Modeling extreme values with a GEV mixture probability distributions

Application to localisation w.r.t. latitude

Pascal Alain Dkengne Sielenou

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```
path <- ".."
```

```
xfun::in_dir(dir = path, expr = source("./src/generate_gev_sample.R"))
xfun::in_dir(dir = path, expr = source("./src/calculate_gev_inverse_cdf.R"))
xfun::in_dir(dir = path, expr = source("./src/estimate_gev_mixture_model_parameters.R"))
xfun::in_dir(dir = path, expr = source("./src/plot_gev_mixture_model_pdf.R"))
xfun::in_dir(dir = path, expr = source("./src/plot_gev_mixture_model_cdf.R"))
xfun::in_dir(dir = path, expr = source("./src/estimate_gev_mixture_model_quantile.R"))
```

```
library(readr)
```

```
Gnss_imar <- xfun::in_dir(dir = path, expr = read_csv("./applications/Gnss_imar.csv"))
```

```
## Rows: 20002 Columns: 25
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## dbl (25): version_major, version_minor, status, timestamp, latitude, longitu...
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
Gnss_map_matching <- xfun::in_dir(dir = path, expr = read_csv("./applications/Gnss_map_matching.csv"))
```

```
## Rows: 20001 Columns: 25
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## dbl (25): version_major, version_minor, status, timestamp, latitude, longitu...
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
timestamp_position <- sapply(Gnss_map_matching$timestamp,
                             function(ts)
                               which.min(abs(ts - Gnss_imar$timestamp)))
```

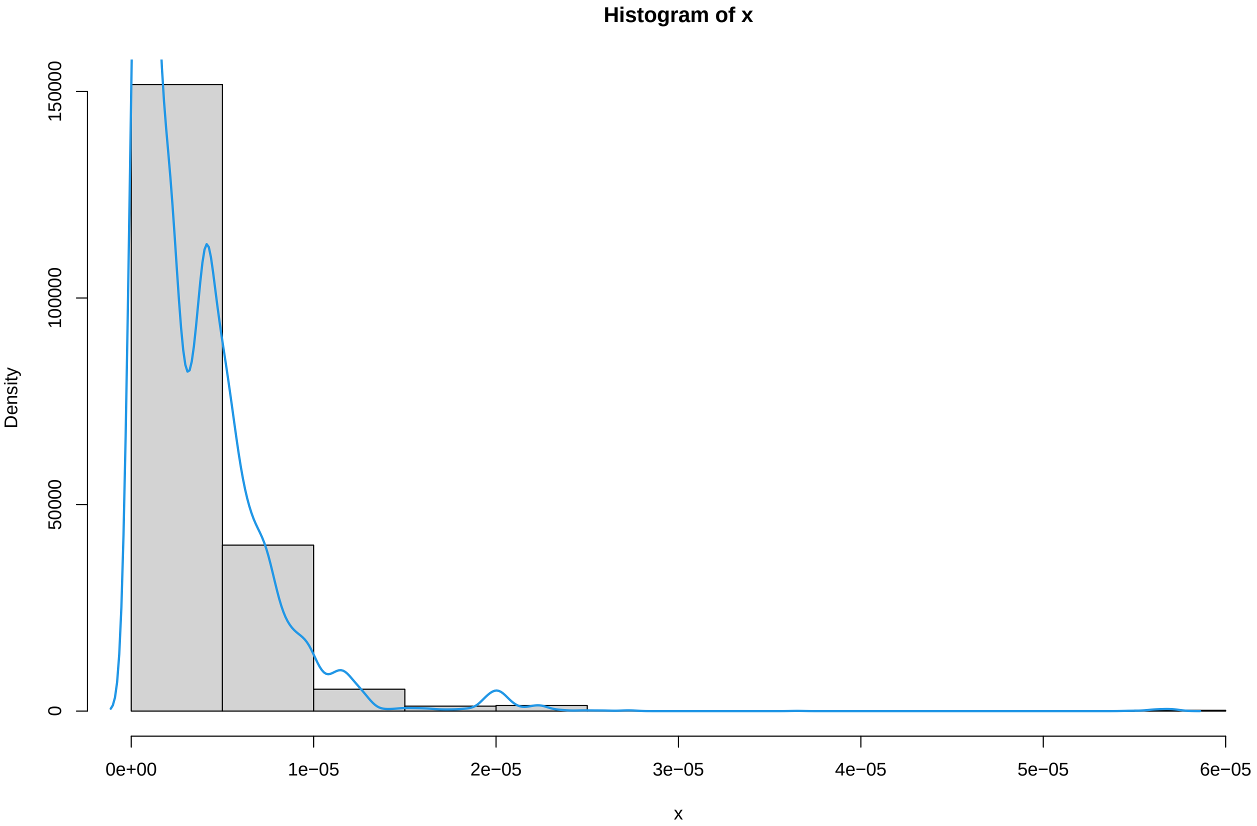
```
latitude_Gnss_map_matching_errors <- Gnss_imar$latitude[-1] - Gnss_map_matching$latitude
```

```
x <- abs(latitude_Gnss_map_matching_errors)
n <- length(x)

n
```

```
## [1] 20001
```

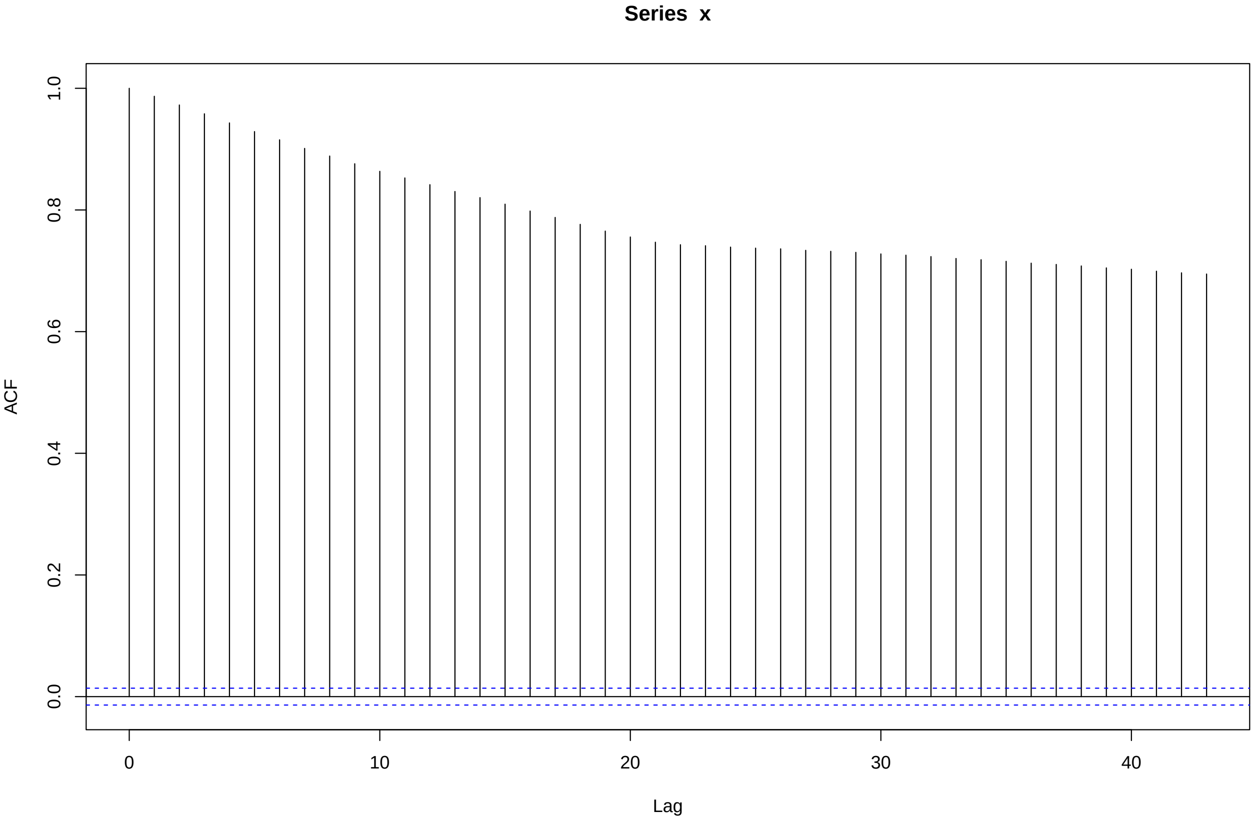
```
# Histogram of all data
```

```
hist(x, prob = TRUE)
lines(density(x),
      lwd = 2,
      col = 4)
```



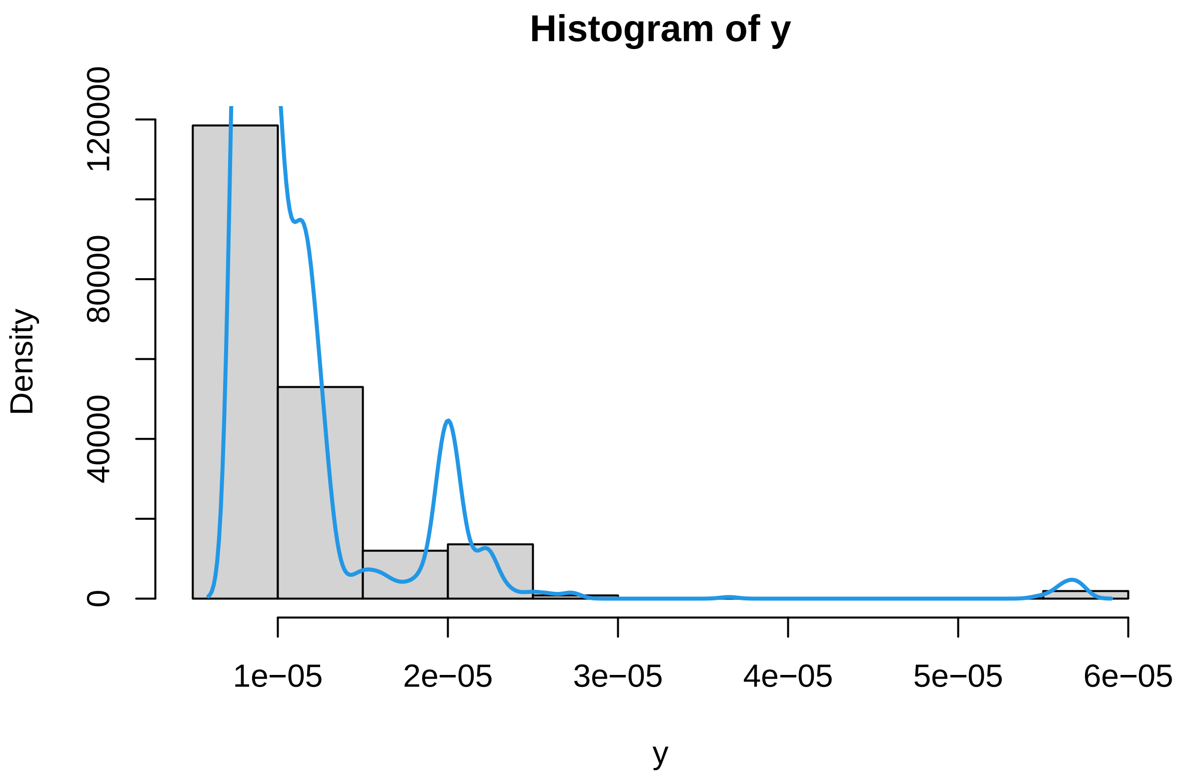
```
# Autocorrelation function of all data
```

```
acf(x)
```



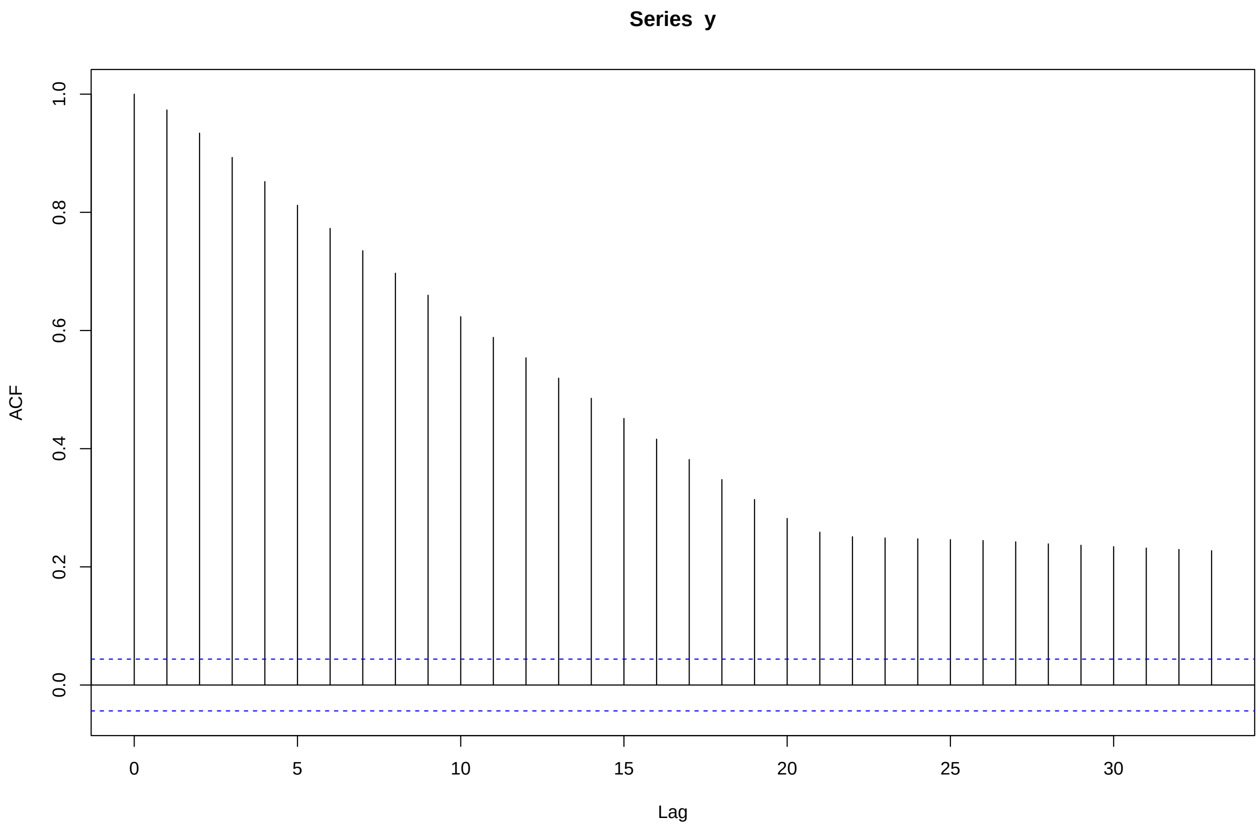
```
# Histogram of the largest data
```

```
nlargest <- 2000
y <- extract_nlargest_sample(x, n = nlargest)
hist(y, prob = TRUE)
lines(density(y),
      lwd = 2,
      col = 4)
```



```
# Autocorrelation function of the largest data
```

```
acf(y)
```



```
y <- extract_nlargest_sample(x, n = nlargest)

# Estimation of gev mixture models
```

```
gev_mixture_model <- suppressWarnings(estimate_gev_mixture_model_parameters(x = x,
                                                                            block_sizes = 10:40,
                                                                            minimum_nblocks = 50,
                                                                            threshold = min(y),
                                                                            nlargest = nlargest,
                                                                            confidence_level = 0.95,
                                                                            use_extremal_index = TRUE,
                                                                            use_lower_threshold = FALSE
                                                                            maximum_iterations = 1500,
                                                                            log_mv = TRUE,
                                                                            log_pw = TRUE,
                                                                            trace = FALSE,
                                                                            method = "MLE"))
```

```
##   Successful convergence.
##   Successful convergence.
```

```
gev_mixture_model$extremal_indexes
```

```
##           10           11           12           13           14 
## 0.05455837083 0.07814452754 0.05455837083 0.05170740715 0.05170740715 
##           15           16           17           18           19 
## 0.05170740715 0.05170740715 0.05170740715 0.05455837083 0.05170740715 
##           20           21           22           23           24 
## 0.04721881682 0.05170740715 0.05170740715 0.05170740715 0.04721881682 
##           25           26           27           28           29 
## 0.04721881682 0.03721457476 0.05170740715 0.04721881682 0.03721457476 
##           30           31           32           33           34 
## 0.03721457476 0.03082972874 0.03082972874 0.03721457476 0.03721457476 
##           35           36           37           38           39 
## 0.04721881682 0.02995362671 0.02995362671 0.03721457476 0.02995362671 
##           40 
## 0.02775272812
```

```
gev_mixture_model$normalized_gev_parameters_object
```

```
##           loc_star      scale_star   shape_star
## 10 7.204883247e-06 4.472925028e-07 0.6227929265
## 11 7.199328856e-06 3.991244189e-07 0.6488347948
## 12 7.049453809e-06 4.653051601e-07 0.5848421745
## 13 7.210542915e-06 3.583251096e-07 0.6619029664
## 14 7.035958749e-06 4.321059457e-07 0.5886015736
## 15 7.052150238e-06 3.803194929e-07 0.6212062314
## 16 6.998935048e-06 3.795404892e-07 0.6174885041
## 17 7.034634239e-06 3.616063896e-07 0.6175609129
## 18 6.926231854e-06 3.953970469e-07 0.5915645418
## 19 6.915102392e-06 3.875266157e-07 0.5925876895
## 20 6.956024919e-06 3.483896094e-07 0.6186291191
## 21 6.746074051e-06 4.221587212e-07 0.5623529452
## 22 6.768368080e-06 3.831641960e-07 0.5910738988
## 23 6.497294032e-06 4.667094802e-07 0.5342340206
## 24 6.845391846e-06 3.655446258e-07 0.5800666620
## 25 6.684087138e-06 3.888379136e-07 0.5906472328
## 26 6.850306268e-06 3.359389676e-07 0.6126085895
## 27 6.599860056e-06 3.815307227e-07 0.5760601330
## 28 6.732053052e-06 3.464419870e-07 0.5848394960
## 29 6.606543135e-06 3.614551140e-07 0.5926173297
## 30 6.501050648e-06 3.837188865e-07 0.5724774309
## 31 6.584695638e-06 3.797454021e-07 0.5637761030
## 32 6.810320570e-06 2.816404155e-07 0.6429576876
## 33 6.483719112e-06 3.521260931e-07 0.6115755204
## 34 6.530733755e-06 3.455700361e-07 0.5997915888
## 35 6.198620224e-06 4.518002761e-07 0.5332420582
## 36 6.557165154e-06 3.651417884e-07 0.5527048038
## 37 6.291181366e-06 4.038476400e-07 0.5354792219
## 38 6.236669796e-06 4.045296029e-07 0.5514645919
## 39 6.803787469e-06 2.517348439e-07 0.6745742644
## 40 6.609191895e-06 2.833765563e-07 0.6162270417
```

```
gev_mixture_model$full_normalized_gev_parameters_object
```

```
##           loc_star      scale_star   shape_star
## 10 6.604053644e-06 7.310007617e-08 0.6227929265
## 11 6.701853828e-06 7.634531124e-08 0.6488347948
## 12 6.399044337e-06 8.491827028e-08 0.5848421745
## 13 6.745391298e-06 5.043987465e-08 0.6619029664
## 14 6.430235446e-06 7.557625589e-08 0.5886015736
## 15 6.537144630e-06 6.039479952e-08 0.6212062314
## 16 6.482970798e-06 6.093849605e-08 0.6174885041
## 17 6.543087878e-06 5.804657030e-08 0.6175609129
## 18 6.377459643e-06 7.076286537e-08 0.5915645418
## 19 6.374181643e-06 6.698363912e-08 0.5925876895
## 20 6.478053967e-06 5.270286037e-08 0.6186291191
## 21 6.137288760e-06 7.980651968e-08 0.5623529452
## 22 6.232670340e-06 6.652724442e-08 0.5910738988
## 23 5.803183614e-06 9.589208063e-08 0.5342340206
## 24 6.322455924e-06 6.220693097e-08 0.5800666620
## 25 6.134232148e-06 6.406758552e-08 0.5906472328
## 26 6.374958770e-06 4.473700719e-08 0.6126085895
## 27 6.057773349e-06 6.925618225e-08 0.5760601330
## 28 6.239031108e-06 5.810328200e-08 0.5848394960
## 29 6.083361160e-06 5.140840911e-08 0.5926173297
## 30 5.932636792e-06 5.831478254e-08 0.5724774309
## 31 6.005854183e-06 5.340842269e-08 0.5637761030
## 32 6.419053563e-06 3.007228516e-08 0.6429576876
## 33 5.984886613e-06 4.705234845e-08 0.6115755204
## 34 6.034615154e-06 4.800227208e-08 0.5997915888
## 35 5.517692693e-06 8.870107829e-08 0.5332420582
## 36 5.991557336e-06 5.252763048e-08 0.5527048038
## 37 5.652252658e-06 6.171459264e-08 0.5354792219
## 38 5.622577289e-06 6.587932936e-08 0.5514645919
## 39 6.465619449e-06 2.361540055e-08 0.6745742644
## 40 6.199840865e-06 3.112338201e-08 0.6162270417
```

```
gev_mixture_model$automatic_weights_pw_shape
```

```
## 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35
##  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
## 36 37 38 39 40
##  0  0  0  1  0
```

```
gev_mixture_model$automatic_weights_pw_scale
```

```
##               10               11               12               13
##  0.000000000e+00  0.000000000e+00  0.000000000e+00  0.000000000e+00
##               14               15               16               17
##  0.000000000e+00  0.000000000e+00  0.000000000e+00  0.000000000e+00
##               18               19               20               21
##  0.000000000e+00  0.000000000e+00  0.000000000e+00  0.000000000e+00
##               22               23               24               25
##  0.000000000e+00  1.000000000e+00  0.000000000e+00  0.000000000e+00
##               26               27               28               29
##  0.000000000e+00  0.000000000e+00  0.000000000e+00  1.110223025e-16
##               30               31               32               33
##  1.110223025e-16  0.000000000e+00  0.000000000e+00  1.110223025e-16
##               34               35               36               37
##  0.000000000e+00  0.000000000e+00  1.110223025e-16  0.000000000e+00
##               38               39               40
##  0.000000000e+00  3.885780586e-16 -5.551115123e-17
```

```
gev_mixture_model$automatic_weights_pw_loc
```

```
##              10              11              12              13              14
## 0.000000000e+00 7.105427358e-14 0.000000000e+00 1.000000000e+00 0.000000000e+00
##              15              16              17              18              19
## 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00
##              20              21              22              23              24
## 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00
##              25              26              27              28              29
## 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00
##              30              31              32              33              34
## 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00
##              35              36              37              38              39
## 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00
##              40
## 0.000000000e+00
```

```
gev_mixture_model$weighted_normalized_gev_parameters_object[3, ]
```

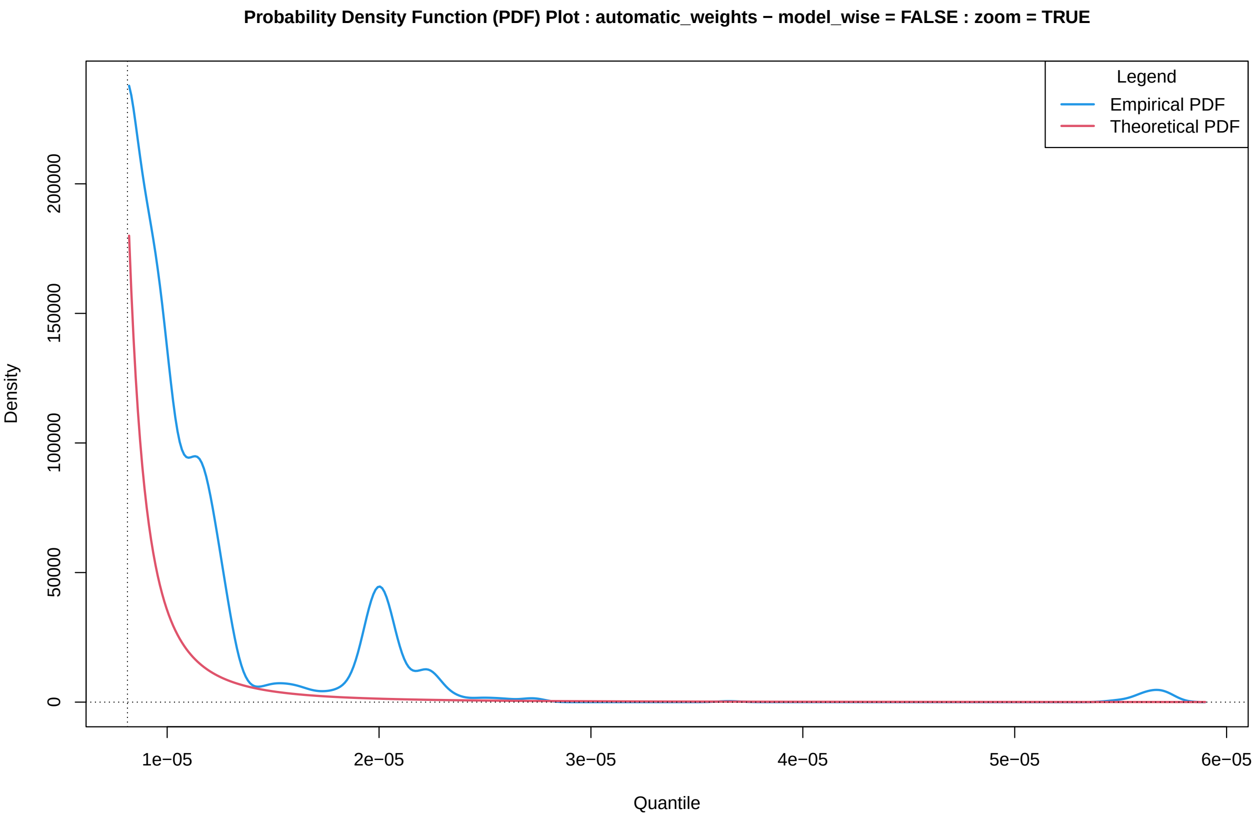
```
##                          loc_star      scale_star   shape_star
## automatic_weights 7.210542915e-06 4.667094802e-07 0.6745742644
```

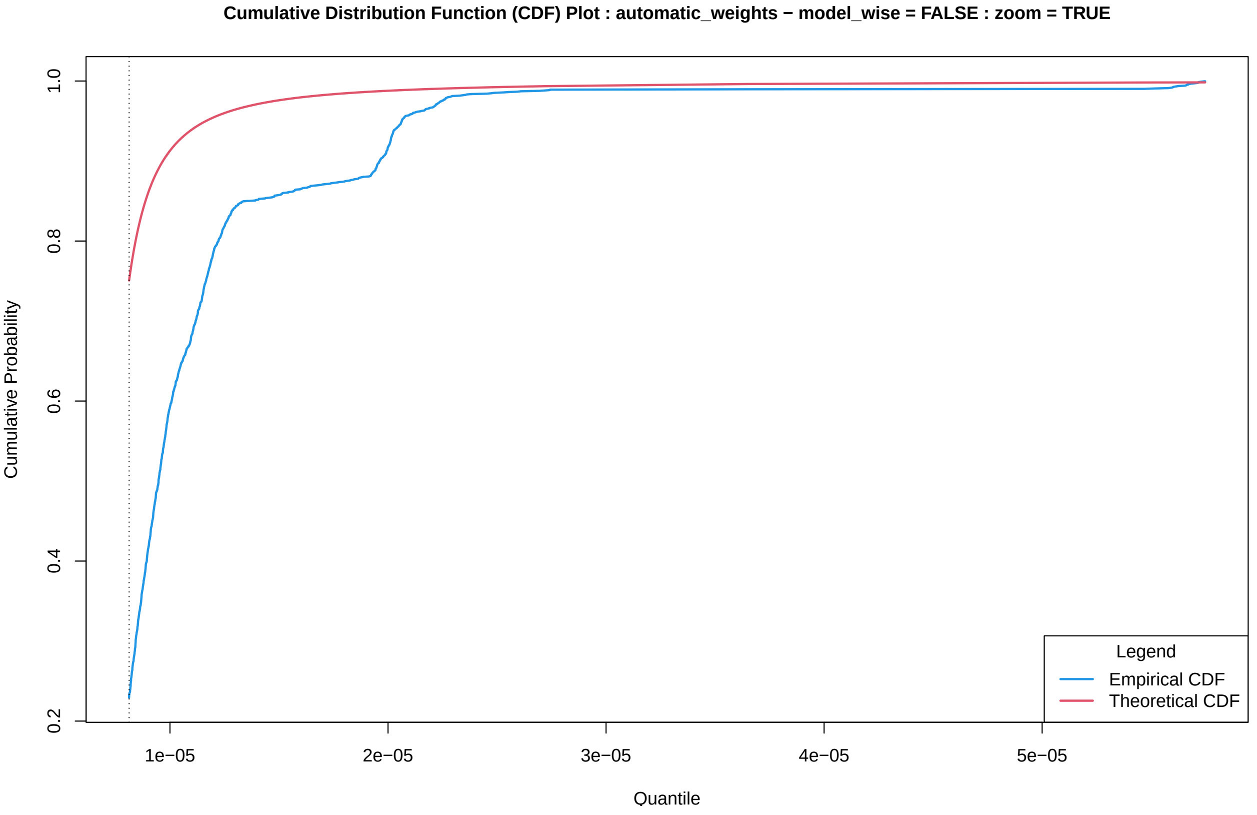
```
gev_mixture_model$automatic_weights_mw
```

```
##               10               11               12               13
##  2.486899575e-14  1.000000000e+00  0.000000000e+00  0.000000000e+00
##               14               15               16               17
##  0.000000000e+00  0.000000000e+00  0.000000000e+00  0.000000000e+00
##               18               19               20               21
##  0.000000000e+00  0.000000000e+00 -8.881784197e-16  0.000000000e+00
##               22               23               24               25
##  0.000000000e+00  0.000000000e+00 -8.881784197e-16 -2.664535259e-15
##               26               27               28               29
##  0.000000000e+00 -8.881784197e-16  0.000000000e+00  0.000000000e+00
##               30               31               32               33
##  1.776356839e-15  8.881784197e-16 -1.776356839e-15  8.881784197e-16
##               34               35               36               37
##  0.000000000e+00  8.881784197e-16  4.440892099e-16  0.000000000e+00
##               38               39               40
## -2.220446049e-15  2.664535259e-15  0.000000000e+00
```

```
# Model diagnostics

## GEV mixture model with respect to parameters
```

```
par(mfrow = c(2, 1))

plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = FALSE,
                           zoom = TRUE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")

plot_gev_mixture_model_cdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = FALSE,
                           zoom = TRUE,
                           xlab = "Quantile",
                           ylab = "Cumulative Probability",
                           main = "Cumulative Distribution Function (CDF) Plot")
```

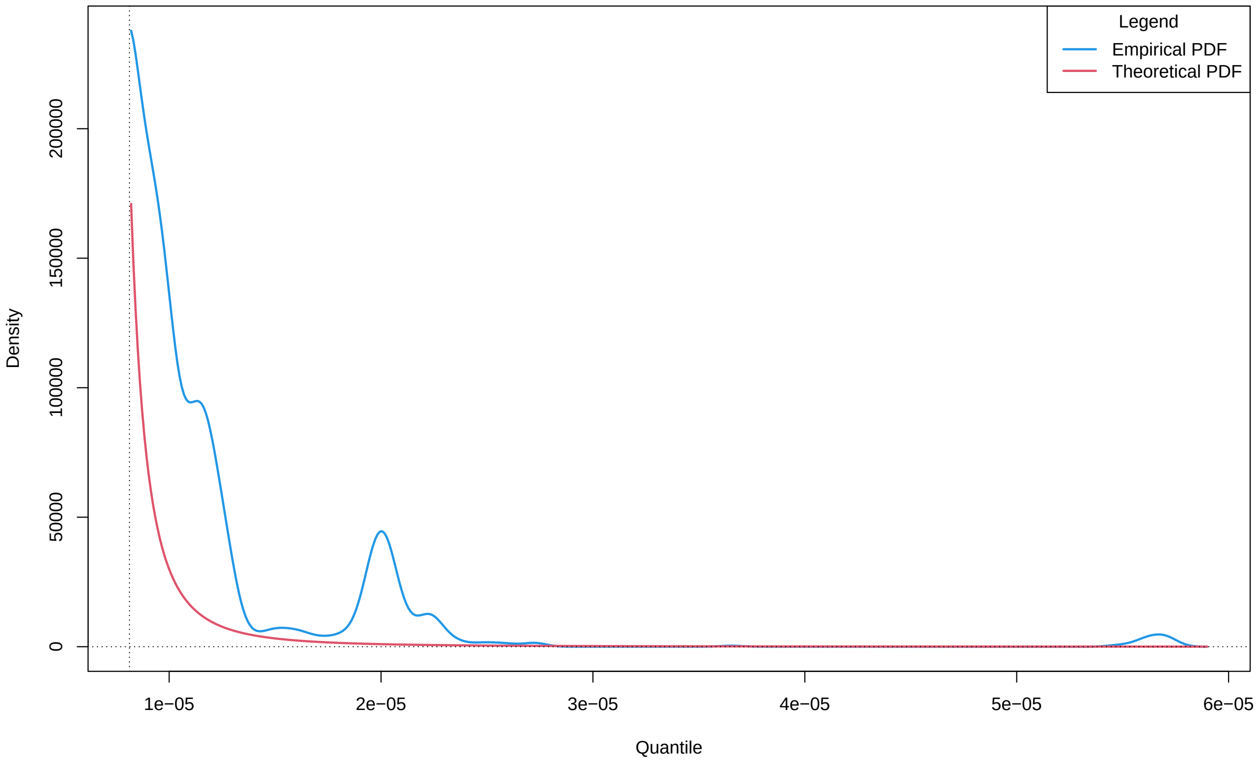

Probability Density Function (PDF) Plot : automatic_weights − model_wise = FALSE : zoom = TRUE
Legend
Empirical PDF
Theoretical PDF
Density
0
50000
100000
150000
200000
1e−05
2e−05
3e−05
4e−05
5e−05
6e−05
Quantile


Cumulative Distribution Function (CDF) Plot : automatic_weights − model_wise = FALSE : zoom = TRUE
Legend
Empirical CDF
Theoretical CDF
Cumulative Probability
0.2
0.4
0.6
0.8
1.0
1e−05
2e−05
3e−05
4e−05
5e−05
Quantile

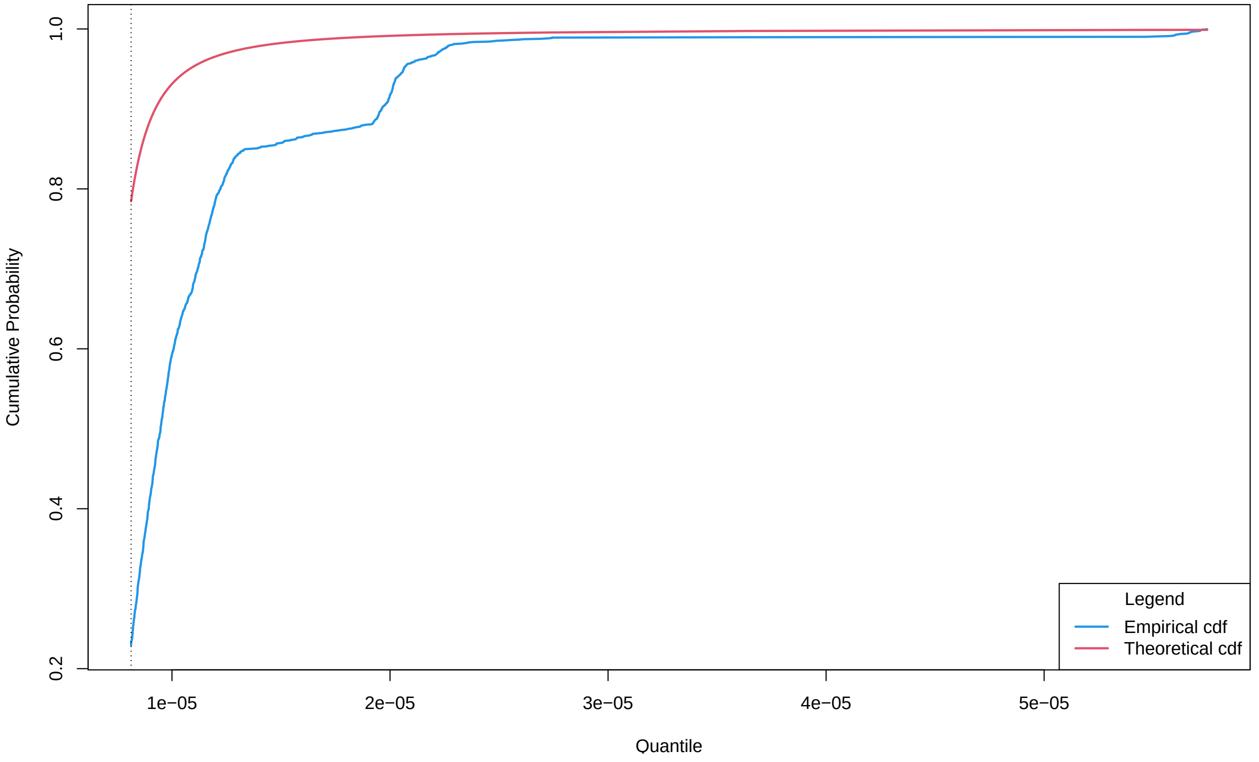
```r
## GEV mixture model with respect to distribution functions
```

```r
par(mfrow = c(2, 1))
plot_gev_mixture_model_pdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = TRUE,
                           zoom = TRUE,
                           xlab = "Quantile",
                           ylab = "Density",
                           main = "Probability Density Function (PDF) Plot")

plot_gev_mixture_model_cdf(gev_mixture_model,
                           type = "automatic_weights",
                           model_wise = TRUE,
                           zoom = TRUE,
                           xlab = "Quantile",
                           ylab = "Cumulative Probability",
                           main = "Cumulative Distribution Function (CDF) Plot")
```

Probability Density Function (PDF) Plot : automatic_weights − model_wise = TRUE : zoom = TRUE



Cumulative Distribution Function (CDF) Plot : automatic_weights − model_wise = TRUE : zoom = TRUE



```
# Estimation of an extreme quantile
```

```
estimator_types <- c("automatic_weights_mw",
                     "pessimistic_weights_mw",
                     "identic_weights_mw",
                     "automatic_weights_pw",
                     "pessimistic_weights_pw",
                     "identic_weights_pw",
                     "empirical",
                     "confidence_interval_mw",
                     "confidence_interval_pw")
```

```
alpha <- 10^(-14)

## Quantile from GEV mixture model with respect to parameters
```

```
rl_pw <- estimate_gev_mixture_model_quantile(gev_mixture_model,
                                             alpha = alpha,
                                             confidence_level = 0.95,
                                             do.ci = TRUE,
                                             estimator_type = estimator_types[4])

rl_pw[2]
```

```
##      estimate
## 1 406.8096229
```

```
## Quantile from GEV mixture model with respect to distribution functions
```

```
rl_mw <- estimate_gev_mixture_model_quantile(gev_mixture_model,
                                             alpha = alpha,
                                             confidence_level = 0.95,
                                             do.ci = TRUE,
                                             estimator_type = estimator_types[1])

rl_mw[2]
```

```
##     estimate
## 1 167.333379
```

```
## Quantiles from equivalent estimated GEV models
```

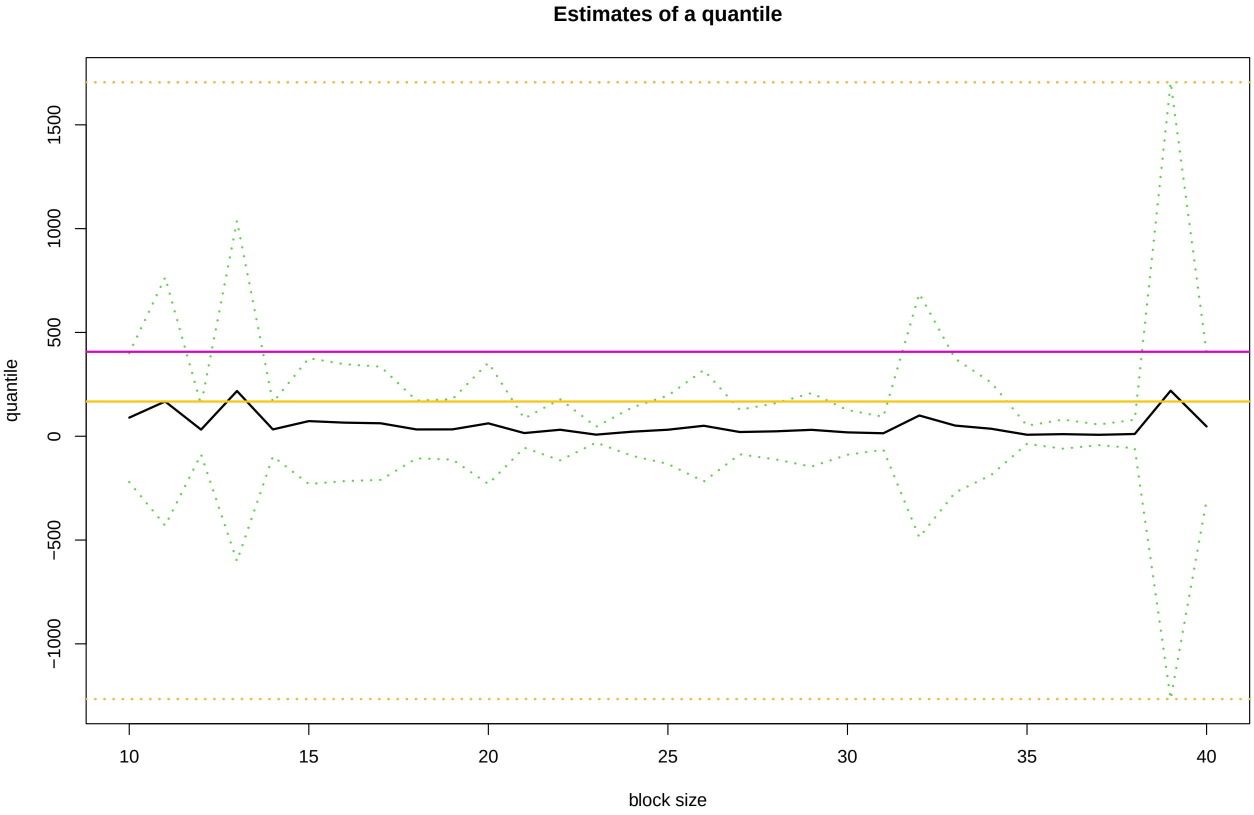
```
est_rl_pw <- suppressWarnings(estimate_gev_mixture_model_quantile(gev_mixture_model,
                                                                  alpha = alpha,
                                                                  confidence_level = 0.95,
                                                                  do.ci = TRUE,
                                                                  estimator_type = estimator_types[9]))

est_rl_pw
```

```
##           lower    estimate       upper
## 10 -220.7372902 89.64621936 400.0297289
## 11 -430.4337365 167.4265078 765.2867521
## 12 -87.44242467 31.88886474 151.2201541
## 13 -601.6230318 217.8741754 1037.371383
## 14 -97.19465297 32.92840487 163.0514627
## 15 -229.8969245  72.8768868 375.6506981
## 16  -216.049197 65.45922787 346.9676528
## 17 -210.2042302 62.49350367 335.1912375
## 18 -106.5171607 32.76075582 172.0386723
## 19 -112.3873136 33.04981904 178.4869517
## 20 -230.1144208 62.05895488 354.2323305
## 21 -54.97696162 15.34771995 85.67240152
## 22 -116.0535791 31.31041761 178.6744143
## 23 -30.40879785 7.697355081 45.80350801
## 24 -94.48358185 21.89378329 138.2711484
## 25 -133.1540156 31.39374515 195.9415059
## 26 -217.8142266 50.46316158 318.7405497
## 27 -87.00188892 20.40927053   127.82043
## 28 -111.5805374 23.74099447 159.0625264
## 29 -146.2947273 30.85278274 208.0002928
## 30 -88.96632082  18.5544738 126.0752684
## 31 -65.20088189 14.37012768 93.94113725
## 32 -487.0168334 99.98923065 686.9952947
## 33 -270.4920762 51.37109554 373.2342672
## 34 -187.0965913 36.12546811 259.3475275
## 35 -36.42302573 7.246961119 50.91694797
## 36 -59.93062563 10.11848715 80.16759993
## 37 -42.83670154 6.897562578  56.6318267
## 38 -57.59770465 10.82582734 79.24935932
## 39 -1265.924392 219.4638422 1704.852077
## 40 -308.3036112 47.15832832 402.6202678
```

```r
## Comparison of estimated quantiles
```

```r
est_rl_pw_range <- range(as.matrix(est_rl_pw))

est_rl_mw <- suppressWarnings(estimate_gev_mixture_model_quantile(gev_mixture_model,
                                                                  alpha = alpha,
                                                                  confidence_level = 0.95,
                                                                  do.ci = TRUE,
                                                                  estimator_type = estimator_types[8]))

est_rl_mw_range <- range(as.matrix(est_rl_mw))
```

```r
matplot(x = rownames(est_rl_pw),
        y = est_rl_pw,
        xlab = "block size",
        ylab = "quantile",
        main = "Estimates of a quantile",
        ylim = range(c(est_rl_pw_range)),
        cex = 1,
        cex.lab = 1,
        cex.axis = 1,
        type = "l",
        lty = c("dotted", "solid", "dotted"),
        lwd = c(2,2,2),
        col = c(3, 1, 3))

abline(h = rl_mw[2], col = 7, lwd = 2)
abline(h = rl_pw[2], col = 6, lwd = 2)
abline(h = est_rl_pw_range, col = 6, lty = "dotted", lwd = 2)
abline(h = est_rl_mw_range, col = 7, lty = "dotted", lwd = 2)
```



```
# Legend:
# yellow: Quantile from GEV mixture model with respect to distribution functions
# pink: Quantile from GEV mixture model with respect to parameters
```