Optimal Redistribution with a Shadow Economy: Online Appendix

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Abstract

The supplementary material contains the derivation of the optimal tax formula using the mechanism design approach (Appendix A) and the Pareto efficiency test of the actual tax schedule in Colombia (Appendix B).

A. Derivation of the optimal tax rates

Below we derive the optimal tax rates in terms of model primitives using the mechanism design approach, i.e. by perturbing an allocation directly subject to the incentive-compatibility constraints. Then we define the sufficient statistics used to derive the optimal tax rates in the main text and show the equivalence between the sufficient statistics approach from the main text and the mechanism design approach.

A.1. A mechanism design approach

Consider an incentive-compatible allocation (y^f, T). By Corollary 1 from Milgrom and Segal (2002) the indirect utility function $V(y^f(\theta,\kappa), T, \theta, \kappa)$ is differentiable with respect to θ almost everywhere. The derivative, by the local incentive-compatibility constraints, is given by

$$\frac{d}{d\theta}V(y^f(\theta,\kappa),T,\theta,\kappa) = \left(\rho^f(\theta)\frac{y^f(\theta,\kappa)}{w^f(\theta)} + \rho^s(\theta)\frac{y^s(\theta,\kappa)}{w^s(\theta)}\right) v'\left(\frac{y^f(\theta,\kappa)}{w^f(\theta)} + \frac{y^s(\theta,\kappa)}{w^s(\theta)}\right)$$
$$\equiv V_\theta(y^f(\theta,\kappa),T,\theta,\kappa),$$

where $\rho^x(\theta) \equiv w^x_\theta(\theta)/w^x(\theta)$ stands for the productivity growth rate in sector $x \in \{f,s\}$. From now on we will suppress for brevity the dependence of V and V_θ on the allocation. Hence, we can represent the indirect utility function in the integral form

$$V(\theta,\kappa) = V(0,\kappa) + \int_0^\theta V_\theta(\theta',\kappa)d\theta'. \tag{1}$$

Take some high-cost worker (θ,∞). We will derive the optimality condition by perturbing formal income of this worker by small dy^f and adjusting the tax paid such that the utility level $V(\theta,\infty)$ is unchanged. This perturbation affects the slope $V_\theta(\theta,\infty)$, which in turn implies via equation (1) a uniform shift of utility levels of all high-cost types above.

Moreover, since all agents face the same tax schedule, we need to adjust the allocation of the low-cost workers as well. We can distinguish three cases. First, the distorted shadow workers can respond by marginally decreasing formal income. Second, the distorted shadow workers can respond by jumping to a discretely lower formal income level. These two cases have identical fiscal impact and lead to the same optimal tax formula. Finally, when $y^f(\theta,\infty) > y^f(\overline{\theta},0)$, all the shadow workers have lower formal income and hence are unaffected by the perturbation.

As mentioned in the main text, we assume that the monotonicity constraints on the formal income schedules and the constraint 3 from Proposition 2 are not binding. Both assumptions can be verified ex post and in all our applications these constraints were slack. Furthermore, we can ignore constraints 4 and 5 from Proposition 2 while deriving the optimality conditions. That is because we start from an incentive-compatible allocation and, as we show below, we do not need to keep track of the size of the intensive margin responses of shadow workers to calculate the implied fiscal loss.

Distortion of formal workers. A formal income perturbation dy^f affects the utility of type (θ, ∞) by $\left(1 - \frac{v'(n(\theta,\infty))}{w^f(\theta)}\right) dy^f$, or equivalently by $T'(y^f(\theta, \infty))dy^f$. We need to adjust the total tax paid by the same amount such that the utility level stays constant. The fiscal impact of doing so is

$$T'(y^f(\theta, \infty))(1 - G_\theta(\tilde{\kappa}(\theta)))f(\theta)dy^f. \tag{2}$$

The impact of this perturbation on the slope of the utility schedule is

$$dV_\theta(\theta, \infty) = \rho^f(\theta)\left(1 - T'(y^f(\theta, \infty))\right)\left(1 + \frac{1}{\varepsilon(\theta, \infty)}\right) dy^f, \tag{3}$$

where $\varepsilon(\theta, \kappa) \equiv \frac{v'(n(\theta,\kappa))}{n(\theta,\kappa)v''(n(\theta,\kappa))}$ is the elasticity of labor supply and $n(\theta, \kappa)$ is the total labor supply of agent (θ, κ). Hence, a perturbation that leads to a change of slope $dV_\theta(\theta, \infty)$ implies a change in tax revenue from the formal workers by

$$\frac{T'(y^f(\theta, \infty))}{1 - T'(y^f(\theta, \infty))}\left(1 + \frac{1}{\varepsilon(\theta, \infty)}\right)^{-1} \frac{1}{\rho^f(\theta)}(1 - G_\theta(\tilde{\kappa}(\theta)))f(\theta)dV_\theta(\theta, \infty). \tag{4}$$

Distortion of shadow workers. Let's consider the case of $y^f(\theta, \infty) \leq y^f(\overline{\theta}, 0)$, otherwise there is no tax loss from the shadow workers. The mapping $\tilde{s}(\theta) \equiv \min\{\theta' \in [\overline{\theta}, \underline{\theta}] \text{ s.t. } y^f(\theta', 0) \geq y^f(\theta, \infty)\}$ indicates which shadow worker is distorted by the perturbation of formal income of the high-cost worker with a productivity type θ. First, suppose that $y^f(\tilde{s}(\theta), 0) = y^f(\theta, \infty)$, so that the distorted shadow worker has the same formal income as the distorted formal worker. A perturbation of formal income dy_2^f affects the utility level of $(\tilde{s}(\theta), 0)$-type worker by $\left(1 - \frac{w^s(\tilde{s}(\theta))}{w^f(\tilde{s}(\theta))}\right) dy_2^f = T'(y^f(\tilde{s}(\theta), 0))dy_2^f$. We need to adjust the tax paid by the same amount, which affects the resource constraint by

$$T'(y^f(\tilde{s}(\theta), 0))G_{\tilde{s}(\theta)}(\tilde{\kappa}(\tilde{s}(\theta)))f(\tilde{s}(\theta))dy_2^f. \tag{5}$$

The slope of the utility schedule of low-cost workers changes by

$$dV_\theta(\tilde{s}(\theta), 0) = \left(\rho^f(\tilde{s}(\theta)) - \rho^s(\tilde{s}(\theta))\right) \frac{w^s(\tilde{s}(\theta))}{w^f(\tilde{s}(\theta))} dy_2^f. \tag{6}$$

The perturbation needs to respect the common tax schedule at higher formal incomes - the slopes of $V(\theta,\infty)$ and $V(\tilde{s}(\theta),0)$ have to change by the same amount, which can be achieved by appropriately adjusting dy_2^f. Then, by using the first-order condition of workers $(\tilde{s}(\theta),0)$, we can express the tax loss as

$$\frac{w^f(\tilde{s}(\theta)) - w^s(\tilde{s}(\theta))}{w^s(\tilde{s}(\theta))}\frac{G_{\tilde{s}(\theta)}(\tilde{\kappa}(\theta))f(\tilde{s}(\theta))}{\rho^f(\tilde{s}(\theta)) - \rho^s(\tilde{s}(\theta)))}dV(\theta,\infty). \tag{7}$$

Second, suppose that $y^f(\tilde{s}(\theta),0) > y^f(\theta,\infty)$, in which case the distorted shadow worker has a higher formal income that the distorted formal worker. In this case there is a discontinuity in the formal income schedule of the low-cost workers at $\tilde{s}(\theta)$. Denote by superscripts $\{-,+\}$ the directional limit of a given variable, e.g. $y^f(\tilde{s}(\theta)^-,0)$ stands for the left limit of formal income of the low-cost workers at $\tilde{s}(\theta)$. From the definition of the mapping $\tilde{s}$ we know that $y^f(\tilde{s}(\theta)^-,0) < y^f(\tilde{s}(\theta)^+,0)$.

The perturbation of the formal income of type (θ,∞) decreased the utility of all workers with formal income above $y^f(\theta,\infty)$, including $\tilde{s}(\theta)$, by $dV_\theta(\theta,\infty)$. It means that the perturbation, absent behavioral responses, leads to a discontinuity at $\tilde{s}(\theta)$ in the utility schedule of the low-cost workers, which is not incentive compatible. The behavioral responses will restore the continuity of $V(\theta,0)$ by adjusting the mapping $\tilde{s}(\theta)$. Denote this adjustment by $d\tilde{s}(\theta)$.

Continuity of $V(\theta,0)$ at $\tilde{s}(\theta)$ means that $V(\tilde{s}(\theta)^-,0) = V(\tilde{s}(\theta)^+,0)$. Suppose that the utility of worker $(\tilde{s}(\theta),0)$ is decreased by dT. Continuity of the utility schedule requires that

$$\begin{aligned} V_\theta(\tilde{s}(\theta)^-,0)d\tilde{s}(\theta) &= V_\theta(\tilde{s}(\theta)^+,0)d\tilde{s}(\theta) - dT \\ \implies d\tilde{s}(\theta) &= \frac{w^f(\tilde{s}(\theta))/w^f(\tilde{s}(\theta))}{\rho^f(\tilde{s}(\theta)) - \rho^s(\tilde{s}(\theta))}\frac{dT}{y^f(\tilde{s}(\theta)^+,0) - y^f(\tilde{s}(\theta)^-,0)}. \end{aligned}$$

This adjustment of $\tilde{s}(\theta)$ is associated with a tax loss

$$\Big(T(y^f(\tilde{s}(\theta)^+,0)) - T(y^f(\tilde{s}(\theta)^-,0))\Big) f(\tilde{s}(\theta))G_{\tilde{s}(\theta)}(\tilde{\kappa}(\theta))d\tilde{s}(\theta). \tag{8}$$

Note that $V(\tilde{s}(\theta)^-, 0) = V(\tilde{s}(\theta)^+, 0)$ implies that

$$\frac{T(\tilde{s}(\theta)^+, 0) - T(\tilde{s}(\theta)^-, 0)}{y^f(\tilde{s}(\theta)^+, 0) - y^f(\tilde{s}(\theta)^-, 0)} = 1 - \frac{w^s(\tilde{s}(\theta))}{w^f(\tilde{s}(\theta))}. \tag{9}$$

Using this result, we can express the tax loss as

$$\frac{w^f(\tilde{s}(\theta)) - w^s(\tilde{s}(\theta))}{w^s(\tilde{s}(\theta))} \frac{G_{\tilde{s}(\theta)}(\tilde{\kappa}(\tilde{s}(\theta)))f(\tilde{s}(\theta))}{\rho^f(\tilde{s}(\theta)) - \rho^s(\tilde{s}(\theta))} dT. \tag{10}$$

Notice the dT is equal to $dV_\theta(\theta, \infty)$. Hence, the tax loss is the same as in the previous case, when $y^f(\tilde{s}(\theta), 0) = y^f(\theta, \infty)$.

Impact on workers with higher formal income. First, suppose that $y^f(\theta, \infty) \leq y^f(\bar{\theta}, 0)$. The perturbation implies a shift $dV_\theta(\theta, \kappa)$ in utility levels of formal workers above type θ and shadow workers above $\tilde{s}(\theta)$. Recall that the marginal social welfare weights are equal to the Pareto weights. The fiscal and welfare impact of such change is

$$\int_\theta^{\tilde{s}(\theta)} \int_{\tilde{\kappa}(\theta')}^{\infty} \big(\lambda(\theta', \kappa) - 1\big)\, dG(\kappa) dF(\theta') dV_\theta(\theta, \infty) \\ + \int_{\tilde{s}(\theta)}^{\bar{\theta}} \int_0^\infty (\lambda(\theta', \kappa) - 1) dG(\kappa) dF(\theta') dV_\theta(\theta, \infty). \tag{11}$$

Note that among the productivity types in the segment $(\theta, \tilde{s}(\theta))$ the high-cost workers are affected by the perturbation, but the low-cost worker are not. Hence, the perturbation changes the threshold $\tilde{\kappa}$ at this segment. Denote by $\tilde{\Delta}T(\theta) \equiv T(y^f(\theta, \infty)) - T(y^f(\theta, 0))$ the tax loss from worker of type θ moving to the shadow economy. The fiscal impact of the change in participation is

$$\int_\theta^{\tilde{s}(\theta)} \tilde{\Delta}T(\theta') g_{\theta'}(\tilde{\kappa}(\theta')) dF(\theta') dV_\theta(\theta, \infty). \tag{12}$$

In the case of $y^f(\theta, \infty) > y^f(\bar{\theta}, 0)$ only the formal workers are affected by a tax reform. The total fiscal and welfare impact on agents with higher formal income is

$$\int_\theta^{\bar{\theta}} \left[\int_{\tilde{\kappa}(\theta')}^{\infty} \big(\lambda(\theta', \kappa) - 1\big)\, dG(\kappa) + \tilde{\Delta}T(\theta') g_{\theta'}(\tilde{\kappa}(\theta')) \right] dF(\theta') dV_\theta(\theta, \infty). \tag{13}$$

Collecting the terms. At the optimum, the total impact of a small perturbation is zero. First, consider the case of $y^f(\theta,\infty) \leq y^f(\bar{\theta},0)$. The sum of the distortion cost of a high-cost worker (4), the distortion cost of the low-cost worker (7) as well as of impacts on the workers with higher formal income (11) and (12) needs to be zero, which results in

$$\begin{aligned}
&\frac{T'(y^f(\theta,\infty))}{1-T'(y^f(\theta,\infty))}\frac{(1-G_\theta(\tilde{\kappa}(\theta)))f(\theta)}{\rho^f(\theta)(1+\varepsilon^{-1}(\theta,\infty))}+\frac{w^f(\tilde{s}(\theta))-w^s(\tilde{s}(\theta))}{w^s(\tilde{s}(\theta))}\frac{G_{\tilde{s}(\theta)}(\tilde{\kappa}(\tilde{s}(\theta))f(\tilde{s}(\theta))}{\rho^f(\tilde{s}(\theta))-\rho^s(\tilde{s}(\theta))}\\
&\quad=\int_\theta^{\tilde{s}(\theta)}\left[\int_{\tilde{\kappa}(\theta')}^\infty(1-\lambda(\theta',\kappa))dG_{\theta'}(\kappa)-g_{\theta'}(\tilde{\kappa}(\theta'))\tilde{\Delta}T(\theta')\right]dF(\theta')\\
&\qquad+\int_{\tilde{s}(\theta)}^{\bar{\theta}}\int_0^\infty(1-\lambda(\theta',\kappa))dG_{\theta'}(\kappa)dF(\theta'). \qquad (14)
\end{aligned}$$

If the perturbation affects no shadow workers ($y^f(\theta,\infty) > y^f(\bar{\theta},0)$), the terms (4) and (13) should sum up to zero, which yields

$$\begin{aligned}
&\frac{T'(y^f(\theta,\infty))}{1-T'(y^f(\theta,\infty))}\frac{(1-G_\theta(\tilde{\kappa}(\theta)))f(\theta)}{\rho^f(\theta)(1+\varepsilon^{-1}(\theta,\infty))}\\
&\quad=\int_\theta^{\bar{\theta}}\left[\int_{\tilde{\kappa}(\theta')}^\infty\big(1-\lambda(\theta',\kappa)\big)\,dG_{\theta'}(\kappa)-g_{\theta'}(\tilde{\kappa}(\theta'))\tilde{\Delta}T(\theta')\right]dF(\theta'). \qquad (15)
\end{aligned}$$

A.2. Definitions of sufficient statistics

$\varepsilon^x(\theta)$ and $\tilde{\varepsilon}^x(\theta)$ stand for the formal income elasticity of workers in sector $x \in \{f,s\}$ with respect to the marginal tax rate along the linear and non-linear tax schedule, respectively. $\varepsilon^x_{w^f}(\theta)$ and $\tilde{\varepsilon}^x_{w^f}(\theta)$ stand for the formal income elasticity of workers in sector $x \in \{f,s\}$ with respect to the gross formal wage along the linear and non-linear tax schedule, respectively. The elasticities of formal workers are derived from the optimality condition $y^f(\theta,\infty) = w^f(\theta)(v')^{-1}\big((1-T'(y^f(\theta,\infty)))w^f(\theta)\big)$, while the elasticities of shadow workers are derived from the optimality condition $(1-T'(y^f(\theta,0)))w^f(\theta) = w^s(\theta)$.

The elasticities of formal workers are

$$\varepsilon^f(y^f(\theta,\infty)) \equiv \frac{v'(n(\theta,\infty))}{n(\theta,\infty)v''(n(\theta,\infty))}, \tag{16}$$

$$\tilde{\varepsilon}^f(y) \equiv \left[\frac{1}{\varepsilon^f(y)} + \frac{T''(y)y}{1-T'(y)}\right]^{-1}, \tag{17}$$

$$\varepsilon^f_{w^f}(y) \equiv 1 + \varepsilon^f(y), \tag{18}$$

$$\tilde{\varepsilon}^f_{w^f}(y) \equiv \frac{\tilde{\varepsilon}^f(y)}{\varepsilon^f(y)}\varepsilon^f_{w^f}(y). \tag{19}$$

The elasticities of shadow workers are

$$\tilde{\varepsilon}^s(y) \equiv \frac{1-T'(y)}{T''(y)y}, \tag{20}$$

$$\tilde{\varepsilon}^s_{w^f}(y^f(\theta,0)) \equiv \left(1 - \frac{\rho^s(\theta)}{\rho^f(\theta)}\right)\tilde{\varepsilon}^s(y^f(\theta,0)). \tag{21}$$

Note that shadow workers have infinite elasticities of formal income along the linear tax schedule: as soon as the net formal wage departs from the shadow wage, the shadow worker either stops supplying formal labor entirely or becomes a formal worker. Nevertheless, elasticities along the non-linear tax schedule are well defined, as long the tax schedule is not locally linear.

Denote the derivative of formal income w.r.t. the productivity type along the non-linear tax schedule as

$$\tilde{y}^f_\theta(\theta,\kappa) \equiv \begin{cases} \tilde{\varepsilon}^f_{w^f}(y^f(\theta,\kappa))\rho^f(\theta)y^f(\theta,\kappa) & \text{if } \kappa \geq \tilde{\kappa}(\theta), \\ \tilde{\varepsilon}^s_{w^f}(y^f(\theta,\kappa))\rho^f(\theta)y^f(\theta,\kappa) & \text{otherwise.} \end{cases} \tag{22}$$

The density of formal workers at formal income $y^f(\theta,\infty)$, scaled by the share of formal workers, is defined as $h^f(y^f(\theta,\infty)) \equiv (1 - G_\theta(\tilde{\kappa}(\theta)))f(\theta)/\tilde{y}^f_\theta(\theta,\infty)$. The density of shadow workers at formal income $y^f(\theta,0)$, scaled by the share of shadow workers, is $h^s(y^f(\theta,0)) \equiv G_\theta(\tilde{\kappa}(\theta))f(\theta)/\tilde{y}^f_\theta(\theta,0)$ and $h^s(y^f) \equiv 0$ for $y^f \notin y^f([\underline{\theta},\bar{\theta}],0)$. The density of formal income is simply $h(y) \equiv h^f(y) + h^s(y)$. The mean elasticity at income level y is $\bar{\varepsilon}(y) \equiv h^f(y)\tilde{\varepsilon}^f(y) + h^s(y)\tilde{\varepsilon}^s(y)$.

Define the elasticity of the density of formal workers with respect to the tax burden of staying formal $\tilde{\Delta}T(\theta)$ as

$$\pi(y^f(\theta,\infty)) \equiv \frac{g_\theta(\tilde{\kappa}(\theta))\tilde{\Delta}T(\theta)}{1-G_\theta(\tilde{\kappa}(\theta))}. \tag{23}$$

Define the average welfare weights of formal and shadow workers at a given formal income as

$$\bar{\lambda}^f(y^f(\theta,\infty)) \equiv \int_{\tilde{\kappa}(\theta)}^{\infty} \lambda(\theta,\kappa)\frac{dG_\theta(\kappa)}{1-G_\theta(\tilde{\kappa}(\theta))}, \quad \bar{\lambda}^s(y^f(\theta,0)) \equiv \int_0^{\tilde{\kappa}(\theta)} \lambda(\theta,\kappa)\frac{dG_\theta(\kappa)}{G_\theta(\tilde{\kappa}(\theta))}. \tag{24}$$

Then the average welfare weight at formal income y is $\bar{\lambda}(y) \equiv \left(h^f(y)\bar{\lambda}^f(y) + h^s(y)\bar{\lambda}^s(y)\right)/h(y)$. Finally, the mapping $\theta \mapsto s(\theta)$ is defined as $s(y^f(\theta,\infty)) \equiv y^f(\tilde{s}(\theta),0)$.

A.3. The equivalence of the mechanism design approach and the sufficient statistics approach

By substituting the terms defined above, we can represent the left-hand sides of (14) and (15) as in the sufficient statistics formulas from Theorem 1. In addition, we can represent the right-hand side of (14) as

$$\int_\theta^{\bar{\theta}} \left[1-\bar{\lambda}^f(y^f(\theta',\infty))\right](1-G_{\theta'}(\tilde{\kappa}(\theta')))dF(\theta') + \int_{\tilde{s}(\theta)}^{\bar{\theta}} \left[1-\lambda^s(y^f(\theta',0))\right]G(\tilde{\kappa}(\theta'))dF(\theta')$$
$$- \int_\theta^{\tilde{s}(\theta)} \frac{g_{\theta'}(\tilde{\kappa}(\theta'))\tilde{\Delta}T(\theta')}{1-G_{\theta'}(\tilde{\kappa}(\theta'))}(1-G_{\theta'}(\tilde{\kappa}(\theta')))dF(\theta'). \tag{25}$$

By changing variables we obtain

$$\int_{y^f(\theta,\infty)}^{\infty} \left[1-\bar{\lambda}^f(y)\right]h^f(y)dy + \int_{y^f(\tilde{s}(\theta),0)}^{\infty} \left[1-\bar{\lambda}^s(y)\right]h^s(y)dy - \int_{y^f(\theta,\infty)}^{y^f(\tilde{s}(\theta),\infty)} \pi(y)h^f(y)dy \tag{26}$$

$$= \int_{y^f(\theta,\infty)}^{\infty} \left[1-\bar{\lambda}(y)\right]h(y)dy - \int_{y^f(\theta,\infty)}^{y^f(\tilde{s}(\theta),\infty)} \pi(y)h^f(y)dy. \tag{27}$$

Finally, note that $y^f(\tilde{s}(\theta),\infty) = s(y^f(\theta,\infty)) + \Delta_0(s(y^f(\theta,\infty)))$, which means that the above is equal the right-hand side of the first formula from Theorem 1. We can express the right-hand side of (15) as the right-hand side of the second formula from Theorem 1 analogously.

B. Pareto efficiency test of the Colombian tax schedule

The top panel of Figure I shows the actual tax schedule in Colombia in 2013. The marginal tax rates are high at the bottom due to the phase-out of transfers, then drop to 22% - the rate of payroll taxation - and then increase as the progressive personal income tax starts at around \$22,000. The marginal tax rate reaches 38% around \$50,000 and the top tax rate of 43% applies to incomes above \$175,000.

We can test the efficiency of the actual Colombian tax by extracting Pareto weights which would rationalize it. If at any income level the average Pareto weights are negative, the tax system is inefficient and the government can increase tax revenue without reducing utility of any agent.[1] To extract the welfare weights, differentiate the first optimal tax formula from Theorem 1 to get

$$\bar{\lambda}(y) = \mathbb{E}(\bar{\lambda}) + \underbrace{\frac{\partial DWL(y)}{\partial y}\frac{1}{h(y)}}_{\text{intensive margin}} - \underbrace{\pi(y)\frac{h^f(y)}{h(y)}}_{\text{extensive margin}}, \tag{28}$$

where $DWL(\cdot)$ stands for the total deadweight loss, i.e. the left-hand side of (**??**), at formal income level y. The mean Pareto weight at a given income level can be explained by three components. The first one is the average Pareto weight across all income levels, equal to 1. The second is the contribution of the intensive margin. The total deadweight loss, including both formal and shadow workers, increases faster at income levels associated with higher Pareto weights. That is because higher $\bar{\lambda}(y)$ reduces the deadweight loss below y and does not affect the deadweight loss above y (see (**??**)). The third component captures the extensive margin: a decision to participate in the shadow economy. Recall that $\pi(y)$ is the elasticity of the density of formal workers with respect to the tax burden of staying formal. The impact of extensive margin is similar to that of

[1] The original test of Pareto efficiency was proposed by Werning (2007). The methodology was further developed and applied by Bourguignon and Spadaro (2012); Brendon (2013); Lorenz and Sachs (2016); Jacobs, Jongen, and Zoutman (2017).

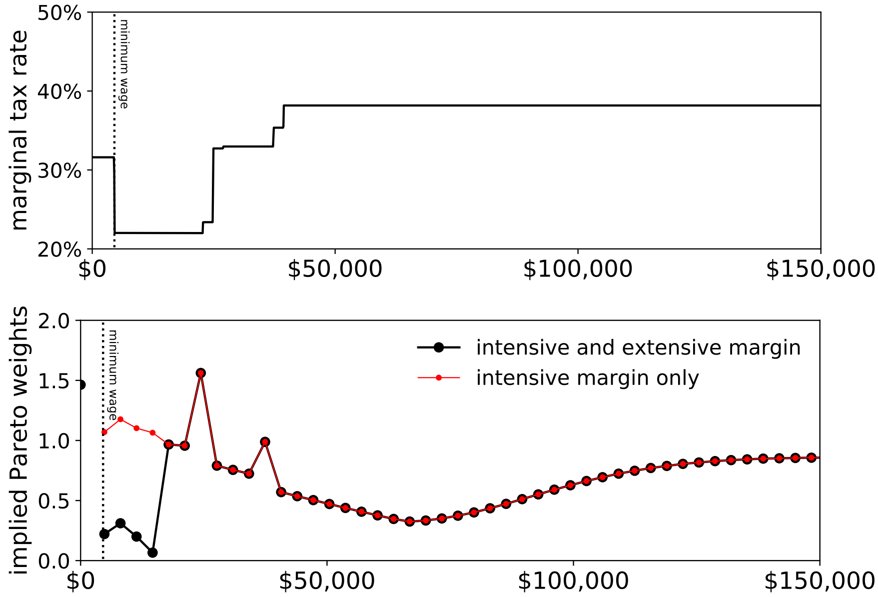
the Pareto weight: it implies a higher derivative of the deadweight loss. Hence, higher $\pi(y)$ means that a smaller part of the increase of deadweight loss remains to be explained by social preferences.

The Pareto weights implicit in the actual tax schedule are presented in the bottom panel of Figure I. We find no evidence of negative Pareto weights - the Colombian tax schedule is Pareto efficient.[2] However, the implicit weights exhibit a peculiar pattern: they are much lower for workers with earnings close to the minimum wage than for workers with slightly higher earnings. For instance, formal workers earning \$13,000 annually have an implied Pareto weight which is seven times smaller than the weight of workers earning \$19,000. Although the income interval of unusually low Pareto weights is relatively small, it contains 28% of all formal workers. None of the workers with formal earnings in this interval has shadow earnings. As shown in the bottom panel of Figure I, the unusually low Pareto weights at low incomes are driven by the extensive margin term in formula (28). If instead we ignore the extensive margin term, the Pareto weights are more regular and decreasing with income at low income levels.[3] A plausible interpretation of this result is that the Colombian tax schedule was set without taking into account the extensive margin incentives for informality. The marginal rate of the actual tax schedule is constant where the Pareto weights are unusually low. The tax schedule which accounts for informality and follows the more intuitive, decreasing schedule of Pareto weights would instead have increasing, rather than flat, marginal rates.

[2] The actual tax schedule is efficient conditional on the value of the minimum wage. Since our framework is not designed to study the minimum wage, we do not wish to evaluate its efficiency.

[3] Pareto weights are also locally increasing when marginal rates of the personal income tax are increasing rapidly. It is unrelated to the informal sector.

Figure I: Income tax in Colombia



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