
##
  iter:
           310
                f-value:
                           0.0099569734652526
                                                pgrad:
                                                         0.0906556118699715
           320
                f-value:
                           0.00979228214291445
   iter:
                                                 pgrad:
                                                          0.0108476933788483
          330
##
                f-value:
   iter:
                          0.00978421633597183
                                                 pgrad:
                                                          0.0040579093305268
           340
                f-value:
                           0.00974909675902864
   iter:
                                                 pgrad:
                                                          0.00550918840495213
          350
                f-value:
                          0.00971155746463015
##
   iter:
                                                 pgrad:
                                                          0.004190856767114
   iter:
          360
                f-value:
                          0.00968203788943579
                                                 pgrad:
                                                          0.00427558522677417
##
  iter:
          370
                f-value:
                          0.00955428909902171
                                                 pgrad:
                                                          0.00607598928263819
           380
  iter:
                f-value:
                           0.00949023032126044
                                                 pgrad:
                                                          0.0125600630827081
          390
                f-value:
                          0.00948267134538066
                                                          0.00407595061868717
## iter:
                                                 pgrad:
           400
##
   iter:
                f-value:
                           0.00945781408452619
                                                 pgrad:
                                                          0.00403181161752098
##
  iter:
          410
                f-value:
                           0.00935787247435729
                                                 pgrad:
                                                          0.00628609036345701
  iter:
          420
                f-value:
                           0.00933766733287337
                                                 pgrad:
                                                          0.00501154861418185
          430
##
   iter:
                f-value:
                          0.00929837645148809
                                                 pgrad:
                                                          0.00385530117457983
##
   iter:
          440
                f-value:
                          0.00902758967056701
                                                          0.00534007439540829
                                                 pgrad:
          450
##
   iter:
                f-value:
                           0.00899464770739329
                                                 pgrad:
                                                          0.00384924629786193
          460
                f-value:
##
  iter:
                           0.0089877249627724
                                                pgrad:
                                                         0.00540091009242072
##
   iter:
          470
                f-value:
                          0.00896543104296427
                                                 pgrad:
                                                          0.00399207915278049
##
          480
                f-value:
                          0.00890627105062749
                                                 pgrad:
                                                          0.00467568167963101
   iter:
           490
                f-value:
                           0.00887403673112892
                                                          0.0042704492690902
   iter:
                                                 pgrad:
          500
                f-value:
##
   iter:
                          0.00880594096149354
                                                          0.00371011798540322
                                                 pgrad:
          510
                f-value:
   iter:
                           0.00877844274696209
                                                 pgrad:
                                                          0.00370117604966963
          520
##
   iter:
                f-value:
                          0.00869187089707189
                                                 pgrad:
                                                          0.00571953678573744
  iter:
          530
                f-value:
                           0.00866522015789732
                                                 pgrad:
                                                          0.00362892424285949
          540
##
  iter:
                f-value:
                           0.00865261380679733
                                                 pgrad:
                                                          0.00353377561876828
          550
##
   iter:
                f-value:
                           0.00846234756077121
                                                 pgrad:
                                                          0.0177336019798146
          560
##
                f-value:
                           0.00846093400613193
                                                 pgrad:
  iter:
                                                          0.0102154377475788
  iter:
          570
                f-value:
                           0.0084538731290676
                                                pgrad:
                                                         0.00317186657770469
          580
##
   iter:
                f-value:
                          0.00843072843507296
                                                 pgrad:
                                                          0.00340045279260959
##
   iter:
          590
                f-value:
                          0.00830136930165625
                                                 pgrad:
                                                          0.0328858244991686
          600
                f-value:
##
   iter:
                          0.00827990332146367
                                                 pgrad:
                                                          0.00442430458588716
  iter:
          610
                f-value:
                           0.00826785496786987
                                                          0.00482756994391503
                                                 pgrad:
          620
                f-value:
## iter:
                          0.00825605125863948
                                                 pgrad:
                                                          0.00332206171599347
##
   iter:
          630
                f-value:
                           0.00822858319925396
                                                 pgrad:
                                                          0.00347546376226748
##
   iter:
          640
                f-value:
                           0.00820358111959129
                                                 pgrad:
                                                          0.00327855678080674
   iter:
          650
                f-value:
                           0.00818881723086623
                                                          0.00324884493960176
                                                 pgrad:
          660
                f-value:
                          0.00816294419635713
   iter:
                                                 pgrad:
                                                          0.003254134782818
##
          670
                          0.00812658592079843
   iter:
                f-value:
                                                 pgrad:
                                                          0.00421154738109353
  iter:
          680
                f-value:
                          0.0081139687737887
                                                pgrad:
                                                         0.00330717222357668
          690
                          0.00808234596829086
## iter:
                f-value:
                                                 pgrad:
                                                          0.00326932739040664
          700
                f-value:
                          0.00786991072605025
                                                 pgrad:
## iter:
                                                          0.0128458319723205
```

```
710
                f-value:
                           0.00786599826096805
## iter:
                                                 pgrad:
                                                          0.00416432388861587
          720
## iter:
                f-value:
                           0.00785015136384478
                                                 pgrad:
                                                          0.00574899874812107
                           0.00784859398616078
   iter:
          730
                f-value:
                                                 pgrad:
                                                          0.00298313103097114
          740
##
   iter:
                f-value:
                           0.00781732081074598
                                                 pgrad:
                                                          0.00310950993599735
##
   iter:
          750
                f-value:
                           0.00779572781439942
                                                 pgrad:
                                                          0.00311393782992786
          760
                f-value:
                           0.00755298010987539
##
   iter:
                                                 pgrad:
                                                          0.00359296700705602
  iter:
          770
                f-value:
                           0.00754005682941537
                                                 pgrad:
                                                          0.00341162063395267
## iter:
          780
                f-value:
                           0.0075343826149095
                                                pgrad:
                                                         0.00344135436968085
##
   iter:
          790
                f-value:
                           0.00750951347028892
                                                 pgrad:
                                                          0.00310164377110195
##
   iter:
          800
                f-value:
                           0.0075033481127835
                                                pgrad:
                                                         0.00309133734139538
   iter:
          810
                f-value:
                           0.00747372373633524
                                                          0.00433290198417824
                                                 pgrad:
          820
##
   iter:
                f-value:
                           0.00747246454016574
                                                 pgrad:
                                                          0.00307169309663829
##
          830
                f-value:
                           0.00744976584349858
   iter:
                                                 pgrad:
                                                          0.00300674186166144
                           0.00743724195953044
##
   iter:
          840
                f-value:
                                                 pgrad:
                                                          0.0030695454511509
          850
##
  iter:
                f-value:
                           0.00742720583246284
                                                 pgrad:
                                                          0.00301935260069625
          860
                f-value:
                           0.00741232528410654
##
   iter:
                                                 pgrad:
                                                          0.00282166139519424
##
          870
                f-value:
                           0.00740067632485398
   iter:
                                                 pgrad:
                                                          0.00283945469459029
          880
                f-value:
                           0.00738869561752151
   iter:
                                                 pgrad:
                                                          0.00284912333742976
          890
                f-value:
                           0.0073707995263364
##
   iter:
                                                pgrad:
                                                         0.00301450161274003
   iter:
          900
                f-value:
                           0.00736456968592381
                                                 pgrad:
                                                          0.00398168142554112
##
   iter:
          910
                f-value:
                           0.00736251490397933
                                                 pgrad:
                                                          0.00286391250883639
          920
  iter:
                f-value:
                           0.00734076098116408
                                                 pgrad:
                                                          0.00294400610660606
          930
  iter:
                f-value:
                           0.00733417198390161
                                                 pgrad:
                                                          0.00285232064313118
##
          940
##
   iter:
                f-value:
                           0.00731488607100594
                                                 pgrad:
                                                          0.00283354571143513
##
  iter:
          950
                f-value:
                           0.00717368786705533
                                                  pgrad:
                                                          0.0465945944990266
   iter:
          960
                f-value:
                           0.00713235819309342
                                                 pgrad:
                                                          0.00456419480804759
          970
##
   iter:
                f-value:
                           0.00713064400583636
                                                 pgrad:
                                                          0.00265048932454603
##
   iter:
          980
                f-value:
                           0.00712647022587269
                                                          0.00254869758652998
                                                 pgrad:
          990
##
   iter:
                f-value:
                           0.00711670810122668
                                                  pgrad:
                                                          0.00263843255181795
          1000
                 f-value:
  iter:
                            0.00711472115155467
                                                  pgrad:
                                                           0.00271743821238585
##
   iter:
           1010
                 f-value:
                            0.00710626246767916
                                                  pgrad:
                                                           0.00259072673578808
##
   iter:
          1020
                 f-value:
                            0.00709195423993049
                                                           0.00636663757304368
                                                  pgrad:
           1030
                 f-value:
                            0.00709040085033405
                                                           0.00271342640956013
   iter:
                                                  pgrad:
          1040
                            0.00708440589180918
                                                           0.00274509437642007
##
   iter:
                 f-value:
                                                  pgrad:
           1050
                 f-value:
   iter:
                            0.00706573812867941
                                                  pgrad:
                                                           0.00259322603213091
          1060
##
   iter:
                 f-value:
                            0.00703469051914232
                                                  pgrad:
                                                           0.0128555374213597
   iter:
           1070
                 f-value:
                            0.00702874458331472
                                                  pgrad:
                                                           0.00260721896437782
          1080
                            0.00702166736158313
##
  iter:
                 f-value:
                                                  pgrad:
                                                           0.00280968801827025
           1090
   iter:
                 f-value:
                            0.00700452194352769
                                                  pgrad:
                                                           0.00288781669584293
          1100
##
                 f-value:
                            0.00700367795207215
   iter:
                                                  pgrad:
                                                           0.00260489301244676
   iter:
          1110
                 f-value:
                            0.00696737270989455
                                                  pgrad:
                                                           0.0304608584276835
          1120
   iter:
                 f-value:
                            0.00694462760457332
                                                  pgrad:
                                                           0.00436452021690392
   iter:
          1130
                 f-value:
                            0.00694317548267491
                                                  pgrad:
                                                           0.00256249656585028
          1140
                 f-value:
   iter:
                            0.00693715890024524
                                                  pgrad:
                                                           0.00254267118523022
  iter:
          1150
                 f-value:
                            0.0069189946699316
                                                          0.00280013145225189
                                                 pgrad:
          1160
##
   iter:
                 f-value:
                            0.00691706131734788
                                                  pgrad:
                                                           0.0026362630902681
##
   iter:
          1170
                 f-value:
                            0.00691038777607087
                                                  pgrad:
                                                           0.00248781825373606
   iter:
          1180
                 f-value:
                            0.00689938592804702
                                                  pgrad:
                                                           0.00249236303470624
   iter:
          1190
                 f-value:
                            0.00689465896678908
                                                           0.00248163182216066
                                                  pgrad:
   iter:
          1200
                 f-value:
                            0.00688259692739267
                                                           0.00476726998661592
                                                  pgrad:
##
          1210
                            0.00688160079496943
   iter:
                 f-value:
                                                  pgrad:
                                                           0.00243925485947227
   iter:
          1220
                 f-value:
                            0.00687381174296956
                                                  pgrad:
                                                           0.00221045168280476
          1230
                 f-value:
                            0.00685421009055642
## iter:
                                                  pgrad:
                                                           0.00396180189585837
          1240
                 f-value:
                            0.00685337924618855
                                                  pgrad:
## iter:
                                                           0.00229701825847178
```

```
## iter: 1250
                f-value:
                           0.00685061998681728
                                                         0.0022599162891162
                                                 pgrad:
          1260
## iter:
                f-value:
                           0.00684694232847875
                                                 pgrad:
                                                         0.00231750108831369
                                                         0.00245077338342314
   iter:
          1270
                f-value:
                           0.00683804936271901
                                                 pgrad:
          1280
                f-value:
  iter:
                           0.00683719604069893
                                                 pgrad:
                                                         0.00226702796292388
                           0.00671468872497997
   iter:
          1290
                f-value:
                                                 pgrad:
                                                         0.0108335687997843
          1300
                f-value:
                           0.00671215440139744
                                                 pgrad:
   iter:
                                                         0.00227438447242267
  iter:
          1310
                f-value:
                           0.00670930133064111
                                                 pgrad:
                                                         0.00212799690918242
  iter:
          1320
                f-value:
                           0.00670838771063075
                                                 pgrad:
                                                         0.00206875418416275
   iter:
          1330
                f-value:
                           0.00670563039129917
                                                 pgrad:
                                                         0.00207685528930829
          1340
   iter:
                f-value:
                           0.00670287787291262
                                                 pgrad:
                                                         0.00205284415479245
  iter:
          1350
                f-value:
                           0.00669953603411477
                                                         0.00207407741733651
                                                 pgrad:
          1360
   iter:
                f-value:
                           0.00669692986367538
                                                 pgrad:
                                                         0.00202948818051979
   iter:
          1370
                f-value:
                           0.00669185579524876
                                                 pgrad:
                                                         0.00222086849489367
          1380
                                                 pgrad:
   iter:
                f-value:
                           0.00668963110871684
                                                         0.00211453779543991
          1390
  iter:
                f-value:
                           0.00668709646587916
                                                 pgrad:
                                                         0.00209953388875805
   iter:
          1400
                f-value:
                           0.00668465008008465
                                                         0.00206169670167089
                                                 pgrad:
          1410
                f-value:
                           0.00668153171787548
                                                         0.00224824492211828
   iter:
                                                 pgrad:
                f-value:
          1420
                           0.00668023499850865
                                                         0.00218311055431153
   iter:
                                                 pgrad:
          1430
                f-value:
                           0.00667780328795542
                                                         0.00216652080679602
  iter:
                                                 pgrad:
   iter:
          1440
                f-value:
                           0.00667442039059133
                                                 pgrad:
                                                         0.00218927262603252
##
   iter:
          1450
                f-value:
                           0.00667076693093889
                                                 pgrad:
                                                         0.00222646880388322
          1460
  iter:
                f-value:
                           0.00665855680375632
                                                 pgrad:
                                                         0.0021766461578745
          1470
  iter:
                f-value:
                           0.00665694251409258
                                                         0.00219747104771939
                                                 pgrad:
   iter:
          1480
                f-value:
                           0.00665050902377035
                                                 pgrad:
                                                         0.00219485150395926
  iter:
          1490
                f-value:
                           0.0066419226296087
                                               pgrad:
                                                        0.00233916240399468
                           0.00663491146389973 pgrad:
  iter:
          1500
                f-value:
                                                        0.00231577080283862
                        0.221052805326838
   iter:
             f-value:
                                           pgrad: 0.909090909090909
   iter:
          10
              f-value:
                       0.0118036848089322 pgrad: 0.0145125328759166
   iter:
          20
              f-value:
                         0.0116143287551212
                                             pgrad:
                                                      0.0101271759629114
              f-value:
  iter:
          30
                         0.0317959407861968
                                             pgrad:
                                                      0.757772160351201
  iter:
          40
              f-value:
                         0.030497157595848
                                             pgrad:
                                                     0.74210028191009
                         0.00983726474549546 pgrad:
   iter:
          50
              f-value:
                                                       0.0359876957449514
                                             pgrad:
              f-value:
                         0.0200810004860273
                                                      0.645795387705607
   iter:
          70
                         0.00879732178394598
                                              pgrad:
                                                       0.0104946097345807
   iter:
              f-value:
                         0.00874162016967584
   iter:
              f-value:
                                               pgrad:
                                                       0.0322650174053616
##
   iter:
          90
              f-value:
                         0.042794158576465 pgrad: 0.874072153974836
   iter:
          100
               f-value:
                          0.00816313719286287 pgrad:
                                                        0.00335527926156728
                          0.0080569688242327
  iter:
          110
               f-value:
                                               pgrad:
                                                       0.00389721454691716
          120
               f-value:
                          0.0105400089112623
   iter:
                                               pgrad:
                                                       0.391196673451353
          130
                          0.00793768255513799
##
  iter:
               f-value:
                                                pgrad:
                                                        0.0122805495917568
                          0.00762067735542764
  iter:
          140
               f-value:
                                                pgrad:
                                                        0.0121740262981412
          150
               f-value:
                          0.0182100969553395
   iter:
                                               pgrad:
                                                       0.636089317253035
   iter:
          160
               f-value:
                          0.00748162067231964
                                                pgrad:
                                                        0.00376072602961973
          170
               f-value:
                          0.0074220555014156
   iter:
                                               pgrad:
                                                       0.00666101627625426
  iter:
          180
               f-value:
                          0.0116991742349984
                                                       0.499285819916741
                                               pgrad:
          190
               f-value:
                          0.0070719943392188
## iter:
                                               pgrad:
                                                       0.0125920364028162
  iter:
          200
               f-value:
                          0.0073021851166327
                                               pgrad:
                                                       0.160000114001862
   iter:
          210
               f-value:
                          0.00912310007605068
                                                pgrad:
                                                        0.375353869166983
                          0.00667781022435975
   iter:
          220
               f-value:
                                                pgrad:
                                                        0.0133596780923304
   iter:
          230
               f-value:
                          0.00666009677341328
                                                pgrad:
                                                        0.00262741257640842
          240
##
               f-value:
                          0.00663583320007166
                                                        0.0120400009087012
   iter:
                                                pgrad:
          250
                                                pgrad:
  iter:
               f-value:
                          0.00924457984391016
                                                        0.396120387458885
## iter:
          260
               f-value:
                          0.00870767789728867
                                                pgrad:
                                                        0.365334497743859
## iter:
          270
               f-value:
                          0.0064680049347368 pgrad: 0.00415466739440539
```

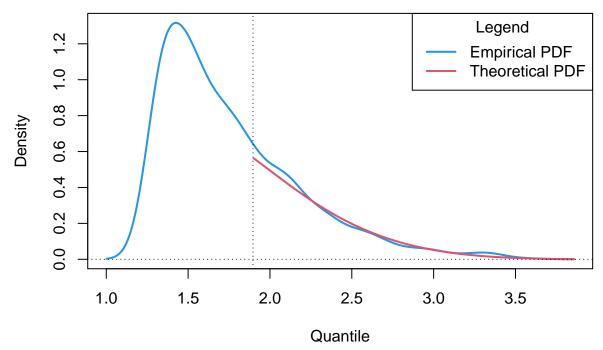
```
280
                f-value:
                          0.0105321677758269
## iter:
                                                pgrad:
                                                         0.494731242234545
          290
## iter:
                f-value:
                          0.0100049610720473
                                                         0.472943703990297
                                                pgrad:
   iter:
          300
                f-value:
                          0.0138186968354243
                                                pgrad:
                                                         0.592761490222173
          310
##
   iter:
                f-value:
                          0.0061948919175648
                                                pgrad:
                                                         0.00281016669244903
##
   iter:
          320
                f-value:
                          0.0061706381348664
                                                pgrad:
                                                         0.00212017979561652
          330
                f-value:
                          0.00616827414035668
                                                          0.00704115651516501
##
   iter:
                                                 pgrad:
  iter:
          340
                f-value:
                          0.0107474908446146
                                                pgrad:
                                                         0.532241659603696
##
  iter:
          350
                f-value:
                          0.0125816701692326
                                                pgrad:
                                                         0.598977576555581
##
   iter:
          360
                f-value:
                          0.00614581950865829
                                                          0.00554042945439337
                                                 pgrad:
##
   iter:
          370
                f-value:
                          0.00608134537245152
                                                 pgrad:
                                                          0.00172849502858019
  iter:
           380
                f-value:
                          0.00609402879224919
                                                          0.00526959780262271
                                                 pgrad:
          390
##
   iter:
                f-value:
                          0.00604952428629928
                                                 pgrad:
                                                          0.0208657086454901
##
          400
                f-value:
                          0.00703458436137554
   iter:
                                                 pgrad:
                                                          0.236470025313701
   iter:
                                                          0.48007895409873
##
          410
                f-value:
                           0.00967751611895991
                                                 pgrad:
          420
##
  iter:
                f-value:
                           0.00594564325411851
                                                 pgrad:
                                                          0.00155625849486292
           430
                f-value:
                          0.00593376934025122
                                                          0.00156743362322442
   iter:
                                                 pgrad:
          440
                f-value:
                          0.00589188095655883
                                                 pgrad:
##
   iter:
                                                          0.0014577545529898
           450
                f-value:
                           0.00660617244255302
   iter:
                                                 pgrad:
                                                          0.201903042528244
          460
                f-value:
                          0.00588814244141845
                                                          0.0233527718840268
##
   iter:
                                                 pgrad:
   iter:
          470
                f-value:
                          0.00587263680761971
                                                 pgrad:
                                                          0.00174480251987907
##
   iter:
          480
                f-value:
                          0.00585885006519701
                                                 pgrad:
                                                          0.0027288524840996
           490
                f-value:
  iter:
                           0.00584693148567402
                                                 pgrad:
                                                          0.00140324245398071
  iter:
          500
                f-value:
                          0.00583211458415129
                                                 pgrad:
                                                          0.00179729239538751
##
##
   iter:
          510
                f-value:
                          0.0058240735793989
                                                pgrad:
                                                         0.00153549349148574
##
  iter:
          520
                f-value:
                          0.00581403709829301
                                                 pgrad:
                                                          0.00149207431537957
  iter:
          530
                f-value:
                          0.00579150670040858
                                                 pgrad:
                                                          0.00135753096851332
          540
                f-value:
                          0.00579293872380642
##
   iter:
                                                 pgrad:
                                                          0.00188229611427171
##
   iter:
          550
                f-value:
                          0.00607142552317501
                                                 pgrad:
                                                          0.129346326173459
          560
                f-value:
##
   iter:
                          0.00575940904734696
                                                 pgrad:
                                                          0.0128624495115464
  iter:
          570
                f-value:
##
                           0.00642533611188862
                                                 pgrad:
                                                          0.206814983979993
##
   iter:
          580
                f-value:
                          0.00570795473615402
                                                 pgrad:
                                                          0.00165186526475258
##
   iter:
          590
                f-value:
                          0.00568797575842208
                                                          0.00233064043127862
                                                 pgrad:
   iter:
           600
                f-value:
                           0.00570354733853615
                                                          0.0371855278927313
                                                 pgrad:
          610
                f-value:
                          0.00574952133191243
##
   iter:
                                                 pgrad:
                                                          0.0616879074652015
          620
                f-value:
                          0.00601261389322443
   iter:
                                                 pgrad:
                                                          0.127203818031167
          630
##
   iter:
                f-value:
                          0.00565570975879268
                                                 pgrad:
                                                          0.021779790465613
  iter:
          640
                f-value:
                           0.00564619888380736
                                                 pgrad:
                                                          0.000828631645381939
          650
                           0.00563992828573555
##
  iter:
                f-value:
                                                 pgrad:
                                                          0.00118204648799514
          660
                f-value:
##
   iter:
                           0.00666098465161958
                                                 pgrad:
                                                          0.216122996464075
##
          670
                f-value:
                          0.00563190437515216
                                                          0.000738429558166387
  iter:
                                                 pgrad:
  iter:
          680
                f-value:
                           0.00562621913280783
                                                 pgrad:
                                                          0.000804985986636586
          690
                f-value:
##
   iter:
                          0.00560374531576061
                                                 pgrad:
                                                          0.000875192584685269
##
   iter:
          700
                f-value:
                          0.00559818463147247
                                                 pgrad:
                                                          0.000759276934696573
          710
                f-value:
##
   iter:
                          0.00556743788763344
                                                 pgrad:
                                                          0.000793419694281705
  iter:
          720
                f-value:
                           0.00555491450030265
                                                          0.000803098527853208
                                                 pgrad:
          730
                f-value:
##
  iter:
                          0.00555372038753195
                                                 pgrad:
                                                          0.00238844477261102
##
   iter:
          740
                f-value:
                          0.00554260298796829
                                                 pgrad:
                                                          0.000964361068904336
##
   iter:
          750
                f-value:
                           0.00553967730095845
                                                 pgrad:
                                                          0.000605217126414814
##
   iter:
          760
                f-value:
                          0.00554355267337512
                                                          0.00183805165059181
                                                 pgrad:
          770
               f-value:
                          0.00553949061690592
                                                          0.00517092436893118
##
   iter:
                                                 pgrad:
##
     Successful convergence
names(gev_mixture_model)
```

[1] "data"

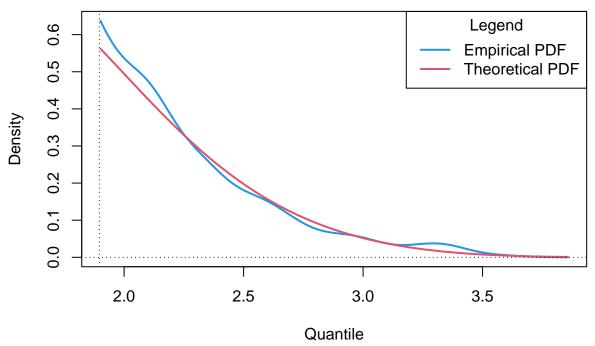
```
##
    [2] "data largest"
##
    [3] "block sizes"
##
    [4] "equivalent block sizes"
##
   [5] "rejected_block_sizes"
##
    [6] "block_maxima_indexes_object"
   [7] "gev models object"
##
   [8] "extremal indexes"
##
##
   [9] "normalized_gev_parameters_object"
##
  [10] "weighted_normalized_gev_parameters_object"
##
  [11] "identic_weights_mw"
  [12] "pessimistic_weights_mw"
  [13] "pessimistic_weights_pw_shape"
   [14] "pessimistic_weights_pw_scale"
  [15] "pessimistic_weights_pw_loc"
  [16] "automatic_weights_mw"
  [17] "automatic_weights_mw_statistics"
  [18] "automatic_weights_pw_shape"
  [19] "automatic weights pw scale"
  [20] "automatic_weights_pw_loc"
  [21] "automatic_weights_pw_statistics"
gev mixture model$block sizes
    [1] 10 11 12 13 14 15 16 17 18 19 20
gev_mixture_model$normalized_gev_parameters_object
##
              loc_star
                              scale_star
                                                shape_star
## 10 1.180399743401083 0.667445127250672 -0.216962155063532
## 11 1.192336224987749 0.652549243358429 -0.214785741907019
  12 1.205849521102029 0.637732832395915 -0.206187493026991
## 13 1.265458737051933 0.619544869157828 -0.206800102014219
  14 1.085029556981179 0.723897224092731 -0.240617936418741
  15 1.227649077478720 0.609748697210775 -0.197155578693372
## 16 1.057345709776979 0.731771178306135 -0.238812440817355
## 17 1.577797006808085 0.425140051982429 -0.117922425530432
## 18 0.974581354409016 0.792468640640622 -0.259466571031497
## 19 0.722007867474274 1.010967499512901 -0.325362305383457
## 20 1.009756155832240 0.814325107408905 -0.280702440943181
gev_mixture_model$weighted_normalized_gev_parameters_object
##
                              loc star
                                             scale star
                                                               shape star
                      1.13620099593666 0.698690042847031 -0.227706835529982
## identic_weights
## pessimistic_weights 1.17806294065337 0.718968633534225 -0.225166146842437
                      1.44716366792221 0.457735077067039 -0.117922425530432
## automatic weights
gev_mixture_model $automatic_weights_mw
##
                                                    12
                                                                     13
                 10
                                  11
##
                 14
                                  15
                                                    16
                                                                     17
##
  0.000000000000000 \ 0.665460411309261 \ 0.00000000000000 \ 0.334539588690739
                                                    20
##
                 18
                                  19
```

```
gev_mixture_model$automatic_weights_mw_statistics
## $function value
## [1] 0.01008067856721
## $gradient_value
## [1] 2.85943454373383e-05
## $function_reduction
## [1] 0.167899530017517
##
## $number_iterations
## [1] 1686
##
## $convergence
## [1] 0
##
## $message
## [1] "Successful convergence"
gev_mixture_model$automatic_weights_pw_statistics
## $function_value
## [1] 0.0055390396255099
##
## $gradient_value
## [1] 8.47456259212009e-06
## $function_reduction
## [1] 0.215513765701328
## $number_iterations
## [1] 3775
##
## $convergence
## [1] 0
##
## $message
## [1] "Successful convergence"
plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = TRUE,
                           zoom = FALSE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")
```

bility Density Function (PDF) Plot : automatic_weights - model_wise = TRUE : zoo



ability Density Function (PDF) Plot : automatic_weights - model_wise = TRUE : zoc



```
gev_mixture_model_parameters <- gev_mixture_model$normalized_gev_parameters_object</pre>
shapes <- gev_mixture_model_parameters$shape_star</pre>
scales <- gev_mixture_model_parameters$scale_star</pre>
locations <- gev_mixture_model_parameters$loc_star</pre>
weights <- gev_mixture_model$automatic_weights_mw</pre>
p < -0.95
q_initial_guesses <- sapply(1:length(weights), function(j) calculate_gev_inverse_cdf(p = p,
                                                                                         loc = locations[j]
                                                                                         scale = scales[j],
                                                                                         shape = shapes[j])
q_initial_guesses
    [1] 2.64175364363720 2.62520974564678 2.62231124733478 2.64039908372025
    [5] 2.62132347314048 2.59841767265660 2.61402861790055 2.64311258093824
   [9] 2.61560096540202 2.64706401297776 2.65051545445380
range(q_initial_guesses)
## [1] 2.5984176726566 2.6505154544538
block_size <- max(gev_mixture_model$block_sizes)</pre>
y <- gev_mixture_model$data_largest
threshold <- find_threshold_associated_with_given_block_size(x = y, block_size = block_size)
```

```
library(evd)
data <- y[y > threshold]
M3 <- fgev(data, prob = 0.95)
МЗ
##
## Call: fgev(x = data, prob = 0.95)
## Deviance: 90.7488023809861
## Estimates
        quantile scale
                                           shape
## 1.922576286634 0.198310461963 0.340437697411
##
## Standard Errors
          quantile
                               scale
                                                 shape
## 0.00975950564206  0.01218405732956  0.06986723101810
##
## Optimization Information
##
    Convergence: successful
##
    Function Evaluations: 65
    Gradient Evaluations: 13
M4 <- fgev(data)
M4
##
## Call: fgev(x = data)
## Deviance: 90.7488026701074
##
## Estimates
      loc
                         scale
                                           shape
## 2.104146565777 0.198314304228 0.340445742676
## Standard Errors
##
              loc
                             scale
                                              shape
## 0.0140109634943 0.0121831202276 0.0698716839473
## Optimization Information
##
    Convergence: successful
##
    Function Evaluations: 67
##
    Gradient Evaluations: 12
Fn <- ecdf(y)
p <- seq(from = Fn(threshold), to = 0.999, length.out = 20)
## [1] 0.70100000000000 0.716684210526316 0.732368421052632 0.748052631578947
## [5] 0.763736842105263 0.779421052631579 0.795105263157895 0.810789473684211
## [9] 0.826473684210526 0.842157894736842 0.857842105263158 0.873526315789474
## [13] 0.889210526315789 0.904894736842105 0.920578947368421 0.936263157894737
## [17] 0.951947368421053 0.967631578947368 0.983315789473684 0.9990000000000000
```

```
quantiles <- calculate_gev_mixture_model_inverse_cdf(p = p*0.1, locations, scales, shapes, weights, ite
quantiles
## [1] 0.876644389282018 0.881406051647020 0.886098608061482 0.890724779734860
   [5] 0.895287127967437 0.899788066650276 0.904229873556944 0.908614700565232
## [9] 0.912944582928962 0.917221447704429 0.921447121422848 0.925623337088754
## [13] 0.929751740574614 0.933833896473419 0.937871293463800 0.941865349235840
## [17] 0.945817415020323 0.949728779759315 0.953600673951837 0.957434273204707
probaility <- calculate_gev_mixture_model_cdf(q = quantiles, locations, scales, shapes, weights)</pre>
probaility
## [1] 0.070100000000000 0.0716684210526316 0.0732368421052630 0.0748052631578947
## [5] 0.0763736842105263 0.0779421052631577 0.0795105263157896 0.0810789473684210
## [9] 0.0826473684210528 0.0842157894736844 0.0857842105263159 0.0873526315789473
## [13] 0.0889210526315789 0.0904894736842103 0.0920578947368420 0.0936263157894736
## [17] 0.0951947368421052 0.0967631578947367 0.0983315789473684 0.0999000000000001
qnorm(p = p)
## [1] 0.527278791439508 0.573019369900915 0.619991842033896 0.668374236427926
  [5] 0.718374645462989 0.770239402477233 0.824264167978341 0.880809286378071
## [9] 0.940321583807025 1.003366190605948 1.070674551002344 1.143219739889629
## [13] 1.222340347465152 1.309956575262244 1.408976539846756 1.524140232935929
## [17] 1.664035860967863 1.847071060314614 2.127622188291079 3.090232306167813
calculate\_gev\_inverse\_cdf(p = p*0.1, loc = 2.52214, scale = 0.5222, shape = 0.1487)
  [1] 2.04704864576757 2.05082608071144 2.05455715735433 2.05824370453814
   [5] 2.06188744538342 2.06549000547116 2.06905292024001 2.07257764168813
##
   [9] 2.07606554445718 2.07951793136610 2.08293603845361 2.08632103958138
## [13] 2.08967405064320 2.09299613342025 2.09628829911788 2.09955151161514
## [17] 2.10278669045481 2.10599471359863 2.10917641996952 2.11233261180067
calculate_gev_inverse_cdf(p = p, loc = 1, scale = 0.5, shape = 0.1)
   [1] 1.54519616882743 1.58097321060849 1.61860059907953 1.65832648480524
##
   [5] 1.70044777858763 1.74532400031750 1.79339635588865 1.84521459754289
## [9] 1.90147580387572 1.96308201987432 2.03122890156810 2.10754769742410
## [13] 2.19434415751605 2.29502593155008 2.41492943609364 2.56309506962005
## [17] 2.75667954702242 3.03469566363171 3.52271486709944 5.97581256378162
```