Optimal Redistribution with a Shadow Economy

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Abstract

We extend the theory of the optimal redistributive taxation by Mirrlees (1971) to economies with an informal labor market. The existence of the shadow economy affects welfare either by enhancing (diminishing) the efficiency of labor allocation or by enlarging (reducing) the scope of possible redistribution. Conditional on the distribution of formal income and marginal social welfare weights, the presence of a shadow economy weakly reduces the optimal marginal tax rates. We apply the theory to Colombia. Conditional on the optimal policy, the Colombian shadow economy is roughly welfare neutral for low to medium strength of redistributive preferences and welfare deteriorating for strongly redistributive social preferences.

JEL Codes: H21, H26, J46.

1. Introduction.

Informal activity, broadly defined as any economic endeavor which evades taxation, accounts for a large fraction of economic activity in both developing and developed economies. The share of informal production in GDP is consistently estimated to be on average above 10% in high income OECD countries and above 30% in developing and transition countries (Schneider and Enste 2000; Schneider, Buehn, and Montenegro 2011). Given this evidence, the informal sector should be considered in the design of fiscal policy. In this paper we extend the theory of the optimal redistributive taxation by Mirrlees (1971) to economies with an informal labor market.

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The shadow economy allows workers to earn additional income which is unobserved by the government. Intuitively, this additional margin of response to taxation makes income redistribution more difficult. On the other hand, informal jobs seem to be less productive and attract mostly the poor.[1] If the informal sector benefits those in need, perhaps it can be useful from a social justice perspective. Our aim is to evaluate these claims within the optimal taxation framework. We pose the following questions:

1. How does the shadow economy affect social welfare?
2. What is the optimal income tax with the shadow economy?

Building on the seminal work of Mirrlees (1971), we develop a framework with heterogeneous agents equipped with distinct formal and shadow productivities. Workers face an idiosyncratic fixed cost of working in the shadow economy, which may reflect either ethical or technological constraints. The government observes only formal incomes and introduces taxation to maximize its redistributive welfare criterion. Importantly, we allow workers to supply labor to the formal and the shadow sectors simultaneously.

Concerning the first question, Kopczuk (2001) provides an example of welfare-improving tax evasion when the workers are heterogeneous with respect to their ability to evade taxes. We explore this result by decomposing the welfare impact of a shadow economy into efficiency and redistribution components. We find that, depending on the productivity loss that different agents suffer from moving to the shadow economy, informality can weaken or strengthen both margins of welfare. In a nutshell, if the productivity loss is low/high for the agents that receive transfers, the shadow economy can raise/reduce social welfare via the efficiency channel. Similarly, if the productivity loss is low/high for the agents that pay high taxes, the shadow economy is likely to limit/expand the scope of feasible redistribution.

To gain the intuition, consider the redistributive impact of the shadow economy. Suppose that the government wants to tax one group of workers (called 'tax payers') and provide transfer to another group (called 'transfer receivers'). From the optimal taxation theory of Mirrlees (1971) we know that the relevant constraint for redistribution is the incentive constraint of tax payers: they cannot be better-off by choosing the income levels associated with transfer receivers. In the presence of the shadow economy, the government may consider using informality to alter this incentive constraint. When the marginal tax rates of transfer receivers are high enough, they will decide to work only informally and have no formal earnings. Then the incentives of tax payers to deviate depend only on their shadow productivity and no longer on their formal productivity. If the tax payers suffer a large productivity loss from informality, the government can tax them more when transfer receivers are working in the informal sector. Conversely, when the productivity loss of tax payers is small, the deviating tax payers could benefit

[1] For instance, we find that in Colombia 58% of main jobs are informal, but the shadow economy accounts for only 31.4% of earnings from all the main jobs.

from informality by supplementing formal earnings with additional shadow income. In that case the shadow economy reduces the scope of possible redistribution.

Now consider the efficiency impact of shifting labor supply from the formal to the shadow economy. We need to compare the effective labor distortions in the two sectors. Distortions in the formal economy are driven by marginal tax rates. Distortions in the shadow economy are coming from the possibly lower productivity in the informal sector. When the productivity loss from informality is sufficiently small in comparison to the tax rates in the formal sector, shifting labor to the shadow economy reduces actual distortions and enhances efficiency.[2] On the other hand, when shadow productivities are very low and the productivity loss from informality is large, staying in the formal economy may benefit labor efficiency even when marginal tax rates are high.

We answer the second question by deriving a novel optimal income tax formula for economies with an informal sector. The tax formula trades off the deadweight loss from raising marginal tax rates with a welfare-adjusted tax revenue gain. When all workers of a given productivity type are formal, our optimal tax formula is consistent with the Diamond (1998) formula from the standard model without a shadow economy. However, when some of these workers are informal, two novel terms emerge. They represent the deadweight loss of distorting shadow workers and the tax revenue impact of increased participation in the shadow economy. Importantly, these terms are not captured by the standard sufficient statistic tax formulas of Saez (2001) or Jacquet, Lehmann, and Van der Linden (2013). To see it concretely, we show that a variation in the marginal tax rate at a given formal income level can cause distortions to shadow workers at a *strictly higher* formal income level. As a result, and in contrast to the standard sufficient statistics formulas, the mean local elasticity is not sufficient to represent the entire deadweight loss of taxation.

We find that the new terms in the tax formula increase the cost of raising marginal tax rates. Hence, conditional on income distribution and marginal social welfare weights, our tax formula yields weakly lower tax rates than the standard sufficient statistic formula. We reach a stronger conclusion when considering top incomes. We show that the shadow economy shrinks the thickness of the upper tail of the formal income distribution. Therefore, the shadow economy reduces the optimal top tax rate both via the modification of the sufficient statistics formula and by affecting the value of the key sufficient statistic.

We use our theory to evaluate informality in Colombia. Although we find that the actual Colombian tax and transfer system is Pareto efficient, the marginal social welfare weights implied by the tax schedule exhibit a curious pattern: they are much lower for workers with low, but positive, formal incomes than for workers with slightly higher

[2] This effect is related to what Porta and Shleifer (2008) call the romantic view on the shadow economy. In this view, associated with works of Hernando de Soto (de Soto (1990, 2000)) and modeled formally by Choi and Thum (2005), the informal sector protects productive firms from harmful regulation.

formal incomes.[3] We show that this pattern is driven entirely by the extensive margin incentives for participating in the shadow economy. A plausible interpretation is that these incentives were not properly taken into account while designing the tax system, which in turn reduced the effective welfare weight placed on low earners.

We compare the tax schedules implied by our tax formula and by the standard sufficient statistics formula. The optimal tax schedules feature low tax rates at low incomes that increase fairly quickly with formal income. Interestingly, the shape of the optimal tax schedule at low incomes is driven by the productivity loss from informality rather than by the social preferences of the planner. As a result, the shape of the tax schedule at low incomes is quite robust to changes in social preferences. The standard sufficient statistic formula, in contrast, implies higher tax rates at low incomes, particularly for strongly redistributive social preferences. Thus, the standard formula leads to an excessively large informal sector. Within the range of social welfare functions we consider, the shadow economy is up to 21.4 p.p. larger in terms of employment than at the optimum, with welfare cost equivalent to up to 6.4% drop in consumption.

Conditionally on the optimal tax policy, the existence of the shadow economy in Colombia is roughly welfare neutral for social preferences with low to medium taste for redistribution, and welfare deteriorating when preferences for redistribution are strong. The informal sector strengthens labor efficiency by providing less productive workers with relatively high shadow productivity and by reducing marginal tax rates in the formal sector. On the other hand, the productivity loss from informality of highly productive workers is low enough to restrict redistribution. These efficiency and redistribution channels tend to cancel each other out as long as the taste for redistribution is weak enough. However, for highly redistributive social preferences the redistribution channel dominates and the presence of the shadow economy leads to a welfare loss of 1.27% of consumption. Practically, it means that even if the informal sector could be shut down without any cost, such policy would not yield substantial welfare gains unless the government had a strong taste for redistribution.

Related literature. Following Allingham and Sandmo (1972), tax evasion has been traditionally studied in a framework with probabilistic audits and penalties, taking a tax rate as given. Andreoni, Erard, and Feinstein (1998) and Slemrod and Yitzhaki (2002) review this strand of literature. We take a complementary approach and study the optimal non-linear tax schedule, conditional on fixed tax evasion abilities of workers. Although we do not model tax audits and penalties explicitly, they are one of the possible justifications for different productivities in the formal and the shadow sector. Under this interpretation, our results on the welfare-improving informal sector can provide insights into the optimal design of tax audits. Some early results from merging

[3] For instance, workers earning formally \$10,000 annually have an implied welfare weight which is four times smaller than the weight of workers earning \$13,000. None of these workers has any additional shadow income.

both optimal taxation and optimal tax compliance policies were derived by Cremer and Gahvari (1996), Kopczuk (2001) and Slemrod and Kopczuk (2002). Kopczuk (2001) shows as well that the standard formula for the optimal linear tax is still valid with tax evasion. In contrast, we show that the standard formula for the optimal *non-linear* tax no longer holds in the presence of a shadow economy.[4]

This paper is related to the literature on optimal taxation with multiple sectors. Rothschild and Scheuer (2014) consider uniform taxation of multiple sectors when agents can work in many sectors simultaneously. Kleven, Kreiner, and Saez (2009), Scheuer (2014) and Gomes, Lozachmeur, and Pavan (2017) study differential taxation of broadly understood sectors (e.g. individual tax filers and couples, employees and entrepreneurs), when agents can belong to one sector only. Jacobs (2015) studies a complementary problem when all agents work in all sectors at the same time. Our analysis differs in that we consider a particular case of differential taxation (only one sector is taxed) when agents face an idiosyncratic fixed cost of participation in one of the sectors. This structure implies that some agents can effectively work in one sector only, while others are unconstrained in supplying labor to two sectors simultaneously. We show that a typical result on the sufficiency of local incentive constraints is no longer valid.[5] We find an alternative set of incentive constraints which ensures global incentive compatibility.

Emran and Stiglitz (2005) and Boadway and Sato (2009) study commodity taxation in the presence of informality. Both papers assume that the commodity tax affects only the formal sector.[6] Hence, it is equivalent to a proportional tax on formal income, provided that formal and shadow goods are perfect substitutes. Under these assumptions our focus on non-linear income tax is without loss of generality. A related literature on optimal taxation with home production (Kleven, Richter, and Sørensen 2000; Olovsson 2015) studies the case of non-perfect substitutability between the market and the home produced goods.

Structure of the paper. In Section 2 we show with a simple model how the shadow economy can be optimal and how it affects welfare. Section 3 is devoted to the main model and the optimal tax formulas. In Section 4 we estimate the main model. In Section 5 we conduct the efficiency test of the Colombian tax schedule, compute the

[4] Our settings is not identical to Kopczuk's, since we consider a fixed cost of shadow employment. In a previous working paper version (Doligalski and Rojas 2016), we show that the standard formula for the optimal non-linear tax is not valid even if we abstract from the fixed cost of shadow employment.

[5] The planner's problem in our setting is an example of multidimensional screening, as agents are heterogeneous with respect to the productivity and the fixed cost of shadow employment. Carroll (2012) shows that the local incentive constraints are sufficient in the multidimensional setting when the appropriately defined space of agents' types is convex. This condition is not satisfied in our setting. The local incentive constraints are insufficient to prevent deviations in both dimensions simultaneously.

[6] In principle, VAT taxation allows to tax the informal firms indirectly if they purchase intermediate goods from the formal firms. De Paula and Scheinkman (2010) show that exactly for this reason informal firms tend to make transactions with other informal firms.

optimal policies and study the welfare impact of the Colombian shadow economy. The last section provides conclusions.

2. A model with two types.

There are two types of individuals, indexed by letters L and H, with strictly positive population shares μ_L and μ_H. They care about consumption c and labor supply n according to the quasilinear utility function $U(c,n) \equiv c - v(n)$. We assume that v is increasing, strictly convex, twice differentiable and satisfies $v'(0) = 0$.

There are two labor markets and, correspondingly, each agent is equipped with two linear production technologies. An agent of type $i \in \{L, H\}$ produces with productivity w_i^f in a formal labor market and with productivity w_i^s in an informal labor market. Any agent can work formally, informally, or in both markets simultaneously. An agent of type i works n_i hours in total, which is the sum of n_i^f hours at the formal job and n_i^s hours in the shadow economy. The formal and the informal income, denoted by y_i^f and y_i^s respectively, is a product of the relevant productivity and the relevant labor supply. We identify type H as the one with higher formal productivity: $w_H^f > w_L^f$. Moreover, in this section we assume that each type's shadow productivity is lower than formal productivity: $\forall_i \; w_i^f > w_i^s$. It means that the shadow economy is inefficient and is never used in the first-best. We relax this assumption when we consider the model with a continuum of types. In this section we do not consider a fixed cost of shadow employment, which is a feature of the full model.

The allocation of resources may involve transfers across types, so one's consumption can be different than the sum of formal and informal income. We capture these flows of resources with a tax T_i. A negative tax is called a transfer, and we are going to use these terms interchangeably.

The social planner knows the structure of the economy, but has limited information about the individual agents. The only variables at the individual level the planner observes are the formal income and the tax paid. We can think about y_i^f and $y_i^f - T_i$ as a pre-tax and an after-tax reported income. Although shadow income is unobserved and cannot be controlled directly, it is influenced by the choice of formal income. Formal earnings affects the marginal disutility of labor, hence, the agent's choice of shadow hours. Two types of income are related according to the following function, implied by the first-order condition with respect to shadow labor:

$$y_i^s\left(y^f\right) \equiv w_i^s \max\left\{\left(v'\right)^{-1}\left(w_i^s\right) - y^f/w_i^f, 0\right\}. \tag{1}$$

When the agent works a sufficient number of hours in the formal sector, the marginal disutility of labor is too high to work additionally in the shadows. However, if the formal hours fall short of $(v')^{-1}(w_i^s)$, the resulting gap is filled with shadow labor.[7]

We assume that the social planner objective is to maximize the utility of the less productive type L. This social welfare function allows us to expose the interesting features of the model in the simplest way. At the end of this section we will derive general properties of all constrained efficient allocations in the model with two types.

2.1. The planner's problem.

The planner solves the maximization problem

$$\max_{\left\{\left(y_i^f, T_i\right)\in\mathbb{R}_+\times\mathbb{R}\right\}_{i\in\{L,H\}}} U\left(c_L, n_L\right) \tag{2}$$

subject to:
(i) a resource constraint

$$\mu_L T_L + \mu_H T_H \geq 0, \tag{3}$$

(ii) definitions of consumption and total labor supply, which incorporate agents' choices of shadow earnings

$$\forall_{i\in\{L,H\}}\ c_i = y_i^f + y_i^s\left(y_i^f\right) - T_i, \tag{4}$$

$$\forall_{i\in\{L,H\}}\ n_i = y_i^f / w_i^f + y_i^s\left(y_i^f\right)/w_i^s, \tag{5}$$

(iii) incentive-compatibility constraints

$$\forall_{i\in\{L,H\}}\ U\left(c_i, n_i\right) \geq U\left(y_{-i}^f + y_i^s\left(y_{-i}^f\right) - T_{-i}, \frac{y_{-i}^f}{w_i^f} + \frac{y_i^s\left(y_{-i}^f\right)}{w_i^s}\right). \tag{6}$$

The incentive compatibility constraints capture the limited information available to the planner. They imply that no agent can be better off by choosing formal income of the other type and optimally adjusting shadow earnings.

We refer to the solution to the planner's problem as the optimum.

2.2. Shadow economy in the optimum.

Lemma 1. *In the optimum the incentive constraint of type H is binding, while the incentive constraint of type L is slack.*

If individual types were known, the planner would redistribute so much from type H to L such that H would be better off lying about the type. Hence, the incentive constraint

[7] The individually optimal shadow labor, accounting for a possible corner solution, follows $\min\{v'(n_i^f + n_i^s) - w_i^s, n_i^s\} = 0$. Using the inverse function of v', taking n_i^s out of the min operator and multiplying both sides by w_i^s yields (1).

of type H restricts welfare in the optimum. As in the standard Mirrlees model, labor distortions can be used to relax the binding incentive constraint. If type i is tempted to mimic type $-i$, distorting number of hours of $-i$ will discourage the deviation. When i is more (less) productive than the other type, decreasing (increasing) number of hours worked by $-i$ will make the deviation less attractive. Lemma 1 tells us that no agent wants to mimic type H, hence the planner has no reason to distort the labor of these agents - the classical result of *no distortions at the top* holds. Since agents supply shadow labor only if formal labor is sufficiently distorted downwards, type H will work only formally.

Corollary 1. *Type H faces no labor distortions and has no shadow earnings.*

On the other hand, the planner can improve social welfare by distorting the labor supply of type L. Stronger distortions relax the binding incentive constraint and allow the planner to redistribute more. When distortions are sufficiently strong, type L will supply shadow labor. The next proposition shows that whether such distortions are optimal depends crucially on the *comparative advantage in shadow labor* of type L. We use the optimality condition derived in Lemma A.1 available in Appendix A. In order to make sure that this condition is well-behaved, we require that v' is convex.[8]

Proposition 1. *Suppose that v' is convex.*
(i) When $w_H^f (v')^{-1} (w_H^s) \geq w_L^f (v')^{-1} (w_L^s)$, type L works in the shadow economy if and only if

$$\left(\frac{w_L^s}{w_L^f} - \frac{w_H^s}{w_H^f}\right)\mu_H \geq \frac{w_L^f - w_L^s}{w_L^f}\mu_L. \tag{7}$$

(ii) When $w_H^f (v')^{-1} (w_H^s) < w_L^f (v')^{-1} (w_L^s)$, condition (7) is a necessary condition for type L to work in the shadow economy. The sufficient condition is given by

$$\left(\frac{w_L^s}{w_L^f} - v'\left(\frac{w_L^f}{w_H^f} (v')^{-1} (w_L^s)\right) \Big/ w_H^f\right)\mu_H \geq \frac{w_L^f - w_L^s}{w_L^f}\mu_L. \tag{8}$$

Inequality (7) is a necessary condition for the shadow economy to be used in the optimum. It compares the marginal benefit and cost of increasing shadow labor of type L. The left-hand side consists of the comparative advantage of type L over type H in the shadow labor multiplied by the population share of type H. The positive comparative advantage means that increasing shadow labor of L relaxes the incentive constraint of H and allows to tax this type more. Intuitively, if H is comparatively worse in shadow labor, mimicking L is more difficult if L works in the informal sector.[9] Welfare gains from the tax hike

[8] In the standard case of isoelastic utility, it implies that the elasticity of the labor supply is not greater than 1.

[9] Relatedly, Jacobs (2015) shows in the multi-sector economy with sector specific taxes that distorting sectoral choices according to agents' comparative advantage relaxes incentive constraints. Kopczuk (2001) proves that, when agents are homogeneous with respect to their ability to evade taxes, the tax

are proportional to the share of type H, since this parameter controls the size of the tax base. On the right hand side, the cost of increasing shadow labor is determined by the productivity loss from using the inferior shadow production, multiplied by the share of distorted types.

Condition (7) is also a sufficient condition for type L to work in the shadow economy if the shadow productivity of H is not much lower than the shadow productivity of L (i.e. $w_H^f (v')^{-1} (w_H^s) \geq w_L^f (v')^{-1} (w_L^s)$ holds). If this is not the case, the planner's problem has two local maxima. A stronger sufficiency condition (8) guarantees that L supplies shadow labor in either of the two potential solutions.

Figure 1 illustrates the proposition on the parameter space (w_H^s, w_L^s). Along the diagonal no type has the comparative advantage, since ratios of shadow and formal productivity of the two types are equal. The optimal shadow economy requires that type L has the comparative advantage in shadow labor, so the shadow economy can be optimal only above the diagonal. Depending on whether $w_H^f (v')^{-1} (w_H^s)$ is higher than $w_L^f (v')^{-1} (w_L^s)$ or not, the inequality (7) is a necessary and sufficient condition for the optimal shadow economy, or we use a stronger sufficient condition (8) instead. Note that the lower bound of the necessity region crosses the vertical axis at the value $\mu_L w_L^f$. As the proportion of type L decreases toward zero, the region where shadow economy is optimal increases, in the limit encompassing all the points where L has a comparative advantage in shadow labor over H.

2.3. Shadow economy and welfare.

We examine the welfare implications of the shadow economy by comparing two allocations. The first allocation, noted with a superscript M, is the optimum of the standard Mirrlees model. We can think about the standard Mirrlees model as a special case of our model in which both w_L^s and w_H^s are equal 0. The second allocation, noted with a superscript SE, involves type L working only in the shadow economy and the maximal incentive-compatible redistribution from H to L. This allocation is the optimum whenever type L optimally works in the shadow economy.[10] The welfare difference between the two allocations can be decomposed in the following way

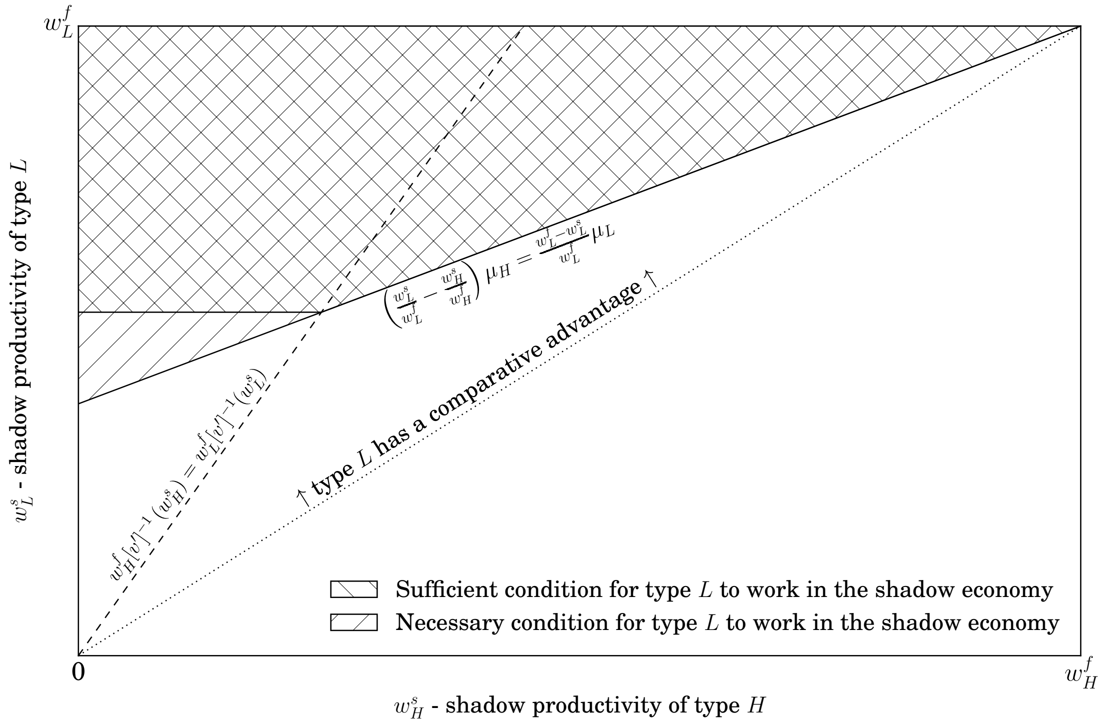
$$\underbrace{U\left(c_L^{SE}, n_L^{SE}\right) - U\left(c_L^M, n_L^M\right)}_{\text{welfare gain}} = \underbrace{U\left(w_L^s n_L^{SE}, n_L^{SE}\right) - U\left(w_L^f n_L^M, n_L^M\right)}_{\text{efficiency gain}} + \underbrace{T_L^M - T_L^{SE}}_{\text{redistribution gain}}.$$

The *efficiency gain* measures the difference in distortions imposed on type L, while the *redistribution gain* describes the change in the level of transfer type L receives.

evasion can be eliminated without any cost by properly adjusting the tax schedule. In our setting it corresponds to the situation in which none of the agents has comparative advantage in shadow labor. Consequently, there can be no gains from shadow labor in that case. Note that the simple model is a special case of Kopczuk's framework, as we do not consider the fixed cost of shadow employment yet.

[10] Optimum of the model with a shadow economy can have no agent working informally. In that case the welfare is necessarily bounded above by the welfare obtained in the standard Mirrlees model.

Figure 1: The optimal shadow economy



Efficiency gain. The distortion imposed on type L in the shadow economy arise from the productivity loss $w_L^f - w_L^s$. By varying w_L^s, this distortion can be made arbitrarily small. On the other hand, the distortion in the standard Mirrlees model comes from the marginal tax rate on formal income. Given redistributive social preferences, it is always optimal to impose a positive tax rate on type L. The efficiency gain captures the difference in distortions between two regimes. A positive efficiency gain means that the shadow economy raises social welfare by providing a *shelter against tax distortions.*

Redistribution gain. The shadow economy improves redistribution if the planner is able to give higher transfer to type L (or equivalently raise higher tax from type H). The difference in transfers can be expressed as

$$T_L^M - T_L^{SE} = \mu_H \left(U\left(w_L^f n_L^M, \frac{w_L^f}{w_H^f} n_L^M \right) - U\left(w_H^s \left(v'\right)^{-1}(w_H^s), \left(v'\right)^{-1}(w_H^s) \right) \right). \quad (9)$$

The scale of redistribution is determined by the production possibility of type H after misreporting. In the standard Mirrlees model deviating type H uses formal productivity and can earn only as much as type L. In the allocation where type L works only informally, type H cannot supply any formal labor, but is unconstrained in supplying shadow labor. A positive redistribution gain means that the shadow economy deters the deviation of type H, helping the planner to tell the two types of agents apart. In this case the informal sector plays a role of *screening device.*

The next proposition introduces thresholds on shadow productivities which allow us to determine the signs of the efficiency and the redistribution gains.

Proposition 2. *Define an increasing function* $\Psi(w^s) \equiv U\left(w^s (v')^{-1}(w^s), (v')^{-1}(w^s)\right)$ *and thresholds:*

$$\bar{w}^s_L \equiv \Psi^{-1}\left(U\left(w^f_L n^M_L, n^M_L\right)\right) \in \left(0, w^f_L\right), \bar{w}^s_H \equiv \Psi^{-1}\left(U\left(w^f_L n^M_L, \frac{w^f_L}{w^f_H} n^M_L\right)\right) \in \left(0, w^f_H\right).$$

The efficiency gain is increasing in w^s_L *and positive iff* $w^s_L > \bar{w}^s_L$. *The redistribution gain is decreasing in* w^s_H *and positive iff* $w^s_H < \bar{w}^s_H$.

Corollary 2. *When both the efficiency and the redistribution gains are positive (negative), the optimum welfare is higher (lower) than the welfare of the standard Mirrlees model.*

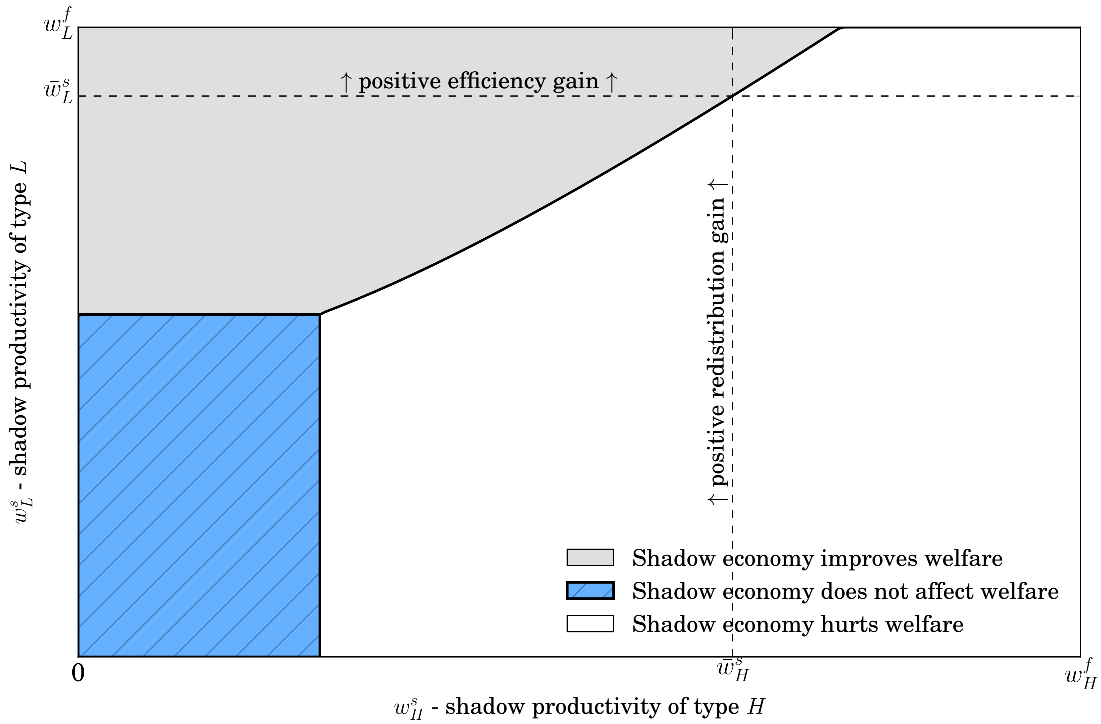
By Proposition 2 when the shadow productivity of type L is large enough, the informal sector improves efficiency of labor allocation. Moreover, when the shadow productivity of type H is low enough, the informal sector strengthens redistribution. Naturally, when the two gains have the same sign, the ranking of total welfare of the two allocations follows.

The proposition is illustrated on Figure 2. Intuitively, the shadow economy does not have to strengthen both redistribution and efficiency simultaneously to be welfare improving. Particularly interesting is the region where the redistribution gain is negative, but the efficiency gain is sufficiently high such that the welfare is higher with the shadow economy. In this case the optimum of the shadow economy model Pareto dominates the optimum of the Mirrlees model. Type L gains, since the welfare is higher with the shadow economy. Type H benefits as well, as the negative redistribution gain implies a lower tax of this type. Figure 2 also shows that when both shadow productivities are low enough, the optimum coincides with the Mirrleesian allocation.

Kopczuk (2001) provides an example in which, starting from the allocation without tax evasion, a marginal increase in evasion yields welfare gains.[11] By the envelope theorem, any resulting adjustment of labor supply has no first-order impact on individual utilities. As a result, the welfare gain found by Kopczuk consists of the redistribution gain only. Since we consider non-marginal changes in tax evasion, we can also track the welfare impact of the adjustment of labor, i.e. the efficiency gain. It allows us to show that the shadow economy can have a non-trivial influence on the efficiency of labor supply, leading not only to a welfare improvement, but in some cases also to a Pareto improvement.

[11] We can recover his example in our framework by setting $w^s_H = 0$ and properly adjusting w^s_L. He presents also a second, more straightforward example of welfare improving tax evasion, which is based on some agents having a distaste for paying taxes. We abstract from agents having preferences directly over tax payments.

Figure 2: Shadow economy and welfare



2.4. General social welfare function.

In this subsection we will derive some properties of the whole Pareto frontier of the two-types model. We consider the planner that maximizes the general social welfare function

$$W \equiv \lambda_L \mu_L U(c_L, n_L) + \lambda_H \mu_H U(c_H, n_H), \tag{10}$$

where the two Pareto weights are non-negative. The maximization is subject to constraints (3) - (6). By Proposition 1, when $\lambda_H = 0$, the comparative advantage of type L in shadow labor was necessary for this type to work in the shadow economy. Proposition 3 generalizes this result.

Proposition 3. *Type $i \in \{L, H\}$ optimally works in the shadow economy only if*

$$\lambda_i > \lambda_{-i} \quad \textit{and} \quad \frac{w_i^s}{w_i^f} > \frac{w_{-i}^s}{w_{-i}^f}.$$

In order to optimally work in the shadow economy, any type $i \in \{L, H\}$ has to satisfy two requirements. First, the planner has to be willing to redistribute resources to type i - the Pareto weight of this type has to be greater than the weight of the other type. The shadow economy can be beneficial only when it relaxes the binding incentive constraints, and the incentive constraint of type $-i$ binds if $\lambda_i > \lambda_{-i}$. Intuitively, if the planner prefers to tax rather than support a given type with a transfer, it is suboptimal to let this type evade taxation. Second, type i needs a comparative advantage in the shadow labor. Otherwise, shifting labor from formal to shadow sector does not relax the incentive constraints of the other type.

What are the sufficient conditions for type i to optimally work in the shadow economy? We can employ the welfare decomposition from the previous subsection. Let's compare the welfare of two allocations; in the first one (denoted by superscript SE) type i works exclusively in the shadow economy and the second one (denoted by superscript M) being the optimum of the standard Mirrlees model.

$$\underbrace{W^{SE} - W^{M}}_{\text{welfare gain}} = \underbrace{\mu_i \lambda_i \left(U\left(w_i^s n_i^{SE}, n_i^{SE}\right) - U\left(w_i^f n_i^M, n_i^M\right)\right)}_{\text{efficiency gain}} + \underbrace{\mu_i \left(\lambda_i - \lambda_{-i}\right)\left(T_i^M - T_i^{SE}\right)}_{\text{redistribution gain}}.$$

The welfare difference can be decomposed into the difference in effective distortions imposed on type i (the efficiency gain) and the difference in transfers received by this type (the redistribution gain). The only essential change in comparison to the simpler case considered before comes from the Pareto weights. The more the planner cares about type $-i$, the less valuable are gains in redistribution in comparison to the gains in efficiency.

Proposition 4. *Suppose that $\lambda_i > \lambda_{-i}$ for some $i \in \{L, H\}$. Define thresholds*

$$\bar{w}_i^s \equiv \Psi^{-1}\left(U\left(w_i^f n_i^M, n_i^M\right)\right) \in \left(0, w_i^f\right), \bar{w}_{-i}^s \equiv \Psi^{-1}\left(U\left(w_i^f n_i^M, \frac{w_i^f}{w_{-i}^f} n_i^M\right)\right) \in \left(0, w_{-i}^f\right),$$

where $\Psi(w^s) \equiv U\left(w^s \left(v'\right)^{-1}(w^s), \left(v'\right)^{-1}(w^s)\right)$. When $w_i^s \geq \bar{w}_i^s$ and $w_{-i}^s \leq \bar{w}_{-i}^s$, where at least one of the inequalities is strict, then type i optimally works in the shadow economy and the optimum welfare is strictly higher than in the standard Mirrlees model.

Proposition 4 introduces thresholds of shadow productivities which allow us to determine the sign of efficiency and redistribution gains for the general social welfare function. When both gains are positive, the shadow economy is obviously optimal. Interestingly, when the planner cares more about formally more productive type H, these agents may end up working in the shadow economy. It may be surprising, since in the standard Mirrlees model the formal labor supply of this type is either undistorted, or distorted upwards, while supplying shadow labor requires a downwards distortion. Nevertheless, if shadow economy magnifies productivity differences between types, it may be in the best interest of type H to supply only informal labor and enjoy higher transfer financed by the other type. The shadow economy in such allocation works as a tax haven, accessible only to the privileged.

3. Model with a continuum of types.

In this section we derive and characterize the optimal tax schedule in the model with a continuum of productivity types and an idiosyncratic fixed cost of shadow employment. The fixed cost can be interpreted either as a technological constraint on tax evasion or a utility cost of violating social norms. It allows us to consider a horizontal heterogeneity, where two agents of the same formal productivity can have different shadow employment opportunities. Incorporating the fixed cost also allows us to have a better fit with the data (see Section **??**).[12]

We maintain the preference structure of the simple model.[13] Importantly, the agents' preferences do not exhibit wealth effects. Individuals have two privately observed characteristics: the productivity parameter θ and the cost parameter κ. $\theta \in [0,1]$ determines the productivity in the formal economy $w^f(\theta)$ and in the shadow economy $w^s(\theta)$. We assume that both productivity functions are non-negative and continuously differentiable with respect to θ and that the formal productivity is strictly increasing. θ has a cumulative distribution function $F(\theta)$ and density $f(\theta)$. The parameter $\kappa \in [0, \infty)$ is a fixed cost of engaging in shadow employment. Conditional on θ, it has cumulative distribution function $G_\theta(\kappa)$ and density $g_\theta(\kappa)$.[14]

To solve the model with a continuum of types, it is useful to recover the *Spence-Mirrlees* single crossing property. This property ensures that higher productivity types choose higher formal incomes.

Lemma 2. *Agents' preferences satisfy strict Spence-Mirrlees single crossing condition if and only if $w^s(\theta)/w^f(\theta)$ is decreasing with θ or $w^s(\theta) = 0$ for all θ.*

The single crossing requires that the comparative advantage in shadow labor is decreasing with the formal productivity. This natural assumption is maintained throughout the theory section. In Section **??** we verify that it holds in the data.

3.1. Implementability.

We characterize the income choices of all agents by focusing on two classes of agents: *low-cost workers* with no fixed cost of shadow employment ($\kappa = 0$) and *high-cost workers* with a prohibitively high fixed cost (denoted by $\kappa = \infty$). We will describe the implementable

[12] In principle, we could introduce a fixed cost of formal employment as well. This would correspond to what Magnac (1991) calls a segmentation approach to informal labor markets, according to which shadow workers are restricted from formal employment by various labor regulations. An alternative, competitive approach is that individuals sort between the two sectors according to their individual advantage, which corresponds more closely to our framework. Magnac (1991) shows that the data on married women in Colombia favors the latter, competitive approach.

[13] Agents' utility function over consumption and labor, net of the fixed cost of shadow employment, is $U(c,n) \equiv c - v(n)$, where v is increasing, strictly convex, twice differentiable and satisfies $v'(0) = 0$.

[14] For the model with a continuum of productivity types, but without the fixed cost of shadow employment, see our earlier working paper Doligalski and Rojas (2016).

income schedules of these agents shortly. For now, take as given the formal income schedule of the low-cost workers $y^f(\cdot, 0)$ and the high-cost workers $y^f(\cdot, \infty)$. Suppose they face income tax schedule $T : \mathbb{R}_+ \to \mathbb{R}$. Their indirect utility function is

$$V(\theta, \kappa) \equiv \max_{y^s \geq 0} U\left(y^f(\theta, \kappa) + y^s - T(y^f(\theta, \kappa)), \frac{y^f(\theta, \kappa)}{w^f(\theta)} + \frac{y^s}{w^s(\theta)}\right) - \kappa \mathbb{1}_{y^s > 0}, \tag{11}$$

where $\kappa \in \{0, \infty\}$. Denote the informal earnings of low-cost workers, implicit in the above definition, by $y^s(\cdot, 0)$.

Let's define a threshold function $\tilde{\kappa}(\theta) \equiv V(\theta, 0) - V(\theta, \infty)$. The threshold $\tilde{\kappa}(\theta)$ is positive when the low-cost worker earns some shadow income and obtains a strictly higher utility than the high-cost worker. Take a worker of an arbitrary type $(\theta, \kappa) \in [0, 1] \times [0, \infty)$. Depending on whether the cost parameter κ is above or below the threshold, the agent behaves either like a high-cost or a low-cost worker:

$$\left(y^f(\theta, \kappa), y^s(\theta, \kappa)\right) = \begin{cases} \left(y^f(\theta, \infty), 0\right) & \text{if } \kappa \geq \tilde{\kappa}(\theta) \\ \left(y^f(\theta, 0), y^s(\theta, 0)\right) & \text{otherwise,} \end{cases} \tag{12}$$

and the indirect utility follows

$$V(\theta, \kappa) = \begin{cases} V(\theta, \infty) & \text{if } \kappa \geq \tilde{\kappa}(\theta) \\ V(\theta, 0) - \kappa & \text{otherwise.} \end{cases} \tag{13}$$

Agents with cost parameter κ below $\tilde{\kappa}(\theta)$ supply some shadow labor (and possibly some formal labor as well) and are called *shadow workers.* The remaining agents work only formally and are called *formal workers.*

We have described the income choices of all agents conditional on the formal income schedules of low- and high-cost workers. Now we will characterize the income choices of these two classes of agents. Without a loss of generality we focus on right-continuous formal income schedules.

Definition 1. *Formal income schedules $y^f(\cdot, \infty)$ and $y^f(\cdot, 0)$ are implementable if there exists a tax schedule $T(\cdot)$ such that, for any $\theta, \theta' \in [0, 1]$ and $\kappa, \kappa' \in \{0, \infty\}$:*

1. *The incentive compatibility constraint of the high-cost worker holds:*

$$V(\theta, \infty) \geq U\left(y^f(\theta', \kappa') - T(y^f(\theta', \kappa')), \frac{y^f(\theta', \kappa')}{w^f(\theta)}\right). \tag{14}$$

2. *The incentive compatibility constraint of the low-cost worker holds:*

$$V(\theta, 0) \geq \max_{y^s \geq 0} U\left(y^f(\theta', \kappa') + y^s - T(y^f(\theta', \kappa')), \frac{y^f(\theta', \kappa')}{w^f(\theta)} + \frac{y^s}{w^s(\theta)}\right). \tag{15}$$

The incentive constraints of the high-cost workers prevent these agents from choosing the formal earnings assigned to any other agent. The incentive constraints of the low-cost workers additionally allow the deviating agent to adjust the shadow earnings.[15]

In the typical optimal taxation or screening model it is enough to restrict attention to *local* incentive constraints, making sure that no agent has incentives to misreport their productivity type *marginally* (see e.g. Fudenberg and Tirole (1991)).[16] The local incentive constraints of a formal worker of type (θ, κ) can be expressed as the standard first-order condition with respect to formal income.[17] It requires that the net marginal return to formal income is equal to the marginal disutility from higher earnings:

$$\left(1 - T'\left(y^f(\theta,\kappa)\right)\right) w^f(\theta) = v'\left(\frac{y^f(\theta,\kappa)}{w^f(\theta)}\right). \tag{16}$$

The local incentive constraint of shadow worker (θ, κ) additionally equalizes the net return to formal and shadow labor:

$$\left(1 - T'\left(y^f(\theta,\kappa)\right)\right) w^f(\theta) = v'\left(\frac{y^f(\theta,\kappa)}{w^f(\theta)} + \frac{y^s(\theta,\kappa)}{w^s(\theta)}\right) = w^s(\theta). \tag{17}$$

Finally, when the formal income schedule of shadow workers is discontinuous at some type (θ_d, κ) then the local incentive constraint makes sure that the utility schedule remains continuous.[18] It means that the *average* net return to formal and shadow labor coincide:

$$\left(1 - \frac{T(y^f(\theta_d,\kappa)) - T(y^f(\theta_d^-,\kappa))}{y^f(\theta_d,\kappa) - y^f(\theta_d^-,\kappa)}\right) w^f(\theta_d) = w^s(\theta_d). \tag{18}$$

The local incentive constraints are not sufficient in our setting. Specifically, there exist formal income schedules of the low-cost workers that are non-decreasing, satisfy local incentive constraints and yet violate some of the *global* incentive constraints. The reason for that is twofold. First, the planner cannot adjust tax schedule for high- and low-cost workers independently, as they face the same tax schedule. Second, when tax

[15] Denote the image of $y^f(\cdot,\cdot)$ by Y. Thus, Y is the set of formal income levels chosen by any agent. The definition of implementability requires that no agent have incentives to deviate from the assigned formal income to any other income from the set Y. It does not, on the other hand, prevent deviations to formal income levels from $\mathbb{R}_+ \setminus Y$. However, these deviations can always be ruled out by modifying the tax schedule at $\mathbb{R}_+ \setminus Y$, which has no utility or resource cost. Hence, in further analysis we ignore these type of deviations.

[16] In our model, the local incentive constraint of type (θ, κ) can be represented as

$$\frac{d}{d\theta'}\left[\max_{y^s \geq 0} U\left(y^f(\theta',\kappa) + y^s - T\left(y^f(\theta',\kappa)\right), \frac{y^f(\theta',\kappa)}{w^f(\theta)} + \frac{y^s}{w^s(\theta)}\right) - \kappa \mathbb{1}_{y^s>0}\right]\Bigg|_{\theta'=\theta} = 0.$$

By Proposition 5, the implementable formal income schedules are non-decreasing and hence differentiable almost everywhere. When one-sided derivatives do not coincide, we require that this condition holds from each side. When the formal income schedule is discontinuous, we require the continuity of the utility schedule: $V(\theta^-, \kappa) = V(\theta, \kappa)$.

[17] Whenever the formal income schedule is locally flat with respect to θ, the local incentive constraint is satisfied automatically. Then (16) and the left equality in (17) hold as $\leq$ inequalities. It happens e.g. when bottom types are not participating in the formal labor market or at the tax kink.

[18] Formal income schedule of formal workers can, in principle, be discontinuous as well. Then the relevant local incentive constraint can be derived from the continuity of the utility schedule $V(\cdot, \infty)$.

rates are such that the net returns to formal and shadow labor are equal, the low-cost workers can shift labor between sectors at no cost. They can make large formal income adjustments which cannot be prevented by local constraints alone.[19] In order to ensure implementability, we need to verrify that, apart from the local incentive constraints, some non-local incenntive constraints hold as well.

Proposition 5. *Formal income schedules $y^f(\cdot, \infty)$ and $y^f(\cdot, 0)$ are implementable if and only if there exists a tax schedule $T(\cdot)$ such that:*

1. *$y^f(\cdot, \infty)$ and $y^f(\cdot, 0)$ are non-decreasing and satisfy local incentive constraints.*
2. *The bottom high-cost agent cannot gain by deviating to any lower formal income.*
3. *The top low-cost agent cannot gain by deviating to any higher formal income.*
4. *Suppose that $y^f(\cdot, 0)$ is discontinuous at θ_d. The agent $(\theta_d, 0)$ cannot gain by deviating to any formal income level from the interval $(y^f(\theta_d^-, 0), y^f(\theta_d, 0))$.*

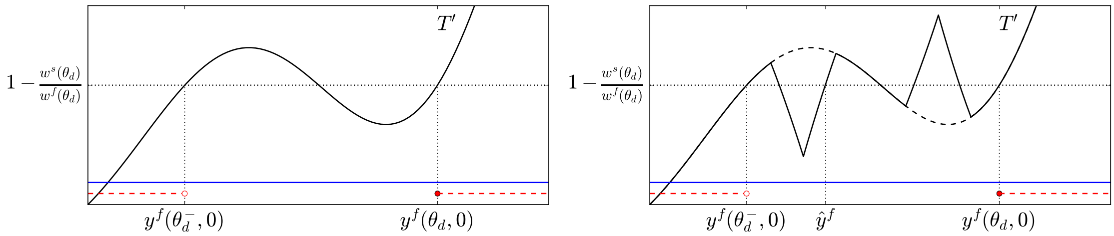
The intuition behind the proof is following. The local incentive constraints are sufficient to prevent deviations of agents *within* their cost class (the class of either high- or low-cost workers). Additional constraints are required to prevent deviations *between* the cost classes. Many of such deviations are already covered by the local incentive constraints, since the images of the two formal income schedules are partially overlapping. Therefore, we need to focus only on deviations to formal income levels which are earned by some high-cost (low-cost) workers but by no low-cost (high-cost) worker. We capture these deviations by imposing non-local incentive constraints for the bottom high-cost agent, top low-cost agent and at each discontinuity point of $y^f(\cdot, 0)$.

Figure 3 shows graphically the insufficiency of local incentive constraints. Consider an interval of low-cost workers that supply shadow labor. By the assumed decreasing comparative advantage $w^s(\theta)/w^f(\theta)$ and the local incentive constraint (15), the marginal tax rate they face is increasing in θ. When the marginal tax rates are not monotone increasing in formal income in the corresponding income range, the formal income schedule of the low-cost workers must be discontinuous, as illustrated at the graph. The local incentive constraint (18) of the agent at the discontinuity $(\theta_d, 0)$ requires this worker to be indifferent between the two formal income levels across the discontinuity: $y^f(\theta_d^-, 0)$ and $y^f(\theta_d, 0)$. In the right panel we modify the marginal tax rates in a way that total tax levels at $y^f(\theta_d^-, 0)$ and $y^f(\theta_d, 0)$ do not change. Thus, the local incentive constraint of agent $(\theta_d, 0)$ still holds. However, this agent has a profitable deviation. Note that the average tax rate between $y^f(\theta_d^-, 0)$ and formal income level $\hat{y}^f$ is below $1 - w^s(\theta)/w^f(\theta)$. It implies that utility from deviation to $\hat{y}^f$ is higher than the utility at $y^f(\theta_d, 0)$. Therefore, the formal income schedule of the low-cost types is not globally incentive compatible.

[19] Kleven, Kreiner, and Saez (2009), Scheuer (2014) and Gomes, Lozachmeur, and Pavan (2017) recover sufficiency of local incentive constraints in two-dimensional settings with two sectors, under assumption that agents can work in one sector at a time. Rothschild and Scheuer (2014) allow workers to supply labor in multiple sectors simultaneously, but the government observes and taxes the sum of all incomes, which implies that the local incentive constraints are sufficient.

Figure 3: Insufficiency of local incentive constraints.

(a) Global incentive constraints hold. (b) Global incentive constraints are violated.



The horizontal lines indicate whether at a given formal income level there are high-cost workers (solid, blue) or low-cost workers (dashed, red). In both panels, agent $(\theta_d, 0)$ *is indifferent between* $y^f(\theta_d^-, 0)$ *and* $y^f(\theta_d, 0)$*. Hence, the local incentive constraint (18) holds. However, in the right panel the worker strictly prefers formal income level* $\hat{y}^f$*, since the average tax rate between* $y^f(\theta_d^-, 0)$ *and* $\hat{y}^f$ *is below* $1 - w^s(\theta_d)/w^f(\theta_d)$*.*

3.2. The planner's problem.

The social planner maximizes the average of individual utilities, weighted with Pareto weights $\lambda(\theta, \kappa)$. We normalize the weights such that $\mathbb{E}\{\lambda(\theta, \kappa)\} = 1$, which implies that the Pareto weights and the marginal social welfare weights coincide.[20] The planner solves

$$\max_{\substack{y^f(\cdot,\infty):[0,1]\to\mathbb{R}_+ \\ y^f(\cdot,0):[0,1]\to\mathbb{R}_+ \\ T:\mathbb{R}_+\to\mathbb{R}}} \int_0^1 \int_0^\infty \lambda(\theta, \kappa) V(\theta, \kappa) dG_\theta(\kappa) dF(\theta) \tag{19}$$

subject to the implementability conditions from Proposition 5 and the budget constraint

$$\int_0^1 \int_0^\infty T(y^f(\theta, \kappa)) dG_\theta(\kappa) dF(\theta) \geq E, \tag{20}$$

where E stands for exogenous government expenditures. By solving the planner's problem for arbitrary Pareto weights, we recover the entire Pareto frontier of the model without wealth effects.[21]

We proceed with the theoretical analysis under the assumptions that the monotonicity constraints on formal income schedules are not binding (i.e. there is no bunching) and that the implementability condition 2. is not binding. We are going to derive the

[20] The marginal social welfare weights describe the welfare impact of marginally increasing consumption of a given type, expressed in the unit of tax revenue (see e.g. Piketty and Saez (2013)). In our environment they are simply $\lambda(\theta, \kappa)/\eta$, where η is the multiplier of the budget constraint. It's easy to show that at the optimum $\eta = \mathbb{E}\{\lambda(\theta, \kappa)\}$.

[21] Suppose that the planner follows the social welfare function of a form $\int_0^1 \int_0^\infty \Gamma\left(V(\theta, \kappa)\right) dG_\theta(\kappa) dF(\theta)$, where Γ is an increasing and differentiable function. Γ is typically assumed to be strictly concave and it can represent either decreasing marginal utility of consumption or the planner's taste for equality. We find the optimal allocation in this case by setting the Pareto weights in the planner's problem according to $\lambda(\theta, \kappa) = \Gamma'(V(\theta, \kappa))$, where V is the indirect utility function at the optimum. In this case the Pareto weights are endogenous, since they explicitly depend on the optimal allocation, and the model needs to be solved iteratively: in each iteration the Pareto weights are updated to reflect the indirect utility function implied by the previous solution of the model.

optimal allocation by performing small perturbations of implementable formal income schedules. As we show in the proof of Theorem 1, the deadweight loss from distorting shadow workers is independent of the magnitude of formal income adjustment they make. It means that we do not need to account for implementability conditions 3. and 4. explicitly in our tax formulas. In the computational algorithm we verify the monotonicity constraints and condition 2. ex post. Furthermore, we numerically optimize with respect to formal income choices of the low-cost workers, which effectively replaces conditions 3. and 4. with a stronger incentive compatibility condition (15).

3.3. Derivation of the optimal tax formulas.

So far we have stated the problem of finding the optimal tax schedule using the mechanism design. It is instructive, however, to think about it in terms of tax perturbations of Saez (2001). In this section we will derive the optimal tax formulas with the tax perturbation approach, by considering a small variation of the marginal tax rate at some formal income level. In Appendix C we derive the tax formulas with the mechanism design approach and we provide the exact correspondence between the two approaches.

From now on we will focus on the endogenous distribution of formal income. Denote the density of formal income by $h(\cdot)$. We can decompose it into the density of formal workers $h^f(\cdot)$ and the density of shadow workers $h^s(\cdot)$, such that at each income level y we have $h(y) = h^f(y) + h^s(y)$.[22]

Suppose that agents face a twice differentiable tax schedule $T(\cdot)$. Consider perturbing the marginal tax rate in the formal income interval $[y, y + dy]$ by a small $d\tau > 0$. This perturbation influences tax revenue via: *(i)* formal income responses of formal and shadow workers, *(ii)* an adjustment of the distribution of formal and shadow workers, *(iii)* mechanical and welfare effects of taxation.

Formal income responses of formal and shadow workers. In response to the increase in the marginal tax rate, the agents with income y or slightly higher will reduce their formal earnings. The income reduction of formal workers is approximately

$$h^f(y)\tilde{\varepsilon}^f(y)y\frac{d\tau dy}{1-T'(y)}, \quad \text{where } \tilde{\varepsilon}^f(y) \equiv \left(\frac{1}{\varepsilon(y)} + \frac{T''(y)y}{1-T'(y)}\right)^{-1}. \tag{21}$$

$\tilde{\varepsilon}^f(y)$ is the elasticity of formal income of formal workers with respect to the marginal tax rate along the non-linear tax schedule. It depends both on $\varepsilon(y)$, the elasticity along the *linear* tax schedule, and the local tax curvature. For instance, the typical isoelastic disutility of labor $v(n) = n^{1+1/\epsilon}$ implies $\varepsilon(y) = \epsilon$. When the tax schedule is non-linear,

[22] Formally, $h^f(\cdot)$ and $h^s(\cdot)$ are not densities, as they do not integrate to 1, but rather to a share of formal and shadow workers in total employment, respectively. Keeping this slight abuse of terminology in mind, we will continue calling them densities.

the income responses depend not only on this structural elasticity $\varepsilon(y)$, but also on the curvature of the tax schedule. When the tax is locally strictly progressive ($T''(y) > 0$), an increase of income in response to a tax rate cut is reduced, as an increase in income leads to higher tax rates due to progressivity. Hence, local progressivity (regressivity) of the tax schedule reduces (increases) the elasticity of income.

What are the formal income responses of shadow workers? Suppose that there are some shadow workers with formal income y. In Appendix C we show that the reduction of formal income of shadow workers is

$$h^s(y)\tilde{\varepsilon}^s(y)y\frac{d\tau dy}{1-T'(y)}, \quad \text{where } \tilde{\varepsilon}^s(y) \equiv \frac{1-T'(y)}{T''(y)y} > \tilde{\varepsilon}^f(y). \tag{22}$$

The elasticity of formal income of *shadow* workers is strictly greater than that of *formal* workers. That is because the elasticity of formal income of shadow workers along the *linear* tax schedule is infinite. Intuitively, shadow workers respond not by adjusting their total labor supply, but rather by reallocating labor between formal and shadow sectors, keeping the total labor supply fixed. For this reason their preferences for total labor do not influence formal income elasticity.

Suppose that a shadow worker of type θ faces a linear tax with tax rate $1-w^s(\theta)/w^f(\theta)$. This agent is indifferent between supplying formal and shadow labor and the optimality condition (17) pins down only the total labor supply, but not its sectoral split. Suppose that the tax rate is increased marginally. Now the return to shadow labor is strictly greater than return to formal labor. Thus, the agent shift the entire labor supply to shadow economy and reduces his formal income all the way to 0. This dramatic reduction of formal income means that the elasticity of formal income of shadow workers along the *linear* tax schedule is infinite. In contrast, if the tax schedule was nonlinear and locally strictly progressive ($T''(y) > 0$), the shadow worker would reduce his formal income only until the marginal tax rate is equal $1 - w^s(\theta)/w^f(\theta)$ again, which implies a finite elasticity.[23]

So far we considered the situation in which there are some shadow workers with formal income y, at which we perturb the marginal tax rate. Since the income schedule of low-cost workers can easily become discontinuous, we also need to discuss the case in which there are no low-cost workers at y, but there are some with strictly higher formal income. In general, the perturbation of $T'(y)$ triggers an income response of the low-cost workers at formal income $s(y)$ defined as

$$s(y) \equiv \min_{\theta}\{y^f(\theta,0) \text{ s.t. } y^f(\theta,0) \geq y\}. \tag{23}$$

When there are some low-cost workers with formal income y, then $s(y) = y$, as in the case discussed above. To consider the other case, suppose that the income schedule of low-cost

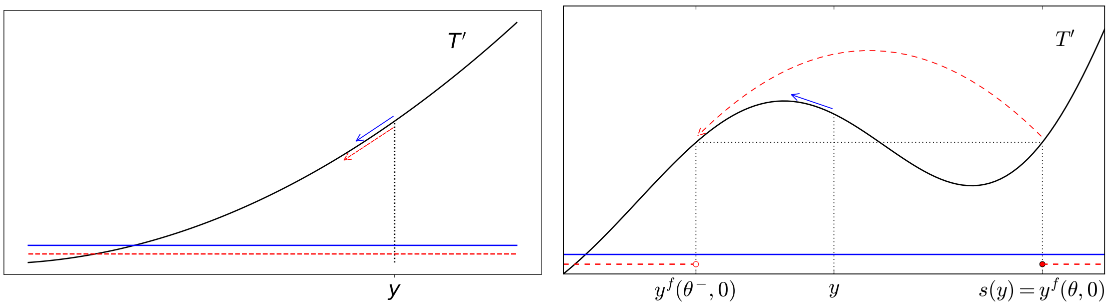
[23]Since the second-order optimality condition of a shadow workers is $T''(y) \geq 0$, we do not need to consider a locally strictly regressive tax. No shadow worker would choose such formal income level.

workers $y^f(\cdot,0)$ has discontinuity at θ. It means that for any $y \in \left(y^f(\theta^-,0), y^f(\theta,0)\right)$ we have $s(y) = y^f(\theta,0)$. Furthermore, by (18) the shadow worker with income $s(y)$ is exactly indifferent between earning $s(y)$ and $y^f(\theta^-,0)$. Consider an increase in $T'(y)$. As the tax burden at $s(y)$ increases, the agent strictly prefers $y^f(\theta^-,0)$ to $s(y)$ and jumps to the lower formal income level.

Figure 5 illustrates the two types of formal income responses of shadow workers. On the left panel, the formal income schedule of the shadow workers is locally continuous at y and the tax schedule is locally strictly progressive. Consequently, the shadow workers respond to an increase of $T'(y)$ by marginally reducing their formal income. On the right panel, the formal income schedule of the shadow workers is discontinuous. In the response to an increase in $T'(y)$ the shadow worker discretely jumps to a lower formal income level.

Figure 5: Formal income responses to the increase of $T'(y)$.

(a) Shadow workers respond marginally. (b) Shadow workers jump.



The horizontal lines indicate whether there are high-cost workers (solid, blue) or low-cost workers (dashed, red) at a given formal income level.

When the formal income schedule of shadow workers is discontinuous, formal income responses of shadow workers to a marginal change in tax rates are non-marginal, discrete. Surprisingly, and very conveniently, we can still use the intensive margin elasticity $\tilde{\varepsilon}^s(\cdot)$ to describe it. The perturbation increases the tax burden at $s(y)$ by $d\tau dy$ and makes some shadow workers discretely decrease their formal income from $s(y)$ to $s(y) - \Delta y$. The measure of shadow workers at income level $s(y)$ that decides to jump is given by $h^s(s(y))ds(y)$. By differentiating (18), we obtain $ds(y) = [T''(s(y))\Delta y]^{-1} d\tau dy$. Therefore, the overall income reduction is

$$\Delta y h^s(s(y))ds(y) = h^s(s(y))\varepsilon^s(s(y))s(y)\frac{d\tau dy}{1 - T'(s(y))}, \tag{24}$$

exactly as in the case when shadow workers adjust income marginally. The intuition is that although formal income elasticity of each individual worker who jumps is infinite, the measure of jumping individuals is small enough such that the overall elasticity at $s(y)$ is finite.

Suppose that $y \le y^f(1,0)$. We can express tax revenue impact of the income responses of formal and shadow workers, no matter whether they are responding marginally or jumping, as

$$-\left(\frac{T'(y)}{1-T'(y)}h^f(y)\tilde{\varepsilon}^f(y)y+\frac{T'(s(y))}{1-T'(s(y))}h^s(s(y))\tilde{\varepsilon}^s(s(y))s(y)\right)d\tau dy. \tag{25}$$

In the remaining case of $y > y^f(1,0)$, no shadow workers are distorted and we need to consider only income responses of formal workers.

Adjustment of the distribution of formal and shadow workers. The perturbation of $T'(y)$ increases tax burden for workers with incomes above y. Consequently, it increases incentives for informality for agents who, conditional on working informally, would have formal earnings below y. Therefore, we need to consider the impact of the perturbation on the distribution of formal and shadow workers.

Define the formal income gap between high- and low-cost agents of a given productivity type as $\Delta_\kappa(y^f(\theta,\kappa)) \equiv y^f(\theta,\infty) - y^f(\theta,0)$. If there are some shadow workers with productivity type θ, $\Delta_\infty(y^f(\theta,\infty))$ tells us by how much the formal $\theta-$worker would decrease his formal income if he worked informally. Conversely, $\Delta_0(y^f(\theta,0))$ tells us by how much the shadow worker of type θ would increase his formal income if he did not work in the shadow economy. If instead all θ-workers are formal, then $\Delta_\infty(y^f(\theta,\infty)) = \Delta_0(y^f(\theta,0)) = 0$.

Suppose that $y \leq y^f(1,0)$. The perturbation of $T'(y)$ increases incentives for informality for formal workers in the income interval $(y, s(y+dy) + \Delta_0(s(y+dy)))$. Workers with income below y are unaffected, since their tax schedule is unchanged. Workers with income above $s(y+dy) + \Delta_0(s(y+dy)$ pay taxes higher by $d\tau dy$ no matter whether they stay formal or move to the shadow economy, so their incentives for informality are unchanged as well. In the derivations that follow we will focus on an subinterval $[y+dy, s(y)+\Delta_0(s(y))]$, since the terms corresponding to the remaining parts of the original interval are are of second order (i.e. are proportional to dy^2) and vanish as we consider an arbitrarily small dy.

Denote the impact of the perturbation on the density of formal workers at income y' by $dh^f(y')$. Also denote the *tax burden of staying formal* at formal income by $\Delta T(y') \equiv T(y') - T(y' - \Delta_\infty(y'))$. It captures the tax revenue loss from a formal worker with income y moving to the shadow economy. The tax revenue impact of the perturbation via the adjustment of the distribution of formal workers is

$$\int_{y+dy}^{s(y)+\Delta_0(s(y))} dh^f(y')\Delta T(y')dy'd\tau dy = -\int_{y+dy}^{s(y)+\Delta_0(s(y))} \pi(y')h^f(y')dy'd\tau dy, \tag{26}$$

where $\pi(y^f(\theta,\infty)) \equiv \frac{g_\theta(\tilde{\kappa}(\theta))}{1-G_\theta(\tilde{\kappa}(\theta))}\Delta T(y^f(\theta,\infty))$ is the elasticity of the density of formal workers at y' with respect to the tax burden of staying formal. Intuitively, the more elastic is the density of formal workers, the higher is the tax revenue loss due to increasing participation in the shadow economy.

In the case of $y > y^f(1,0)$, all shadow workers have formal incomes below y. Consequently, the tax perturbation increases incentives for informality for all incomes greater than $y + dy$, rather than only up to $s(y) + \Delta_0(s(y))$.

Mechanical and welfare impact. Consider the tax schedule at incomes above $y + dy$. The perturbation keeps the tax rate fixed, while increasing the tax level by $d\tau dy$. An increase in the total tax on the one hand increases the tax revenue. On the other it reduces utility of agents with higher incomes, resulting in the welfare loss. Denote the average Pareto weight at a given formal income level by $\bar{\lambda}(\cdot)$. Ignoring the second order terms, the mechanical and welfare impact of the perturbation is

$$\int_{y+dy}^{\infty} (1 - \bar{\lambda}(y'))h(y')dy'd\tau dy. \tag{27}$$

Optimal tax formulas. Optimality requires that no small tax perturbation can increase the welfare-adjusted tax revenue. Hence, the sum of all impacts of the tax perturbation: (25), (26) and (27), needs to be zero for any $d\tau$ and an arbitrary small dy.

Theorem 1. *Suppose that bunching does not occur. For $y \leq y^f(1,0)$, the optimal tax rate satisfies*

$$\frac{T'(y)}{1 - T'(y)} h^f(y)\varepsilon^f(y)y + \frac{T'(s(y))}{1 - T'(s(y))} h^s(s(y))\varepsilon^s(s(y))s(y)$$

$$= \int_y^{\infty} \left[1 - \bar{\lambda}(y)\right] h(y)dy - \int_y^{s(y)+\Delta_0(s(y))} \pi(y')h^f(y')dy'. \tag{28}$$

For $y > y^f(1,0)$, the optimal tax rate satisfies

$$\frac{T'(y)}{1 - T'(y)} h^f(y)\varepsilon^f(y)y = \int_y^{\infty} \left[1 - \bar{\lambda}(y') - \pi(y')\right] h(y')dy'. \tag{29}$$

Tax formula (28) equates the *deadweight loss* from distorting the formal workers and the shadow workers on the left-hand side, with the *tax revenue gain* from higher tax on formal incomes above y net of the tax loss from increased participation in the shadow economy on the right-hand side. The second tax formula (29) captures the case when no low-cost workers are distorted by a perturbation of the tax rate at the given formal income level.

The deadweight loss of both formal and shadow workers increases in *(i)* the marginal tax rate, as the reduction in formal income implies a higher tax loss if it is taxed at the higher rate, *(ii)* the density of formal income and *(iii)* the formal income reduction per worker in response to higher tax rate, i.e. the product of formal income and the elasticity at this income level. There are two important differences between deadweight losses of formal and shadow workers. First, the distorted shadow workers may have formal income that strictly higher than y, while the distorted formal workers are always exactly at y. Second, conditional on the local progressivity of the tax schedule, shadow workers are more elastic than formal workers.

The tax revenue gain in formulas (28) and (29) consists of two terms. The first one summarizes the mechanical and welfare impact from increased taxation of all workers with higher formal income. A perturbation in the marginal tax rate increases their taxes and reduces welfare proportionally to their Pareto weight. The second term captures the tax revenue cost of increased participation in the shadow economy. Note that in formula (1) the participation in the shadow economy of workers with formal income above $s(y)+\Delta_0(s(y))$ is unchanged. Even if they decided to work in the shadow economy, they would still pay higher taxes. Thus, they have no additional incentives for informality. In contrast, when formula (29) applies, the perturbation of the marginal tax rate increases incentives for informality for all workers with higher formal income. That is the case because, conditional on working the shadow economy, all agents have formal income lower than y.

3.4. How does a shadow economy affect optimal tax rates?

Let's first consider an income level y where $\Delta_\infty(y) = 0$, which means that all workers of productivity type θ, where $y^f(\theta, \infty) = y$, are formal. In this case the optimal tax formula (28) collapses into Diamond (1998) formula from the standard Mirrlees model expressed with sufficient statistics:

$$\frac{T'(y)}{1-T'(y)} h(y)\bar{\varepsilon}(y)y = \int_y^\infty \left[1-\bar{\lambda}(y')\right] h(y')dy', \tag{30}$$

where $\bar{\varepsilon}(y)$ stands for the mean elasticity of formal income at y.[24] It is important to note that in this particular case not only the sufficient statistics formulas coincide, but so do the values of the sufficient statistics.[25] The optimal tax rate at this income level is thus *exactly* as in the standard model without the shadow economy. This result holds regardless of whether there are shadow workers at higher or lower income levels.

To see why all the terms related to shadow economy vanish, first note that no shadow worker is distorted. More specifically, the distorted low-cost worker is formal, which implies that $h^s(s(y)) = 0$. Second, $\Delta_\infty(y) = 0$ implies both that $s(y) = y$ and $\Delta_0(y) = 0$.

[24] Diamond (1998) derived the optimal tax formula in the Mirrlees model without wealth effects. Saez (2001) expressed the optimal tax formula with sufficient statistics. Acknowledging that, we will refer to (30) as 'Diamond formula'.

[25] This statement ceases to be true once Pareto weights are allowed to depend on on allocation.

As a result, the integral capturing the impact of tax perturbation on the distribution of formal income disappears. Since there is no formal worker with income *above* y who, conditional on starting some shadow job, would choose formal income *below* y, varying $T'(y)$ does not affect extensive margin incentives for shadow employment of any agent.

What if $\Delta_\infty(y) > 0$? First, consider the case of $s(y) = y$, which means that there are some shadow workers with formal income y. Then the formula (28) can be written as

$$\frac{T'(y)}{1-T'(y)} h(y)\bar{\varepsilon}(y)y = \int_y^\infty \left[1 - \bar{\lambda}(y')\right] h(y')dy' - \int_y^{y+\Delta_0(y)} \pi(y')h^f(y')dy'. \tag{31}$$

Since the distorted formal and shadow workers have exactly the same formal earnings, we can combine their individual deadweight losses into one term: the average deadweight loss at income y, as in the Diamond formula. The optimal formula differs from Diamond's due to the additional term on the right-hand side. It stands for the tax revenue loss from increased participation in the shadow economy. Whenever the marginal tax rates at the interval $[y - \Delta_\infty(y), y + \Delta_0(y)]$ are positive, the additional term is positive as well. Then the increased participation in the shadow economy is costly for the tax revenue and the tax formula leads to *lower* marginal tax rates than the Diamond formula does.[26]

We can also compare (31) to the tax formula by Jacquet, Lehmann, and Van der Linden (2013), derived in the model with both intensive and extensive margin of labor supply. The difference is in the nature extensive responses. While in their model agents can either work or withdraw entirely from the labor market, in our model agents switch from working only formally to working in the shadow economy, while very often retaining some formal earnings. It limits the size of the term related to extensive responses for two reasons. First, if the total tax is increasing with formal income, joining the shadow economy and keeping some formal income implies a lower tax revenue loss than leaving the formal economy entirely. Second, the tax perturbation at y does not increase incentives for shadow employment for all workers with higher incomes, but only for those who would earn less than y formally if they started the shadow employment. Given that, the tax formula (29) implies the marginal tax rates that are weakly higher than the ones of Jacquet, Lehmann, and Van der Linden (2013).

Note that when $y > y(1, 0)$ holds, the optimal tax formula is given by (29), which can be expressed similarly to (31), the only difference being the extensive margin term that extends to all the formal income above y, rather than being limited by $y + \Delta_0(y)$. The conclusions based on formula (31) extend to this case as well.

Finally, suppose that $\Delta_\infty(y) > 0$ and $s(y) > y$. It means that the distorted shadow workers have strictly higher formal income than the distorted formal workers. We can

[26]Suppose on the contrary that the marginal tax rates at this interval are negative. Then there is a *fiscal benefit* of pushing workers to the shadow economy, as they end up paying higher taxes. In that case the optimal tax rates will be higher than the ones implied by the Diamond formula.

express the formula (28) as

$$\frac{T'(y)}{1-T'(y)}h(y)\bar{\varepsilon}(y)y = \int_y^\infty \left[1-\bar{\lambda}(y')\right]h(y')dy' - \int_y^{s(y)+\Delta_0(s(y))} \pi(y')h^f(y')dy' \\ - \frac{T'(s(y))}{1-T'(s(y))}h^s(s(y))\tilde{\varepsilon}^s(s(y))s(y). \quad (32)$$

In this case the Diamond formula misses not only the impact on increased participation in the shadow economy, as before, but also the deadweight loss of shadow workers. Since the distorted shadow workers have formal income that is higher than y, their deadweight loss is not included in the average deadweight loss at y. As long as $T'(s(y))$ is positive, the deadweight loss of shadow workers contribute to lower tax rates than according to the Diamond formula.

We conclude this section by considering optimal taxation of top incomes.

Proposition 6 (**Tax rate at the top**). *Suppose that (i) formal income distribution has a Pareto tail:* $\lim_{y\to\infty}\frac{yh(y)}{1-H(y)} = \alpha$, *(ii) the fixed cost of shadow employment has a Pareto tail:* $\forall_\theta \lim_{\kappa\to\infty}\frac{\kappa g_\theta(\kappa)}{1-G_\theta(\kappa)} = \gamma$, *(iii) the mean labor elasticity at the top converges to* $\bar{\varepsilon}(\infty)$, *(iv) the mean Pareto weight at the top converges to* $\bar{\lambda}(\infty)$.

The top net-of-tax rate according to the Diamond formula is $1-T'_D(\infty) \equiv \frac{\alpha\bar{\varepsilon}(\infty)}{1-\bar{\lambda}(\infty)+\alpha\bar{\varepsilon}(\infty)}$. *The optimal top net-of-tax rate* $1-T'(\infty)$ *satisfies:*

$$1-T'(\infty) = \begin{cases} 1-T'_D(\infty) & \text{if } 1-T'_D(\infty) \geq w^s(1)/w^f(1), \\ w^s(1)/w^f(1) & \text{if } 1-T'_D(\infty) < w^s(1)/w^f(1) \text{ and } \gamma \geq \bar{\gamma}, \\ \frac{\alpha\bar{\varepsilon}(\infty)}{1-\bar{\lambda}(\infty)+\alpha\bar{\varepsilon}(\infty)-\pi(\infty)} & \text{if } 1-T'_D(\infty) < w^s(1)/w^f(1) \text{ and } \gamma \leq \bar{\gamma}, \end{cases} \quad (33)$$

for some threshold $\bar{\gamma} > 0$.

Suppose that the Pareto weight at the top is below the average Pareto weight: $1 > \bar{\lambda}(\infty)$. It implies that the top tax rate and the elasticity $\pi(\infty)$, capturing the fiscal cost of pushing workers to the shadow economy, are both positive.

When the top tax rate implied by the Diamond formula is low enough, it is optimal with a shadow economy as well. In this case the top *net-of-tax* rate is increasing in the labor elasticity at the top and the welfare weight at the top, and it is decreasing in the thickness of the upper tail of the formal income distribution $1/\alpha$.

In the opposite case, when the Diamond top tax rate would push some of the top workers to the shadow economy, the planner has incentives to decrease the top net-of-tax rate at least to $w^s(1)/w^f(1)$. At this level, an additional marginal decrease in the top net-of-tax rate has a discrete cost due to a displacement of a fraction of top workers to the shadow economy. This fraction increases with γ, the inverse Pareto parameter of the fixed cost distribution. High γ means that there are many workers with relatively low fixed cost of shadow employment. Hence, for sufficiently high γ the optimal net-of-tax rate will be equal to $w^s(1)/w^f(1)$. When γ is sufficiently low, it is worth it to decrease the top net-of-tax rate even further and bear the cost of displacing a fraction of workers to the shadow economy.

The analysis so far took the distribution of formal income as given. In the case of the top tax rate we can also determine how the shadow economy affects the thickness of the upper tail of formal income.

Lemma 3. *Suppose that formal productivity distribution has a Pareto tail:*

$$\lim_{\theta \to 1} \frac{f(\theta) w^f(\theta)}{1 - F(\theta)} \left(\frac{dw^f(\theta)}{d\theta} \right)^{-1} = \alpha_w. \tag{34}$$

Under assumptions of Proposition 6, the distribution of formal income has a Pareto tail with parameter

$$\alpha = \begin{cases} \frac{\alpha_w}{1+\bar{\varepsilon}(\infty)} & \text{if } 1 - T'(\infty) \geq w^s(1)/w^f(1), \\ \frac{\alpha_w}{1+\bar{\varepsilon}(\infty)} + \gamma & \text{otherwise.} \end{cases} \tag{35}$$

When there are shadow workers among the most productive types, the thickness of upper tail of formal income distribution is reduced: the Pareto parameter of formal income increases by γ. Therefore, the difference in sufficient statistics reinforces the difference in tax formulas. In this case the optimal top tax rate with shadow economy is lower than in the standard model both because of the extensive margin of shadow labor at the top and because the upper tail of income distribution is thinner.

4. Estimation

Although we have expressed the optimal tax formulas in terms of sufficient statistics, some of these statistics are very local in nature. We know from the previous section that shadow workers are very responsive to the shape of the tax schedule. As a result, the density of shadow workers at some formal income level, even if reliably estimated, is of limited use unless we know exactly how this density changes as we vary the income tax. To overcome this obstacle, we follow the suggestion of Chetty (2009) and use the structural model to extrapolate the values of sufficient statistics out of sample.

We estimate the model with the data from the Colombian household survey, which allows us to reliably extract information on individual informal incomes at the main job. We find that 58% of workers in Colombia in 2013 were employed informally at the main job, a result consistent with other indicators of informal employment in Colombia. Among 14 Latin American countries surveyed by the International Labour Organization, Colombia was ranked 9th in terms of share of informal employment in total workforce (ILO 2014). It shows that Colombia is not an outlier with respect to the size of the informal sector.

The rest of this section is structured as follows. First, we introduce our identification strategy. Second, we discuss the sample selection and recovery of informal incomes from the household survey. Finally, we present results of the estimation.

4.1. Modeling assumptions.

Suppose that productivity in each sector $j \in \{s, f\}$ features a constant, sector specific growth rate ρ^j with respect to the productivity type θ:

$$\log\left(w^j(\theta)\right) = \log\left(w^j(0)\right) + \rho^j\theta, \quad j \in \{s, f\}. \tag{36}$$

The above assumption is not restrictive for the unconditional distribution of formal wages, as long as we are free to choose any distribution of the productivity types $F(\theta)$. This assumption, however, restricts the joint distribution of formal and shadow wages. The comparative advantage in the shadow economy becomes

$$\frac{w^s(\theta)}{w^f(\theta)} = \frac{w^s(0)}{w^f(0)} \exp\left\{\left(\rho^s - \rho^f\right)\theta\right\}. \tag{37}$$

Heckman and Honore (1990) and French and Taber (2011) show that the data on wages and the sector the worker participates is in general not sufficient to identify the productivity profiles. Heckman and Honore (1990) also prove that the model can be identified with additional regressors that affect the location parameters of the skill distribution. We follow this approach. Denote the vector of regressors, which includes worker's and job's characteristics, by X.[27] We assume that the regressors are useful in predicting the worker's productivity type:

$$\theta \sim N(X\beta, \sigma_\theta^2), \tag{38}$$

where β is a vector of parameters. We obtain $F(\theta)$ using (38) and a kernel density estimation of the $X\beta$ distribution. To capture the right tail of the wage distribution, we fit a Pareto distribution with parameter α_w to the top 1% of formal wages.

The fixed cost of shadow employment κ follows a generalized Pareto distribution with density

$$g_\theta(\kappa) = \frac{1}{\sigma_\kappa\left(w^f(\theta) - w_\kappa\right)^{\alpha_\kappa}}\left(1 + \frac{\kappa}{\sigma_\kappa\left(w^f(\theta) - w_\kappa\right)^{\alpha_\kappa}}\right)^{-2}, \tag{39}$$

where parameters σ_κ, α_κ and w_κ determine how the distribution of fixed cost is affected by the productivity type θ. Finally, we assume that agents' preferences over labor supply

[27] In our estimation the vector X contains typical regressors from Mincerian wage equations such as age, gender, education level and experience. We also include job and firm characteristics, such as the task performed by the worker and the size of the firm. See the detailed description in the Appendix XXX.

follow

$$v(n) = \Gamma \frac{n^{1+1/\varepsilon}}{1+1/\varepsilon}, \tag{40}$$

where ε is the common elasticity of labor supply.

Together, assumptions (36), (38), (39) and (40) identify the model. We estimate the model by Maximum Likelihood. The details of the estimation can be found in Appendix D.

4.2. Data description

Our estimates are based on the 2013 wave of the household survey by the official statistical agency of Colombia (DANE). In particular, the survey contains data on *(i)* gross wages and hours at the main job, which we use to compute workers' productivities, *(ii)* contributions to social security at the main job, allowing us to determine whether the main job is formal or informal, *(iii)* workers and job characteristics, such as age, gender, education, experience, industry and tasks at the main job, which we include in the vector of regressors X.

We identify the main job of a given worker as informal if the worker reports not contributing to mandatory social security programs. Since social security contributions are paid jointly with payroll taxes and the withheld part of the personal income tax, a worker who contributes to the social security system is automatically subject to income taxation. Thus, this approach is particularly well suited for our exercise. Detecting informality via social security contributions is broadly consistent with the methodology of the International Labour Organization (ILO 2013) and is used by the Ministry of Labor of Colombia (ILO 2014), as well as by Mora and Muro (2017) and Guataquí R., García S., and Rodríguez A. (2010) in the studies of Colombia.[28] We further assume that survey respondents correctly reveal their gross income from the main job, regardless of whether the main job is formal or informal. Other papers making this assumption include Meghir, Narita, and Robin (2015) for Brazil and López García (2015) for Chile.[29]

[28] Official statistical agency of Colombia (DANE) follows an alternative measure of informality based on size of the establishment, status in employment and educational level of workers. They find that 57.3% and 56.7% of workers were informal in the first two quarters of 2013 (ILO 2014), which is very close to 58% we find for the entire 2013.

[29] In contrast, several papers made an opposite assumption by treating gross income reported in surveys as a gross *formal* income (e.g. studies of UK by Pissarides and Weber (1989) and Lyssiotou, Pashardes, and Stengos (2004), and a study of Russia by Gorodnichenko, Martinez-Vazquez, and Peter (2009)). There are several justifications for our approach. First, respondents to the Colombian household survey are not asked explicitly about their shadow or formal income. Rather, the informality status of income is indirectly inferred from the reported affiliation to the social insurance programs. Second, given that more than half of workers in Colombia are informal, the stigma associated with revealing their informal incomes is likely to be much lower than e.g. in UK, where the shadow economy is much smaller. Finally, if survey respondents reported only their formal income, it would imply that 58% of respondents misreported their social security status, falsely claiming not contributing to social security, which is unlikely and would invalidate the ILO methodology of measuring informality.

Table 1: Parameter estimates

preferences		productivity schedules					distributions of θ and κ			
ε	Γ	$w^s(0)$	ρ^s	$w^f(0)$	ρ^f	α_w	σ_θ	σ_κ	α_κ	w_κ
0.358	0.497	0.011	2.99	0.0085	4.15	2.4	0.51	0.59	0.05	0.008

We restrict attention to individuals aged 24-50 years without children (26,000 individuals). We choose this sample because these workers face the tax and transfer schedule which is not means-tested and does not depend on choices absent from our modeling framework, such as number of children or college attainment. Hence, consistently with the model, the actual tax system in our sample can be represented as a function of income only. We additionally exclude individuals with formal earnings at the main job below the level implies by the minimum wage at the full time job, whose reported income is likely to be erroneous. In our sample, 24% of individuals have a main job that is informal.

The income tax and transfer schedule in the sample consists of three components: *(i)* a flat payroll tax with 22% rate,[30] *(ii)* a personal income tax with the rate which increases from 0% to 27% for annual incomes above the equivalent of 25,000 USD, *(iii)* transfers and social programs that peak at zero formal income and vanish as income approaches the minimum wage for a full time worker (approximately 3000 USD annually). Other tax instruments, such as the VAT or the corporate income tax paid by formal firms, may affect gross wages and hence the workers' labor choices. We indirectly capture these impacts by using empirical gross wages.

4.3. Results

Table 1 presents the estimates of parameters of preferences, productivity schedules and distributions of types. The estimated vector β is reported in Appendix D.

The left panel of Figure 7 presents the estimated productivity profiles and the density of productivity types. The bottom 8% of workers are more productive in the shadow sector while the median worker is 32% more productive formally. The density of types is approximately normal, which means that sectoral wages are distributed approximately log-normally.

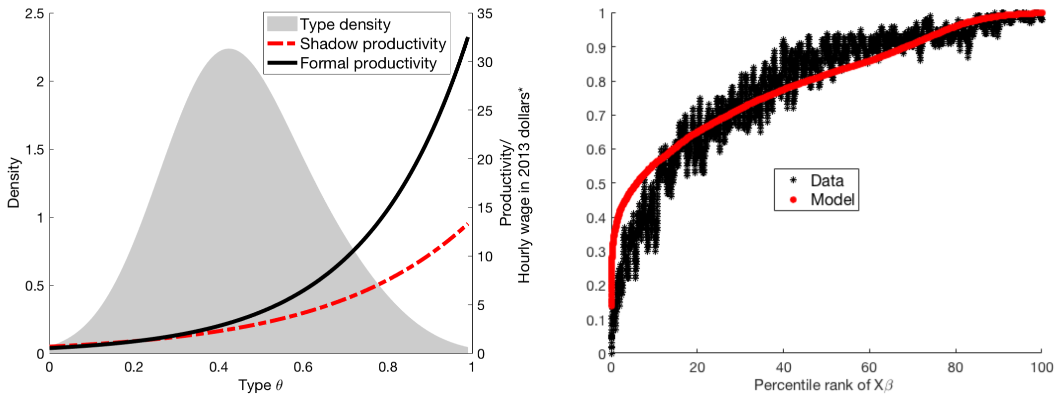
We find the elasticity of 0.358, close to the value of 0.33 suggested by Chetty (2012). Together with the estimated Pareto tail of formal wages, it implies that the formal income distribution at the top is Pareto distributed with parameter 1.767.[31] Saez (2001) finds that in US the upper tail of income is Pareto distributed with parameter 2, implying a slightly thinner tail than in Colombia.

[30] In line with the Congressional Budget Office of the US, we assume that the incidence of all payroll taxes and social security contributions is on workers.

[31] Formal income follows Pareto distribution with parameter $\alpha_w/(1+\varepsilon)$.

Figure 7: Estimation results

(a) Productivity profiles and type distribution (b) Probability of having a formal main job



Our estimates verify that the single crossing condition, assumed in the theoretical analysis, holds in the data. The single crossing condition requires that the comparative advantage in the shadow economy $w^s(\theta)/w^f(\theta)$ is decreasing in type. Under our functional assumption (36), it is equivalent to a positive difference in the growth rates of formal and shadow productivities: $\rho^f > \rho^s$. We find that this difference is positive and statistically different from zero at 1% level of significance.[32]

The right panel of Figure 7 shows the estimated probability of having a main job in a formal sector for each percentile of $X\beta$. The probability of having formal main job is strictly increasing and ranges from 14% to 99%. To illustrate the fit of the model we also plot the share of shadow workers in a rolling window of 200 workers centered around each observed $X\beta$ in the sample. The model is tracks the data well, showing that our parametric specification is compatible with the observed sorting of workers across sectors.

5. Quantitative section.

First we derive the social preferences implicit in the actual Colombian tax schedule. Then we analyze the optimal tax schedules given various strength of a redistributive motive of the social planner. Finally we examine the welfare consequences of the existence of the shadow economy in Colombia.

5.1. Social preferences implicit in the actual tax schedule.

The optimal tax formula can be used to extract social preferences, i.e. Pareto weights at each income level, implicit in a given tax schedule. This methodology is typically used

[32]The estimate of the difference is 1.16 with a standard error of 0.002. We obtained the standard errors from the information matrix.

to study the Pareto efficiency of the actual tax, as a negative Pareto weight would mean that the tax system is inefficient and the government can increase tax revenue without reducing utility of any agent.[33]

To extract the welfare weights, differentiate the optimal tax formula (28) evualuated at some formal income level y to get

$$\bar{\lambda}(y) = \mathbb{E}(\bar{\lambda}) + \underbrace{\frac{\partial DWL(y)}{\partial y}\frac{1}{h(y)}}_{\text{intensive margin}} - \underbrace{\pi(y)\frac{h^f(y)}{h(y)}}_{\text{extensive margin}} , \tag{41}$$

where $DWL(\cdot)$ stands for the total deadweight loss, i.e. the left-hand side of (28), at formal income level y. The mean Pareto weight at a given income level can be explained by three components. The first one is the average Pareto weight across all income levels, equal to 1. The second is the contribution of the intensive margin. The total deadweight loss, including both formal and shadow workers, increases faster at income levels associated with higher Pareto weights. That is because higher $\bar{\lambda}(y)$ reduces the deadweight loss below y and does not affect the deadweight loss above y (see equation (28)). The third component captures the extensive margin: a decision to participate in the shadow economy. Recall that $\pi(y)$ is the elasticity of the density of formal workers with respect to the tax burden of staying formal. The impact of extensive margin is similar to that of the Pareto weight: it implies a higher derivative of the deadweight loss. Hence, higher $\pi(y)$ means that a smaller part of the increase of deadweight loss remains to be explained by social preferences.

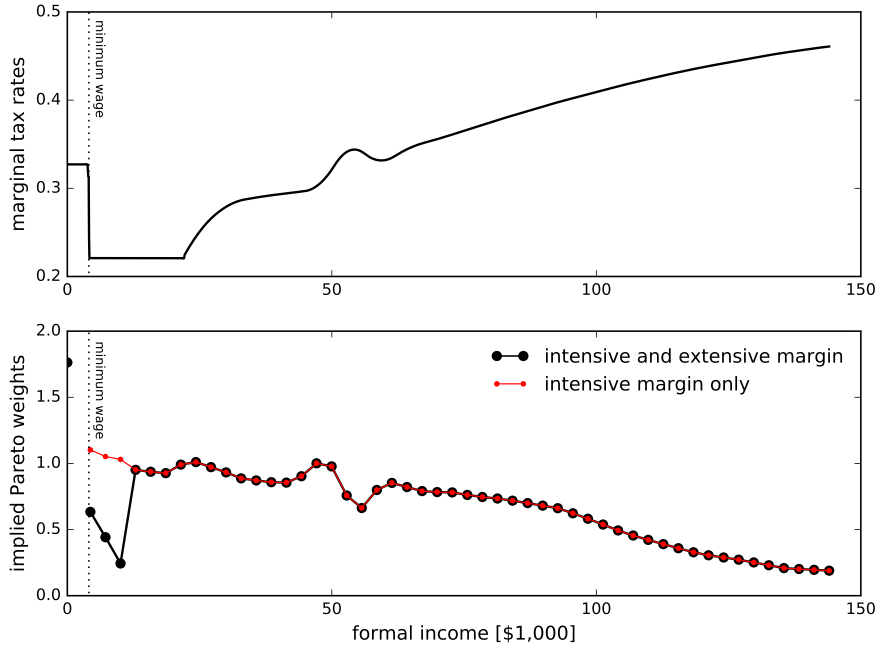
Figure 9 shows the actual tax schedule in Colombia in 2013 as well as the implied Pareto weights. The marginal tax rates are high at the bottom due to the phase-out of transfers, then drop to 22% - the rate of payroll taxation - and then increase as the personal income tax starts at around \$22,000. The personal income tax is roughly progressive, but highly irregular: it is a step function of more than 70 steps of varying length. To abstract from inefficiencies associated with multiple small notches which are not the focus of the paper, we approximate this part of the Colombian tax schedule with cubic splines.[34]

We find no evidence of negative Pareto weights - the smoothed Colombian tax schedule is Pareto efficient. However, the implicit weights exhibit a peculiar pattern: they are much lower for workers with earnings close to the minimum wage than for workers with slightly higher earnings. For instance, formal workers earning \$10,000 annually have an implied Pareto weight which is *four times smaller* than the weight of workers earning \$13,000. Although the income interval of unusually low Pareto weights is relatively

[33] The original test of Pareto efficiency was proposed by Werning (2007). The methodology was further developed and applied by Bourguignon and Spadaro (2012); Brendon (2013); Lorenz and Sachs (2016); Jacobs, Jongen, and Zoutman (2017).

[34] The minimum wage is another source of inefficiency. Since our framework is not designed to address this issue, we do not optimize with respect to the the minimum wage level.

Figure 9: Income tax in Colombia



small, it contains 34% of all formal workers. None of the workers with formal earnings in this interval has shadow earnings.

Unusually low Pareto weights at low incomes are driven by the incentives to participate in the shadow economy, i.e. the extensive margin term in formula (41). If instead we consider only the intensive margin term (see the red line in the bottom panel of Figure 9), the Pareto weights look much more regular. In particular, they are decreasing with income at low income levels.[35] The plausible interpretation of this result is that the Colombian tax schedule was set without taking into account the extensive margin incentives for informality. Note that the marginal rate of the actual tax schedule is constant where the Pareto weights are unusually low. The tax schedule which accounts for informality and follows the more intuitive, decreasing schedule of Pareto weights would instead have increasing, rather than flat, marginal rates.

5.2. Optimal tax schedules.

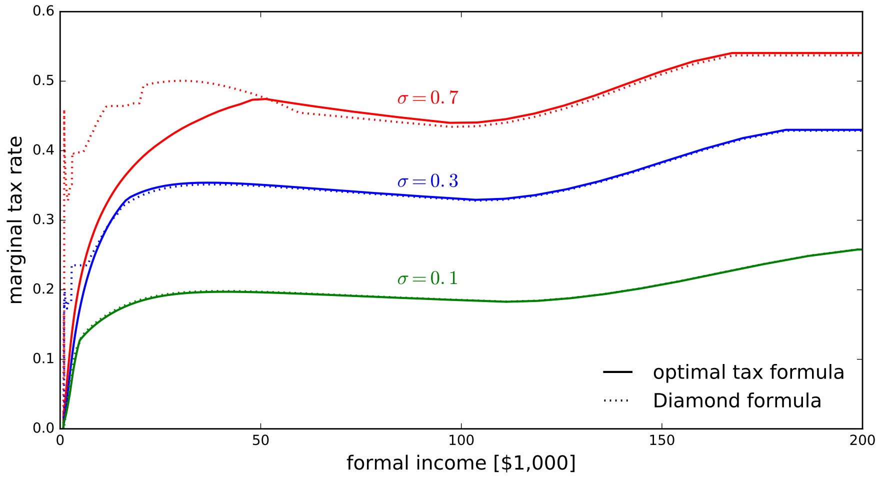
Consider the following social welfare function

$$\int_0^{\theta^{top}} \int_0^{\infty} \frac{\tilde{V}(\theta,\kappa)^{1-\sigma}}{1-\sigma} dG_\theta(\kappa) dF(\theta) + \lambda^{top} \int_{\theta^{top}}^{1} \int_0^{\infty} \tilde{V}(\theta,\kappa) dG_\theta(\kappa) F(\theta). \qquad (42)$$

where $\tilde{V}(\theta,\kappa)$ is the utility shifted by a constant to ensure positive values. The planner has a taste for redistribution which increases in parameter σ. The planner is willing to

[35] Pareto weights are also locally increasing when marginal rates of the personal income tax are increasing rapidly. It has nothing to do with the informal sector.

Figure 11: Optimal tax schedules



redistribute both between different productivity types and between different cost types, to the extend that they obtain different utility level. It implies that the implicit Pareto weight placed on a shadow θ-worker will be lower than the weight of a formal θ-worker. We find the optimal income tax for various values of σ. In each case we require the same net tax revenue as the one generated by the actual Colombian tax. To facilitate the computations, we assume that the Pareto weights are constant in the upper tail.[36]

The optimal policies are depicted on Figure 11. We also show how the tax schedule would look like if the planner followed the Diamond formula instead of the optimal tax formula. Since the income distribution is endogenous to the tax policy, we find the tax schedules iteratively: a tax schedule implies an income distribution which, together with the tax formula, results in a new tax schedule. The schedules we present are the fixed points of this mapping.[37] Note that each tax formula can have, in principle, multiple fixed points. In practice, we find multiple solutions only for the Diamond formula. All of these solutions share a common feature: an excessive size of the shadow economy. We report the one which yields the highest welfare. It also happens to have the smallest informal sector out of all the solutions to the Diamond formula.

The optimal tax schedules feature low marginal tax rates at low income levels which increase fairly quickly with income. What is more, the tax schedules at low income levels are very similar for different values of σ. At low incomes the net-of-tax rates are below the comparative advantage $w^s(\cdot)/w^f(\cdot)$ and some workers supply shadow labor. From section 3 we know that in this case the planner reduces the marginal tax rates. To put it differently, the planner shifts the net-of-tax rates toward the comparative advantage of workers at the given income level. The tax schedules at low incomes are

[36] We set the Pareto weight at the upper tail, λ^{top}, to the value of the implicit Pareto weight of θ^{top} agent, equal $\tilde{V}(\theta^{top}, \infty)^{-\sigma}$. If all θ^{top}-workers are formal, as is the case in our numerical exercises, the implied schedule of Pareto weights is then continuous.

[37] Rothschild and Scheuer (2016) call these fixed points 'Self-confirming Policy Equilibria'.

Table 2: Comparison of allocations implied by the optimal and the Diamond formula

	$\sigma = 0.1$		$\sigma = 0.3$		$\sigma = 0.7$	
	optimal	Diamond	optimal	Diamond	optimal	Diamond
share of shadow workers	1.2%	1.4%	2.1%	6.7%	4.5%	25.9%
share of shadow income	0.1%	0.1%	0.2%	1.1%	0.6%	11.1%
welfare gain rel. to Diamond	0.0%		0.1%		6.4%	

similar, since they follow closely the schedule of comparative advantage. The optimal tax rates at middle and higher incomes increase with σ. The U-shape pattern of marginal tax rates, detected by Diamond (1998) and Saez (2001) for the US, is less pronounced here. That is mainly because the shadow economy reduces the tax rates at low income levels in comparison to the standard model.

The discrepancy between tax schedules implied by the optimal and the Diamond formulas depends on the redistributive taste of the planner. For the lowest considered value of σ, the two tax schedules are virtually identical. As σ increases, the Diamond formula implies higher rates at low income levels. As we discussed in Section 3, the Diamond formula captures neither the entire deadweight loss nor the extensive margin responses of shadow workers. It leads to higher marginal tax rate at income levels where some agents are tempted by the informal employment, which in this case happens at low income levels. Furthermore, higher marginal tax rates displace larger fraction of workers to the shadow economy, shrinking the density of formal income and reducing the deadweight loss of taxation. The endogeneity of income distribution reinforces the upward bias of taxes under the Diamond formula.

At middle and high income levels all agents remain formal and the Diamond policies converge to the optimal ones. The Diamond formula leads to an excessive redistribution, which reduces utility levels and implicit welfare weights in this income range. As a result, the Diamond planner slightly undershoots the tax rates in the middle of the income distribution in comparison to the optimal ones.

Table 2 compares the size of shadow economy implied either by the optimal or by the Diamond tax formula. The optimal fraction of shadow workers increases with σ from 1.2% to 4.5%. The share of income earned in the shadow economy is proportionally smaller, as only the least productive agents work informally. The Diamond formula, which prescribes higher tax rates at low income levels, leads to an excessive informal employment: the share of shadow workers varies from 1.4% up to 25.9%. The welfare loss from using suboptimal tax formula, expressed in terms of consumption, increases steeply with σ from close to 0% up to 6.4% for the most redistributive planner.

5.3. Welfare impact of the shadow economy.

Does the Colombian shadow economy have a potential to improve welfare? We generalize the notions of efficiency and redistribution gains from Section 2 to a model with a continuum of types and a fixed cost of shadow employment. Suppose that all variables with superscript ^{M} correspond to the optimum of the Mirrlees model without the shadow economy:[38]

$$\underbrace{V(\theta,\kappa) - V^M(\theta)}_{\text{total gain of agent }(\theta,\kappa)} =$$
$$\underbrace{V(\theta,\kappa) + T(y^f(\theta,\kappa)) - V^M(\theta) - T^M(y^M(\theta))}_{\text{efficiency gain of agent }(\theta,\kappa)} + \underbrace{T^M(y^M(\theta)) - T(y^f(\theta,\kappa))}_{\text{redistribution gain of agent }(\theta,\kappa)}. \quad (43)$$

As before, the redistribution gain captures the change in the level of transfers, while the efficiency gain measures the reduction of distortions in the shadow economy allocation relative to the Mirrleesian allocation. Note that the fixed cost of shadow employment becomes another source of distortion in the shadow economy allocation.

Figure 12 depicts the welfare decomposition for the high-cost and the low-cost workers when $\sigma = 0.3$. The high-cost workers with low to medium productivity type benefit modestly from lower marginal tax rates, but suffer from lower transfers with the shadow economy. For sufficiently productive high-cost workers the redistribution gain becomes positive, as lower marginal tax rates at low incomes imply lower tax burden at higher incomes. In the case of the low-cost workers, the low productivity types work only in the shadow economy and gain on the efficiency side due to high shadow productivity, but lose due to lower transfers. For the middle types the picture is reversed. These agents work simultaneously in both sectors and choose formal income level with relatively high marginal tax rates, which implies high distortions, but low total taxes. Finally, the most productive low-cost workers stay fully formal and gain from lower total taxes.

In Table 3 we aggregate efficiency and redistribution gains with welfare weights and type densities and express them in consumption equivalent terms, relative to the Mirrleesian allocation. The sum of the two aggregate gains is the total welfare gain from the existence of the shadow economy.

The shadow economy improves aggregate efficiency, but restricts redistribution. There are two effects at play. First, the least productive agents are actually more productive in the shadow economy, which improves efficiency. Second, the marginal tax rates with shadow economy are weakly lower. On the one hand it strengthens efficiency, on the other it restricts redistribution. The sum of the two effects depends on the redistributive taste of the planner. For σ close to zero only the first effect is present and the shadow economy modestly improves welfare. As σ increases, the redistribution gain falls faster

[38] Specifically, we take the implied welfare weights from the optimum with the shadow economy and use them to derive the optimal Mirrleesian allocation.

Figure 12: Welfare decomposition ($\sigma = 0.3$)

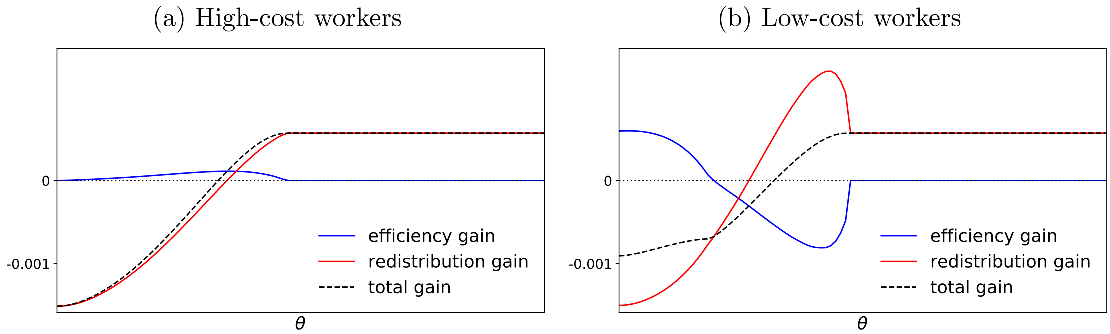


Table 3: Potential welfare gain from the existence of the shadow economy

	$\sigma = 0.1$	$\sigma = 0.3$	$\sigma = 0.7$
aggregate efficiency gain	0.02%	0.23%	2.68%
aggregate redistribution gain	-0.0%	-0.25%	-3.95%
total welfare gain	0.01%	-0.02%	-1.27%

than the efficiency gain increases, leading to a substantial welfare loss for planners with strong redistributive preferences.

6. Conclusions

In this paper we study the optimal income taxation when agents have access to the informal labor market, the income from which is unobserved by the government. We show that the optimal tax formulas with a shadow economy contain additional terms which typically lead to weakly lower marginal tax rates. In our calibrated model of Colombia the marginal tax rates are lower particularly at low income levels. In contrast, the standard tax formula implies higher tax rates at low income levels and displaces an excessive share of workers to the shadow economy, which can lead to a large welfare loss. We also show that, from the theoretical perspective, the shadow economy has an ambiguous impact on welfare. Depending on the distribution of formal and shadow productivities, the shadow economy can either improve or deteriorate welfare through two channels: labor efficiency and redistribution. We find that Colombian shadow economy strengthens efficiency of labor supply at the expense of possible redistribution. For low to medium strength of redistributive preferences the two channels roughly cancel out and the shadow economy is welfare neutral. For strongly redistributive governments the redistribution channel dominates and the shadow economy reduces social welfare.

Our analysis could be extended in several directions. First, suppose that the government can use audits and penalties to differentially affect tax evasion opportunities of different agents.[39] The optimal design of tax audits could, rather than minimizing overall tax evasion, tailor individual evasion opportunities to maximize the welfare improving potential of the shadow economy. Second, informal activity is closely related to home production. In some developed countries home production may be quantitatively more important than tax evasion. When a home produced good is a perfect substitute for a market income, our framework can be directly applied to study this phenomenon.

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[39] For instance, conducting tax audits at medium levels of formal income restricts tax evasion of highly productive agents, but not of low productivity workers who would never choose such income level. See Cremer and Gahvari (1996) for the analysis of tax audits in the optimal taxation model with two types.

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A. Additional lemmas and proofs of Section 2

Proof of Lemma 1. Denote the incentive constraint of type i by IC_i. Suppose that at the optimum IC_H does not bind. Increase y_H^f and T_H by a small $\epsilon > 0$ and decrease T_L by $\frac{\mu_H}{\mu_L}\epsilon$. Welfare increases, so it could not be the optimum in the first place.

Suppose that IC_L is binding. If $T_H \geq T_L$, the planner can increase tax revenue while keeping welfare constant by assigning (y_H^f, T_H) to all agents. If $T_H < T_L$, the same is possible by assigning (y_L^f, T_L) to all agents. These pooling allocations can be improved in terms of welfare by moving to *laissez-faire*, where taxes are zero and labor supply is undistorted. In *laissez-faire* both incentive constraints are slack. □

Lemma A.1. *At the optimum the following condition holds*

$$\min\left\{\frac{v'\left(n_L\right)}{w_L^f} - \left(\mu_L + \mu_H \frac{v'\left(n_{H,L}\right)}{w_H^f}\right), n_L^f\right\} = 0, \tag{44}$$

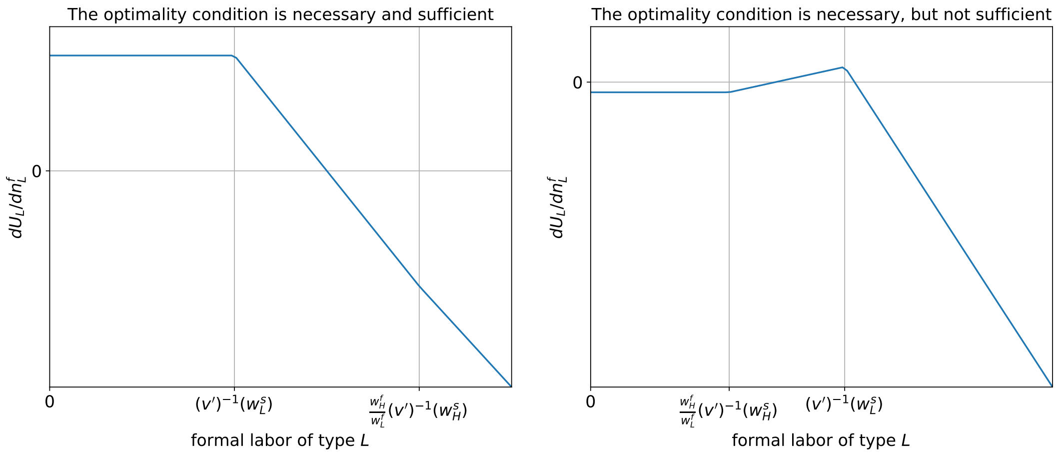
where $n_{H,L} \equiv y_L^f / w_H^f + y_H^s\left(y_L^f\right)/w_H^s$ is the labor supply of type H pretending to be L.

Suppose that v' is convex. If $w_H^f \left(v'\right)^{-1}\left(w_H^s\right) \geq w_L^f \left(v'\right)^{-1}\left(w_L^s\right)$ then (44) is a sufficient condition for the optimum.

Proof of Lemma A.1. Use the resource constraint to express tax paid by H as a function of tax paid by L: $T_H = -\frac{\mu_L}{\mu_H} T_L$. Consider changing formal labor supply of L by dn_L^f and adjusting the T_L such that the incentive constraint of H is satisfied as equality. It means that

$$\frac{dT_L}{\mu_H} = \left(w_L^f - \frac{w_L^f}{w_H^f} v'\left(n_{H,L}\right)\right) dn_L^f. \tag{45}$$

Figure 14: Sufficiency of the optimality condition (44).



This perturbation affects utility of L by

$$dU(c_L, n_L) = \left(w_L^f - v'(n_L)\right) dn_L^f - dT_L = \left(w_L^f \left(\mu_L + \mu_H \frac{v'(n_{H,L})}{w_H^f}\right) - v'(n_L)\right) dn_L^f. \tag{46}$$

Optimum requires that either $dU(c_L, n_L) = 0$ or $dU(c_L, n_L) < 0$ and $n_L^f = 0$, which results in (44). Sufficiency of (44) depends on the second order terms, which are calculated in Table 4. The terms are signed under the assumption of v' being convex. We need to consider two cases: (i) If $w_H^f (v')^{-1}(w_H^s) \geq w_L^f (v')^{-1}(w_L^s)$, then $dU(c_L, n_L)/dn_L^f$ is non-increasing in n_L^f and the optimality condition (44) is sufficient. (ii) If $w_H^f (v')^{-1}(w_H^s) < w_L^f (v')^{-1}(w_L^s)$, then $dU(c_L, n_L)/dn_L^f$ is non-monotone and the optimality condition (44) can potentially point at local extrema which are not the global maximum. These two cases are shown on Figure 14.

Table 4: Second order derivative of welfare with respect to the perturbation

The case of $w_H^f (v')^{-1}(w_H^s) \geq w_L^f (v')^{-1}(w_L^s)$			
$n_L^f \in$	$[0, (v')^{-1}(w_L^s))$	$\left((v')^{-1}(w_L^s), \frac{w_H^f}{w_L^f}(v')^{-1}(w_H^s)\right)$	$\left(\frac{w_H^f}{w_L^f}(v')^{-1}(w_H^s), \infty\right)$
$\frac{d^2U(c_L,n_L)}{dn_L^{f2}} =$	0	$-v''\left(n_L^f\right) < 0$	$\mu_H \left(\frac{w_L^f}{w_H^f}\right)^2 v''\left(\frac{w_L^f}{w_H^f} n_L^f\right) - v''\left(n_L^f\right) < 0$
The case of $w_H^f (v')^{-1}(w_H^s) < w_L^f (v')^{-1}(w_L^s)$			
$n_L^f \in$	$[0, \frac{w_H^f}{w_L^f}(v')^{-1}(w_H^s))$	$\left(\frac{w_H^f}{w_L^f}(v')^{-1}(w_H^s), (v')^{-1}(w_L^s)\right)$	$\left((v')^{-1}(w_L^s), \infty\right)$
$\frac{d^2U(c_L,n_L)}{dn_L^{f2}} =$	0	$\mu_H \left(\frac{w_L^f}{w_H^f}\right)^2 v''\left(\frac{w_L^f}{w_H^f} n_L^f\right) > 0$	$\mu_H \left(\frac{w_L^f}{w_H^f}\right)^2 v''\left(\frac{w_L^f}{w_H^f} n_L^f\right) - v''\left(n_L^f\right) < 0$

□

Proof of Proposition 1. Inequality (7) means that the optimality condition (44) is satisfied at $n_L^f = 0$. From Figure 14 it is clear that otherwise it is never optimal for L to work in the shadow economy. Furthermore, when $w_H^f (v')^{-1} (w_H^s) \geq w_L^f (v')^{-1} (w_L^s)$, the optimality condition (44) has a unique solution at $n_L^f = 0$.

Suppose that $w_H^f (v')^{-1} (w_H^s) < w_L^f (v')^{-1} (w_L^s)$. Inequality (8) means that the maximum of $dU (c_L, n_L) / dn_L^f$, which is attained at $n_L^f = (v')^{-1} (w_L^s)$, is non-positive (see Figure 14). Therefore, it is optimal to reduce n_L^f all the way to zero. $\square$

Proof of Proposition 2. First we will express the utility of type L in the optimum of the standard Mirrlees model. From the incentive constraint of H and the resource constraint we get

$$T_L^M = \mu_H \left(U \left(w_L^f n_L^M, \frac{w_L^f}{w_H^f} n_L^M \right) - U \left(w_H^f n_H^M, n_H^M \right) \right). \tag{47}$$

Plugging it into the utility of type L yields

$$U \left(c_L^M, n_L^M \right) = U \left(w_L^f n_L^M, n_L^M \right) - \mu_H \left(U \left(w_L^f n_L^M, \frac{w_L^f}{w_H^f} n_L^M \right) - U \left(w_H^f n_H^M, n_H^M \right) \right). \tag{48}$$

Using the same steps, we can express the utility of type L in the shadow economy model, when L works only in the shadow economy, as

$$U \left(c_L^{SE}, n_L^{SE} \right) = U \left(w_L^s n_L^{SE}, n_L^{SE} \right) - \mu_H \left(U \left(w_H^s n_{H,L}^{SE}, n_{H,L}^{SE} \right) - U \left(w_H^f n_H^{SE}, n_H^{SE} \right) \right), \tag{49}$$

where $n_L^{SE} = [v]^{-1}(w_L^s)$ and $n_{HL}^{SE} = [v]^{-1}(w_H^s)$. Since there are no distortions at the top and no wealth effects, $n_H^M = n_H^{SE}$. The difference in utility of L between the two allocations is

$$\left[U \left(w_L^s n_L^{SE}, n_L^{SE} \right) - U \left(w_L^f n_L^M, n_L^M \right) \right] + \mu_H \left[U \left(w_L^f n_L^M, \frac{w_L^f}{w_H^f} n_L^M \right) - U \left(w_H^s n_{H,L}^{SE}, n_{H,L}^{SE} \right) \right]. \tag{50}$$

The first term (the efficiency gain) is independent of w_H^s and depends on w_L^s only through $U(w_L^s n_L^{SE}, n_L^{SE})$. Hence, it is increasing in w_L^s and is positive iff $w_L^s > \bar{w}_L^s$. The second term (the redistribution gain) is independent of w_L^s and depends on w_H^s only through $-U(w_H^s n_{H,L}^{SE}, n_{H,L}^{SE})$. Hence, it is decreasing in w_H^s and positive when $w_H^s < \bar{w}_H^s$.

Now we will show that the optimum of the standard Mirrlees is not incentive compatible with the shadow economy when the redistribution gain is non-positive. It means that when both gains are negative, the optimum of the shadow economy model necessarily yields lower welfare than the standard Mirrlees model. Assume that $w_H^s \geq \bar{w}_H^s$. We will show that $n_{HL}^{SE} > \frac{w_L^f}{w_H^f} n_L^M$. Suppose the contrary, it follows that

$$U \left(w_L^f n_L^M, \frac{w_L^f}{w_H^f} n_L^M \right) \geq U \left(w_H^f n_{HL}^{SE}, n_{HL}^{SE} \right) > U \left(w_H^s n_{HL}^{SE}, n_{HL}^{SE} \right). \tag{51}$$

The first inequality comes from the fact that $\frac{w_L^f}{w_H^f}n_L^M$ is below the efficient level of labor supply of type H, so lowering the labor of this type even further to n_{HL}^{SE} will decrease the utility of H. The second inequality follows from $w_H^f > w_H^s$. This sequence of inequalities implies that the redistribution gain is strictly positive. Hence, if the redistribution gain is non-positive, $n_{HL}^{SE} > \frac{w_L^f}{w_H^f}n_L^M$ holds. Note that $n_{HL}^{SE} > \frac{w_L^f}{w_H^f}n_L^M$ means that the optimal allocation of the standard Mirrlees model is not incentive compatible with the shadow economy - deviating type H would supply some additional shadow labor. □

Proof of Proposition 3. Suppose that $\lambda_i \leq \lambda_{-i}$. Note that the incentive constraint of at most one type can bind - otherwise the planner could improve welfare by moving to *laissez-faire*, as was shown in the proof of Lemma 1. The incentive constraint of type $-i$ does not restrict the optimum welfare, since the planner can relax it and potentially improve welfare by increasing T_i and reducing T_{-i}. Hence, only the incentive constraint of i can be binding (in a sense of restricting the optimum welfare) in the optimum. Suppose that $n_i^s > 0$. Decrease n_i^s and increase n_i^f by the same amount and increase T_i such that the utility of i is unchanged. The planner has additional resources without decreasing welfare.

Suppose that $\lambda_i > \lambda_{-i}$, so that the incentive constraint of $-i$ binds. Perturb n_i^f and adjust T_i along the binding incentive constraint:

$$\frac{dT_i}{\mu_{-i}} = \left(w_i^f - \frac{w_i^f}{w_{-i}^f}v'\left(n_{-i,i}\right)\right)dn_i^f, \tag{52}$$

where $n_{-i,i}$ is the total labor supply of the deviating type $-i$. This perturbation affects social welfare by

$$dW = \lambda_i\mu_i\left[\left(w_i^f - v'\left(n_i\right)\right)dn_i^f - dT_i\right] + \lambda_{-i}\mu_{-i}\frac{\mu_i}{\mu_{-i}}dT_i \tag{53}$$

$$= \lambda_i\mu_i w_i^f\left(\left(1 - \frac{v'\left(n_i\right)}{w_i^f}\right) + \left(\frac{\lambda_{-i}}{\lambda_i} - 1\right)\mu_{-i}\left(1 - \frac{v'\left(n_{-i,i}\right)}{w_{-i}^f}\right)\right)dn_i^f \tag{54}$$

Suppose that n_i^f is low enough such that $v'\left(n_i\right) = w_i^s$. Furthermore, assume that $\frac{w_{-i}^s}{w_{-i}^f} \geq \frac{w_i^s}{w_i^f}$. It follows that

$$1 - \frac{w_i^s}{w_i^f} \geq 1 - \frac{w_{-i}^s}{w_{-i}^f} \geq 1 - \frac{v'\left(n_{-i,i}\right)}{w_{-i}^f} > \left(1 - \frac{\lambda_{-i}}{\lambda_i}\right)\mu_{-i}\left(1 - \frac{v'\left(n_{-i,i}\right)}{w_{-i}^f}\right), \tag{55}$$

which means that $dW > 0$ - increasing formal labor supply of i is welfare improving. Therefore, it is never optimal to decrease n_i^f so much that type i starts working in the shadow economy. □

Proof of Proposition 4. First we will show how to derive the redistribution gain (the efficiency gain is straightforward). Since there are no distortions imposed on type $-i$,

$$\mu_{-i}\lambda_{-i}\left[U\left(c_{-i}^{SE},n_{-i}^{SE}\right)-U\left(c_{-i}^{M},n_{-i}^{M}\right)\right]=\mu_{-i}\lambda_{-i}\left[T_{-i}^{M}-T_{-i}^{SE}\right]=-\mu_{i}\lambda_{-i}\left[T_{i}^{M}-T_{i}^{SE}\right]. \tag{56}$$

Add it to a redistributive component present in the utility of type i: $\mu_i\lambda_i\left[T_i^M-T_i^{SE}\right]$, to get a redistribution gain. Further, using the binding incentive constraint of $-i$, we can express this gain as

$$\mu_i\mu_{-i}\left(\lambda_i-\lambda_{-i}\right)\left(U\left(w_i^f n_i^M,\frac{w_i^f}{w_{-i}^f}n_i^M\right)-\Psi\left(w_{-i}^s\right)\right). \tag{57}$$

The thresholds are derived by setting each gain to zero. Note that both gains are monotone in a relevant shadow productivity. Hence, when $w_i^s\geq\bar{w}_i^s$ and $w_{-i}^s\leq\bar{w}_{-i}^s$, where at least one inequality is strict, both gains are non-negative and there is a welfare gain from shadow economy in comparison to the standard Mirrlees model. Since the standard Mirrlees model constitutes an upper bound on welfare that can be obtained without shadow employment, type i optimally works in the shadow economy. $\square$

B. Proofs of Section 3

Proof of Lemma 2. The strict *Spence-Mirrlees* single crossing condition holds if, keeping the formal income level fixed, the marginal rate of substitution $v'\left(\frac{y^f}{w^f(\theta)}+\frac{y^s}{w^s(\theta)}\right)\Big/w^f(\theta)$ is decreasing with θ. For formal workers it follows from the convexity of v. For shadow workers we have $v'(n)=w^s(\theta)$ and the single-crossing follows from $w^s(\theta)/w^f(\theta)$ being decreasing. $\square$

Proof of Proposition 5. Given the single crossing condition, the necessity of non-decreasing formal income schedules and local incentive constraints follows from Theorem 7.2 in Fudenberg and Tirole (1991). By Theorem 7.3 in Fudenberg and Tirole (1991), non-decreasing formal income schedule and local incentive constraints are sufficient to prevent deviations *within* the cost class, i.e. deviations of some high-cost (low-cost) worker to formal income level earned by another high-cost (low-cost) worker. We will show that conditions 2. - 4. are sufficient to prevent deviations *between* the cost classes.

Denote the image of formal income schedule of types with fixed cost $\kappa\in\{0,\infty\}$ by $Y(\kappa)\equiv\{y\in\mathbb{R}_+:\exists_{\theta\in[0,1]}y^f(\theta,\kappa)=y\}$. Deviations between the cost classes may arise if the formal income schedules of the two classes do not have identical images: $Y(0)\neq Y(\infty)$. The difference in images may occur when suprema or infima of the two sets do not coincide: either $y^f(0,0)<y^f(0,\infty)$ or $y^f(1,0)<y^f(1,\infty)$. Conditions 2. and 3. take care of these possibilities. Alternatively, one of the income schedules can exhibit a discontinuous jump between formal income values where the other schedule remains continuous. Condition 4. prevents potential deviations when $y^f(\cdot,0)$ is discontinuous,

while $y^f(\cdot,\infty)$ remains continuous, i.e. when there is $y \in (y^f(0,\infty), y^f(1,0))$ such that $y \notin Y(0)$ and $y \in Y(\infty)$.[40] Below we show that the reverse situation never happens: when there is $y \in (y^f(0,\infty), y^f(1,0))$ such that $y \in Y(0)$, then always $y \in Y(\infty)$.

We will show that for any θ we can find $\tilde{\theta}$ such that $y^f(\tilde{\theta},\infty) = y^f(\theta,0)$. Take some implementable income schedules $y^f(\cdot,0)$ and $y^f(\cdot,\infty)$, the corresponding tax schedule $T(\cdot)$ and any θ such that $y^f(\theta,0) > y^f(0,\infty)$ and $y^s(\theta,0) > 0$. Consider a productivity type $\tilde{\theta}$ such that

$$\frac{v'(y^f(\theta,0)/w^f(\tilde{\theta}))}{w^f(\tilde{\theta})} = \frac{w^s(\theta)}{w^f(\theta)}. \tag{58}$$

We will show that $y^f(\tilde{\theta},\infty) = y^f(\theta,0)$. It means that at any formal income level above $y^f(0,\infty)$ which is chosen by some low-cost worker there is also some high-cost worker.[41]

Consider indifference curves of agents $(\theta,0)$ and $(\tilde{\theta},\infty)$ in the (y^f,T) space, depicted in Figure 15. The indifference curve of the low-cost θ-worker and the high-cost $\tilde{\theta}$-worker are tangential at formal income $y^f(\theta,0)$. Furthermore, the indifference curve of the low-cost worker is a straight line whenever this agent supplies shadow labor, while the indifference curve of the high-cost worker is strictly concave. Finally, the two indifference curves never cross. Otherwise, the indifference curves of agents $(\tilde{\theta},\infty)$ and (θ,∞) would cross more than once and the single crossing condition would be violated. Altogether, it means that $y^f(\theta,0)$ is the incentive-compatible formal income choice of the high-cost $\tilde{\theta}$-worker. Suppose on the contrary that agent $(\tilde{\theta},\infty)$ prefers some $\tilde{y}^f \neq y^f(\theta,0)$. This is a profitable deviation for agent $(\theta,0)$ as well, since his indifference curve is weakly higher. It contradicts the original assumption of implementability of $y^f(\cdot,0)$. $\square$

Proof of Proposition 6. Define a *net deadweight loss* as a the difference between the total deadweight loss (the left-hand side of the tax formula) and the tax revenue gain (the right-hand side of the tax formula), divided by a share of income above a given income level. Below we write the net deadweight loss at the top according to the Diamond formula as a function of the top tax rate $T'(\infty)$:

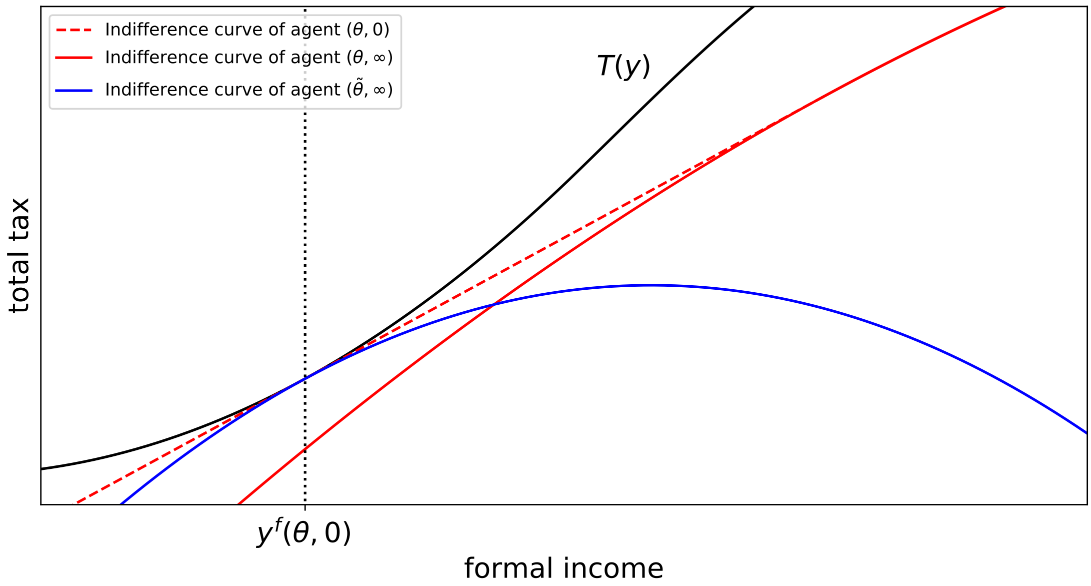
$$\begin{aligned}\lim_{y\to\infty} \frac{T'(y)}{1-T'(y)} \frac{h(y)y}{1-H(y)} \bar{\varepsilon}(y) - \mathbb{E}\{(1-\bar{\lambda}(y)) \mid y' > y\} \\ = \frac{T'(\infty)}{1-T'(\infty)} \alpha\bar{\varepsilon}(\infty) - (1-\bar{\lambda}(\infty)) \equiv NDWL_D(T'(\infty)).\end{aligned} \tag{59}$$

The optimal top tax rate according to the Diamond formula, which we denote by $T'_D(\infty)$, is given implicitly by $NDWL_D(T'_D(\infty)) = 0$.

[40] Note that, by the single crossing, it is sufficient to impose additional constraints only on particular types: top, bottom, or type at the discontinuity. If these constraints hold, other types are not tempted by a deviations, since incentive-compatible income schedules are non-decreasing.

[41] If $w^f(0) > 0$, we need to make sure that $\tilde{\theta}$ always exists. Suppose on the contrary that $v'(y^f(\theta,0)/w^f(0))/w^f(0) < w^s(\theta)/w^f(\theta)$, so that there is no $\tilde{\theta} \geq 0$ which satisfies (58). It means that if agent $(\theta,0)$ prefers $y^f(\theta,0)$ to $y^f(0,\infty)$, so does agent $(0,\infty)$. It is a contradiction, since income schedule $y^f(\cdot,\infty)$ is implementable.

Figure 15: Indifference curves.



We will derive the net deadweight loss at the top in the model with a shadow economy: $NDWL_{SE}(T'(\infty))$. When $T'(\infty) < 1 - w^s(1)/w^f(1)$ we have $NDWL_{SE}(T'(\infty)) = NDWL_D(T'(\infty))$. In the opposite case $T'(\infty) > 1 - w^s(1)/w^f(1)$ the highest formal income level of the low-cost workers $y^f(1, 0)$ is finite and the relevant tax formula is (29). We can show that

$$\lim_{\theta \to 1} \frac{\Delta T(y^f(\theta, \infty))}{\tilde{\kappa}(\theta)} = (1 + \bar{\varepsilon}(\infty)) \frac{T'(\infty)}{1 - T'(\infty)} \left(\left(\frac{w^s(1)/w^f(1)}{1 - T'(\infty)} \right)^{1+\bar{\varepsilon}} - 1 \right)^{-1} \equiv \delta(T'(\infty)), \tag{60}$$

which implies that the elasticity of the density of formal workers at the top converges to

$$\pi(y^f(\theta, \infty)) = \frac{g_\theta(\tilde{\kappa}(\theta))\tilde{\kappa}(\theta)}{1 - G_\theta(\tilde{\kappa}(\theta))} \frac{\Delta T(y^f(\theta, \infty))}{\tilde{\kappa}(\theta))} \to \gamma \delta(T'(\infty)) \equiv \pi(\infty). \tag{61}$$

Now, $NDWL_{SE}(T'(\infty))$ can be written as

$$\lim_{y \to \infty} \left\{ \frac{T'(y)}{1 - T'(y)} \bar{\varepsilon}(y) \frac{yh(y)}{1 - H(y)} - \mathbb{E}\{1 - \bar{\lambda}(y') - \pi(y') \mid y' \geq y\} \right\} \tag{62}$$

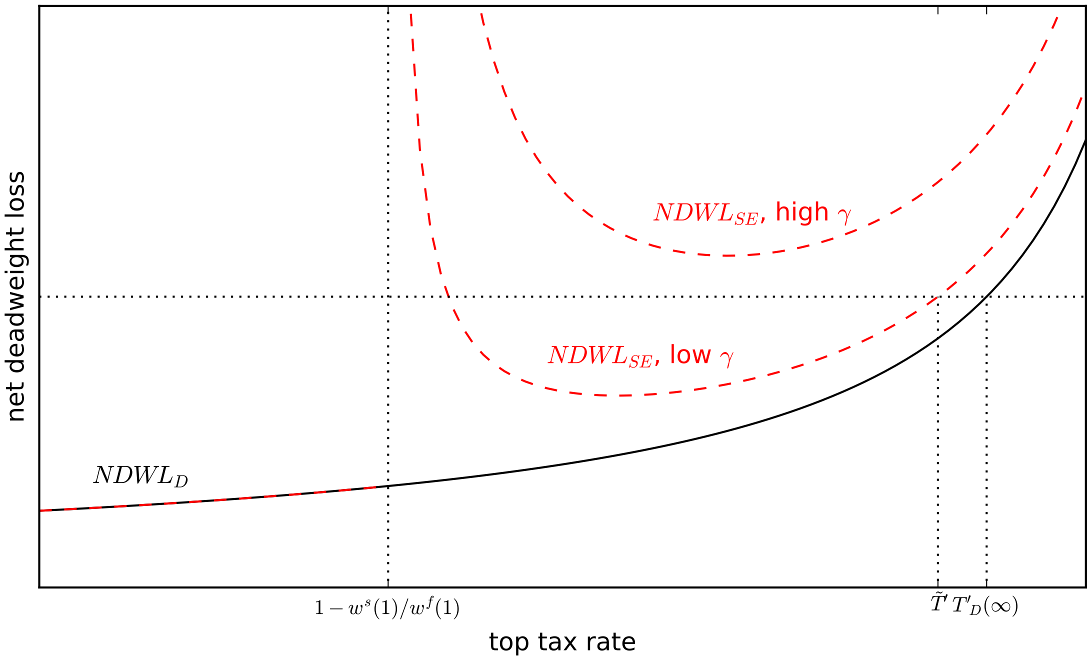
$$= \frac{T'(\infty)}{1 - T'(\infty)} \alpha \bar{\varepsilon}(\infty) - (1 - \bar{\lambda}(\infty) - \pi(\infty)) \equiv NDWL_{SE}(T'(\infty)) \tag{63}$$

Thus, for $T'(\infty) > 1 - w^s(1)/w^f(1)$ we have

$$NDWL_{SE}(T'(\infty)) = NDWL_M(T'(\infty)) + \gamma \delta(T'(\infty)). \tag{64}$$

Note that *(i)* $\delta(T'(\infty)) > 0$ for $T'(\infty) \in (1 - w^s(1)/w^f(1), 1)$ and *(ii)* $\delta(T'(\infty))$ diverges to infinity as $T'(\infty)$ approaches $1 - w^s(1)/w^f(1)$ from the right (see Figure 16). As

Figure 16: Finding the optimal top tax rate.



Net deadweight loss is the difference between the deadweight loss and the tax revenue gain - negative net deadweight loss indicates gains from increasing the tax rate. $NDWL_D$ is the net deadweight loss according to the Diamond formula, $NDWL_{SE}$ is the net deadweight loss with the shadow economy (see the proof of Proposition 6 for formal definitions). When γ is high, the optimal top tax rate is $1 - w^s(1)/w^f(1)$. When γ is low, the optimal top tax rate is $\tilde{T}'$.

γ converges to 0, $NDWL_{SE}(T'(\infty))$ is arbitrarily close to $NDWL_D(T'(\infty))$ for any $T'(\infty) > 1 - w^s(1)/w^f(1)$. Hence, for sufficiently small γ the optimal top tax rate is given by $\tilde{T}'$, defined implicitly by $NDWL_{SE}(\tilde{T}') = 0$.[42] On the other hand, when γ is sufficiently large, the optimal top tax rate is $1 - w^s(1)/w^f(1)$.

More formally, suppose that $T'_D(\infty) > 1 - w^s(1)/w^f(1)$. The optimal top tax rate solves

$$\min_{T'(\infty) \geq 1 - \frac{w^s(1)}{w^f(1)}} \int_{1-\frac{w^s(1)}{w^f(1)}}^{T'(\infty)} NDWL_D(\tau)d\tau + \gamma \int_{1-\frac{w^s(1)}{w^f(1)}}^{T'(\infty)} \delta(\tau)d\tau. \tag{65}$$

Suppose that at some γ the solution is $T'(\infty) = 1 - w^s(\theta)/w^f(\theta)$. Since $\delta(T'(\infty)) > 0$, this tax rate is optimal also for higher values of γ. It proves the existence of threshold $\bar{\gamma}$. $\square$

Proof of Lemma 3. Since the labor elasticity converges to $\bar{\varepsilon}(\infty)$, the disutility from labor converges to $\tilde{v}(n) = c_0 + \frac{c_1}{1+1/\bar{\varepsilon}(\infty)} n^{1+1/\bar{\varepsilon}(\infty)}$ for some constants c_0, c_1. If all top workers

[42]In this case there are at least two solutions to $NDWL_{SE}(T'(\infty)) = 0$, a local maximum and a local minimum. If there are only two solutions, the higher one is the maximum.

are formal, the distribution of formal income satisfies

$$\lim_{y\to\infty}\frac{1-H(y)}{h(y)y}=\lim_{\theta\to 1}\frac{1-F(\theta)}{f(\theta)w^f(\theta)}\frac{dw^f(\theta)}{d\theta}\frac{dy^f(\theta,\infty)}{dw^f(\theta)}\frac{w^f(\theta)}{y^f(\theta,\infty)}=\frac{1+\bar{\varepsilon}(\infty)}{\alpha_w}. \tag{66}$$

When there are some shadow workers among the top productivity types, we have

$$\lim_{y\to\infty}\frac{1-H(y)}{h(y)y}=\lim_{\theta\to 1}\frac{1-\int_0^\theta(1-G_{\theta'}(\tilde{\kappa}(\theta')))dF(\theta')}{(1-G_\theta(\tilde{\kappa}(\theta)))\,f(\theta)w^f(\theta)}\frac{dw^f(\theta)}{d\theta}\frac{dy^f(\theta,\infty)}{dw^f(\theta)}\frac{w^f(\theta)}{y^f(\theta,\infty)}. \tag{67}$$

One can show that threshold $\tilde{\kappa}(\theta)$ is asymptotically proportional to $w^f(\theta)^{1+\bar{\varepsilon}(\infty)}$:

$$\lim_{\theta\to 1}\frac{\tilde{\kappa}(\theta)}{w^f(\theta)^{1+\bar{\varepsilon}(y^f(\theta,\infty))}}=\frac{1}{c_1^{\bar{\varepsilon}(\infty)}(1+\bar{\varepsilon}(\infty))}\left(\left(\frac{w^s(1)}{w^f(1)}\right)^{1+\bar{\varepsilon}(\infty)}-\left(1-T'(\infty)\right)^{1+\bar{\varepsilon}(\infty)}\right) \tag{68}$$

Consequently, $1-G_\theta(\tilde{\kappa}(\theta))$ is asymptotically proportional to $w^f(\theta)^{-\gamma(1+\bar{\varepsilon}(\infty))}$ and

$$\begin{aligned}\lim_{\theta\to 1}\frac{1-\int_0^\theta(1-G_{\theta'}(\tilde{\kappa}(\theta')))dF(\theta')}{(1-G_\theta(\tilde{\kappa}(\theta)))\,f(\theta)w^f(\theta)}\frac{dw^f(\theta)}{d\theta}&=\lim_{w^f\to\infty}\frac{\int_{w^f}^\infty 1/(w)^{1+\alpha_w+\gamma(1+\bar{\varepsilon}(\infty))}dw}{1/(w^f)^{1+\alpha_w+\gamma(1+\bar{\varepsilon}(\infty))}w^f}\\&=\frac{1}{\alpha_w+\gamma(1+\bar{\varepsilon}(\infty))}.\end{aligned} \tag{69}$$

Plugging it into (67), we get $\lim_{y\to\infty}\frac{1-H(y)}{h(y)y}=\frac{1+\bar{\varepsilon}(\infty)}{\alpha_w+\gamma(1+\bar{\varepsilon}(\infty))}$. □

C. Derivation of the optimal tax rates.

In Lemma C.1 we derive the optimal tax rates in terms of model primitives, using the mechanism design approach. Then we define the sufficient statistics used to derive the optimal tax rates in the main text and show the equivalence between he sufficient statistics approach and the mechanism design approach.

Lemma C.1 (Optimal tax formulas). *Suppose that bunching does not occur. Denote the sectoral productivity growth rates by $\rho^x(\theta)\equiv w^x_\theta(\theta)/w^x(\theta), x\in\{f,s\}$; the elasticity of labor supply by $\varepsilon(\theta,\kappa)\equiv\frac{v'(n(\theta,\kappa))}{n(\theta,\kappa)v''(n(\theta,\kappa))}$, where $n(\theta,\kappa)$ is the total labor supply of agent (θ,κ); the tax loss from worker of productivity type θ joining the shadow economy by $\tilde{\Delta}T(\theta)\equiv T(\theta',\infty)-T(\theta',0)$. Define $\tilde{s}(\theta)\equiv\min\{\theta'\in[0,1]$ s.t. $y^f(\theta',0)\geq y^f(\theta,\infty)\}$.*

When $y^f(\theta,\infty) \le y^f(1,0)$, the optimal tax rate satisfies

$$\frac{T'(y^f(\theta,\infty))}{1-T'(y^f(\theta,\infty))}\frac{(1-G_\theta(\tilde{\kappa}(\theta)))f(\theta)}{\rho^f(\theta)(1+\varepsilon^{-1}(\theta,\infty))}+\frac{w^f(\tilde{s}(\theta)-w^s(\tilde{s}(\theta))}{w^s(\tilde{s}(\theta))}\frac{G_{\tilde{s}(\theta)}(\tilde{\kappa}(\tilde{s}(\theta))f(\tilde{s}(\theta))}{\rho^f(\tilde{s}(\theta))-\rho^s(\tilde{s}(\theta))}$$
$$=\int_\theta^{\tilde{s}(\theta)}\left[\int_{\tilde{\kappa}(\theta')}^\infty(1-\lambda(\theta',\kappa))dG_{\theta'}(\kappa)-g_{\theta'}(\tilde{\kappa}(\theta'))\tilde{\Delta}T(\theta')\right]dF(\theta')$$
$$+\int_{\tilde{s}(\theta)}^1\int_0^\infty(1-\lambda(\theta',\kappa))dG_{\theta'}(\kappa)dF(\theta'). \quad (70)$$

When $y^f(\theta,\infty) > y^f(1,0)$, the optimal tax rate satisfies

$$\frac{T'(y^f(\theta,\infty))}{1-T'(y^f(\theta,\infty))}\frac{(1-G_\theta(\tilde{\kappa}(\theta)))f(\theta)}{\rho^f(\theta)(1+\varepsilon^{-1}(\theta,\infty))}$$
$$=\int_\theta^1\left[\int_{\tilde{\kappa}(\theta')}^\infty\big(1-\lambda(\theta',\kappa)\big)\,dG_{\theta'}(\kappa)-g_{\theta'}(\tilde{\kappa}(\theta'))\tilde{\Delta}T(\theta')\right]dF(\theta'). \quad (71)$$

Proof of Lemma C.1.

By Corollary 1 from Milgrom and Segal (2002) the value function $V(\theta,\kappa)$ is differentiable with respect to θ almost everywhere. The derivative is given by

$$\frac{dV(\theta,\kappa)}{d\theta}=\left(\rho^f(\theta)\frac{y^f(\theta,\kappa)}{w^f(\theta)}+\rho^s(\theta)\frac{y^s(\theta,\kappa)}{w^s(\theta)}\right)v'\left(\frac{y^f(\theta,\kappa)}{w^f(\theta)}+\frac{y^s(\theta,\kappa)}{w^s(\theta)}\right)\equiv V_\theta(\theta,\kappa), \quad (72)$$

where $\rho^x(\theta)\equiv w^x_\theta(\theta)/w^x(\theta)$ stands for the productivity growth rate in sector $x\in\{f,s\}$. Hence, we can represent the value function in the integral form

$$V(\theta,\kappa)=V(0,\kappa)+\int_0^\theta V_\theta(\theta',\kappa)d\theta'. \quad (73)$$

Take some high-cost worker (θ,∞). We will derive the optimality condition by perturbing formal income of this worker by small dy^f and adjusting the tax paid such that the utility level $V(\theta,\infty)$ is unchanged. This perturbation affects the slope $V_\theta(\theta,\infty)$, which in turn implies via equation (73) a uniform shift of utility levels of all high-cost types above.

Moreover, since all agents face the same tax schedule, we need to adjust the allocation of the low-cost workers as well. We can distinguish three cases. First, when $y^f(\tilde{s}(\theta),\infty)=y^f(\theta,\infty)$, the distorted shadow workers respond by marginally decreasing formal income. Second, when $y^f(\tilde{s}(\theta),\infty)>y^f(\theta,\infty)$, the distorted shadow workers respond by jumping to a discretely lower formal income level. These two cases have identical fiscal impact and lead to the same optimal tax formula. Finally, when $y^f(\theta,\infty)>y^f(1,0)$, all the shadow workers have lower formal income and hence are unaffected by the perturbation.

Distortion of formal workers. A formal income perturbation dy^f affects the utility of type (θ, ∞) by $\left(1 - \frac{v'(n(\theta,\infty))}{w^f(\theta)}\right) dy^f$, or equivalently by $T'(y^f(\theta, \infty))dy^f$. We need to adjust the total tax paid by the same amount such that the utility level stays constant. The fiscal impact of doing so is

$$T'(y^f(\theta, \infty))(1 - G_\theta(\tilde{\kappa}(\theta)))f(\theta)dy^f. \tag{74}$$

The impact of this perturbation on the slope of the utility schedule is

$$dV_\theta(\theta, \infty) = \rho^f(\theta)\left(1 - T'(y^f(\theta, \infty))\right)\left(1 + \frac{1}{\varepsilon(\theta, \infty)}\right) dy^f. \tag{75}$$

Hence, a perturbation that leads to a change of slope $dV_\theta(\theta, \infty)$ implies a change in tax revenue from the formal workers by

$$\frac{T'(y^f(\theta, \infty))}{1 - T'(y^f(\theta, \infty))}\left(1 + \frac{1}{\varepsilon(\theta, \infty)}\right)^{-1} \frac{1}{\rho^f(\theta)}(1 - G_\theta(\tilde{\kappa}(\theta)))f(\theta)dV_\theta(\theta, \infty). \tag{76}$$

Distortion of shadow workers. Let's consider the case of $y^f(\theta, \infty) \leq y^f(1, 0)$, otherwise there is no tax loss from the shadow workers. First, suppose that $y^f(\tilde{s}(\theta), 0) = y^f(\theta, \infty)$. A perturbation of formal income dy_2^f affects the utility level of $(\tilde{s}(\theta), 0)$-type worker by $\left(1 - \frac{w^s(\tilde{s}(\theta))}{w^f(\tilde{s}(\theta))}\right) dy_2^f = T'(y^f(\tilde{s}(\theta), 0))dy_2^f$. We need to adjust the tax paid by the same amount, which affects the resource constraint by

$$T'(y^f(\tilde{s}(\theta), 0))G_{\tilde{s}(\theta)}(\tilde{\kappa}(\tilde{s}(\theta)))f(\tilde{s}(\theta))dy_2^f. \tag{77}$$

The slope of the utility schedule of low-cost workers changes by

$$dV_\theta(\tilde{s}(\theta), 0) = \left(\rho^f(\tilde{s}(\theta)) - \rho^s(\tilde{s}(\theta))\right)\frac{w^s(\tilde{s}(\theta))}{w^f(\tilde{s}(\theta))}dy_2^f. \tag{78}$$

The perturbation needs to respect the common tax schedule at higher formal incomes - the slopes of $V(\theta, \infty)$ and $V(\tilde{s}(\theta), 0)$ have to change by the same amount, which can be achieved by appropriately adjusting dy_2^f. Then, by using the first-order condition of workers $(\tilde{s}(\theta), 0)$, we can express the tax loss as

$$\frac{w^f(\tilde{s}(\theta)) - w^s(\tilde{s}(\theta))}{w^s(\tilde{s}(\theta))}\frac{G_{\tilde{s}(\theta)}(\tilde{\kappa}(\theta))f(\tilde{s}(\theta))}{\rho^f(\tilde{s}(\theta)) - \rho^s(\tilde{s}(\theta))))}dV(\theta, \infty). \tag{79}$$

Second, suppose that $y^f(\tilde{s}(\theta), 0) > y^f(\theta, \infty)$. In this case there is a discontinuity at $\tilde{s}(\theta)$ in the formal income schedule of the low-cost workers. Denote by superscripts $\{-, +\}$ the directional limit of a given variable, e.g. $y^f(\tilde{s}(\theta)^-, 0)$ stands for the left limit of formal income of the low-cost workers at $\tilde{s}(\theta)$. From the definition of the mapping s we know that $y^f(\tilde{s}(\theta)^-, 0) < y^f(\theta^+, 0)$.

The perturbation in the formal income of (θ, ∞) decreased the utility of all workers with formal income above $y^f(\theta,\infty)$, including $\tilde{s}(\theta)$, by $dV_\theta(\theta,\infty)$. It means that the perturbation, absent behavioral responses, leads to a discontinuity at $\tilde{s}(\theta)$ in the utility schedule of the low-cost workers, which is not incentive compatible. The behavioral responses will restore the continuity of $V(\theta,0)$ by adjusting the mapping $\tilde{s}(\theta)$. Denote this adjustment by $d\tilde{s}(\theta)$.

Continuity of $V(\theta,0)$ at $\tilde{s}(\theta)$ means that $V(\tilde{s}(\theta)^-,0) = V(\tilde{s}(\theta)^+,0)$. Suppose that the utility of worker $(\tilde{s}(\theta),0)$ is decreased by dT. Continuity of the utility schedule requires that

$$V_\theta(\tilde{s}(\theta)^-,0)d\tilde{s}(\theta) = V_\theta(\tilde{s}(\theta)^+,0)d\tilde{s}(\theta) - dT$$
$$\implies d\tilde{s}(\theta) = \frac{w^f(\tilde{s}(\theta))/w^f(\tilde{s}(\theta))}{\rho^f(\tilde{s}(\theta)) - \rho^s(\tilde{s}(\theta))}\frac{dT}{y^f(\tilde{s}(\theta)^+,0) - y^f(\tilde{s}(\theta)^-,0)}.$$

This adjustment of $\tilde{s}(\theta)$ is associated with a tax loss

$$\Big(T(y^f(\tilde{s}(\theta)^+,0)) - T(y^f(\tilde{s}(\theta)^-,0))\Big)\, f(\tilde{s}(\theta))G_{\tilde{s}(\theta)}(\tilde{\kappa}(\theta))d\tilde{s}(\theta). \tag{80}$$

Note that $V(\tilde{s}(\theta)^-,0) = V(\tilde{s}(\theta)^+,0)$ implies that

$$\frac{T(\tilde{s}(\theta)^+,0) - T(\tilde{s}(\theta)^-,0)}{y^f(\tilde{s}(\theta)^+,0) - y^f(\tilde{s}(\theta)^-,0)} = 1 - \frac{w^s(\tilde{s}(\theta))}{w^f(\tilde{s}(\theta))}. \tag{81}$$

Using this result, we can express the tax loss as

$$\frac{w^f(\tilde{s}(\theta)) - w^s(\tilde{s}(\theta))}{w^s(\tilde{s}(\theta))}\frac{G_{\tilde{s}(\theta)}(\tilde{\kappa}(\tilde{s}(\theta)))f(\tilde{s}(\theta))}{\rho^f(\tilde{s}(\theta)) - \rho^s(\tilde{s}(\theta))}dT. \tag{82}$$

Notice the dT is equal to $dV_\theta(\theta,\infty)$. Hence, the tax loss is the same as in the previous case, when $y^f(\tilde{s}(\theta),0) = y^f(\theta,\infty)$.

Impact on workers with higher formal income. First, suppose that $y^f(\theta,\infty) \le y^f(1,0)$. The perturbation implies a shift $dV_\theta(\theta,\kappa)$ in utility levels of formal workers above type θ and shadow workers above $\tilde{s}(\theta)$. Recall that the marginal social welfare weights are equal to the Pareto weights. The fiscal and welfare impact of such change is

$$\int_\theta^{\tilde{s}(\theta)}\int_{\tilde{\kappa}(\theta')}^{\infty}\big(\lambda(\theta',\kappa) - 1\big)\, dG(\kappa)dF(\theta')dV_\theta(\theta,\infty)$$
$$+ \int_{\tilde{s}(\theta)}^{1}\int_0^\infty(\lambda(\theta',\kappa) - 1)dG(\kappa)dF(\theta')dV_\theta(\theta,\infty). \tag{83}$$

Note that among the productivity types in the segment $(\theta,\tilde{s}(\theta))$ the high-cost workers are affected by the perturbation, but the low-cost worker are not. Hence, the perturbation changes the threshold $\tilde{\kappa}$ at this segment. Denote by $\tilde{\Delta}T(\theta) \equiv T(y^f(\theta,\infty)) - T(y^f(\theta,0))$

the tax loss from worker of type θ moving to the shadow economy. The fiscal impact of the change in participation is

$$\int_{\theta}^{\tilde{s}(\theta)} \tilde{\Delta}T(\theta')g_{\theta'}(\tilde{\kappa}(\theta'))dF(\theta')dV_{\theta}(\theta,\infty). \tag{84}$$

In the case of $y^f(\theta,\infty) > y^f(1,0)$ only the formal workers are affected by a tax reform. The total fiscal and welfare impact on agents with higher formal income is

$$\int_{\theta}^{1}\left[\int_{\tilde{\kappa}(\theta')}^{\infty}\big(\lambda(\theta',\kappa)-1\big)\,dG(\kappa)+\tilde{\Delta}T(\theta')g_{\theta'}(\tilde{\kappa}(\theta'))\right]dF(\theta')dV_{\theta}(\theta,\infty). \tag{85}$$

Collecting the terms. At the optimum, the total impact of a small perturbation is zero. First, consider the case of $y^f(\theta,\infty) \leq y^f(1,0)$. The sum of the distortion cost of a high-cost worker (76), the distortion cost of the low-cost worker (79) as well as of impacts on the workers with higher formal income (83) and (84) needs to be zero, which results in tax formula (70). If the perturbation affects no shadow workers ($y^f(\theta,\infty) > y^f(1,0)$), the terms (76) and (85) should sum up to zero, which yields tax formula (71). This concludes the proof of Lemma C.1.

Definitions of sufficient statistics.

$\varepsilon^x(\theta)$ and $\tilde{\varepsilon}^x(\theta)$ stand for the formal income elasticity of workers in sector $x \in \{f,s\}$ with respect to the marginal tax rate along the linear and nonlinear tax schedule, respectively. $\varepsilon^x_{w^f}(\theta)$ and $\tilde{\varepsilon}^x_{w^f}(\theta)$ stand for the formal income elasticity of workers in sector $x \in \{f,s\}$ with respect to the gross formal wage along the linear and nonlinear tax schedule, respectively. The elasticities of formal workers are derived from the optimality condition $y^f(\theta,\infty) = w^f(\theta)(v')^{-1}\big((1-T'(y^f(\theta,\infty)))w^f(\theta)\big)$, while the elasticities of shadow workers are derived from the optimality condition $(1-T'(y^f(\theta,0)))w^f(\theta) = w^s(\theta)$.

The elasticities of formal workers are

$$\varepsilon^f(y^f(\theta,\infty)) \equiv \frac{v'(n(\theta,\infty))}{n(\theta,\infty)v''(n(\theta,\infty))}, \tag{86}$$

$$\tilde{\varepsilon}^f(y) \equiv \left[\frac{1}{\varepsilon^f(y)}+\frac{T''(y)y}{1-T'(y)}\right]^{-1}, \tag{87}$$

$$\varepsilon^f_{w^f}(y) \equiv 1+\varepsilon^f(y), \tag{88}$$

$$\tilde{\varepsilon}^f_{w^f}(y) \equiv \frac{\tilde{\varepsilon}^f(y)}{\varepsilon^f(y)}\varepsilon^f_{w^f}(y). \tag{89}$$

The elasticities of shadow workers are

$$\tilde{\varepsilon}^s(y) \equiv \frac{1 - T'(y)}{T''(y)y}, \tag{90}$$

$$\tilde{\varepsilon}^s_{w^f}(y^f(\theta, 0)) \equiv \left(1 - \frac{\rho^s(\theta)}{\rho^f(\theta)}\right) \tilde{\varepsilon}^s(y^f(\theta, 0)). \tag{91}$$

Note that shadow workers have infinite elasticities of formal income along the linear tax schedule: as soon as the net formal wage departs from the shadow wage, the shadow worker either stops supplying formal labor entirely or becomes a formal worker. Nevertheless, elasticities along the non-linear tax schedule are well defined, as long the tax schedule is not locally linear.

Denote the derivative of formal income w.r.t. the productivity type along the non-linear tax schedule as

$$\tilde{y}^f_\theta(\theta, \kappa) \equiv \begin{cases} \tilde{\varepsilon}^f_{w^f}(y^f(\theta, \kappa))\rho^f(\theta)y^f(\theta, \kappa) & \text{if } \kappa \geq \tilde{\kappa}(\theta), \\ \tilde{\varepsilon}^s_{w^f}(y^f(\theta, \kappa))\rho^f(\theta)y^f(\theta, \kappa) & \text{otherwise.} \end{cases} \tag{92}$$

The density of formal workers at formal income $y^f(\theta, \infty)$, scaled by the share of formal workers, is defined as $h^f(y^f(\theta, \infty)) \equiv (1 - G_\theta(\tilde{\kappa}(\theta)))f(\theta)/\tilde{y}^f_\theta(\theta, \infty)$. The density of shadow workers at formal income $y^f(\theta, 0)$, scaled by the share of shadow workers, is $h^s(y^f(\theta, 0)) \equiv G_\theta(\tilde{\kappa}(\theta))f(\theta)/\tilde{y}^f_\theta(\theta, 0)$ and $h^s(y^f) \equiv 0$ for $y^f \notin y^f([0, 1], 0)$. The density of formal income is simply $h(y) \equiv h^f(y) + h^s(y)$. The mean elasticity at income level y is $\bar{\varepsilon}(y) \equiv h^f(y)\tilde{\varepsilon}^f(y) + h^s(y)\tilde{\varepsilon}^s(y)$.

Define the elasticity of the density of formal workers with respect to the tax burden of staying formal $\tilde{\Delta}T(\theta)$ by

$$\pi(y^f(\theta, \infty)) \equiv \frac{g_\theta(\tilde{\kappa}(\theta))\tilde{\Delta}T(\theta)}{1 - G_\theta(\tilde{\kappa}(\theta))}. \tag{93}$$

Define the average welfare weights of formal and shadow workers at a given formal income as

$$\bar{\lambda}^f(y^f(\theta, \infty)) \equiv \int_{\tilde{\kappa}(\theta)}^{\infty} \lambda(\theta, \kappa)\frac{dG_\theta(\kappa)}{1 - G_\theta(\tilde{\kappa}(\theta))}, \quad \bar{\lambda}^s(y^f(\theta, 0)) \equiv \int_0^{\tilde{\kappa}(\theta)} \lambda(\theta, \kappa)\frac{dG_\theta(\kappa)}{G_\theta(\tilde{\kappa}(\theta))}. \tag{94}$$

Then the average welfare weight at formal income y is $\bar{\lambda}(y) \equiv \left(h^f(y)\bar{\lambda}^f(y) + h^s(y)\bar{\lambda}^s(y)\right)/h(y)$.

The equivalence of the mechanism design and the sufficient statistics approaches.

Note that $s(y^f(\theta, \infty)) = y^f(\tilde{s}(\theta, 0))$. By substituting the terms defined above, we can represent the left-hand sides of (70) and (71) as in the sufficient statistics formulas (28) and (29), respectively.

Let's focus on the tax revenue gain (the right-hand side of the tax formulas). We can represent the tax revenue gain (the right-hand side of (70)) as

$$\int_{\theta}^{1} \left[1-\bar{\lambda}^f(y^f(\theta',\infty))\right](1-G_{\theta'}(\tilde{\kappa}(\theta')))dF(\theta')+\int_{\tilde{s}(\theta)}^{1} \left[1-\lambda^s(y^f(\theta',0))\right]G(\tilde{\kappa}(\theta'))dF(\theta')$$
$$-\int_{\theta}^{\tilde{s}(\theta)} \frac{g_{\theta'}(\tilde{\kappa}(\theta'))\tilde{\Delta}T(\theta')}{1-G_{\theta'}(\tilde{\kappa}(\theta'))}(1-G_{\theta'}(\tilde{\kappa}(\theta')))dF(\theta'). \quad (95)$$

By changing variables we obtain the right-hand side of (28):

$$\int_{y^f(\theta,\infty)}^{\infty} \left[1-\bar{\lambda}^f(y)\right]h^f(y)dy+\int_{y^f(\tilde{s}(\theta),0)}^{\infty} \left[1-\bar{\lambda}^s(y)\right]h^s(y)dy-\int_{y^f(\theta,\infty)}^{y^f(\tilde{s}(\theta),\infty)} \pi(y)h^f(y)dy \quad (96)$$

$$=\int_{y^f(\theta,\infty)}^{\infty} \left[1-\bar{\lambda}(y)\right]h(y)dy-\int_{y^f(\theta,\infty)}^{y^f(\tilde{s}(\theta),\infty)} \pi(y)h^f(y)dy. \quad (97)$$

Finally, note that $y^f(\tilde{s}(\theta),\infty) = s(y^f(\theta,\infty)) + \Delta_0(s(y^f(\theta,\infty)))$. We can express the right-hand side of (71) as the right-hand side of (29) analogously.

D. Empirical appendix

This appendix describes the estimation of the model. First we describe the data and how we recover wages and sector participation. Second, we describe the identifying assumptions and the corresponding likelihood function. Last we present the parameter estimates.

Data. We use the 2013 wave of the household survey by the official statistical agency of Colombia (DANE). We restrict attention to individuals aged 24-50 years without children (26,000 individuals). We choose this sample, since these workers face the tax and transfer schedule which is not means-tested and does not depend on choices absent from our modeling framework, such as number of children or college attainment.

The information we use in the estimation is given by $\{w_i, f_i, X_i, s_i\}_{i=1}^N$ where w_i is the hourly wage of worker i before taxes; f_i an indicator variable for having a main job in the formal sector; X_i a vector of worker characteristics; s_i the sampling weights and N the total number of observations in our sample. The indicator variable f_i is set equal to one if the worker reports to be affiliated to all three items of social security: pension system, health insurance and labor accidents insurance. A fraction (about 3%) of workers also have a second job. If the first job is formal we cannot identify if the worker's second job

Table 5: Variables included in X_i

Variable	Description	Values
Individual characteristics		
Gender	Dummy variable equal to 1 for women	0-1
Age	Age of the worker	16-90
Age^2	Age squared	
Educ	Number of education years	0-26
Degree	Highest degree achieved (No degree to Doctorate)	1-5
Work	Number of months worked in the last year	1-12
Exper	Number of months worked in the last job	0-720
1stJob	Dummy for the first job (1 if it is the first job)	0-1
Job characteristics		
S-Man	Dummy for the manufacturing sector	0-1
S-Fin	Dummy for financial intermediation	0-1
S-Ret	Dummy for the sales and retailers sector	0-1
B-city	Dummy for a firm in one of the two largest cities	0-1
Size	Categories for the number of workers	1-9
Lib	Dummy for a liberal occupation	0-1
Admin	Dummy for an administrative task	0-1
Seller	Dummy for sellers and related	0-1
Services	Dummy for a service task	0-1
Worker-firm relationship		
Union	Dummy for labor union affiliation (1 if yes)	0-1
Agency	Dummy for agency hiring (1 if yes)	0-1
Senior	Number of months of the worker in the firm	0-720

is shadow or formal. Therefore $f_i = 1$ indicates formality of the main job and does not imply that the worker is exclusively formal. We compute the hourly wage before taxes w_i as the ratio of the income and worked hours at the main job the month prior to the survey. If the worker is identified to be formal at the main job we include the statutory payroll taxes that are paid by the employer in the computation of the pre-tax income at the main job. Variables included in vector X_i are listed in Table 5.

Likelihood function. We make four assumptions:

1. the parametric form of productivity profiles (equation (36)),
2. the relation between observables X and produtivity type θ (equation (38)),
3. the parametric form of the distribution of fixed cost ((39)),
4. iso-elastic utility function (equation (40)).

Using these assumptions we can decompose the probability of the realization $\{w_i, f_i, X_i\}$

into three elements:

$$P(w = w_i, f = f_i, X = X_i; B) = P(X_i) \times P(w = w_i \mid X_i; B) \times P(f = f_i \mid w_i; B)$$

where

$$B = \left(\beta, \varepsilon, \Gamma, w^s(0), \rho^s, w^f(0), \rho^f, \sigma_\theta, \sigma_\kappa, w_\kappa\right)$$

and the elements correspond to:

- $P(X_i) = s_i$ is the sampling weight assigned in the survey. Measures how representative is the observation at the population level.
- $P(w = w_i \mid X_i; B)$ is the probability that a worker with characteristics X_i has the wage w_i at the sector j where she is participating. Let $\epsilon_i = \theta_i - X_i\beta$, then $\epsilon_i \sim N(0, \sigma_\theta^2)$ and we have

$$\begin{aligned} P(w = w_i \mid X_i; B) &= P\left(w_i = w^j(0)\exp\left\{\rho^j\left(X_i\beta + \epsilon_i\right)\right\}\right) \\ &= P\left(\epsilon_i = \log(w_i) - \log(w^j(0)) - \rho^j X_i\beta\right) \\ &= \frac{1}{\sqrt{2\pi\sigma_\theta^2}}\exp\left\{-\frac{\left(\log(w_i) - \log(w^j(0)) - \rho^j X_i\beta\right)^2}{2\sigma_\theta^2}\right\} \end{aligned}$$

  If the worker belongs to the top 1% then $P(w = w_i)$ is given by the Pareto distribution with parameter α_w.
- $P(f = f_i \mid w_i; B)$ is the probability that a worker with observed wage w_i at sector j has a participation decision in the formal sector given by f_i. Note that the observed wage at the given sector reveals the type of the worker as follows:

$$\theta_i = \frac{1}{\rho^j}\log\left(\frac{w_i}{w^j(0)}\right)$$

  Given the type of the worker there are two possibilities: i) The participation cost is above the threshold $\tilde{\kappa}(\theta_i)$ and formal income is given by $y^f(\theta_i, \infty)$, or ii) the participation cost is below $\tilde{\kappa}(\theta_i)$ and formal income $y^f(\theta_i, 0)$. Then, considering these two possibilities we can write the participation probability as

$$P(f = f_i \mid w_i; B) = \begin{cases} G_{\theta_i}\left(\tilde{\kappa}(\theta_i)\right) I_{(y^f(\theta_i,0)>0)} + \left[1 - G_\theta\left(\tilde{\kappa}(\theta_i)\right)\right] I_{(y^f(\theta_i,\infty)>0)} & \text{if } f_i = 1 \\ G_{\theta_i}\left(\tilde{\kappa}(\theta_i)\right) I_{(y^f(\theta_i,0)=0)} + \left[1 - G_\theta\left(\tilde{\kappa}(\theta_i)\right)\right] I_{(y^f(\theta_i,\infty)=0)} & \text{if } f_i = 0 \end{cases}$$

  where G_{θ_i} is the cummulative distribution of κ for worker i and I is an indicator function.

Parameter estimates. Parameter estimates are reported in Table 6.

Table 6: Parameter estimates

preferences		productivity schedules					distributions of θ and κ			
ε	Γ	$w^s(0)$	ρ^s	$w^f(0)$	ρ^f	α_w	σ_θ	σ_κ	α_κ	w_κ
0.358	0.497	0.011	2.99	0.0085	4.15	2.4	0.51	0.59	0.05	0.008

β individual characteristics								β worker-firm		
Gender	Age	Age^2	Educ	Degree	Work	Exper	1stJob	Union	Agency	Senior
-0.10	0.02	-.0002	0.02	0.13	0.03	.001	.007	0.07	-0.15	0.001

β job characteristics								
S-Man	S-Fin	S-Ret	B-city	Size	Lib	Admin	Seller	Services
-0.07	0.21	-0.14	0.15	0.03	0.42	-0.03	-0.08	-0.19