1 Distribuciones condicionales

Before, we wrote

$$P(X \in S)$$

Now, we will write

$$P(X \in S|A) \coloneqq \frac{P(\{w : X(w) \in S\} \cap A)}{P(A)}$$

En general,

$$X = (X_1, \ldots, X_d)$$

$$P(X_i \in A|X_j \in B) = \frac{P(X_i) \in A, X_j \in B}{P(X_j \in B)}$$

1.1 Independence of random variable

X, Y are independent random variables random variables if

$$\begin{aligned} \forall A, B : P(x \in A|y \in B) = P(x \in A) \\ \forall A, B : P(y \in B|x \in A) = P(y \in B) \\ \forall A, B : P(x \in A, y \in B) = P(x \in A)P(y \in B) \end{aligned}$$

Definition 1. *We say that*

$$X \sim Y$$

if the distribution of X equals the distribution of Y.

1.2 Recap

Definition 2. *Let X be a discrete random variable. We define de expected value of X as*

$$E[X] = \sum x \cdot P(X = x)$$

We note that

$$\sum (x - E[X]) \cdot P(X = x) = \sum x \cdot P(X = x) - E[X] \sum P(X = x) = 0$$

Definition 3. *For any $g : X \to Y$,*

$$E[g(X)] = \sum g(x)P(x = X)$$

Theorem 1. *E is linear.*

$$E(\alpha X_1 + \beta X_2) = \alpha E[X_1] + \beta E[X_2]$$

Proof.

$$\begin{aligned}
E(\alpha X_1 + \beta X_2) &= \sum_{(x_1, x_2)} (\alpha x_1 + \beta x_2) P(X = (x_1, x_2)) \\
&= \sum_{x_1} \sum_{x_2} (\alpha x_1 + \beta x_2) P(x_1 = X_1, x_2 = X_2) \\
&= \sum_{x_1} \sum_{x_2} \alpha x_1 P(x_1 = X_1, x_2 = X_2) + \sum_{x_1} \sum_{x_2} \beta x_2 P(x_1 = X_1, x_2 = X_2) \\
&= \sum_{x_1} \alpha x_1 P(x_1 = X_1) + \sum_{x_2} \beta x_2 P(x_2 = X_2)
\end{aligned}$$

□