Optimal Redistribution with a Shadow Economy

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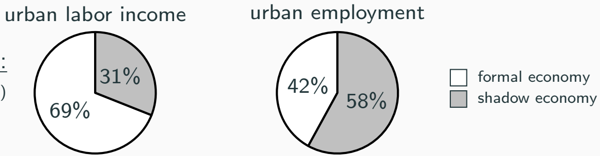
A shadow economy/informal sector: economic activity which evades taxation.

The shadow economy is **large** in many low and middle income countries.

- Share in urban emp. in 90s: Latin America 54%, SE Asia 70% (source: OECD).

Colombia in 2013:

(source: own calculations)



Why is the shadow economy important for income redistribution?

- Underreporting income makes income taxation more difficult.
- Poor workers engage more often in informal activity.

What is the optimal income tax with a shadow economy?

What is the optimal size of the shadow economy? How does it affect welfare?

This paper

Environment

1. Distribution of workers with heterogenous productivities.
2. Private information about individual productivity.
3. Formal labor market (observed) and shadow labor market (hidden).

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Results

The theory of optimal income taxation with a shadow economy.

- The shadow economy can play two welfare enhancing roles:
    - a *shelter against tax distortions* and a *screening device*.
- Novel optimal tax formula.
- Quantitative results for Colombia [not today, work in progress].

Technical contribution: solution of the screening model where local incentive constraints are not sufficient.

Related literature & contribution

Models of the shadow economy

Rauch (1991); Amaral and Quintin (2006); Albrecht, Navarro, and Vroman (2009); Meghir, Narita, and Robin (2015).

This paper: focus on the workers' sectoral choice and workers' heterogeneity.

Taxation and tax evasion

Allingham and Sandmo (1972); Kopczuk (2001); Alvarez-Parra and Sánchez (2009); Waseem (2013); Frías, Kumler, and Verhoogen (2013); Pappadá and Zylberberg (2015).

This paper: the optimal non-linear income tax.

Optimal non-linear income taxation

Mirrlees (1971); Diamond (1998); Saez (2001); Gomes, Lozachmeur, and Pavan (2014); Rothschild and Scheuer (2014).

This paper: optimal tax formula in environment where a part of income is unobserved.

Plan of the presentation

Intro

Simple model

Optimal tax formula

Workers

Two types of workers $i \in \{H, L\}$ with population shares μ_H and μ_L.

Workers have a quasilinear utility function over consumption and labor

$$U(c, n) = c - v(n),$$

where $v'' > 0$ and $v'(0) = 0$.

Worker i can supply labor to

$$\text{formal labor market} \xleftarrow{n_i^f} \text{Worker } i \xrightarrow{n_i^s} \text{shadow labor market}$$

formal labor market
productivity w_i^f
income $y_i^f \equiv w_i^f n_i^f$

shadow labor market
productivity w_i^s
income $y_i^s \equiv w_i^s n_i^s$

Total labor supply is $n_i \equiv n_i^f + n_i^s$.

Assumptions on productivities

Ordering of types: Type H is more productive formally: $w_H^f > w_L^f$.

Simplifying assumption: Each type is more productive formally $\forall_i \; w_i^f > w_i^s$.

$\Longrightarrow$ in the first best nobody works in the shadow economy.

- we drop this assumption in the full model.

The social planner

The planner observes only the formal income of each worker.

Direct revelation mechanism:

1. The planner chooses formal income y_i^f and taxes T_i for each type i.
2. Workers make type reports and are assigned formal income and taxes.
3. Workers choose their shadow income.

Consumption of worker of type i after truthful report is

$$c_i = y_i^f + y_i^s - T_i.$$

Simplifying assumption: the planner maximizes the utility of type L.

The planner's problem

The planner maximizes the social welfare function

$$\max_{\left(y_i^f, T_i\right)_{i\in\{H,L\}}} U(c_L, n_L)$$

subject to the resource constraint

$$\mu_L T_L + \mu_H T_H \geq 0,$$

workers' choice of shadow labor

$$\forall_{i\in\{H,L\}} \ n_i^s = \arg\max_{n^s} U\left(y_i^f - T_i + w_i^s n^s, \frac{y_i^f}{w_i^f} + n^s\right),$$

and incentive compatibility constraints, preventing type misreporting

$$\forall_{i,j\in\{H,L\}} \ U\left(c_i, n_i\right) \geq \max_{n^s} U\left(y_j^f - T_j + w_i^s n^s, \frac{y_j^f}{w_i^f} + n^s\right).$$

H does not work in the shadow economy.

Lemma

In the optimum the incentive constraint of type H is binding, while the incentive constraint of type L is slack.

Planner wants to redistribute from H to $L \implies IC$ constraint of H binds.

Corollary (No distortions at the top)

Type H faces no labor distortions and has no shadow earnings.

No distortions of H and $w_H^f > w_L^S \implies$ no shadow labor of H.

Shadow economy in the optimum.

Assumption 1. v'' is non-decreasing.

Assumption 2. $w_H^f \left[v'\right]^{-1}(w_H^s) \geq w_L^f \left[v'\right]^{-1}(w_L^s)$.

Proposition

Under Assumption 1, type L optimally supplies shadow labor only if

$$\left(\frac{w_L^s}{w_L^f} - \frac{w_H^s}{w_H^f}\right)\mu_H \geq \frac{w_L^f - w_L^s}{w_L^f}\mu_L.$$

Under Assumptions 1 and 2, this condition is both necessary and sufficient.

Consider marginally decreasing n_L^f and increasing n_L^s

Benefit: more redistribution due to the relaxed IC constraint.
Cost: output loss due to lower productivity.

We derive an alternative sufficient condition when Assumption 2 does not hold.

Shadow economy in the optimum.

Assumption 1. $\lambda_H = 0$ and v'' is non-decreasing.

Assumption 2. $w_H^f \left[v'\right]^{-1}(w_H^s) \geq w_L^f \left[v'\right]^{-1}(w_L^s)$.

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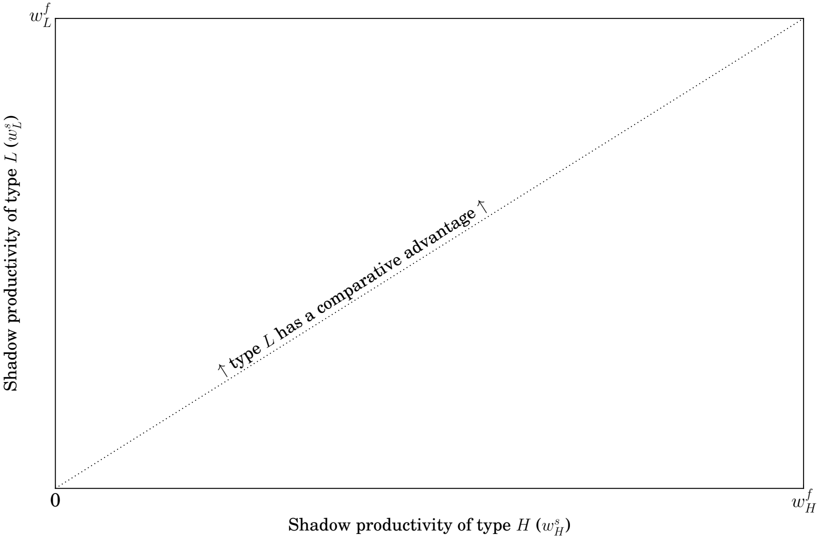

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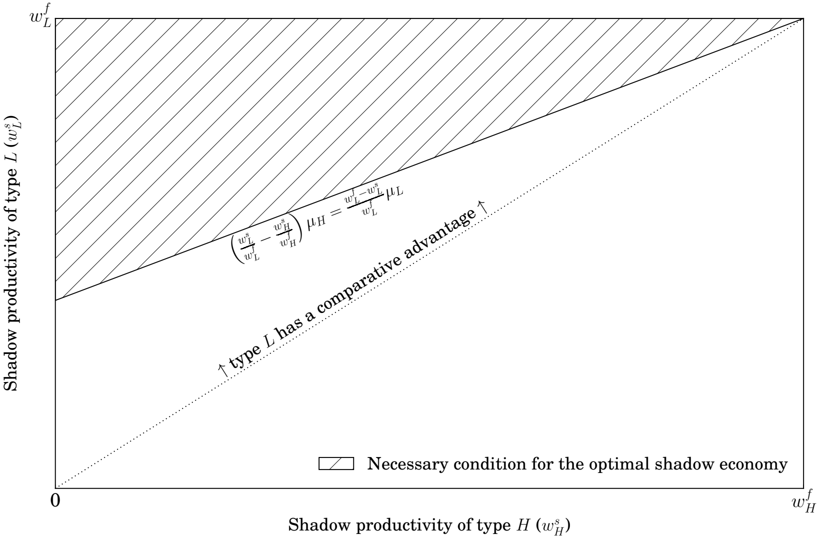

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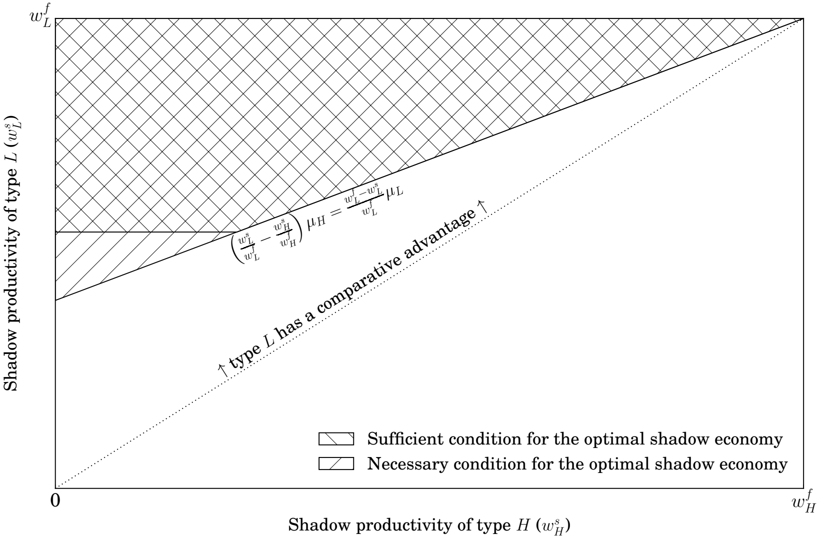


Shadow economy and welfare.

Let's compare two allocations:

$$\begin{aligned} SE &: \text{ allocation in which type } L \text{ works only in the shadows,} \\ M &: \text{ optimum of the standard Mirrlees model } (\forall_i w_i^s = 0). \end{aligned}$$

We can decompose the welfare difference between the two allocations

$$U\left(w_L^s n_L^{SE} - T_L^{SE}, n_L^{SE}\right) - U\left(w_L^f n_L^M - T_L^M, n_L^M\right)$$

into

$$\underbrace{U\left(w_L^s n_L^{SE}, n_L^{SE}\right) - U\left(w_L^f n_L^M, n_L^M\right)}_{\text{efficiency gain}} \quad + \quad \underbrace{T_L^M - T_L^{SE}}_{\text{redistribution gain}}.$$

Efficiency gain.

$$U\left(w_L^s n_L^{SE}, n_L^{SE}\right) - U\left(w_L^f n_L^M, n_L^M\right)$$

Efficiency gain measures the difference in distortions between M and SE.

- Distortions in M: positive tax rate on formal income.
- Distortions in SE: lower shadow productivity.

Efficiency gain is strictly increasing in w_L^s, positive for $w_L^s > \bar{w}_L^s \in (0, w_L^f)$.

Positive efficiency gain $\rightarrow$ shadow economy as a *shelter against tax distortions*.

Redistribution gain.

$$T_L^M - T_L^{SE}$$

Redistribution gain is a difference in transfers of type L.

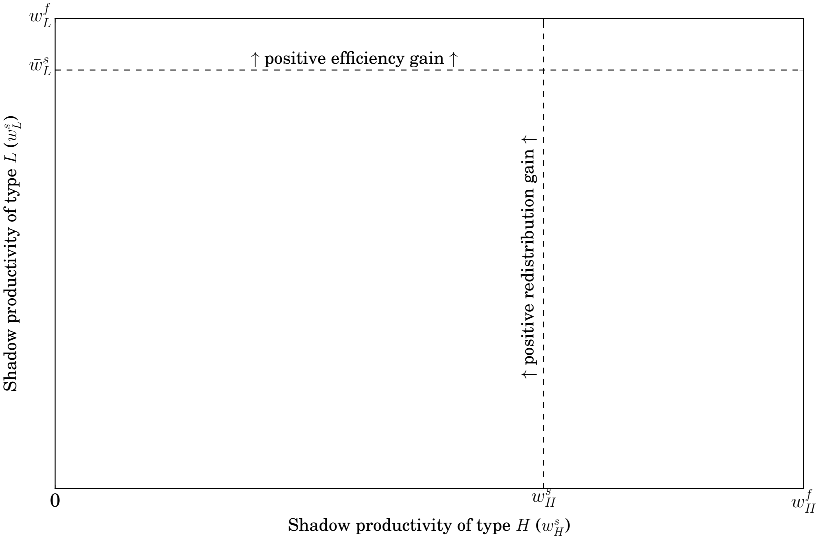
It depends on the utility of H from mimicking L:

- Utility of mimicker in M: $U\left(y_L^M - T_L^M, y_L^M / w_H^f\right)$
- Utility of mimicker in SE: $\max_{n^s} U\left(w_H^s n^s - T_L^{SE}, n^s\right)$

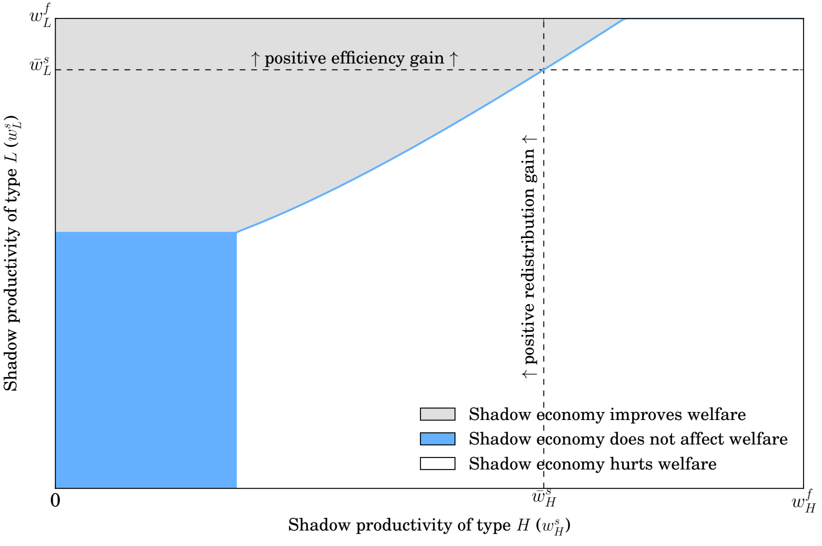
Redistribution gain is strictly decreasing in w_H^s, positive for $w_H^s < \bar{w}_H^s \in (0, w_H^f)$.

Positive redistribution gain $\rightarrow$ shadow economy as a *screening device*.

Shadow economy and welfare.



Shadow economy and welfare.



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Continuum of types

Workers differ in productivity and in a fixed cost of shadow employment.

A continuum of productivity types $\theta \in [0, 1]$ distributed with $F(\theta)$, $f(\theta)$.

θ determines formal productivity $w^f(\theta)$ and shadow productivity $w^s(\theta)$.

- Ordering of types: $w^f(\theta)$ is increasing in θ.
- Assumption for single-crossing: $w^s(\theta)/w^f(\theta)$ is non-increasing in θ.

An idiosyncratic fixed cost of shadow employment $\kappa \in [0, \infty)$.

- Conditional on θ, κ is distributed with $G_\theta(\kappa)$, $g_\theta(\kappa)$.

The optimal tax formula

The optimal tax formula can be expressed as

$$\begin{aligned}&\frac{\tau(\theta)}{1-\tau(\theta)} h^f\left(y^f(\theta,\infty)\right) y^f(\theta,\infty)\tilde{\varepsilon}_{y^f,1-\tau}(\theta,\infty)\\&\quad+\left(\frac{w^f(s(\theta))}{w^s(s(\theta))}-1\right) h^s\left(y^f(s(\theta),0)\right) y^f(s(\theta),0)\tilde{\varepsilon}_{y^f,1-\tau}(s(\theta),0)\\&\quad\quad=\int_{y^f(\theta,\infty)}^{\infty}\left[1-\bar{\lambda}(y)\right] h(y)dy-\int_{y^f(\theta,\infty)}^{y^f(s(\theta),\infty)} \pi(y)h^f(y)dy, \quad (1)\end{aligned}$$

where

h^f, h^s, h	density of formal income of formal workers, shadow workers and all workers, respectively
$\tilde{\varepsilon}_{y^f,1-\tau}(\theta,\kappa)$	elasticity of $y^f(\theta,\kappa)$ with respect to the marginal tax rate along the non-linear tax schedule
$\pi(y^f(\theta,\infty))$	the elasticity of formality of workers with productivity type θ with respect to $\Delta T(\theta)$
$\bar{\lambda}(y)$	the mean Pareto weight of all workers at formal income y by $\bar{\lambda}(y)$
$s(\theta)$	a productivity type of low-cost workers distorted by $\tau(\theta)$

Tax rate at the top

Proposition

Suppose that (i) the labor elasticity at the top converges to ε, (ii) there is a Pareto tail in formal productivity: $\lim_{\theta\to 1} \frac{1-F(\theta)}{f(\theta)}\rho^f(\theta) = 1/\alpha$, (iii) there is a Pareto tail in the fixed cost of shadow employment: $\forall_\theta \lim_{\kappa\to\infty} \frac{1-G_\theta(\kappa)}{\kappa g_\theta(\kappa)} = 1/\gamma$, (iv) Pareto weights at the top converge to $\lambda(1)$.

The optimal top tax rate $\tau(1)$ satisfies

$$\frac{\tau(1)}{1-\tau(1)} = \begin{cases} (1+1/\varepsilon)(1-\lambda(1))/\alpha & \textit{if } 1-\bar{\tau} \geq \frac{w^s(1)}{w^f(1)}, \\ (1+1/\varepsilon)\left(1-\lambda(1)-\delta\right)/\alpha' & \textit{otherwise,} \end{cases}$$

where $\delta \equiv \gamma \frac{(1+\varepsilon)(1-\bar{\tau})^\varepsilon \tau(1)}{(w^s(1)/w^f(1))^{1+\varepsilon} - (1-\tau(1))^{1+\varepsilon}} > 0$ and $\alpha' \equiv (1+\varepsilon)\gamma + \alpha$.

Top tax rate is either the same or lower than in Mirrlees (1971). Two reasons:

- Higher rate increases the share of shadow workers at the top: δ.
- Distribution of formal income has a thinner tail: $1/\alpha' < 1/\alpha$.

Conclusions

Although the shadow economy allows for income underreporting, it can be useful as a *shelter against tax distortions* or as a *screening device*.

We derive a novel optimal tax formula.

Shadow economy weakly reduces the optimal top tax rate.

For other tax predictions, we need a calibrated model [work in progress].

The optimal shadow economy: full results

Proposition

Suppose that v'' is nondecreasing.

(i) When $w_H^f [v']^{-1}(w_H^s) \geq w_L^f [v']^{-1}(w_L^s)$, type L works in the shadow economy if and only if

$$\left(\frac{w_L^s}{w_L^f} - \frac{w_H^s}{w_H^f}\right)\mu_H \geq \frac{w_L^f - w_L^s}{w_L^f}\mu_L. \tag{2}$$

(ii) When $w_H^f [v']^{-1}(w_H^s) < w_L^f [v']^{-1}(w_L^s)$, (2) is a necessary condition for type L to work in the shadow economy. The sufficient condition is given by

$$\left(\frac{w_L^s}{w_L^f} - v'\left(\frac{w_L^f}{w_H^f}[v']^{-1}(w_L^s)\right) \Big/ w_H^f\right)\mu_H \geq \frac{w_L^f - w_L^s}{w_L^f}\mu_L. \tag{3}$$