Modeling extreme values with a GEV mixture probability distributions

Application to localisation w.r.t. longitude

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```
path <- ".."
```

```
xfun::in_dir(dir = path, expr = source("./src/generate_gev_sample.R"))
xfun::in_dir(dir = path, expr = source("./src/calculate_gev_inverse_cdf.R"))
xfun::in_dir(dir = path, expr = source("./src/estimate_gev_mixture_model_parameters.R"))
xfun::in_dir(dir = path, expr = source("./src/plot_gev_mixture_model_pdf.R"))
xfun::in_dir(dir = path, expr = source("./src/plot_gev_mixture_model_cdf.R"))
xfun::in_dir(dir = path, expr = source("./src/estimate_gev_mixture_model_quantile.R"))
```

```
library(readr)
```

```
Gnss_imar <- xfun::in_dir(dir = path, expr = read_csv("./applications/Gnss_imar.csv"))
```

```
## Rows: 20002 Columns: 25
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## dbl (25): version_major, version_minor, status, timestamp, latitude, longitu...
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
Gnss_map_matching <- xfun::in_dir(dir = path, expr = read_csv("./applications/Gnss_map_matching.csv"))
```

```
## Rows: 20001 Columns: 25
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## dbl (25): version_major, version_minor, status, timestamp, latitude, longitu...
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

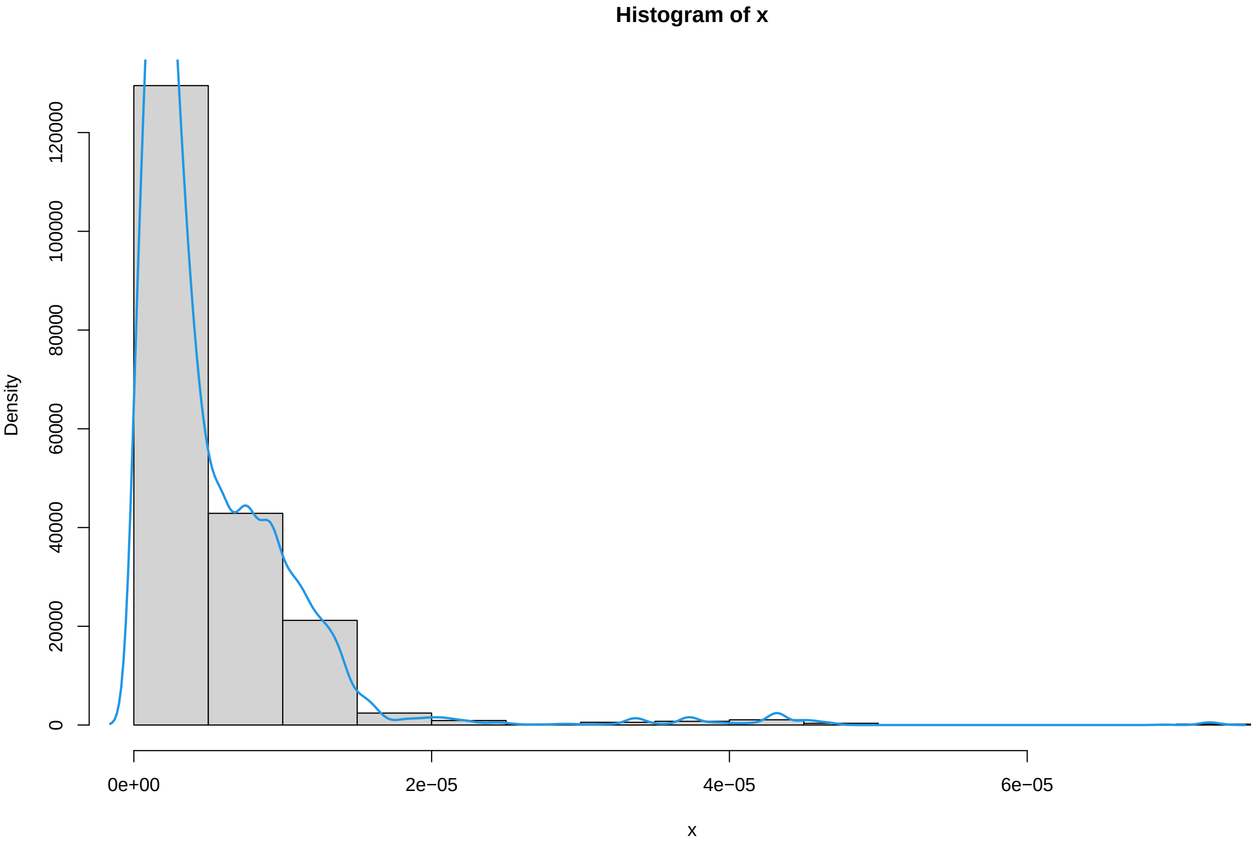
```
longitude_Gnss_map_matching_errors <- Gnss_imar$longitude[-1] - Gnss_map_matching$longitude
```

```
x <- abs(longitude_Gnss_map_matching_errors)
n <- length(x)

n
```

```
## [1] 20001
```

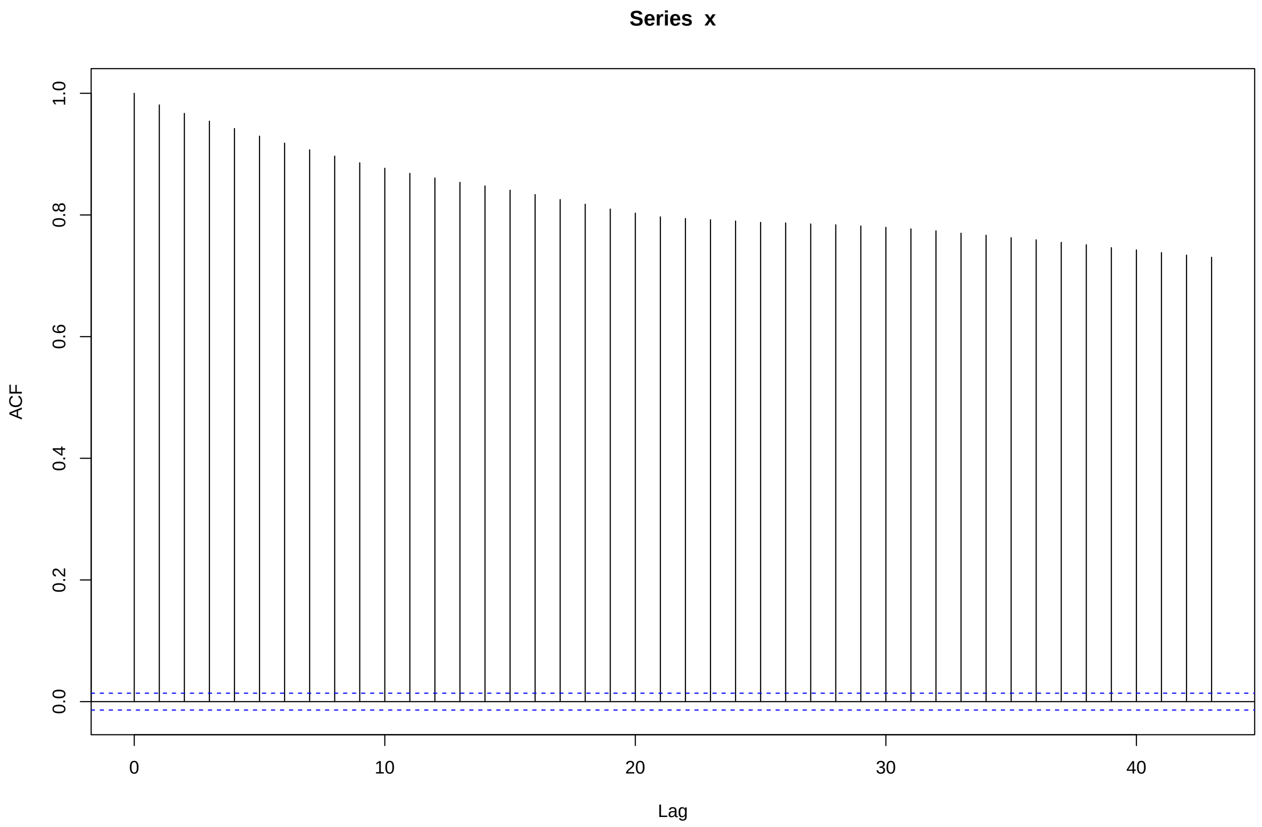
```
# Histogram of all data
```

```
hist(x, prob = TRUE)
lines(density(x),
      lwd = 2,
      col = 4)
```



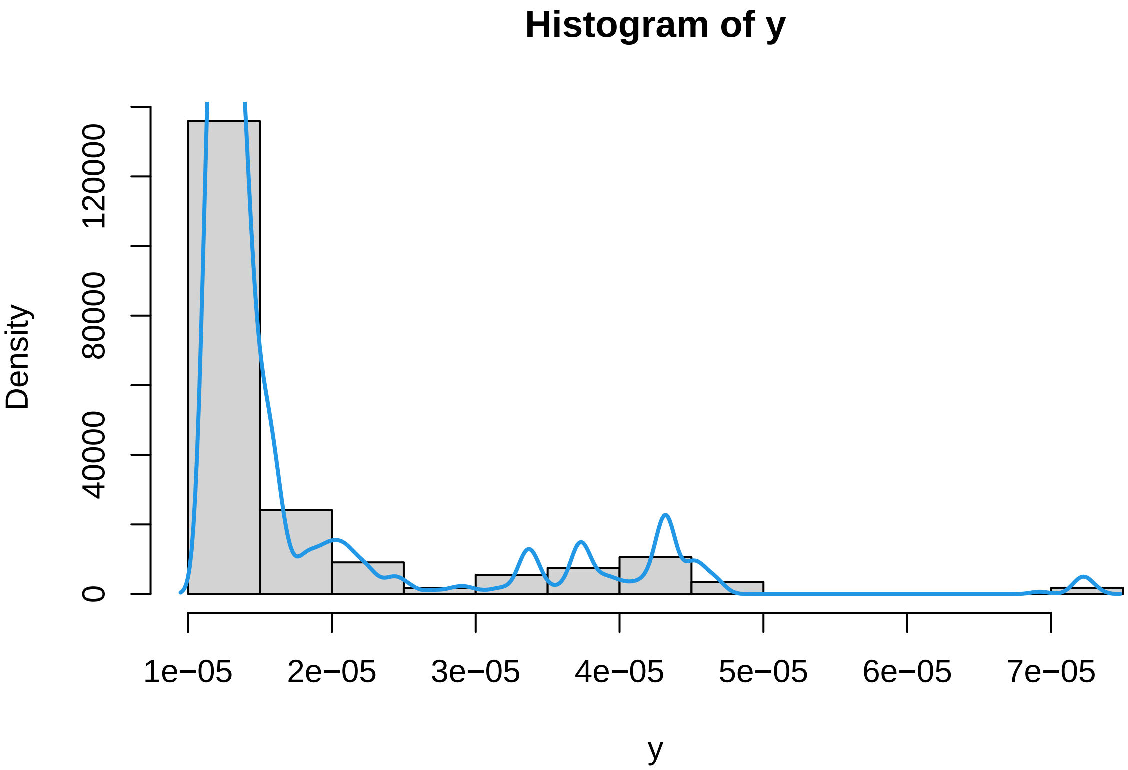
```
# Autocorrelation function of all data
```

```
acf(x)
```



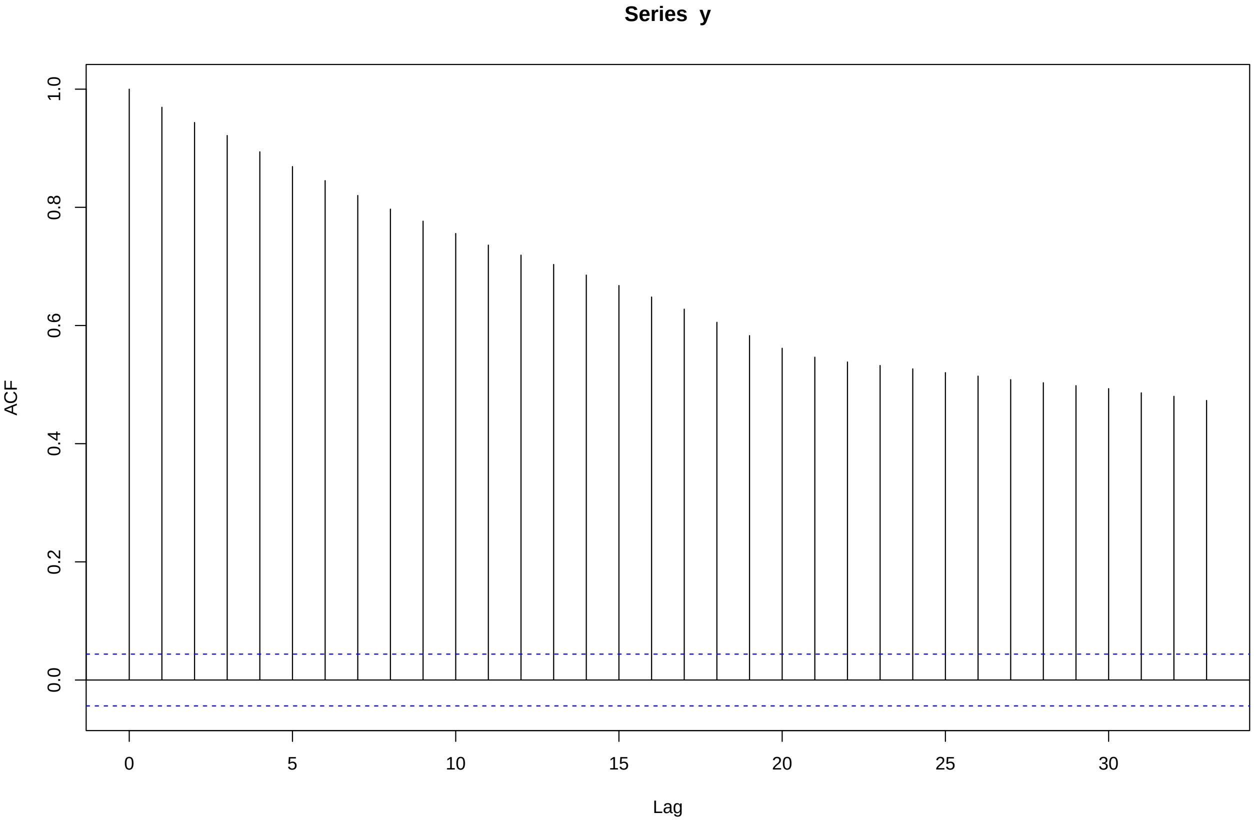
```
# Histogram of the largest data
```

```
nlargest <- 2000
y <- extract_nlargest_sample(x, n = nlargest)
hist(y, prob = TRUE)
lines(density(y),
      lwd = 2,
      col = 4)
```



```
# Autocorrelation function of the largest data
```

```
acf(y)
```



```
y <- extract_nlargest_sample(x, n = nlargest)

# Estimation of gev mixture models
```

```
gev_mixture_model <- suppressWarnings(estimate_gev_mixture_model_parameters(x = x,
                                                                            block_sizes = 10:40,
                                                                            minimum_nblocks = 50,
                                                                            threshold = min(y),
                                                                            nlargest = nlargest,
                                                                            confidence_level = 0.95,
                                                                            use_extremal_index = TRUE,
                                                                            use_lower_threshold = FALSE
                                                                            maximum_iterations = 1500,
                                                                            log_mv = TRUE,
                                                                            log_pw = TRUE,
                                                                            trace = FALSE,
                                                                            method = "MLE"))
```

```
##   Successful convergence.
##   Successful convergence.
```

```
gev_mixture_model$extremal_indexes
```

```
##            10            11            12            13            14 
## 0.05340727771 0.03421679345 0.05340727771 0.05340727771 0.05340727771 
##            15            16            17            18            19 
## 0.04163206040 0.03313610154 0.03090271250 0.03421679345 0.03090271250 
##            20            21            22            23            24 
## 0.04163206040 0.03313610154 0.03165934408 0.03165934408 0.03165934408 
##            25            26            27            28            29 
## 0.03669113495 0.03165934408 0.03882452751 0.03669113495 0.03669113495 
##            30            31            32            33            34 
## 0.03165934408 0.02461729364 0.02614513731 0.03165934408 0.02614513731 
##            35            36            37            38            39 
## 0.02461729364 0.03165934408 0.02239998851 0.02277324991 0.02277324991 
##            40 
## 0.02277324991
```

```
gev_mixture_model$normalized_gev_parameters_object
```

```
##           loc_star      scale_star   shape_star
## 10 1.118425017e-05 3.689861448e-07 0.8775166949
## 11 1.115388837e-05 3.369452585e-07 0.8830703642
## 12 1.107849734e-05 3.408728285e-07 0.8709342811
## 13 1.108198444e-05 3.105473832e-07 0.8804293344
## 14 1.099750070e-05 3.100500806e-07 0.8769978235
## 15 1.106445810e-05 2.829672839e-07 0.8823880809
## 16 1.109383739e-05 2.433532173e-07 0.9151407604
## 17 1.090427577e-05 3.032639969e-07 0.8435206379
## 18 1.096477131e-05 2.475740949e-07 0.8882366631
## 19 1.085639316e-05 2.613145975e-07 0.8785151116
## 20 1.084028562e-05 2.843624375e-07 0.8383304760
## 21 1.095672588e-05 2.208401509e-07 0.9056342085
## 22 1.087282048e-05 2.423235289e-07 0.8701367064
## 23 1.089410916e-05 2.223323535e-07 0.8868842511
## 24 1.093645869e-05 1.925758760e-07 0.9359241819
## 25 1.081687565e-05 2.259039728e-07 0.8667340229
## 26 1.088434586e-05 1.972530773e-07 0.9028741883
## 27 1.093823762e-05 1.764990106e-07 0.9344045661
## 28 1.073588973e-05 1.991065582e-07 0.9157044866
## 29 1.068275782e-05 2.169616806e-07 0.8941103808
## 30 1.068444812e-05 2.239283264e-07 0.8753635363
## 31 1.088574829e-05 1.671275669e-07 0.9262894276
## 32 1.098154317e-05 1.553572762e-07 0.9252778334
## 33 1.076954153e-05 1.831614213e-07 0.8817846758
## 34 1.039575561e-05 2.691369479e-07 0.8002701405
## 35 1.066680764e-05 1.755110424e-07 0.9216080879
## 36 1.048790160e-05 2.160446714e-07 0.8612322331
## 37 1.069669898e-05 1.640864827e-07 0.9254615793
## 38 1.095741713e-05 1.338209625e-07 0.9689244251
## 39 1.099368552e-05 1.488659170e-07 0.9211229694
## 40 1.077857612e-05 1.547977205e-07 0.9261009816
```

```
gev_mixture_model$full_normalized_gev_parameters_object
```

```
##           loc_star      scale_star   shape_star
## 10 1.079591275e-05 2.821358045e-08 0.8775166949
## 11 1.079170020e-05 1.710761710e-08 0.8830703642
## 12 1.071761898e-05 2.657149498e-08 0.8709342811
## 13 1.075600264e-05 2.354344318e-08 0.8804293344
## 14 1.067103829e-05 2.374325218e-08 0.8769978235
## 15 1.076317794e-05 1.712126561e-08 0.8823880809
## 16 1.083968417e-05 1.076724516e-08 0.9151407604
## 17 1.056389688e-05 1.614737104e-08 0.8435206379
## 18 1.069995296e-05 1.235272617e-08 0.8882366631
## 19 1.057296626e-05 1.231977757e-08 0.8785151116
## 20 1.052469394e-05 1.979231374e-08 0.8383304760
## 21 1.072401897e-05 1.009281647e-08 0.9056342085
## 22 1.060813655e-05 1.201233010e-08 0.8701367064
## 23 1.065514873e-05 1.040210964e-08 0.8868842511
## 24 1.073882583e-05 7.606508620e-09 0.9359241819
## 25 1.057109321e-05 1.287597652e-08 0.8667340229
## 26 1.067554597e-05 8.733044418e-09 0.9028741883
## 27 1.075842365e-05 8.480015512e-09 0.9344045661
## 28 1.052899560e-05 9.652676823e-09 0.9157044866
## 29 1.045273564e-05 1.129645706e-08 0.8941103808
## 30 1.044109048e-05 1.090192239e-08 0.8753635363
## 31 1.071115747e-05 5.405937181e-09 0.9262894276
## 32 1.081940356e-05 5.333084980e-09 0.9252778334
## 33 1.057171573e-05 8.721668394e-09 0.8817846758
## 34 1.007765430e-05 1.456996598e-08 0.8002701405
## 35 1.048263540e-05 5.776416505e-09 0.9216080879
## 36 1.024986981e-05 1.104402734e-08 0.8612322331
## 37 1.052466814e-05 4.878547356e-09 0.9254615793
## 38 1.082284178e-05 3.427623590e-09 0.9689244251
## 39 1.083703180e-05 4.568585178e-09 0.9211229694
## 40 1.061646022e-05 4.662021290e-09 0.9261009816
```

```
gev_mixture_model$automatic_weights_pw_shape
```

```
##              10              11              12              13              14
## 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00
##              15              16              17              18              19
## 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00
##              20              21              22              23              24
## 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00 9.947598301e-14
##              25              26              27              28              29
## 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00
##              30              31              32              33              34
## 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00
##              35              36              37              38              39
## 0.000000000e+00 0.000000000e+00 0.000000000e+00 1.000000000e+00 0.000000000e+00
##              40
## 0.000000000e+00
```

```
gev_mixture_model$automatic_weights_pw_scale
```

```
##               10               11               12               13
##  1.000000000e+00  0.000000000e+00  0.000000000e+00  0.000000000e+00
##               14               15               16               17
##  0.000000000e+00  0.000000000e+00  0.000000000e+00  0.000000000e+00
##               18               19               20               21
##  0.000000000e+00 -8.881784197e-16  0.000000000e+00  8.881784197e-16
##               22               23               24               25
##  0.000000000e+00  8.881784197e-16  1.776356839e-15  0.000000000e+00
##               26               27               28               29
## -1.332267630e-15  1.332267630e-15  1.776356839e-15 -8.881784197e-16
##               30               31               32               33
## -1.776356839e-15  1.332267630e-15  4.440892099e-16  4.440892099e-16
##               34               35               36               37
##  0.000000000e+00  0.000000000e+00 -8.881784197e-16 -8.881784197e-16
##               38               39               40
## -8.881784197e-16  2.442490654e-15 -6.661338148e-16
```

```
gev_mixture_model$automatic_weights_pw_loc
```

```
## 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35
##  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
## 36 37 38 39 40
##  0  0  0  0  0
```

```
gev_mixture_model$weighted_normalized_gev_parameters_object[3, ]
```

```
##                          loc_star      scale_star   shape_star
## automatic_weights 1.109383739e-05 3.689861448e-07 0.9689244251
```

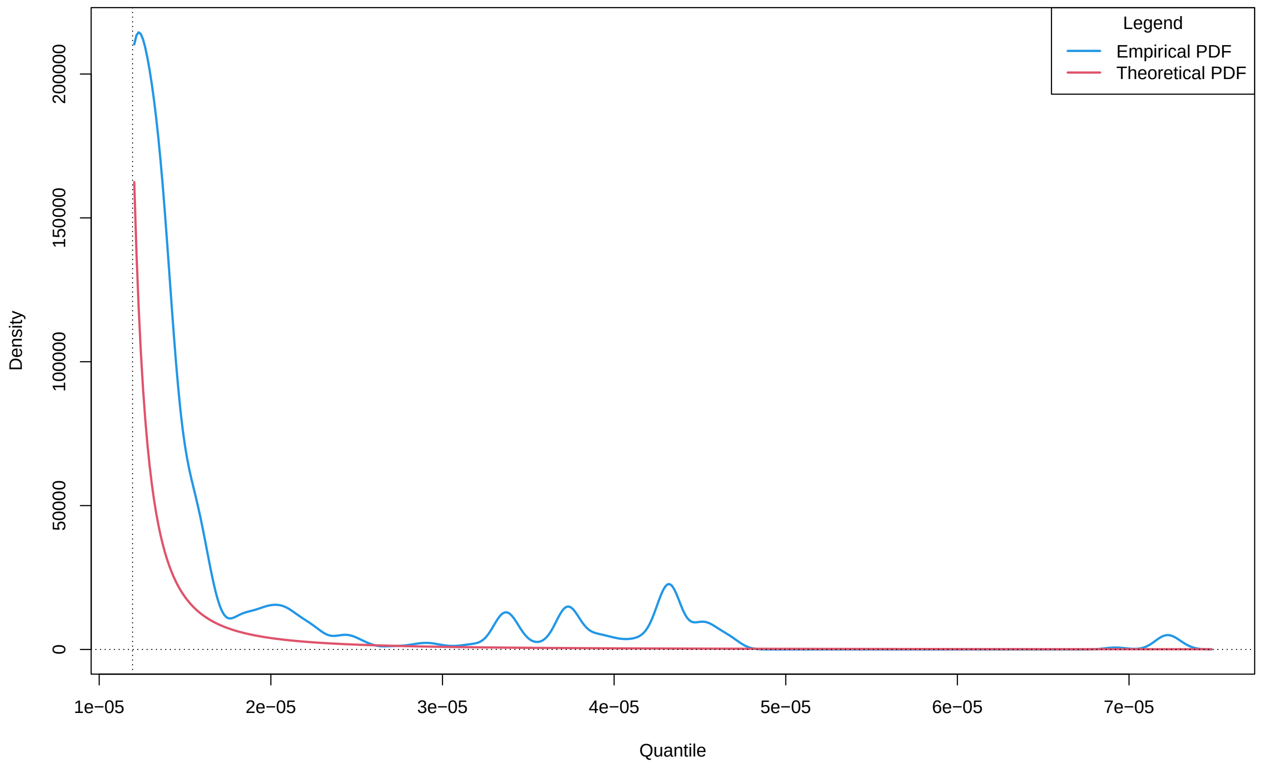
```
gev_mixture_model$automatic_weights_mw
```

```
##               10               11               12               13
##  1.000000000e+00  0.000000000e+00  0.000000000e+00  0.000000000e+00
##               14               15               16               17
##  0.000000000e+00  0.000000000e+00  0.000000000e+00  0.000000000e+00
##               18               19               20               21
##  0.000000000e+00 -8.881784197e-16  0.000000000e+00  0.000000000e+00
##               22               23               24               25
##  8.881784197e-16  0.000000000e+00  8.881784197e-16 -1.776356839e-15
##               26               27               28               29
##  8.881784197e-16 -8.881784197e-16  0.000000000e+00  8.881784197e-16
##               30               31               32               33
##  1.776356839e-15  8.881784197e-16  8.881784197e-16  1.776356839e-15
##               34               35               36               37
##  8.881784197e-16  8.881784197e-16  0.000000000e+00 -1.332267630e-15
##               38               39               40
## -8.881784197e-16  4.440892099e-16 -8.881784197e-16
```

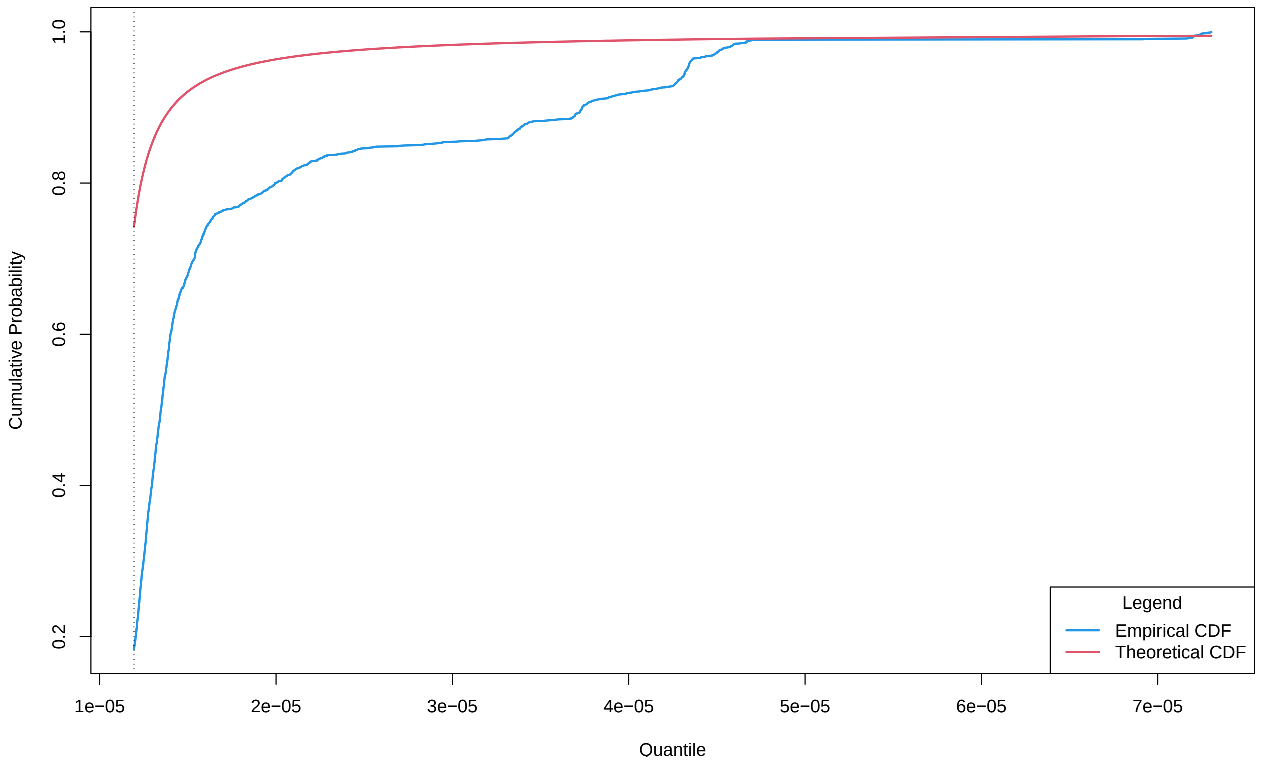
```r
# Model diagnostics

## GEV mixture model with respect to parameters
```

```r
par(mfrow = c(2, 1))

plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = FALSE,
                           zoom = TRUE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")

plot_gev_mixture_model_cdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = FALSE,
                           zoom = TRUE,
                           xlab = "Quantile",
                           ylab = "Cumulative Probability",
                           main = "Cumulative Distribution Function (CDF) Plot")
```

Probability Density Function (PDF) Plot : automatic_weights − model_wise = FALSE : zoom = TRUE

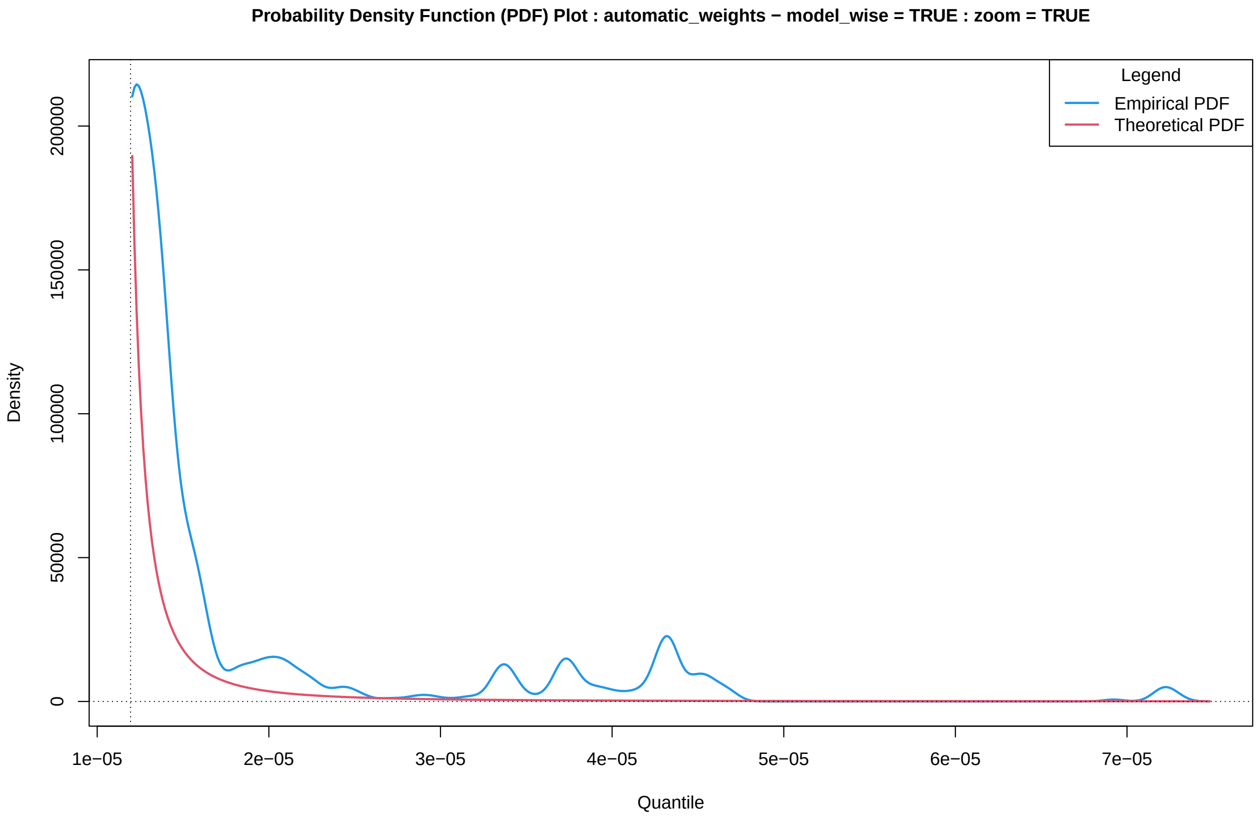


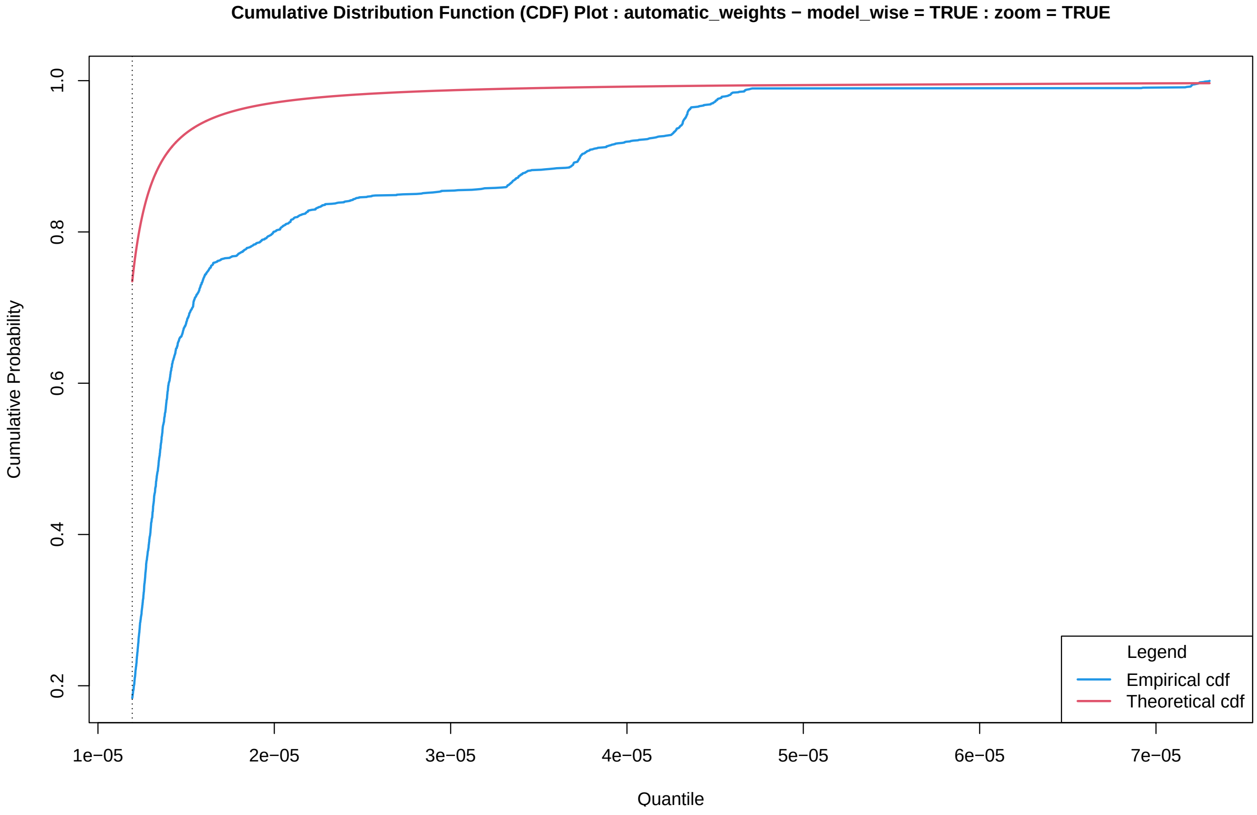
Cumulative Distribution Function (CDF) Plot : automatic_weights − model_wise = FALSE : zoom = TRUE



```
## GEV mixture model with respect to distribution functions
```

```
par(mfrow = c(2, 1))
plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = TRUE,
                           zoom = TRUE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")

plot_gev_mixture_model_cdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = TRUE,
                           zoom = TRUE,
                           xlab = "Quantile",
                           ylab = "Cumulative Probability",
                           main = "Cumulative Distribution Function (CDF) Plot")
```


Probability Density Function (PDF) Plot : automatic_weights − model_wise = TRUE : zoom = TRUE
Density
0
50000
100000
150000
200000
1e−05
2e−05
3e−05
4e−05
5e−05
6e−05
7e−05
Quantile
Legend
Empirical PDF
Theoretical PDF


Cumulative Distribution Function (CDF) Plot : automatic_weights − model_wise = TRUE : zoom = TRUE
Cumulative Probability
0.2
0.4
0.6
0.8
1.0
1e−05
2e−05
3e−05
4e−05
5e−05
6e−05
7e−05
Quantile
Legend
Empirical cdf
Theoretical cdf

```
# Estimation of an extreme quantile
```

```
estimator_types <- c("automatic_weights_mw",
                     "pessimistic_weights_mw",
                     "identic_weights_mw",
                     "automatic_weights_pw",
                     "pessimistic_weights_pw",
                     "identic_weights_pw",
                     "empirical",
                     "confidence_interval_mw",
                     "confidence_interval_pw")
```

```
alpha <- 10^(-14)

## Quantile from GEV mixture model with respect to parameters
```

```
rl_pw <- estimate_gev_mixture_model_quantile(gev_mixture_model,
                                             alpha = alpha,
                                             confidence_level = 0.95,
                                             do.ci = TRUE,
                                             estimator_type = estimator_types[4])

rl_pw[2]
```

```
##      estimate
## 1 1501781.868
```

```
## Quantile from GEV mixture model with respect to distribution functions
```

```
rl_mw <- estimate_gev_mixture_model_quantile(gev_mixture_model,
                                             alpha = alpha,
                                             confidence_level = 0.95,
                                             do.ci = TRUE,
                                             estimator_type = estimator_types[1])

rl_mw[2]
```

```
##      estimate
## 1 107434.0117
```

```
## Quantiles from equivalent estimated GEV models
```

```
est_rl_pw <- suppressWarnings(estimate_gev_mixture_model_quantile(gev_mixture_model,
                                                                  alpha = alpha,
                                                                  confidence_level = 0.95,
                                                                  do.ci = TRUE,
                                                                  estimator_type = estimator_types[9]))

est_rl_pw
```

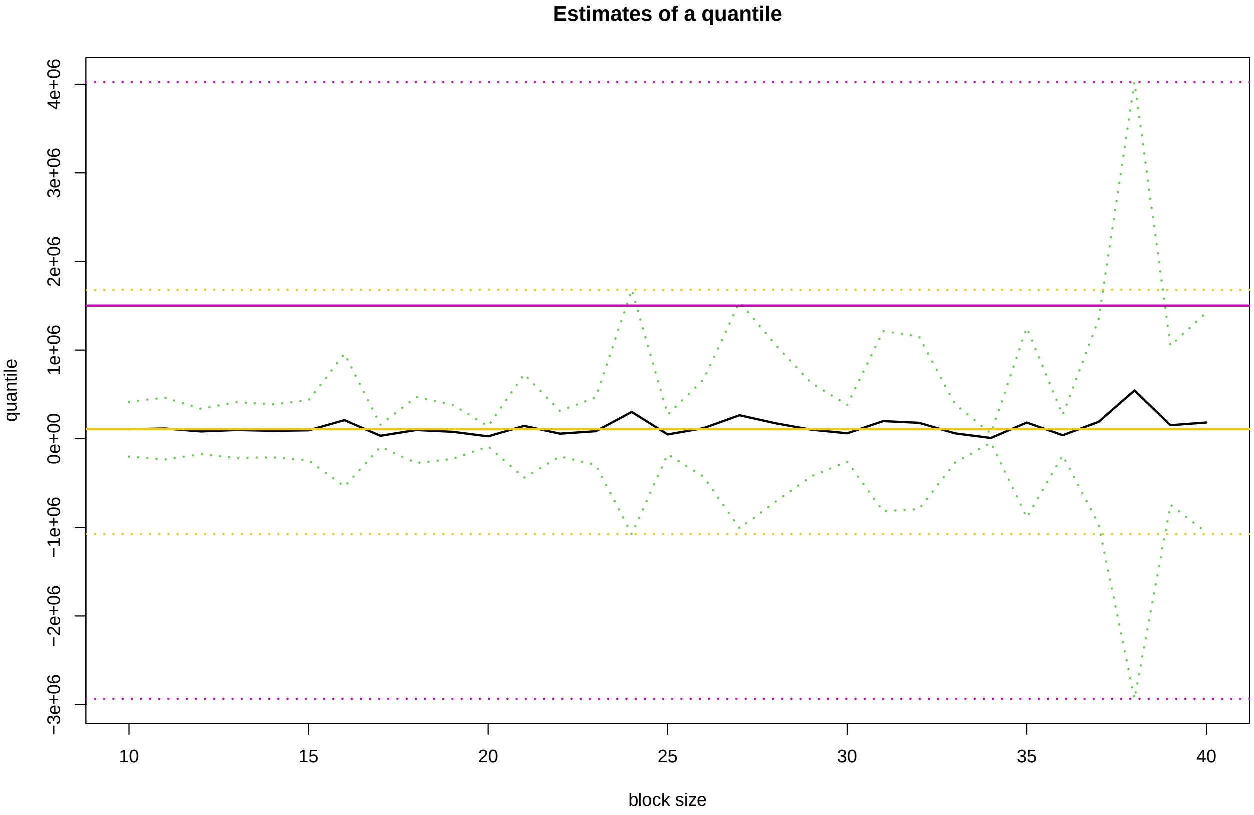
```
##           lower    estimate       upper
## 10 -201668.8737 107507.2682 416683.4101
## 11  -233773.333 115206.1453 464185.6236
## 12 -174126.4116 82175.05904 338476.5297
## 13 -214967.9565 98399.70279  411767.362
## 14 -210482.8871  88997.3798 388477.6467
## 15 -244332.4891  94867.1689 434066.8269
## 16  -539857.768 209684.2489 959226.2658
## 17 -92380.79748 33225.78282 158832.3631
## 18 -273297.6671 98226.42759 469750.5223
## 19 -229265.4518 78357.65445 385980.7607
## 20  -87033.2076 26837.43497 140708.0775
## 21 -439750.3595 144661.0446 729072.4487
## 22 -199502.2001 57090.40527 313683.0106
## 23 -296895.2762 84840.68818 466576.6526
## 24 -1076456.231 302247.0054 1680950.242
## 25 -176122.6373 48257.36164 272637.3606
## 26 -428811.6149 119327.2232 667466.0613
## 27 -1011322.711 265118.8569 1541560.424
## 28 -711686.4875 174371.4065   1060429.3
## 29 -424186.9611 101955.3848 628097.7308
## 30 -259360.1769  61323.4848 382007.1465
## 31 -817710.3677 198627.6507 1214965.669
## 32 -793737.0884  179332.027 1152401.142
## 33 -269858.6264 60347.07665 390552.7797
## 34  -43294.8111 8515.913838 60326.63877
## 35 -887890.8513  182238.623 1252368.097
## 36 -188744.0205  39392.9306 267529.8817
## 37 -965206.8434 190413.8389 1346034.521
## 38 -2935131.969 544793.0703  4024718.11
## 39 -745178.3317 152424.3564 1050027.044
## 40 -1058363.567 182976.1078 1424315.783
```

```
## Comparison of estimated quantiles
```

```
est_rl_pw_range <- range(as.matrix(est_rl_pw))

est_rl_mw <- suppressWarnings(estimate_gev_mixture_model_quantile(gev_mixture_model,
                                                                  alpha = alpha,
                                                                  confidence_level = 0.95,
                                                                  do.ci = TRUE,
                                                                  estimator_type = estimator_types[8]))

est_rl_mw_range <- range(as.matrix(est_rl_mw))
```

```
matplot(x = rownames(est_rl_pw),
        y = est_rl_pw,
        xlab = "block size",
        ylab = "quantile",
        main = "Estimates of a quantile",
        ylim = range(c(est_rl_pw_range)),
        cex = 1,
        cex.lab = 1,
        cex.axis = 1,
        type = "l",
        lty = c("dotted", "solid", "dotted"),
        lwd = c(2,2,2),
        col = c(3, 1, 3))

abline(h = rl_mw[2], col = 7, lwd = 2)
abline(h = rl_pw[2], col = 6, lwd = 2)
abline(h = est_rl_pw_range, col = 6, lty = "dotted", lwd = 2)
```

```r
abline(h = est_rl_mw_range, col = 7, lty = "dotted", lwd = 2)
```



```r
# Legend:
# yellow: Quantile from GEV mixture model with respect to distribution functions
# pink: Quantile from GEV mixture model with respect to parameters
```