

Optimal Redistribution with a Shadow Economy

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Abstract

We extend the theory of optimal redistributive taxation to economies with an informal labor market. The optimal tax formula contains two novel terms capturing reported income responses of informal workers on an intensive and an extensive margin. Both terms decrease the optimal tax rates. We estimate the model with Colombian data and show that the reduction of tax rates relative to the best-performing standard tax formula can be quantitatively large, reaching 25 pp and leading to a 1.9% welfare gain. We also provide a novel decomposition of the welfare impact of the shadow economy into an efficiency and a redistribution components. Conditional on the optimal tax policy, the Colombian shadow economy benefits efficiency at the expense of redistribution. Consequently, the presence of the informal sector reduces welfare only when preferences for redistribution are strong.

Keywords: shadow economy, informal labor market, income taxation, redistribution.

JEL Codes: H21, H26, J46.

1. Introduction

Informal activity, broadly defined as any economic endeavor which evades taxation, accounts for a large fraction of economic activity in both developing and developed

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economies. The share of informal production in GDP is consistently estimated to be on average above 10% in high income OECD countries and above 30% in developing and transition countries, in extreme cases reaching up to 70% (Schneider and Enste 2000; Schneider, Buehn, and Montenegro 2011). Globally, 2 billion workers are employed informally (ILO 2018). The shadow economy allows workers to earn additional income which is unobserved by the government. Intuitively, this additional margin of response to taxation makes income redistribution more difficult. On the other hand, the informal jobs seem to be less productive and attract mostly the poor.¹ If the informal sector benefits those in need, perhaps it can be useful from the social welfare perspective. Our aim is to evaluate these claims within an optimal taxation framework. We pose the following questions:

1. What is the optimal income tax schedule in the presence of a shadow economy?
2. How does a shadow economy affect social welfare?

Concerning the first question, we find that the shadow economy substantially reduces optimal tax rates over most of the income distribution. This result is valid when the comparison is made with respect to either the standard tax formulas applied in the model with the shadow economy or the optimal tax schedule in the model where the shadow economy does not exist.

To answer the second question, we decompose the social welfare impact of the informal sector into an efficiency and a redistribution components. We analytically show that, conditional on the optimal policy, the shadow economy can harm or enhance welfare on either of the two dimensions. Using the calibrated model, we find that the shadow economy in Colombia benefits labor efficiency at the expense of possible redistribution. As a result, the presence of the Colombian informal sector reduces social welfare only when social preferences for redistribution are strong.

Building on the seminal work of Mirrlees (1971), we consider a framework with heterogeneous agents equipped with distinct formal and shadow productivities. Workers also face an idiosyncratic fixed cost of working in the shadow economy, which may reflect either ethical or technological constraints. The government observes only formal incomes and introduces taxation to maximize its redistributive welfare criterion. Importantly, we allow workers to supply labor to the formal sector and the shadow sector simultaneously.

Our first theoretical contribution is a novel sufficient statistics optimal tax formula for economies with an informal sector. The tax formula contains two new terms which capture the deadweight loss of taxation due to shadow workers' responses on the extensive margin (getting an informal job) and the intensive margin (shifting hours between a formal and an informal job). Importantly, these terms are not fully accounted for in the standard sufficient statistic formulas from the models with the intensive margin of

¹For instance, focusing on the main jobs, we find that the shadow economy in Colombia accounts for 58% of jobs and 55% of hours but for only 31.4% of earnings.

labor supply only (Diamond 1998, Saez 2001) or both the intensive and the participation margins (Jacquet, Lehmann, and Van der Linden 2013). To see it concretely, note that according to the standard formulas and absent wealth effects workers respond on the intensive margin only when the marginal tax rate at their formal income level is changed. We show that shadow workers can respond on the intensive margin to a tax rate perturbation which happens at a strictly lower formal income level.² Such responses are not captured by the standard formulas. Furthermore, the extensive margin responses are typically modelled as binary: working or not working. In our setting it would correspond to allowing agents to work only formally or only informally. We generalize the extensive margin responses by allowing workers to retain some formal earnings when getting an informal job.

We analytically examine how the shadow economy affects the optimal tax rates in two ways. First, we fix the distribution of formal income and compare our optimal tax formula to the standard formulas. We find that the terms corresponding to the shadow workers' responses on the intensive and the extensive margins reduce the optimal tax rate in comparison to the standard formulas. Second, we fix model primitives, such as the distribution of productivities in the formal sector, and compare the optimal top tax rate in the model with and without the shadow economy. This comparison is more challenging since the income distribution is allowed to freely adjust to the tax policy. We analytically show that the optimal top tax rate in the model with a shadow economy is lower both due to the new responses of shadow workers and due to the endogenous adjustment of the income distribution. In particular, once the top tax rate exceeds a certain tipping point, a large fraction of top earners moves to the shadow economy and the thickness of the upper tail of the formal income distribution drops discretely. Given a thinner tail of formal earnings, it is optimal to set the top tax rate at a lower level.

Our second theoretical contribution is a novel decomposition of the welfare impact of the shadow economy. We compare the optimal allocations when a shadow economy is present and when it is costlessly shut down. The difference between these two allocations can be expressed as a sum of an *efficiency gain* and a *redistribution gain*. In a simplified framework we analytically derive the comparative statics of both gains and show that, depending on the joint distribution of formal and shadow productivities, the informal sector can harm or enhance welfare on either of the two dimensions. Our result sheds light on non-trivial welfare implications of informality. Kopczuk (2001) provides an example of welfare-improving tax evasion in which, according to our decomposition, the tax evasion allows for more redistribution at the cost of efficiency. It may suggest that welfare gains from the informal sector arise only due to a more equitable division of a

²The intuition is as follows. Shadow workers choose their formal and shadow labor supply such that the net returns to both are equal. When the marginal tax rates are non-monotone, there may be multiple formal income levels which satisfy this condition and some of them will constitute local optima. Increasing the marginal tax rate between the two locally optimal formal income levels affects utility in the higher one, but not in the lower one. As a result, it may trigger a jump to the lower local optimum.

smaller pie. We show that this is not the case. Specifically, when the shadow economy augments efficiency but restricts redistribution, a costless shutdown of the informal sector could reduce utility of all agents. In this case the presence of the shadow economy is Pareto improving.

To gain intuition on the welfare impact of the shadow economy, consider the efficiency gain first. If the productivity loss from moving to a shadow economy is low/high for agents that face high marginal tax rates, the shadow economy will raise/reduce labor efficiency. When agents do not lose much of their productivity by working informally, the shadow sector effectively shelters them from tax distortions of the formal economy.³ Conversely, when the productivity loss is large, the distortions implied by the lower shadow productivity may well dominate tax distortions. Now consider the redistribution gain. If agents who pay high total taxes suffer a low/high productivity loss for moving to the informal sector, then the shadow economy is likely to limit/expand the scope of a possible redistribution. When the productivity loss of these agents is small, they can reduce their formal earnings and adjust their shadow earnings at low cost, which limits redistribution. Conversely, when their shadow productivity is low, they are less tempted by low formal incomes, since their final consumption would be much lower. As a result, they are willing to maintain high formal earnings even when taxes are high.

We quantify the importance of our theoretical results with the model estimated with Colombian data. Colombia is an attractive case study for two reasons. First, it has a large informal sector: we find that 58% of main jobs are informal. Second, the level of informality in Colombia is very close to the average for the whole Latin America. We extract the information on formal and shadow incomes from the household survey and estimate the model by maximum likelihood. The model replicates well the empirical sorting of workers between the formal sector and the informal sector.

In the first quantitative exercise we compare the performance of the optimal tax formula with the standard tax formulas. In contrast to our theoretical results, here we allow for the endogenous adjustment of the income distribution. We find that the [Diamond \(1998\)](#) formula, which does not account for any extensive margin responses, prescribes *very high* tax rates which at low income levels can exceed the optimal ones by more than 70 percentage points. As a result, the shadow economy doubles in size relative to the optimum with catastrophic welfare consequences. In contrast, the [Jacquet et al. \(2013\)](#) formula, which accounts for responses on the formal participation margin, approximates the optimal tax schedule well when the preferences for redistribution are weak. When instead the preferences for redistribution are strong, this formula also leads to excessively high tax rates. In particular, tax rates above the median formal income are too high by up to 25 percentage points. The reason is that in this part of the income distribution the agents who join the shadow economy maintain a fraction of their formal earnings

³This effect is related to what [Porta and Shleifer \(2008\)](#) call the romantic view on the shadow economy. In this view, associated with works of Hernando de Soto ([de Soto 1990, 2000](#)) and modeled formally by [Choi and Thum \(2005\)](#), the informal sector protects productive firms from harmful regulation.

and, hence, are not properly accounted for in the standard formula. With strongly redistributive preferences the welfare loss from using [Jacquet et al. \(2013\)](#) formula rather than the optimal formula is equivalent to 1.9% drop in consumption. We conclude that the welfare gains from using the optimal tax formula are very large when the social welfare function places a high weight on equality.

In the second quantitative exercise we compare the optimal allocation with the *Mirrleesian* allocation, defined as the optimum in the otherwise identical economy in which the informal sector does not exist. While at low income levels the Mirrleesian tax rates increase rapidly, the optimal rates with the shadow economy are relatively constant. Consequently, the optimal tax rates are strictly lower than Mirrleesian rates over most of the income distribution. We find that the shadow economy enhances the social welfare when the preferences for redistribution are weak (by approx. 1% of consumption) and it reduces the social welfare when the preferences for redistribution are strong (by approx. 3% of consumption). Our welfare decomposition implies that the informal sector in Colombia strengthens labor efficiency by providing less productive workers with relatively high shadow productivity and by reducing marginal tax rates in the formal sector. On the other hand, lower marginal tax rates reduce income redistribution and this effect is dominant when the social welfare function places a high weight on equality. Our results point out that even if the informal sector could be shut down at no cost, such policy would bring welfare gains only if the government had a strong preference for redistribution.

Related literature. Following [Allingham and Sandmo \(1972\)](#), tax evasion has been studied in a framework with probabilistic audits and penalties, taking a tax rate as given. [Andreoni, Erard, and Feinstein \(1998\)](#) and [Slemrod and Yitzhaki \(2002\)](#) review this strand of literature. We take a complementary approach and study the optimal non-linear tax schedule conditional on fixed tax evasion abilities of workers. Although we do not model tax audits and penalties explicitly, they are one of the possible justifications for different productivities in the formal and the shadow sector. Under this interpretation, our results on the welfare-improving informal sector can provide insights into the optimal design of tax audits. Some early results from merging both optimal taxation and optimal tax compliance policies were derived by [Cremer and Gahvari \(1996\)](#), [Kopczuk \(2001\)](#) and [Slemrod and Kopczuk \(2002\)](#). [Kopczuk \(2001\)](#) also shows that the standard formula for the optimal linear tax is still valid with tax evasion. In contrast, we show that the standard formula for the optimal *non-linear* tax no longer holds in the presence of a shadow economy.⁴ [Beaudry, Blackorby, and Szalay \(2009\)](#) study redistribution with informal sector when both formal income and formal hours worked are observed. We instead maintain the Mirrleesian assumption of unobserved hours worked.

⁴Our settings is not identical to Kopczuk's, since we consider a fixed cost of shadow employment. In a previous working paper version ([Doligalski and Rojas 2016](#)), we show that the standard formula for the optimal non-linear tax is not valid even if we abstract from the fixed cost of shadow employment.

This paper is closely related to the literature on the optimal taxation with multiple sectors. [Rothschild and Scheuer \(2014\)](#) consider uniform taxation of multiple sectors when agents can work in many sectors simultaneously. [Kleven, Kreiner, and Saez \(2009\)](#), [Scheuer \(2014\)](#) and [Gomes, Lozachmeur, and Pavan \(2017\)](#) study differential taxation of broadly understood sectors (e.g. individual tax filers and couples, employees and entrepreneurs), when agents can belong to one sector only. [Jacobs \(2015\)](#) studies a complementary problem when all agents work in all sectors at the same time. Our analysis differs in that we consider a particular case of differential taxation (only one sector is taxed) when agents face an idiosyncratic fixed cost of participation in one of the sectors. This structure implies that some agents can effectively work in one sector only, while others are unconstrained in supplying labor to two sectors simultaneously. We show that a typical result on the sufficiency of local incentive constraints is no longer valid.⁵ We find an alternative set of incentive constraints which ensures global incentive compatibility.

[Emran and Stiglitz \(2005\)](#) and [Boadway and Sato \(2009\)](#) study commodity taxation in the presence of informality. Both papers assume that the commodity tax affects only the formal sector.⁶ Hence, it is equivalent to a proportional tax on formal income, provided that formal and shadow goods are perfect substitutes. Under these assumptions our focus on non-linear income tax is without loss of generality. A related literature on optimal taxation with home production ([Kleven, Richter, and Sørensen 2000](#); [Olovsson 2015](#)) studies the case of non-perfect substitutability between market and home produced goods.

Structure of the paper. In [Section 2](#), using a simplified framework, we analytically characterize the efficiency and the redistribution impacts of the shadow economy. In [Section 3](#) we derive the optimal tax formula and compare it with the standard formulas. [Section 4](#) is devoted to the quantitative exploration of our theoretical results. The last section provides conclusions.

2. Welfare impact of the shadow economy

In this section we decompose the welfare impact of the presence of the shadow economy into redistribution and efficiency gains. We consider a simplified version of the full

⁵The planner's problem in our setting is an example of multidimensional screening, as agents are heterogeneous with respect to the productivity and the fixed cost of shadow employment. [Carroll \(2012\)](#) shows that the local incentive constraints are sufficient in the multidimensional setting when the appropriately defined space of agents' types is convex. This condition is not satisfied in our setting. The local incentive constraints are insufficient to prevent deviations in both dimensions simultaneously.

⁶In principle, VAT taxation covers informal firms indirectly if they purchase intermediate goods from the formal firms. [De Paula and Scheinkman \(2010\)](#) show that exactly for this reason informal firms tend to make transactions with other informal firms.

model which allows us to characterize analytically comparative statics of both welfare components. Specifically, we consider an economy with two types of workers, no fixed cost of shadow employment and no possibility of working simultaneously in the formal and the informal sectors.

There are two types of individuals, indexed by L and H , with population shares μ_L and $\mu_H = 1 - \mu_L$. They care about consumption c and labor supply n according to a quasilinear utility function $U(c, n) \equiv c - v(n)$, where v is increasing, strictly convex, twice differentiable and satisfies $v'(0) = 0$. While the assumption of the linear utility from consumption allows for an easy exposition, it is straightforward to generalize the results from this section to concave utilities from consumption.

There are two labor markets and, correspondingly, each agent is equipped with two linear production technologies. An agent of type $i \in \{L, H\}$ produces with productivity w_i^f in a formal labor market and with productivity w_i^s in an informal labor market. Income in each sector is given by $y_i^x = w_i^x n_i^x$, where n_i^x denotes labor supply in sector $x \in \{f, s\}$. We identify type H as the one with higher formal productivity: $w_H^f > w_L^f$. Moreover, in this section we assume that each type is more productive formally: $\forall_i w_i^f > w_i^s$. It implies that the shadow economy is inefficient and is never used in the first-best, when the planner can observe individual types. We relax this assumption when we consider the full model.

2.1. The planner's problem

The social planner observes only the formal income of each individual. Furthermore, the planner can transfer resources between agents with taxes T_i . We can think about y_i^f and $y_i^f - T_i$ as a pre-tax and an after-tax reported income. It is convenient to express agents' choices of shadow income as a function of their formal income:

$$y_i^s(y^f) \equiv w_i^s v'^{-1}(w_i^s) \times \mathbf{1}(y^f = 0). \quad (1)$$

If agents have any formal earnings, their shadow earnings are zero. If instead they have no formal earnings, they are unconstrained in choosing their shadow income. Given this function, we can specify agents' consumption $c_i = y_i^f + y_i^s(y_i^f) - T_i$ and labor supply $n_i = y_i^f/w_i^f + y_i^s(y_i^f)/w_i^s$, conditional on a truthful revelation of types.

The social planner maximizes the sum of utilities weighted with Pareto weights λ_i

$$W = \max_{\{(y_i^f, T_i) \in \mathbb{R}_+ \times \mathbb{R}\}_{i \in \{L, H\}}} \lambda_L \mu_L U(c_L, n_L) + \lambda_H \mu_H U(c_H, n_H) \quad (2)$$

subject to a resource constraint

$$\mu_L T_L + \mu_H T_H \geq 0, \quad (3)$$

and incentive-compatibility constraints

$$U(c_i, n_i) \geq U\left(y_{-i}^f + y_i^s(y_{-i}^f) - T_{-i}, \frac{y_{-i}^f}{w_i^f} + \frac{y_i^s(y_{-i}^f)}{w_i^s}\right) \quad i \in \{H, L\}. \quad (4)$$

The incentive compatibility constraints capture the limited information available to the planner. They imply that no agent can be better off by choosing formal income of the other type and, if this income level is zero, freely adjusting shadow earnings.

Lemma 1. *Suppose that $\lambda_i > \lambda_{-i}$. In the optimum,*

- *type $-i$ faces no labor distortions and does not work in the shadow economy.*
- *type i faces labor distortions and may work in the shadow economy.*

Proof. See [Appendix A](#). □

Lemma 1 is a generalization of the classic *no distortion at the top* result. When $\lambda_i > \lambda_{-i}$, the planner wants to redistribute from type $-i$ to type i . The incentive constraint of type $-i$ will bind, and hence the planner cannot improve the allocation by distorting labor of type $-i$. Since an agent works in the shadow economy only if his formal labor is sufficiently distorted downwards (and equal to zero), the agent $-i$ will never work in the shadow economy in the optimum. On the other hand, distorting the labor choice of type i relaxes the binding incentive constraint and allows for more redistribution. Hence, type i can potentially work in the shadow economy in the optimum.

2.2. Welfare decomposition

Suppose that $\lambda_i > \lambda_{-i}$, such that the planner wants to redistribute resources from type $-i$ to type i . There are two candidate allocations for the optimum: a *Mirrleesian allocation* in which type i works formally (denoted with superscript M) and a *shadow economy allocation* in which type i works informally (denoted with superscript SE). Note that the *Mirrleesian allocation* is also the optimum in the setting without the shadow economy. We examine the welfare impact of the shadow economy by comparing these two allocations.

Proposition 1. *Suppose that $\lambda_i > \lambda_{-i}$. The welfare difference between the shadow economy allocation and the Mirrleesian allocation can be decomposed in the following way*

$$\underbrace{W^{SE} - W^M}_{\text{welfare impact}} = \underbrace{\lambda_i \mu_i \left(U(w_i^s n_i^{SE}, n_i^{SE}) - U(w_i^f n_i^M, n_i^M) \right)}_{\text{efficiency gain}} + \underbrace{(\lambda_i - \lambda_{-i}) \mu_i (T_i^M - T_i^{SE})}_{\text{redistribution gain}},$$

where

- the efficiency gain is increasing with w_i^s and is positive when $w_i^s > \bar{w}_i^s$,
- the redistribution gain is decreasing with w_{-i}^s and is positive when $w_{-i}^s < \bar{w}_{-i}^s$,
- the productivity thresholds satisfy $\bar{w}_i^s < w_i^f$ and $\bar{w}_{-i}^s < w_{-i}^f$.

Proof. See [Appendix A](#). □

Proposition 1 decomposes the welfare impact of the shadow economy into an *efficiency gain*, measuring the difference in distortions imposed on type i , and a *redistribution gain*, capturing the change in the level of transfers received by type i .

Efficiency gain. In the shadow economy allocation, type i supplies the efficient level of labor to the inefficient shadow sector. In the Mirrleesian allocation, due to the distortions imposed by the planner, type i supplies an inefficient amount of labor to the efficient formal sector. The relative inefficiency of the shadow sector depends on the productivity difference $w_i^f - w_i^s$. When this difference is sufficiently small ($w_i^s > \bar{w}_i^s$), distortions in the shadow sector are smaller than distortions in the formal sector and the shadow economy improves the efficiency of labor allocation. Intuitively, in this case the shadow economy provides a shelter against tax distortions. If instead the shadow economy distortions are large ($w_i^s < \bar{w}_i^s$), the efficiency impact of the informal sector will be negative.

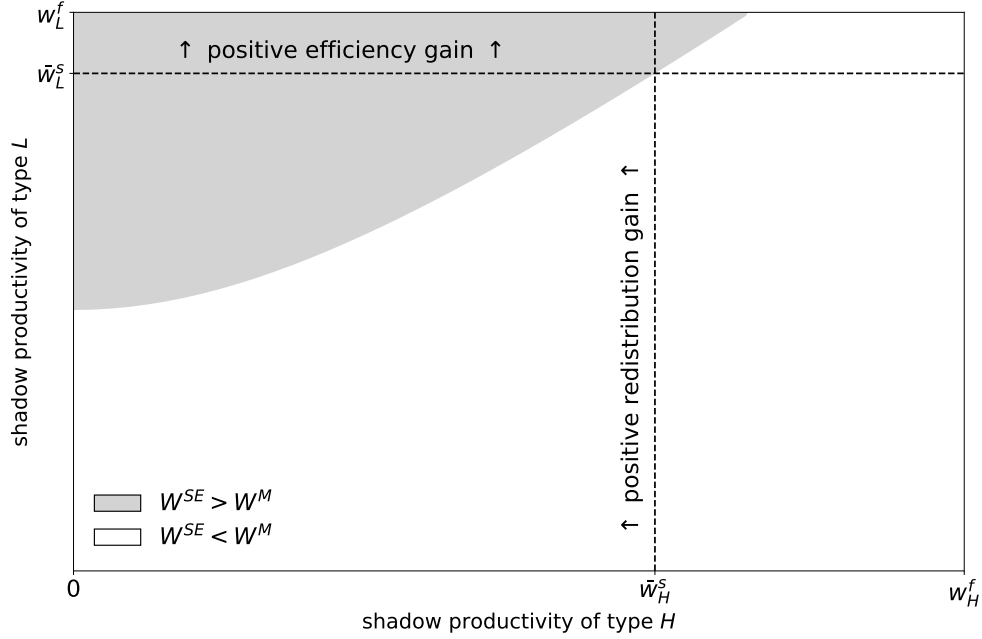
Redistribution gain. The shadow economy improves redistribution if the planner is able to provide type i with a higher transfer (or equivalently raise a higher tax from type $-i$). The scale of redistribution is determined by the payoff of type $-i$ from misreporting. In the Mirrleesian allocation the deviating worker works formally and can earn only as much as type i . In the shadow economy allocation the deviating worker cannot supply any formal labor, but is unconstrained in supplying shadow labor. As the shadow productivity of type $-i$ increases, the payoff from misreporting in the shadow economy allocation rises and the redistribution is reduced. On the other hand, when w_{-i}^s is sufficiently low ($w_{-i}^s < \bar{w}_{-i}^s$), the shadow economy deters the deviation of type $-i$, helping the planner to tell the two types of agents apart. In this case the informal sector is used as a screening device.

Proposition 1 is illustrated in [Figure 1](#), where we assume that the planner maximizes the utility of type L : $\lambda_H = 0$. Intuitively, the shadow economy does not have to strengthen both redistribution and efficiency simultaneously to be welfare improving. Particularly interesting is the region where the redistribution gain is negative, but the efficiency gain is sufficiently high such that welfare is higher with the shadow economy. In this case the shadow economy allocation Pareto dominates the Mirrleesian allocation. Type L gains, since the welfare is higher with the shadow economy. Type H benefits as well, as the negative redistribution gain implies a lower tax burden on this type.

[Kopczuk \(2001\)](#) provides an example in which, starting from the allocation without

tax evasion, a marginal increase in evasion yields welfare gains.⁷ According to our decomposition, in his example welfare improves due to greater redistribution, but at the cost of efficiency. It may suggest that the shadow economy improves welfare by allowing for more even division of a smaller aggregate output. We show that such a scenario is only one of many possibilities. The shadow economy can reduce redistribution, while still being welfare-improving, in which case all agents benefit from the presence of the shadow economy.

Figure 1: Welfare impact of the shadow economy



3. The model with a continuum of types

In this section we derive and characterize the optimal tax schedule in the model with a continuum of productivity types and an idiosyncratic fixed cost of shadow employment. The fixed cost can be interpreted either as a technological constraint on tax evasion or a utility cost of violating social norms. The idiosyncratic fixed cost allows two agents of the same formal productivity to have different shadow employment opportunities, which is an important feature of the data.⁸ For the model with a continuum of productivity types, but without the fixed cost of shadow employment, see the earlier working paper version (Doligalski and Rojas 2016).

⁷Kopczuk (2001) also presents a second example of welfare-improving tax evasion in which some agents have a distaste for paying taxes. We abstract from agents having preferences directly over tax payments.

⁸In Section 4 we show that observable individual characteristics alone are not sufficient to explain empirical informality patterns (see the second panel of Figure 5).

We maintain the quasilinear preference structure from the simple model.⁹ This assumption, which follows [Diamond \(1998\)](#), does not prevent us from studying redistributive tax schedules, since we characterize the entire Pareto frontier which is invariant to any increasing transformation of the utility function. Hence, our results are applicable also with utility functions $\mathbb{G}(c - v(n))$, where \mathbb{G} is a strictly increasing and concave function. Nevertheless, this approach rules out income effects. The impact of the income effects on the optimal tax schedules is well understood since [Saez \(2001\)](#) and the analysis can be easily extended in this direction.

Individuals have two privately observed characteristics: a productivity type θ and a cost type κ . The productivity type θ determines the productivity in the formal economy $w^f(\theta)$ and in the shadow economy $w^s(\theta)$. We assume that both productivity functions are non-negative and continuously differentiable with respect to θ and that the formal productivity is strictly increasing. θ is drawn from $[\underline{\theta}, \bar{\theta}]$, $\bar{\theta} \leq \infty$, according to a cumulative distribution function $F(\theta)$ and a density $f(\theta)$. The cost type κ is a fixed cost of engaging in shadow employment. Conditional on θ , it is drawn from $[0, \infty)$ according to a cumulative distribution function $G_\theta(\kappa)$ and a density $g_\theta(\kappa)$.¹⁰

To solve the model with a continuum of types, it is useful to recover the *Spence-Mirrlees* single crossing property. This property ensures that formal income is increasing in productivity type θ .

Lemma 2. *Agents' preferences satisfy a strict Spence-Mirrlees single crossing condition if and only if $w^s(\theta)/w^f(\theta)$ is strictly decreasing with θ or $w^s(\theta) = 0$ for all θ .*

Proof. In [Appendix B](#). □

The single crossing requires that the comparative advantage in shadow labor is decreasing with formal productivity. This natural assumption is maintained throughout this section. In [Section 4](#) we verify that it holds for Colombia.

⁹The agents' utility function over consumption and labor, net of the fixed cost of shadow employment, is $U(c, n) \equiv c - v(n)$, where v is increasing, strictly convex, twice differentiable and satisfies $v'(0) = 0$.

¹⁰In principle, we could introduce a fixed cost of formal employment as well. This would correspond to what [Magnac \(1991\)](#) calls a segmentation approach to informal labor markets, according to which shadow workers are restricted from formal employment by various labor regulations. An alternative, competitive approach is that individuals sort between the two sectors according to their individual advantage, which corresponds more closely to our framework. [Magnac \(1991\)](#) shows that the data on married women in Colombia favor the latter, competitive approach. Similarly, [Pratap and Quintin \(2006\)](#) find no support for a cost of entry into the formal sector in Argentina. Furthermore, they show that for the workers with low productivity the informal wage is larger than the formal wage, which could be explained by the entry cost in the informal sector.

3.1. Incentive compatibility

Suppose that agents face a tax schedule T . The indirect utility an agent (θ, κ) derives from formal earnings y , given an optimal choice of shadow earnings, is

$$V(y, T, \theta, \kappa) \equiv \max_{y^s \geq 0} U \left(y + y^s - T(y), \frac{y}{w^f(\theta)} + \frac{y^s}{w^s(\theta)} \right) - \kappa \mathbb{1}_{y^s > 0}. \quad (5)$$

An *allocation* consists of an assignment of formal income to all types $y^f : [\underline{\theta}, \bar{\theta}] \times [0, \infty) \rightarrow \mathbb{R}_+$ and a tax schedule $T : \mathbb{R}_+ \rightarrow \mathbb{R}$.¹¹ An allocation (y^f, T) is *incentive-compatible* if given the tax schedule T the assignment of formal earnings y^f maximizes each agent's utility, i.e. if for all θ and κ

$$V(y^f(\theta, \kappa), T, \theta, \kappa) \geq V(y', T, \theta, \kappa) \text{ for all } y' \in \mathbb{R}_+. \quad (6)$$

We can characterize incentive-compatible allocations by focusing on two classes of agents: *low-cost workers* with no fixed cost of shadow employment ($\kappa = 0$) and *high-cost workers* with a prohibitively high fixed cost (denoted by $\kappa = \infty$). We will describe incentive-compatible formal income schedules of these agents shortly. For now, take as given the formal income schedule of the low-cost workers $y^f(\cdot, 0)$ and of the high-cost workers $y^f(\cdot, \infty)$ and suppose that they are incentive-compatible given a tax schedule T . Denote the informal earnings of the low-cost workers, implicit in the definition of their indirect utility, by $y^s(\cdot, 0)$.

Define a *formality threshold* $\tilde{\kappa}(\theta) \equiv V(y^f(\theta, 0), T, \theta, 0) - V(y^f(\theta, \infty), T, \theta, \infty)$, where for brevity we suppress the dependence on the allocation. This threshold is positive when the low-cost workers earn some shadow income and obtain a strictly higher utility than the high-cost workers of the same productivity type. Take a worker of an arbitrary type (θ, κ) . Depending on whether the cost parameter κ is above (resp. below) the formality threshold $\tilde{\kappa}(\theta)$, this agent chooses earnings like a high-cost (resp. a low-cost) worker of the same productivity type:

$$(y^f(\theta, \kappa), y^s(\theta, \kappa)) = \begin{cases} (y^f(\theta, \infty), 0) & \text{if } \kappa \geq \tilde{\kappa}(\theta) \\ (y^f(\theta, 0), y^s(\theta, 0)) & \text{otherwise.} \end{cases} \quad (7)$$

The agents with a fixed cost κ above the formality threshold $\tilde{\kappa}(\theta)$ work only formally and are called *formal workers*. The agents with a cost below the threshold supply some shadow labor (and possibly some formal labor as well) and are called *shadow workers*.

We have described an incentive-compatible assignment of formal income to all agents conditional on the formal income schedules of the low-cost and the high-cost workers. Now we will characterize the income choices of these two classes of agents. Without

¹¹ Without loss of generality we focus on tax schedules with prohibitively high values at income levels which do not belong to the image of $y^f(\cdot, \cdot)$. It rules out deviations to formal income levels which are not earned by any agent.

loss of generality we focus on formal income schedules which are right-continuous.¹² In the typical optimal taxation or screening model it is enough to restrict attention to *local incentive-compatibility*, making sure that no agent has incentives to misreport his productivity type marginally (see e.g. Fudenberg and Tirole 1991).¹³ Whereas in our setting local incentive-compatibility constraints are not sufficient for the global incentive-compatibility, as we will demonstrate soon, they are still very useful. When the allocation is locally differentiable, the local incentive-compatibility constraints can be expressed as intuitive first-order conditions with respect to formal income. Suppose that the choice of formal earnings is interior.¹⁴ The first-order condition of the high-cost worker with productivity type θ is

$$\left(1 - T' \left(y^f(\theta, \infty) \right)\right) w^f(\theta) = v' \left(\frac{y^f(\theta, \infty)}{w^f(\theta)} \right), \quad (8)$$

which means that the net marginal return to formal income is equal to the marginal disutility from higher formal earnings. If the low-cost worker of productivity type θ does not work in the shadow economy, he will choose the same earnings as his high-cost relative. If the low-cost worker instead works informally, his first-order condition additionally equalizes the marginal net return to formal and shadow earnings:

$$\left(1 - T' \left(y^f(\theta, 0) \right)\right) w^f(\theta) = v' \left(\frac{y^f(\theta, 0)}{w^f(\theta)} + \frac{y^s(\theta, 0)}{w^s(\theta)} \right) = w^s(\theta). \quad (9)$$

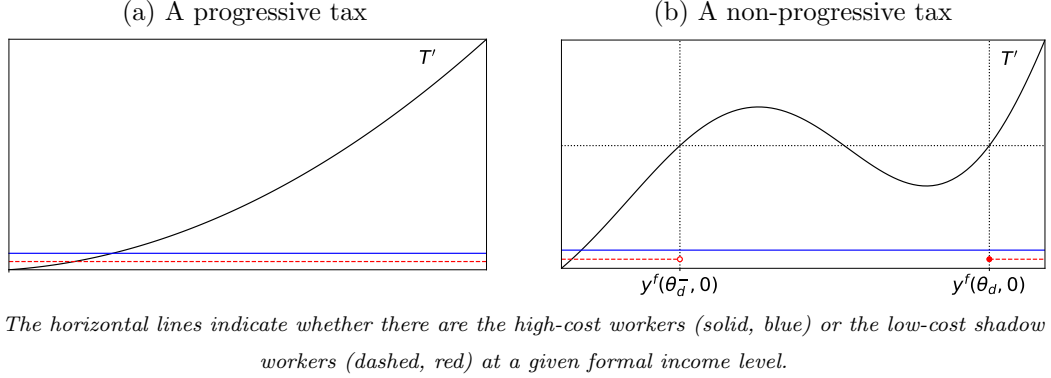
Notice that the net-of-tax rate of an agent of productivity type θ with both formal and shadow earnings is equal to his comparative advantage in shadow labor $w^s(\theta)/w^f(\theta)$. Furthermore, by the single-crossing assumption $w^s(\theta)/w^f(\theta)$ is strictly decreasing with θ . This implies that the marginal tax rate faced by shadow workers increases strictly with their productivity type. Note that the tax schedule need not be strictly progressive, i.e. with strictly increasing marginal rates. Rather, shadow workers choose formal earnings in such a way that higher productivity types always face higher marginal tax rates. If the tax schedule has a region of decreasing marginal tax rates then the low-cost workers who work informally will simply skip this income range and their formal income schedule will be discontinuous (see Figure 2).

¹²Effectively, it means that the shadow worker who indifferent between two levels of formal earnings will choose the higher level.

¹³The local incentive-compatibility imposes two requirements. First, the indirect utility $V(y^f(\theta, \kappa), T, \theta, \kappa)$ needs to be continuous with respect to the productivity type θ . Second, when the income schedule $y^f(\cdot, \kappa), \kappa \in \{0, \infty\}$, is differentiable, which happens almost everywhere, the allocation needs to satisfy $\frac{d}{d\theta'} V(y^f(\theta', \kappa), T, \theta, \kappa) \Big|_{\theta'=\theta} = 0$, which can be expressed as the first-order conditions in the main text.

¹⁴When the choice of formal earnings is not interior, then (8) and the left equality in (9) hold as \leq inequalities. It can happen at zero formal earnings or at the tax kink.

Figure 2: Continuity of $y^f(\cdot, 0)$ and tax progressivity.



Since local tax regressivity may lead to a discontinuity in the formal income schedule of the low-cost workers, we need to include the local incentive-compatibility constraint for this case. Suppose that $y^f(\cdot, 0)$ jumps discontinuously at θ_d . The local incentive-compatibility constraint ensures that the low-cost agent of type θ_d is indifferent between the two discontinuously different income levels. It implies that not only the marginal but also the *average* net returns to formal and shadow earnings coincide:

$$\left(1 - \frac{T(y^f(\theta_d, \kappa)) - T(y^f(\theta_d^-, \kappa))}{y^f(\theta_d, \kappa) - y^f(\theta_d^-, \kappa)}\right) w^f(\theta_d) = w^s(\theta_d). \quad (10)$$

In the one-dimensional taxation or screening model the local incentive-compatibility constraints together with income monotonicity requirement are sufficient for the global incentive compatibility. This result has been extended to some environments with multidimensional heterogeneity.¹⁵ Yet, it does not apply in our setting. Specifically, there exist formal income schedules of the low-cost and the high-cost workers that are increasing and satisfy the local incentive-compatibility constraints and yet violate some of the *global* incentive constraints (see the proof of [Proposition 2](#) for the graphical example). There are two reasons for this. First, the planner cannot adjust the tax schedule for high and low-cost workers independently, as they face the same tax schedule. Second, when tax rates are such that the net returns to formal and shadow labor are equal, the low-cost workers can shift labor between sectors at no cost. As a result, they can make large formal income adjustments which cannot be prevented by local constraints alone. In order to ensure incentive-compatibility, we reinforce the standard requirements with three additional non-local incentive-compatibility constraints.

Proposition 2. *An allocation (y^f, T) is incentive-compatible if, and only if,*

¹⁵[Kleven et al. \(2009\)](#), [Scheuer \(2014\)](#) and [Gomes et al. \(2017\)](#) recover sufficiency of local incentive constraints in two-dimensional settings with two sectors, under the assumption that agents can work in one sector at a time. [Rothschild and Scheuer \(2014\)](#) allow workers to supply labor in multiple sectors simultaneously, but the government observes and taxes the sum of all incomes, which implies that the local incentive constraints are sufficient.

1. $y^f(\cdot, \infty)$ and $y^f(\cdot, 0)$ are increasing and satisfy local incentive-compatibility constraints.
2. $y^f(\theta, \cdot)$ is consistent with the formality threshold and satisfies (7) for all θ .
3. The type (θ, ∞) cannot gain by deviating to any lower formal income.
4. The type $(\bar{\theta}, 0)$ cannot gain by deviating to any higher formal income.
5. Suppose that $y^f(\cdot, 0)$ is discontinuous at θ_d . The type $(\theta_d, 0)$ cannot gain by deviating to any formal income from the interval $(y^f(\theta_d^-, 0), y^f(\theta_d, 0))$.

Proof. In [Appendix B](#). □

The intuition behind the proof is following. The local incentive-compatibility constraints prevent deviations *within* their cost class (the class of either the high-cost or the low-cost workers) *between* different productivity types. The formality threshold prevents deviation *within* the productivity type *between* different cost types. Additional constraints are required to prevent simultaneous deviations between the cost types and the productivity types. Many such deviations are already covered by the local incentive-compatibility constraints, since the images of the two formal income schedules are partially overlapping. Therefore, we need to focus only on deviations to formal income levels which are earned by some high-cost (low-cost) workers but by no low-cost (high-cost) worker. We capture these deviations by imposing non-local incentive constraints for the least productive high-cost agent, the most productive low-cost agent and at each discontinuity point of $y^f(\cdot, 0)$.

Whereas multidimensional screening problems where the local incentive-compatibility constraints are insufficient are notorious for intractability, that is not the case with our model. The non-local incentive-compatibility constraint 3 can be verified ex post and - if it is violated - incorporated directly to the planner's problem, in a manner analogue to the monotonicity requirement in the standard one-dimensional problem. The non-local constraints 4 and 5 are even less problematic as they do not affect the optimal tax formulas.¹⁶ Nevertheless, these constraints become important in the numerical application of the optimal tax formulas.

3.2. The planner's problem

The social planner maximizes the average of individual utilities, weighted with Pareto weights $\lambda(\theta, \kappa)$. We normalize the weights such that $\mathbb{E}\{\lambda(\theta, \kappa)\} = 1$ which implies that

¹⁶When deriving the optimal tax rates we show that the fiscal cost of intensive margin responses of shadow workers is independent of the magnitude of formal income adjustment: for the government budget constraint it does not matter whether the shadow workers adjust formal income marginally or jump to a discreetly lower formal income level. Given that we do not need to keep track of the exact formal income responses of the shadow workers, we can derive the optimal tax formulas ignoring constraints 4 and 5.

the Pareto weights and the marginal social welfare weights coincide.¹⁷ The planner solves

$$\max_{\substack{y^f : [\underline{\theta}, \bar{\theta}] \times [0, \infty) \rightarrow \mathbb{R}_+ \\ T : \mathbb{R}_+ \rightarrow \mathbb{R}}} \int_{\underline{\theta}}^{\bar{\theta}} \int_0^\infty \lambda(\theta, \kappa) V(y^f(\theta, \kappa), T, \theta, \kappa) dG_\theta(\kappa) dF(\theta) \quad (11)$$

subject to the incentive-compatibility constraints from [Proposition 2](#) and the budget constraint

$$\int_{\underline{\theta}}^{\bar{\theta}} \int_0^\infty T(y^f(\theta, \kappa)) dG_\theta(\kappa) dF(\theta) \geq E, \quad (12)$$

where E stands for exogenous government expenditures. By solving the planner's problem for arbitrary Pareto weights, we recover the entire Pareto frontier of the model without wealth effects.¹⁸

We proceed with the theoretical analysis under the standard assumption that the monotonicity constraints on formal income schedules are not binding, which means that there is no bunching along the productivity dimension alone. We rule out this bunching pattern because it is well understood from the one-dimensional models ([Mussa and Rosen 1978](#); [Ebert 1992](#)) and it happens rarely.¹⁹ Crucially, we allow for all other bunching patterns, in particular the bunching of agents with simultaneously different cost and productivity types which happens when there are formal and shadow workers at the same formal income level. We also assume that the non-local incentive-compatibility constraint 3 from [Proposition 2](#) is not binding. We verify ex post that both assumptions are true in all our quantitative exercises. Non-local incentive-compatibility constraint 4 and 5 do not affect the optimal tax formulas and as such are relevant only for computing the optimal tax schedules.

3.3. Derivation of the optimal tax formulas

So far we have stated the problem of finding the optimal tax schedule using the mechanism design approach. It is instructive, however, to think about it in terms of tax perturbations as in [Saez \(2001\)](#). In this section we will derive the optimal tax formulas

¹⁷The marginal social welfare weights describe the welfare impact of marginally increasing consumption of a given type, expressed in units of tax revenue (see e.g. [Piketty and Saez 2013](#)). In our environment they are simply $\lambda(\theta, \kappa)/\eta$, where η is the multiplier of the budget constraint. It is easy to show that at the optimum $\eta = \mathbb{E}\{\lambda(\theta, \kappa)\}$.

¹⁸Suppose that the planner follows the social welfare function $\int_0^1 \int_0^\infty \Gamma(V(\theta, \kappa)) dG_\theta(\kappa) dF(\theta)$, where Γ is an increasing and differentiable function. Γ is typically assumed to be strictly concave and it can represent either decreasing marginal utility of consumption or the planner's taste for equality. We find the optimal allocation in this case by setting the Pareto weights in the planner's problem according to $\lambda(\theta, \kappa) = \Gamma'(V(\theta, \kappa))$, where V is the indirect utility function at the optimum. In this case the Pareto weights are endogenous, since they explicitly depend on the optimal allocation, and the model needs to be solved iteratively: in each iteration the Pareto weights are updated to reflect the indirect utility function implied by the previous solution of the model.

¹⁹This type of bunching is more important in the setting without the fixed cost of shadow employment and we study it in detail in the earlier working paper version ([Doligalski and Rojas 2016](#)).

with the tax perturbation approach by considering a small variation of the marginal tax rate at some formal income level. In Online Appendix A we derive the tax formulas using mechanism design and provide the exact correspondence between the two approaches.

From now on we will focus on the endogenous distribution of formal income. Denote the density of formal income by $h(\cdot)$. We can decompose it into the density of formal workers $h^f(\cdot)$ and the density of shadow workers $h^s(\cdot)$, such that at each income level y we have $h(y) = h^f(y) + h^s(y)$.²⁰ Take some incentive-compatible allocation with twice-differentiable tax schedule T and perturb the marginal tax rate in the formal income interval $[y, y + dy]$ by a small $d\tau > 0$. This perturbation influences tax revenue via: (i) intensive margin responses of formal and shadow workers, (ii) extensive margin responses due to workers changing their informality status, (iii) mechanical and welfare effects.

Intensive margin responses of formal and shadow workers. In response to the increase in the marginal tax rate, the agents with income y or slightly higher will reduce their formal earnings. The income reduction of formal workers is standard and equal approximately to

$$h^f(y)\tilde{\varepsilon}^f(y)y\frac{d\tau dy}{1 - T'(y)}, \quad \text{where } \tilde{\varepsilon}^f(y) \equiv \left(\frac{1}{\varepsilon(y)} + \frac{T''(y)y}{1 - T'(y)} \right)^{-1}. \quad (13)$$

$\tilde{\varepsilon}^f(y)$ is the elasticity of formal income of formal workers with respect to the marginal tax rate along the non-linear tax schedule. It depends both on $\varepsilon(y)$, the elasticity along the *linear* tax schedule, and the local tax curvature. For instance, the typical isoelastic disutility of labor $v(n) = n^{1+1/\epsilon}$ implies $\varepsilon(y) = \epsilon$. When the tax schedule is non-linear, the income responses depend not only on this structural elasticity $\varepsilon(y)$, but also on the local progressivity of the tax schedule. In particular, when the tax is locally strictly progressive ($T''(y) > 0$), an income increase in response to a tax rate cut is reduced, as a higher income leads to a higher tax rate. Hence, the local progressivity (regressivity) of the tax schedule reduces (increases) the elasticity of income.

What are the formal income responses of shadow workers? Suppose that the perturbation triggers a formal income adjustment of shadow workers with formal earnings $s(y)$.²¹ In particular, suppose that $s(y) = y$, which means that there are some shadow workers with formal income y . In Online Appendix A we show that the reduction of formal income of shadow workers is equal to

$$h^s(y)\tilde{\varepsilon}^s(y)y\frac{d\tau dy}{1 - T'(y)}, \quad \text{where } \tilde{\varepsilon}^s(y) \equiv \frac{1 - T'(y)}{T''(y)y} > \tilde{\varepsilon}^f(y). \quad (14)$$

The elasticity of the formal income of *shadow* workers is strictly greater than that of

²⁰Formally, $h^f(\cdot)$ and $h^s(\cdot)$ are not densities as they integrate to shares of formal and shadow workers in total employment, respectively, rather than to 1. Keeping this slight abuse of terminology in mind, we will continue calling them densities.

²¹One can show that $s(y) \equiv \min_{\theta} \{y^f(\theta, 0) \text{ s.t. } y^f(\theta, 0) \geq y\}$.

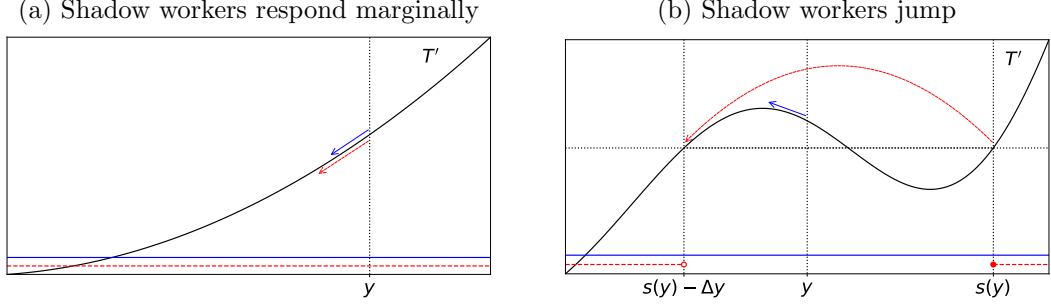
formal workers: $\tilde{\varepsilon}^s(y) > \tilde{\varepsilon}^f(y)$. This is because the elasticity of shadow workers along the *linear* tax schedule is infinite. Suppose that a shadow worker of type θ faces a linear tax with tax rate $1 - w^s(\theta)/w^f(\theta)$. This agent is indifferent between supplying formal and shadow labor and the first-order condition (9) pins down only the total labor supply but not its sectoral split. Suppose that the tax rate is increased marginally. Now the return to shadow labor is strictly greater than the return to formal labor. Thus, the agent shifts the entire labor supply to the shadow economy and reduces his formal income all the way to zero. This dramatic reduction of formal income means that the elasticity of formal income of shadow workers along the *linear* tax schedule is infinite. In contrast, if the tax schedule were non-linear and locally strictly progressive ($T''(y) > 0$), the shadow worker would reduce his formal income only until the marginal tax rate is again equal $1 - w^s(\theta)/w^f(\theta)$, which implies a finite elasticity.²²

So far we considered the situation in which there are some shadow workers with formal income y at which we perturb the marginal tax rate. Since the income schedule of low-cost shadow workers can become discontinuous when the tax schedule is not progressive everywhere, we need to examine the case in which there are no low-cost workers at y , but there are some with strictly higher formal income. Suppose that $s(y)$, a formal income level at which shadow workers respond on the intensive margin to perturbation, is greater than y . This happens when the formal income schedule of the low-cost workers is discontinuous and there are no low-cost workers at the formal income interval $(s(y) - \Delta y, s(y))$, where $s(y) - \Delta y \leq y \leq s(y)$. By the local incentive-compatibility constraint (10) the shadow worker with formal earnings $s(y)$ is exactly indifferent between earning $s(y)$ and $s(y) - \Delta y$. Consider an increase in $T'(y)$. As the tax burden at $s(y)$ increases, the agent strictly prefers $s(y) - \Delta y$ to $s(y)$ and jumps to the lower level of earnings.

Figure 3 illustrates the two types of formal income responses of shadow workers. On the left panel, the formal income schedule of the low-cost shadow workers is locally continuous at y and the tax schedule is locally strictly progressive. Consequently, the shadow workers respond to an increase of $T'(y)$ by marginally reducing their formal income. On the right panel, the formal income schedule of the shadow workers is discontinuous. In the response to an increase in $T'(y)$ the shadow worker discretely jumps to a lower formal income level.

²²Since the second-order optimality condition of shadow workers is $T''(y) \geq 0$, we do not need to consider a locally strictly regressive tax. No shadow worker would choose such a formal income level.

Figure 3: Formal income responses to the increase of $T'(y)$.



The horizontal lines indicate whether there are the high-cost workers (solid, blue) or the low-cost shadow workers (dashed, red) at a given formal income level. The arrows represent the formal income responses of the high-cost workers (solid, blue) and the low-cost, shadow workers (dashed, red).

When the formal income schedule of shadow workers is discontinuous, formal income responses of shadow workers to a marginal change in tax rates are non-marginal, discrete. Surprisingly and very conveniently, they can be still described with the intensive margin elasticity $\tilde{\varepsilon}^s(\cdot)$. The perturbation increases the tax burden at $s(y)$ by $d\tau dy$ and makes some shadow workers discretely decrease their formal income from $s(y)$ to $s(y) - \Delta y$. The measure of shadow workers at income level $s(y)$ that decides to jump is given by $h^s(s(y))ds(y)$. By differentiating (10) we obtain $ds(y) = [T''(s(y))\Delta y]^{-1}d\tau dy$.²³ Therefore, the overall income reduction is exactly as in the case when shadow workers adjust income marginally and is independent of the size of the income jump Δy :

$$\Delta y h^s(s(y))ds(y) = h^s(s(y))\tilde{\varepsilon}^s(s(y))s(y) \frac{d\tau dy}{1 - T'(s(y))}, \quad (15)$$

The intuition is that, although each jumping individual makes a non-marginal income reduction Δy , the measure of jumping individuals is inversely proportional to the size of the reduction. As a result, the overall income reduction is independent of Δy and such that the elasticity at $s(y)$ is finite and exactly the same as if the shadow workers adjusted income marginally.

Therefore, when there are some low-cost workers with formal income above y , we can express the tax revenue impact of the intensive margin responses of formal and shadow workers, no matter whether they are responding marginally or jumping, as

$$- \left(\frac{T'(y)}{1 - T'(y)} h^f(y) \tilde{\varepsilon}^f(y) y + \frac{T'(s(y))}{1 - T'(s(y))} h^s(s(y)) \tilde{\varepsilon}^s(s(y)) s(y) \right) d\tau dy. \quad (16)$$

In the remaining case when there are no low-cost workers at or below y the second term in the bracket is set to zero.

²³Denote $s(y) - \Delta y$ as \tilde{y} . We can rewrite (10) as $(s(y) - \tilde{y})T'(s(y)) - (T(s(y)) - T(\tilde{y})) = 0$. Perturb the tax level at $s(y)$ by $d\tau dy$. By totally differentiating this equation we find that $\Delta y T''(s(y))ds(y) - d\tau dy = 0$.

Extensive margin responses. Let's define the formal income gap between the high-cost and the low-cost workers in two ways. $\Delta_\infty(y')$ tells us by how much the formal worker with income y' would decrease his formal income if he had a lower realization of the fixed cost and worked informally. $\Delta_0(y')$ tells us by how much the shadow worker with formal income y' would increase his formal earnings if he did not work in the shadow economy.

The perturbation of $T'(y)$ increases the tax burden for workers with incomes above y . Consequently, it increases incentives for informality for agents who, conditional on working informally, would earn less than y in the formal sector. Suppose that there are some low-cost workers with formal income above y . The perturbation of $T'(y)$ increases incentives for informality for formal workers in the income interval $(y, s(y + dy) + \Delta_0(s(y + dy)))$. Workers with income below y are unaffected, since their tax schedule is unchanged. Workers with income above $s(y + dy) + \Delta_0(s(y + dy))$ pay taxes higher by $d\tau dy$ no matter whether they stay formal or move to the shadow economy, so their incentives for informality are unchanged as well. In the following derivations we focus on a subinterval $[y + dy, s(y) + \Delta_0(s(y))]$, since the terms corresponding to the remaining parts of the original interval are of second order (i.e. they are proportional to dy^2) and vanish as we consider an arbitrarily small dy .

Denote the impact of the perturbation on the density of formal workers at income y' by $dh^f(y')$. Also denote the *tax burden of staying formal* at formal income by $\Delta T(y') \equiv T(y') - T(y' - \Delta_\infty(y'))$. It captures the tax revenue loss when a formal worker with income y starts supplying informal labor. The tax revenue impact of the perturbation via the adjustment of the distribution of formal workers is

$$\int_{y+dy}^{s(y)+\Delta_0(s(y))} dh^f(y') \Delta T(y') dy' d\tau dy = - \int_{y+dy}^{s(y)+\Delta_0(s(y))} \pi(y') h^f(y') dy' d\tau dy, \quad (17)$$

where $\pi(y')$ is the elasticity of the density of formal workers at y' with respect to the tax burden of staying formal. Intuitively, the more elastic is the density of formal workers, the higher is the tax revenue loss due to increased participation in the shadow economy.

In the case when all the low-cost workers have formal incomes below y , all shadow workers have formal incomes below y . Consequently, the tax perturbation increases incentives for informality at all formal income levels above $y + dy$, rather than only up to $s(y) + \Delta_0(s(y))$.

Mechanical and welfare impact. Consider the tax schedule at incomes above $y + dy$. The perturbation keeps the tax rate fixed, while increasing the tax level by $d\tau dy$. On the one hand, an increase in the tax level mechanically raises the tax revenue. On the other hand, it reduces utility of agents with higher incomes, resulting in a welfare loss. Denote the average Pareto weight at a given formal income level y by $\bar{\lambda}(y)$. Ignoring the

second-order terms, the combined mechanical and welfare impact of the perturbation is

$$\int_{y+dy}^{\infty} (1 - \bar{\lambda}(y')) h(y') dy' d\tau dy. \quad (18)$$

Optimal tax formulas. Optimality requires that no small tax perturbation can increase welfare-adjusted tax revenue. Hence, the sum of all the impacts of the tax perturbation: (16), (17) and (18), needs to be zero for any $d\tau$ and an arbitrary small dy .

Theorem 1. *Suppose that the bunching along the productivity dimension alone does not occur. When some low-cost workers have formal income greater than or equal to y , the optimal tax rate satisfies*

$$\begin{aligned} & \frac{T'(y)}{1 - T'(y)} h^f(y) \tilde{\varepsilon}^f(y) y + \frac{T'(s(y))}{1 - T'(s(y))} h^s(s(y)) \tilde{\varepsilon}^s(s(y)) s(y) \\ &= \int_y^{\infty} [1 - \bar{\lambda}(y)] h(y) dy - \int_y^{s(y) + \Delta_0(s(y))} \pi(y') h^f(y') dy'. \end{aligned} \quad (19)$$

When all the low-cost workers have formal income below y , the optimal tax rate satisfies

$$\frac{T'(y)}{1 - T'(y)} h^f(y) \tilde{\varepsilon}^f(y) y = \int_y^{\infty} [1 - \bar{\lambda}(y') - \pi(y')] h(y') dy'. \quad (20)$$

Tax formula (19) equates the *deadweight loss* from distorting the formal workers and the shadow workers on the left-hand side, with the *tax revenue gain* from higher tax on formal incomes above y net of the tax loss from increased participation in the shadow economy on the right-hand side. The deadweight loss of both formal and shadow workers increases in (i) the marginal tax rate, as the reduction in formal income implies a higher tax loss if it is taxed at the higher rate, (ii) the density of formal income and (iii) the formal income reduction per worker in response to a higher tax rate, i.e. the product of formal income and the elasticity at this income level. There are two important differences between the deadweight loss terms of formal and shadow workers. First, unlike the distorted formal workers, the distorted shadow workers may have formal income that is strictly higher than y . Second, conditional on the local progressivity of the tax schedule, the formal income of shadow workers is more elastic than the income of formal workers. The second tax formula (20) captures the case when no low-cost workers and, hence, no shadow workers are distorted by the tax rate perturbation.

The tax revenue gain in formulas (19) and (20) consists of two terms. The first one summarizes the mechanical and welfare impact from increased taxation of all workers with higher formal income. A perturbation in the marginal tax rate increases their taxes and reduces welfare proportionally to their Pareto weights. The second term captures the tax revenue cost of increased participation in the shadow economy. Note that in

formula (19) the participation in the shadow economy of workers with formal income above $s(y) + \Delta_0(s(y))$ is unchanged. If these workers decided to work in the shadow economy, their formal income would be high enough such that they would still pay higher taxes. Thus, they have no additional incentives for informality. In contrast, when formula (20) applies, the perturbation of the marginal tax rate increases incentives for informality for all workers with higher formal income. That is because all affected agents, if they worked informally, would have formal income below the level at which the tax rate is perturbed.

3.4. How does a shadow economy affect optimal tax rates?

We examine the impact of a shadow economy on the optimal tax rates in two ways. First, we fix the formal income distribution and other sufficient statistics and compare the tax schedules implied by the optimal tax formulas with a shadow economy, given by equations (19) and (20), and by the standard tax formulas which do not explicitly address informality. In this comparison we implicitly allow the model primitives - the productivity and the fixed cost distributions - to differ between different tax formulas, since they need to give rise to the same formal income distribution. This comparison is most informative for choosing tax policy based on the given, observed formal income distribution.²⁴ Second, we compare the optimal top tax rate with and without a shadow economy for given model primitives while allowing the formal income distribution to adjust. This comparison is useful for the counterfactual analysis: it informs us how the optimal top tax rate would change if we could costlessly shut down the informal sector.

3.4.1. Comparison for a fixed formal income distribution

Take as given the formal income distribution, the schedule of average Pareto weights at each formal income level, intensive and extensive margin elasticities and other sufficient statistics required to compute the optimal tax rates with the shadow economy. We will compare the tax formulas from Theorem 1 with the tax formula of Diamond (1998) ('Diamond formula')²⁵ from the model with the intensive margin of labor supply only and the tax formula of Jacquet et al. (2013) ('Jacquet et al. formula') with the intensive and the participation margins of labor supply. The Diamond formula is

$$\frac{T'_D(y)}{1 - T'_D(y)} h(y) \bar{\varepsilon}(y) y = \int_y^\infty [1 - \bar{\lambda}(y')] h(y') dy', \quad (21)$$

²⁴ Scheuer and Werning (2017) conduct a similar comparison in the model with superstar effects. They focus on the upper bound of the constrained efficient tax rates, which is equivalent to assuming that the social planner has Rawlsian preferences. We instead allow for arbitrary social preferences, which are captured by the schedule of average Pareto weights at each income level.

²⁵ Diamond (1998) derived the optimal tax formula in the Mirrlees model without wealth effects. Saez (2001) expressed the optimal tax formula with sufficient statistics.

where $\bar{\varepsilon}(y)$ is the average formal income elasticity at the formal earnings y . Hence, the Diamond formula accounts only for the intensive margin responses of formal earnings at the income level where the tax rate is perturbed. The more general Jacquet et al. formula is

$$\frac{T'_J(y)}{1 - T'_J(y)} h(y) \bar{\varepsilon}(y) y = \int_y^\infty [1 - \bar{\lambda}(y')] h(y') dy' - \int_y^\infty \pi(y') \mathbb{1}_{\Delta_\infty(y')=y'} h^f(y') dy'. \quad (22)$$

where $\pi(y') \mathbb{1}_{\Delta_\infty(y')=y'}$ is the elasticity of formal labor market participation at formal income y' with respect to the tax level $T_J(y')$. The indicator function makes sure that only the extensive margin responses which reduce formal earnings to zero are accounted for.²⁶ Hence, the Jacquet et al. formula accounts for the intensive margin responses at formal earnings at the income level where the tax rate is perturbed and formal participation responses at higher income levels.

Proposition 3. *Fix the schedule of average Pareto weights $\bar{\lambda}(\cdot)$, the distribution of formal income $h^f(\cdot)$ and $h^s(\cdot)$, intensive margin elasticities $\bar{\varepsilon}^f(\cdot)$ and $\bar{\varepsilon}^s(\cdot)$, extensive margin elasticities $\pi(\cdot)$, formal income gaps $\Delta_\infty(\cdot)$ and $\Delta_0(\cdot)$ and the mapping $y \mapsto s(y)$. Suppose that (i) bunching along the productivity dimension alone does not occur and (ii) $\pi(y) \geq 0$ and $T'(y) > 0$ for all y . Then $T'(y) \leq T'_J(y) \leq T'_D(y)$.*

Proof. In [Appendix B](#). □

Once we fix the schedule of Pareto weights, the formal income distribution and all other sufficient statistics, we obtain a clear ordering of the tax formulas. At each income level the optimal marginal tax rates are below the Jacquet et al. tax rates, which in turn are below the Diamond tax rates. The intuition is simple. The optimal tax formula with a shadow economy correctly incorporates the entire fiscal cost of raising tax rates at the given income level. The Jacquet et al. formula instead is missing some of the intensive margin responses (the responses of shadow workers that happen at a higher formal income level) and some of the extensive margin responses (the responses after which a worker maintains some formal income) and, hence, prescribes tax rates which are (weakly) too high. The Diamond formula is also missing some of the intensive margin responses and, in addition, does not account for any extensive margin responses. As a result, the Diamond tax rates are (weakly) higher than these implied by the other two formulas. Note that at some income levels the three tax rates may coincide. It happens always when all workers of a given productivity type are formal (i.e. when $\Delta_\infty(y) = 0$), since then the tax perturbation triggers no intensive margin responses of shadow workers nor any extensive margin responses.

²⁶It is easy to show that $y' > s(y) + \Delta_\infty(s(y))$ implies that $\Delta_\infty(y') < y'$. Therefore, the effective range over which the extensive margin responses are integrated is not larger in the Jacquet et al. formula than in the formula (19).

3.4.2. Comparison for fixed primitives

Let's take as given model primitives: the distribution of productivity and cost types, the productivity schedules and the schedule of Pareto weights. In the following proposition we compare the optimal top tax rate with a shadow economy $T'(\infty)$ with the Mirrleesian top tax rate $T'_M(\infty)$, i.e. the optimal top rate when all shadow productivities are set to zero. In contrast to the previous comparison, here we allow the formal income distribution and all the other sufficient statistics to endogenously adjust to the top tax rate. Let's first determine how a top tax rate influences the shape of the upper tail of the formal income distribution with a shadow economy.

Lemma 3. *Suppose that (i) the formal productivity distribution has a Pareto tail: $\lim_{\theta \rightarrow \bar{\theta}} \frac{f(\theta)w^f(\theta)}{1-F(\theta)} \left(\frac{dw^f(\theta)}{d\theta} \right)^{-1} = \alpha$, (ii) the fixed cost of shadow employment has a Pareto tail: $\forall \theta \lim_{\kappa \rightarrow \infty} \frac{\kappa g_\theta(\kappa)}{1-G_\theta(\kappa)} = \gamma$, (iii) the primitive labor elasticity is constant and equal to ε . Denote the tail parameter of the formal income distribution by $\alpha_y \equiv \lim_{y \rightarrow \infty} \frac{h(y)y}{1-H(y)}$. The tail parameter satisfies*

$$\alpha_y = \begin{cases} \frac{\alpha}{1+\varepsilon} & \text{if } 1 - T'(\infty) \geq w^s(\bar{\theta})/w^f(\bar{\theta}), \\ \frac{\alpha}{1+\varepsilon} + \gamma & \text{otherwise.} \end{cases} \quad (23)$$

Proof. In [Appendix B](#). □

The tail parameter α_y describes the *thinness* of the upper tail of the formal income distribution. When the top tax rate is sufficiently low, none of the most productive types work in the shadow economy and the thinness of the formal income tail is exactly the same as in the standard Mirrlees model. As soon as the top tax rate crosses a tipping point $1 - w^s(\bar{\theta})/w^f(\bar{\theta})$ a positive fraction of top earners moves to the shadow economy. As a result, the thinness of the upper tail increases discretely by γ , the tail parameter of the fixed cost distribution. Intuitively, if γ is high, there are many workers with a low fixed cost of shadow employment who reduce their formal income and join the shadow economy. If instead γ is low, there are few workers with a low fixed cost of shadow employment and the formal income distribution is less affected.

Proposition 4. *Suppose that the assumptions of [Lemma 3](#) hold and additionally the Pareto weight $\lambda(\theta, \kappa)$ converges to $\lambda \in [0, 1)$ as $\theta \rightarrow \bar{\theta}$ for all κ . Then $T'(\infty) \leq T'_M(\infty)$. In particular, there exists a threshold $\tilde{\gamma} > 0$ such that*

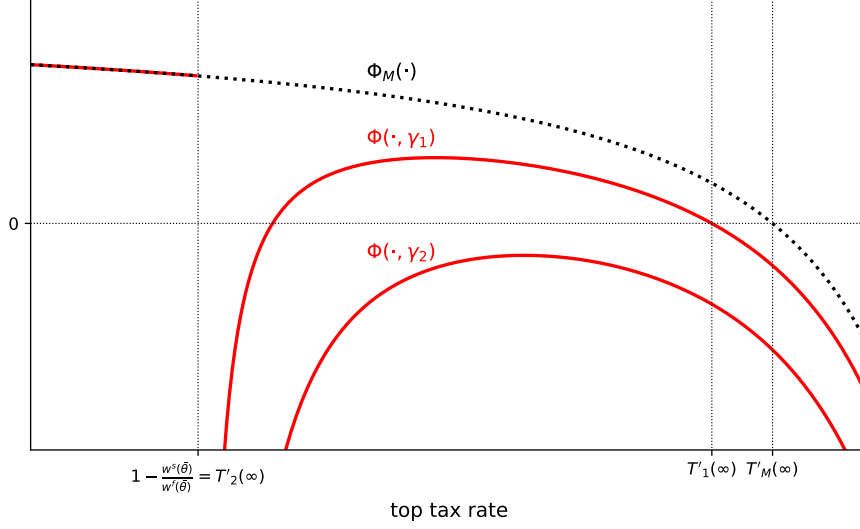
$$1 - T'(\infty) = \begin{cases} 1 - T'_M(\infty) & \text{if } 1 - T'_M(\infty) \geq w^s(\bar{\theta})/w^f(\bar{\theta}), \\ w^s(\bar{\theta})/w^f(\bar{\theta}) & \text{if } 1 - T'_M(\infty) < w^s(\bar{\theta})/w^f(\bar{\theta}) \text{ and } \gamma \geq \tilde{\gamma}, \\ \frac{\alpha\varepsilon/(1+\varepsilon) + \gamma\varepsilon}{1 - \lambda + \alpha\varepsilon/(1+\varepsilon) + \gamma\varepsilon - \gamma\delta(T'(\infty))} & \text{if } 1 - T'_M(\infty) < w^s(\bar{\theta})/w^f(\bar{\theta}) \text{ and } \gamma \leq \tilde{\gamma}, \end{cases} \quad (24)$$

where $1 - T'_M(\infty) \equiv \frac{\alpha\varepsilon/(1+\varepsilon)}{1 - \lambda + \alpha\varepsilon/(1+\varepsilon)}$ and $\delta(T'(\infty)) > 0$.

Proof. In [Appendix B](#). □

The shadow economy leads to a (weakly) lower optimal top tax rate, conditional on other primitives of the economy. First, suppose that the Mirrleesian top tax rate is below the tipping point $1 - w^s(\bar{\theta})/w^f(\bar{\theta})$. Then the optimal rate with a shadow economy and the Mirrleesian rate coincide. Second, suppose the opposite: the Mirrleesian rate is above the tipping point and would push some top productivity types to the shadow economy (this case is illustrated in [Figure 4](#)). From [Lemma 3](#) we know that even a marginal increase of the top tax rate above the tipping point entails a non-marginal fiscal cost, as a thinness of the upper tail of formal income distribution is increased by γ . In [Figure 4](#) it is represented as a discontinuous drop of the marginal social benefit of increasing the top tax rate at the tipping point. Hence, when γ is sufficiently large the top tax rate is optimally set exactly at the tipping point $1 - w^s(\bar{\theta})/w^f(\bar{\theta})$, i.e. at the highest level which does not give incentives for informality at the top. In contrast, when γ is relatively low, the benefits of higher tax rate dominate the cost of the increased thinness of the income tail and some top workers will optimally work in the shadow economy. The optimal rate still falls short of the Mirrleesian rate for two reasons. First, since the upper tail of formal income distribution is thinner, the gains from increasing the top tax rate are reduced (the terms $\gamma\varepsilon$ in the numerator and the denominator). Second, increasing the top tax rate is more costly due to the top productivity types who respond on the extensive margin and join the shadow economy (the term $-\delta(T'(\infty))$ in the denominator).

Figure 4: Determining the optimal top tax rate



$\Phi_M(\tau)$ is the marginal social benefit of increasing the top tax rate from the level τ in the standard Mirrlees model. $\Phi(\tau, \gamma)$ is marginal social benefit of increasing the top tax rate in the model with a shadow economy when the distribution of the fixed cost has a tail parameter γ . We consider two values of the tail parameter of the cost distribution: γ_1 and γ_2 , $\gamma_2 > \tilde{\gamma} > \gamma_1$, where $\tilde{\gamma}$ is a threshold from [Proposition 4](#). $T'_k(\infty)$ is the optimal top tax rate with γ_k , $k \in \{1, 2\}$.

4. Quantitative analysis

In this section we explore the quantitative importance of our theoretical results. We estimate the model with a continuum of types using the household survey from Colombia. Colombia is a good representative of Latin America, a region with high informality. Using the estimated model, we first compare the allocations implied by the optimal tax formula and the standard tax formulas. Second, we examine the welfare impact of the Colombian shadow economy. Additional quantitative results are available in Online Appendix B, where we conduct the test of Pareto efficiency of the actual tax schedule in Colombia.

4.1. Estimation

Although we have expressed the optimal tax rates in terms of sufficient statistics, some of these statistics are very local in nature. Shadow workers in particular are very responsive to the shape of the tax schedule. As a result, the density of shadow workers at some formal income level, even if reliably estimated, is of limited use unless we know exactly how it changes as we vary the income tax. To overcome this obstacle, we follow the suggestion of [Chetty \(2009\)](#) and estimate the structural model to extrapolate the values of sufficient statistics out of sample.

We estimate the continuum of types model using the survey data from Colombia. Whereas our estimation strategy can be applied to a wide set of countries, we focus

a region with high informality which is sufficiently developed to use a non-linear income tax and transfer schedule: Latin America. Among the Latin American countries Colombia is a very attractive candidate since its informality rate is very close to the mean and the median for the whole region.²⁷

Below we explain how we identify informality in the data and introduce our estimation strategy. The detailed descriptions of the data and of the estimation procedure are provided in [Appendix C](#).

Identifying informality. We identify the main job of a given worker as informal if the worker reports not contributing to the mandatory social security programs. Since social security contributions are paid jointly with payroll taxes and the withheld part of the personal income tax, a worker who contributes to the social security is automatically subject to income taxation. Thus, this approach is particularly well suited for our exercise.²⁸ We find that 58% of all workers in Colombia in 2013 were employed informally at the main job, a result consistent with other indicators of informality in Colombia.²⁹ The average wage in the informal sector is about half of the average wage in the formal sector and the distribution of wages in the two sectors overlap significantly (see [Figure 10](#) in [Appendix C](#)).

Sample selection. We restrict attention to individuals aged 24-50 years without children (34,000 individuals). We choose this sample, since these workers face a tax and transfer schedule which is not means-tested and does not depend on choices absent from our modeling framework, such as number of children or college attainment.

Estimation strategy. The main challenge in estimating the model is identifying the joint distribution of formal and shadow productivities. For each worker we observe the hourly wage at the main job, which we interpret as productivity, and a sector of the main job, which can be either formal or informal. Crucially, we do not observe the counterfactual productivity in the sector in which the worker *is not employed* at the main job. [Heckman and Honore \(1990\)](#) and [French and Taber \(2011\)](#) show that the data on wages and the sector in which workers' participate is in general not sufficient to identify the sectoral productivity profiles, since workers self-select to a sector in which they are

²⁷Based on [ILO \(2018\)](#), the national share of informal employment in total employment in Latin America has a mean of 58.3% and a median of 59%, while it is equal to 60.6% in Colombia. This result differs slightly from our estimate of the size of the informal sector due to a different time period considered.

²⁸Detecting informality via social security contributions is broadly consistent with the methodology of the International Labour Organization ([ILO 2013](#)) and is used by the Ministry of Labor of Colombia ([ILO 2014](#)), as well as by [Mora and Muro \(2017\)](#) and [Guataquí, García, and Rodríguez \(2010\)](#) in the studies of Colombia.

²⁹The official statistical agency of Colombia (DANE) follows an alternative measure of informality based on size of the establishment, status in employment and educational level of workers. They find that 57.3% and 56.7% of workers were informal in the first two quarters of 2013 ([ILO 2014](#)), which is very close to 58% we find for the entire 2013.

more productive. Heckman and Honore (1990) prove that the model can be identified with additional regressors which influence wages. We follow this approach. Denote the vector of regressors, which includes worker's and job's characteristics, by X .³⁰ We assume that X is informative about the worker's productivity type: $\theta \sim N(X\beta, \sigma_\theta^2)$, where a vector β and a scalar σ_θ are parameters to be estimated. This assumption allows us to match similar individuals who, due to different realizations of the fixed cost of shadow employment, ended up having the main job in different sectors. Given that, we can infer a counterfactual shadow productivity of each formal worker from the observed shadow productivity of the matched shadow workers, and *vice versa*.

Additionally, we assume that (i) the sectoral log-productivity schedules $\log w^f(\cdot)$ and $\log w^s(\cdot)$ are affine with respect to the productivity type, (ii) the fixed cost of shadow employment κ is drawn from a generalized Pareto distribution, the parameters of which are allowed to vary with the productivity type θ , (iii) disutility from labor is given by $v(n) = \Gamma \frac{n^{1+1/\varepsilon}}{1+1/\varepsilon}$, implying a constant intensive margin labor elasticity ε which we fix to 0.33 following Chetty (2012). The support of the productivity type $[\theta, \bar{\theta}]$ is normalized to $[0, 1]$. We obtain the density of the productivity type $F(\theta)$ with kernel density estimation and we fit a Pareto tail to the distribution of top wages. Given these assumptions, we formulate the likelihood function and estimate the model using maximum likelihood. The likelihood function and parameter estimates are available in Appendix C.

Estimation results. The left panel of Figure 5 presents the estimated productivity profiles and the density of productivity types. The bottom 25% of workers are more productive in the shadow sector while the median worker is 6% more productive formally. We find that the comparative advantage in the shadow economy decreases with the productivity type.³¹ Thus, as assumed in the theoretical analysis, the single crossing condition holds. The density of productivity types in the main part of the distribution is approximately normal, which means that sectoral wages are distributed approximately log-normally with a Pareto tail.

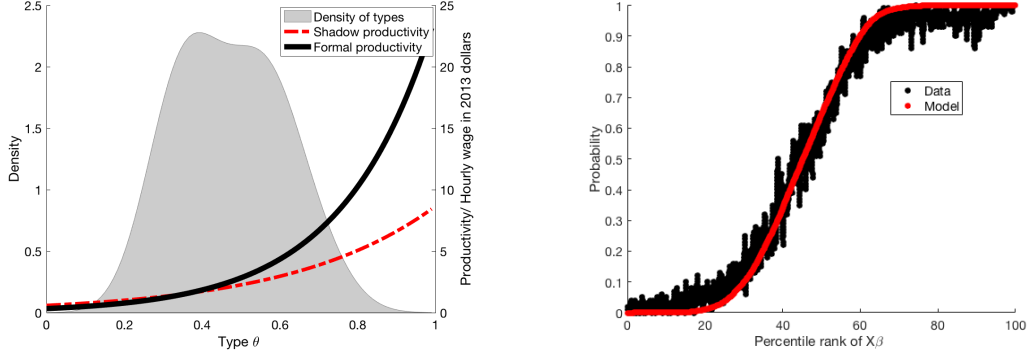
The right panel of Figure 5 shows the estimated probability of having a main job in a formal sector for each percentile of $X\beta$. The probability of having a formal main job is increasing and covers the whole range from 0% to 100%. To illustrate the fit of the model we also plot the share of shadow workers in a rolling window of 200 workers centered around each observed $X\beta$ in the sample. The model tracks the data well, showing that

³⁰In our estimation the vector X contains typical regressors from Mincerian wage equations such as age, gender, education level and experience. Following Pratap and Quintin (2006) we also include job and firm characteristics, such as the task performed by the worker and the size of the firm. This way we focus on wage differences across sectors that are not sensitive to the sector composition. Pratap and Quintin (2006) emphasize the importance of the establishment size to explain the differences on average wages across the formal and informal sector.

³¹Under our parametric assumptions, the comparative advantage in the shadow economy follows $w^s(\theta)/w^f(\theta) = w^s(0)/w^f(0) \exp\{(\rho^s - \rho^f)\theta\}$ and is decreasing when $\rho^s - \rho^f < 0$. The point estimate of $\rho^s - \rho^f$ is -1.74 with a standard error of 0.08.

Figure 5: Estimation results

- (a) Productivity profiles and type distribution (b) Probability of having a formal main job



(a) Kernel density estimation of the type distribution obtained from the observed $X\beta$ in our sample (left axis).

(b) The data counterpart is the fraction of individuals with a formal main job in a rolling window of 200 workers centered around each $X\beta$ in our sample.

our parametric specification is compatible with the observed sorting of workers across sectors.

4.2. Optimal tax schedules and comparison to standard tax formulas

In this subsection we derive the optimal tax schedules for Colombia and compare them with the outcomes of the standard tax formulas. We assume that Pareto weights follow $\lambda(\theta) = r(1 - F(\theta))^{r-1}$ as in [Rothschild and Scheuer \(2013\)](#). The parameter $r \geq 1$ captures the strength of the redistributive preferences and is equal to the Pareto weight placed on the least productive agents, while the weights of the most productive agents always converge to 0. We consider three cases of social preferences: weakly redistributive ($r = 1.1$), moderately redistributive ($r = 1.4$) and strongly redistributive ($r = 1.8$).³²

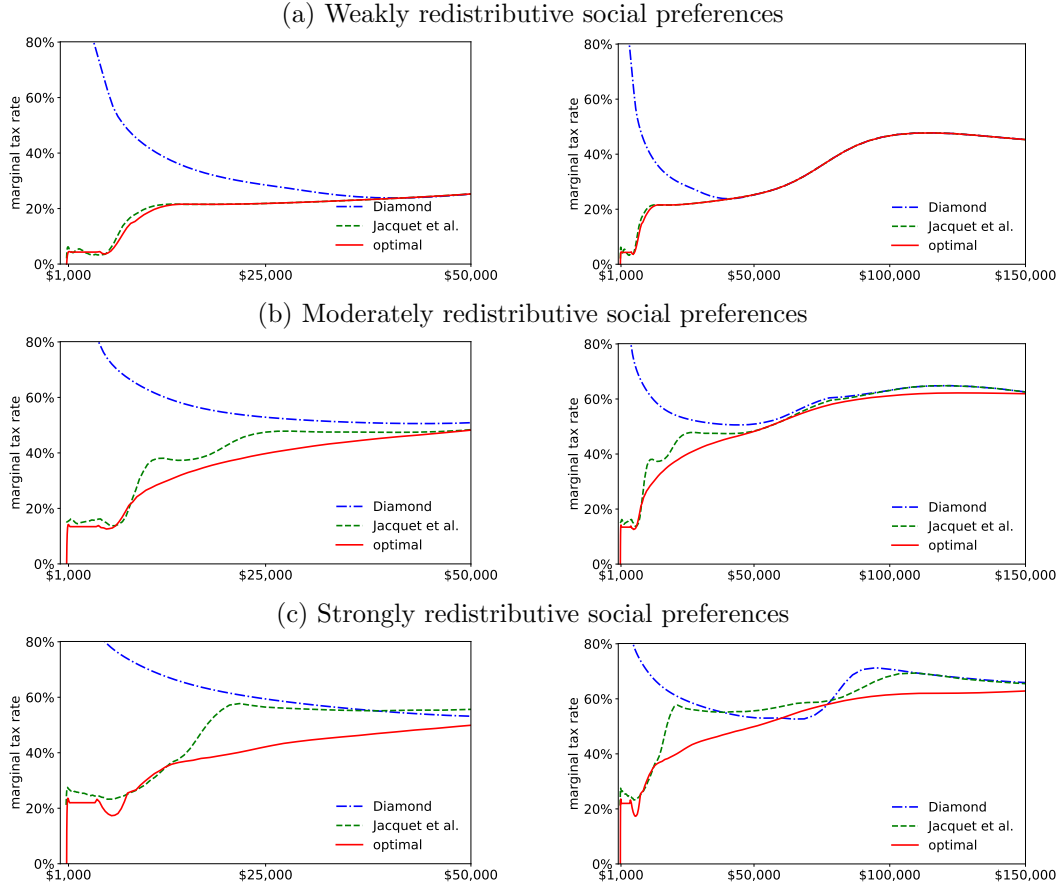
Since the distribution of income is endogenous to tax policy, we find tax schedules implied by each formula iteratively: a tax schedule implies an income distribution which, together with a formula, results in a new schedule. The tax schedules we present are the fixed points of this mapping.³³ In principle, each tax formula can have multiple fixed points. In practice, we find multiple solutions only for the Diamond and Jacquet et al. formulas. In each case we report the solution which yields the highest welfare. Each tax schedule is required to generate the same revenue as the actual Colombian income tax.

[Figure 6](#) depicts the optimal tax schedules and the tax schedules implied by the Diamond and the Jacquet et al. formulas. The optimal tax schedules are always close to fully progressive: the marginal tax rates tend increase with income. The rates are low and roughly constant at low income levels. They start to rise close to the median income

³²The Pareto weights placed on the 90th percentile of θ are in each case approximately 0.9, 0.6 and 0.3, respectively.

³³[Rothschild and Scheuer \(2016\)](#) call these fixed points ‘Self-confirming Policy Equilibria’.

Figure 6: Tax schedules implied by each tax formula



‘Diamond’, ‘Jacquet et al.’ and ‘optimal’ stand for the tax schedules implied by the respective tax formula. With moderately redistributive social preferences and the optimal tax schedule the 50th, 95th and 99th percentiles of formal income in the model with a shadow economy are approx. \$10,000, \$41,000 and \$79,000, respectively.

Table 1: Aggregate statistics of allocations implied by the tax formulas

social preferences	tax formula	share of shadow workers	share of shadow income	welfare loss
weakly redistributive	optimal	29.18%	8.19%	0%
	Jacquet et al.	29.83%	8.25%	0.0%
	Diamond	67.41%	33.19%	13.51%
moderately redistributive	optimal	37.27%	12.01%	0%
	Jacquet et al.	41.56%	14.41%	0.74%
	Diamond	74.79%	47.72%	23.65%
strongly redistributive	optimal	43.7%	15.86%	0%
	Jacquet et al.	48.85%	20.2%	1.89%
	Diamond	76.4%	51.59%	22.77%

The welfare loss is expressed as a proportional change of consumption required to reach the value of the social welfare function in the optimal allocation.

(approx \$10,000) and increase continuously up to the top income tail. A stronger taste for redistribution shifts the schedule of tax rates up while roughly preserving this shape.

The tax schedules implied by the Diamond formula are very different. They all feature very high marginal tax rates at low income levels, approaching 100% at the lowest incomes. The tax rates are decreasing through the most of the income distribution and increase again as they approach the top income tail. The Diamond tax schedules differ so much from the optimal ones since this formula does not recognize agents' responses on the extensive margin and, consequently, prescribes excessively high tax rates. Furthermore, as the low and medium productivity agents with relatively high Pareto weights join the shadow economy and reduce their formal incomes to zero, the perceived welfare cost of increasing tax rates at low income levels falls even further, leading to even higher tax rates.

The Jacquet et al. formula approximates the optimal tax schedule well when the social preferences are weakly redistributive. When preferences for redistribution are stronger, this formula leads to excessively high marginal tax rates, particularly above the median formal income. This formula recognizes the extensive margin responses which reduce the formal income to zero, which are crucial at low income levels. However, the formula misses two other types of responses. First, some shadow workers at low income levels respond on the intensive margin by jumping between two discretely different levels of shadow earnings. Not accounting for these responses leads to tax rates at low income levels which are too high by up to 1.5 points and 4 points for the moderately and strongly redistributive social preferences, respectively. Second, the agents with above median formal income who join the shadow economy retain some formal earnings. These responses are also missing from the Jacquet et al. formula. As a result, the tax rates above the median income are set too high by up to 15 points and 25 points for the moderately and strongly redistributive social preferences, respectively.

We conclude that the shadow economy have considerable impact on the optimal tax schedule not only at low incomes, but also in the upper half of the income distribution. The two standard formulas lead to tax rates which are typically too high. The Jacquet et al. formula approximates the optimal schedule better than the Diamond formula, particularly for weakly redistributive social preferences and at low income levels. While **Proposition 3** predicts such order of tax rates for a fixed distribution of income, in this exercise we allow the distribution of income to endogenously adjust to the tax schedule.

The aggregate statistics implied by the tax schedules are reported in **Table 1**. The optimal fraction of shadow workers increases with the strength of redistributive preferences from 29.2% up to 43.7%. The share of shadow income behaves similarly but at a much lower level, as only the least productive agents work informally. The Diamond formula effectively doubles the share of shadow workers relative to the optimum. As a result, the Diamond formula leads to a catastrophic welfare loss. The Jacquet et al. formula roughly replicates the optimum and implies no noticeable welfare loss when preferences

for redistribution are weak. When the social preferences are moderately or strongly redistributive, the Jacquet et al. formula increases the share of shadow workers by 4 to 5 percentage points relative to the optimum. As the strength of redistributive preferences increases, so does the welfare loss of using the Jacquet et al. formula. In the most redistributive case the welfare loss is equivalent to a 1.9% consumption drop.

4.3. Welfare impact of the shadow economy

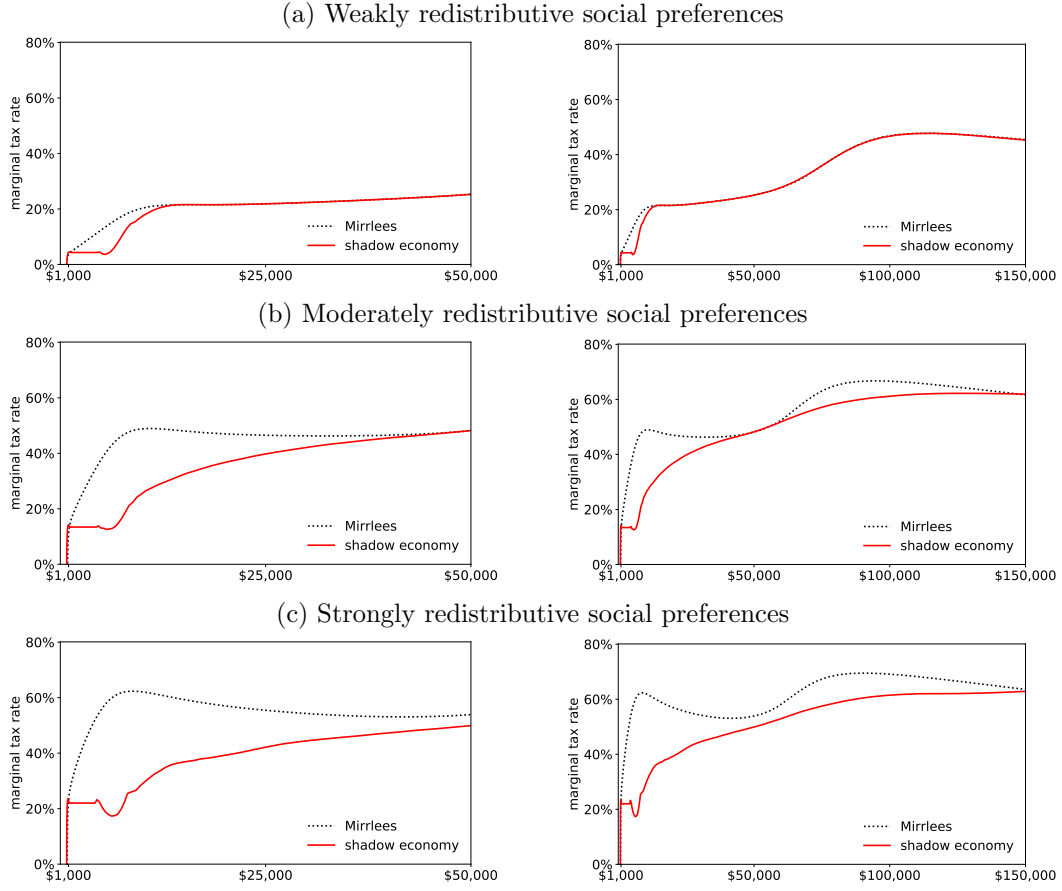
In this subsection we compare the optimal tax schedule with an informal sector with the optimal tax schedule in the otherwise identical economy where the informal sector does not exist. [Figure 7](#) illustrates the difference. The schedule without the shadow economy - the *Mirrleesian* tax schedule - features marginal tax rates which quickly increase with income at low income levels. In contrast, the tax rates with shadow economy are relatively constant at the low income levels and start rising only close to the median income. For the weakly redistributive social preferences, the two tax schedules coincide above the median income. When the social preferences for redistribution are strong, the shadow economy leads to a lower tax rates at virtually all income levels, apart from the very top. Given the functional form of the productivity schedules [Proposition 4](#) implies the shadow economy does not affect the top tax rate.

[Table 2](#) shows how the Colombian shadow economy affects the social welfare. When the preferences for redistribution are weak, the shadow economy improves welfare by an amount equivalent to a 1% increase in consumption. For moderate strength of redistributive preferences the welfare gain from the shadow economy is equivalent to a 0.25% increase in consumption. In contrast, when the preferences for redistribution are strong, the shadow economy reduces welfare by an amount equivalent to approximately 3% drop in consumption.

To understand the forces underlying these results we apply the welfare decomposition developed in [Section 2](#). The welfare impact of the shadow economy can be decomposed into the sum of the *redistribution gain*, which captures the influence of the adjusted level of taxes, and the *efficiency gain*, which captures the influence of adjusted labour supply in both sectors, including the fixed cost of shadow employment.³⁴ The decomposition, reported in [Table 2](#), shows that the shadow economy strengthens efficiency while restricting redistribution. There are two channels driving these results. First, the least productive agents are more productive in the shadow economy than in the formal economy, which increases the efficiency gain. Second, the shadow economy leads to lower tax rates. On the one hand, it implies lower labour distortions in the formal sector, which

³⁴Take the optimal allocation from the model with a shadow economy (y^f, T) and the Mirrleesian allocation (y_M, T_M) . The redistribution gain is formally defined as $\int_{\theta}^{\bar{\theta}} \int_0^{\infty} \lambda(\theta) [T_M(y_M(\theta)) - T(y^f(\theta, \kappa))] dG_{\theta}(\kappa) dF(\theta)$, while the efficiency gain is the difference between the welfare impact and the redistribution gain.

Figure 7: Optimal tax schedules with and without a shadow economy



‘Mirrlees’ stands for a Mirrleesian tax schedule in the model without a shadow economy. ‘Optimal’ stand for the optimal tax schedules in the model with a shadow economy. With moderately redistributive social preferences and the optimal tax schedule the 50th, 95th and 99th percentiles of formal income in the model with a shadow economy are approx. \$10,000, \$41,000 and \$79,000, respectively.

contributes to higher efficiency gain. On the other hand, it substantially reduces redistribution. Naturally, the lower level of income redistribution hurts more when preferences for redistribution are strong and in this case the negative redistribution gain dominates the positive efficiency gain.

Table 2: Welfare impact of the shadow economy

social preferences	welfare impact	efficiency gain	redistribution gain
weakly redistributive	0.99%	1.06%	-0.07%
moderately redistributive	0.25%	4.22%	-3.96%
strongly redistributive	-2.93%	7.84%	-10.78%

The welfare impact is expressed as a proportional change of consumption in the Mirrleesian allocation required to reach the value of the social welfare function in the shadow economy allocation.

5. Conclusions

This paper studies the optimal income taxation when agents can earn incomes in a shadow economy which are unobserved by the government. We show that the optimal tax formula with a shadow economy contains additional terms, capturing intensive and extensive margin responses of shadow workers, which lead to lower tax rates. We quantitatively demonstrate that the tax rate reduction relative to the best-performing standard tax formula is substantial, reaching 25 percentage points, and leads to large welfare gains. We also show that, depending on the distribution of formal and shadow productivities, the shadow economy can improve or deteriorate social welfare through two channels: efficiency and redistribution. We find that the shadow economy in Colombia strengthens efficiency of labor supply at the expense of possible redistribution. For weak to moderate social preferences for redistribution the efficiency channel dominates and the shadow economy increases social welfare. For strongly redistributive social preferences the redistribution channel becomes dominant and the shadow economy reduces social welfare. These results are important because they highlight the non-trivial welfare implications of informality. To reduce informality is a common policy objective, included for instance among the Sustainable Development Goals.³⁵ We instead caution against unconditional implementation of policies aimed at reducing informality.

Our analysis could be extended in several directions. First, suppose that the government can use audits and penalties to differentially affect tax evasion opportunities of different agents.³⁶ The optimal design of tax audits could, rather than minimizing overall tax evasion, tailor individual evasion opportunities to maximize the welfare improving potential of the shadow economy. Second, informal activity is closely related to home production. In some developed economies home production may be quantitatively more

³⁵Sustainable Development Goal 8 (Promote sustained, inclusive and sustainable economic growth, full and productive employment and decent work for all), target indicator 8.3.1 (Proportion of informal employment in non-agriculture employment, by sex), see [UN General Assembly \(2017\)](#). For another example, consider the Programme for the Promotion of Formalization in Latin America and the Caribbean (FORLAC) run by the International Labour Organization.

³⁶For instance, conducting tax audits at medium levels of formal income restricts tax evasion of highly productive agents, but not of low productivity workers who would never choose such income level. See [Cremer and Gahvari \(1996\)](#) for the analysis of tax audits in the optimal taxation model with two types.

important than tax evasion. When a home produced good is a perfect substitute for market income, our framework can be directly applied to study home production.

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A. Proofs from Section 2

Proof of Lemma 1. The planner can increase social welfare by transferring consumption from type $-i$ to type i , so at the optimum the incentive constraint of $-i$ will bind and the incentive constraint of i will be slack. Denote the undistorted level of formal income of type $-i$ by $y_{-i}^{f*} \equiv w_{-i}^f v'^{-1}(w_{-i}^f)$. If $y_{-i}^f \neq y_{-i}^{f*}$, the planner can extract more resources without violating the incentive constraint by setting $y_{-i}^f = y_{-i}^{f*}$ and increasing T_{-i} to keep the utility level of type $-i$ constant. Since $y_{-i}^{f*} > 0$, type $-i$ will not work in the shadow economy.

To see that the planner optimally distorts the labor supply of type i , notice that a marginal adjustment of y_i^f , starting from the undistorted level y_i^{f*} , has no direct impact on welfare of type i by the Envelope Theorem. However, the distortion in a correct direction will reduce the payoff of $-i$ from misreporting, relax the incentive constraint and, hence, allow for more redistribution. In particular, if $w_i^f < w_{-i}^f$ ($w_i^f > w_{-i}^f$), a marginal decrease (increase) of y_i^f will relax the incentive constraint. \square

Proof of Proposition 1. The difference in the utility level of type i between the two allocations is

$$U(c_i^{SE}, n_i^{SE}) - U(c_i^M, n_i^M) = U(w_i^s n_i^{SE}, n_i^{SE}) - U(w_i^f n_i^M, n_i^M) + T_i^M - T_i^{SE}. \quad (25)$$

The difference in utility level of type $-i$ is

$$U(c_{-i}^{SE}, n_{-i}^{SE}) - U(c_{-i}^M, n_{-i}^M) = T_{-i}^M - T_{-i}^{SE} = -\frac{\mu_i}{\mu_{-i}} (T_i^M - T_i^{SE}), \quad (26)$$

where the first equality follows from Lemma 1, since in the two allocations the labor supply of $-i$ is undistorted, and the second equality follows from the resource constraint. Using both utility differences, we can decompose $W^{SE} - W^M$ as stated in the proposition.

Define an function $\Psi(w) \equiv U(wv'^{-1}(w), v'^{-1}(w))$, equal to the utility level of an individual with productivity w who supplies labor efficiently and receives no transfers. The efficiency term can be restated as $\lambda_i \mu_i (\Psi(w_i^s) - U(w_i^f n_i^M, n_i^M))$. Since Ψ is an increasing function, the efficiency gain is increasing in w_i^s and changes sign at $\bar{w}_i^s \equiv \Psi^{-1}(U(w_i^f n_i^M, n_i^M))$. To see that $\bar{w}_i^s < w_i^f$, note that since n_i^M is distorted, $\Psi(w_i^f) > U(w_i^f n_i^M, n_i^M)$.

To characterize the redistribution term, note that, due to the binding incentive constraints, we have

$$U(c_{-i}^{SE}, n_{-i}^{SE}) - U(c_{-i}^M, n_{-i}^M) = \Psi(w_{-i}^s) - T_{-i}^{SE} - U(w_i^f n_i^M, w_i^f n_i^M / w_{-i}^f) + T_i^M. \quad (27)$$

Combining it with (26), we find that $T_i^M - T_i^{SE} = \mu_{-i} (U(w_i^f n_i^M, w_i^f n_i^M / w_{-i}^f) - \Psi(w_{-i}^s))$. It implies that the redistribution term is decreasing in w_{-i}^s and changes sign at $\bar{w}_{-i}^s \equiv$

$\Psi^{-1}\left(U\left(w_i^f n_i^M, w_i^f n_i^M / w_{-i}^f\right)\right)$. $\bar{w}_{-i}^s < w_{-i}^f$ holds, since $U\left(w_i^f n_i^M, w_i^f n_i^M / w_{-i}^f\right) < \Psi(w_{-i}^f)$ due to the optimal distortion of n_i^M . \square

B. Proofs from Section 3

Proof of Lemma 2. The strict *Spence-Mirrlees* single crossing condition holds if, keeping the formal income level fixed, the marginal rate of substitution $v'\left(\frac{y^f}{w^f(\theta)} + \frac{y^s}{w^s(\theta)}\right) / w^f(\theta)$ is strictly decreasing with θ . For formal workers it follows from the strict convexity of v . For shadow workers we have $v'(n) = w^s(\theta)$ and the single-crossing follows from $w^s(\theta)/w^f(\theta)$ being strictly decreasing. \square

Proof of Proposition 2. Given the single crossing condition, the necessity of constraint 1 (i.e. increasing formal income schedule and local incentive-compatibility constraints) follows from Theorem 7.2 in [Fudenberg and Tirole \(1991\)](#). By Theorem 7.3 in [Fudenberg and Tirole \(1991\)](#), constraint 1 is sufficient to prevent deviations *within* the cost class, i.e. deviations of some high-cost (low-cost) worker to formal income level earned by another high-cost (low-cost) worker. Additionally, constraint 2 is clearly necessary and sufficient to prevent deviations between different cost types for a fixed productivity type. Below we will first show that constraints 3-5 are sufficient to prevent simultaneous deviations *between* the cost types and the productivity types. Finally, we will provide a graphical example of the insufficiency of local incentive-compatibility constraints.

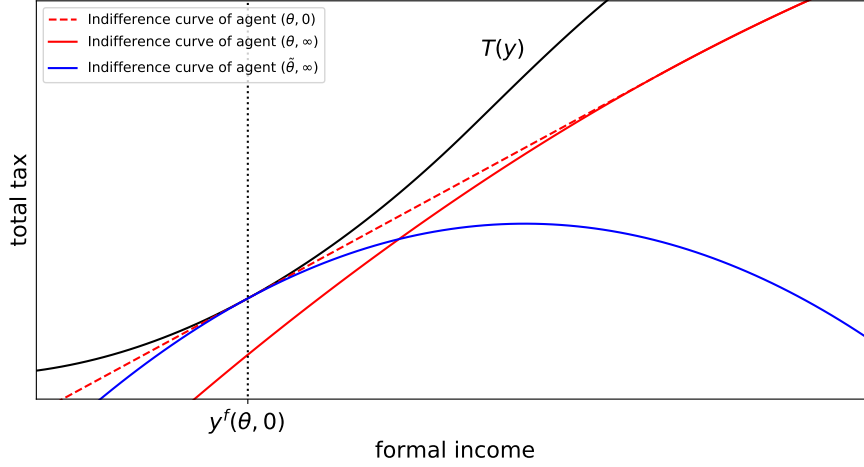
Denote the image of formal income schedule of types with fixed cost $\kappa \in \{0, \infty\}$ by $Y(\kappa) \equiv \{y \in \mathbb{R}_+ : \exists_{\theta \in [\underline{\theta}, \bar{\theta}]} y^f(\theta, \kappa) = y\}$. Deviations between the cost classes may arise if the formal income schedules of the two classes do not have identical images: $Y(0) \neq Y(\infty)$. The difference in images may occur when suprema or infima of the two sets do not coincide: either $y^f(\underline{\theta}, 0) < y^f(\underline{\theta}, \infty)$ or $y^f(\bar{\theta}, 0) < y^f(\bar{\theta}, \infty)$. Constraints 3 and 4 take care of these deviations. Alternatively, one of the income schedules can exhibit a discontinuous jump where the other schedule remains continuous. Condition 5 prevents potential deviations when $y^f(\cdot, 0)$ is discontinuous and $y^f(\cdot, \infty)$ remains continuous.³⁷ Below we show that the reverse situation never happens: when there is $y \in (y^f(\underline{\theta}, \infty), y^f(\bar{\theta}, 0))$ such that $y \in Y(0)$, then always $y \in Y(\infty)$.

We will show that for any θ we can find $\tilde{\theta}$ such that $y^f(\tilde{\theta}, \infty) = y^f(\theta, 0)$. Take some incentive-compatible allocation (y^f, T) and choose any θ such that $y^f(\theta, 0) > y^f(\underline{\theta}, \infty)$ and $y^s(\theta, 0) > 0$. Consider a productivity type $\tilde{\theta}$ such that

$$\frac{v'(y^f(\theta, 0)/w^f(\tilde{\theta}))}{w^f(\tilde{\theta})} = \frac{w^s(\theta)}{w^f(\theta)}. \quad (28)$$

³⁷Note that it is sufficient to impose additional constraints only on particular types: $(\underline{\theta}, 1)$, $(\bar{\theta}, 0)$, or a type at the discontinuity. If these constraints hold, no other type is tempted by a deviations due to a single-crossing condition.

Figure 8: Indifference curves.



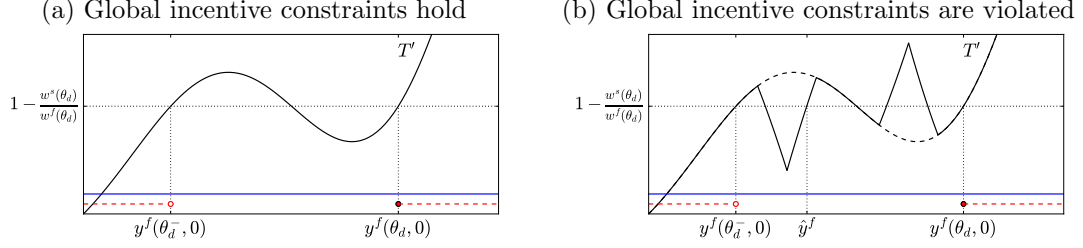
We will show that $y^f(\tilde{\theta}, \infty) = y^f(\theta, 0)$. It means that at any formal income level above $y^f(0, \infty)$ which is chosen by some low-cost worker there is also some high-cost worker.³⁸

Consider indifference curves of agents $(\theta, 0)$ and $(\tilde{\theta}, \infty)$ depicted in Figure 8. The indifference curve of the low-cost θ -worker and the high-cost $\tilde{\theta}$ -worker are tangential at formal income $y^f(\theta, 0)$. Furthermore, the indifference curve of the low-cost worker is a straight line whenever this agent supplies shadow labor, while the indifference curve of the high-cost worker is strictly concave. Finally, the indifference curves of agents $(\theta, 0)$ and $(\tilde{\theta}, \infty)$ never cross. Otherwise, the indifference curves of agents $(\tilde{\theta}, \infty)$ and (θ, ∞) would cross more than once and the single crossing condition would be violated. Altogether, it means that $y^f(\theta, 0)$ is the incentive-compatible formal income choice of the high-cost $\tilde{\theta}$ -worker. Suppose on the contrary that agent $(\tilde{\theta}, \infty)$ prefers some $\tilde{y}^f \neq y^f(\theta, 0)$. This is a profitable deviation for agent $(\theta, 0)$ as well, since his indifference curve is weakly higher. It contradicts the original assumption of implementability of $y^f(\cdot, 0)$.

To see the insufficiency of local incentive-compatibility constraints, consider Figure 9. Consider an interval of low-cost workers that supply shadow labor. By the assumed decreasing comparative advantage $w^s(\theta)/w^f(\theta)$ and the local incentive constraint (9), the marginal tax rate they face is increasing in θ . When the marginal tax rates are not monotone increasing in formal income, the formal income schedule of the low-cost workers must be discontinuous, as illustrated in the figure. The local incentive constraint of the agent at the discontinuity $(\theta_d, 0)$, given by equation (10), requires this worker to be indifferent between the two formal income levels across the discontinuity: $y^f(\theta_d^-, 0)$ and $y^f(\theta_d, 0)$. In the right panel we modify the marginal tax rates in a way that total tax levels at $y^f(\theta_d^-, 0)$ and $y^f(\theta_d, 0)$ do not change. Thus, the local incentive constraint

³⁸If $w^f(\underline{\theta}) > 0$, we need to make sure that $\tilde{\theta}$ always exists. Suppose on the contrary that $v'(y^f(\theta, 0)/w^f(\underline{\theta}))/w^f(\underline{\theta}) < w^s(\theta)/w^f(\theta)$, so that there is no $\tilde{\theta} \geq \underline{\theta}$ which satisfies (28). One can then show that it implies that if agent $(\theta, 0)$ prefers $y^f(\theta, 0)$ to $y^f(\underline{\theta}, \infty)$, so does agent $(\underline{\theta}, \infty)$. It is a contradiction, since $y^f(\theta, 0) > y^f(\underline{\theta}, \infty)$ and the allocation is incentive-compatible.

Figure 9: Insufficiency of the local incentive-compatibility constraints



The horizontal lines indicate whether at a given formal income level there are high-cost workers (solid, blue) or low-cost workers (dashed, red). In both panels agent $(\theta_d, 0)$ is indifferent between $y^f(\theta_d^-, 0)$ and $y^f(\theta_d, 0)$. Hence, the local incentive constraint (10) holds. However, in the right panel the worker strictly prefers formal income level \hat{y}^f , since the average tax rate between $y^f(\theta_d^-, 0)$ and \hat{y}^f is below $1 - w^s(\theta_d)/w^f(\theta_d)$.

of agent $(\theta_d, 0)$ still holds. However, this agent has a profitable deviation to \hat{y}^f . The average tax rate between $y^f(\theta_d^-, 0)$ and \hat{y}^f is below $1 - w^s(\theta)/w^f(\theta)$, which implies that the utility from deviation to \hat{y}^f is higher than the utility at $y^f(\theta_d, 0)$. Therefore, the new allocations is not globally incentive-compatible since it violates constraint 5 from Proposition 2. It is easy to construct similar examples of locally incentive-compatible allocations violating constraints 3 and 4. \square

Proof of Proposition 3. By assumptions made, the fiscal impact of the extensive margin responses and of the intensive margin responses on marginal tax rates is non-negative. We can distinguish four cases:

1. There are some low-cost workers above y and
 - a) $\Delta_\infty(y) = 0$: $T'(y) = T'_J(y) = T'_D(y)$,
 - b) $\Delta_\infty(y) > 0$ and $s(y) = y$: $T'(y) \leq T'_J(y) \leq T'_D(y)$,
 - c) $\Delta_\infty(y) > 0$ and $s(y) > y$: $T'(y) < T'_J(y) \leq T'_D(y)$.
2. There are no low-cost workers above y : $T'(y) \leq T'_J(y) \leq T'_D(y)$.

Consider these cases successively. 1a) $\Delta_\infty(y) = 0 \Leftrightarrow \Delta_0(y) = 0 \implies s(y) = y \wedge h^s(y) = 0$, which means that all workers at y are formal, there are no intensive margin responses of shadow workers and there are no extensive margin responses. Both (19) and the Jacquet et al. formula collapse into the Diamond formula. 1b) Since $s(y) = y$, the average intensive margin elasticity at y is sufficient to capture the intensive margin responses of formal and shadow workers. However, the Diamond formula captures none of the extensive margin responses, while the Jacquet et al. formula captures only a fraction of them. 1c) Neither the Diamond nor the Jacquet et al. formula capture the intensive margin responses of the shadow workers, since they happen at income level higher than y . Analogous to the previous case with respect to the extensive margin responses. In the case 2, the two standard tax formulas correctly capture the intensive

margin responses, but, analogously to the two previous cases, they (partially) miss the extensive margin responses. \square

Proof of Lemma 3. If all top workers are formal (i.e. $1 - T'(\infty) \geq w^s(\bar{\theta})/w^f(\bar{\theta})$), the distribution of formal income satisfies

$$\lim_{y \rightarrow \infty} \frac{1 - H(y)}{h(y)y} = \lim_{\theta \rightarrow \bar{\theta}} \frac{1 - F(\theta)}{f(\theta)w^f(\theta)} \frac{dw^f(\theta)}{d\theta} \frac{dy^f(\theta, \infty)}{dw^f(\theta)} \frac{w^f(\theta)}{y^f(\theta, \infty)} = \frac{1 + \varepsilon}{\alpha}. \quad (29)$$

When there are some shadow workers among the top productivity types, we have

$$\lim_{y \rightarrow \infty} \frac{1 - H(y)}{h(y)y} = \lim_{\theta \rightarrow \bar{\theta}} \frac{1 - \int_0^\theta (1 - G_{\theta'}(\tilde{\kappa}(\theta'))) dF(\theta')}{(1 - G_\theta(\tilde{\kappa}(\theta))) f(\theta)w^f(\theta)} \frac{dw^f(\theta)}{d\theta} \frac{dy^f(\theta, \infty)}{dw^f(\theta)} \frac{w^f(\theta)}{y^f(\theta, \infty)}. \quad (30)$$

One can show that the formality threshold $\tilde{\kappa}(\theta)$ is asymptotically proportional to $w^f(\theta)^{1+\varepsilon}$:

$$\lim_{\theta \rightarrow \bar{\theta}} \frac{\tilde{\kappa}(\theta)}{w^f(\theta)^{1+\varepsilon}} = \frac{1}{1 + \varepsilon} \left(\left(\frac{w^s(\bar{\theta})}{w^f(\bar{\theta})} \right)^{1+\varepsilon} - (1 - T'(\infty))^{1+\varepsilon} \right). \quad (31)$$

Consequently, $1 - G_\theta(\tilde{\kappa}(\theta))$ is asymptotically proportional to $w^f(\theta)^{-\gamma(1+\varepsilon)}$ and

$$\begin{aligned} \lim_{\theta \rightarrow \bar{\theta}} \frac{1 - \int_0^\theta (1 - G_{\theta'}(\tilde{\kappa}(\theta'))) dF(\theta')}{(1 - G_\theta(\tilde{\kappa}(\theta))) f(\theta)w^f(\theta)} \frac{dw^f(\theta)}{d\theta} &= \lim_{w^f \rightarrow \infty} \frac{\int_{w^f}^\infty 1/(w)^{1+\alpha+\gamma(1+\varepsilon)} dw}{1/(w^f)^{1+\alpha+\gamma(1+\varepsilon)} w^f} \\ &= \frac{1}{\alpha + \gamma(1 + \varepsilon)}. \end{aligned} \quad (32)$$

Plugging it into (30), we get $\lim_{y \rightarrow \infty} \frac{1-H(y)}{h(y)y} = \frac{1+\varepsilon}{\alpha+\gamma(1+\varepsilon)}$. \square

Proof of Proposition 4. If all the top productivity workers, including all the cost types, are formal then the optimal tax formula (??) in the limit as $\theta \rightarrow \bar{\theta}$ implies

$$\frac{T'(\infty)}{1 - T'(\infty)} \frac{\alpha\varepsilon}{1 + \varepsilon} = 1 - \lambda \implies 1 - T'(\infty) = \frac{\alpha\varepsilon/(1 + \varepsilon)}{1 - \lambda + \alpha\varepsilon/(1 + \varepsilon)}. \quad (33)$$

This happens either if there is no shadow economy or if the shadow economy exists, but the top workers have not incentives to work informally: $(1 - T'(\infty))w^f(\bar{\theta}) \geq w^s(\bar{\theta})$.

Suppose on the contrary that $(1 - T'(\infty))w^f(\bar{\theta}) < w^s(\bar{\theta})$, which means that some top productivity workers work informally. In Lemma 3 we determined the tail parameter of the productivity of formal workers:

$$\begin{aligned} \lim_{\theta \rightarrow \bar{\theta}} \frac{1 - \int_0^\theta (1 - G_{\theta'}(\tilde{\kappa}(\theta'))) dF(\theta')}{(1 - G_\theta(\tilde{\kappa}(\theta))) f(\theta)w^f(\theta)} \frac{dw^f(\theta)}{d\theta} &= \lim_{w^f \rightarrow \infty} \frac{\int_{w^f}^\infty 1/(w)^{1+\alpha+\gamma(1+\varepsilon)} dw}{1/(w^f)^{1+\alpha+\gamma(1+\varepsilon)} w^f} \\ &= (\alpha + \gamma(1 + \varepsilon))^{-1}. \end{aligned} \quad (34)$$

Furthermore, define the following function

$$\lim_{\theta \rightarrow \bar{\theta}} \frac{\Delta T(y^f(\theta, \infty))}{\tilde{\kappa}(\theta)} = (1+\varepsilon) \frac{T'(\infty)}{1-T'(\infty)} \left(\left(\frac{w^s(\bar{\theta})/w^f(\bar{\theta})}{1-T'(\infty)} \right)^{1+\varepsilon} - 1 \right)^{-1} \equiv \delta(T'(\infty)), \quad (35)$$

where $\delta(\tau) > 0$ for $\tau > 1 - w^s(\bar{\theta})/w^f(\bar{\theta})$ and $\delta(\tau)$ diverges to $+\infty$ as τ converges to $1 - w^s(\bar{\theta})/w^f(\bar{\theta})$ from the right. Then the elasticity of the density of formal workers at the top converges to

$$\frac{g_\theta(\tilde{\kappa}(\theta))\tilde{\kappa}(\theta)}{1 - G_\theta(\tilde{\kappa}(\theta))} \frac{\Delta T(y^f(\theta, \infty))}{\tilde{\kappa}(\theta)} \rightarrow \gamma \delta(T'(\infty)). \quad (36)$$

As $\theta \rightarrow \bar{\theta}$ the optimal tax formula (??) implies

$$\begin{aligned} \frac{T'(\infty)}{1-T'(\infty)} \left(\frac{\alpha\varepsilon}{1+\varepsilon} + \gamma\varepsilon \right) &= 1 - \lambda - \gamma\delta(T'(\infty)) \\ \implies 1 - T'(\infty) &= \frac{\alpha\varepsilon/(1+\varepsilon) + \gamma\varepsilon}{1 - \lambda + \alpha\varepsilon/(1+\varepsilon) + \gamma\varepsilon - \gamma\delta(T'(\infty))}. \end{aligned} \quad (37)$$

To characterize the top tax rate, define the following auxiliary functions

$$\Phi_M(\tau) \equiv 1 - \lambda - \frac{\tau}{1-\tau} \frac{\alpha\varepsilon}{1+\varepsilon}, \quad (38)$$

$$\Phi(\tau, \gamma) \equiv \begin{cases} \Phi_M(\tau) & \text{if } \tau \leq 1 - w^s(\bar{\theta})/w^f(\bar{\theta}), \\ 1 - \lambda - \gamma\delta(\tau) - \frac{\tau}{1-\tau} \left(\frac{\alpha\varepsilon}{1+\varepsilon} + \gamma\varepsilon \right) & \text{if } \tau > 1 - w^s(\bar{\theta})/w^f(\bar{\theta}). \end{cases} \quad (39)$$

$\Phi_M(\tau)$ is the marginal social benefit of increasing the top tax rate from the level τ in the standard Mirrlees model. $\Phi_M(\cdot)$ is strictly decreasing, strictly concave and naturally $\Phi_M(T'_M(\infty)) = 0$. $\Phi(\tau, \gamma)$ is the marginal social benefit of increasing the top tax rate from the level τ in the model with a shadow economy when the fixed cost distribution has a tail parameter γ . $\Phi(\cdot, \gamma)$ is discontinuous at $1 - w^s(\bar{\theta})/w^f(\bar{\theta})$, where for any positive γ it falls down to $-\infty$. On the interval $(1 - w^s(\bar{\theta})/w^f(\bar{\theta}), 1)$ the function $\Phi(\cdot, \gamma)$ is, for any positive γ , first increasing and then decreasing, strictly concave and bounded from above by $\Phi_M(\cdot)$.

Suppose that $T'_M(\infty) > 1 - w^s(\bar{\theta})/w^f(\bar{\theta})$. The optimal top tax rate $T'(\infty)$ satisfies

$$T'(\infty) = \arg \max_{\tau \geq 1 - \frac{w^s(\bar{\theta})}{w^f(\bar{\theta})}} \int_{1 - \frac{w^s(\bar{\theta})}{w^f(\bar{\theta})}}^{\tau^*} \Phi(\tau, \gamma) d\tau \quad (40)$$

$$= \arg \max_{\tau \geq 1 - \frac{w^s(\bar{\theta})}{w^f(\bar{\theta})}} \int_{1 - \frac{w^s(\bar{\theta})}{w^f(\bar{\theta})}}^{\tau^*} \Phi_M(\tau) d\tau - \gamma \int_{1 - \frac{w^s(\bar{\theta})}{w^f(\bar{\theta})}}^{\tau^*} \left(\delta(\tau) + \frac{\tau\varepsilon}{1-\tau} \right) d\tau. \quad (41)$$

There are two possible candidates for the optimal top tax rate: (i) $1 - w^s(\bar{\theta})/w^f(\bar{\theta})$ and (ii) $\tilde{\tau}$ which satisfies $\Phi(\tilde{\tau}, \gamma) = 0$ and $\partial\Phi(\tau, \gamma)/\partial\tau|_{\tau=\tilde{\tau}} < 0$ (see [Figure 4](#)). Suppose

that at some γ the solution is equal to $1 - w^s(\bar{\theta})/w^f(\bar{\theta})$. Since $\delta(\tau) + \frac{\tau\varepsilon}{1-\tau} > 0$ for all $\tau > 1 - w^s(\bar{\theta})/w^f(\bar{\theta})$, the solution is unchanged for any higher values of γ . It proves the existence of threshold $\tilde{\gamma}$. \square

C. Estimation details.

First we describe the data and explain how we recover wages and sectoral participation. Second, we list the identifying assumptions and formulate the likelihood function. Last we present the parameter estimates.

Data. We use the 2013 wave of the household survey by the official statistical agency of Colombia (DANE). We restrict attention to individuals aged 24-50 years without children (34,000 individuals). We choose this sample, since these workers face a tax and transfer schedule which is not means-tested and does not depend on choices absent from our modeling framework, such as a number of children or college attainment.

The information we use in the estimation is given by a sample $\left\{ \omega_i, \ell_i^f, x_i, s_i \right\}_{i=1}^N$ of the random variables $\{ \mathcal{W}, I^f, X \}$, where \mathcal{W} is the hourly wage of worker before taxes; I^f an indicator variable for having a main job in the formal sector; X a vector of worker characteristics; and s_i the sampling weight of observation i and N the total number of observations in our sample. The indicator variable I^f is set equal to one if the worker reports to be affiliated to all three components of social security: pension system, health insurance and labor accidents insurance. A fraction (about 3%) of workers also have a second job. If the first job is formal we cannot identify if the worker's second job is shadow or formal. Therefore I^f indicates formality of the main job and does not imply that the worker is exclusively formal.

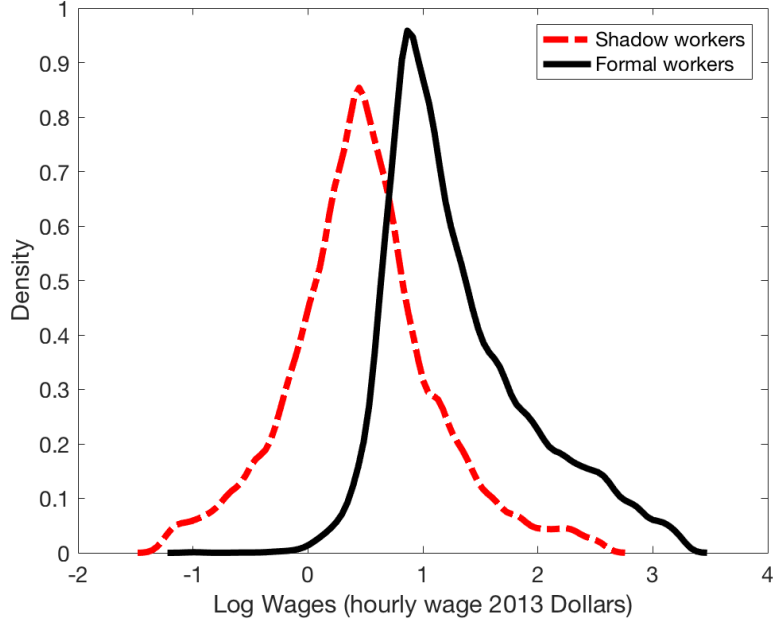
We use two questions of the survey to construct our measure of the hourly wage \mathcal{W} . First, the worker is asked what was her income at the main job last month. Second, what is the number of hours she 'normally' works at that job. We use the ratio of the reported income and hours in those questions to compute our measure of the hourly wage. Since the 'normal' number of hours need not to correspond to last month's number of hours we use our measure as a noisy measure of productivity in the model.³⁹ If the worker is identified to be formal at the main job we include the statutory payroll taxes that are paid by the employer in the computation of the pre-tax income at the main job. In [Figure 10](#) the distribution of log-wages is presented for each sector. Variables included in vector X are listed in [Table 3](#).

³⁹We further assume that survey respondents correctly reveal their gross income from the main job, regardless of whether the main job is formal or informal. Other papers making this assumption include [Meghir, Narita, and Robin \(2015\)](#) for Brazil and [López García \(2015\)](#) for Chile.

Table 3: Variables included in X

Variable	Description	Values
<i>Individual characteristics</i>		
Gender	Dummy variable equal to 1 for women	0-1
Age	Age of the worker	16-90
Age ²	Age squared	
Educ	Number of education years	0-26
Degree	Highest degree achieved (No degree to Doctorate)	1-5
Work	Number of months worked in the last year	1-12
Exper	Number of months worked in the last job	0-720
1stJob	Dummy for the first job (1 if it is the first job)	0-1
<i>Job characteristics</i>		
S-Man	Dummy for the manufacturing sector	0-1
S-Fin	Dummy for financial intermediation	0-1
S-Ret	Dummy for the sales and retailers sector	0-1
B-city	Dummy for a firm in one of the two largest cities	0-1
Size	Categories for the number of workers	1-9
Lib	Dummy for a liberal occupation	0-1
Admin	Dummy for an administrative task	0-1
Seller	Dummy for sellers and related	0-1
Services	Dummy for a service task	0-1
<i>Worker-firm relationship</i>		
Union	Dummy for labor union affiliation (1 if yes)	0-1
Agency	Dummy for agency hiring (1 if yes)	0-1
Senior	Number of months of the worker in the firm	0-720

Figure 10: Density of observed log-wages in the formal and the informal sectors



Kernel density estimation of the wage distribution obtained from the observed wages in our sample.

Modeling assumptions. We assume that productivity in the participating sector is equal to the measured hourly wage \mathcal{W} plus a normally distributed measurement error $u \sim N(0, \sigma_u)$. Also, productivity in each sector $j \in \{s, f\}$ features a constant, sector specific growth rate ρ^j with respect to the productivity type θ :

$$\log(w^j(\theta)) = \log(w^j(0)) + \rho^j \theta, \quad j \in \{s, f\}. \quad (42)$$

The above assumption is not restrictive for the unconditional distribution of formal wages, as long as we are free to choose any distribution of the productivity types $F(\theta)$. This assumption, however, restricts the joint distribution of formal and shadow wages. The comparative advantage in the shadow economy becomes

$$\frac{w^s(\theta)}{w^f(\theta)} = \frac{w^s(0)}{w^f(0)} \exp\left\{\left(\rho^s - \rho^f\right)\theta\right\}. \quad (43)$$

The fixed cost of shadow employment κ follows a generalized Pareto distribution with density

$$g_\theta(\kappa) = \frac{1}{\sigma_\kappa (w^f(\theta) - w_\kappa)^{\alpha_\kappa}} \left(1 + \frac{\kappa}{\sigma_\kappa (w^f(\theta) - w_\kappa)^{\alpha_\kappa}}\right)^{-2}, \quad (44)$$

where parameters σ_κ , α_κ and w_κ determine how the distribution of the fixed cost is affected by the productivity type θ .

Proposition 5. *The model given by (42), (44) and an unrestricted distribution of types $F(\theta)$ is not identified with data on workers wages and sectoral choice.*

Proof. The model is not identified as any distribution of wages could have been generated by a version of the model where participation costs are irrelevant and all workers are sorted only according to their relative productivities. We assume the empirical marginal tax rates are non-negative and bounded away from 100%.

Consider the following parametrization of the model: $w^s(0) = \bar{w}$, $w^f(0) = \underline{w}^2/\bar{w}$, $\rho^s = -\rho^f = 2\ln(\underline{w}) - 2\ln(\bar{w})$, where \bar{w} is an upper bound on the support of wages and $\underline{w} \in (0, 1)$ is a lower bound. The support of θ is $[0, 1]$ and the distribution of the fixed cost is collapsed to zero. Under this parametrization formal productivity is increasing in type θ , shadow productivity decreasing, and they cross at productivity equal to \underline{w} for type $\theta = 0.5$.

Let $F_{W,s}$ be the cumulative density of wages of the participants in the shadow sector, $F_{W,f}$ that of the participants in the formal sector and μ_s the mass of individuals in the shadow sector. Any joint distribution of (w, I^f) can be replicated by setting the cumulative distribution of types as follows:

$$F(\theta) = \begin{cases} \mu_s F_{W,s}(\bar{w} \exp\{\rho^s \theta\}) & \text{if } \theta \in [0, 0.5] \\ \mu_s + (1 - \mu_s) F_{W,f}\left(\frac{\underline{w}^2}{\bar{w}} \exp\{\rho^f \theta\}\right) & \text{if } \theta \in (0.5, 1] \end{cases}$$

Finally, to guarantee that workers with $\theta \in (0.5, 1]$ self-select to be formal workers, set the lower bound \underline{w} to be the product of the lowest observed formal wage and the lowest possible net-of-tax rate: $\underline{w} = \min(w^f) \times \min_{y \geq 0} \{1 - T'(y)\}$. It guarantees that the after-tax formal wage is never below the shadow wage. \square

Proposition 5 is a particular instance of the results of [Heckman and Honore \(1990\)](#) and [French and Taber \(2011\)](#), where it is shown that the data on wages and the sectoral participation is in general not sufficient to identify the productivity profiles. [Heckman and Honore \(1990\)](#) also prove that the model can be identified with additional regressors that affect the location parameters of the skill distribution. Motivated by this approach we include a vector of regressors X that can potentially convey information about the workers productivity and assume the following relationship:

$$\theta \sim N(X\beta, \sigma_\theta^2), \quad (45)$$

where β is a vector of parameters. We obtain $F(\theta)$ using (45) and a kernel density estimation of the $X\beta$ distribution. To capture the right tail of the wage distribution, we fit a Pareto distribution with parameter α_w to the top 1% of formal wages. Finally, we assume that agents' preferences over labor supply follow

$$v(n) = \Gamma \frac{n^{1+1/\varepsilon}}{1 + 1/\varepsilon}, \quad (46)$$

where ε is the common elasticity of labor supply which we fixed at 0.33 following [Chetty](#)

(2012). Together, assumptions (42), (44), (45) and (46) identify the model. We estimate the model by Maximum Likelihood.

Likelihood function. We can decompose the mixed joint density of a given realization $\{\omega, \iota^f, x\}$ of the random variables $\{\mathcal{W}, I^f, X\}$ into three elements:

$$f_{\mathcal{W}, I^f, X}(\omega, \iota^f, x; B) = P(X = x) \times P_{I^f|X}(I^f = \iota^f \mid X = x; B) \times f_{\mathcal{W}|I^f, X}(\omega \mid \iota^f, x; B)$$

where B is the vector of parameters

$$B = \left(\beta, \varepsilon, \Gamma, \gamma_0^s, \gamma_1^s, \gamma_0^f, \sigma_\theta, \sigma_u, \sigma_\kappa, w_\kappa \right)$$

and the elements correspond to:

- $P(X = x_i)$ is the sampling weight s_i .
- $P_{I^f|X}(I^f = \iota^f \mid X = x; B)$ is the probability that someone with characteristics x takes the participation decision ι^f . The decision to participate in the formal sector ι^f depends on the productivity type θ and the participation cost κ . Let $\mathbf{i}(\theta, \kappa)$ denote the optimal participation decision. Then this probability can be rewritten as

$$\begin{aligned} P_{I^f|X}(I^f = \iota^f \mid X = x; B) &= \int_0^1 P_{I^f|\theta}(I^f = \iota^f \mid \theta; B) f_{\theta|X}(\theta \mid x; B) d\theta \\ &= \int_0^1 \left(\int_0^\infty I_{(\mathbf{i}(\theta, \kappa) = \iota^f)} g_\theta(\kappa) d\kappa \right) f_{\theta|X}(\theta \mid x; B) d\theta \\ &= \int_0^1 \left((\iota^f + (-1)^{\iota^f} G_\theta(\tilde{\kappa})) \right) f_{\theta|X}(\theta \mid x; B) d\theta \end{aligned}$$

where $I_{(a)}$ is an indicator function that takes the value of 1 if the condition a is satisfied; $\tilde{\kappa}$ is the threshold value of the participation cost; $f_{\theta|X}$ is given by a normal distribution $N(X\beta, \sigma_\theta)$; and $g_\theta(\tilde{\kappa})$ is given by (44).

- $f_{\mathcal{W}|I^f, X}(\omega \mid \iota^f, x; B)$ is the likelihood that a worker with characteristics x and observed participation ι^f has a measured wage of ω . This probability can be written as

$$f_{\mathcal{W}|I^f, X}(\omega \mid \iota^f, x; B) = \int_0^1 f_{\mathcal{W}|I^f, \theta}(\omega \mid \iota^f, \theta; B) f_{\theta|I^f, X}(\theta \mid \iota^f, x; B) d\theta$$

where

$$f_{\mathcal{W}|I^f, \theta}(\omega \mid \iota^f, \theta; B) = \begin{cases} N(\log(\omega) - \log(w^f(0) - \rho^f \theta, \sigma_u)) & \text{if } \iota^f = 1 \\ N(\log(\omega) - \log(w^s(0) - \rho^s \theta, \sigma_u)) & \text{else} \end{cases}$$

and

$$f_{\theta|I^f,X}(\theta | \iota^f, x; B) = \frac{P_{I^f|\theta}(I^f = \iota^f | \theta; B) f_{\theta|X}(\theta | x; B)}{P_{I^f|X}(I^f = \iota^f | X = x; B)}$$

Parameter estimates. The parameter estimates are reported in [Table 4](#). The estimated density of types as well as the fit of the model along the shadow economy participation margin are shown in [Figure 5](#) in the main text.

Table 4: Parameter estimates

preferences		productivity schedules					distributions of θ and κ				
ε	Γ	$w^s(0)$	ρ^s	$w^f(0)$	ρ^f	α_w	σ_θ	σ_κ	α_κ	w_κ	σ_u
0.33	0.032	0.006	2.90	0.003	4.64	2.25	0.09	1.38	0.88	0.018	0.53
(-)	(8e-4)	(1e-4)	(.06)	(1e-4)	(.06)	(.03)	(2e-3)	(0.03)	(.01)	(2e-4)	(3e-3)
β individual characteristics							β worker-firm				
Gender	Age	Age ²	Educ	Degree	Work	Exper	1stJob	Union	Agency		
-0.08	0.04	-5e-4	0.02	0.05	0.02	6e-5	2e-4	0.17	-0.015		
(2e-3)	(1e-4)	(6e-6)	(5e-4)	(1e-3)	(7e-4)	(6e-5)	(2e-5)	(4e-3)	(3e-4)		
β job characteristics											
S-Man	S-Fin	S-Ret	B-city	Size	Lib	Admin	Seller	Services	Senior		
-0.04	0.14	-0.012	0.10	0.11	0.25	-5e-3	4e-3	-0.02	7e-4		
(1e-3)	(3e-3)	(3e-4)	(3e-3)	(2e-3)	(6e-3)	(1e-4)	(1e-4)	(6e-4)	(1e-5)		

Standard errors are reported in brackets. Standard errors are obtained by Case Resampling Bootstrap using 150 draws.