Hedging with Human Capital in Venture Capital Contracts

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January 22, 2014

Excessive risk taking by entrepreneurs?

- With relative risk aversion of 2, the monetary payoff to a venture-backed entrepreneur has a certainty equivalent slightly greater than 0 (Hall and Woodward 2010)

- Q: Are entrepreneurs necessarily nonrational to accept those contracts?
- A: No, because of...
  - Non-concavity of the value function (Hopenhayn and Vereshchagina 2009)
  - Exogenously lower risk aversion (Kihlstrom and Laffont 1979)
  - Endogenously lower risk aversion (Cressy 2000, Polkovnichenko 2003)

Nonmonetary payoffs

- There may be nonmonetary payoffs that
  - enhance future income of entrepreneur and work as a hedge against the risk of venture failure
  - e.g. experience, prestige, skills.
- Those nonmonetary payoffs (further human capital) may depend on the entrepreneur's action $\rightarrow$ a moral hazard problem.

- Why would optimal contract prescribe accumulation of human capital?
  - Under DARA utility, human capital decreases risk aversion of the entrepreneur, which reduces the cost of providing incentives.

Contribution

- Explain the apparent risk taking behaviour of entrepreneurs with nonmonetary payoffs.
- Propose a model that matches styles fact of the venture capital industry
    - Riskiness of monetary compensation, backloaded compensation, financing rounds
- Identify an additional role of risk-free compensation in a moral hazard models.
- Propose a novel moral hazard model, solution of which does not rely on first order approach.

Model

- Standard moral hazard model: agent's effort affects distribution function of outcomes over a fixed support.
- Proposed moral hazard model: agent's effort affects the support of the distribution of output, while probabilities are fixed.
- Advantages
  - Does not rely on the first order approach.
  - Intuitive intepretation: the agent chooses a stake in a lottery with fixed probabilities.

Timing

1. Risk neutral investor provides resources I.
2. Risk averse entrepreneur with utility u invests resources in project capital k and human capital h with linear technology: $k + (h - h_0) \leq I$.
3. Venture's value $y(k)$ is revealed and collected.
4. Entrepreneur receives compensation from the investor $\tau(y)$ and labor income wh.

Venture

- The value of the venture is a random variable $y = \alpha k$, where
  - $k \in [0, 1]$ is project capital
  - α is a random variable with support $A \in \mathbb{R}_+$
    - $\min A = 0$ with a positive mass/density
    - $\max A = \bar{\alpha}$.

Assumption: $E\alpha > w > 1$.

Contract

- A feasible contract consists of provided resources $I \geq 0$ and a function of the monetary compensation $\tau : [0, \bar{\alpha}] \to \mathbb{R}_+$.
- An incentive feasible contract is a feasible contract that additionally includes scalars denoting the recommended size of investment in each stock: $k \in [0;1]$, $h \geq h_0$ and that satises the incentive compatibility constraint
$(k, h) \in \arg\max_{k,h} \mathbb{E}u\left[\tau + wh\right]$ s.t. $I \geq k + h$

- Assumption: *τ is non-negative and does not depend on h.*

Entrepreneur's problem

$$\max_{k\in[0,1],h\geq h_0} p_0 u\left[\tau\left(0\right)+wh\right]+p_1 u\left[\tau\left(\bar{\alpha}k\right)+wh\right]+p_2 u\left[\tau\left(2\bar{\alpha}k\right)+wh\right]$$

s.t.

$$I \quad \geq \quad k+\left(h-h_0\right)$$

Investor's problem

$$\Pi = \max_{\substack{k \in [0,1], h \geq h_0 \\ \tau : [0, \bar{\alpha}] \to \mathbb{R}_+}} E\{\alpha k - \tau(\alpha k)\} - k - (h - h_0)$$

- s.t. incentive compatibility constraint

$$\forall_{\tilde{k} \in [0,1]} U_k \geq U_{\tilde{k}}$$

where
$$U_{\tilde{k}} = p_0 u \left[\tau(0) + w\left(h + k - \tilde{k}\right)\right] + p_1 u \left[\tau\left(\bar{\alpha}\tilde{k}\right) + w\left(h + k - \tilde{k}\right)\right] + p_2 u \left[\tau\left(2\bar{\alpha}\tilde{k}\right) + w\left(h + k - \tilde{k}\right)\right]$$

First-best

Suppose that the investor controls the allocation of resources. The first best contract involves:

- maximal project capital accumulation $k = 1$,
- no monetary compensation to the entrepreneur $\forall_y \tau(y) = 0$,
- no human capital accumulation $h = h_0$.

First-best cannot be approximated

Proposition

The profits in the moral hazard case are lower than profits in the first-best by at least $\max\{\mathbb{E}\alpha - 1, w\} > 0$.

- Mirrlees (1999) result does not apply
  - $\tau(y) \geq 0$ and the investor is unable to punish entrepreneur severly.

Discretization

Assumption

α takes a finite number of values, here $\alpha = \begin{cases} 0 & \textit{with prob. } p_0 \\ \bar{\alpha} & \textit{with prob. } p_1 \\ 2\bar{\alpha} & \textit{with prob. } p_2 \end{cases}$.

Lemma

Under the above assumption, the solution to the moral hazard problem can always be attained with a 'discrete 'compensation function

$$\tau(y) = \begin{cases} \tau_2 & \textit{if } y = 2\bar{\alpha}k \\ \tau_1 & \textit{if } y = \bar{\alpha}k \\ \tau_0 & \textit{if } y = 0 \\ 0 & \textit{otherwise} \end{cases}.$$

Discretization

- Investor's problem can be equivalently defined as

$$\Pi = \max_{\substack{k \in [0,1],\, h \geq h_0 \\ (\tau_i)_{i=0}^{2} \geq 0}} \mathbb{E}\alpha k - \sum_{m=0}^{2} p_m \tau_m - k - (h - h_0)$$

s.t.

$$U_k \geq \max\left\{U_0, U_{\frac{1}{2}k}, U_{2k}\right\}$$

- Entrepreneur chooses between four different stakes in a venture lottery.

Properties of optimal contract

Proposition

In the optimal contract the following holds:

1. *The constraint $U_k \geq U_{2k}$ is never binding.*
2. $\tau_2 \geq \tau_1$.
3. τ_0 *is equal to 0, while* τ_2 *is greater than 0 when* $k > 0$.

- The most risky lottery U_{2k} is never optimal.
- The monetary compensation is nondecreasing, conditional on project capital k.

Finding optimal contract in 2 steps

A cost function $C(k)$ is a solution to the problem of minimizing cost of implementing the project capital k

$$C(k) = \min_{\tau_1 \geq 0, \tau_2 \geq 0, h \geq h} k + (h - h_0) + p_0\tau_0 + p_1\tau_1 + p_2\tau_2$$

s.t.

$$U_k \geq \max\left\{U_0, U_{\frac{1}{2}k}, U_{2k}\right\}$$

Let's look for optimal contract in 2 steps:

1. For each level of project capital $k \in [0,1]$, find the minimal cost $C(k)$ of implementing k .
2. Choose the most profitable k: $\max_{k\in[0,1]} E\{\alpha k - C(k)\}$.

Binding constraints

Proposition

Fix $k > 0$. Keeping p_0 constant, there are two regions based on the value of p_2, determined by a treshold value $\bar{p}_2 \in (0, 1)$.

- *For $p_2 \leq \bar{p}_2$, only the constraint $U_k \geq U_0$ is binding and $\tau_2 = \tau_1 > 0$.*
- *For $p_2 > \bar{p}_2$, the constraint $U_k \geq U_{\frac{1}{2}k}$ is binding and $\tau_2 > \tau_1 > 0$.*

- If p_2 is low, entrepreneur is tempted only by pure risk-free income.
- If p_2 is high, entrepreneur is tempted by a small stake in the lottery.

Necessary condition for $h > h_0$

Proposition

Suppose that $U_k \geq U_0$ is the only binding constraint. In the optimum $h > h_0$ only if u is exhibits decreasing absolute risk aversion.

- Trading higher h for lower τ can work only when u is DARA.
  - When u is DARA, higher h decreases absolute risk aversion.
  - With lower absolute risk aversion, entrepreneur demands less incentives to engage in the venture lottery.
  - Hence, the investor can lower the lottery prize τ.

Necessary condition for $h > h_0$

Proof

- Suppose that only $U_k \geq U_0$ binds and consider a problem of minimizing cost of implementing a given k.
- Set τ such that the incentive constraint is satisfied at h_0.
- Consider increasing h and decreasing τ such that the incentive constraint is satisfied. It is profitable only if

$$\frac{w}{u'\left[\tau + wh_0\right]}\left(p_0 u'\left[wh_0\right] + \left(p_1 + p_2\right) u'\left[\tau + wh_0\right] - u'\left[w\left(h_0 + k\right)\right]\right) > 1$$

- However, $\frac{\partial}{\partial h} LHS = w \frac{\partial}{\partial h} \frac{u'[wh_0]}{u'[wh_0+\tau]} \left(p_0 - \frac{u'[w(h_0+k)]}{u'[wh_0]}\right) \geq 0$ when $\frac{\partial}{\partial c} \frac{u'[c]}{u'[c+\epsilon]} = \frac{u'[c]}{u'[c+\epsilon]} \left(\frac{u''[c]}{u'[c]} - \frac{u''[c+\epsilon]}{u'[c+\epsilon]}\right) \geq 0$ for $\epsilon > 0$.
- It leads to $\tau = 0$, which contradicts the incentive constraint.

Sufficient conditions for $h > h_0$

Proposition

Suppose that $U_k \geq U_0$ is the only binding constraint and $\frac{w-1}{w} \geq p_0 \geq \frac{u'[w(h_0+k)]}{u'[wh_0]}$ holds. If u is DARA, in the optimum $h > h_0$. If u is not DARA, the solution does not exist.

Proof

- By assumption the perturbation is profitable at h_0: $p_0 u'[wh_0] - u'[w(h_0+k)] > u'[wh_0+\tau]\left(p_0 - \frac{w-1}{w}\right)$
- Moreover, LHS is positive, while RHS is negative.
- The sign of LHS can be changed by increasing h if u is DARA.
    - By DARA $p_0 \geq \frac{u'[w(h_0+k)]}{u'[wh_0]} > \frac{u''[w(h_0+k)]}{u''[wh_0]}$, while $\frac{\partial}{h} LHS < 0 \Leftrightarrow p_0 > \frac{u''[w(h_0+k)]}{u''[wh_0]}$.

Examples of optimality conditions

CRRA(σ)

$$(wh)^{-\sigma}\left(p_0-\left(\frac{wh}{w\,(h+k)}\right)^{\sigma}+\left(p_1+p_2-\frac{1}{w}\right)\left(\frac{wh}{\tau+wh}\right)^{\sigma}\right)\leq 0.$$

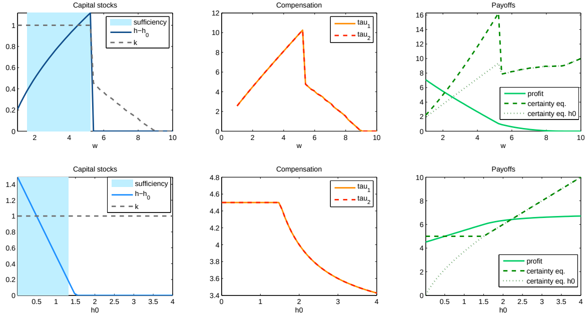
Optimality condition can be satisfied by increasing h.

CARA(γ)

$$e^{-\gamma h\tau}\left(p_0+\left(p_1+p_2-\frac{1}{w}\right)e^{-\gamma\tau}-e^{-\gamma wk}\right)\leq 0.$$

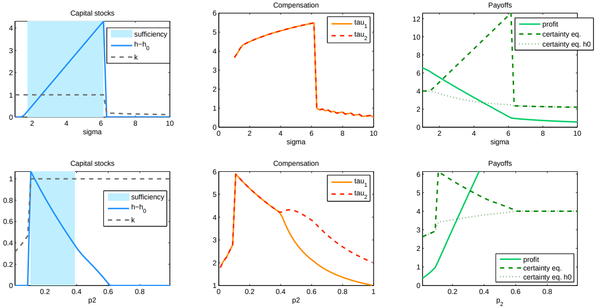
h does not affect the sign of LHS.

Comparative statics in w and h_0



$$u = \frac{c^{1-\sigma}}{1-\sigma}, \sigma = 2, w = 2, p_0 = p_1 = p_2 = \frac{1}{3}, h_0 = 1, \bar{\alpha} = 10,$$

Comparative statics in σ and p_2



$$u = \frac{c^{1-\sigma}}{1-\sigma}, w = 2, \sigma = 2, p_0 = p_1 = \frac{1-p_2}{2}, h_0 = 1, \bar{\alpha} = 10$$

Standard moral hazard

$$\max_{\substack{k>0,\, h\geq h_0 \\ (\tau_i)_{i=1}^{I}\geq 0}} \sum_{i=1}^{I} p_i\left(k\right)\left(y_i-\tau_i\right)-k-\left(h-h_0\right)$$

s.t.

$$\forall_{\tilde{k}\in[0,1]} \sum_{i=1}^{I} p_i\left(k\right) u\left[\tau_i+wh\right] \geq \sum_{i=1}^{I} p_i\left(\tilde{k}\right) u\left[\tau_i+w\left(h+k-\tilde{k}\right)\right]$$

- If FOA is valid, then only one incentive constraint

$$\sum_{i=1}^{I} p_i'\left(k\right) u\left[\tau_i+wh\right] \geq w\sum_{i=1}^{I} p_i\left(k\right) u'\left[\tau_i+wh\right].$$

Necessary and sufficient conditions for $h > h_0$

Proposition

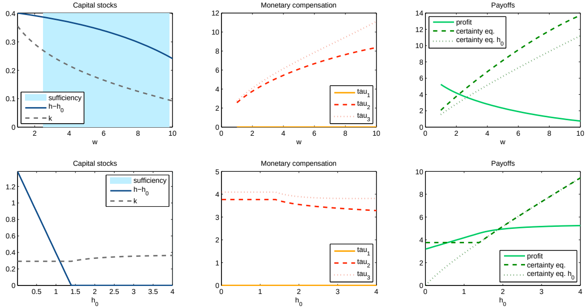
Fix k. Suppose that FOA is valid.

1. *When u is CARA, in the optimum* $h = h_0$.
2. *In the optimum* $h > h_0$ *only if* $-\frac{u'''[wh_0]}{u''[wh_0]} > -\frac{1}{w}\frac{p_0'}{p_0}$.
3. *When* $p_0 \leq \frac{w-1}{w}$, *in the optimum* $h > h_0$ *only if u is DARA and* $-\frac{u'''[wh_0]}{u''[wh_0]} > -\frac{1}{w}\frac{p_0'}{p_0}$.

Proposition

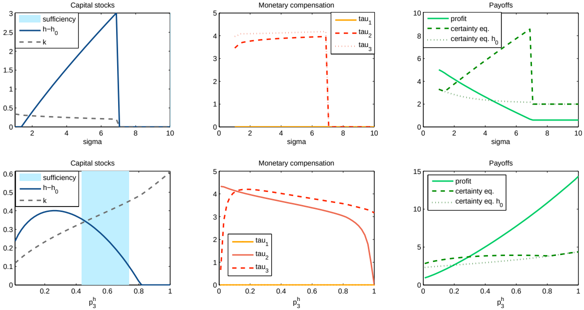
Fix k. Suppose that FOA is valid, u exhibits DARA and the optimum exists. In the optimum $h > h_0$, *if* $-\frac{u''[wh_0]}{u'[wh_0]} > -\frac{1}{w}\frac{p_0'}{p_0}$ *and* $p_0 \leq \frac{w-1}{w}$.

Comparative statics in w and h_0



$$p_i(k) = e^{-\rho k} p_i^l + \left(1 - e^{-\rho k}\right) p_i^h, p^l = (0.95, 0.04, 0.01), p^h = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

Comparative statics in σ and p_2



$$p_i(k) = e^{-\rho k} p_i^l + \left(1 - e^{-\rho k}\right) p_i^h, p^l = (0.95, 0.04, 0.01), p^h = \left(\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}\right)$$

Summary

- Venture capital contracts are likely to be less risky to the entrepreneurs because of nonmonetary payoffs.
- Those nonmonetary payoffs can lead to moral hazard problem.
- Even in that case, human capital can be employed in order to minimize the cost of providing incentives to the entrepreneur.
  - It is likely to happen when the initial human capital is low, while the return on human capital is high.
- The model can extended to match stylized fact of the venture capital industry
  - Backloaded compensation, financing rounds $\rightarrow$ check the paper!