Hedging with Human Capital in Venture Capital Contracts

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Abstract

The aim of this paper is to identify a new role for the provision of insurance in the principal - agent framework, when the principal wants the agent to engage in some risky, yet potentially profitable, activity. When the agent's preferences exhibit decreasing absolute risk aversion, the principal may provide some risk-free income in order to decrease risk aversion of the agent and reduce the cost of incentives. In other words, more insurance makes it easier to persuade the agent to perform a risky action. This idea is applied to the case of an entrepreneur obtaining financing from a venture capital fund. Hall and Woodward (2009) show that a certainty equivalent of monetary payoffs to entrepreneurs in such contracts is close to 0. However, there is empirical evidence that entrepreneurs gain also in terms of higher future reemployment wages. This paper discusses the moral hazard problem in which the entrepreneur can divert resources provided to the venture by the investor by accumulating a personal human capital. When the entrepreneur's preferences exhibit decreasing absolute risk aversion, the investor may allow for the human capital accumulation in order to strategically control the risk aversion of the agent and reduce the cost of providing incentives. The human capital accumulation due to this incentive reason is likely, when the entrepreneur's human capital is valuable and initially scarce.

1 Introduction

The aim of this paper is to identify a new role for the provision of insurance in the principal - agent framework. Suppose that the principal wants the agent to engage in some risky, yet potentially profitable, activity. The principal can affect the agent's decision only through the design of a contract. However, the contract cannot depend directly on the choice of the agent, but rather on its imperfect signal. It is a standard result in the moral hazard literature that in the optimal contract the compensation of the agent is high when the signal indicates that the agent behaved according to the principal's plan, and low otherwise. Hence, the optimal contract exposes the agent to risk. The optimal incentive scheme may also involve some insurance - a risk-free income for the agent. This can happen even when the participation constraint of the agent is not considered. Since the principal persuades the agent to engage in a risky activity and the agent is risk averse, the principal has to pay an additional risk premium. If the agent's preferences exhibit decreasing absolute risk aversion, the principal can control the level of risk aversion. Specifically, by providing more insurance, the principal lowers the risk aversion of the agent, and so reduces the required risk premium. Therefore, the

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provision of insurance allows the investor to provide the incentives in a cheaper manner. This idea is applied to the case of entrepreneurs obtaining financing from venture capital funds.

Hall and Woodward (2009) show that an entrepreneur backed by venture capital receives a monetary payoff so risky that under a reasonable level of risk aversion it has a certainty equivalent only slightly bigger than zero. However, Mokyr (2010) points out that the calculation of benefits and risks of entrepreneurship should include also nonmonetary payoffs. Suppose that entrepreneurs, in the process of managing an innovative start-up, gain some value, relevant experience or skills that enhance their value on the job market. It is plausible that those nonmonetary gains are not as risky as the monetary payoffs and hence can constitute some, though most likely imperfect, insurance against the risk of venture failure. Such benefits can strongly alter the risk-adjusted valuation of the compensation to the risk averse entrepreneur. There is evidence (Evans and Leighton (1989), Hamilton (2000)) that employees that leave their jobs and enter entrepreneurship receive, after returning to salary job, wages not lower than employees that stay at the salary job for the whole time. This happens even though the latter group observes wage increase with tenure. It indicates that the entrepreneurs obtain some value that outweighs the shorter tenure at the job.[1] This additional value can be simply a by-product of involvement in the venture, such as prestige or experience. On the other hand, it can also be an asset, the acquisition of which can, to a certain extent, be controlled, such as skills. Let's call this asset human capital. Suppose that acquiring the human capital involves a cost of resources or time. Thus, a trade-off emerges, in which resources or time can be allocated to either developing the venture or acquiring the entrepreneur's human capital. Benefits of the entrepreneur's human capital are different for the entrepreneur and the venture capitalist, since the investor cannot claim the future labor income of the entrepreneur. The entrepreneur and the investor face different trade-offs, so this becomes a moral hazard setting.

This paper describes the contract between the risk averse entrepreneur and the risk neutral investor. The investor provides resources that can be spent on accumulation of two types of assets. The first one is the project capital, which determines the value of the venture. The innovative venture is a risky business, so the investment in the project capital bears significant amount of risk. The second investment opportunity is the personal human capital of the entrepreneur, which determines the wage the entrepreneur can get at the labor market. The human capital bears no risk and is an efficient mean of providing safe income for entrepreneur, as it yields higher return than money. It is assumed that the entrepreneur decides upon the allocation of resources, while the investor can affect this decision only through design of the contract. The contract cannot depend directly on the investment decision of the entrepreneur, which is either unobservable or uncontractible, but it can depend on the realized value of the venture. In the optimal contract, the entrepreneur receives higher income in the states which indicate that the entrepreneur behaved in line with the will of the investor, and lower income otherwise. Apart from this reward scheme, the optimal contract may include the risk-free income, so the human capital gain. There could be several reasons for this. The total gain for the entrepreneur from engaging in the venture may have to be sufficiently high due to the outside option. Alternatively, the human capital may also have a positive impact on the outcome of the venture. I show that there is another reason for the positive human capital accumulation. When the entrepreneur's preferences exhibit decreasing absolute risk aversion, the additional insurance through the risk-free income

[1] In both studies all self-employed are identified as entrepreneurs. Levine and Rubinstein (2013) criticize this approach by showing that the group of self-employed consist of two types of individuals. The first type are innovators engaged in risky ventures, while the second are small business owners that did not succeed as salary workers. It suggests that entrepreneurs that can benefit from managing an innovative start-up are confused with not productive small business owners, and hence the return to entrepreneurship in terms of reemployment wage may be underestimated.

may reduce the cost of providing incentives. Since in the model the cheapest mean of providing a certain income to the entrepreneur is the human capital, this is the additional rationale for the human capital accumulation. In order to isolate this particular effect, throughout most of the paper it is assumed that the human capital does not affect the profitability of the venture and that the participation constraint of the entrepreneur is not binding. Then the first-best case, in which the investor has full control over the allocation of resources, involves no accumulation of the human capital. In the optimal contract in the moral hazard case the human capital accumulation is used in order to strategically manipulate the risk aversion of the entrepreneur and minimize the cost of incentives. I provide necessary and sufficient conditions for this to happen. Intuitively, the positive accumulation of the human capital is likely to be a part of the optimal contract when the initial level of the human capital of the entrepreneur is low and the return on investment in the human capital is high.

There are other explanations to the apparent excessive risk-taking of the entrepreneurs. Vereshchagina and Hopenhayn (2009) show that discrete choice between entrepreneurship and a salary job can lead to local convexity of the value function. In this environment, some lotteries, interpreted as entrepreneurial projects, are welfare improving. Kihlstrom and Laffont (1979) claim that people with inborn lower risk aversion self-select to entrepreneurship. Cressy (2000) points out that, with a utility function exhibiting decreasing absolute risk aversion, wealthier agents are endogenously less risk averse. Wealth provides an insurance in case of failure and induces entrepreneurs to take more risk. This argument was extended by Polkovnichenko (2003), who was able to explain most of the private equity puzzle with insurance provided by human capital, interpreted as the value of the future labor earnings. In his model, human capital is exogenous and the entrepreneurs self-finance their projects. In contrast to that paper, here the human capital is an endogenous variable determined in the optimal contract. Moreover, the project is externally financed by the venture capital fund, which justifies the use of the principal agent framework.

This paper also proposes a new model of moral hazard. Typically it is assumed that the agent, through the unobservable action, affects the probability distribution of output over the fixed support. I will refer to this approach as the standard model.[2] In this paper, the agent's effort, i.e. investment in the project capital, affects the support of the distribution of output. In other words, by varying the level of effort, the agent scales the distribution of output. The entrepreneur chooses the project capital investment, which is equivalent to choosing a stake in the lottery. Apart from the intuitive interpretation, the proposed framework is attractive because it does not require the first order approach to be valid. The standard model with continuous effort can be analytically solved under the assumption that the agent is tempted only by the local deviations from the prescribed action. It puts a tight constraint on problems that can be analyzed.[3] In contrast, the proposed approach allows to consider models in which large, nonlocal deviations are binding, e.g. not exerting any effort at all. The solution to this problem is in general an infinite dimensional object, since the monetary compensation of entrepreneur is a real function over an interval. Moreover, there is a continuum of incentive compatibility constraints. In order to solve the model, I assume that the venture can produce only a finite number of outcomes. Then Lemma 1 demonstrates that the optimal contract simplifies the continuous choice of effort of the entrepreneur by practically making it discrete - only a finite number of deviations will be tempting for the entrepreneur, so only a finite number of incentive-compatibility constraints will be

[2] For the review of this approach, see e.g. Bolton and Dewatripont (2005).

[3] Rogerson (1985) proved the validity of the first order approach in the static model with additive disutility from effort under the monotone likelihood ratio and the convex distribution function conditions. For the validity of the first order approach in other types of principal-agent model, see Abraham, Koehne, and Pavoni (2011).

binding. The optimal contract, originally of infinite dimentionality, can be represented by a finite number of scalars. Intuitively, the investor is able to reverse-engineer and prevent most deviations. However, there is a finite number of deviations that cannot be easily prevented and the corresponding incentive-compatibility constraints will be binding in the optimal contract. Besides the finiteness of the set of outcomes, which is also a feature of the standard model, this approach requires no additional assumptions on the probability distribution. I characterize the optimal contract in the case, when the venture lottery can take 3 values. Without any additional assumptions, the compensation of the agent is nondecreasing in the output of the venture, conditional on the agent choosing an incentive compatible action.

The following two sections describe the setting and the properties of the optimal contract. Section 4 analytically and numerically explains the role of human capital in the optimal contract. Section 5 demonstrates that those results are robust across models - the human capital accumulation works in the similar manner in the standard model as well. A few extensions of the proposed model that allow it to match stylized facts of the venture capital industry are presented in Section 6.

2 Model

The model is static. The risk neutral investor provides the risk averse entrepreneur with resources that can be invested in the risky venture or in the risk-free human capital of entrepreneur. After the venture value is realized, the entrepreneur receives a monetary compensation, goes to the job market and earns a wage on the human capital.

The entrepreneur is endowed with no wealth, with some initial stock of human capital h_0 and with an idea for a risky, but potentially profitable business. The entrepreneur's preferences over consumption are described by the expected utility function $\mathbb{E}u[c]$, where u is a twice differentiable, strictly increasing and strictly concave function. In order to finance the venture, the entrepreneur joins forces with a risk neutral investor with deep pockets. The investor provides the entrepreneur with resources I and the monetary compensation τ, which may depend on the value of the venture. The entrepreneur can spent resources on accumulation of two stocks: project capital k and human capital h. The accumulation of both stocks is linear. The project capital influences the value of the venture in a way specified below. The income of entrepreneur in the job market is equal to wh, where w is a unit wage on the human capital. Therefore, the consumption of the entrepreneur is the sum of a potentially risky income from the venture τ and a risk-free wage on human capital on the job market.

The value of the venture is equal to $y = \alpha k$, where α is a random productivity of project capital. α is drawn from the set $A \subseteq \mathbb{R}_+$. The minimum of A exists and is equal to 0, so in the worst case the project capital is worthless. The probability density, or the probability mass function, is positive in this point. The maximum of A exists and is denoted by $\bar{\alpha}$. In order to avoid the cases of no solution due to unbounded accumulation of k, it is assumed that k belongs to the segment $[0,1]$. Notice that it implies that, without conditioning on the choice of k, y can take any value from the set $[0,\bar{\alpha}]$. I assume that $\mathbb{E}\alpha > w > 1$, which means that the expected return on the project capital is greater than return on the human capital, which in turn is greater then the cost of resources. If the first inequality does not hold, the investor cannot persuade the entrepreneur to accumulate any project capital at all without suffering losses. The second assumption means that it is cheaper to provide a risk-free income to the entrepreneur with the human capital than with money. It also implies that if the entrepreneur had some initial monetary wealth, this money would be spent on human capital accumulation.

To close the model, I assume that the outside option of the entrepreneur is to go to the job market with the initial level of human capital h_0. It implies that through the main body of the paper the participation constraint of the entrepreneur is slack - there is no depreciation of h and the entrepreneur goes to the job market no matter if the contract is signed or not. However, joining the venture can result in higher level of human capital. The fact that the entrepreneur earns a wage on the general human capital in period 1 does not imply that the entrepreneur leaves the project after the value is created. It rather indicates that the role of the entrepreneur changes from the innovator that develops a new technology to the CEO who manages an established company.

Some of the above assumptions are relaxed in Section 6. Specifically, it is shown that the framework is equivalent to a model in which the entrepreneur decides upon the allocation of personal effort or time, rather than resources. The model can be applied to the case when the success of the venture affects positively the human capital of the entrepreneur. Adding a time dimension to the problem reconciles the model with two common features of the venture capital contracts. First, due to a binding participation constraint, a compensation of the entrepreneur is backloaded. Secondly, if the entrepreneur can leave the venture prematurely, the investor has incentives to provide resources to the venture gradually, in a sequence of financing rounds.

2.1 Contract

The contract that can depend only on the realized value of the venture. Specifically, τ cannot directly depend on unobservable k and h. Note that h is unobservable in the venture, but not on the job market. This assumption means that the investor cannot make the payments to the entrepreneur depended on the entrepreneur's subsequent income. Additionally, non-negativity of monetary payoffs to the entrepreneur is imposed. The investor is limited in the ability to punish the entrepreneur, because the consumption of the entrepreneur cannot be lower than wh_0. In other words, the investor cannot legally claim future labor income of entrepreneur. Given those assumptions, the compensation of the entrepreneur is a function from a set of possible output values to the non-negative real numbers $\tau : [0, \bar{\alpha}] \to \mathbb{R}_+$.

Definition A *feasible contract* consists of an amount of provided resources $I \geq 0$ and a monetary compensation function $\tau : [0, \bar{\alpha}] \to \mathbb{R}_+$.

Definition An *incentive feasible contrac*t is a feasible contract that additionally includes the recommended size of each capital stock: $k \in [0,1]$, $h \geq h_0$ that satisfy the incentive compatibility constraint $(k,h) \in \arg\max_{k,h} \mathbb{E}u[\tau + wh]$ s.t. $I \geq k + h$.

2.2 First-best

If the investor controls the investment decision within the venture[4], the optimal contract is a solution to the following profit maximization problem

[4]In this environment it is not enough that the investor observes the action of the entrepreneur, since the non-negativity of τ severely restricts investor's ability to punish deviations.

$$\Pi^{fb} = \max_{\substack{k\in[0,1],h\geq h_0\\ \tau:[0,\bar{\alpha}]\to\mathbb{R}_+}} \mathbb{E}\{\alpha k-\tau(\alpha k)\}-k-(h-h_0)$$

$$\text{s.t.} \quad \mathbb{E}u[\tau(\alpha k)+wh] \quad \geq \quad u[wh_0]$$

Note that minimizing cost of the investor by setting h equal to h_0 and τ always to 0 makes the participation constraint satisfied as equality. It means that this constraint is always slack[5]. The first-best allocation involves the maximal project capital accumulation: $k=1$, no human capital accumulation: $h=h_0$ and no monetary payments to the entrepreneur $\forall_y \tau(y)=0$.

3 Optimal contract under moral hazard

The following section describes the optimal contract under moral hazard, i.e. when the entrepreneur controls the allocation of the resources within the venture. The set of contracts available to the investor is restricted by the incentive compatibility constraints. First, it is shown that the first-best allocation can be neither attained, nor approximated in the environment with moral hazard. Further, it is demonstrated how to derive an optimal contract when α has a finite support, and the properties of the optimal contract are described.

3.1 The first-best cannot be approximated

The optimal contract in the moral hazard case is a solution to the following problem

$$\Pi = \max_{\substack{k\in[0,1],h\geq h_0\\ \tau:[0,\bar{\alpha}]\to\mathbb{R}_+}} \mathbb{E}\{\alpha k-\tau(\alpha k)\}-k-(h-h_0) \tag{1}$$

s.t.

$$\mathbb{E}\{u[\tau(y)+wh]\,|k^e=k\} \quad \geq \quad \mathbb{E}\left\{u\left[\tau(y)+w\left(h+k-\tilde{k}\right)\right]|k^e=\tilde{k}\right\},\forall\tilde{k}\in[0,1]$$

$\mathbb{E}\left\{u\left[\tau(y)+w\left(h+k-\tilde{k}\right)\right]|k^e=\tilde{k}\right\}$ denotes the expected utility of the entrepreneur when the provided resources are $k+h-h_0$ and the project capital chosen by entrepreneur, k^e, is equal $\tilde{k}$. The constraint means that the expected utility of the entrepreneur under the investment decision prescribed by the investor is not lower than any other possible investment strategy. In other words, the investment in the capital stocks are made in concordance with self-interest of the agent, who takes the monetary compensation and amount of resources as given.

Proposition 1. *The profits in the moral hazard case are lower than profits in the first-best by at least* $\max\{\mathbb{E}\alpha-1,w\}>0$.

Proof Let's examine the maximization problem similar to (1), but with the relaxed incentive compatibility constraint: the sole deviation available to the entrepreneur is not accumulating the project capital at all. Such

[5] I'll come back to this issue in Section 6, when the investor will be equipped with more instruments and hence able to make this constraint binding.

problem is following

$$\Pi^r = \max_{\substack{k\in[0,1],h\geq h_0\\ \tau:[0,\bar{\alpha}]\to\mathbb{R}_+}} \mathbb{E}\{\alpha k - \tau(\alpha k)\} - k - (h-h_0)$$

$$\text{s.t.} \quad \mathbb{E}u[\tau(\alpha k)+wh] \quad \geq \quad u[\tau(0)+w(h+k)].$$

Since this is a relaxed problem, profits Π^r are not smaller than Π, profits of the full problem. Let's consider the difference between Π^{fb}, profits at the first-best allocation, and Π^r. The total difference in profits is decomposed into three differences.

$$\Pi^{fb}-\Pi^r \quad = \quad \underbrace{(\mathbb{E}\alpha-1)(1-k^r)}_{\delta_k} \quad + \quad \underbrace{h^r-h_0}_{\delta_h} + \quad \underbrace{\mathbb{E}\{\tau(\alpha k^r)\}}_{\delta_\tau}$$

Notice that each δ is bounded below by 0. If all of them are simultaneously equal to zero, the first best allocation is attained. Notice that with the Jensen's inequality we can bound from above the left hand side of the relaxed incentive constraint: $u[\delta_\tau+w(\delta_h+h_0)] > \mathbb{E}u[\tau(\alpha k)+wh]$. The right hand side of the constraint can we rewritten as $u\left[\tau(0)+w(\delta_h+h_0)-w\left(\frac{\delta_k}{\mathbb{E}\alpha-1}-1\right)\right]$. Then the necessary condition for the relaxed constraint to hold is

$$\delta_\tau+\frac{w}{\mathbb{E}\alpha-1}\delta_k > \tau(0)+w$$

Since $\tau(0)$ is non-negative and w is positive, it is impossible to simultaneously decrease both differences δ_τ and δ_k to 0. Therefore Π^{fb} is strictly greater than Π^r. Suppose that the investor can freely choose δ_τ and δ_k, as long as they satisfy the above inequality. If $\mathbb{E}\alpha-1\geq w$, the investor would choose $\delta_\tau=0$ and $\delta_\tau > \mathbb{E}\alpha-1$, while in the case $w>\mathbb{E}\alpha-1$, $\delta_\tau > w$ and $\delta_k=0$ would be chosen. It implies that $\Pi^{fb}-\Pi^r > \max\{\mathbb{E}\alpha-1,w\} > 0$. Therefore, any incentive feasible allocation of the relaxed problem, and hence the full problem, is bounded away in terms of profits from the first best allocation. **Q.E.D.**

Mirrlees (1999) showed that when the utility function of the agent is unbounded below and the probability of the worst outcome is falling rapidly with the agent's effort, the first-best contract can be approximated arbitrarily well in the moral hazard setting. The compensation scheme that achieves this result consists of a severe punishment in the case of the worst outcome and the first-best allocation elsewhere. The agent is trying to avoid the punishment by exerting effort. Since the probability of the punishment is sufficiently sensitive to the level of effort, the probability of the worst outcome becomes arbitrarily small. Such outcome is not possible in the considered framework - the moral hazard allocation is bounded away from the first-best allocation. Notice that none of the Mirrlees assumptions holds here: the utility of the entrepreneur is bounded below by $u[wh_0]$, while the probability of the bad outcome, $\alpha=0$, is fixed and non negligible.

3.2 Optimal contract

In order to solve for the optimal contract, a finite support of the productivity shock is assumed.

Assumption 1 $A=\left\{0,\frac{\bar{\alpha}}{2},\bar{\alpha}\right\}$, $\bar{\alpha}>0$.

Let's denote the probabilities of productivity realizations by, respectively, p_0, p_1 and p_2. Under Assumption 1, the investor can easily detect most of the deviations from the prescribed investment plan. When the entrepreneur invests according to the will of the investor, only three values of output are possible: 0, $\frac{\bar{\alpha}}{2}k$ or $\bar{\alpha}k$, where k is the level of project capital chosen by the investor. Hence, it is intuitive that the investor will not compensate the entrepreneur, should any other value of output realize.

Lemma 1. *Under Assumption 1, the solution to the moral hazard problem (1) is attained with a compensation function of a form* $\tau(y) = \begin{cases} \tau_m \in \mathbb{R}_+ & \text{if } y = \frac{m}{2}\bar{\alpha}k,\ m \in \{0,1,2\} \\ 0 & \text{otherwise} \end{cases}$*, where k is a chosen level of the project capital.*

Proof Let's define the optimization problem analogous to (1), in which τ is restricted to be of a form specified above. The "discretized" problem is given by

$$\Pi^d = \max_{\substack{k \in [0,1], h \geq h_0 \\ (\tau_m)_{m=0}^2 \in \mathbb{R}_+^3}} \mathbb{E}\alpha k - \sum_{m=0}^{2} p_m \tau_m - k - (h - h_0) \tag{2}$$

s.t.

$$\mathbb{E}\{u[\tau(y) + wh] | k^e = k\} \quad \geq \quad \mathbb{E}\left\{u\left[\tau(y) + w\left(h + k - \tilde{k}\right)\right] | k^e = \tilde{k}\right\}, \forall \tilde{k} \in \left\{0, \tfrac{1}{2}k, 2k\right\}$$

Note that, in comparison to the problem (1), the number of incentive constraints is reduced from continuum to 3. There are only 3 potential deviations that the investor has to take care of. It is because when $k^e \notin \left\{0, \frac{1}{2}k, k\right\}$, then τ is always 0, and hence the gain of any such deviation is dominated by the payoff of investing all resources in h, i.e. $k^e = 0$. I will show that $\Pi = \Pi^d$.

Take any scalars k and h and function τ and that are incentive feasible in (1). It implies a discretized compensation function as in lemma 1, with $\tau_m = \tau(m\bar{\alpha}k)$ $m \in \{0,1,2\}$. Then $\left(k, h, (\tau_m)_{m=0}^2\right)$ is incentive feasible in (2). It satisfies all constraints, since the expected utility from obeying the investor's decision is the same in both problems, while the gain from deviating in the discretized problem (2) is no bigger than in the original problem (1). In addition, the profits of both problems are the same, since both compensation functions, conditional on project investment k, are the same. Hence, $\Pi^d \geq \Pi$.

Take any $\left(k, h, (\tau_m)_{m=0}^2\right)$ that is incentive feasible in problem (2). Set

$$\tau(y) = \begin{cases} \tau_m \in \mathbb{R}_+ & \text{if } y = \frac{m}{2}\bar{\alpha}k,\ m \in \{0,1,2\} \\ 0 & \text{otherwise} \end{cases}$$

Note that $(k, h, \tau(y))$ satisfies three constraints of problem (1), which are present in problem (2). It also satisfies the remaining constraints, because for any other deviation $\tilde{k}$ the following holds $E\{u[\tau(y) + w(h+k)] | k = 0\} = u[\tau_0 + w(h+k)] > u\left[\tau_0 + w\left(h + \bar{k} - \tilde{k}\right)\right] \geq E\left\{u[\tau(y) + wh] | k = \tilde{k}\right\}$. It means that all the remaining constraints are dominated by the constraint that is already satisfied. Any potential solution $\left(k, h, (\tau_m)_{m=0}^2\right)$ that is incentive feasible in (2) induces an incentive feasible potential solution $(k, h, \tau(y))$ in (1). Same as before, profits generated by those potential solutions are equal. Hence, $\Pi \geq \Pi^d$. Together with a previous result, it implies that $\Pi = \Pi^d$. **Q.E.D.**

Due to the discrete structure of the productivity shock, the investor is able to figure out most of the possible deviations of the entrepreneur. In order to prevent them, the investor reduces the monetary payoff to 0. However, there are some deviations that, in the case of lucky productivity realization, look like an implementation of the investor's plan. Those deviations correspond to no accumulation of the project capital, accumulation of half of the prescribed project capital, and, possible in the case of $k \leq \frac{1}{2}$ and $I \geq 2k$, accumulation of twice as much project capital. According to Lemma 1, those are the only potentially binding incentive constraints. In other words, thanks to the structure of the productivity shock, the investor is practically able to discretize the decision problem of the entrepreneur.

Let's denote the incentive compatibility constraint corresponding to the investment of xk, where k is a level of project capital chosen by investor, by μ_x. By Lemma 1 we can consider only constraints μ_0, $\mu_{\frac{1}{2}}$ and μ_2. In addition, denote the expected utility of the entrepreneur from choosing project capital $\tilde{k}$, $\mathbb{E}\left\{u\left[\tau(y)+w\left(h+k-\tilde{k}\right)\right]|k^e=\tilde{k}\right\}$, by $U_{\tilde{k}}$. Then U_k stands for the expected utility derived from complying with investor's instructions, while U_0, $U_{\frac{1}{2}k}$ and U_{2k} stand for values of all the potentially tempting deviations from the prescribed plan. The following proposition provides some characterization of the optimal contract.

Proposition 2. *In the optimal contract the following holds:*

1. *The constraint μ_2 is never binding.*

2. $\tau_2 \geq \tau_1$.

3. τ_0 *is equal to 0, while* τ_2 *is greater than 0 when* $k > 0$.

Proof

1. Assume that μ_2 binds in the solution. If $\tau_1 < \tau_2$, then the principal can decrease τ_2 and increase τ_1 in such a way that profits increase while U_k does not fall. Simultaneously, U_{2k} decreases, so this constraint cannot be binding. If $\tau_1 \geq \tau_2$, then it is impossible for this constraint to be binding, since $U_k > U_{2k}$. Hence, this constraint is always slack and we can remove it from the problem.

2. If $\tau_2 < \tau_1$, then the principal can always decrease τ_1 and increase τ_2 in such a way that profits increase while U_k does not fall. Simultaneously, $U_{\frac{1}{2}k}$ decreases and U_0 stays constant, so the constraints are not violated. Hence, in the optimum $\tau_2 \geq \tau_1$.

3. First note that $\tau_0 \leq \max\{\tau_1, \tau_2\} = \tau_2$, since otherwise constraint μ_0 is violated. Suppose that $\tau_0 > \tau_1$. Then $U_0 > U_{\frac{1}{2}k}$ and there is a profitable deviation (increasing τ_1, decreasing τ_0) such that U_k is constant and U_0 falls. Hence, $\tau_0 \leq \tau_1 \leq \tau_2$. Now consider decreasing compensation in all states by the same amount such that τ_0 is 0. This drop in compensation is replaced by increasing h by $\frac{\tau_0}{w}$. Note that the utility obtained at each choice of k is unchanged. However, the investor saves $\tau_0 \frac{w-1}{w} \geq 0$. To see that τ_2 is positive whenever there is any project capital accumulation, note that the constraint μ_0 implies that the following holds $p_2\tau_2 + p_1\tau_1 > wk$. Since $\tau_2 \geq \tau_1$, we have that $\tau_2 > \frac{wk}{p_1+p_2}$. **Q.E.D.**

Proposition 2 shows that the optimal contract exposes the entrepreneur to some risk - τ_0, the monetary payoff in the case of venture failure, is zero, while τ_2, the monetary payoff in the case of venture success, is positive. Imperfect risk-sharing provides incentives for the entrepreneur to accumulate the project capital. In addition, it is the cheapest for the investor to provide a risk-free compensation via the human capital, therefore it could be expected that the lowest monetary payoff to the entrepreneur will be equal to 0. Proposition 2 implies that the monetary payoff, conditional on complying with the investor's plan, is nondecreasing.[6] The entrepreneur facing the optimal contract has to choose between three possibilities with varying risky and risk-free payoffs. Firstly, the entrepreneur can comply with the investor by taking a full bid in lottery of the venture and receive some risk-free income via human capital. Secondly, the entrepreneur can deviate by taking a smaller bid in a different lottery, which pays some reward only in the case of the highest output realization, and receiving more of the risk-free income. Thirdly, there is an option of rejecting all lotteries and receiving the highest risk-free payoff. The fourth option of taking a very high bid in the lottery is clearly inferior.

In order to provide some further description of the optimal contract, the cost function $C(k)$ of implementing a given level of the project capital k is defined and the investor's problem is divided into two stages.

Definition The cost of implementing a level of project capital k is given by the value of the following problem

$$C(k) = \min_{h \geq h_0, (\tau_m)_{m=0}^{2} \in \mathbb{R}_+^3} p_0\tau_0 + p_1\tau_1 + p_2\tau_2 + k + h - h_0 \tag{3}$$

$$\text{s.t.} \quad U_k \quad \geq \quad U_{\tilde{k}},\ \tilde{k} \in \left\{0, \tfrac{1}{2}k\right\}$$

Now we can decompose the problem (2) into two stages[7]

1. For each $k \in [0,1]$, obtain $C(k)$ by solving the cost minimization problem (3)
2. Choose k in order to maximize profits: $\max_{k \in [0,1]} \mathbb{E}\{\alpha k - C(k)\}$, and take $\left(h, (\tau_m)_{m=0}^{2}\right)$ from the solution of the corresponding cost minimization problem.

Consider the first stage of the investor's problem. By fixing k, it is possible to get a sharper description of the optimal contract.

Proposition 3. *Fix $k > 0$. Keeping p_0 constant, there are two regions based on the value of p_2, determined by a threshold value $\bar{p}_2 \in (0,1)$.*

- *For $p_2 \leq \bar{p}_2$, only the constraint μ_0 is binding and $\tau_2 = \tau_1 > 0$.*
- *For $p_2 > \bar{p}_2$, the constraint $\mu_{\frac{1}{2}}$ is binding and $\tau_2 > \tau_1 > 0$.*

Proof Let's vary p_1 and p_2 while keeping p_0 constant. I will show that if at some value of p_2 the constraint $\mu_{\frac{1}{2}}$ is binding, it is also binding for all higher values of p_2. Consider a relaxed cost minimization problem for a given k, similar to (3) but only with the constraint μ_0. Let's denote the variables corresponding to the solution of this relaxed problem with a superscript r. Then $U_k^r = U_0^r$, since it is the only constraint, and $\tau_1^r = \tau_2^r = \tau^r$, because there is no way of providing more incentives by differentiating those monetary payoffs. Note that, keeping p_0 constant, variation in p_2 neither changes the solution (h^r, τ^r), nor the values U_k^r and U_0^r. Define $U_{\frac{1}{2}k}^r(p_2) = (1-p_2)\, u\left[w\left(h^r + \frac{1}{2}k\right)\right] + p_2 u\left[\tau^r + w\left(h^r + \frac{1}{2}k\right)\right]$ as an expected utility from

[6] In the standard model with a valid first order approach, such result requires a monotone likelihood ratio condition (e.g.Hölmstrom (1979), Abraham, Koehne, and Pavoni (2011)). In the considered model, this assumption is violated e.g. if $p_2 > p_1$. However, the Proposition 2 still holds.

[7] Grossman and Hart (1983) use the same decomposition in the standard model to derive the properties of the optimal contract without assuming that the first order approach is valid.

deviating by accumulating just $\frac{1}{2}k$ of the project capital, evaluated at (h^r, τ^r) and expressed as a function of p_2. When $U^r_{\frac{1}{2}k}(p_2) < U^r_k$, then the solution to the relaxed problem is also a solution to the full problem. However, when $U^r_{\frac{1}{2}k}(p_2) > U^r_k$, then the solution to the relaxed problem violates the constraint $\mu_{\frac{1}{2}}$, hence the constraint $\mu_{\frac{1}{2}k}$ is binding in the full problem. Note that $U^r_{\frac{1}{2}k}(p_2)$ is increasing in p_2 and $U^r_{\frac{1}{2}k}(0) < U^r_k$. Additionally, $U^r_k < U^r_{\frac{1}{2}k}(1)$, so for sufficiently small p_0 is it true that $U^r_k < U^r_{\frac{1}{2}k}(1-p_0)$. It implies that, given p_0, there is a threshold value of p_2, below which the constraint $\mu_{\frac{1}{2}}$ is not binding and above which this constraint is binding. This threshold can be calculated by finding p_2 that solves $U^r_{\frac{1}{2}k}(p_2) = U^r_k$ and is equal $\bar{p}_2 = \frac{u[w(h^r+k)] - u\left[w\left(h^r+\frac{1}{2}k\right)\right]}{u\left[\tau^r + w\left(h^r+\frac{1}{2}k\right)\right] - u\left[w\left(h^r+\frac{1}{2}k\right)\right]} \in (0,1)$.

What remains to be shown is that whenever $p_2 > \bar{p}_2$, it is true that $\tau_2 > \tau_1 > 0$. Note that by Proposition 2 $\tau_2 \geq \tau_1$, so it is sufficient to exclude the case that $\tau_2 = \tau_1$. Suppose on the contrary that $p_2 > \bar{p}_2$ and solution to the problem (3) involves $\tau_2 = \tau_1 = \tau$. Since p_2 is in the region of binding constraint $\mu_{\frac{1}{2}}$, τ has to be greater than τ^r, hence $U_k = U_{\frac{1}{2}k} > U_0$. Consider a deviation consisting in decreasing τ_1 by δ and increasing τ_2 by $x\delta$ in such a way that $U_k = U_{\frac{1}{2}k}$ is preserved. The change in profit of such deviation is $-(xp_2 - p_1)\,\delta$. On the other hand, $\frac{\partial U_k}{\partial \delta} = \frac{\partial U_{\frac{1}{2}k}}{\partial \delta}$ evaluated at $\delta = 0$ implies that $(xp_2 - p_1)\,u'[\tau + wh] = -p_2 u'\left[\tau + w\left(h + \frac{1}{2}k\right)\right]$, which means that the profit's change is positive. It contradicts the statement that the solution to the problem (3) under $p_2 > \bar{p}_2$ involves $\tau_2 = \tau_1$. Therefore, $\tau_2 > \tau_1$. Finally, $\tau_1 > 0$ because otherwise $U_0 > U_{\frac{1}{2}k}$. **Q.E.D.**

Proposition 3 says that when the probability of the high productivity realization p_2 is low, the only tempting deviation is not accumulating the project capital at all. In this case it is optimal to pay a flat compensation to the entrepreneur whenever the output of the venture is positive, since it can never happen when the entrepreneur deviates. However, as p_2 increases, the accumulation of half of the project capital becomes more attractive and may eventually become a binding constraint. This deviation is tempting for high values of p_2, since with this probability, if the agent invest just half of the prescribed project capital, the entrepreneur receives a monetary compensation τ_1. When the constraint $\mu_{\frac{1}{2}}$ is binding, the investor optimally decreases τ_1 in order to reduce $U_{\frac{1}{2}k}$, and compensates it with higher τ_2.

The characterization above treats k as fixed. If we allow k to vary as well, it might not be true that there are only two regions determined by value of p_2. For instance, as p_2 increases, the constraint $\mu_{\frac{1}{2}}$ may start to bind, but as p_2 increases further the investor decides to increase k and the constraint $\mu_{\frac{1}{2}}$ ceases to bind. However, it cannot happen when k is already maximal.

Lemma 2. *Suppose that for some value $\hat{p}_2$ it is optimal to set $k = 1$ and only the constraint μ_0 is binding. Keeping p_0 constant, there are two regions based on the value of p_2, determined by the threshold value $\bar{p}_2 \in (0,1)$*

- *For $\hat{p}_2 \leq p_2 \leq \bar{p}_2$, only the constraint μ_0 is binding, $\tau_2 = \tau_1 > 0$ and $k = 1$.*
- *For $\bar{p}_2 < p_2$, the constraint $\mu_{\frac{1}{2}}$ is binding, $\tau_2 > \tau_1 > 0$ and $k \leq 1$.*

Proof As noted in the proof of the Proposition 3, variation in p_2 does not affect the solution to the cost minimization problem (3) when only the constraint μ_0 is binding. However, it can affect the optimal choice of k. Increasing p_2 raises the expected return from the project capital while leaving other elements of the problem unchanged, hence k is nondecreasing in p_2 in this case. It is assumed that $k = 1$, so the increase in p_2 does not change the solution to the problem (2) as long as the constraint $\mu_{\frac{1}{2}}$ is not binding. , the constraint $\mu_{\frac{1}{2}}$ starts binding as soon as p_2 crosses the threshold $\bar{p}_2 = \frac{u[w(h^r+k)]-u\left[w\left(h^r+\frac{1}{2}k\right)\right]}{u\left[\tau^r+w\left(h^r+\frac{1}{2}k\right)\right]-u\left[w\left(h^r+\frac{1}{2}k\right)\right]}$. The value of the deviation $U^r_{\frac{1}{2}k}$, defined as in the proof of the Proposition 3, increases in p_2, while U^r_k is constant in p_2, hence further increases in p_2 will keep the constraint $\mu_{\frac{1}{2}}$ binding. However, now increase in p_2 raises both the expected return to project capital and the cost of providing incentives to the entrepreneur, hence k may not stay at the maximal level. **Q.E.D.**

4 Accumulation of human capital

In this section the necessary and sufficient conditions for the accumulation of human capital are derived. Then, the model is solved numerically and comparative statics with respect to the key parameters are presented.

4.1 Necessary and sufficient conditions for $h > h_0$.

Let's consider the first stage of the investor's problem, so treat k as fixed. Further, consider the case in which only the constraint μ_0 is binding.[8] In this case the monetary compensation is positive and constant whenever $y > 0$. Denote this monetary compensation by τ. The necessary optimality condition of the cost minimization problem (3) with respect to human capital is

$$w\mu_0 \left(p_0 u'[wh] + (p_1+p_2)\,u'[\tau + wh] - u'[w(h+k)]\right) \leq 1. \tag{4}$$

The left hand side expresses the gain from relaxing the incentive-compatibility constraint via human capital, which in the optimum cannot be greater than the resource cost of increasing human capital, equal to 1. The condition (4) can be satisfied as inequality in the corner solution $h = h_0$. The contract will involve a positive accumulation of human capital only if increasing h allows the investor to reduce τ. It may happen, if u exhibits decreasing absolute risk aversion.

Definition A utility function u exhibits *decreasing absolute risk aversion* (DARA) if and only if $\forall_{0\leq c_1<c_2}$ $-\frac{u''[c_1]}{u'[c_1]} > -\frac{u''[c_2]}{u'[c_2]}$.[9]

DARA property is satisfies e.g. by a CRRA utility function. When DARA is satisfied, a higher risk-free income via h decreases the absolute risk aversion of the entrepreneur. The agent is more willing to engage in risky lotteries, or, in other words, is willing to accept a given lottery with a reduced prize. The investor can utilize that to reduce the incentive payment τ. Proposition 4 shows that DARA is a necessary condition for the human capital accumulation.

Proposition 4. *Fix k and suppose that μ_0 is the only binding constraint. In the optimum $h > h_0$ only if u is DARA.*

Proof Consider the first order condition of the investor's problem with respect to human capital 4. Suppose that τ is set at the optimal level, so the optimality condition with respect to τ holds: $\mu_0 = \frac{1}{u'[wh+\tau]}$. The corner solution $h = h_0$ is not optimal if and only if

[8] It is not a particularly strong assumption, e.g. it holds under the benchmark calibration for the uniform distribution $\forall_{i\in\{0,1,2\}} p_i = \frac{1}{3}$.

[9] Similarly, we can define a constant and an increasing absolute risk aversion by replacing the inequality sign in the definition of DARA with, respectively, $=$ and $<$.

$$\frac{1}{u'[wh_0+\tau]}\left(p_0u'[wh_0]-u'[w(h_0+k)]\right)>p_0-\frac{w-1}{w}. \tag{5}$$

The optimal plan involves the accumulation of human capital if it can make this condition hold as equality, i.e. if the left hand side is decreasing with h. It is the case if

$$\frac{u'[wh]}{u'[wh+\tau]}\left(p_0-\frac{u'[w(h+k)]}{u'[wh]}\right)>\frac{u'[wh+\varepsilon]}{u'[w(h+\varepsilon)+\tau]}\left(p_0-\frac{u'[w(h+\varepsilon+k)]}{u'[wh+\varepsilon]}\right).$$

Suppose that $\frac{u'[wh]}{u'[wh+\tau]}\leq\frac{u'[wh+\varepsilon]}{u'[w(h+\varepsilon)+\tau]}$. This is the case if $\frac{\partial}{\partial c}\frac{u'[wh]}{u'[wh+\varepsilon]}\geq 0$ for $\varepsilon>0$. It can be shown that $\frac{\partial}{\partial c}\frac{u'[wh]}{u'[wh+\varepsilon]}=\frac{u'[wh]}{u'[wh+\varepsilon]}\left(\frac{u''[wh]}{u'[wh]}-\frac{u''[wh+\varepsilon]}{u'[wh+\varepsilon]}\right)$. Then $\frac{\partial}{\partial c}\frac{u'[wh]}{u'[wh+\varepsilon]}\geq 0$ implies that $-\frac{u''[wh+\varepsilon]}{u'[wh+\varepsilon]}\geq-\frac{u''[wh]}{u'[wh]}$, i.e. u exhibits a nondecreasing absolute risk aversion. In this case it is impossible to satisfy the optimality condition by accumulating human capital. Note that the expression on the left hand side is a multiplication of two elements that are increasing in h. On the other hand, $\frac{u'[wh]}{u'[wh+\tau]}>\frac{u'[wh+\varepsilon]}{u'[w(h+\varepsilon)+\tau]}$ implies that u is DARA, and the expression on the left hand side of the optimality condition is decreasing in h. **Q.E.D.**

The necessary condition for $h>h_0$ is derived by assuming that the optimality condition with respect to the human capital is not satisfied at h_0 and examining under which conditions increasing human capital can help to satisfy it. Note that in fact the accumulation of human capital requires that u is DARA only locally, around the consumption level wh_0. Since, in the case of utility functions typically used in economics, this property do not vary with consumption level, this remark is omited from the statement of the proposition. The Proposition 5 states sufficient conditions under which the optimality condition is not satisfied at h_0, which imply the sufficient conditions for the human capital accumulation.

Proposition 5. *Fix k and suppose that μ_0 is the only binding constraint and $\frac{w-1}{w}\geq p_0\geq\frac{u'[w(h_0+k)]}{u'[wh_0]}$ holds. If u is DARA, in the optimum $h>h_0$. If u is not DARA, the solution does not exist for a given k.*

Proof Consider the optimality condition (4) evaluated at h_0

$$\left(p_0u'[wh_0]-u'[w(h_0+k)]\right)\leq u'[wh_0+\tau]\left(p_0-\frac{w-1}{w}\right). \tag{6}$$

Two assumed inequalities imply that the left hand side is positive, while the right hand side is negative, which violates the optimality condition. The right hand side is affected by τ, however the investor is unable to change the sign of the expression on the right hand side by changing the monetary compensation. The only remaining instrument at the investor's disposal is human capital. Notice that the following holds $p_0\geq\frac{u'[w(h_0+k)]}{u'[wh_0]}>\frac{u''[w(h_0+k)]}{u''[wh_0]}$. The first inequality is just an assumption of the proposition, while the second one is an implication of DARA. Thus, $p_0u''[wh_0]-u''[w(h_0+k)]<0$ and the left hand side of (6) is decreasing in h. Note that the derivative of the left hand side of (6) is negative at least until the left hand side becomes negative. Then the optimality condition can be satisfied as an equality by further increasing h or τ. **Q.E.D.**

The sufficiency conditions of Proposition 5 are conditions under which the corner allocation $h=h_0$ violates the optimality, combined with a necessary DARA property. Consider the bounds on p_0 implied by the inequality conditions of of Proposition 5. The upper bounds $\frac{w-1}{w}$ converges to 1 as w increases, while, under the Inada condition, the lower bound $\frac{u'[w(h_0+k)]}{u'[wh_0]}$ converges to 0 as h_0 decreases toward 0. Therefore, the accumulation of the human capital is likely in the optimum when the human capital is valuable and scarce. Results of this section are illustrated by the following two examples.

Example (CRRA) Suppose that $u(c)=\frac{c^{1-\sigma}-1}{1-\sigma}$, the utility function that satisfies DARA. The optimality condition with respect to human capital (4) can be expressed as

$$(wh)^{-\sigma}\left(p_0-\left(\frac{wh}{w(h+k)}\right)^{\sigma}+\left(p_1+p_2-\frac{1}{w}\right)\left(\frac{wh}{\tau+wh}\right)^{\sigma}\right)\leq 0.$$

Note that for any positive τ and k, the expression in the brackets converges to p_0 as h goes to 0, while the limit of the expression in the brackets is equals $-\frac{1}{w}$ as h goes to infinity. In other words, the investor can satisfy the optimality condition by altering the level of entrepreneur's human capital.

Example (CARA) Suppose that the utility function satisfies constant absolute risk aversion: $u(c)=-e^{-\gamma c}$. The optimality condition (4) is as follows

$$e^{-\gamma h\tau}\left(p_0+\left(p_1+p_2-\frac{1}{w}\right)e^{-\gamma\tau}-e^{-\gamma wk}\right)\leq 0.$$

The level of human capital cannot affect the sign of the expression. Therefore, if the inequality conditions from Proposition 5 are satisfied, there is no allocation that satisfies the necessary optimality conditions. The incentive compatibility constraint μ_0 under CARA utility is $p_0+(p_1+p_2)e^{-\gamma\tau}\leq e^{-\gamma wk}$, which implies that $p_0<e^{-\gamma wk}$. On the other hand, the inequality condition from Proposition 5 says $p_0\geq e^{-\gamma wk}$. Therefore, if the sufficient conditions are satisfied, then for a given k there is no incentive feasible contract.

4.2 Numerical analysis

The model is solved numerically in two steps. First, the continuous choice of k is discretized with a linear grid with 1000 points raging from 0 to 1. Second, for each grid point of k the cost minimization problem (3) is solved. Finally, the profit maximizing point on the grid of k is chosen. The benchmark parametrization employs the CRRA utility function $u=\frac{c^{1-\sigma}-1}{1-\sigma}$ with a relative risk aversion $\sigma=2$. The initial stock of human capital h_0 equals 1, while the wage on the market for human capital w is 2. $\bar{\alpha}$, the parameter governing the productivity of project capital, is set to 20 and the probability distribution $(p_i)_{i=0}^{2}$ is uniform with probability of each productivity realizing equal to $\frac{1}{3}$. The following graphs were created by varying one of the parameters, while keeping the others at the benchmark value. For the benchmark parametrization only the μ_0 constraint is binding, however it changes as parameters vary. In each figure rows correspond to a one particular parameter varying, while columns describe capital stocks, monetary compensation of the entrepreneur, and payoffs to both parties: profits of the investor and certainty equivalent of the contract to the entrepreneur. The last column shows also a variable "certainty eq. h0", which is a certainty equivalent of the contract evaluated at the initial level of human capital, disregarding the accumulation of h. The shaded region in the first column informs that the sufficiency conditions for $h>h_0$ are satisfied.

Figure 1 illustrates the observation that the accumulation of human capital is likely, when the compensation of human capital w is high and when the initial stock of the human capital h_0 is low. However, for very high values of w there is no accumulation of h, since the cost of preventing the entrepreneur from diverting the resources is too high. In those cases the investor finds it optimal to decrease the size of the venture k so much that the incentive compatibility holds without any accumulation of human capital. Figure 2 shows that the contract varies with a relative risk aversion σ in a similar way as with w - increase in risk aversion calls for more hedging via human capital, until the cost of incentives is too high and the investor cuts the size

Figure 1: Comparative statics in w and h_0

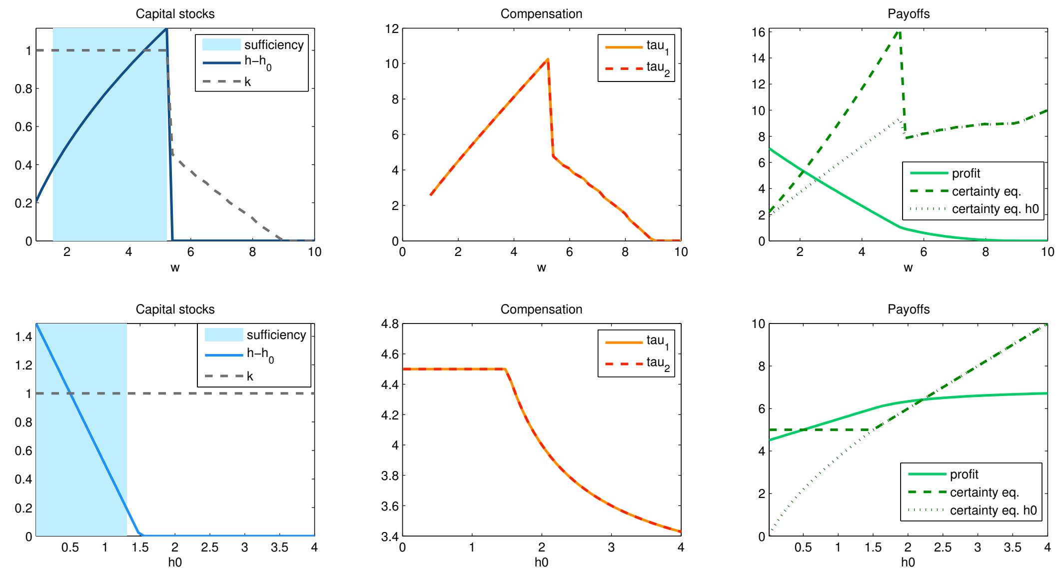
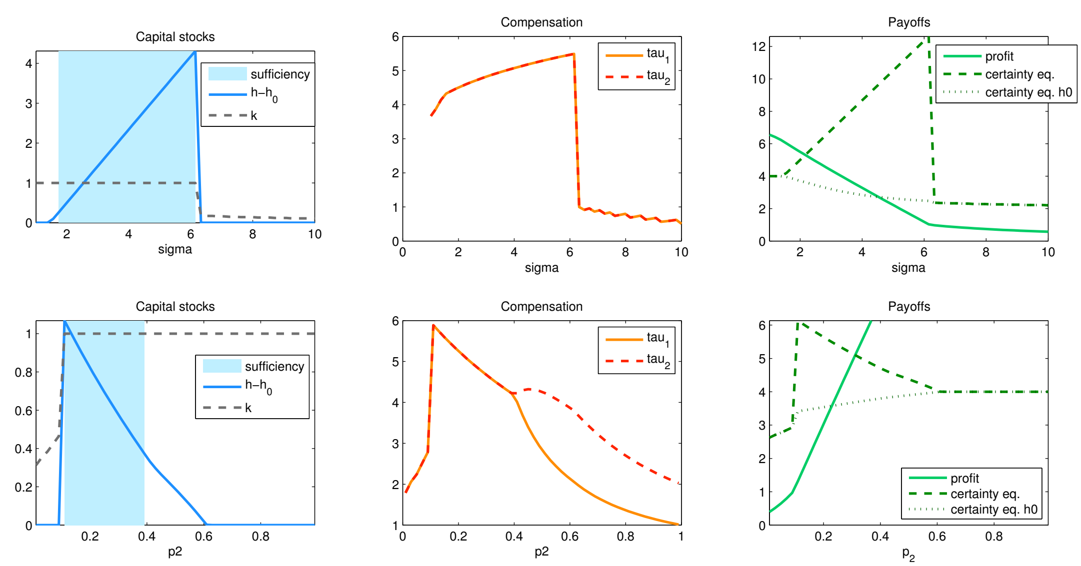


Figure 2: Comparative statics in σ and p_2



of the venture. The second row of Figure 2 presents how contract properties vary with p_2, the probability of high output realization, while keeping p_1 and p_2 equal at the level $\frac{1-p_2}{2}$. As p_2 increases, the potential profitability of the venture increases, and so the size of the venture represented by a project capital k. When k reaches the maximum, the investor employs human capital in order to keep the incentive compatibility of the entrepreneur satisfied. As p_2 increases further, the venture becomes less risky and the entrepreneur demands less hedge - the human capital h decreases. The shaded area represents the region of the parameter space where, by Proposition 5, the accumulation of human capital is assured. The graphs show that the sufficient conditions are not necessary - there is some human capital accumulation outside the shaded area. Through most of the graphs τ_1 and τ_2 are equal, which means that only μ_0 constraint is binding. However, on the graph showing how the monetary compensation varies with p_2, monetary compensation at some point becomes strictly increasing $\tau_2 > \tau_1$, hence the constraint $\mu_{\frac{1}{2}}$ starts to bind. Note that also when this constraint is binding there is some positive accumulation of human capital. The last column shows that whenever there is a positive accumulation of the human capital, the utility gain from the contract to the entrepreneur is underestimated if only monetary payoffs are considered. The gap between the true and the perceived value of the contract to the entrepreneur is especially high for low levels of the initial human capital.

5 Human capital in the standard moral hazard model

In this section sufficient and necessary conditions for the human capital accumulation are derived for the standard model. Furthermore, the numerical comparative statics with key parameters are provided.

5.1 Sufficient and necessary conditions for $h > h_0$.

Suppose that, instead of controlling the support of the venture's output, the entrepreneur controls the probability distribution of outcomes on a fixed support. In other words, the project capital k affects the probability distribution, but not the output realization at each random state. The number of potential output realizations is $\mathbb{I}$. Denote the output realization by $(y_i)_{i=1}^{\mathbb{I}}$, probabilities of each state realizing by $(p_i(k))_{i=1}^{\mathbb{I}}$, and the corresponding monetary payments to the entrepreneur by $(\tau_i)_{i=1}^{\mathbb{I}}$. The problem of the investor is following

$$\max_{\substack{k>0, h \geq h_0 \\ (\tau_i)_{i=1}^{\mathbb{I}} \geq 0}} \sum_{i=1}^{\mathbb{I}} p_i(k)(y_i - \tau_i) - k - (h - h_0) \tag{7}$$

s.t.

$$\sum_{i=1}^{\mathbb{I}} p_i(k) u[\tau_i + wh] \quad \geq \quad \sum_{i=1}^{\mathbb{I}} p_i(\tilde{k}) u\left[\tau_i + w\left(h + k - \tilde{k}\right)\right], \forall \tilde{k} \in [0,1]$$

Problems of this type are typically solved with a first order approach (FOA). In the following analysis, it is assumed that FOA is valid[10]. It means that it is possible to replace the set of original incentive compatibility constraints with just one constraint, specifying that the first order condition of the entrepreneur is satisfied. It is equivalent to assuming that only the local deviation matters, so it is enough that the agent has no incentives to deviate from the principal's plan marginally. The first order condition of the entrepreneur is

[10] For the sufficient conditions of FOA in standard moral hazard problems, see e.g. Rogerson (1985) and Abraham, Koehne, and Pavoni (2011).

following

$$\sum_{i=1}^{I} p_i'(k)\, u[\tau_i + wh] \geq w \sum_{i=1}^{I} p_i(k)\, u'[\tau_i + wh]. \tag{8}$$

The inequality can be strict only if $h = h_0$. Consider a problem of minimizing the cost of implementation of a given level of project capital k, analogous to (3). Proposition 6 states the necessary condition for the accumulation of human capital in the optimum.

Proposition 6. *Fix k. Suppose that FOA is valid and u''' exists.*

1. *When u is CARA, in the optimum $h = h_0$.*
2. *In the optimum $h > h_0$ only if $-\frac{u'''[wh_0]}{u''[wh_0]} > -\frac{1}{w}\frac{p_0'}{p_0}$.*
3. *When $p_0 \leq \frac{w-1}{w}$, in the optimum $h > h_0$ only if u is DARA and $-\frac{u'''[wh_0]}{u''[wh_0]} > -\frac{1}{w}\frac{p_0'}{p_0}$.*

Proof

1. When u is CARA, the incentive compatibility constraint (8) can be expressed as

   $$\sum_{i=1}^{I} p_i'(k)\, u[\tau_i] \geq w \sum_{i=1}^{I} p_i(k)\, u'[\tau_i],$$

   hence it does not depend on h. It means that the investor obtains no gain from allowing for the accumulation of human capital, but suffers the cost, so $h = h_0$.

2. Consider the necessary optimality condition with respect to h

   $$w\mu \left(\sum_i p_i' u'[\tau_i + wh] - w \sum_i p_i u''[\tau_i + wh] \right) \leq 1.$$

   The inequality can be strict only if optimally $h = h_0$. We can use the optimality conditions with respect to τ_i

   $$\mu \left(p_i' u'[\tau_i + wh] - w p_i u''[\tau_i + wh] \right) \leq p_i.$$

   For all states with interior choice of τ_i this inequality holds as an equality. Without a loss of generality we can assume that only for one productivity realization the choice of τ_i is not interior, i.e. this condition holds as a strict inequality. If there are more of states where it happens, we can pool them all together in one state, labeled for simplicity with $i = 0$[11]. Then, using the above inequality for states with the interior choice of τ_i in the optimality condition with respect to h yields

   $$\mu \left(p_0' u'[wh] - w p_0 u''[wh] \right) \leq p_0 - \frac{w-1}{w}.$$

[11] Suppose that $\mathcal{C}$ is a set of indices of states for which the optimality condition w.r.t. monetary compensation is satisfies as a strict inequality. Then $p_0 = \sum_{i \in \mathcal{C}} p_i$ and $p_0' = \sum_{i \in \mathcal{C}} p_i'$.

The solution can be interior with $h > h_0$ only if the initial level of human capital h_0 is not optimal

$$\mu\left(p_0' u'[wh_0] - w p_0 u''[wh_0]\right) > p_0 - \frac{w-1}{w}.$$

The optimality condition can be satisfied only if $p_0' u'[wh] - w p_0 u''[wh]$ is decreasing in h. It implies that the following has to hold for $h = h_0$ and $\varepsilon > 0$

$$\frac{u''[wh_0] - u''[wh_0 + \varepsilon]}{u'[wh_0] - u'[wh_0 + \varepsilon]} < \frac{1}{w}\frac{p_0'}{p_0}.$$

By the mean value theorem we can express the ratio on the right hand side using derivatives of higher order.

$$-\frac{u'''[wh_0]}{u''[wh_0]} > -\frac{1}{w}\frac{p_0'}{p_0}.$$

This inequality has to hold so that the left hand side of the optimality condition is decreasing with h.

3. Now consider the case of $p_0 \leq \frac{w-1}{w}$. If the initial level of human capital h_0 is not optimal, then

$$\mu\left(p_0' - w p_0 \frac{u''[wh_0]}{u'[wh_0]}\right) > \left(p_0 - \frac{w-1}{w}\right)\frac{1}{u'[wh_0]}.$$

Suppose that u exhibits an increasing absolute risk aversion. In this case, the left hand side is increasing with h. However, due to strict concavity of u and the assumption $p_0 \leq \frac{w-1}{w}$ the right hand side is decreasing in h. Therefore, the optimality condition cannot be satisfied by increasing h, whenever u exhibits an increasing absolute risk aversion and $p_0 \leq \frac{w-1}{w}$. The analysis from the previous point still applies, so it is also necessary that $-\frac{u'''[wh_0]}{u''[wh_0]} > -\frac{1}{w}\frac{p_0'}{p_0}$. **Q.E.D.**

Even though the necessary conditions in both models are related[12], none is implied by the other. Specifically, the necessary conditions of the standard moral hazard model do not rule out the human capital accumulation when u exhibits an increasing absolute risk aversion. Conversely, if $h_0 > -\left(\frac{p_0'}{p_0}\right)^{-1}$ at the some level of k, then there exists a positive parameter of relative risk aversion for which CRRA utility function satisfies the necessary condition from the Proposition 5, but not in the standard moral hazard model.

Proposition 7. *Fix k. Suppose that FOA is valid, u exhibits DARA, u''' exists and the optimum exists. In the optimum $h > h_0$, if $-\frac{u''[wh_0]}{u'[wh_0]} > -\frac{1}{w}\frac{p_0'}{p_0}$ and $p_0 \leq \frac{w-1}{w}$.*

Proof Recall the optimality condition with respect to h, used in the proof of the Proposition 6

$$\mu\left(p_0' u'[wh] - w p_0 u''[wh]\right) \leq p_0 - \frac{w-1}{w}.$$

Under the assumptions made, the left hand side of this condition is positive, while the right hand side is negative. Moreover, the assumption that u is DARA implies that the necessary condition from the Proposition 6 is also satisfied: $\frac{\partial}{\partial c} - \frac{u''[c]}{u'[c]} < 0 \Leftrightarrow -\frac{u'''[c]}{u''[c]} > -\frac{u''[c]}{u'[c]}$. It is assumed that $-\frac{u''[wh_0]}{u'[wh_0]} > -\frac{1}{w}\frac{p_0'}{p_0}$ holds, hence

[12] Prudence ($u''' > 0$) is a necessary, but not a sufficient condition for decreasing absolute risk aversion.

the necessary condition $-\frac{u'''[wh_0]}{u''[wh_0]} > -\frac{1}{w}\frac{p_0'}{p_0}$ holds as well. The necessary condition implies that the left hand side is decreasing in h. However, the solution exists only if the left hand side can decrease sufficiently to equalize the right hand side. Notice that under $p_0 \leq \frac{w-1}{w}$ and $-\frac{u''[wh_0]}{u'[wh_0]} > -\frac{1}{w}\frac{p_0'}{p_0}$ there is no solution for CARA utility. The absolute risk aversion is constant, so it cannot decrease to the level $-\frac{1}{w}\frac{p_0'}{p_0}$ required to set the left hand side of the optimality condition to 0. **Q.E.D.**

The condition $p_0 \leq \frac{w-1}{w}$, present also among sufficient conditions in the Proposition 5, is satisfied when the return to human capital is sufficiently high. The other condition puts a lower bound on the absolute risk aversion of the entrepreneur at the consumption level wh_0. This lower bound, similarly as in Proposition 5, is satisfied when h_0 is low.

5.2 Numerical analysis

The standard model is solved by assuming that FOA is valid and verifying this assumption ex post. The benchmark parametrization mimics the parametrization of the model in the Section 4 in the utility function and parameters σ, h_0 and w. Number of output realizations is set to 3, with output realizations being $\{0, 10, 20\}$. The density function is $p_i(k) = e^{-\rho k} p_i^l + \left(1 - e^{-\rho k}\right) p_i^h \; \forall_{i \in \{1,2,3\}}$, where $p^l = (0.95, 0.04, 0.01)$ and $p^h = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. The parameter ρ is set to value such that in the first best, when the principal decides upon the investment, it is optimal to accumulate exactly a unit of the project capital.

Figures 3 and 4 show how contract changes with one parameter varying, while others are set at the benchmark value. The second row of Figure 2 presents how contract properties vary with p_3^h, the probability of high output realization, while keeping p_1^h and p_2^h equal at the level $\frac{1-p_3^h}{2}$. The contract changes with parameters in a qualitatively similar manner as in the case of the other model. The accumulation of human capital is common, also outside the region of parameters where the sufficiency conditions hold. In the graph showing how monetary compensation changes with p_3^h , for low values of p_3^h the monetary compensation is not monotone: $\tau_2 > \tau_3$. It is the case because the monotone likelihood ratio condition is not satisfied: $\frac{p_i'}{p_i}$ is not monotone increasing with i. Abraham, Koehne, and Pavoni (2011) proved that in the model with moral hazard with hidden savings, when the utility function exhibits nonincreasing absolute risk aversion and FOA is valid, the compensation of the agent is increasing in the ratio $\frac{p_i'}{p_i}$. The same result holds here and the proof follows the same logic. Hence, the graph indicates that $\frac{p_2'}{p_2} > \frac{p_3'}{p_3}$ for low values of p_3^h.

6 Extensions

This section shows how to generalize the simple model and extend it in order to match a number of features of real life venture capital contracts. First, it is shown that the problem of the entrepreneur can be stated as a time or effort allocation problem, rather than the resource allocation problem. Secondly, the assumption that labor income of entrepreneur after venture is independent of the venture success is dropped - a successful entrepreneur can do better in the job market than an unsuccessful one. Furthermore, there are stylized fact of the venture capital industry that cannot be addressed in the simple model due to a static framework: a backloaded compensation of the entrepreneur and the financing rounds. It is shown how those features can be accommodated into a more general, dynamic model.

Figure 3: Comparative statics in w and h_0

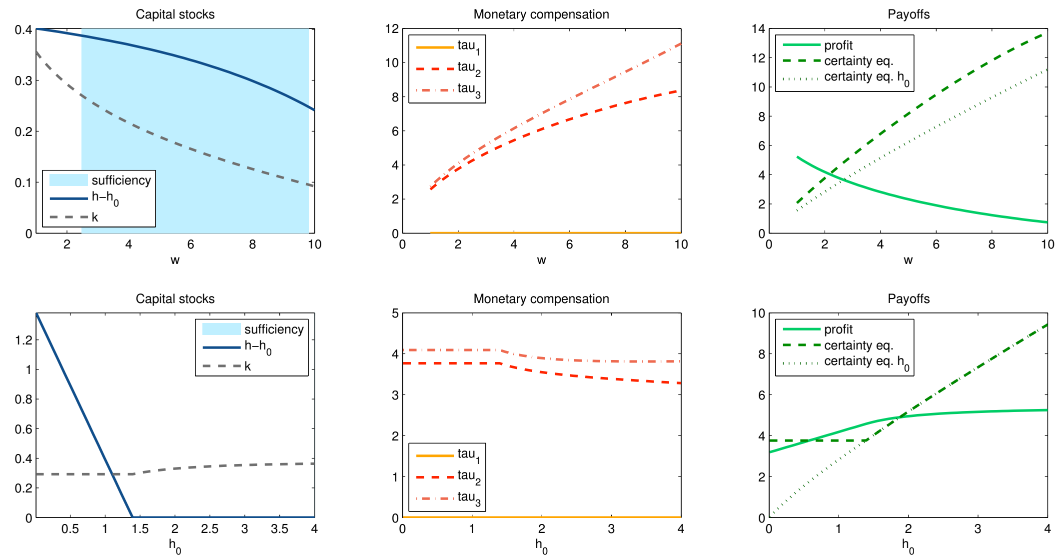
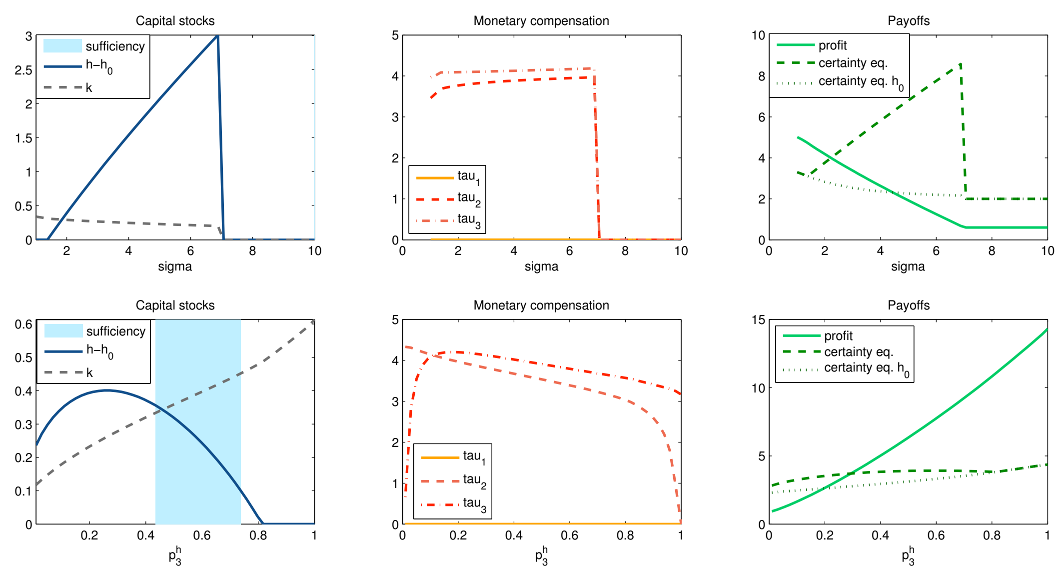


Figure 4: Comparative statics in σ and p_2



6.1 Effort rather than resources

Suppose that the entrepreneur can spend a unit endowment of effort on working for the venture e_k or building a stock of human capital e_h. There is no disutility from effort. Moreover, it is assumed that the accumulation of the project and the human capital depends positively on the amount of effort devoted to each activity, as well as on the amount of resources provided to the venture I. The entrepreneur faces the effort constraint: $1-e_k-e_h \geq 0$ as well as the functions of the capital stocks accumulation: $k = f(e_k, I)$ and $h-h_0 = g(e_h, I)$. If the functions f and g depend solely on the product of the effort and resources, i.e. $k = \hat{f}(e_k \times I)$ and $h-h_0 = \hat{g}(e_h \times I)$, then this environment is isomorphic to the one in which the investment in the capital stocks is purely a resource allocation decision. For instance, if $k = e_k I$ and $h-h_0 = e_h I$, then by multiplying the effort constraint with I we obtain the resource constraint considered in the paper: $I-k-(h-h_0) \geq 0$.

6.2 Human capital gain for successful entrepreneurs

So far it was assumed that the only gain from the venture success is the monetary output. Suppose now that the venture success rewards entrepreneur also with additional human capital. Assume that whenever the project capital k is greater than some $\bar{k} \in (0,1)$ and the venture produces a positive output, the entrepreneur receives additional, fixed amount of human capital h_v. Note that for sufficiently low $\bar{k}$ and high h_v the discretized problem (2) with three possible deviations does not describe the optimal contract, because the additional deviation, consisting in accumulating exactly $\bar{k}$ of the project capital, will be binding. For simplicity, assume that it is not the case[13]. Then the optimal contract is a solution to the following problem

$$\max_{k,h,(\tau_m)_{m=0}^{2},} \mathbb{E}\alpha k - \sum_{m=0}^{2} p_m \tau_m - k - (h-h_0) \tag{9}$$

s.t.

$$\forall_{\tilde{k}\in\left\{0,\frac{1}{2}k\right\}} \quad \mathbb{E}\left\{u\left[\tau(y)+w\left(h+\mathbf{1}_{h_v}h_v\right)\right]|k^e=k\right\} \quad \geq \quad \mathbb{E}\left\{u\left[\tau(y)+w\left(h+k-\tilde{k}+\mathbf{1}_{h_v}h_v\right)\right]|k^e=\tilde{k}\right\}$$

where $\mathbf{1}_{h_v}$ is an indicator function equal to 1 when $k^e \geq \bar{k}$ and $\alpha > 0$, and equal to 0 otherwise. This problem is very similar to the original problem (2).

Consider any incentive feasible contract of the original problem and denote its elements with superscript o. If at the original problem only the constraint μ_0 was binding, the same level of project capital k^o can be implemented in the extended problem (9) with a compensation $\forall_i \tau_i = \tau_i^0 - w\mathbf{1}_{h_v}h_v$ and a risk-free human capital $h = h^o$. Note that such compensation results in the identical consumption structure of the entrepreneur in the two problems, so if k^o was incentive compatible in the original problem (2), it is also incentive compatible in the extended problem (9), and vice versa. However, the cost of implementing k^o in the extended problem can be smaller than in the original problem, hence the optimal choice of project capital may differ: $k \geq k^o$. In the other case, when the incentive feasible contract of the original problem is characterized by a binding constraint $\mu_{\frac{1}{2}}$, the same holds as long as $\tau_1^o - w\mathbf{1}_{h_v}h_v \geq 0$. The problem may arise when this simple way of translating an incentive feasible contract from original problem into the extended problem produces negative monetary compensation. In this case, the solution to the extended problem involves $\tau_1 = 0 > \tau_1^o - w\mathbf{1}_{h_v}h_v$ and $\tau_2 < \tau_2^o - w\mathbf{1}_{h_v}h_v$. Therefore, apart from this small correction of Proposition 3, all the results hold also

[13] In other words, it is assumed that $u[w(h+k)] \geq p_0 u\left[w\left(h+k-\bar{k}\right)\right]+(1-p_0)u\left[w\left(h+k-\bar{k}+h_v\right)\right]$. It is sufficient to have $h_v \leq \bar{k}$.

in this environment.

6.3 Backloaded compensation

Consider the case of the entrepreneur that derives utility not only from the consumption after the venture is resolved, but also from consumption during the work for the venture. Suppose that the entrepreneur cannot save or borrow. For simplicity assume that neither the investor nor the agent discounts the future and denote the payment to the entrepreneur during the work in the venture by τ_{-1}. The optimal contract in this environment is the solution to the following problem

$$\max_{k,h,(\tau_m)_{m=-1}^{2},} \quad \mathbb{E}\alpha k - \tau_{-1} - \sum_{m=0}^{2} p_m \tau_m - k - (h - h_0) \tag{10}$$

s.t.

$$\begin{aligned} \mathbb{E}\{u[\tau(y)+wh]\,|k^e = k\} &\geq \mathbb{E}\left\{u\left[\tau(y)+w\left(h+k-\tilde{k}\right)\right]|k^e = \tilde{k}\right\}, \forall \tilde{k} \in \left\{0, \tfrac{1}{2}k, 2k\right\} \\ u[\tau_{-1}] + \mathbb{E}\{u[\tau(y)+wh]\,|k^e = k\} &\geq 2u[wh_0] \end{aligned}$$

In this environment, as long as u satisfies $\lim_{c\to 0} u'[c] = \infty$, the participation constraint is binding, since the investor can control the provision of utility with τ_{-1}, without distorting incentives to accumulate the project capital. Note that any incentive feasible contract from the original problem (2), complemented with τ_{-1} such that the participation constraint is satisfied, is incentive feasible in the extended problem (10), and vice versa. As a consequence of the constraint μ_0, $\mathbb{E}\{u[\tau(y)+wh]\,|k^e = k\}$ is greater than $u[wh_0]$. In addition, the constraint μ_0 implies that $p_1\tau_1 + p_2\tau_2 > wk$. On the other hand, the binding participation constraint implies that $u[wh_0] > u[\tau_{-1}]$ and hence $wh_0 > \tau_{-1}$. Therefore, if $k > h_0$, it is true that $p_1\tau_1 + p_2\tau_2 > \tau_{-1}$, i.e. the compensation of the entrepreneur is backloaded. It is consistent with the observation of Hall and Woodward (2009), who point out that entrepreneurs backed by the venture capital receive a below market level salary during the life of the venture and a high expected payoff at the venture exit.

6.4 Financing rounds

Venture capital funds usually provide resources in a sequence of financing rounds (Gompers (1995)). Financing rounds can be explained by a gradual learning about the value of the venture, as in Bergemann and Hege (1998). Such structure of financing is also consistent with an interaction of the resource accumulation decision of the entrepreneur and the limited commitment of the entrepreneur to the venture. Consider the setting from the paragraph above, in which the entrepreneur derives utility also from the period of work for the venture. Suppose that after receiving a payment τ_{-1} the entrepreneur can spend the resources on the accumulation of the human capital and leave the venture prematurely. When leaving, the entrepreneur loses the right to the compensation in the final period $\tau(y)$ and incurs a temporary cost κ. The cost can be interpreted as a legal punishment of breaking the contract, or the wage loss due to the initial involvement in the venture. This deviation is accounted by the following enforcement constraint in the investor's problem

$$u[\tau_{-1}] + \mathbb{E}\{u[\tau(y)+wh]\,|k^e = k\} \geq u[\tau_{-1} + w(h+k) - \kappa] + u[w(h+k)]$$

The incentive constraint with respect to the project capital investment still has to hold in the final period. Specifically, suppose that the parameters are such that the μ_0 constraint is binding, which means that in the absence of the enforcement constraint $\mathbb{E}\{u[\tau(y)+wh]|k^e=k\}=u[w(h+k)]$. Given this equality, the enforcement constraint can be costlessly satisfied as long as $I=k+h\leq\frac{\kappa}{w}$. It means that, if the resource provided to the venture are sufficiently high $\left(I>\frac{\kappa}{w}\right)$, then the investor has to increase the compensation of the entrepreneur in the final period over what is required by incentive compatibility: $\mathbb{E}\{u[\tau(y)+wh]|k^e=k\}>u[w(h+k)]$. There is a trade-off between the size of investment and the compensation of the entrepreneur. For some intermediate values of the project capital productivity $\bar{\alpha}$ it will result in the project investment lower than in the pure moral hazard case, considered in the Section 3. However, if the investor keeps the provision of resources at the small level $\left(I\leq\frac{\kappa}{w}\right)$, the trade-off between the size of investment and the cost of compensation does not appear. It suggests that in some cases it is optimal for the investor to split the resource provision in the smaller rounds, if only the cost of doing this is not prohibitively large. Consider the environment in which in the first-best all resources are provided optimally in the final period, and there is an interest rate cost of delaying the provision of resources. Then the first round will generally be the smallest, since it bears the highest interest rate cost. The similar problem was considered by Neher (1999), where the entrepreneur cannot commit not to renegotiate the contract after the resources are sunk. The financing rounds due to a limited commitment problem appear also in Cooley, Marimon, and Quadrini (2004), with one important difference: the venture is a temporary project in which the value is created in the final period, as opposed to the infinitely lived firm modeled by Cooley, Marimon, and Quadrini (2004). This feature, together with the positive interest rate, implies that the resources are provided later rather than earlier. It is consistent with findings of Gompers (1995), who documents that financing rounds are on average increasing.

7 Summary

This paper argues that in order to evaluate the payoff to entrepreneurs, it is not enough to consider only the monetary compensation. Furthermore, it is shown that nonmonetary compensation can play an important role in managing incentives, since it can be used as a hedge against the risk of venture failure. The investor may allow for the accumulation of the personal human capital of the entrepreneur in order to decrease the agent's risk aversion and reduce the cost of providing incentives. In other words, more insurance, which in this model is provided by the human capital, makes it easier to persuade the risk averse entrepreneur to engage in the risky venture. The human capital accumulation due to this incentive reason is likely when the entrepreneur's human capital is valuable and initially scarce. Generally, similar provision of insurance can take place in any principal-agent relation, in which the principal wants to persuade the agent to take more risk. For instance, an unintended use of insurance that spurs risky behavior was described by Peltzman (1975), who showed that the mandatory seat belts legislation leads to more risky driving.

On the methodological side, different approach to moral hazard problems is proposed. In contrast to the standard moral hazard problem with continuous choice, here the agent controls the support of the distribution function of outcome, rather than controlling the probability distribution over the fixed support. A solution method, which does not rely on the first order approach, is demonstrated for the case when outcomes can take a finite number of values. In the future, it would be interesting to examine the residual case, in which the support the distribution of outcome is of infinite cardinality.

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