Modeling extreme values with a GEV mixture probability distributions

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```
# library(xfun)
```

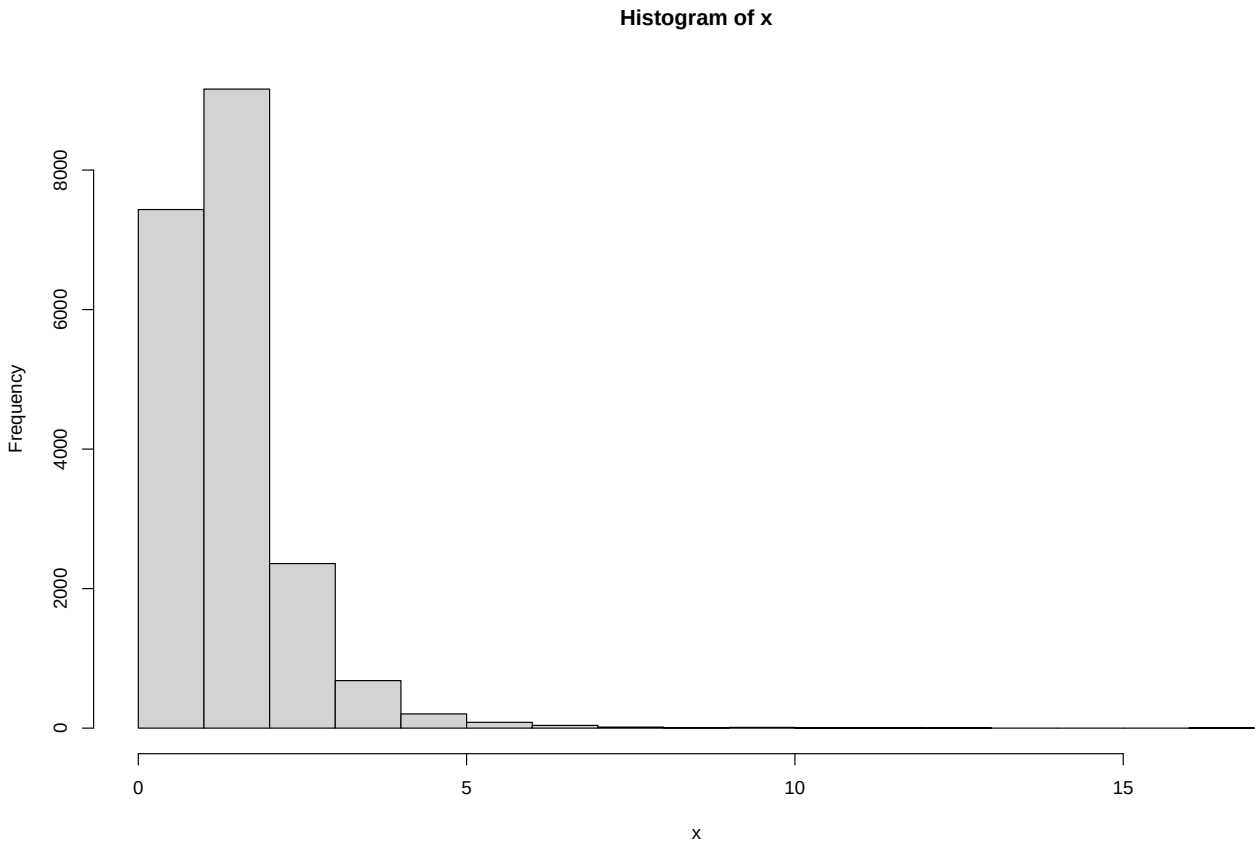
```
path <- ".."
```

```
xfun::in_dir(dir = path, expr = source("./src/generate_gev_sample.R"))
xfun::in_dir(dir = path, expr = source("./src/calculate_gev_inverse_cdf.R"))
xfun::in_dir(dir = path, expr = source("./src/estimate_gev_mixture_model_parameters.R"))
xfun::in_dir(dir = path, expr = source("./src/plot_gev_mixture_model_pdf.R"))
xfun::in_dir(dir = path, expr = source("./src/plot_several_standardized_block_maxima_mean.R"))
xfun::in_dir(dir = path, expr = source("./src/estimate_gev_mixture_model_quantile.R"))
```

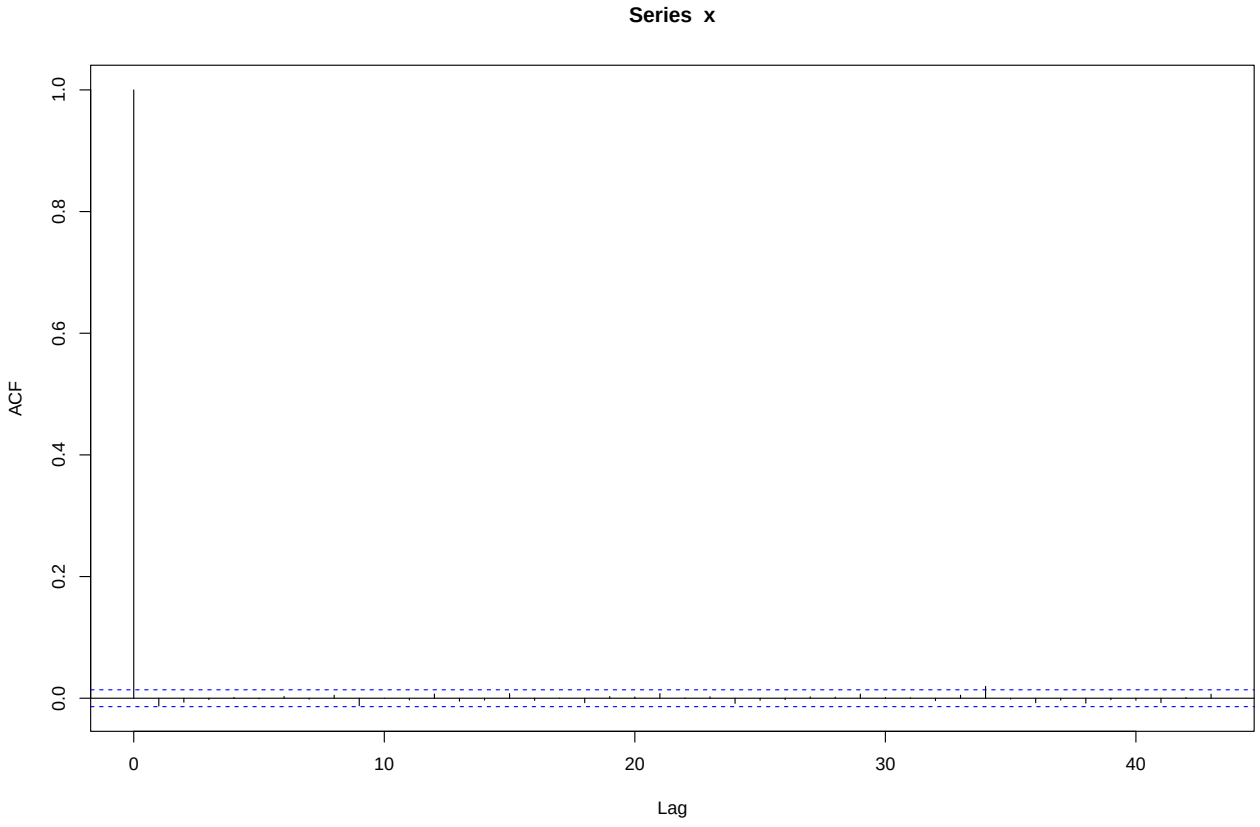
```
n <- 20000
```

```
loc <- 1
scale <- 0.5
shape <- +0.2
set.seed(1122)
x <- generate_gev_sample(n = n, loc = loc, scale = scale, shape = shape)
```

```
hist(x)
```


Histogram of x
Frequency
0
2000
4000
6000
8000
0
5
10
15
x

```r
acf(x)
```


Series x
ACF
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
Lag

```r
nlargest <- 2000

#
y <- extract_nlargest_sample(x, n = nlargest)
```

```r
gev_mixture_model <- estimate_gev_mixture_model_parameters(x,
                                                           nsloc = NULL,
                                                           std.err = FALSE,
                                                           block_sizes = NULL,
                                                           minimum_nblocks = 50,
                                                           threshold = NULL,
                                                           nlargest = nlargest,
                                                           confidence_level = 0.95,
                                                           log_mv = TRUE,
                                                           log_pw = TRUE,
                                                           trace = FALSE)
```

```
##   Successful convergence.
##   Unsuccessful convergence.
```

```r
names(gev_mixture_model)
```

```
##  [1] "data"
##  [2] "data_largest"
##  [3] "block_sizes"
##  [4] "equivalent_block_sizes"
##  [5] "rejected_block_sizes"
##  [6] "block_maxima_indexes_object"
##  [7] "gev_models_object"
##  [8] "extremal_indexes"
##  [9] "normalized_gev_parameters_object"
## [10] "weighted_normalized_gev_parameters_object"
## [11] "identic_weights_mw"
## [12] "pessimistic_weights_mw"
## [13] "pessimistic_weights_pw_shape"
## [14] "pessimistic_weights_pw_scale"
## [15] "pessimistic_weights_pw_loc"
## [16] "automatic_weights_mw"
## [17] "automatic_weights_mw_statistics"
## [18] "automatic_weights_pw_shape"
## [19] "automatic_weights_pw_scale"
## [20] "automatic_weights_pw_loc"
## [21] "automatic_weights_pw_statistics"
```

```r
gev_mixture_model$block_sizes
```

```
##  [1] 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34
## [26] 35 36 37 38 39 40
```

```r
gev_mixture_model$normalized_gev_parameters_object
```

```
##             loc_star        scale_star         shape_star
## 10 2.642403293700052 0.752127333100304 0.1935587282348927
## 11 2.546138058101965 0.787571463335049 0.1784257439409826
## 12 2.448741918369274 0.870026459583803 0.1493436703142377
## 13 2.255331098013337 1.008786408425567 0.1070485804267197
```

```
## 14 2.296912341240774 0.932702759199047 0.1306329520566816
## 15 2.429055725993036 0.847379499351585 0.1588713003574188
## 16 2.360661176339023 0.946939523225669 0.1220956111689579
## 17 2.169932954903481 0.995337339942381 0.1156599763903668
## 18 1.943932126962672 1.095935783820315 0.0911006326587247
## 19 1.638912124648246 1.253305730929867 0.0594632396237253
## 20 1.664601630802802 1.313317791178084 0.0446559805488399
## 21 1.922381505766038 1.074060243751823 0.0961433872378516
## 22 1.726253986447777 1.242306658108995 0.0587374482691914
## 23 1.581957384976938 1.252901421161388 0.0642171437919959
## 24 1.274898841824530 1.446256750647600 0.0263580995060073
## 25 1.615358764901647 1.234856291940610 0.0641951702496065
## 26 1.489736848164863 1.293977722061287 0.0571202483782339
## 27 1.709527600929698 1.212274427650484 0.0667434626977947
## 28 1.167417911854821 1.484845837983629 0.0196819470591896
## 29 1.182051564932219 1.513373325406365 0.0146705437269663
## 30 1.799216079521679 1.157524838302045 0.0782459349333187
## 31 1.307937241575498 1.371611870258447 0.0398375941076987
## 32 1.109823845111127 1.481391829889365 0.0284722809938165
## 33 1.395072464599815 1.331686676946735 0.0517239525997982
## 34 0.965750876277890 1.497649513843024 0.0280773272658508
## 35 1.861873177058224 1.113904482257353 0.0843882542220052
## 36 2.241202228014492 1.009365461994762 0.1002426565249170
## 37 0.871286797773485 1.493956882085667 0.0301000887396550
## 38 0.965812263584733 1.446608719979474 0.0403375480828114
## 39 1.872994133216964 1.218716144548067 0.0576162382314594
## 40 1.275156784355862 1.446540960170076 0.0263399097595917
```

```
gev_mixture_model$weighted_normalized_gev_parameters_object
```

```
##                            loc_star       scale_star         shape_star
## identic_weights     1.73330105645042 1.19765290809932 0.0769066339386873
## pessimistic_weights 1.97885488373038 1.24659267874985 0.0792013640170796
## automatic_weights   2.52883459686618 0.80225298727108 0.1712386489999690
```

```
gev_mixture_model$automatic_weights_mw_statistics
```

```
## $function_value
## [1] 0.00152910690520403
##
## $gradient_value
## [1] 1.73080077283116e-05
##
## $function_reduction
## [1] 0.0212986540509239
##
## $number_iterations
## [1] 1838
##
## $convergence
## [1] 0
##
## $message
## [1] "Successful convergence"
```

```
gev_mixture_model$automatic_weights_pw_statistics
```

```
## $function_value
## [1] 0.00160502397901242
##
## $gradient_value
## [1] 0.000296510001158934
##
## $function_reduction
## [1] 0.0216228515066112
##
## $number_iterations
## [1] 9006
##
## $convergence
## [1] 1
##
## $message
## [1] "Maximum number of iterations exceeded"
```

```
gev_mixture_model$automatic_weights_mw
```

```
##                 10                 11                 12                 13
## 0.5659539963357884 0.3650004054915857 0.0000000000000000 0.0000000000000000
##                 14                 15                 16                 17
## 0.0000000000000000 0.0000000000000000 0.0000000000000000 0.0000000000000000
##                 18                 19                 20                 21
## 0.0000000000000000 0.0000000000000000 0.0000000000000000 0.0000000000000000
##                 22                 23                 24                 25
## 0.0000000000000000 0.0000000000000000 0.0000000000000000 0.0000000000000000
##                 26                 27                 28                 29
## 0.0000000000000000 0.0000000000000000 0.0000000000000000 0.0000000000000000
##                 30                 31                 32                 33
## 0.0000000000000000 0.0000000000000000 0.0000000000000000 0.0000000000000000
##                 34                 35                 36                 37
## 0.0000000000000000 0.0000000000000000 0.0000000000000000 0.0690455981726258
##                 38                 39                 40
## 0.0000000000000000 0.0000000000000000 0.0000000000000000
```

```
gev_mixture_model$pessimistic_weights_pw_shape
```

```
##                 10                 11                 12                 13
## 0.0362079991134801 0.0356641891434439 0.0346419372196888 0.0332073061542551
##                 14                 15                 16                 17
## 0.0339997879634937 0.0349735701105632 0.0337107557224597 0.0334945022211841
##                 18                 19                 20                 21
## 0.0326819183343640 0.0316641325754101 0.0311987277493950 0.0328471414674281
##                 22                 23                 24                 25
## 0.0316411593596272 0.0318150191933412 0.0306330482910234 0.0318143201123490
##                 26                 27                 28                 29
## 0.0315900306339890 0.0318954956896628 0.0304292185470182 0.0302771069244869
##                 30                 31                 32                 33
## 0.0322644908532094 0.0310487618096963 0.0306978806241459 0.0314200206097837
##                 34                 35                 36                 37
## 0.0306857587756950 0.0324632795445305 0.0329820671049334 0.0307478915650622
```

```
##                  38                  39                  40
## 0.0310642886426129 0.0316057028549566 0.0306324910887108
```

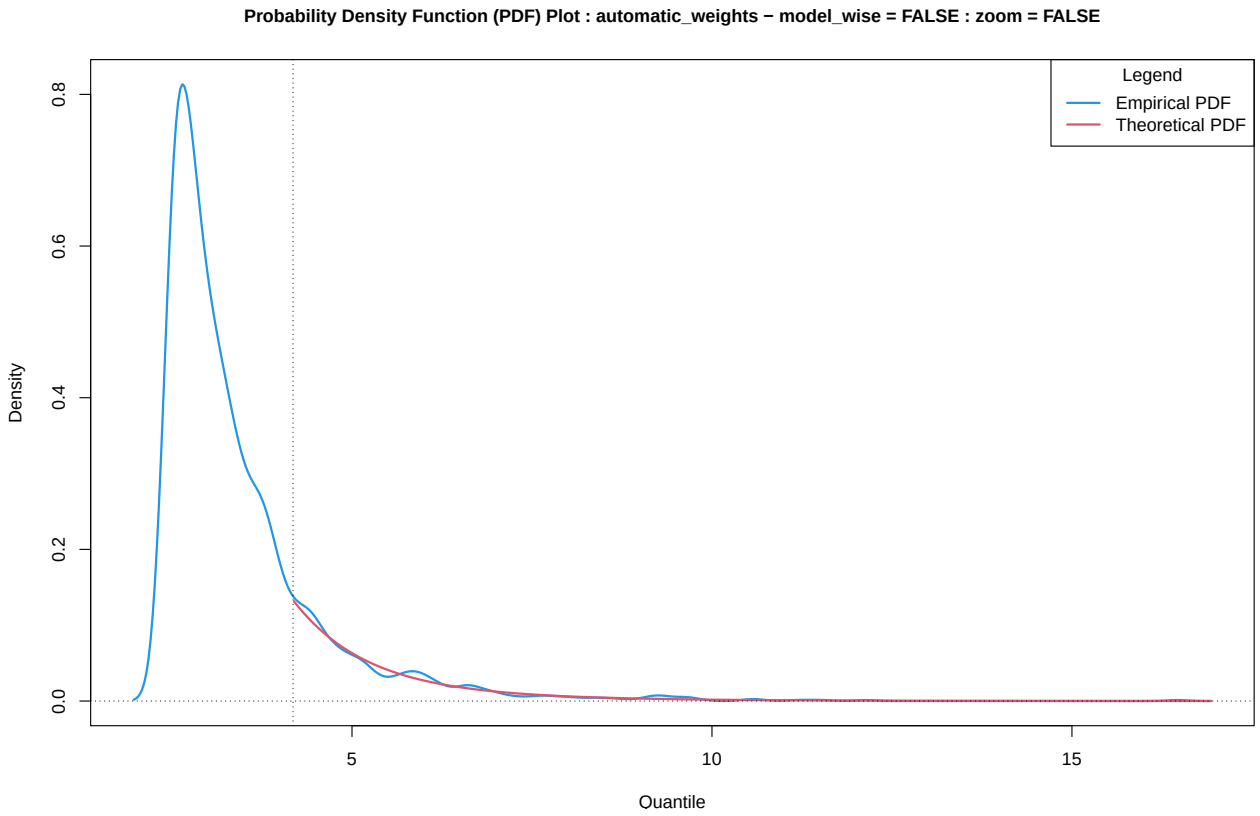
```
gev_mixture_model$pessimistic_weights_pw_scale
```

```
##                  10                  11                  12                  13
## 0.0201536514969441 0.0208807904257873 0.0226754901976806 0.0260506980146790
##                  14                  15                  16                  17
## 0.0241421895522401 0.0221677305805204 0.0244883544954232 0.0257026858583200
##                  18                  19                  20                  21
## 0.0284228653282011 0.0332669324978230 0.0353244707369622 0.0278078512073783
##                  22                  23                  24                  25
## 0.0329030320372886 0.0332534850706931 0.0403469137681483 0.0326588033269398
##                  26                  27                  28                  29
## 0.0346478569994410 0.0319295713594808 0.0419342951687878 0.0431478021020319
##                  30                  31                  32                  33
## 0.0302284336463692 0.0374448814927969 0.0417897036277742 0.0359793380830500
##                  34                  35                  36                  37
## 0.0424746602391220 0.0289382033349170 0.0260657871326266 0.0423181061859656
##                  38                  39                  40
## 0.0403611171438602 0.0321359165138373 0.0403583823749102
```

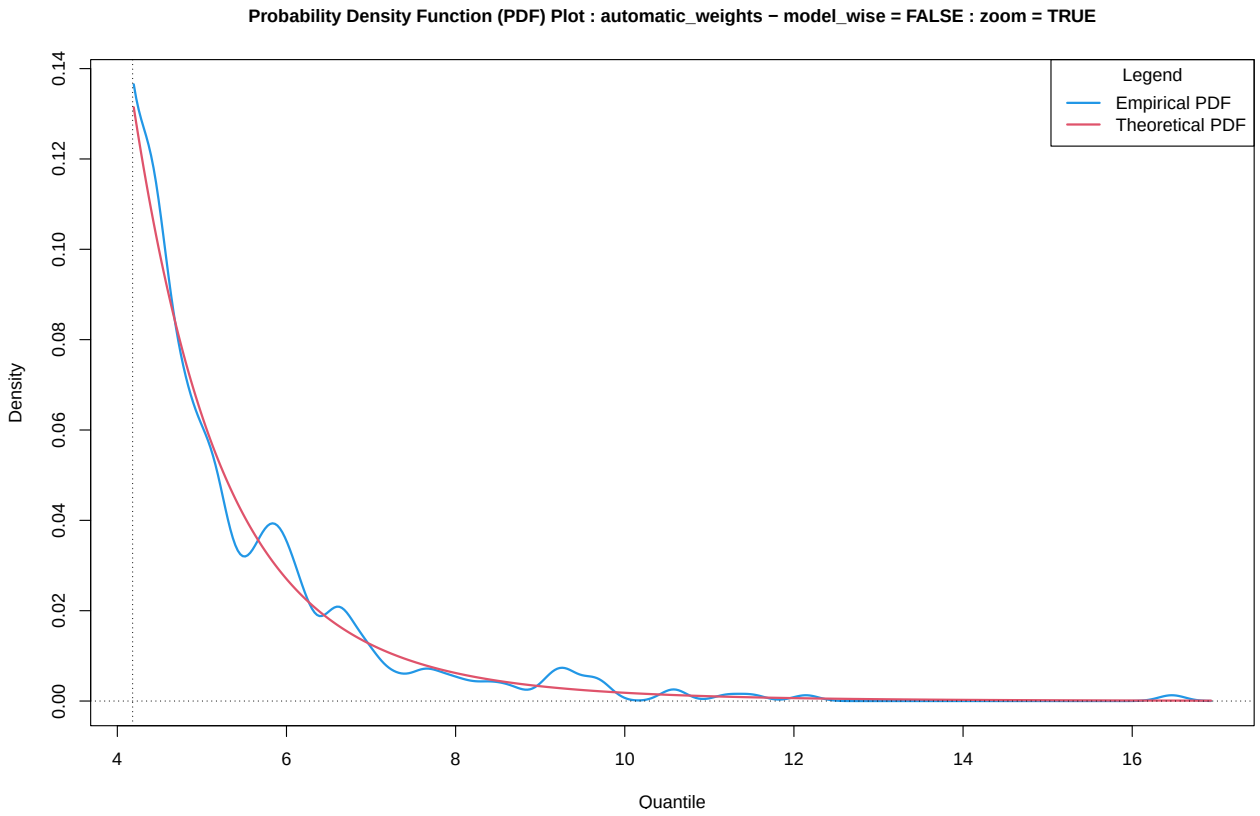
```
gev_mixture_model$pessimistic_weights_pw_loc
```

```
##                  10                  11                  12                  13
## 0.0706822524552801 0.0641952538055033 0.0582377133141203 0.0479962229663740
##                  14                  15                  16                  17
## 0.0500340394521149 0.0571024456984215 0.0533275134573627 0.0440675717062291
##                  18                  19                  20                  21
## 0.0351534705415042 0.0259119265234449 0.0265862151070086 0.0344039962387739
##                  22                  23                  24                  25
## 0.0282768997491983 0.0244773599481934 0.0180057304114358 0.0253087449573837
##                  26                  27                  28                  29
## 0.0223210029103443 0.0278078630026863 0.0161708321274071 0.0164092103932335
##                  30                  31                  32                  33
## 0.0304171715061619 0.0186105470028225 0.0152658005255615 0.0203049294414555
##                  34                  35                  36                  37
## 0.0132175051382824 0.0323839975709137 0.0473228587113132 0.0120260847445627
##                  38                  39                  40
## 0.0132183165502309 0.0327461485885080 0.0180103754541679
```

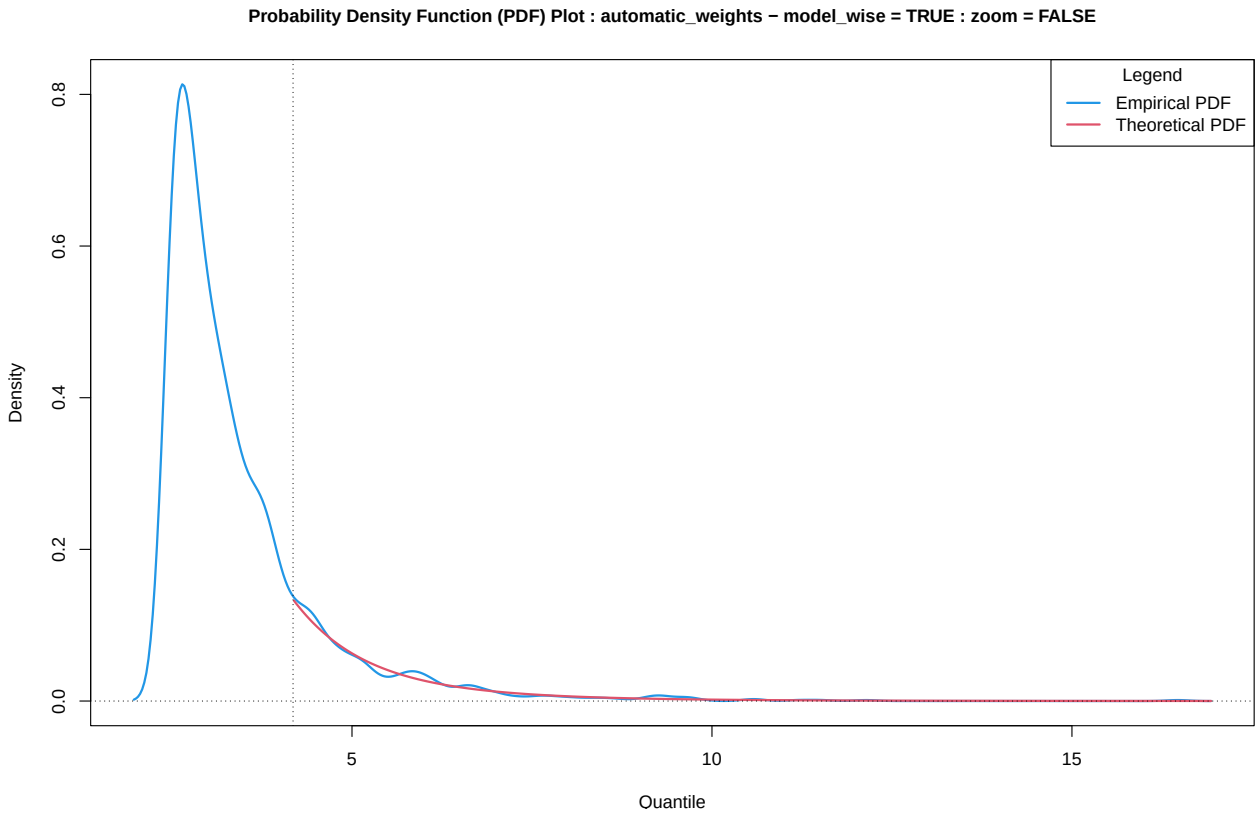
```
plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = FALSE,
                           zoom = FALSE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")
```



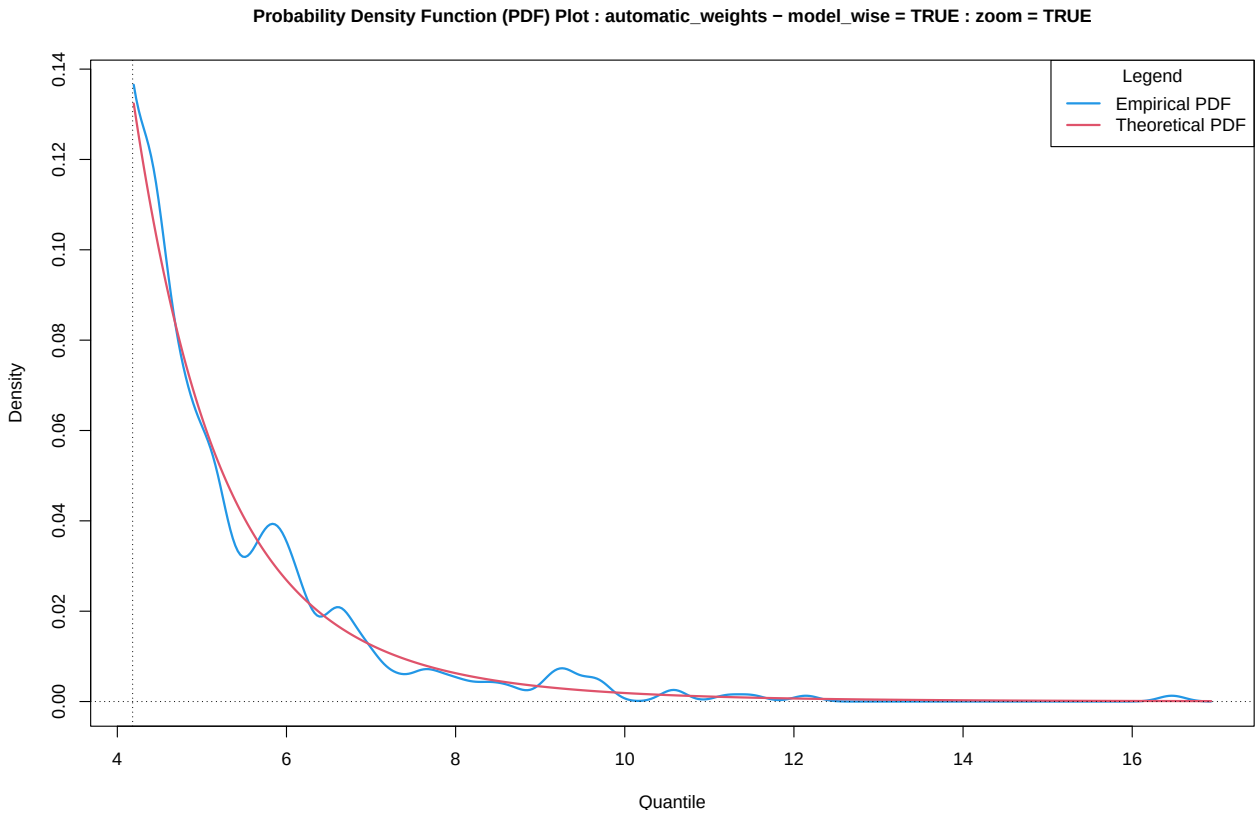

```
plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = FALSE,
                           zoom = TRUE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")
```




```
plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = TRUE,
                           zoom = FALSE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")
```




```
plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = TRUE,
                           zoom = TRUE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")
```




```
estimator_types <- c("automatic_weights_mw",
                     "pessimistic_weights_mw",
                     "identic_weights_mw",
                     "automatic_weights_pw",
                     "pessimistic_weights_pw",
                     "identic_weights_pw",
                     "empirical",
                     "confidence_interval_mw",
                     "confidence_interval_pw")
```

```
alpha <- 10^(-14)
```

```
rl_mw <- estimate_gev_mixture_model_quantile(gev_mixture_model,
                                             alpha = alpha,
                                             confidence_level = 0.95,
                                             do.ci = TRUE,
                                             estimator_type = estimator_types[1])

rl_mw
```

```
##   lower         estimate upper
## 1    NA 1164.07918658847    NA
```

```
rl_pw <- estimate_gev_mixture_model_quantile(gev_mixture_model,
                                             alpha = alpha,
                                             confidence_level = 0.95,
                                             do.ci = TRUE,
                                             estimator_type = estimator_types[4])
```

```
rl_pw
```

```
##   lower         estimate upper
## 1    NA 786.320539051601    NA
```

```
rl_empirical <- estimate_gev_mixture_model_quantile(gev_mixture_model,
                                                    alpha = alpha,
                                                    confidence_level = 0.95,
                                                    do.ci = TRUE,
                                                    estimator_type = estimator_types[7])

rl_empirical
```

```
##   lower         estimate upper
## 1    NA 16.4691752920578    NA
```

```
true_rl <- calculate_gev_inverse_cdf(p = 1 - alpha, loc = loc, scale = scale, shape = shape)
true_rl
```

```
## [1] 1576.14563730748
```

```
est_rl_pw <- estimate_gev_mixture_model_quantile(gev_mixture_model,
                                                 alpha = alpha,
                                                 confidence_level = 0.95,
                                                 do.ci = TRUE,
                                                 estimator_type = estimator_types[9])

est_rl_pw
```

```
##                lower         estimate            upper
## 10 -2484.64098212084 1274.19026529215 5033.02151270513
## 11  -1841.2937155674 911.007968219406 3663.30965200622
## 12  -925.72628590654 505.345773381006 1936.41783266855
## 13 -335.306618304781 225.002733353635  785.31208501205
## 14 -638.545542831646 351.496096165355 1341.53773516236
## 15 -1347.04049154573 616.931963044948 2580.90441763563
## 16 -519.760475803188 294.362156133265 1108.48478806972
## 17 -506.578894321681 267.983697975371 1042.54629027242
## 18 -284.466033955132 173.816252146402 632.098538247936
## 19 -135.696049779433 105.538602466583 346.773254712598
## 20 -87.4861155562066 84.2002721822366  255.88665992068
## 21 -335.947782046034  189.33371670804 714.615215462113
## 22 -142.997076187259 103.301810996719 349.600698180698
## 23 -185.473221191269 115.440328235464 416.353877662197
## 24 -61.4296027536465 67.1834067135052 195.796416180657
## 25 -185.806070475747 113.797727812177 413.401526100102
## 26 -175.810554563404 104.062655693533  383.93586595047
## 27 -206.798739673656 117.470953659791 441.740646993237
## 28 -54.8216983838722 61.7049156973922 178.231529778657
## 29 -47.8989912662825 58.0590712765047 164.017133819292
## 30 -292.689534659356 140.915367242746 574.520269144848
## 31 -112.743814785851 80.3339379845447 273.411690754941
## 32 -101.149232422002 71.0867695480135 243.322771518028
## 33 -180.182678810933 96.7381474310665 373.658973673066
## 34 -103.367974605884 71.2364568665077 245.840888338899
## 35 -351.396930164208  153.70460374184 658.806137647889
```

```
## 36 -485.980628984292 194.532569237435 875.045767459163
## 37 -110.286256789582 73.4409637464035 257.168184282389
## 38 -162.265774368551 85.0582562186526 332.382286805856
## 39 -166.927635394425 99.3982156228285 365.724066640082
## 40 -88.8781132169206 67.1747748736211 223.227662964163
```

```
est_rl_pw_range <- range(as.matrix(est_rl_pw))
est_rl_pw_range
```

```
## [1] -2484.64098212084  5033.02151270513
```

```
est_rl_mw <- estimate_gev_mixture_model_quantile(gev_mixture_model,
                                                 alpha = alpha,
                                                 confidence_level = 0.95,
                                                 do.ci = TRUE,
                                                 estimator_type = estimator_types[8])

est_rl_mw
```

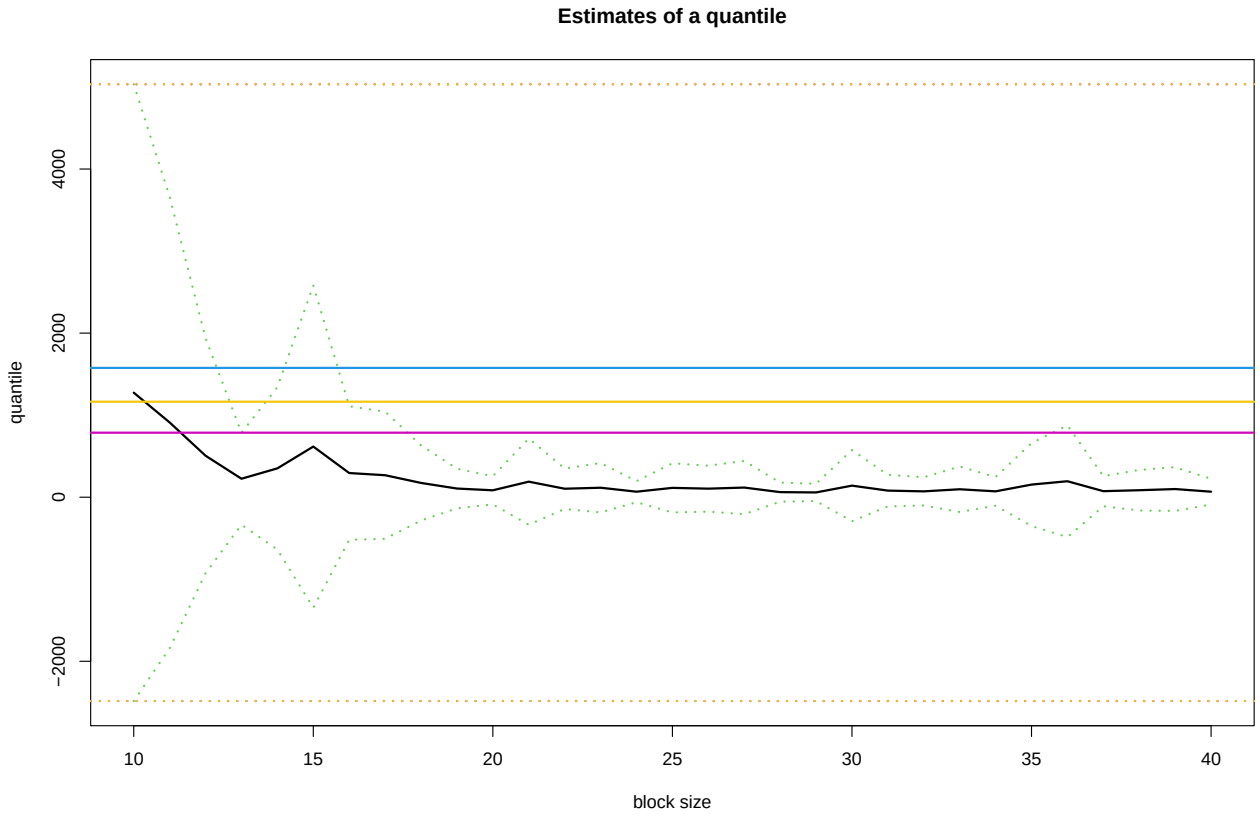
```
##                lower         estimate            upper
## 10 -2484.64098212084 1274.19026529215 5033.02151270513
## 11  -1841.2937155674 911.007968219406 3663.30965200622
## 37 -110.286256789582 73.4409637464035 257.168184282389
```

```
est_rl_mw_range <- range(as.matrix(est_rl_mw))
est_rl_mw_range
```

```
## [1] -2484.64098212084  5033.02151270513
```

```
matplot(x = rownames(est_rl_pw),
        y = est_rl_pw,
        xlab = "block size",
        ylab = "quantile",
        main = "Estimates of a quantile",
        ylim = range(c(est_rl_pw_range, true_rl)),
        cex = 1,
        cex.lab = 1,
        cex.axis = 1,
        type = "l",
        lty = c("dotted", "solid", "dotted"),
        lwd = c(2,2,2),
        col = c(3, 1, 3))

abline(h = true_rl, col = 4, lwd = 2)
abline(h = rl_mw[2], col = 7, lwd = 2)
abline(h = rl_pw[2], col = 6, lwd = 2)
abline(h = est_rl_pw_range, col = 6, lty = "dotted", lwd = 2)
abline(h = est_rl_mw_range, col = 7, lty = "dotted", lwd = 2)
```


Estimates of a quantile
quantile
4000
2000
0
−2000
10
15
20
25
30
35
40
block size