

LC.index

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1 LC.Definitions

```
module LC.Definitions where

import LC.Variables
import LC.Terms
import LC.Domains
import LC.Environments
import LC.Semantics
```

2 LC.Domains

```
module LC.Domains where

open import Function
  using (Inverse; _  $\leftrightarrow$  _ ) public
open Inverse {{ ... }}
  using (to; from) public

postulate
  D $\infty$  : Set
postulate
  instance iso : D $\infty$   $\leftrightarrow$  (D $\infty$   $\rightarrow$  D $\infty$ )

variable d : D $\infty$ 
```

3 LC.Environments

```
module LC.Environments where

open import LC.Variables
open import LC.Domains
open import Data.Bool using (if _ then _ else _)

Env = Var → D∞

variable ρ : Env

_ [ _ / _ ] : Env → D∞ → Var → Env
ρ [ d / v ] = λ v' → if v == v' then d else ρ v'
```

4 LC.Semantics

```
module LC.Semantics where

open import LC.Variables
open import LC.Terms
open import LC.Domains
open import LC.Environments

[[ _ ]] : Exp → Env → D $\infty$ 
-- [[ e ]]  $\rho$  is the value of e with  $\rho$  giving the values of free variables

[[ var v      ]]  $\rho$  =  $\rho$  v
[[ lam v e    ]]  $\rho$  = from (  $\lambda$  d  $\rightarrow$  [[ e ]] ( $\rho$  [ d / v ]) )
[[ app e1 e2 ]]  $\rho$  = to ( [[ e1 ]]  $\rho$  ) ( [[ e2 ]]  $\rho$  )
```

5 LC.Terms

```
module LC.Terms where

open import LC.Variables

data Exp : Set where
  var _ : Var → Exp          -- variable value
  lam  : Var → Exp → Exp      -- lambda abstraction
  app  : Exp → Exp → Exp      -- application

variable e : Exp
```

6 LC.Tests

```
{-# OPTIONS --rewriting --confluence-check #-}

open import Agda.Builtin.Equality
open import Agda.Builtin.Equality.Rewrite

module LC.Tests where

open import LC.Domains
open import LC.Variables
open import LC.Terms
open import LC.Semantics

open import Relation.Binary.PropositionalEquality using (refl)
open Inverse using (inversel; inverser)

to-from-elim :  $\forall \{f\} \rightarrow \text{to} (\text{from } f) \equiv f$ 
to-from-elim = inversel iso refl

from-to-elim :  $\forall \{d\} \rightarrow \text{from} (\text{to } d) \equiv d$ 
from-to-elim = inverser iso refl

{-# REWRITE to-from-elim #-}

-- The following proofs are potentially unsound,
-- due to rewriting using the postulated iso

-- ( $\lambda x1. x1$ )x42 = x42
check-id :
   $\llbracket \text{app} (\text{lam } (x\ 1) (\text{var } x\ 1)) (\text{var } x\ 42) \rrbracket \equiv \llbracket \text{var } x\ 42 \rrbracket$ 
check-id = refl

-- ( $\lambda x1. x42$ )x0 = x42
check-const :
   $\llbracket \text{app} (\text{lam } (x\ 1) (\text{var } x\ 42)) (\text{var } x\ 0) \rrbracket \equiv \llbracket \text{var } x\ 42 \rrbracket$ 
check-const = refl

-- ( $\lambda x0. x0\ x0$ )( $\lambda x0. x0\ x0$ ) = ...
-- check-divergence :
--    $\llbracket \text{app} (\text{lam } (x\ 0) (\text{app} (\text{var } x\ 0) (\text{var } x\ 0))) (\text{lam } (x\ 0) (\text{app} (\text{var } x\ 0) (\text{var } x\ 0))) \rrbracket$ 
--    $\equiv \llbracket \text{var } x\ 42 \rrbracket$ 
-- check-divergence = refl

-- ( $\lambda x1. x42$ )( $(\lambda x0. x0\ x0)(\lambda x0. x0\ x0)$ ) = x42
check-convergence :
   $\llbracket \text{app} (\text{lam } (x\ 1) (\text{var } x\ 42)) (\text{app} (\text{lam } (x\ 0) (\text{app} (\text{var } x\ 0) (\text{var } x\ 0))) (\text{lam } (x\ 0) (\text{app} (\text{var } x\ 0) (\text{var } x\ 0)))) \rrbracket$ 
```

```

≡ [ [ var x 42 ] ]
check-convergence = refl

-- (λx1.x1)(λx1.x42) = λx2.x42
check-abs :
  [ [ app (lam (x 1) (var x 1))
        (lam (x 1) (var x 42)) ] ]
    ≡ [ [ lam (x 2) (var x 42) ] ]
check-abs = refl

-- (λx1.(λx42.x1)x2)x42 = x42
check-free :
  [ [ app (lam (x 1)
              (app (lam (x 42) (var x 1))
                    (var x 2)))
        (var x 42) ] ] ≡ [ [ var x 42 ] ]
check-free = refl

```


7 LC.Variables

```
module LC.Variables where

open import Data.Bool using (Bool)
open import Data.Nat using ( $\mathbb{N}$ ;  $\equiv^b$  _)

data Var : Set where
  x :  $\mathbb{N}$   $\rightarrow$  Var -- variables

variable v : Var

_ == _ : Var  $\rightarrow$  Var  $\rightarrow$  Bool
x n == x n' = (n  $\equiv^b$  n')
```

8 LC.index

```
{-# OPTIONS --rewriting --confluence-check #-}  
  
module LC.index where  
  
import LC.Definitions  
import LC.Domains  
import LC.Environments  
import LC.Semantics  
import LC.Terms  
import LC.Tests  
import LC.Variables
```