Applied Topology in Poznań 2025
Book of Abstracts

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CENTRE IN TOPOLOGICAL DATA ANALYSIS

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im. Adama Mickiewicza
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This is the book of abstracts for the Applied Topology in Poznań 2025 conference. The plenary talks and contributed talks of session 1 will take place in Aula A. The contributed talks session 2 will take place in Aula B. The poster session will take place in the main hall.

Contents

Abstracts

1 Monday

1.1 Plenary Speakers (Aula A)

Quasi Zigzag Persistence: A Topological Framework for Analyzing Time-Varying Data

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Abstract. In this talk, we present Quasi Zigzag Persistent Homology (QZPH) as a framework for analyzing time-varying data by integrating multiparameter persistence and zigzag persistence. To this end, we introduce a stable topological invariant that captures both static and dynamic features at different scales. We present an algorithm to compute this invariant efficiently. We show that it enhances the machine learning models when applied to tasks such as sleep-stage detection, demonstrating its effectiveness in capturing the evolving patterns in time-evolving datasets.

Joint work with Shreyas Samaga

Topological Data Analysis and the structure and function of physical systems

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Abstract. Topological Data Analysis is by now an established methodology to deal with a very wide variety of data-intensive problems. This talk will present an overview of the very wide range of possible applications to the study of physical system, order and disorder, time evolution and others. Furthermore, there are growing links with the foundational methodologies of AI, most notably neural networks, and we will provide examples of how geometry and topology can contribute to the study of their behaviour.

Topological Feature Selection for Time Series Data

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Abstract. I will present old and new tools of applied topology for feature selection on vector-valued time series data. To start, we employ persistent homology and sliding window embeddings to quantify the coordinated dynamics of time series. Next, I will describe an algorithm for gradient

descent to assign scores, or weights, to the variables of the time series based on their contribution to the dynamics as quantified by persistent homology; the result will be a convex combination of a subset of the variables. In this setting, persistence vineyards are piecewise linear, and I will give a simple formula for the derivatives of the vines. I will demonstrate our method of topological feature selection with synthetic data and also using C. elegans neuronal data.

This is joint work with Johnathan Bush (James Madison University).

1.2 Contributed Talks and Poster Talks: Parallel Session 1 (Aula A)

1.2.1 First Block:

On the topology of concurrent systems

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Abstract. As amply demonstrated in the literature, concepts and methods from algebraic topology can be profitably employed in concurrency theory, the field of computer science that studies systems of simultaneously executing processes. A powerful combinatorial-topological model for concurrent systems is given by higher-dimensional automata, i.e., pointed labeled precubical sets. The purpose of this talk is to show that the topology of an HDA model of a concurrent system can be arbitrarily complex. More precisely, I will show that for every connected polyhedron there exists a concurrent system that admits an HDA model with the same homotopy type as the polyhedron. This is joint work with T. Kahl and R. Lopes (Journal of Applied and Computational Topology, vol. 9, 2025).

The multiple points of maps from sphere to Euclidean space

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Abstract. It is obtained some sufficient conditions to guarantee the existence of multiple points of maps from S^m to $\mathbb{R}^d$. Our main tool is the ideal-valued index of G-space defined by E. Fadell and S. Husseini. We obtain more detailed relative positional relationship of multiple points. It is proved that for a continuous real value function $f : S^m \to \mathbb{R}$ such that $f(-p) = -f(p)$, if $m+1$ is a power of 2, then there are $m+1$ points $p_1, \ldots, p_{m+1}$ in S^m such that $f(p_1) = \cdots = f(p_{m+1})$, where $p_1, \ldots, p_{m+1}$ are linearly dependent and any m points of $p_1, \ldots, p_{m+1}$ are linearly independent. As a generalization of Hopf's theorem, we also prove that for any continuous map $f : S^m \to \mathbb{R}^d$, if $m > d$, then there exists a pair of mutually orthogonal points having the same image in addition to the antipodal points.

Classifying Neural Stimuli Using Combinatorics and Topology

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Abstract. We introduce an algorithm for classifying neural stimuli in microscale connectomes (brain networks) using spike train data. Our main contribution is the development of a dimensionality reduction and noise filtration technique on spike train data. Our approach utilises a mixture of combinatorial and topological metrics on directed graphs to extract important neighbourhoods, and then combines the functional data on those neighbourhoods with another metric to derive a vector in $\mathbb{R}^{50}$, which we feed into a classification pipeline.

We tested a variety of metrics, including: number of vertices, clustering coefficients, Euler characteristic and Betti numbers of directed flag complex, and spectral properties. We applied our dataset to the Blue Brain reconstruction of the neocortical column of a young rat, consisting of approximately 31,000 neurons. We injected a random sequence of eight neuronal stimuli, each with 500 repetitions, into the connectome. On this dataset we obtained a classification accuracy of up to **88%**.

We finish by exploring why certain metrics give better classification accuracy. To this end we examine the link between complexity and efficiency, and hypothesise that complex neighbourhoods are highly robust, containing redundancy in case synapses or neurons are damaged, but this reduces efficiency. We define complexity in two ways, a combinatorial and a topological approach, and use classification accuracy as a proxy for efficiency. To validate our hypothesis, we show that if we first restrict to low complexity neighbourhoods, and then apply our classification pipeline we consistently get a better accuracy.

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1.2.2 Second Block:

Temporomandibular joint activity recognition using persistent homology

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Abstract. Human activity recognition (HAR) is a well-established research area with numerous applications in healthcare as assistive technology. This study is part of a project aimed at developing a fertility monitoring device. We focus on recognizing activities based on the movement of the temporomandibular joint using a single accelerometer.

We have designed a device and system capable of recording accelerometer signals, each labeled with one of the following activities: eating, drinking, speaking, or other activities. In this poster, we present an approach utilizing persistent homology as a preprocessing step for activity recognition through machine learning.

Endomorphisms, Persistent Homology and the Conley Index via Quiver Representations

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Abstract. We introduce a method for the computation of the homomorphism inuced in persistent homology by a continuous map, building on ideas from previous approaches [1, 2, 4]. The method combines a continuous function $f\colon X \to Y$ with the inclusion map $\iota\colon X \to Y$, allowing for the computation of the Conley index map, as discussed in [3]. Given topological spaces X and Y, along with sampled data $f|_S$ representing the restriction of f to a finite subset $S \subset X$, we propose a framework based on quiver representations for the computation of an endomorphism in persistent homology. We introduce the notion of approximate index pairs based on the sampled map, and we apply our framework to make an attempt to compute a persistence version of the Conley index.

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Distance Matrices of Sampled Curves, Persistent Homology, and Dynamics

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Abstract. Persistent homology has become a useful tool in time series analysis. Recently, it has been observed that the persistent homology of distance matrices of time series carries interesting dynamical information. Starting with a time series of length n, the *recurrence plot* is a black-and-white image with $n \times n$ pixels, where the pixel at position (i, j) is black if the distance between the corresponding time series points is less than a threshold ϵ and white otherwise. A filtration is then obtained by varying ϵ.

In this poster, I present a formal relationship between the persistent homology of a distance matrix and the persistent homology of the corresponding (multivariate) time series in the form of

a degree-one chain map of filtered complexes. The idea is that each entry in a distance matrix corresponds to an edge in the Vietoris–Rips or Čech complex. Provided the time series is sampled densely enough, this edge can be completed to a cycle by following the time series.

This chain map provides an interpretation of recurrence plot analysis using persistent homology. It also has interesting properties, for example, it induces a surjective map from the degree-zero homology of the distance complex to the degree-one homology of the time series. This can be applied to the computation of maps appearing in the definition of the cycling signature, a recent tool for time series analysis.

Using and Extending the Topological Morphology Descriptor (TMD) for Microglia Analysis

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Abstract. The Topological Morphology Descriptor (TMD) encodes the spatial structure of a rooted geometric tree – such as a neuron or a glial cell – into a barcode via an algorithm related to Persistent Homology, coupling branch topology with spatial embedding to yield a compact yet discriminative topological signature. Since its introduction in 2018, the TMD has been extended in various directions, including a collaboration between EPFL, KTH, and the Siegert Microglia–Neuron Interaction Lab at ISTA. An overview of this project wil be given, including variance-reduction techniques for TMD, dimensionality reduction and reconstruction using Variational Autoencoders, and a generalization of the TMD to filtrations of pairs of spaces with inclusion relation.

Bone apparent strength prediction via integration of topological data analysis with bone morphometry

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Abstract. Accurate bone strength prediction is essential for assessing fracture risk, particularly in aging populations and individuals with osteoporosis. Although bone mineral density remains the clinical standard, it inadequately captures the complex structural characteristics of trabecular bone that influence mechanical behavior. This study investigates the use of topological data analysis (TDA) to characterize trabecular micro-architecture from high-resolution micro-computed tomography scans. We extract topological features, specifically those derived from persistent homology, and combine them with conventional morphometric descriptors to train machine learning models for bone strength prediction. Models based solely on topological features outperform those using traditional morphometrics, highlighting TDA's ability to capture biomechanically relevant structure. In particular, internal voids often dismissed as imaging noise, proved to be the most predictive. While limited by dataset size and class imbalance, these results suggest that TDA offers a promising approach for advancing osteoporosis risk assessment.

Topological potentials driving simulated Tobacco Mosaic Virus dimerization

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Abstract. The self-assembly of proteins into functional complexes is partially driven and stabilized by short-ranged non-polar effects. Models based on molecular geometry can correctly identify the most stable, final structure of an assembly. However, they often fail to find it efficiently in simulations. The central problem being addressed here is that short-range geometric models provide a rugged energy landscape where simulations can become kinetically trapped in non-functional arrangements. We show that augmenting the morphometric approach to solvation free energy with a 'topological potential' term dramatically improves simulation performance. Our main result, demonstrated on the Tobacco Mosaic Virus dimer, is an order of magnitude increase in the success rate of finding the correct, assembly-competent structure compared to the geometric model alone. This finding demonstrates that the kinetic barriers inherent in a short-range potential can be effectively lowered by a long-range term that is not based on traditional electrostatics, but on the global topology of the system. This shows that topological data analysis can be used not just to analyze molecular configurations, but as an active component within an energy function to guide a simulation. This principle of synergistic geometric-topological potentials offers a strategy for creating more efficient molecular simulations.

1.3 Contributed Talks and Poster Talks: Parallel Session 2 (Aula B)

1.3.1 First Block

Using topological methods to classify porous metal structures as sponges and foams

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Abstract. Foams and metal sponges are two idealized ends of a spectrum of metal porous materials. In foams, the gas intermixed with the structure forms distinct, unconnected cells, completely separated from each other by the structure's elements. In sponges, the air forms an interconnected continuous network intertwined with the structural one. However, the terminology used by scientific communities is not always consistent. For example, in the metallic community, the word "foam" is used informally for every porous material. This might stem in practice largely from the fact that many protocols used to prepare porous metal materials result in structures intermediate in terms of properties between classical foams and sponges. We are proposing the usage of concepts defined in topology to quantify the properties of the intermediate porous metallic structures and to define a precise "sponginess score" measuring how close a given structure is to an ideal sponge vs foam. We explore the usefulness of the proposed score on numerically generated structures and analyze how the defined sponginess score predicts mechanical properties of the porous materials."

Predicting mechanical properties of porous materials using topological data analysis

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Abstract. Porous metals are increasingly important in technology. Due to their tunable mechanical properties, they are promising candidates in various emerging applications such as metallic scaffolds for load-bearing bones and lightweight structures for transport technologies. The purpose of this study is to create topological descriptors of porous materials that allow a fast prediction of their mechanical properties. Currently, the main focus is on Young's modulus. The topological properties of an object do not change when the object is rotated, while Young's module may depend on direction. To construct direction-aware descriptors, we encoded direction-dependent information in filtration values. We combine topological data analysis with theoretical models based on material porosity for better results. In this talk, I will present new topological descriptors of porous materials and discuss the effectiveness of regression models based on them.

Topology and Crime

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Abstract. What can the topology of neighborhoods in land use and socioeconomic geometry tell us about the criminogenic conditions within the physical neighborhoods of a city? As our setting, we use Stockholm, which is subdivided into so-called DeSO (demographical statistical) areas that remain relatively stable over time. In these areas, we have publicly available land use (e.g., number of libraries per unit area, proportion of multi-family housing, etc.) and socioeconomic (e.g., income levels, etc.) data available. We view feature vectors from these data as inhabitants of a distance space and use a recently developed geometric exploration framework (Agerberg, Chachólski, Ramanujam, PMLR, 2023) to extract point clouds from this space. Given a point cloud associated with each DeSO's input data, we compute persistent homology and perform spectral clustering to find groupings of neighborhoods to be studied further. The geometric extraction process is parameterized on a choice of "test distributions." In this work, we take the view that rather than making an a priori choice on what test distribution to use, we instead try to learn which distributions lead to the best clusterings. The clusters are then compared using crime data. Some of the computations on persistence curves have been accelerated using the 'masspcf' Python package (Wehlin, 2024).

This is joint work with Patricia Brantingham, Vania Ceccato, Wojciech Chachólski, and Ioannis Ioannidis.

1.3.2 Second Block

Holes in Latent Space: Topological Signatures of Adversarial Influence

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Abstract. Understanding how adversarial conditions affect language models requires techniques that capture both global structure and local detail within high-dimensional activation spaces. We propose persistent homology (PH), a tool from topological data analysis, to systematically characterize multiscale latent space dynamics in LLMs under two distinct attack modes—backdoor fine-tuning and indirect prompt injection. By analyzing six state-of-the-art LLMs, we show that adversarial conditions consistently compress latent topologies, reducing structural diversity at smaller scales while amplifying dominant features at coarser ones. PH's inherent robustness to noise allows it to capture stable, meaningful geometric patterns that achieve perfect classification performance (ROC-AUC = 1.00) when distinguishing normal from adversarial activations—demonstrating the fundamental significance of these topological signatures across layers, architectures, and model sizes. The consistency of these results aligns with adversarial effects emerging deeper in the network. To capture finer-grained mechanisms underlying these shifts, we introduce a neuron-level PH framework that quantifies how information flows and transforms within and across layers. Together, our findings demonstrate that PH offers a principled and unifying approach to interpreting representational dynamics in LLMs, particularly under distributional shift.

Spectral systems and connection matrices

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Abstract. Spectral sequences and systems are tools from algebraic topology widely known and used in the study of homology and fibrations. These usually arise form an exact couple system, having a filtration on a space associated. Parallel to this, connection matrices were developed in the realm of applied topology for qualitative study of dynamical systems. We have a Morse decomposition of a system, and the relative homologies being the Conley index provide us information about it. These two wildly different ideas turn out to be two faces of the same coin. We see how these connection matrices are at its core just a spectral system, and how given an exact couple system one can build a connection matrix and thus a space that generates it with the relative homologies.

Quantum computing and persistence in topological data analysis

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Abstract. Topological data analysis (TDA) aims to extract noise-robust features from a data set by examining the number and persistence of holes in its topology. We show that a computational

[1] This abstract represents joint work with Inés García-Redondo, Qiquan Wang, Haim Dubossarsky, and Anthea Monod.

problem closely related to a core task in TDA - determining whether a given hole persists across different length scales in the filtration - is $\mathbf{BQP}_1$-hard and contained in $\mathbf{BQP}$ [1]. This result implies a superpolynomial quantum speedup for this problem under standard complexity-theoretic assumptions. Our approach relies on encoding the persistence of a hole in a variant of the guided sparse Hamiltonian problem, where the guiding state is constructed from a harmonic representative of the hole.

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Minimal flat-injective presentations and their algorithmic challenge

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Abstract. Flat-injective presentations were first defined by Miller [1] under the name 'flange presentation' as an alternative to free presentations for multiparameter persistence. Unlike free presentations they mimic the combinatorial properties of the barcode, which is a full invariant for one-parameter persistence modules, in the multiparameter case by describing a graded $k[X_1, \ldots, X_n]$-module M as the image of a homomorphism $\varphi\colon F \to E$, where F is flat and E is injective. The main feature from a computational perspective is that any flat-injective presentation can be described by a monomial matrix, which is a matrix with rows and columns labeled by births and deaths, respectively, of the underlying module.

We first show that any finitely supported and finitely generated persistence module admits a flat-injective presentation whose monomial matrix resembles its transition maps. Then we reduce the resulting flat-injective presentation by operations on its monomial matrix. Particularly, we investigate the admissible matrix operations and the restrictions that come with the extra duality inherent to flat-injective presentations. Lastly, we discuss what the algorithmic challenges are when minimizing flat-injective presentations.

This is joint work with Anastasios Stefanou.

References

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2 Tuesday/ Frank Lutz Memorial Session

2.1 Plenary speakers (Aula A)

On the Alexander-Oda conjecture

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Abstract. Two PL homeomorphic complexes are related by stellar subdivisions, and inverses. One of the oldest conjectures of PL topology, due to Alexander in the 1930s, is that one can reshuffle these moves so that stellar subdivisions precede inverses. I will discuss the recent proof of this conjecture.

Frontiers of sphere recognition in practice

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Abstract. Sphere recognition is known to be undecidable in dimensions five and beyond, and no polynomial time method is known in dimensions three and four. Here we report on positive and negative computational results with the goal to explore the limits of sphere recognition from a practical point of view. An important ingredient are randomly constructed discrete Morse functions. Joint work with Davide Lofano, Frank H. Lutz and Mimi Tsuruga. In memoriam, I will highlight some of Frank's contributions to combinatorial and applied topology.

Random Simple Homotopy Theory

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Abstract. Discrete Morse Theory is a tool to understand simplicial complexes up to homotopy. It was introduced by Forman in 1999, though the main ideas go back to a 1939 paper by Whitehead. We discuss computational approaches via randomness, and drawbacks thereof. To bypass these drawbacks, we present a new strategy, closer in spirit to Whitehead's original work. We implement an algorithm RSHT (Random Simple-Homotopy) to study the simple-homotopy types of simplicial complexes, with a particular focus on contractible spaces and on finding substructures in higher-dimensional complexes. The RSHT algorithm combines elementary simplicial collapses with pure elementary expansions. It also generalizes the well-known "bistellar flip" algorithm for manifolds. At the moment, we do not know of a single contractible complex whose contractibility cannot be shown via the RSHT algorithm – a strong advantage with respect to discrete Morse theory.

This is joint work with Frank Lutz and his students Crystal Lai, Davide Lofano. It was completed the same week of Frank's sudden demise.

2.2 Contributed Talks: Parallel Session 1 (Aula A)

2.2.1 First Block

Universality for finite topological spaces

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Abstract. Finite topological (or more generaly, Alexandroff) spaces have become a focal point within computational and applied topology, largely because of the development of computational tools such as persistent homology for the topological analysis of noisy data. We introduce a topological hyperspace that plays the role of a *universal* object for all finite (Alexandroff) spaces and explore its principal features. This construction supplies a common framework for comparing finite spaces, thereby providing a universal setting in which certain algebraic–topological invariants of compact metric spaces can be studied.

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Persistent Hochschild homology of directed graphs

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Abstract. There is currently an active interest in homotopy and homology theories in the world of graphs and directed graphs; discrete homotopy theory, magnitude homology and path homology are few prominent themes. Many of these also have incarnations in topological data analysis and persistent homology as tools for network analysis. We extend the use of Hochschild homology of directed graphs in persistence setting. We "lift" Hochschild homology to higher degrees via so called connectivity structures, of which I will present functorial examples. To get an efficiently computable pipeline we have to resort to non-functorial constructions. To remedy this leads to defining the reachability category of a directed graph, and associated persistent reachability homology. Our recent machine learning results on classifying patients of neurodegenerative diseases shows that this pipeline performs much better than traditional simplicial homology of the directed flag complexes. If time permits, I'll briefly discuss an algebraic result that recently introduced commuting algebras are Morita equivalent to incidence algebras. This is a joint work with Luigi Caputi and Nicholas Meadows.

Homotopy and the Promised Constraint Satisfaction Problem

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The work of the author was supported by the OP JAK Project MSCAFellow5_MUNI (CZ.02.01.01/00/22_010/0003229).

Abstract. The Hom complex is a combinatorial construction that captures the structure of multimorphisms of discrete mathematical objects such as graphs. They were originally devised and motivated by Lovázs work on the Kneser conjecture and used by Babson and Kozlov [5] to produce lower bounds on the chromatic number.

One can see that the Hom complex captures a certain notion of homotopy of graphs [3]. From this point of view, the Hom complex is a "space of homotopy classes of graph morphisms". As such they can be used to analyse the space of graph morphisms.

Recently, Hom complexes were used to a geat effect in the context of approximate graph colouring - in instance of the Promised Constraint Satsisfaction Problem (PCSP) by Wrochna and Živný [7] as well as the myself, Avvakumov, Opršal, Tasinato, Nakajima and Wagner [1, 4] to determine hardness of promised colourings of 3 and 4 colourable graphs as well as in promised linear colourings of hypergraphs.

Abstractly, the Hom complex is a functor Hom: $\mathsf{Grph} \times \mathsf{Grph} \to \mathsf{Top}$ that assigns a topological space to a pair of graphs and it has been shown by Csorba and Matsushita [2, 6] that for any finite topological space X with a $\mathbb{Z}_2$ action, there exists a graph G such that $\mathrm{Hom}(K_2, G)$ is equivariantly homotopy equivalent to X.

I will discuss the notion of homotopy captured by the Hom complex, applications in PCSP, the alternatives to the Hom complex contruction and give an overview of a current joint research with Vokřínek and Fuiji, where we extend the results of Csorba and Matsushita to digraphs with varied group actions ($\mathbb{Z}_n$, finite groups) and more general relational structures.

References

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2.2.2 Second Block

Certifying robustness via topological representations

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Abstract. Deep learning models are known to be vulnerable to small malicious perturbations producing so-called adversarial examples. Vulnerability to adversarial examples is of particular concern in the case of models developed to operate in security- and safety-critical situations. As a consequence, the study of robustness properties of deep learning models has recently attracted significant attention.

In this talk we discuss how the stability results for the invariants of Topological Data Analysis can be exploited to design machine learning models with robustness guarantees. We propose a neural network architecture that can learn discriminative geometric representations of data from persistence diagrams. The learned representations enjoy Lipschitz stability with a controllable Lipschitz constant. In adversarial learning, this stability can be used to certify robustness for samples in a dataset, as we demonstrate on synthetic data.

Bregman geometries — a tool for Topological Data Analysis and Machine Learning

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Abstract. Many constructions in computational geometry focus on Euclidean geometry. This is not surprising: it is the most natural choice with many practical applications, especially in dimension three. However, many modern applications rely on geometries which are not only non-Euclidean but also non-metric.

In my talk I will dive into geometries induced by Bregman divergences, and in particular by various notions of entropy. I will outline how geometric and topological methods based on these geometries can be used in topological data analysis and machine learning, where such concepts are commonplace.

Explaining machine learning models with topological data analysis

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Abstract. In some fields such as medical applications, or financial sector it is crucial to systematically explain the machine learning models used in the decision-making process. Applying topological data analysis (TDA) tools such as BallMapper [1] or ClusterGraph [2] to the data set can reveal more details about its local structure. Combining these TDA tools with explainable AI methods such as SHAP [3] or LIME [4] leads to a better understanding not only of the data set but also of the complex models built on them. However, since the computation of SHAP values is slow and LIME is based on synthetical data, which can be unreliable, we can use local regression methods applied at hand to explain the impact of the features on the output of the model. This approach has shown its usefulness on a number of synthetic data sets with known ground truth as well as real data examples showing the potential application to real life problems. The presentation is based on a joint work with Paweł Dłotko, Rafał Topolnicki, and Simon Rudkin.

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2.3 Contributed Talks: Parallel Session 2 (Aula B)

2.3.1 First Block

(Sequential) topological complexity of aspherical spaces and $\mathcal{A}$-genus

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Abstract. The *topological complexity* of a space X is a homotopy invariant originally introduced by M. Farber in order to study the instability of the motion planning problem from robotics. It has close connections with other classic homotopy invariants (such as Lusternik-Schnirelmann category and sectional category), as well as interesting generalizations, like the rth-sequential topological complexity for $r \geq 2$.

One of the most important open problems in the field of topological robotics is the characterization, in algebraic terms, of the topological complexity of $K(G, 1)$-spaces, for G a torsion-free group. In this talk we will make a quick overview of some recent developments on this problem, and then we will introduce a characterization of (sequential) topological complexities of aspherical spaces in terms of another homotopy invariant, the A-genus in the sense of M. Clapp and D. Puppe. While

still topological in nature, this characterization allows to refine some of the already known bounds, as well as to find new ones for both sectional category and sequential topological complexities of aspherical spaces by exploiting certain properties of $\mathcal{A}$-genus. We will conclude by introducing some new ideas about notions of category-like invariants with respect to proper actions of groups.

Lusternik-Schnirelmann-type theorems for topological complexity

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Abstract. The Lusternik-Schnirelmann (LS) category of a space was introduced to obtain a lower bound on the number of critical points of a C^1-function on a given manifold. Related to LS category and motivated by topological robotics, the topological complexity (TC) of a space is a numerical homotopy invariant whose topological properties are still being explored. While the definition of TC is closely related to LS category, the connections of sequential and parametrized TC to critical point theory have not been fully explored yet. In this talk, I will present some lower bounds on numbers of critical points of functions that are given by TC as well as its sequential and parametrized versions. This is joint work with Maximilian Stegemeyer.

2-stratifolds and 3-manifolds polynomials

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Abstract. While studying the tri-genus of 3-manifolds [1] we found the first examples of 2-stratifolds. In an informal way to say it, a 2-stratifold is a finite 2-dimensional simplicial complex that outside a link is a finite collection of surfaces. We present several results about 2-stratifolds [2]. We will mention two applications where 2-stratifolds appear [3], [4]. This part of the talk is joint work with F. González-Acuña and W. Heil. Now, let D = (S; A, B) is a 3-manifold Heegaard Diagram (HD) of genus n. Let us call G = A U B the Heegaard graph of D where V = A intersection B are the vertices of G and E = G − V are the edges of G. For any D where S - G is a finite collection of open disks, we can define a polynomial p(D) of D where p is any graph polynomial of G as those defined in [5] (Tutte, Ribbon, ...). We propose a way to use these graph polynomials to define 3-manifolds polynomials. These polynomials can be calculated for Lens spaces. This second part of the talk is joint work J.A. Frías, F. Manjarrez and J. L. León.

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2.3.2 Second Block

Persistence Spheres: Linear Representations of Persistence Diagrams

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Abstract. In this talk, we present ongoing work on a novel functional representation of Persistence Diagrams (PDs). Inspired by the approach of Dogas and Mandaric (2024), we model PDs as scalar fields on the sphere via the lift zonoid representation of finite measures. Unlike their method, however, our construction yields an operator that is stable with respect to the 1-Wasserstein distance. Beyond providing a stable vectorization of PDs, this operator is also linear with respect to measure addition and positive scalar multiplication.

A Euclidean Embedding for Computing Persistent Homology with Gaussian Kernels

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Abstract. Computing persistent homology of large datasets using Gaussian kernels is useful in the domains of topological data analysis and machine learning as shown by Phillips, Wang and Zheng [SoCG 2015]. However, unlike in the case of persistent homology computation using the Euclidean distance or the k-distance, using Gaussian kernels involves significantly higher overhead, as all distance computations are in terms of the Gaussian kernel distance which is computationally more expensive. Further, most algorithmic implementations (e.g. Gudhi, Ripser, etc.) are based on Euclidean distances, so the question of finding a Euclidean embedding - preferably low-dimensional - that preserves the persistent homology computed with Gaussian kernels, is quite important. We consider the Gaussian kernel power distance (GKPD) given by Phillips, Wang and Zheng. Given an n-point dataset and a relative error parameter $\varepsilon \in (0, 1)$, we show that the persistent homology of the Čech filtration of the dataset computed using the GKPD can be approximately preserved using

$O\left(\varepsilon^{-2}\log n\right)$ dimensions, under a high stable rank condition. Our results also extend to the Delaunay filtration and the (simpler) case of the weighted Rips filtrations constructed using the GKPD.

Our proof utilizes the embedding of Chen and Phillips [ALT 2017], based on the Random Fourier Functions of Rahimi and Recht [NeurIPS 2007], together with two novel ingredients, (i) a new decomposition of the squared radii of Čech simplices computed using the GKPD, in terms of the pairwise GKPDs between the vertices, and (ii) a new concentration inequality for sums of cosine functions of Gaussian random vectors, which we call Gaussian cosine chaoses.

This is joint work with J.D. Boissonnat and appeared in ESA 2024.

Minimal updates for dynamic barcodes and representative cycles

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Abstract. Persistent homology and topological summaries of datasets have become a mainstay in methods of applied topology. Obtaining the barcode can be computationally expensive, so it is useful to have methods that provide minimal updates if the associated filtration undergoes minimal changes. We provide an algorithm with a minimal amount of column operations when simplices are removed from the filtration. Our method also updates the representative cycles of homological classes, and is provided in an implementation built on top of the PHAT software. Together with known methods for adding and swapping simplices in filtrations, our work completes the framework for efficiently updating topological summaries in dynamic settings. This is joint work with Barbara Giunti.

3 Wednesday

3.1 Plenary speakers (Aula A)

Conley index methods in data-driven dynamics

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Abstract. Conceptual models for most physical systems are based on a continuum; values of the states of a system are assumed to be real numbers. At the same time science is increasingly becoming data driven and thus based on finite information. Traditional, analytical analysis of dynamical systems is possible only when the mathematical model of the dynamics is available. Frequently, in particular in experimental sciences, in biological, in sociological sciences or in medicine, this is rarely the case. Instead, scientists collect data by experiments. This suggests the need for tools that seamlessly and systematically provide information about continuous structures from finite data and accounts for the rapid rise in use of methods from topological data analysis. In this context sampled dynamics attracts interest of scientists. Not surprisingly, however, there are significant challenges associated with understanding of the data dynamics.

The aim of the talk is to present recent developments in the Conley index theory for multivalued maps and its applications to the study of dynamical systems known only from data. Moreover, we will tackle Gaussian process surrogate modeling in sampled dynamics.

Geometric construction of Kashiwara crystals on multiparameter persistence

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Abstract. We establish a geometric construction of Kashiwara crystals on the irreducible components of the varieties of multiparameter persistence modules. Our approach differs from the seminal work of Kashiwara and Saito, as well as subsequent related works, by emphasizing commutative relations rather than preprojective relations for a given quiver. Furthermore, we provide explicit descriptions of the Kashiwara operators in the fundamental cases of 1- and 2-parameter persistence modules, offering concrete insights into the crystal structure in these settings.

Topology and Dynamical Systems Meets Cardiology: Bridging Mathematics and Clinical Insights.

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Abstract. The human cardiovascular system displays complex, nonlinear behavior that reflects its ability to adapt to changing conditions. While traditional linear tools used in cardiology have proven effective in many diagnostic contexts, they are often insufficient to capture the subtle irregularities and complex patterns inherent in heart rate variability. In this talk, we explore how methods from topology and symbolic dynamics can be effectively applied to cardiology — particularly for predicting and characterizing disorders such as syncope, congestive heart failure, obstructive sleep apnea, hypertension, transient ischemic attacks, etc. We focus especially on topological techniques such as persistent homology, as well as methods from symbolic dynamics, demonstrating the ability of these approaches to reveal underlying structures in physiological signals such as ECG, blood pressure, and respiration.

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4 Thursday

4.1 Plenary speakers (Aula A)

Maximizing the Utility of TDA

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Abstract. I will talk about some research directions I am pursuing around the use of TDA, including applications to neural networks and feature generation.

Data geometry and homology

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Abstract. Successful data analysis relies on representing data through objects that are amenable to statistical methods. In recent years, there has been an explosion of applications where homological representations have played a significant role. In this talk, I will introduce one such representation called stable rank, and present several novel approaches for using it to encode geometric information and analyze data. I will provide several illustrative examples of how to use stable ranks to find meaningful results in biological data.

Multi-Parameter Persistent Homology is Practical

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Abstract. Multi-parameter persistent homology (MPH) is an active research branch of topological data analysis. Recent years have witnessed an extending layer of efficient algorithms for various problems in MPH providing tools for its practical usage.

We will discuss two recent results in this context: First, we will demonstrate how to extend Delaunay filtrations, also known as alpha filtrations, to the case of point clouds with a real valued

function (joint work with Angel Alonso, Tung Lam, and Mike Lesnick). Second, we will introduce graphcodes, a novel way to faithfully represent 2-parameter persistence modules which allows both for employing MPH in machine learning applications and for faster algorithmic approaches in the context of decomposing persistence modules (joint work with Florian Russold).

4.2 Contributed Talks: Parallel Session 1 (Aula A)

4.2.1 First Block

Ball Mapper Analysis for High-Resolution Time-of-Flight Mass Spectrometry

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Abstract. This study introduces an approach combining two-dimensional gas chromatography (GC×GC), high-resolution time-of-flight mass spectrometry (HR-TOF-MS), and topological data analysis (TDA) to identify and analyze unique chemical signatures in samples. Employing the ball mapper algorithm, we manage the high-dimensional data generated by HR-TOF-MS, simplifying the complex dataset into a representative set of chemical compounds. This method allows us to visualize and analyze chemical diversity and relationships in a more accessible two-dimensional form, facilitating the identification of distinct chemical profiles unique to a sample from different samples. The application of this technique not only enhances our understanding of the compositional variations in samples but also demonstrates the potential of TDA in analytical chemistry for complex mixture analysis such as authentication and environmental studies. Our findings provide a new perspective on the standard analytical approaches, offering advancements in the field of computational analysis in chemometrics.

Spatial Analysis of Malignant-Immune Cell Interactions in the Tumor Microenvironment Using Topological Data Analysis

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Abstract. The spatial interactions between malignant and immune cells in the tumor microenvironment (TME) play a crucial role in cancer biology and treatment response. Understanding these interactions is essential for predicting prognosis and evaluating the effectiveness of immunotherapy. Conventional methods, which focus on local spatial features, often struggle to achieve robust analysis due to the complex and heterogeneous cellular distributions. We propose a Topological Data Analysis (TDA)-based framework using both global and local spatial features between malignant and immune cells. For the global perspective, we introduce Topological Malignant Clusters (TopMC), which use persistence diagrams from TDA to capture the spatial organization of malignant cells with heterogeneous distributions. This approach quantifies tumor-immune cell infiltration at a global scale by

computing distances from the boundaries of the TopMC. Local interactions are evaluated through the density of malignant cells. Using high-resolution multiplex immunofluorescence imaging in Diffuse Large B Cell Lymphoma, we integrate distance and density measures into a distance-density space to analyze patterns of malignant-immune cell interactions in the TME. This study shows the robustness of the proposed approach to variations in cell distribution, enabling consistent analysis irrespective of whether images are acquired from malignant-enriched or border regions of the tumors. In addition, we elucidate the correlation between spatial patterns of immune phenotypes relative to TopMC and patient survival probability.

Artificial Intelligence and Topology for autonomous wheelchairs

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Abstract. In this talk, we will introduce the European project REXASI-PRO (REliable & eXplAinable Swarm Intelligence for People with Reduced mObility), an interdisciplinary initiative focused on the development of artificial intelligence systems that are not only safe, trustworthy, and explainable, but also energy-efficient and environmentally sustainable. The project's main application is the deployment of a fleet of autonomous wheelchairs capable of navigating safely within closed environments such as hospitals and nursing homes. We will briefly present the contributions of the CIMAgroup research group from the University of Seville, which has concentrated on two key areas within the project. First, the exploration of data-centric techniques aimed at reducing energy consumption during the training of machine learning models. Second, the use of computational topology to analyze the behavior of various navigation algorithms, with the goal of identifying those that produce more orderly and efficient movement patterns across different scenarios.

4.2.2 Second Block

ClusterGraph: a topological bridge between dimensionality reduction and clustering

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Abstract. Understanding the structure of high-dimensional data is a central challenge in data analysis. Dimensionality reduction approaches, such as PCA, t-SNE and UMAP, tackle this problem by mapping the data into a lower dimensional space. This projection methods work by trying to preserve the local structure, but they can distort global relationships. Clustering techniques, on the other hand, group together points that are similar on the original space, but the returned partition does not contain any geometrical information.

In this talk, we will present ClusterGraph, a novel topological framework that enhances the output of any clustering algorithm by constructing a graph where nodes represent clusters and edge weights reflect inter-cluster distances in the original high-dimensional space. This structure preserves global

relationships without forcing the data into a low-dimensional Euclidean embedding. This approach enables both data visualization and compression, and introduces a metric distortion score to quantify fidelity to the intrinsic data geometry. We will discuss pruning strategies that simplify the graph while maintaining structural integrity, and demonstrate how ClusterGraph compares to traditional dimensionality reduction techniques on both synthetic and real-world datasets.

Exploring Cliquegrams and Facegrams for Modeling Phylogenetic Networks and Rater Agreement

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Joint work with Paweł Dłotko and Anastasios Stefanou.

Abstract. The cliquegram and facegram models introduced in [1] provide a combinatorial framework for representing phylogenetic networks through filtered (maximal) cliques / face-sets. These models extends traditional dendrograms by capturing higher-order interactions, enabling the modeling of complex evolutionary processes. In this work, we extend the facegram framework by highlighting its connection to persistent homology, compare different invariants of the facegram in addition to the mergegram and consider their usefulness.

As a practical application, we employ facegrams (and cliquegrams) to assess inter-rater reliability for several raters. Inter-rater reliability refers to the degree of agreement or consistency among multiple raters or evaluators when making judgments or assessments about the same subjects. This measure is crucial for ensuring the validity and reliability of subjective evaluations in fields such as psychology, medicine, and social sciences. In particular, we are considering the problem of finding homogeneous subgroups of raters [2]. Under certain approximations, we can construct cliquegrams / facegrams from rater annotations. This is similar to what community detection algorithms do, just for the task of identifing subgroups of raters exhibiting high agreement, thus quantifying consensus among raters but also uncovering structures between the different raters.

The integration of topological data analysis with community detection offers different perspectives on evaluating rater agreement.

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The Multineighbor Complex of a Random Graph

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Abstract. Random graphs and simplicial complexes provide models for understanding how large-scale structure and topology emerge from simple local rules. The classical Erdős–Rényi model describes random graphs built by independently including edges, and its phase transitions—such as the sudden emergence of connectivity—are by now textbook examples of probabilistic combinatorics. In higher dimensions, the Linial–Meshulam model offers a natural extension: simplicial complexes built from complete skeletons by randomly adding top-dimensional faces.

In this talk, I will present a construction that bridges these two paradigms: the *multineighbor complex* of a graph, where simplices are formed by sets of vertices sharing a certain minimal number of common neighbors. Starting from an Erdős–Rényi graph, we obtain a simplicial complex whose topology—perhaps surprisingly—resembles that of a Linial–Meshulam complex with carefully chosen parameters. This perspective reveals how high-dimensional topological randomness can arise from purely pairwise connectivity.

I will describe key results showing when and how this correspondence emerges and highlight the role of correlation between faces in the multineighbor model. Along the way, we'll see how this construction offers a combinatorially intuitive and topologically rich framework, with implications for both probabilistic topology and applied data analysis.

No prior familiarity with random topology will be assumed—just a basic acquaintance with graphs, simplicial complexes, and probablity.

This is joint work with Eric Babson

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4.3 Contributed Talks: Parallel Session 2 (Aula B)

4.3.1 First Block

Symmetries and Chern classes: from Combinatorial Geometry to Algebraic and Applied Topology

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Abstract. In this talk, we will present three problems coming from distinct areas of mathematics: Tverberg-Vrećica conjecture (combinatorial geometry), the existence of linearly independent complex line fields on manifolds (algebraic and differential topology), and the calculation of persistent equivariant cohomology of the circle (applied topology). We will discuss how Chern classes arise in the methods used to their resolution and pose several intriguing questions for future research. The talk is based on several works with different groups of coauthors (P. Soberón; B. Schutte; H. Adams, E. Lagoda, M. Moy & A. De Saha).

Motion planning on Peano continua

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Abstract. A compact and connected metric space that is also locally connected is called *Peano continuum*. One-dimensional Peano continua can be viewed as generalized graphs (though they also include more complex examples like the Sierpinski carpet or the Menger sponge). We will study the complexity of motion planning on one-dimensional Peano continua and show that it is completely determined by the subspace of wild points. (A point $x \in X$ is *wild* if X is not semi-locally simply connected at x).

This is a joint work with J. Brazas.

Vertex-minimal triangulations of spaces with a given fundamental group

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Abstract. Given a finitely presented group G we consider the problem of finding the minimum number n such that there is a simplicial complex K on n vertices with $\pi_1(K) \cong G$. During the talk we will discuss how the fundamental groups of complexes on 8 vertices can be classified using computer calculations. One of our motivations for this study is the question on the minimal triangulation of the Poincaré sphere.

4.3.2 Second Block

Spatiotemporal persistence landscapes

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Abstract. To apply persistent homology to time series, frequently either one-parameter or zigzag persistent homology are used. We combine this two approaches and obtain an algebraic structure that we call the extended zigzag module. For this structure, we propose spatiotemporal persistence landscapes, an invariant to visualize features that are persistent simultaneously in space and time. This is an adaptation of the classical persistence landscapes that are known from the one-parameter as well as the multi-parameter case, where the main difference is that the generalized rank introduced by Kim and Memoli (2021) is used. By sending the extended zigzag module to a three-parameter persistence module, the interleaving distance can be defined for this case, in analogy to the definition of the interleaving distance for zigzag modules. It turns out that the spatiotemporal persistence landscapes are stable with respect to the interleaving distance. If time permits, some applications will be shown.

Dualities in multiparameter persistent homology

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Abstract. In one-parameter persistent homology computation, computing the barcode of the relative cohomology $H^\bullet(\cup K, K)$ using the clearing algorithm and then converting this barcode to one for the absolute homology $H^\bullet(K)$ is by far the most performant way of computing barcodes. This is particularly true if K is a Vietoris-Rips complex, due to the fast growth of the number of q-dimensional simplices with q. Underderlying is a well-known correspondence between the barcodes of $H^\bullet(K)$ and $H^\bullet(\cup K, K)$. We provide a generalization of this correspondence to the N-parameter case, for N arbitrary, and identify parts of this correspondence as a shadow of Grothendieck local duality. For the case $N = 2$, we derive an algorithm for computing minimal free resolutions of $H^\bullet(K)$ via first computing a resolution of the persistent cohomology and converting the resolution. This algorithm has shown superior performance over the direct approach for the case of function-Rips complexes.

Nonlocal Operators in Fractional Calculus: Functional and Kernel-Based Classifications

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Abstract. n fractional calculus, we extended classical differentiation and integration to noninteger orders due to the nonlocal property, enabling the modeling of systems with memory and anomalous dynamics. This review presents a comprehensive classification of fractional derivatives based on two fundamental perspectives: the nature of their memory kernels (singular vs. nonsingular) and their behavior within functional analysis frameworks.

Our Survey is based on singular vs. nonsingular involved fractional derivatives, such as the Riemann-Liouville, Caputo, Hadamard, Hilfer, Katugampola operators, characterized by singular power-law kernels, Logarithmic singularity, Generalized power-law, and scaled power-law. On the other hand, we have a contrast with modern formulations like the Caputo-Fabrizio, Atangana–Baleanu derivatives, and Prabhakar, which employ nonsingular exponential or Mittag-Leffler

kernels or 3-parameter Mittag-Leffler to regularize memory effects, as well as with a combination singular kernel. We then examine how these operators act on various function spaces, including Lp, Hölder, Hilber, and Sobolev spaces. We also discuss their implications for operator theory, weak formulations, and well-posedness in fractional differential equations. In addition, we describe the properties of each fractional derivative.

This classification highlights the analytical distinctions among fractional derivatives and provides a functional foundation for selecting the appropriate operators in mathematical modeling. The work aims to clarify fractional derivatives' theoretical landscape and encourage a deeper exploration of nonlocal operators in applied and pure mathematical circumstances.

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5 Friday

5.1 Plenary speakers (Aula A)

Prediction and Reconstruction of Dynamical Systems A Mathematical Point of View

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Abstract. A fundamental problem in experimental sciences is the problem of reconstructing and predicting the future of an observed dynamical system. In recent years together with Krzysztof Barański and Adam Śpiewak, we have studied this problem from a mathematical point of view within the framework of delayed observations. I will discuss various aspects of our research, including the probabilistic Takens theorem, the Schroer–Sauer–Ott–Yorke conjectures, the prediction delay dimension threshold and, if time permits, algorithmic aspects.

Recovering Intrinsic Low-Dimensional Structures via Convex Non-Smooth Regularization

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Abstract. tba

The Shape of Relations: From Knot Invariants to Cancer Genomics

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Abstract. Topological Data Analysis (TDA) provides a powerful framework for extracting structure from complex data by studying its shape. This talk presents recent work on visualizing maps between high-dimensional spaces to detect correlations between datasets, alongside new adaptations of TDA to settings where representative sampling is impossible. This includes the integration of TDA with machine learning methodologies, particularly in contexts where traditional sampling is impractical, to analyze infinite datasets effectively. Time permitting, we will also talk about relations defined on three or more sets, including a generalization of the Dowker's theorem and provide applications to knot theory and comparative cancer genomics.

5.2 Contributed Talks: Parallel Session 1 (Aula A)

5.2.1 First Block

Computing Connection Matrices of Conley Complexes via Algebraic Morse Theory

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Abstract. The Morse-Conley complex is a central object in the application of homological algebra to analysing dynamical systems, as well as information compression in topological data analysis. Given a poset-graded chain complex, its Morse-Conley complex is the optimal chain-homotopic reduction of the complex that respects the poset grading. We adapt the algebraic Morse theory of Sköldberg for based complexes to our setting, and give a purely algebraic derivation of the Conley complex as a consequence of homological perturbation theory. Using this framework, we express the Conley complex's boundary operator in a closed algebraic formula, which reduces the problem of computing the connection matrix to solving a matrix inverse problem.

This talk features joint work with Álvaro Torras Casas and Ulrich Pennig in "Computing Connection Matrices of Conley Complexes via Algebraic Morse Theory" (arXiv:2503.09301).

Conley-Morse persistence barcode: homological signature of a combinatorial bifurcation

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Abstract. Bifurcation is one of the major topics in the theory of dynamical systems. It characterizes the nature of qualitative changes in parameterized dynamical systems. In this project, we study combinatorial bifurcations within the framework of combinatorial multivector field theory–a young but already well-established theory providing a combinatorial model for continuous-time dynamical systems. We introduce the Conley-Morse persistence barcode, a compact algebraic descriptor of combinatorial bifurcations. The barcode captures structural changes in a dynamical system at the level of Morse decompositions and provides a characterization of the nature of observed transitions in terms of the Conley index. The construction of Conley-Morse persistence barcode builds upon ideas from topological data analysis (TDA). Specifically, we consider a persistence module obtained from a zigzag filtration of topological pairs (formed by index pairs defining the Conley index) over a poset. Using gentle algebras, we prove that this module decomposes into simple intervals (bars) and compute them with algorithms from TDA known for processing zigzag filtrations.
The talk is based on: https://arxiv.org/abs/2504.17105

Transition Matrix without Continuation in the Conley Index Theory

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Abstract. Given a one-parameter family of flows over a parameter interval Λ, assuming there is a continuation of Morse decompositions over Λ, Reineck defined a singular transition matrix to show the existence of a connection orbit between some Morse sets at some parameter points in Λ. This presentation aims to extend the definition of a singular transition matrix in cases where there is no continuation of Morse decompositions over the parameter interval. This extension will help study the bifurcation associated with the change of Morse decomposition from a topological dynamics viewpoint.

5.2.2 Second Block

The persistent Laplacian of non-branching complexes

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Abstract. Non-branching matrices are real matrices with entries in $\{-1, 0, 1\}$, where each row contains at most two non-zero entries. Such matrices naturally arise in the study of Laplacians of pseudomanifolds and cubical complexes. We show that a basis for the kernel of a non-branching matrix can be computed in near-linear time. Especially, the basis has the special property that the supports of all the column vectors in it are disjoint with each other. Building on this result, we show

that the up persistent Laplacian can be computed in near-linear time for a pair of such spaces and its eigenvalues can be more efficiently computed via computing singular values. In addition to that, we analyze the arithmetic operations of up persistent Laplacian with respect to a non-branching filtration. Furthermore, we show that the up persistent Laplacian of q-non-branching simplicial complexes can be represented as the Laplacian of an associated hypergraph, thus providing a higher-dimensional generalization of the Kron reduction, as well as a Cheeger-type inequality. Finally, we highlight the efficiency of our method on image data.

5.3 Contributed Talks: Parallel Session 2 (Aula B)

5.3.1 First Block

Crystallizations of small covers over the n-simplex Δ^n and the prism $\Delta^{n-1} \times I$

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Abstract. A crystallization of a PL manifold is an edge-colored graph that corresponds to a contracted triangulation of the manifold, facilitating the study of its topological and combinatorial properties. A small cover over a simple convex n-polytope P^n is a closed n-manifold with a locally standard $\mathbb{Z}_2^n$-action such that its orbit space is homeomorphic to P^n. In this article, we study the crystallizations of small covers over the n-simplex Δ^n and the prism $\Delta^{n-1} \times I$. It is known that the small cover over the n-simplex Δ^n is $\mathbf{RP}^n$. For every $n \geq 2$, we prove that $\mathbf{RP}^n$ has a unique $2n$-vertex crystallization. We also demonstrate that there are exactly $1 + 2^{n-1}$ D-J equivalence classes of small covers over the prism $\Delta^{n-1} \times I$, where $n \geq 3$. For each $\mathbb{Z}_2$-characteristic function of $\Delta^{n-1} \times I$, we construct a $2^{n-1}(n+1)$-vertex crystallization of the small cover $M^n(\lambda)$ with regular genus $1 + 2^{n-4}(n^2 - 2n - 3)$, where $n \geq 4$. The regular genus of closed PL n-manifolds extends the notions of the genus of surfaces and the Heegaard genus of 3-manifolds to higher dimensions. Classifying PL n-manifolds based on regular genus is a fundamental problem in combinatorial topology. The classification of orientable PL 4-manifolds up to regular genus 5 is known. In this article, we construct four orientable and four non-orientable $\mathbb{RP}^3$-bundles over S^1 up to D-J equivalence with regular genus 6.

Intersection of Multicurves on Higher Genus Surfaces

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Let S be a surface of genus g with s punctures (possibly with boundary). A well-known way to give coordinates to multicurves—defined as disjoint unions of essential simple closed curves on S, up to isotopy—is through Dehn-Thurston coordinates or train track coordinates.

In the special case where S is the n-punctured disk D_n, a particularly elegant way to describe multicurves is given by the *Dynnikov's coordinates.* One of the main advantages of Dynnikov's coordinates is that they come with the *update rules*, which describe the action of each braid generator of the n-braid group B_n (the mapping class group of D_n) *in terms of Dynnikov's coordinates.*

This yields a piecewise-tropical polynomial representation of the braid group B_n, which has been used to solve many interesting dynamical and combinatorial problems involving curves and surface homeomorphisms.

In this talk, I will briefly present joint work with Daniele Alessandrini, which generalizes the Dynnikov's coordinate system and the corresponding update rules to *higher genus surfaces.* I will then describe an *efficient algorithm* for computing the geometric intersection number of two multicurves on such surfaces. The method will be illustrated—especially in the case of *non-orientable surfaces*—using these generalized coordinates and update rules.

GEM THEORY AND COLORED TENSOR MODELS IN QUANTUM GRAVITY

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Abstract. *Gem theory* enables to represent compact PL manifolds of arbitrary dimension by colored triangulations, or equivalently by their dual edge-colored graphs (called GEMs, i.e. *Graphs Encoding Manifolds*).

In this context, combinatorially defined PL invariants *regular genus* and *G-degree* play a relevant role: the first one extends to higher dimension the classical notion of Heegaard genus for 3-manifolds, the second one arises within theoretical physics, from the theory of random tensors as an approach to quantum gravity in dimension greater than two.

The contribution will summarize the general results concerning the two invariants, in relation with other topological and PL invariants, as well as their behavior with respect to connected sums. In particular, in dimension 4, the classification of all compact PL manifolds with regular genus at most one and of all compact PL manifolds with G-degree at most 18 will be presented, together with its extension to regular genus two and to G-degree 24 in case of empty or connected boundary.

Since the G-degree drives the $1/N$ expansion of the correlation function of the free energy in the tensor models setting, the above results can be applied to yield a better understanding of some of the most significant non-null terms of the $1/N$ expansion.