Optimal Income Taxation and Commitment on the Labor Market

Paweł Doligalski*

University of Bristol

December 3, 2018

Abstract

I study the optimal income taxation when workers and firms can commit to maintain their employment relationships. The commitment power allows firms to offer labor contracts which minimize the workers' tax burden and provide insurance. The main finding is that when both workers and firms have any positive commitment power, the tax schedule cannot be too regressive, as otherwise wages would be inefficiently randomized to reduce the expected tax paid by workers. I also show that workers' insurance depends only on the total commitment power on the labor market, but not on its split between workers and firms. I calibrate the model to the US income distribution. The threat of wage randomization reduces the optimal marginal tax rates at low income levels by up to 40 percentage points and in certain cases makes the optimal tax schedule fully linear.

Keywords: income tax, redistribution, insurance within firm, commitment.

JEL Codes: H21, H26, J31.

1. Introduction

The literature on the optimal income taxation, starting from Mirrlees (1971), studies tax systems which efficiently redistribute labor income and insure human capital risk.[1] While a lot of emphasis has been placed on the realistic modeling of income inequality, much

*Department of Economics, University of Bristol, The Priory Road Complex, Priory Road, BS8 1TU, Bristol, the United Kingdom, email: pawel.doligalski@bristol.ac.uk. I am grateful for useful comments of Árpád Ábrahám, Pavel Brendler, Piotr Dworczak, Francesco Giovannoni, Mike Golosov, Piero Gottardi, Tai-Wei Hu, Paweł Kopiec, Dirk Krueger, Albert Marcet, Ramon Marimon, Nicola Pavoni, Dominik Sachs and Jan Werner. All errors remain my own.

[1] The original, static Mirrleesian literature is concerned with redistribution (Mirrlees 1971; Diamond 1998; Saez 2001) while the dynamic Mirrleesian literature focuses on insuring individual productivity risk (Golosov, Kocherlakota, and Tsyvinski 2003; Golosov, Tsyvinski, and Werning 2007; Kocherlakota 2010; Farhi and Werning 2013; Golosov, Troshkin, and Tsyvinski 2016).

less attention has been paid to interactions between workers and employers. Typically, it has been assumed that workers supply labor on the spot markets, taking the hourly wage as given. It is at odds with the prevalence of long-term employment relationships,[2] often featuring complex compensation structures.[3] The purpose of this paper is to examine how relaxing the assumption of spot labor markets affects the optimal income tax.

Allowing for employment relationships with flexible compensation structures matters for income taxation for at least two reasons. First, there is an empirical evidence that compensation structures are designed to reduce workers' tax burden. A good example is adjusting the timing of payments in anticipation of a tax reform: stock options are exercised early to avoid tax rate hikes (Goolsbee 2000) while bonuses and wages are delayed until more favorable tax regime is in place (Kreiner, Leth-Petersen, and Skov 2014, 2016). When workers and employers respond to taxes not only by altering hours, but also by fine-tuning wage structures, raising tax revenue is more difficult and income redistribution may suffer. Second, it was shown that firms provide insurance by reducing the variability of workers' compensation relative to the variability of their output (Guiso, Pistaferri, and Schivardi 2005; Lagakos and Ordonez 2011). Taking this into account may mean that the insurance role of the income tax is less important. Crucially, neither of these two phenomena can be studied in the standard Mirrleesian taxation framework with spot labor markets.

I determine which labor contracts are possible by specifying *labor market commitment*. When firms and workers are commited to maintain the employment relationship, they can credibly agree on arbitrary paths of wages and hours. The above-mentioned within firm insurance and tax avoidance appear then naturally as a part of the optimal labor contract. In contrast, when neither workers nor firms can commit, wages need to closely follow employees' output and the model is equivalent to the standard taxation framework with spot labor markets.[4] Thus, I extend the Mirrleesian taxation framework by allowing for positive commitment power of workers and firms and a richer space of possible labor contracts. This approach to modeling employment relationship is closely related to the theory of implicit labor contracts (Baily 1974; Azariadis 1975) and insurance under

[2] In January 2018 in the US the median tenure at a job was 4.2 years for all employees and 10.1 years for employees 55-64 years old (Bureau of Labor Statistics 2018).

[3] Murphy (1999) reports that a CEO of a S&P 500 Industrials company with above median sales in 1996 was on average compensated with stock options (39%), a salary (24%) and bonuses (19%), while the remaining 18% of compensation stood for other miscellaneous forms of pay. Not only top executives have complex compensation structures: over 80% of stock options in S&P 500 companies were granted to managers and employees below the five top executives (Hall and Murphy 2003). Going beyond large companies, 37% of observations of workers in wage and salary jobs in PSID have a variable pay component: a bonus, a commission or a piece rate (Lemieux, MacLeod, and Parent 2009).

[4] Some degree of the labor market commitment is evident, given the existence of the within-firm insurance and tax avoidance. Another piece of evidence comes from the literature documenting that the credit constrained firms implicitly borrow from their employees by offering low initial wages with high growth rate, which requires a positive commitment power of firms (Michelacci and Quadrini 2005, 2009; Guiso, Pistaferri, and Schivardi 2012). Brandt and Hosios (1996) find that implicit lending within firm can take as well, indicating a positive commitment power of workers.

limited commitment (Harris and Holmstrom 1982; Thomas and Worrall 1988; Krueger and Uhlig 2006).

The primary finding is that when *both* workers and firms have *any positive* commitment power, the income tax cannot be too regressive: the marginal tax rates cannot fall with income too rapidly. If the tax schedule was excessively regressive, agents could exploit this nonlinearity by adding a mean-preserving spread to wages - a phenomenon I call *wage randomization* - to reduce the expected tax paid by a worker. With sufficiently strong regressivity, the gain due to the lower expected tax dominates the utility loss due to higher consumption risk and additional randomness in labor income benefits workers. Note that tax systems which encourage wage randomization are inefficient: a locally linear tax schedule would generate the same tax revenue without giving incentives for randomization and, hence, with lower consumption risk. Crucially, wage randomization - adding a mean preserving spread of wages while keeping the labor supply fixed - requires a positive commitment power of both a worker and a firm. If the worker was unable to commit, he could not credibly promise to stay with the firm once his compensation falls below his output. Similarly, without commitment the firm would fire the worker rather than pay him more than what he produces. Therefore, this form of tax avoidance requires two-sided commitment and is absent from the standard Mirrleesian framework.

I derive a novel *no-randomization constraint* which prevents wage randomization. It requires that a measure of the local tax regressivity cannot exceed the coefficient of absolute risk aversion, which has a simple intuition. A worker's attitude toward income risk depends on two factors: the shape of the tax schedule, which determines how income is transmitted to consumption, and the attitude towards the consumption risk, which is governed by the coefficient of the absolute risk aversion. When the tax schedule is locally regressive (i.e. the after-tax income is convex in the pre-tax income), income risk reduces the expected tax payments and lifts the expected after-tax income. Thus, when the tax regressivity is strong enough relative to the coefficient of the absolute risk aversion, the worker actually benefits from income risk. One can show that when the no-randomization constraint is violated then the worker's absolute aversion to income risk is negative - the worker effectively becomes an income risk lover. Furthermore, the no-randomization constraint is a necessary condition for the efficiency of a tax schedule whenever both a worker and a firm have any positive commitment power, *regardless* of any additional frictions. The optimal taxation literature typically assumes that wages, conditional on a realized productivity, are deterministic. The no-randomization constraint can be used to test whether the tax schedules derived under this assumption will remain efficient once stochastic wages are allowed.

The labor market commitment is fundamental for the insurance within firm. When both workers and firms can commit fully, workers are always fully insured within firms, while the government limits its role to redistribution. When the commitment is imperfect, the insurance in general will be imperfect as well. I show that the degree of insurance workers receive depends only on the *total commitment power* on the labor market, but not on how it is distributed between workers and firms. Any asymmetry in the commitment power between workers and firms can be alleviated by shifting wages across time, which in turn can be isolated from workers' consumption by trading a risk-free asset or by introducing age-dependent taxes.[5] As a result, the full insurance outcome can be reached even under full one-sided commitment, i.e. when only one side of the market can commit fully. In contrast to insurance, the possible redistribution depends not on the *total* but on the *minimal commitment power*. It is the minimum of the commitment power of a worker and of a firm which determines the magnitude of the feasible wage randomization and, hence, the severity of tax avoidance the worker can engage in. The lower the minimal commitment power is, the more can the government redistribute. In particular, the government's ability to redistribute is maximized in the one-sided commitment case when either workers or firms cannot commit at all.

These results have important implications for the design of labor market institutions which can affect the commitment power of workers and firms. For instance, raising firing costs increases the commitment power of firms, while the abolition of non-compete clauses in employment contracts reduces the commitment of workers. A properly executed reform which promotes workers mobility (e.g. by abolishing the non-compete clauses in employment contracts) while strengthening the commitment power of firms (e.g. by raising the firing costs) can restrict tax avoidance and allow for more redistribution without sacrificing insurance within firms.

I demonstrate the empirical relevance of these findings in two ways. First, I show that the actual tax and transfer code in US violates the no-randomization constraint around the phase-out region of the Earned Income Tax Credit. The gains from randomizing wages are potentially large. Second, I calibrate the model to the US income distribution. Diamond (1998) and Saez (2001) show that the optimal tax rates without commitment are U-shaped: high at low incomes, low in the middle and high again for top earners. Once I allow for any positive commitment of both workers and firms, the implied tax regressivity at low income levels violates the no-randomization constraint for any realistic specification of risk aversion. Consequently, optimal tax schedules with two-sided commitment have a much lower tax rate at the bottom (e.g. lower by 40 percentage points with a log utility and a Utilitarian government) and the pace at which this rate falls with income is much subdued. In the particularly striking case of risk neutral agents and a Rawlsian government, the optimal tax schedule becomes *fully linear* and consistent with the theory of the optimal linear taxation (Sheshinski 1972; Piketty and Saez 2013). Though the qualitative impact of no-randomization constraint on the shape of the optimal tax schedule is robust across social welfare functions, the welfare cost

[5]For instance, it is well known that wages are delayed (backloaded) when a firm can commit but a worker cannot: Harris and Holmstrom (1982); Burdett and Coles (2003); Krueger and Uhlig (2006). These papers do not find full insurance (though Harris and Holmstrom (1982) mention this possibility), since they restrict the workers' ability to borrow outside of the insurance relationship.

of reduced regressivity strongly depends on the government objective. The Rawlsian government suffers most: 2.6% of consumption when agents are risk neutral, 0.9% when agents have log utility and less than that as the risk aversion grows. In contrast, welfare losses of the Utilitarian government are negligible for all specifications of risk aversion, as very few workers with lowest incomes were gaining from the strong tax regressivity at the bottom.

The paper closest in spirit is Chetty and Saez (2010) who study how the optimal linear tax formula changes when private wage insurance is allowed, which they also restrict to be linear. They find that the sufficient statistics tax formula does not change when the private insurance does not suffer from agency frictions. I show that once we drop both linearity assumptions this result no longer holds: the sufficient statistics tax formula needs to be combined with the novel no-randomization constraint to prevent excessive tax regressivity and wage randomization. A related strand of papers study optimal income taxation with private insurance which operates not via wages, but private asset trades (Ábrahám, Koehne, and Pavoni 2016; Chang and Park 2017). Golosov and Tsyvinski (2007) allow for both general compensation structures and private asset markets, but do not characterize the impact of flexible pay structures on the optimal tax schedule, focusing rather on correcting a pecuniary externality arising in their setting. Others study nonlinear income taxation with firms in the models of competitive screening (Stantcheva 2014), monopolistic screening (da Costa and Maestri 2017), hours constraints (Werquin 2016) and labor search (Hungerbühler, Lehmann, Parmentier, and Van der Linden 2006; Golosov, Maziero, and Menzio 2013; Yazici and Sleet 2017), yet they do not consider more general compensation structures. Labor market commitment in this paper is treated as exogenous. The commitment power can become endogenous to government policy in the context of insurance via asset trades or informal family insurance, giving rise to a trade-off between public and private insurance (Attanasio and Rıos-Rull 2000; Krueger and Perri 2011). This trade-off does not emerge in the context of wage insurance, which is the focus of my paper, since the value of the outside option - being employed in another firm - is affected by the public policy in the same way as the value of staying with the current employer. Varian (1980) and Stiglitz (1982) study the optimal taxation with exogenously stochastic incomes. Here, wages can be stochastic *endogenously* in response to the shape of the tax schedule.

The framework of this paper is an example of the mechanism design with hidden information (productivity) and hidden action (labor supply).[6] Myerson (1982) proves the revelation principle with hidden information and hidden action under restriction to pure reporting strategies. However, I show that under two-sided commitment only mixed reporting strategies are relevant. A technical contribution of this paper is to prove the

[6]The standard Mirrleesian framework is a mechanism design with hidden information only: knowing hidden productivity, one can infer the labor supply from the observed labor income. In contrast, in my framework the knowledge of the productivity is not enough to infer the labor supply, since the labor income is not necessarily equal to the product of the productivity and the labor supply.

revelation principle in such settings while allowing for mixed reporting. More generally, this result demonstrates the importance of mixed reporting when both types and actions are hidden.

The paper is structured as follows. Section 2 introduces a static framework and characterizes the regressivity restriction due to the labor market commitment. Section 3 extends the model to multiple periods, generalizes the results on redistribution from the static case and provides additional results on insurance. Section 4 examines the importance of the regressivity restriction in the context of the actual and the optimal tax and transfer schedule in the US. The last section concludes. Omitted proofs are located in the Appendix.

2. Static framework: commitment and tax regressivity

This section is devoted to the impact of the labor market commitment on income redistribution in the static framework. In particular, I derive and characterize the no-randomization constraint. In the end of this section I use the no-randomization constraint to test whether restricting attention to deterministic wages is without loss of generality in other optimal taxation models.

2.1. Economic environment

Workers and firms. There is a continuum of workers, which are also called agents, with a utility function over consumption c and labor supply n given by $U(c,n) = u(c) - v(n)$, where u and v are strictly increasing and twice differentiable, $u''(c) \leq 0$ and $v''(n) > 0$. Agents draw productivity from $\Theta \subseteq \mathbb{R}_+$ according to a probability measure μ_Θ. Agents' output is given by the product of their productivity and the labor supply.

There is a continuum of identical, risk neutral firms. Each firm can hire at most one worker. A firm provides its employee with a production technology, collects output and pays labor income. I do not specify firms' ownership since in equilibrium firms make zero profits. There is no asymmetric information between firms and their employees: each firm observes the productivity and the labor supply of its employee.

A planner. On top of the economy there is a social planner who observes consumption and labor income of each worker. The planner does not observe productivity or labor supply of any agent, which is the fundamental assumption of the Mirrleesian models. In addition, the planner does not observe any worker's output. In the Mirrleesian models output is always equal to income and as such is observed by the planner. In my framework output and income can deviate from each other, which implies that output (and labor supply) become an action hidden from the planner.

The planner chooses a mechanism which collects messages from agents, allocates them consumption and labor income and gives them labor supply recommendations. A *message space* is a set of possible messages. A *mechanism* specifies *(i)* a message space M, *(ii)* a *consumption function* $c : M \to \mathbb{R}_+$, *(iii)* a *labor income function* $y : M \to \mathbb{R}$ and *(iv)* a *labor recommendation* $n : M \to \mathbb{R}_+$. Since the labor supply is unobserved by the planner, the agents are not bound to follow the labor recommendation, nevertheless such formulation simplifies setting up a planner's problem. In the main text I restrict the attention to deterministic mechanisms. In Online Appendix A I formulate a general mechanism design problem and show that there are no gains from stochastic mechanisms (Lemma A.3).

Given a mechanism (M, c, y, n), firms and workers will agree upon a *labor contract* which determines the report sent to the planner and the labor supply choice. Agents can sent random reports by using *randomization devices*, which are arbitrary probability spaces. More concretely, a labor contract specifies *(i)* a *reporting strategy*, which consists of a *reporting randomization device* with a sample space $\hat{R}_m$ and a *reporting function* $\hat{m} : \Theta \times \hat{R}_m \to M$, and *(ii)* a *labor strategy* $\hat{n} : \Theta \times \hat{R}_m \to \mathbb{R}_+$. Given a mechanism, a labor contract effectively specifies the choice of earnings and labor supply for all types. Note that while the labor strategy can depend on the reporting randomization device, allowing agents to coordinate reports and labor choices, there is no separate randomization device for labor. In fact, conditional on a report sent, agents never have incentives to randomize labor supply (see Lemma A.2 in Online Appendix A).

A labor market equilibrium. The labor market is governed by the following assumptions. First, workers enter the labor market after their productivity is drawn. Second, once a worker and a firm meet, the firm observes the worker's productivity - there is no asymmetric information between a worker and a firm. Third, the labor market is competitive: firms take as given the reservation utility of workers and the competition drives profits to zero. The competitive labor market allows me to abstract from the profit taxation which is not a focus of this paper. Fourth, labor contracts can be terminated by either party before the report is sent to the planner. A worker can quit subject to a *quiting cost* $\kappa \geq 0$ and immediately sign a contract with another firm, and a firm can fire its worker subject to a *firing cost* $\phi \geq 0$. These costs capture the commitment power of each side of the market. When a worker and a firm separate, the firm disappears and is replaced by a new firm.[7]

Fix some mechanism (M, c, y, n). Take some reporting strategy of this mechanism $\hat{\rho}$ and denote by $\mathbb{E}_{\hat{\rho}}\{\cdot\}$ the expectation operator taking into account that the reports are generated with $\hat{\rho}$.[8] The firm chooses the labor contract $(\hat{\rho}, \hat{n})$ to maximize the expected

[7]In this way I make sure that only one operating firm knows a type of a given worker and, hence, the government is unable to extract this information from the firm.

[8]More formally, consider a labor contract $(\hat{\rho}, \hat{n})$, where $\hat{\rho} = ((R_m, \mathcal{B}_{R_m}, \mu_{R_m}), \hat{m})$. The expectations of some function $g(c, y, n, \theta)$ taken with respect to the probability measure μ_Θ and the reporting

profits $\mathbb{E}_{\hat{\rho}}\{\theta\hat{n}-y\}$ subject to two types of constraints. First, the contract needs to provide a worker of type θ with his reservation utility, which is denoted by $\bar{U}(\theta)$:

$$\mathbb{E}_{\hat{\rho}}\{U(c,\hat{n}) \mid \theta\} \geq \bar{U}(\theta) \text{ for all } \theta \in \Theta. \tag{1}$$

Second, the contract can give no incentives to terminate the employment relationship prematurely - the contract needs to satisfy the following *limited commitment constraints*:

$$\kappa \geq \mathbb{E}_{\hat{\rho}}\{\theta\hat{n}-y \mid \theta, r_m\} \geq -\phi \text{ for all } (\theta, r_m) \in \Theta \times R_m. \tag{LCC}$$

The left inequality bounds the expected profits above the quitting cost. If the profits were larger than the quitting cost, another firm could cover the quitting cost of the worker, poach him and make positive profits. The right inequality bounds the expected losses below the firing cost. If losses exceed the firing cost, the firm would be better off firing the worker rather than continuing employment. Note that the contract can be terminated only until the report is sent to the planner. Once the report is sent, the worker and the firm stick together. This formulation of commitment is analogous to Harris and Holmstrom (1982) and Krueger and Uhlig (2006), where the outside option to the current employment or insurance relationship is a relationship with a different firm.

By the assumption of the competitive labor market, in equilibrium the reservation utility $\bar{U}(\cdot)$ is such that the expected profit from hiring any worker is zero. Given that, it is convenient to express the firm's problem in its dual form, where the firm is maximizing its employee's expected utility

$$\max_{\hat{\rho},\hat{n}} \mathbb{E}_{\hat{\rho}}\{U(c,\hat{n})\} \tag{2}$$

subject to the limited commitment constraints (LCC) and the *zero profit constraints*

$$\mathbb{E}_{\hat{\rho}}\{\theta\hat{n}-y \mid \theta\} = 0 \text{ for all } \theta \in \Theta. \tag{ZPC}$$

An *equilibrium of a mechanism* is defined as a labor contract which solves the above firm's problem corresponding to this mechanism.

The revelation principle. Now we can specify various properties of mechanisms. A mechanism (M, c, y, n) with an associated equilibrium $(\hat{\rho}, \hat{n})$ is *feasible* if $\mathbb{E}_{\hat{\rho}}\{\theta\hat{n}-c\} \geq 0$. A mechanism is *direct* if a message space is equal to the space of productivities: $M = \Theta$. A *truthful reporting strategy* ρ^* has a reporting function $m(\theta) = \theta$. A direct mechanism is *incentive-compatible* if it has an equilibrium with the truthful reporting strategy and

strategy $\hat{\rho}$ are defined as

$$\mathbb{E}_{\hat{\rho}}\{g(c,y,\hat{n},\theta)\} \equiv \int\int g(c(\hat{m}(\theta,r_m)), y(\hat{m}(\theta,r_m)), \hat{n}(\theta,r_m), \theta) d\mu_{\Theta}(\theta) d\mu_{R_m}(r_m).$$

the recommended labor strategy. A direct mechanism is *incentive-feasible* if it is both incentive-compatible and feasible.

The revelation principle allows us to restrict attention to direct mechanisms. The standard proof of the revelation principle (e.g. Myerson 1979) does not apply in the current setting, since agents have both hidden types (productivity) and hidden actions (labor supply). Myerson (1982) extends the revelation principle to such environments under assumption that agents use only pure reporting strategies. However, as I will soon show, under two-sided commitment any binding deviation to a pure reporting strategy is strictly dominated by a deviation to some mixed reporting strategy. I extend the revelation principle to settings with hidden types and hidden actions when the mixed reporting strategies are allowed.

Lemma 1 (Revelation principle)**.** *For any feasible mechanism there exists a direct, incentive-feasible mechanism with the same equilibrium assignment of consumption, labor income and labor supply.*

Proof. A special case of a more general Lemma A.1 in Online Appendix A. ∎

From now on, if the message space of the mechanism is not specified, the mechanism is direct.

2.2. The planner's problem.

The social planner chooses a direct, incentive-feasible mechanism to maximize the weighted average of expected utilities of all agents. Social preferences are formalized with a probability measure $\tilde{\mu}_\Theta$ over Θ, where the ratio $\tilde{\mu}_\Theta(A)/\mu_\Theta(A)$ captures the relative preference given to types from some measurable set $A \subseteq \Theta$. Denote by $\tilde{\mathbb{E}}\{\cdot\}$ the expectation operator corresponding to $\tilde{\mu}_\Theta$. The planner maximizes the social welfare function subject to a feasibility and an incentive-compatibility constraints

$$\max_{c,y,n} \tilde{\mathbb{E}}\left\{U(c,n)\right\} \text{ s.t.} \tag{3}$$

$$\mathbb{E}\left\{\theta n - c\right\} \geq 0, \tag{4}$$

$$(\rho^*, n) \in \max_{\hat{\rho},\hat{n}} \mathbb{E}_{\hat{\rho}}\left\{U(c,\hat{n})\right\} \text{ s.t. ZPC and LCC.} \tag{5}$$

The incentive-compatibility constraint means that the truthful reporting strategy and a labor supply recommendation constitute an equilibrium of the chosen mechanism. A mechanism which solves the planner's problem for some probability measure $\tilde{\mu}_\Theta$ is *incentive-efficient*.

2.3. No commitment or one-sided commitment

When at least one of the parties on the labor market cannot commit ($\min\{\kappa, \phi\} = 0$) and the attention is restricted to deterministic mechanisms, this framework is equivalent to the static Mirrlees model.

Lemma 2. *Suppose that* $\min\{\kappa, \phi\} = 0$*. In any equilibrium of any deterministic mechanism income is almost surely equal to output for all agents.*

Proof. Suppose for instance that $\kappa = 0$ (the other case is analogous). Consider a deterministic mechanism with a labor income function y and an associated equilibrium (ρ, n). By LCC we have $\mathbb{E}_\rho\{\theta n - y \mid \theta, r_m\} \leq 0$ for all $(\theta, r_m) \in \Theta \times R_m$. By ZPC we have

$$\mathbb{E}_\rho\{\theta n - y \mid \theta\} = \mathbb{E}_\rho\{\mathbb{E}_\rho\{\theta n - y \mid \theta, r_m\} \mid \theta\} = 0 \text{ for all } \theta \in \Theta. \tag{6}$$

The expectation of a non-positive random variable is zero only if the variable is equal to zero almost surely. Thus, income is equal to output almost surely. ■

With no commitment or a one-sided commitment and a deterministic mechanism each worker is always paid exactly what he produces. Thus, the framework simplifies to the static Mirrlees. Restriction to stochastic mechanisms is not problematic, since there are no gains from using stochastic mechanisms (see Lemma A.3 in Online Appendix A). Nevertheless, when the mechanism is stochastic, my framework and the Mirrleesian framework are no longer equivalent, as here firms are able to provide some insurance against income shocks conditional on the report sent.[9]

2.4. Two-sided commitment

Now consider the situation in which both workers and firms have some positive commitment power ($\min\{\kappa, \phi\} > 0$). The following lemma shows that only stochastic deviations from truthful reporting are relevant.

Lemma 3. *Suppose* $\min\{\kappa, \phi\} > 0$*. Suppose that given a mechanism* (c, y, n) *some initial type* θ *is indifferent between reporting truthfully and reporting some other type* θ' *with certainty, where* $y(\theta) \neq y(\theta')$*. Then type* θ *strictly prefers randomizing reports between* θ *and* θ' *relative to truthtelling.*

[9] Suppose that an agent, after drawing a productivity $\theta = 1$ and making a truthful report, is allocated income equal to 2 with probability 1/2 and to 0 otherwise. Since the report has already been made, the agent and his firm are commited to stick together. Furthermore, the disutility from labor is strictly convex and the agent strictly prefers to smooth labor supply across risky states. Therefore, in equilibrium the worker will supply labor of 1 and have output of 1 with certainty, which satisfies ZPC. Thus, the firm can provide some insurance against a stochasticity of the mechanism and this is the reason why there is no benefit from using stochastic mechanisms in this framework (Lemma A.3 in Online Appendix A). In contrast, Stiglitz (1982) demonstrated that the optimal mechanism of the static Mirrlees model is sometimes stochastic, as the planner exploits different attitudes toward risk to extract hidden types.

Proof of Lemma 3. Suppose for instance that $y(\theta) > y(\theta')$. From the premise we have $u(c(\theta)) - v(y(\theta)/\theta) = u(c(\theta')) - v(y(\theta')/\theta)$. Add the utility from truthtelling to both sides and divide by 2 to get:

$$\begin{aligned}(u(c(\theta)) - v(y(\theta)/\theta)) &= 1/2\left[u(c(\theta)) - v(y(\theta)/\theta)\right] + 1/2\left[u\left(c\left(\theta'\right)\right) - v\left(y\left(\theta'\right)/\theta\right)\right] \qquad (7)\\ &= 1/2\left[u(c(\theta)) + u\left(c\left(\theta'\right)\right)\right] - 1/2\left[v\left(y(\theta)/\theta\right) + v\left(y\left(\theta'\right)/\theta\right)\right]\\ &< 1/2\left[u(c(\theta)) + u\left(c\left(\theta'\right)\right)\right] - 1/2\left[v\left(\frac{y(\theta)-\epsilon}{\theta}\right) + v\left(\frac{y(\theta')+\epsilon}{\theta}\right)\right].\end{aligned}$$

The second row is a rearrangement of terms. The last row describes utility from randomizing reports θ and θ' with equal probability and reducing variation in output between reports by ϵ, where $\epsilon \in (0, \min\{\kappa, \phi\}]$ such that LCC are not violated. Introduction of ϵ amounts to a mean-preserving contraction of labor supply. Given that $-v$ is strictly concave, it strictly benefits the agent. ■

By Lemma 3, if any incentive-compatibility constraint with respect to a pure reporting strategy was binding, agents would like to deviate using mixed reporting. This contrasts with the well-known result of Myerson (1979) that in the mechanism design with hidden information restricting attention to pure reporting strategies is sufficient. Mixed reporting strategies become relevant in this setting because of the interaction between a hidden information and a hidden action: agents can randomize reports and average out the labor supply between the reports. Since agents' utility is strictly concave in labor supply, the averaging of labor yields a utility gain. A simple implication of Lemma 3 is that with two-sided commitment the income function needs to be continuous: any discontinuity of income would trigger a randomization of reports across the discontinuity.

Corollary 1. *Suppose* $\min\{\kappa, \phi\} > 0$. *Suppose that* Θ *is an interval in* $\mathbb{R}_+$. *For any incentive-compatible mechanism* (c, y, n) *the income function* y *is continuous.*

We are going to simplify the incentive-compatibility constraint in two steps. First, define an *incentive-compatibility constraint in mixed reporting strategies* for type θ mimicking type θ' as

$$u(c(\theta')) - u(c(\theta)) - \left(y(\theta') - y(\theta)\right) v'(y(\theta)/\theta)/\theta \leq 0. \qquad (8)$$

This constraint prevents stochastic misreporting by making sure that an agent who reveals his type truthfully cannot gain by marginally increasing the probability of reporting some type. As is shown in the following lemma, these constraints are stronger than the standard incentive constraints in pure reporting strategies and are sufficient to prevent any misreporting. Furthermore, these constraints are also necessary when the relevant limited commitment constraints are slack.

Lemma 4. *Type* θ *has no incentives to mimic type* θ' *with any positive probability if* (8) *holds. Type* θ *has no incentives to mimic type* θ' *with any positive probability, where* $|y(\theta) - y(\theta')| < \min\{\kappa, \phi\}$, *if and only if* (8) *holds.*

Proof of Lemma 4. First, consider incentives to misreport under full two-sided commitment ($\kappa = \phi = \infty$). In this case the equilibrium output of every agent is equal to his average income. Then the equilibrium reporting strategy of some agent θ generates a probability measure over reports which solves

$$\max_{\sigma \in \Delta_\Theta} \int u\left(c\left(\theta'\right)\right) d\sigma\left(\theta'\right) - v\left(\left(\int y\left(\theta'\right) d\sigma\left(\theta'\right)\right/\theta\right), \tag{9}$$

where Δ_Θ is the space of probability distributions over type reports. Since v is convex, it is a concave maximization problem and the first-order conditions are sufficient. In particular, the first-order condition with respect to the probability of reporting θ', evaluated at the truthful reporting strategy, is exactly (8).

The payoff from misreporting under full two-sided commitment is an upper bound on the payoff from misreporting under partial two-sided commitment ($\min\{\kappa, \phi\} < \infty$), as LCC under partial commitment are tighter. Hence, if (8) holds, then type θ has no incentives to report θ' with any positive probability also under partial commitment. Furthermore, when $|y(\theta) - y(\theta')| < \min\{\kappa, \phi\}$, then LCC are slack when type θ considers randomizing report between θ and θ'.[10] Therefore, the marginal incentives to report θ' are identical as in the full two-sided commitment case and (8) is also necessary to prevent misreporting. ■

Now we can further simplify the set of incentive constraints by focusing on incentives to misreport locally. In the mechanism design with hidden information only we can summarize the incentive-compatibility constraints with respect to pure reporting strategies with the local incentive-compatibility constraints and the income monotonicity requirement (see e.g. Fudenberg and Tirole (1991)). The following theorem shows that the incentive-compatibility under two-sided commitment requires an additional *no-randomization constraint.*

Theorem 1. *Suppose* $\min\{\kappa, \phi\} > 0$. *Suppose that* Θ *is an interval in* $\mathbb{R}_+$. *The mechanism* (c, y, n), *where* $n(\theta) = y(\theta)/\theta$ *for all* θ, *is incentive-compatible if and only if it satisfies*

1. ***income monotonicity:*** $y(\cdot)$ *is increasing,*
2. ***local incentive-compatibility constraints:***

$$U(c(\theta'), y(\theta')/\theta') - U(c(\theta), y(\theta)/\theta) = \int_\theta^{\theta'} \frac{y(\tilde\theta)}{\tilde\theta^2} v'\left(\frac{y(\tilde\theta)}{\tilde\theta}\right) d\tilde\theta \text{ for all } \theta \text{ and } \theta',$$

3. ***no-randomization constraint:*** $v'\left(\frac{y(\theta)}{\theta}\right)/\theta$ *is decreasing with* θ.

[10] Suppose that type θ randomizes reports between θ and θ' and denote the expected income by $\bar y$. The optimal labor supply under full two-sided commitment is $\bar y/\theta$ regardless of the report. Note that for $\tilde\theta \in \{\theta, \theta'\}$ we have $|\bar y - y(\tilde\theta)| \le |y(\theta) - y(\theta')| < \min\{\kappa, \phi\} \implies \kappa > \bar y - y(\tilde\theta) > -\phi$. Hence, LCC are slack given this labor strategy.

Proof. First I will show the sufficiency of conditions 1 to 3. Consider some θ and θ', $\theta' > \theta$. Notice that

$$v(y(\theta')/\theta') - v(y(\theta)/\theta)) = \int_{\theta}^{\theta'} y'(\tilde{\theta})v'\left(\frac{y(\tilde{\theta})}{\tilde{\theta}}\right)/\tilde{\theta} - \frac{y(\tilde{\theta})}{\tilde{\theta}^2}v'\left(\frac{y(\tilde{\theta})}{\tilde{\theta}}\right)d\tilde{\theta} \tag{10}$$

Rearranging the local incentive constraint, we get

$$\begin{aligned}
u(c(\theta')) - u(c(\theta)) &= v(y(\theta')/\theta') - v(y(\theta)/\theta)) + \int_{\theta}^{\theta'} \frac{y(\tilde{\theta})}{\tilde{\theta}^2}v'\left(\frac{y(\tilde{\theta})}{\tilde{\theta}}\right)d\tilde{\theta} &(11)\\
&= \int_{\theta}^{\theta'} y'(\tilde{\theta})v'\left(\frac{y(\tilde{\theta})}{\tilde{\theta}}\right)/\tilde{\theta}d\tilde{\theta} &(12)\\
&\leq \int_{\theta}^{\theta'} y'(\tilde{\theta})v'\left(\frac{y(\theta)}{\theta}\right)/\theta d\tilde{\theta} &(13)\\
&= (y(\theta') - y(\theta))v'\left(\frac{y(\theta)}{\theta}\right)/\theta, &(14)
\end{aligned}$$

where the inequality follows from the no-randomization constraint and income monotonicity. It follows that the incentive-compatibility constraint in mixed reporting strategies for type θ mimicking type θ' holds. By Lemma 4, this is sufficient for incentive-compatibility.

The necessity of income monotonicity follows from the standard screening problem (Theorem 7.2 in Fudenberg and Tirole (1991)). To see the necessity of the no-randomization constraint, suppose that for some θ and θ', $\theta' > \theta$, we have $v'\left(\frac{y(\theta')}{\theta'}\right)/\theta' > v'\left(\frac{y(\theta)}{\theta}\right)/\theta$. By Corollary 1 y is continuous and we can choose θ' such that $0 < |y(\theta) - y(\theta')| < \min\{\kappa, \phi\}$. Income monotonicity implies that

$$(y(\theta') - y(\theta))\left(v'\left(\frac{y(\theta')}{\theta'}\right)/\theta' - v'\left(\frac{y(\theta)}{\theta}\right)/\theta\right) > 0. \tag{15}$$

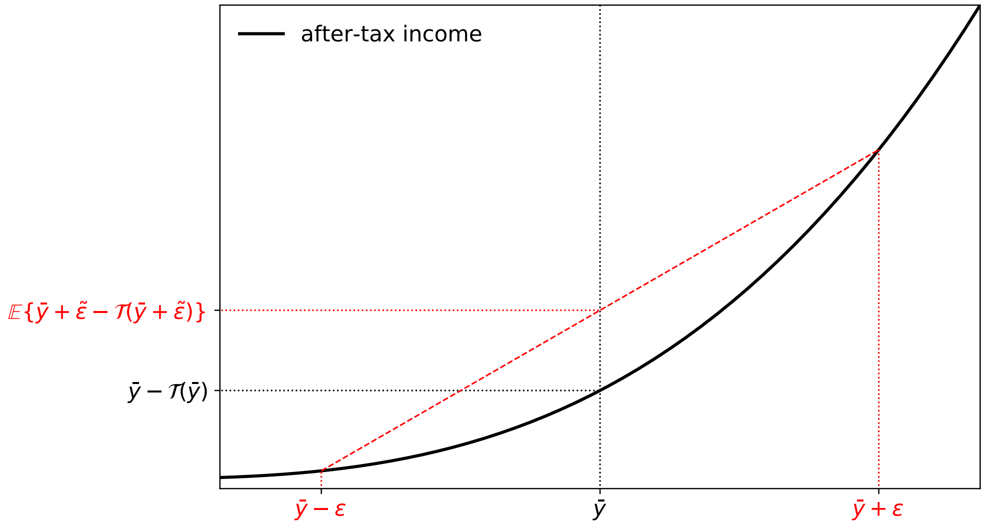
Now sum the incentive-compatibility constraints in mixed reporting strategies with for θ mimicking θ' and *vice versa* to get

$$(y(\theta') - y(\theta))\left(v'\left(\frac{y(\theta')}{\theta'}\right)/\theta' - v'\left(\frac{y(\theta)}{\theta}\right)/\theta\right) \leq 0. \tag{16}$$

Therefore, one of the incentive-compatibility constraints in mixed reporting strategies is violated. By Lemma 4, either θ or θ' has incentives to misreport. ■

Theorem 1 implies that we can summarize the entire impact of the two-sided commitment on incentive-compatible allocations with a single requirement: the no-randomization constraint. The remaining of this section devoted to characterizing this constraint in detail. Let's define a *tax schedule* $\mathcal{T}$ corresponding to some mechanism (c, y, n) as $\mathcal{T}(y(\theta)) = y(\theta) - c(\theta)$ for all θ. It allows us to express the no-randomization constraint

Figure 1: Wage randomization with a regressive tax schedule



A deterministic wage $\bar{y}$ compared to a randomized wage $\bar{y}+\tilde{\varepsilon}$ (in red), where $\tilde{\varepsilon}$ is a random variable taking values $\pm\varepsilon$ with equal probabilities.

in a more transparent way.

Claim 1. *Take some incentive-compatible mechanism (c, y, n). Suppose that the corresponding tax schedule $\mathcal{T}$ is twice differentiable. The no-randomization constraint can be expressed as*

$$-\frac{u''(\bar{y}-\mathcal{T}(\bar{y}))}{u'(\bar{y}-\mathcal{T}(\bar{y}))} \geq \frac{-\mathcal{T}''(\bar{y})}{(1-\mathcal{T}'(\bar{y}))^2} \text{ for all } \bar{y} \in y[\Theta]. \tag{17}$$

The no-randomization constraint (17) requires that the measure of local tax regressivity (the right-hand side) cannot exceed the coefficient of absolute risk aversion (the left-hand side). It has a simple intuition. A worker's attitude toward income risk depends on two factors: the shape of the tax schedule, which determines how income is transmitted to consumption, and the attitude towards the consumption risk, which is governed by the coefficient of the absolute risk aversion. When the tax schedule is locally regressive (i.e. the after-tax income is convex in the pre-tax income), income risk reduces the expected tax payments and lifts the expected after-tax income (see Figure 1). Thus, when the tax regressivity is strong enough relative to the coefficient of absolute risk aversion, the worker actually benefits from income risk. One can show that in this case the worker's absolute aversion to income risk is negative - the worker effectively becomes an income risk lover.[11]

Although the additional income risk benefits the agent, it is never efficient. By Lemma A.3 (in Online Appendix A) we know that stochastic mechanisms cannot improve upon the deterministic ones. Intuitively, rather than introducing additional income risk, the

[11] Given a tax schedule $\mathcal{T}$, define the utility level at income y as $\tilde{u}(y) \equiv u(y-\mathcal{T}(y))$. It is easy to see that the agent is averse to income risk ($\tilde{u}''(y) \leq 0$) if and only if the no-randomization constraint holds.

planner instead could made the tax schedule locally linear. The impact on the expected tax revenue is identical in both cases, yet the agent is not exposed to the consumption risk.

How restrictive is the no-randomization constraint? When agents are risk neutral, only progressive (i.e. convex) tax schedules are incentive-compatible. When agents are risk averse, some regressivity is allowed. Note that the regressivity measure has a squared net-of-tax rate in the denominator. Consequently, the upper bound on regressivity is particularly tight when marginal tax rates are high. In Section 4 I show that this constraint is robustly violated in the context of the US both by the actual tax and transfer schedule and by the optimal Mirrleesian tax schedule.

2.5. No-randomization constraint as an efficiency test of the tax schedule

To understand the generality of the no-randomization constraint, it is useful to derive it with the following simple argument. Suppose an individual earning $\bar{y}$ considers adding a mean-preserving spread to his wage such that the income becomes $\bar{y} \pm \varepsilon$ with equal probabilities for some small $\varepsilon > 0$. Note that the labor supply is kept constant. The utility gain from this income lottery relative to a deterministic income, up to second order terms, is

$$\frac{1}{2}u(\bar{y}+\varepsilon-\mathcal{T}(\bar{y}+\varepsilon))+\frac{1}{2}u(\bar{y}-\varepsilon-\mathcal{T}(\bar{y}-\varepsilon))-u(\bar{y}-\mathcal{T}(\bar{y}))$$
$$\approx \frac{1}{2}\varepsilon^2\left[(1-\mathcal{T}'(\bar{y}))^2u''(\bar{y}-\mathcal{T}(\bar{y}))-\mathcal{T}''(\bar{y})u'(\bar{y}-\mathcal{T}(\bar{y}))\right].$$

It is easy to see that the individual has incentives to engage in wage randomization when the no-randomization constraint (17) is violated. As argued above, such wage randomization is inefficient. Therefore, the no-randomization constraint is a necessary condition for efficiency of the tax schedule whenever individuals are able to add a mean-preserving spread to their wages. This, in turn, is possible when both firms and workers have any positive commitment power, *regardless* of additional frictions. For instance, in the labor search models both firms and workers have a positive commitment power whenever the wage is in the interior of the bargaining set. Then neither a worker nor a firm would have incentives to break the employment relationship if a wage fell or rose slightly. Thus, a mean-preserving spread of a wage is possible and the no-randomization constraint is relevant. Alternatively, consider a moral hazard model where wages are already risky as they depend on the stochastic output. Under two-sided commitment the firm could introduce a mean-preserving spread to a wage conditional on each output realization and in this way reap the gains from tax regressivity.

The literature on non-linear income taxation which explicitly models employment relationships typically assumes fixed, deterministic wages rather than allowing for more general wage lotteries. As an example, consider two papers on the optimal non-linear taxation with labor search frictions, both of which find the optimal tax to be regressive. Golosov, Maziero, and Menzio (2013) study optimal taxation in the directed search framework in the spirit of Moen (1997) with ex ante identical agents. The income tax is optimally increasing and regressive to finance the unemployment benefit while encouraging the search for more productive jobs. One can show that the optimal tax regressivity is mild enough not to violate the no-randomization constraint.[12] In this case restricting attention to fixed wages is without loss of generality - agents would not engage in the inefficient wage randomization if stochastic compensation was possible. Another approach is by Yazici and Sleet (2017) who derive the optimal income tax in the modified on-the-job search framework of Burdett and Mortensen (1998) with ex ante heterogeneity. Whereas the complex optimal tax formula does not allow for the analytical examination of regressivity, the quantitative exercise reveals that the marginal tax rates are high at low income levels and fall very rapidly with income. It strongly suggests that some agents would have incentives to randomize wages and, hence, the tax schedule would be inefficient if the more general wage lotteries were allowed. More generally, wage randomization is likely to be important in settings with ex ante heterogeneity, where different social preferences toward redistribution can yield a variety of shapes of the optimal tax schedule.

3. Dynamic framework: commitment, redistribution and insurance

In this section I extend the static setting of Section 2 to multiple periods. First I describe the dynamic environment, focusing on the differences with respect to the static case. Then I characterize the incentive-efficient mechanisms under no commitment, full two-sided commitment and partial commitment.

3.1. Economic environment and the planner's problem

Workers and firms. A continuum of workers live for $T \geq 1$ periods, where $T \leq \infty$. By setting $T = 1$ we go back to the static case of Section 2. Agents' per-period utility over consumption c and labor supply n is $u(c) - v(n)$, where u and v are strictly increasing and twice differentiable, $u''(c) \leq 0$ and $v''(n) > 0$. Agents discount future with $\beta < 1$. Take

[12]The optimal tax rate is given by equation 23 in Golosov, Maziero, and Menzio (2013). It implies that at the optimum

$$\left(1 - T_e^{*\prime}(w)\right) u'(c(w)) = k \left[B_u^* + k/u'(c(w))\right]^{-1}, \tag{18}$$

where w is a wage, T_e^* is the optimal labor income tax schedule, $c(w)$ is the consumption of a worker with wage w, k is the fixed cost of searching and B_u^* is the optimal unemployment benefit. The marginal tax rate is always below 100%, which means that consumption is increasing with w and the right-hand side is decreasing with w. Thus, workers are averse to the income risk and the no-randomization constraint is satisfied.

some history of consumption $c^T = (c_1^T, ..., c_T^T) \in \mathbb{R}_+^T$ and labor supply $n^T = (n_1^T, ..., n_T^T) \in \mathbb{R}_+^T$. The implied lifetime utility is

$$U(c^T, n^T) \equiv \sum_{t=1}^{T} \beta^{t-1} \left(u(c_t^T) - v(n_t^T) \right).$$

In each period each an agent draws productivity from $\Theta \subseteq \mathbb{R}_+$. Equivalently, we can think about each agent drawing a productivity history $\theta^T = (\theta_1^T, ..., \theta_T^T) \in \Theta^T$ that is revealed over time. Draws of productivity histories are independent across agents and are distributed according to a probability measure μ_{Θ^T}. I impose no restrictions on the measure μ_{Θ^T}: the individual productivity process is arbitrary and can involve fixed effects as well as permanent and transitory components.[13]

There is a continuum of identical, risk neutral firms. Each firm can hire at most one worker. Take some history of labor income $y^T = (y_1^T, ..., y_T^T) \in \mathbb{R}^T$, labor supply $n^T = (n_1^T, ..., n_T^T) \in \mathbb{R}_+^T$ and productivity $\theta^T = (\theta_1^T, ..., \theta_T^T) \in \Theta^T$. The implied continuation profit in period t is

$$\pi_t(y^T, n^T, \theta^T) \equiv \sum_{s=t}^{T} \beta^{s-1} \left(\theta_s^T n_s^T - y_s^T \right).$$

A planner. The social planner chooses a mechanism which specifies *(i)* a message space M, *(ii)* a *consumption function* $c : M^T \to \mathbb{R}_+^T$, *(iii)* a *labor income function* $y : M^T \to \mathbb{R}^T$ and *(iv)* a *labor supply recommendation* $n : M^T \to \mathbb{R}_+^T$, where c_t, y_t and n_t are m^t-measurable for any $t \leq T$.[14] The measurability assumptions guarantee that a mechanism's outcomes or recommendations do not depend on future messages. Given a mechanism (M, c, y, n), a labor contract consists of *(i)* a *reporting strategy* $\hat{\rho}$, which contains a *reporting randomization device* with a sample space $\hat{R}_m$ and a *reporting function* $\hat{m} : \Theta^T \times \hat{R}_m^T \to M^T$, and *(ii)* a *labor strategy* $\hat{n} : \Theta^T \times R_m^T \to N^T$, where $\hat{m}_t$ and $\hat{n}_t$ are $(\theta^t \times \hat{r}_m^t)-$measurable for any $t \leq T$.

A labor market equilibrium. The assumptions about the labor market are analogous to the ones made in Section 2 with the following clarifications. First, workers enter the labor market after their *initial* productivity is drawn. Second, labor contracts can be terminated by either party *in any period* before the report is sent to the planner, subject to a quitting cost κ and a firing cost ϕ. When a worker and a firm separate, the firm

[13] Alternatively, we can interpret this setting as in Harris and Holmstrom (1982) where workers have a fixed, initially unknown productivity. A worker and his employer learn symmetrically about the worker's productivity from stochastic signals and θ_t^T is interpreted as a posterior mean productivity in period t.

[14] I will define measurability in a general case. Consider three measurable spaces $(\Omega_1, \mathcal{B}_{\Omega_1})$, $(\Omega_2, \mathcal{B}_{\Omega_2})$ and $(\Omega', \mathcal{B}_{\Omega'})$. A function $g : \Omega_1^T \times \Omega_2^T \to \Omega'$ is $(\omega^{t_1} \times \omega^{t_2})-$measurable if for all $A \in \mathcal{B}_{\Omega'}$ there is $B_1 \in \mathcal{B}_{\Omega_1^{t_1}}$ and $B_2 \in \mathcal{B}_{\Omega_2^{t_2}}$ such that $g^{-1}(A) = B_1 \times \Omega_1^{T-t_1} \times B_2 \times \Omega_2^{T-t_2}$. To understand measurability with respect to a single measurable space, set $(\Omega_2, \mathcal{B}_{\Omega_2}) = (\varnothing, \{\varnothing\})$.

disappears and is replaced by a new firm. Since all firms are identical and there are no gains from workers' reallocation, I focus on equilibria without separations.

Fix a mechanism (M, c, y, n). Take some reporting strategy $\hat{\rho}$ and denote by $\mathbb{E}_{\hat{\rho}}\{\cdot\}$ the corresponding expectation operator.[15] The equilibrium labor contract $(\hat{\rho}, \hat{n})$ needs to satisfy the dynamic versions of the zero profit constraints and limited commitment constraints[16]

$$\mathbb{E}_{\hat{\rho}}\left\{\pi_1(y, \hat{n}, \theta^T) \mid \theta_1^T = \theta\right\} = 0 \text{ for all } \theta \in \Theta, \tag{ZPC$_d$}$$

$$\kappa \geq \mathbb{E}_{\hat{\rho}}\left\{\pi_t(y, \hat{n}, \theta^T) \mid \theta^t, \hat{r}_m^t\right\} \geq -\phi \text{ for all } t \leq T \text{ and all } (\theta^t, \hat{r}_m^t) \in \Theta^t \times \hat{R}_m^t. \tag{LCC$_d$}$$

The dynamic limited commitment constraints make sure that neither a worker nor a firm has incentives to terminate the contract *in any period*. If the continuation profits were larger than the quitting cost, another firm could profitably steal the worker. Similarly, if losses exceed the firing cost, the firm would be tempted to fire the worker.

The planner's problem. The planner maximizes the average of individual lifetime utilities, weighted according to the probability measure $\tilde{\mu}_\Theta$, subject to a feasibility and incentive-compatibility constraints

$$\max_{c,y,n} \tilde{\mathbb{E}}\left\{U(c, n)\right\} \quad \text{s.t.} \tag{19}$$

$$\mathbb{E}\left\{\sum_{t=1}^{T} \beta^{t-1}(\theta_t n_t - c_t)\right\} \geq 0, \tag{20}$$

$$(\rho^*, n) \in \max_{\hat{\rho}, \hat{n}} \mathbb{E}_{\hat{\rho}}\left\{U(c, \hat{n})\right\} \text{ subject to ZPC}_d \text{ and LCC}_d. \tag{21}$$

The dynamic feasibility constraint implies that the planner can effectively borrow and lend at the gross interest rate $1/\beta$. The purpose of this assumption is to avoid an incomplete consumption insurance purely due to a time trend in expected productivity.

3.2. No labor market commitment

Similarly as in the static model, also in the dynamic case the no commitment scenario is equivalent to the Mirrleesian framework. Differently than in the static model, the one-

[15] More formally, consider a labor contract $(\hat{\rho}, \hat{n})$, where $\hat{\rho} = ((\hat{R}_m, \mathcal{B}_{\hat{R}_m}, \mu_{\hat{R}_m}), \hat{m})$. The expectations of some function $g(c^T, y^T, n^T, \theta^T)$ are defined as

$$\mathbb{E}_{\hat{\rho}}\left\{g(c, y, \hat{n}, \theta^T)\right\} \quad \equiv \quad \int\int g(c(\hat{m}(\theta^T, \hat{r}_m^T)), y(\hat{m}(\theta^T, \hat{r}_m^T)), \hat{n}(\theta^T, \hat{r}_m^T), \theta^T) d\mu_{\Theta^T}(\theta^T) d\mu_{\hat{R}_m^T}(\hat{r}_m^T).$$

[16] Sometimes the abbreviated notation for conditional expectations is used:

$$\mathbb{E}_{\hat{\rho}}\left\{\pi_t(y, \hat{n}, \theta^T) \mid \theta^t, \hat{r}_m^t\right\} \equiv \mathbb{E}_{\hat{\rho}}\left\{\pi_t(y, \hat{n}, \theta^T) \mid (\theta_1^T, ..., \theta_t^T) = \theta^t, (\hat{r}_{m1}^T, ..., \hat{r}_{mt}^T) = \hat{r}_m^t\right\}.$$

sided commitment case ($\min\{\kappa, \phi\} = 0$) is in general not equivalent to the Mirrleesian framework when $T > 1$. That is the case because, as we will see, firms are able to provide some insurance to their workers even under one-sided commitment.

Lemma 5. *Suppose that $\phi = \kappa = 0$. In any equilibrium of any deterministic mechanism income is equal to output at every productivity history.*

3.3. Full two-sided commitment

Suppose that both a worker and a firm can credibly promise not to terminate the employment relationship under any circumstances ($\kappa = \phi = \infty$). Consequently, we can drop the limited commitment constraints. The following lemma characterizes the equilibrium labor strategy.

Lemma 6. *Suppose $\kappa = \phi = \infty$. Fix a productivity history θ^T. The equilibrium labor supply in period $t \leq T$ depends only on (i) the current productivity θ_t^T, (ii) the initial productivity θ_1^T and (iii) the expected lifetime labor income of the agent with initial productivity θ_1^T.*

Since there is no threat of separation under full two-sided commitment, the labor contract maximizes this worker's expected utility subject only to the zero profit condition, which requires that the expected lifetime output and income are equal. Equivalently, the equilibrium labor supply minimizes the expected disutility of producing the expected lifetime income. It means that in the equilibrium the marginal disutilities of production $v'(n_t)/\theta_t$ are equalized across all histories and time periods. The level at which these marginal disutilities are equalized depends both on the initial expected lifetime income and on the initial productivity, which may be informative about the future productivity process of the worker.

3.3.1. Insurance under full two-sided commitment

Let's define a notion of insurance which allows us to easily incorporates utility functions which are not strictly concave in consumption. A mechanism (c, y, n) involves *full insurance* if

$$u'\left(c_1(\theta^T)\right) = u'\left(c_t(\theta^T)\right) \text{ for all } \theta^T \text{ and all } t \leq T. \tag{22}$$

Theorem 2. *Suppose $\kappa = \phi = \infty$. All incentive-efficient mechanisms involve full insurance.*

To understand Theorem 2, it is instructive to consider the optimal allocations in the dynamic Mirrlees model first. Abstracting from redistributive preferences, the planner has two goals: insuring consumption and providing incentives for efficient labor supply. Crudely speaking, labor is supplied efficiently if the labor supply increases with the worker's productivity.[17] In the dynamic Mirrlees model these two goals cannot be achieved at the same time: if consumption does not vary with productivity, neither does income nor output and the labor supply is actually decreasing with productivity. Conversely, the labor supply will be efficient if each agent simply consumes his output, but then there is no insurance. Optimally, the planner chooses a middle ground: a partial consumption insurance and a pattern of history dependent labor distortions.

Things look differently in the full two-sided commitment case. From Lemma 6 we know that the equilibrium labor strategy depends solely on the expected lifetime income of each initial type. Hence, the planner can eliminate the entire labor income risk without introducing additional labor distortions by simply replacing the labor earnings at each contingency with its expected value for that period. As agents are not exposed to the labor income risk, they do not face any consumption risk either. Intuitively, since firms have more information than the planner, the planner outsources the entire insurance provision to firms.

3.3.2. Redistribution under full two-sided commitment

Given the results so far it easy to see that the taxation problem under full two-sided commitment is effectively static. Theorem 2 says that in all incentive-efficient mechanisms each worker has a constant consumption (as long as the utility is strictly concave). Lemma 6 in turn implies that we can focus attention on mechanisms where each worker has a constant labor income. Therefore, we can relate this dynamic problem to the static taxation problem from Section 2. To do that, denote the average disutility from labor of some initial type θ with average per-period income $\bar{y}$ by $\tilde{v}(\bar{y}, \theta)$. One can show that $\tilde{v}(\cdot, \theta)$ inherits the properties of the per-period disutility from labor v: it is strictly increasing, strictly convex and differentiable. Furthermore, define the *single-crossing condition*: $\tilde{v}_y(y, \theta)$ is decreasing with θ when $y \geq 0$ and is decreasing strictly when $y > 0$. In the static model, the single-crossing condition is automatically implied by agents' preferences. In the dynamic framework this condition means that the marginal disutility of average lifetime earnings is decreasing with initial productivity. Intuitively, that is the case if the initial productivity is not negatively correlated with future productivities. A sufficient condition for that is the initial productivity which shifts the distribution of productivities in all future periods according to the first-order stochastic dominance.

Claim 2. *Single-crossing condition holds if for any θ and θ' from Θ, $\theta > \theta'$, it is true that $\left\{\theta_t^T \mid \theta_1^T = \theta\right\} \succeq_{FOSD} \left\{\theta_t^T \mid \theta_1^T = \theta'\right\}$ for all $t \leq T$.*

Given the single-crossing condition, we can proceed as in Section 2 and simplify the incentive-constraints to three familiar requirements.

[17] Abstracting from redistribution, the first-best allocation is characterized by the full insurance $u'(c_t(\theta^T)) = u'(c_1(\theta^T))$ and the efficient labor supply $u'(c_t(\theta^T)) = v'(n_t(\theta^T))/\theta_t^T$ for all $t \leq T$ and all $\theta^T \in \Theta^T$.

Theorem 3. *Suppose that $\kappa = \phi = \infty$. Suppose further that Θ is an interval in $\mathbb{R}_+$, $\tilde{v}(y(\theta), \theta)$ is absolutely continuous in θ for all nondecreasing $y(\cdot)$ and the single-crossing condition holds. The mechanism (c, y, n) such that $c(\theta_t^T) = \bar{c}(\theta_1^T)$ and $y(\theta_t^T) = \bar{y}(\theta_1^T)$ for all $t \leq T$ is incentive-compatible if and only if it satisfies*

1. ***income monotonicity**: $\bar{y}(\cdot)$ is increasing,*
2. ***local incentive constraints**: $u(\bar{c}(\theta')) - \tilde{v}(\bar{y}(\theta'), \theta') = u(\bar{c}(\theta)) - \tilde{v}(\bar{y}(\theta), \theta) - \int_\theta^{\theta'} \tilde{v}_\theta(\bar{y}(\tilde{\theta}), \tilde{\theta}) d\tilde{\theta}$ for all θ and θ',*
3. ***no-randomization constraint**: $\tilde{v}_y(\bar{y}(\theta), \theta)$ is decreasing with θ.*

The dynamic optimal taxation problem under full two-sided commitment is equivalent to the static optimal taxation problem under two-sided commitment with a modified disutility from labor. In particular, the regressivity restriction due to a threat of wage randomization, which we discussed in Section 2, applies in this dynamic setting as well.

3.4. Partial commitment

In this section I characterize incentive-efficient mechanisms when commitment is limited ($\min\{\kappa, \phi\} < \infty$): either a firm or a worker would break the contract subject to sufficiently strong incentives. Partial commitment is common in modeling labor markets and insurance relationships. For instance, Harris and Holmstrom (1982) and Krueger and Uhlig (2006) assume one-sided commitment, where firms can commit fully and agents cannot commit at all. Labor search models typically operate under partial commitment which in this context effectively means that the match surplus is positive. In Thomas and Worrall (1988) the commitment is partial and stochastic due to random outside options of a worker and a firm. My framework can be extended to include a random variation to the quiting cost and the firing cost in a straightforward manner as long as the realized costs are publicly observable.

3.4.1. Insurance under partial commitment

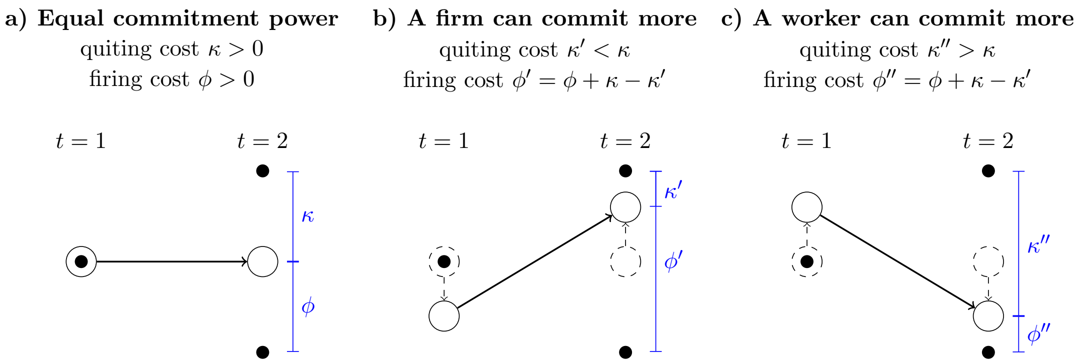
Let's first abstract from redistribution by assuming that all agents are initially identical. Denote by μ_{Θ_1} the probability measure over the set of initial types implied by μ_{Θ^T}.[18]

Assumption 1. *$T \geq 2$ and there exists $\theta \in \Theta$ such that $\mu_{\Theta_1}(\{\theta\}) = \tilde{\mu}_\Theta(\{\theta\}) = 1$.*

Proposition 1. *Suppose that Assumption 1 holds. Consider an mechanism (c, y, n) which is incentive-feasible when the quitting and firing costs are κ and ϕ respectively and $\min\{\kappa, \phi\} < \infty$. There exists a labor income function $\tilde{y}$ such that the mechanism $(c, \tilde{y}, n)$ is incentive-feasible when the quitting and firing costs are κ' and ϕ' respectively, where $\kappa' + \phi' = \kappa + \phi$ and either $\kappa - \kappa'$ or $\phi - \phi'$ is finite.*

[18]Denote the σ-algebra over Θ by $\mathcal{B}_\Theta$. Then $\mu_{\Theta_1}(A) = \mu_{\Theta^T}(A \times \Theta^{T-1})$ for all $A \in \mathcal{B}_\Theta$.

Figure 2: The distribution of commitment and the intertemporal income shifting



Income is represented by ○ *and output is represented by* ●. *There are two time periods and two possible productivity realizations in the second period. The total commitment power is constant and just enough to eliminate the income risk for the chosen labor strategy.*

What matters for the within-firm insurance is the *total commitment power* $\kappa + \phi$. It is irrelevant who can commit more within a firm: an employer or an employee. Any imbalance in the commitment power can be corrected by shifting labor income forward or backward in time, as illustrated in the Figure 2. When the firm can commit more, wage payments are *backloaded* - shifted toward later periods - and in early periods the worker is paid less than his output (see panel b of Figure 2). The worker effectively places a deposit in the firm which discourages quitting early. Analogously, when the worker can commit more, wage payments are *frontloaded* - shifted toward early periods - and it is the firm who effectively gives a deposit to the worker (see panel c of Figure 2). Crucially, this intertemporal income shifting allows the firm to satisfy the limited commitment constraints without introducing additional income risk or changing the labor strategy.

If the income is shifted between periods, how come agents can maintain the same consumption allocation? The simplest intuition is that the agents could undo these intertemporal shifts in income by trading a risk-free bond subject to a sufficiently loose borrowing limit. Since such asset trades are not contingent on productivity and, hence, are within the information set of the planner, the planner could set up an indirect mechanism allowing for such asset trades. The revelation principle informs us that the resulting allocation could also be implemented as a direct mechanism, where the planner directly assigns final consumption to the agents. Alternatively, the planner could replicate the allocation with the asset trades via the age-dependent taxes.

Define *full one-sided commitment* as a situation in which only one side of the labor market can commit fully, i.e. when the quitting cost κ and the firing cost ϕ are such that $\max\{\kappa, \phi\} = \infty$ and $\min\{\kappa, \phi\} < \infty$. One can show that such arrangement is equivalent, up to the labor income function, to the two-sided commitment case.

Corollary 2. *Suppose that Assumption 1 holds, Θ is bounded and $v'(\infty) = \infty$. For any mechanism (c, y, n) that is incentive-efficient under full two-sided commitment there exists a labor income function $\tilde{y}$ such that the mechanism $(c, \tilde{y}, n)$ is incentive-efficient under full one-sided commitment.*

When one side of the labor market can commit fully, the total commitment power is infinite and, hence, equal to the total commitment power under full two-sided commitment. In the spirit of Proposition 1, these two cases are thus equivalent. The additional assumptions of Corollary 2 ensure that income shifts required to satisfy the limited commitment constraints are finite. Consequently, we can apply Theorem 2 to economies with no initial differences and full one-sided commitment to discover that all incentive-efficient mechanisms involve full consumption insurance. A specific case of Corollary 2 was noted by Harris and Holmstrom (1982) (see their footnote 5) who observed that the equilibrium of the insurance market with full commitment of firms and no commitment of workers achieves full consumption insurance when workers can borrow sufficiently. The borrowing allows workers to maintain constant consumption while their income is backloaded. An inability of workers to borrow (or of the government to introduce age-dependent taxes) is the reason why some models with full one-sided commitment fail to achieve full insurance. For instance, Krueger and Uhlig (2006) find that full insurance is achieved only in the long run. If in their model the agents were able to borrow sufficiently, or if there was a government able to introduce age-dependent taxes, the equilibrium would involve full insurance from the initial period.

3.4.2. Redistribution under partial commitment

Let's bring back the initial differences of workers and the redistributive preferences of the planner. Define a useful class of labor contracts available to agents: a labor contract $(\hat{\rho}, \hat{n})$ *does not randomize in the initial period* if $\hat{m}_1(\theta^T, \hat{r}_m^T) = \hat{m}_1(\theta^T, \hat{r}_m'^T)$ and $\hat{n}_1(\theta^T, \hat{r}_m^T) = \hat{n}_1(\theta^T, \hat{r}_m'^T)$ for all θ^T and all $\hat{r}_m^T, \hat{r}_m'^T \in \hat{R}_m^T$. With a slight abuse of notation I will then suppress the redundant, second argument of the reporting and labor strategies. An example of a labor contract which does not randomize in the initial period is an equilibrium contract of any incentive-compatible mechanism, i.e. the truthful reporting strategy ρ^* combined with the labor supply recommendation specified in the mechanism. Naturally, any contract which does not satisfy the above condition is said to *randomize in the initial period.*

Lemma 7. *Suppose* $\min\{\kappa, \phi\} > 0$. *Suppose that given a mechanism (c, y, n) some initial type θ is indifferent between (ρ^*, n) and $(\hat{\rho}, \hat{n})$, where $(\hat{\rho}, \hat{n})$ does not randomize in the initial period and $n_1(\theta) \neq \hat{n}_1(\theta)$. Then the equilibrium labor contract randomizes in the initial period.*

Suppose $\min\{\kappa, \phi\} = 0$. *Suppose that given a mechanism (c, y, n) workers prefer (ρ^*, n)*

to any relevant labor contract which does not randomize in the initial period. Then the mechanism is incentive-compatible.

Lemma 7 is a generalization of the similar result from the static model (Lemma 3). In the truthful equilibrium the initial continuation profits are zero by the zero profit constraints. It means that as long as both the quitting cost and the firing cost are strictly positive, the initial limited commitment constraints of a firm and a worker are slack. Then the agent can improve upon any pure misreporting by randomizing initial reports and slightly reducing the variation of the initial labor supply between the reports. On the other hand, if either the quiting or the firing cost is zero then no averaging of labor supply between random reports is possible, as it would violate one of the limited commitment constraints. Without labor averaging the agent cannot gain by randomizing reports initially.

Proposition 2. *Take some incentive-feasible mechanism (c, y, n) when the quitting cost is κ and the firing cost is ϕ. There exists a labor income function $\tilde{y}$ such that $(c, \tilde{y}, n)$ is incentive-feasible when the quitting cost is κ' and the firing cost is ϕ' where $\kappa'+\phi' = \kappa+\phi$, either $\kappa - \kappa'$ or $\phi - \phi'$ is finite and $\min\{\kappa', \phi'\} \leq \min\{\kappa, \phi\}$.*

Without initial differences between agents, incentive-compatibility of the given consumption function and labor supply recommendation depends only on the *total commitment power* $\kappa + \phi$. Once we consider the initial differences and the redistributive motive of the planner, what matters is also the *minimal commitment power* $\min\{\kappa, \phi\}$. If one side of the labor market has no commitment power, by Lemma 7 the planner does not need to consider randomizing reports in the initial period. As the minimal commitment power rises, the ability of agents to average labor between random reports grows, which increases incentives for random misreporting.

An implication of Proposition 2 is that, keeping the total commitment power constant, the value of the social welfare function at the incentive-efficient mechanism increases as the minimal commitment power decreases. It is particularly clear once we compare the incentive-efficient mechanisms under the full two-sided commitment ($\kappa = \phi = \infty$) and under the full one-sided commitment ($\max\{\kappa, \phi\} = \infty, \min\{\kappa, \phi\} = 0$). By Lemma 7, in the former case the redistribution is constrained by threat of random misreporting in the initial period. In the latter case agents cannot gain by random misreporting, as no averaging of labor between reports is possible. Consequently, the relevant incentive-compatibility constraints are slacker and allow for more redistribution.

4. Labor market commitment and income taxation in US

In this section I investigate the practical relevance of the regressivity restriction due to the labor market commitment. First, I document that the US tax code violates the no-randomization constraint. The tax regressivity is caused by the phase-out of Earned Income Tax Credit and gains from randomization are potentially large. Second, I calibrate the model to match the US income distribution and compare the optimal tax schedules with and without two-sided commitment.

4.1. Actual tax schedule

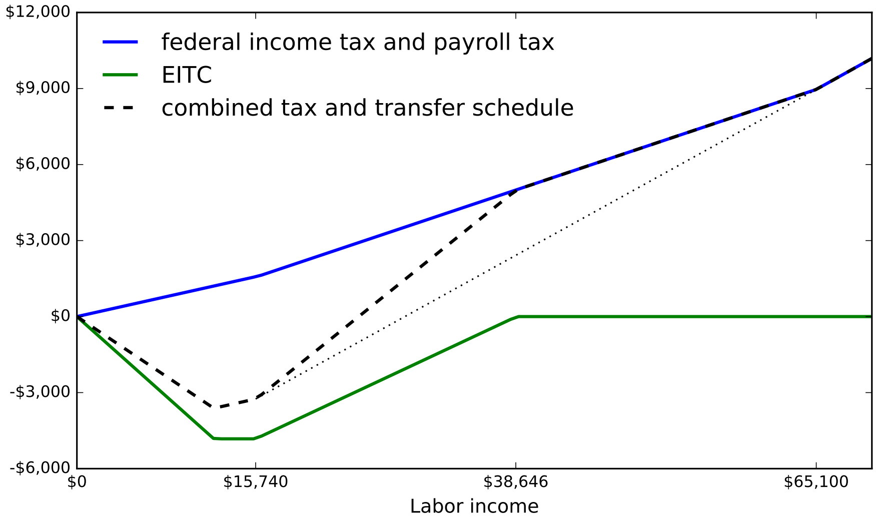
Figure 3 presents the US tax and transfer schedule faced by a jointly filling married couple with two children. For clarity, I focus on a single social transfer program: earned income tax credit (EITC), the largest cash-transfer program for low income families (Eissa and Hoynes 2004). As labor income increases EITC is first phased in at 40% rate and later phased out at 21% rate. The phase-out region ends at $38,646 where the effective marginal tax rate falls discontinuously by 21 percentage points.[19] Thus, an agent with income around $38,646 would gain by randomizing wages. In particular, a risk neutral agent could gain an equivalent to $2,563, slightly more than half of the maximal tax credit and 7.6% of the disposable income (see Table 1). Gains from randomization fall quickly with the degree of risk aversion, yet remain non-negligible: agents with a log utility could gain up to 1% of disposable income, while agents with the coefficient of relative risk aversion of 4 could gain up to 0.2%.

Given potentially large gains from randomizing wages, it is natural to ask whether it can can be observed in the data. To the best of my knowledge, this question has not been addressed in the literature. Lemieux, MacLeod, and Parent (2009) report that more than one third of jobs in PSID include a variable pay component, for instance bonuses. The difficulty, however, is in separating wage variability due to incentive reasons, such as performance pay, and wage randomization for pure tax avoidance reasons. A potential way to disentangle performance pay from wage randomization is to use changes in tax progressivity. A reduction of progressivity which does not make the tax locally regressive should affect the performance pay but not the wage randomization. Alternatively, one could focus on self-employed individuals who do not face agency problems. Furthermore, a stochastic variation of wages can be approximated by a deterministic wage path over time. For instance, instead of randomizing with equal probability between a high wage and a low wage over two periods, a firm could offer the low wage initially and the high wage afterwards. Thus, the wage randomization could be disguised as the tenure effect on wages. In this case one could check whether the tenure effect is stronger at income levels where the tax schedule is regressive.

There are at least two reasons why wage randomization may be hard to find in the data.

[19] EITC creates similar tax regressivity for households with different structure as well, although the income thresholds and the size of the tax credit differ. Other social programs are unlikely to reduce the regressivity of the tax and transfer schedule. Tax child credit phases out above $110,000 and does not affect the effective marginal tax rates in the considered income range. Food stamps for a household of four in 2008 were phased out at $27,560 which would make the schedule even more regressive.

Figure 3: Tax regressivity due to Earned Income Tax Credit



The tax and transfer schedules are for an employed married couple with at least two children, filling jointly in 2008 (source: Saez (2010) and the IRS website). The tax and transfer schedule is regressive at $38,645. The dotted line shows how the wage randomization of the risk neutral worker effectively convexifies (i.e. makes progressive) the tax and transfer schedule.

First, as Table 1 indicates, large gains from randomization require income variation in the range of thousands of dollars annually. It may be the case that either the workers around the phase-out region of EITC or their employers cannot credibly accept a wage variation of such magnitude. Lagakos and Ordonez (2011) note that both workers and firms can commit more in industries where firm-specific human capital is important. As such industries tend to employ more skilled workers, the wage randomization may become relevant only at higher income levels. Second, agents may face optimization frictions preventing them from choosing the compensation structure optimally. Chetty (2012) finds that the adjustment cost of 1% of after-tax earnings can reconcile the widespread estimates of Hicksian elasticity of labor supply. Optimization frictions of the same magnitude would explain a lack of wage randomization around the phase-out region of EITC for the coefficient of relative risk aversion of one or higher.

4.2. Optimal tax schedules

This subsection examines how the labor market commitment affects the optimal tax schedule in the model economy calibrated to match the US income distribution. I will compare the optimal allocation in the static model under two-sided commitment ($\min\{\kappa, \phi\} > 0$) with the optimal allocation when one of the parties has no commitment power ($\min\{\kappa, \phi\} = 0$). The allocation under no commitment is also the optimum of the standard Mirrlees model.

Agents have utility from consumption $u(c) = c^{1-\sigma}/(1-\sigma)$ with a constant coefficient of relative risk aversion σ and a disutility from labor $n^{1+1/\varepsilon}/(1+1/\varepsilon)$. When agents are

Table 1: Gains from wage randomization around the phase-out region of EITC

	coefficient of relative risk aversion				
	$\sigma = 0$	$\sigma = 1$	$\sigma = 2$	$\sigma = 3$	$\sigma = 4$
gain, absolute	\$2,563	\$334	\$163	\$106	\$78
gain, relative	7.6%	1%	0.5%	0.3%	0.2%
randomization	+\$26,454; -\$22,906	±\$6,507	±\$3,253	±\$2,152	±\$1,627

For risk averse agents, the table reports an income lottery with two equally likely outcomes $\$38,645 \pm \varepsilon$, where ϵ is chosen to maximize gains from randomization. For risk neutral agents, the table reports the optimal income randomization which is asymmetric. Absolute (relative) gains are expressed as a absolute (proportional) increase of consumption of an agent who does not randomize required to bring his utility to the level of the randomizing agent. Monetary variables are reported in 2008 dollars.

risk neutral the parameter ε captures the intensive margin labor supply elasticity and, following Chetty (2012), I set it to 0.33. I use the actual tax and transfer schedule of Heathcote, Storesletten, and Violante (2017). The income distribution implied by the model under the empirical tax schedule matches the cross-sectional income distribution in the US. Specifically, incomes up to \$150,000 are distributed lognormally, while higher incomes have are distributed according to the pattern of the income hazard ratio from Diamond and Saez (2011) and feature a Pareto tail above \$475,000 (see details in Online Appendix B). The distribution of productivities is then extracted from the empirical income distribution and the empirical tax schedule by applying the agents' first-order condition as in Saez (2001).

I compute the optimal tax schedules for two social welfare functions: Rawlsian, which places the entire welfare weight on the least productive agent ($\tilde{\mu}_\Theta(\{\min \Theta\}) = 1$), and Utilitarian, which weights utilities of all agents equally ($\tilde{\mu}_\Theta = \mu_\Theta$). The optimal tax rates under two-sided commitment are computed with the algorithm described in Online Appendix B. Computed tax schedules generate the same net tax revenue as the empirical tax schedule.

Rawlsian planner. The two-sided commitment imposes a regressivity restriction which is particularly severe when agents are risk neutral. When additionally the optimal marginal tax rates under no commitment are U-shaped, as argued by Diamond (1998) and Saez (2001), and the social planner is Rawlsian, then the optimal tax schedule under two-sided commitment becomes fully linear.

Proposition 3. *Suppose that (i) agents are risk neutral: $\sigma = 0$, (ii) the planner is Rawlsian, (iii) the optimal marginal tax rates without commitment are U-shaped: $T'(y)$ is quasi-convex with local maximum at 0 and a local minimum at some positive, finite income, (iv) the mean income is finite. Then the optimal tax under two-sided commitment is linear.*

Proof. Under the assumptions made the no-randomization constraint implies that the marginal tax rates cannot fall with income. Since without commitment the tax rates first fall and then rise, under two-sided commitment the tax rates will be either constant everywhere or constant until a certain income level and then rise. The former case happens when the optimal linear tax rate is weakly greater than the optimal top tax rate without commitment, i.e. when (see e.g. Piketty and Saez (2013))

$$\frac{1}{1+\varepsilon} \geq \frac{1}{1+\varepsilon a} \Leftrightarrow a \geq 1, \tag{23}$$

where a is the tail parameter of the income distribution. Note that when the mean income is finite the tail parameter of income distribution is greater than 1. ∎

The proposition is illustrated on Figure 4. For the US economy, where the optimal tax rates under no commitment are U-shaped and the mean income is finite, the optimal Rawlsian tax schedule with risk neutral agents under two-sided commitment is identical to the optimal *linear* tax schedule (Sheshinski 1972; Piketty and Saez 2013). Therefore, in this particular case, the two-sided commitment eliminates all gains from nonlinear taxation. In the counterfactual case, when the top income tail is so thick that the average income in the economy is infinite, the the top tax rate without commitment is elevated above the optimal linear rate. Then the optimal tax rates under two-sided commitment are constant until a certain income level, from which they converge upwards to the top tax rate under no commitment.

When agents are risk averse, the no-randomization constraint allows a certain degree of local tax regressivity. As a result, the U shape of tax rates under no commitment is less pronounced, but not fully eliminated under two-sided commitment (see Figure 5). The main difference with respect to the schedule under no commitment is that the marginal tax rates are substantially lower at very low income levels. For instance when $\sigma = 1$ the tax rates at lowest incomes are reduced by more than 15 percentage points. If they were any higher, the tax rates at middle incomes would be elevated, increasing tax distortions where the density of income is high. The impact of two-sided commitment on the shape of the tax schedule is weakening with the coefficient of the relative risk aversion, which happens for two reasons. First, more risk averse agents require a more regressive tax to have incentives to randomize incomes, which relaxes the no-randomization constraint. Second, when agents are more risk averse, stronger income effects lift tax rates at all income levels (Saez 2001). Consequently, the U shape is flattened and the regressivity of the tax schedule at low income levels is reduced. The regressivity restriction implied by the no-randomization constraint is therefore less binding. The welfare loss due to two-sided commitment is quickly decreasing with the coefficient of the relative risk aversion σ from a large 2.6% for risk neutral agents to a modest 0.1% for $\sigma = 3$ (see Table 2).

Table 2: Welfare loss from two-sided commitment

	coefficient of relative risk aversion				
	$\sigma = 0$	$\sigma = 1$	$\sigma = 2$	$\sigma = 3$	$\sigma = 4$
Rawlsian	2.6%	0.9%	0.3%	0.1%	0.1%
Utilitarian	0.0%	0.0%	0.0%	0.0%	0.0%

The welfare loss is expressed as a proportional consumption change required to reach the value of the social welfare function under no commitment.

Utilitarian planner. The tax rates under no commitment are U-shaped when the planner is Utilitarian, but the income interval where the tax rates are decreasing is smaller relative to the Rawlsian case (see Figure 6).[20] As a result, the impact of the two-sided commitment on the marginal tax rates is confined to the lowest incomes. Although the interval of tax regressivity is small, the rates decrease there very rapidly and the impact of the no-randomization constraint is powerful: when $\sigma = 1$, the marginal tax rates at lowest income levels fall by more than 40 percentage points. The no-randomization constraint eliminates the U shape almost entirely, leading to the optimal tax schedule that is close to being progressive everywhere. However, since the two-sided commitment affects the schedule of the marginal tax rates only at the lowest incomes where density is very low, its social welfare cost is negligible (see Table 2).

5. Conclusions

This paper shows that the labor market commitment on the one hand allows firms to insure their workers, but on the other it may enable wage randomization which bounds tax regressivity and restricts redistribution. The regressivity restriction, governed by the novel no-randomization constraint, has a powerful impact on the shape of the optimal tax schedule, reducing the marginal tax rates at low income levels by up to 40 percentage points. The threat of wage randomization is relevant also in other models featuring two-sided commitment, regardless of additional agency or search frictions. Hence, the no-randomization constraint can be used to test whether the common assumption of deterministic wages is valid in these settings.

Policy recommendations which can be drawn from these results are two-fold. First, a tax and transfer schedule should not be excessively regressive as otherwise workers will inefficiently expose themselves to income risk to avoid taxes. Second, the labor market commitment is in general endogenous and depends on the labor market institutions such as unemployment insurance, regulations of non-compete clauses, firing restrictions and mandated severance packages. According to the results of this paper, a labor market

[20] I do not plot the Utilitarian tax schedule for risk neutral agents, since then the planner has no redistributive objective and the optimal tax rates are constant at zero.

Figure 4: Optimal Rawlsian tax schedule with risk neutral agents

(a) US calibration (b) Calibration with the infinite mean income

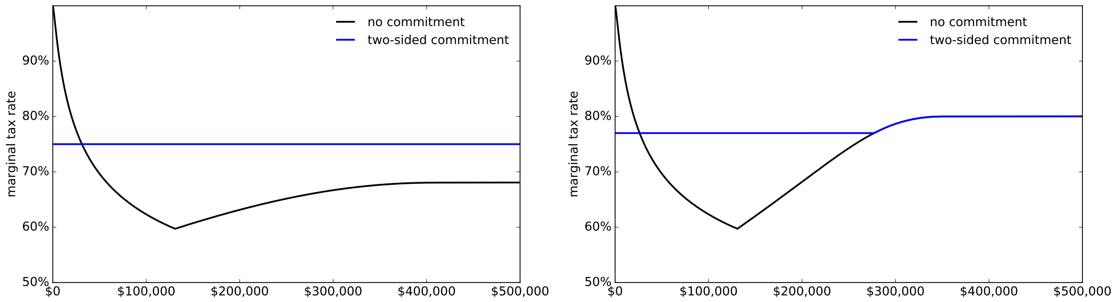


Figure 5: Optimal Rawlsian tax schedule with risk averse agents, US calibration

(a) $\sigma = 1$ (b) $\sigma = 3$

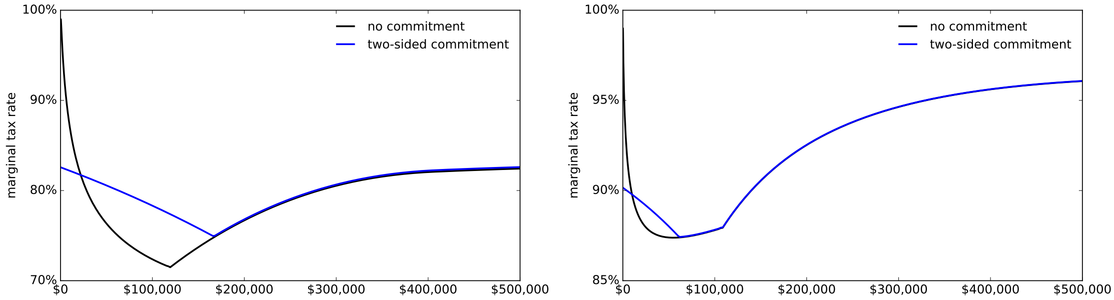
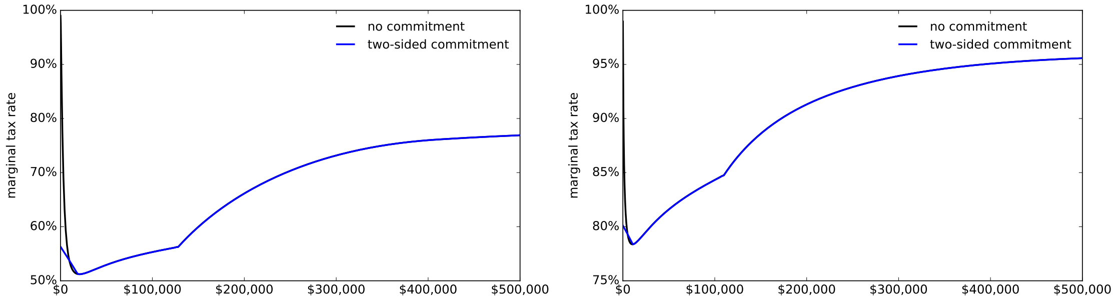


Figure 6: Optimal Utilitarian tax schedule with risk averse agents, US calibration

(a) $\sigma = 1$ (b) $\sigma = 3$



reform which promotes workers' mobility while strengthening the commitment power of firms could allow for more redistribution via the income tax without sacrificing insurance within firms.

References

Ábrahám, Á., S. Koehne, and N. Pavoni (2016): "Optimal income taxation when asset taxation is limited," *Journal of Public Economics*, 136, 14–29.

Attanasio, O., and J.-V. Rios-Rull (2000): "Consumption smoothing in island economies: Can public insurance reduce welfare?," *European Economic Review*, 44(7), 1225–1258.

Azariadis, C. (1975): "Implicit Contracts and Underemployment Equilibria," *Journal of Political Economy*, 83(6), 1183–1202.

Baily, M. N. (1974): "Wages and employment under uncertain demand," *The Review of Economic Studies*, 41(1), 37–50.

Brandt, L., and A. J. Hosios (1996): "Credit, Incentives, and Reputation: A Hedonic Analysis of Contractual Wage Profiles," *Journal of Political Economy*, pp. 1172–1226.

Burdett, K., and M. Coles (2003): "Equilibrium Wage-Tenure Contracts," *Econometrica*, 71(5), 1377–1404.

Burdett, K., and D. T. Mortensen (1998): "Wage differentials, employer size, and unemployment," *International Economic Review*, pp. 257–273.

Bureau of Labor Statistics (2018): "Employee Tenure in 2018," *Economic News Releases*.

Chang, Y., and Y. Park (2017): "Optimal Taxation with Private Insurance," Discussion paper.

Chetty, R. (2012): "Bounds on elasticities with optimization frictions: A synthesis of micro and macro evidence on labor supply," *Econometrica*, 80(3), 969–1018.

Chetty, R., and E. Saez (2010): "Optimal Taxation and Social Insurance with Endogenous Private Insurance," *American Economic Journal. Economic Policy*, 2(2), 85.

da Costa, C. E., and L. J. Maestri (2017): "Optimal Mirrleesian Taxation in Noncompetitive Labor Markets," .

Diamond, P., and E. Saez (2011): "The case for a progressive tax: from basic research to policy recommendations," *Journal of Economic Perspectives*, 25(4), 165–90.

Diamond, P. A. (1998): "Optimal income taxation: an example with a U-shaped pattern of optimal marginal tax rates," *American Economic Review*, pp. 83–95.

Eissa, N., and H. W. Hoynes (2004): "Taxes and the labor market participation of married couples: the earned income tax credit," *Journal of Public Economics*, 88(9-10), 1931–1958.

Farhi, E., and I. Werning (2013): "Insurance and taxation over the life cycle," *The Review of Economic Studies*, 80(2), 596–635.

Fudenberg, D., and J. Tirole (1991): "Game theory, 1991," *Cambridge, Massachusetts*, 393(12), 80.

Golosov, M., N. Kocherlakota, and A. Tsyvinski (2003): "Optimal Indirect and Capital Taxation," *Review of Economic Studies*, pp. 569–587.

Golosov, M., P. Maziero, and G. Menzio (2013): "Taxation and Redistribution of Residual Income Inequality," *Journal of Political Economy*, 121(6).

Golosov, M., M. Troshkin, and A. Tsyvinski (2016): "Redistribution and Social Insurance," *American Economic Review*, 106(2), 359–386.

Golosov, M., and A. Tsyvinski (2007): "Optimal Taxation with Endogenous Insurance Markets," *The Quarterly Journal of Economics*, 122(2), 487–534.

Golosov, M., A. Tsyvinski, and I. Werning (2007): "New Dynamic Public Finance: A User's Guide," *NBER Chapters*, pp. 317–388.

Goolsbee, A. (2000): "What Happens When You Tax the Rich? Evidence from Executive Compensation," *Journal of Political Economy*, 108(2), 352–378.

Guiso, L., L. Pistaferri, and F. Schivardi (2005): "Insurance within the Firm," *Journal of Political Economy*, 113(5).

——— (2012): "Credit within the Firm," *Review of Economic Studies*, 80(1), 211–247.

Hall, B. J., and K. J. Murphy (2003): "The Trouble with Stock Options," *Journal of Economic Perspectives*, 17(3), 49–70.

Harris, M., and B. Holmstrom (1982): "A theory of wage dynamics," *The Review of Economic Studies*, 49(3), 315–333.

Heathcote, J., K. Storesletten, and G. L. Violante (2017): "Optimal tax progressivity: An analytical framework," *The Quarterly Journal of Economics*, 132(4), 1693–1754.

Hungerbühler, M., E. Lehmann, A. Parmentier, and B. Van der Linden (2006): "Optimal redistributive taxation in a search equilibrium model," *The Review of Economic Studies*, 73(3), 743–767.

Kocherlakota, N. R. (2010): *The New Dynamic Public Finance*. Princeton University Press.

Kreiner, C. T., S. Leth-Petersen, and P. E. Skov (2014): "Year-End Tax Planning of Top Management: Evidence from High-Frequency Payroll Data," *The American Economic Review*, 104(5), 154–158.

——— (2016): "Tax reforms and intertemporal shifting of wage income: Evidence from Danish monthly payroll records," *American Economic Journal: Economic Policy*, 8(3), 233–57.

Krueger, D., and F. Perri (2011): "Public versus private risk sharing," *Journal of Economic Theory*, 146(3), 920–956.

Krueger, D., and H. Uhlig (2006): "Competitive risk sharing contracts with one-sided commitment," *Journal of Monetary Economics*, 53(7), 1661–1691.

Lagakos, D., and G. L. Ordonez (2011): "Which workers get insurance within the firm?," *Journal of Monetary Economics*, 58(6), 632–645.

Lemieux, T., W. B. MacLeod, and D. Parent (2009): "Performance pay and wage inequality," *The Quarterly Journal of Economics*, 124(1), 1–49.

Michelacci, C., and V. Quadrini (2005): "Borrowing from Employees: Wage Dynamics with Financial Constraints," *Journal of the European Economic Association*, pp. 360–369.

——— (2009): "Financial Markets and Wages," *The Review of Economic Studies*, pp. 795–827.

Milgrom, P., and I. Segal (2002): "Envelope theorems for arbitrary choice sets," *Econometrica*, 70(2), 583–601.

Mirrlees, J. A. (1971): "An exploration in the theory of optimum income taxation," *The review of economic studies*, pp. 175–208.

Moen, E. R. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105(2), 385–411.

Murphy, K. J. (1999): "Executive compensation," *Handbook of Labor Economics*, 3, 2485–2563.

Myerson, R. (1979): "Incentive Compatibility and the Bargaining Problem," *Econometrica*, 47(1), 61–73.

Myerson, R. B. (1982): "Optimal coordination mechanisms in generalized principal-agent problems," *Journal of Mathematical Economics*, 10(1), 67–81.

Piketty, T., and E. Saez (2013): "Optimal Labor Income Taxation," in *Handbook of Public Economics*, vol. 5, pp. 391–474. Elsevier.

Saez, E. (2001): "Using Elasticities to Derive Optimal Income Tax Rates," *Review of Economic Studies*, 68(1), 205–29.

——— (2010): "Do Taxpayers Bunch at Kink Points?," *American Economic Journal: Economic Policy*, pp. 180–212.

Sheshinski, E. (1972): "The optimal linear income-tax," *The Review of Economic Studies*, 39(3), 297–302.

Stantcheva, S. (2014): "Optimal Income Taxation with Adverse Selection in the Labour Market," *The Review of Economic Studies*, p. rdu005.

Stiglitz, J. E. (1982): "Self-selection and Pareto efficient taxation," *Journal of Public Economics*, 17(2), 213–240.

Thomas, J., and T. Worrall (1988): "Self-enforcing wage contracts," *The Review of Economic Studies*, 55(4), 541–554.

Varian, H. R. (1980): "Redistributive taxation as social insurance," *Journal of Public Economics*, 14(1), 49–68.

Werquin, N. (2016): "Income taxation with frictional labor supply," Discussion paper.

Yazici, H., and C. Sleet (2017): "Taxation, Redistribution and Frictional Labor Supply," Discussion paper.

A. Proofs from Section 2

Proof of Corollary 1. Suppose y is discontinuous. From the standard screening problem we know that any incentive-compatible y is increasing (e.g. Theorem 7.2 in Fudenberg and Tirole (1991)). Hence, the set of discontinuity points of y is countable, all discontinuities are jumps and y has directional limits everywhere. Suppose that there is a discontinuity at θ: $y(\theta) > \lim_{\theta'\uparrow\theta} y(\theta')$. Note that $U(c(\theta), y(\theta)/\theta) = \lim_{\theta'\uparrow\theta} U(c(\theta'), y(\theta')/\theta)$, since otherwise either θ or a marginally lower type would prefer to misreport. By the reasoning of Lemma 3, for sufficiently small $\theta-\theta' > 0$ type θ prefers to randomize reports between θ and θ', which contradicts incentive-compatibility. ■

Proof of Claim 1. By the local incentive-compatibility constraint we have

$$v'\left(\frac{y(\theta)}{\theta}\right)/\theta = \frac{c'(\theta)}{y'(\theta)}u'(c(\theta)) = \left(1-\mathcal{T}'(y(\theta)\right)u'\left(y(\theta)-\mathcal{T}(y(\theta)\right). \tag{24}$$

Then $v'\left(\frac{y(\theta)}{\theta}\right)/\theta$ is decreasing when $d\left[v'\left(\frac{y(\theta)}{\theta}\right)/\theta\right]/d\theta \leq 0$, that is when

$$y'(\theta)\left[\left(1-\mathcal{T}'(y(\theta)\right)^2 u'\left(y(\theta)-\mathcal{T}(y(\theta))\right) - \mathcal{T}''\left(y(\theta)\right)u'\left(y(\theta)-\mathcal{T}(y(\theta))\right)\right] \leq 0. \tag{25}$$

Note that $y'(\theta) > 0$ is implied by the differentiability of $\mathcal{T}$. ■

B. Proofs from Section 3

Proof of Lemma 5. Consider a deterministic mechanism (c, y, n) with a labor income function y and an associated equilibrium (ρ, n). Take any $(\theta^T, r_m^T) \in \Theta^T \times R_m^T$ and consider its partial history (θ^t, r_m^t) for $t < T$. Then

$$\underbrace{\mathbb{E}_\rho\left\{\pi_t(y,n,\theta^T) \mid \theta^t, r_m^t\right\}}_{=0 \text{ by } \mathrm{LCC}_d} = \theta_t^T n_t(\theta^T, r_m^T) - y_t(m(\theta^T, r_m^T)) + \beta\, \underbrace{\mathbb{E}_\rho\left\{\pi_{t+1}(y,n,\theta^T) \mid \theta^t, r_m^t\right\}}_{=0 \text{ by } \mathrm{LCC}_d}. \tag{26}$$

By LCC_d the left-hand side is zero. The second term on the right-hand side is zero by LCC_d as well, since it can be rewritten as $\mathbb{E}_\rho\left\{\mathbb{E}_\rho\left\{\pi_{t+1}(y,n,\theta^T) \mid \theta^{t+1}, r_m^{t+1}\right\} \mid \theta^t, r_m^t\right\}$. This proves the claim for non-terminal periods. For the terminal period note that there is no second term on the right-hand side. ■

Proof of Lemma 6. Given a mechanism (c, y, n) and a reporting strategy $\hat\rho$, the labor strategy solves

$$\min_{\hat n} \mathbb{E}_{\hat\rho}\left\{\sum_{t=1}^T \beta^{t-1} v(\hat n_t)\right\} \text{ s.t. } \mathbb{E}_{\hat\rho}\left\{\sum_{t=1}^T \beta^{t-1}(\theta_t^T \hat n_t - y_t) \mid \theta_1^T = \theta\right\} = 0 \text{ for all } \theta \in \Theta. \tag{27}$$

This is a convex minimization problem. The first-order conditions are

$$\frac{v'(\hat n_t)}{\theta_t^T} = \lambda(\theta_1^T) \text{ for all } t \leq T \text{ and all } (\theta^T, r_m^T) \in \Theta^T \times R_m^T, \tag{28}$$

where $\lambda(\theta)$ is a Lagrange multiplier of ZPC_d of the initial type θ.[21] It is easy to show that the multiplier for each initial type is unique.[22] The equilibrium labor supply depends only on the current productivity and the value of the Lagrange multiplier. The multiplier in turn depends only on the initial productivity θ_1^T, which determines the productivity process faced by the agent at the moment of signing the contract, and the expected lifetime income of the agent with the initial productivity θ_1^T. ■

Proof of Theorem 2. Consider some incentive-feasible mechanism $\Psi = (c, y, n)$ which does not involve full insurance. Construct a new mechanism $\Psi' = (c', y', n)$ in the following way. First, for each initial productivity $\theta \in \Theta$ find a constant consumption level $\bar c(\theta)$ which yields the same utility from consumption as the original mechanism:

$$\sum_{t=1}^T \beta^{t-1} u(\bar c(\theta)) = \mathbb{E}_{\rho^*}\left\{\sum_{t=1}^T \beta^{t-1} u(c_t) \mid \theta_1^T = \theta\right\}. \tag{30}$$

Set c' equal $\bar c(\theta)$ at all periods and for all histories θ^T such that $\theta_1^T = \theta$. Second, choose income function y' equal to the expected income at each time period, given the initial

[21] When corner solutions are possible, the optimality condition becomes $\min\{\lambda(\theta_t^T) - v'(n_t), n_t\} = 0$.

[22] Expressing labor supply at each partial history with (28) and plugging into the zero profit constraint we get

$$\mathbb{E}_{\hat\rho}\left\{\sum_{t=1}^T \beta^{t-1}\left(\theta_t^T v'^{-1}\left(\theta_t^T \lambda(\theta_1^T)\right) - y_t\right) \mid \theta_1^T = \theta\right\} = 0 \text{ for all } \theta \in \Theta. \tag{29}$$

Since the left-hand side is strictly increasing with the multiplier, the multiplier is unique.

productivity:

$$y'_t(\theta^T) = \mathbb{E}_{\rho^*}\left\{y_t \mid \theta_1^T = \theta\right\} \text{ for all } t \leq T \text{ and all } \theta^T \in \Theta^T. \tag{31}$$

By Lemma 6, keeping the reporting strategy fixed, the agent will choose the same labor strategy in mechanisms Ψ and Ψ', since the expected lifetime incomes of initial productivity types are unchanged. Consequently, the two mechanisms evaluated at the truthful reporting strategy yield the same expected utility: $\mathbb{E}_{\rho^*}\{U(c', n)\} = \mathbb{E}_{\rho^*}\{U(c, n)\}$. Since the agents are risk averse, additional insurance they receive in Ψ' frees some resources to the planner.

I will show that the mechanism Ψ' is incentive-compatible. Suppose that there are strategies (ρ', n') which satisfy ZPC with respect to Ψ' and give higher expected utility than truthtelling: $\mathbb{E}_{\rho'}\{U(c', n')\} > \mathbb{E}_{\rho^*}\{U(c', n)\}$. Construct a reporting strategy ρ such that the first period reports are the same as the first period reports of ρ'. Suppose that the realized first period report was $\hat{\theta}$. Reports implied by ρ in consecutive periods have the same distribution as the distribution of productivity histories with initial productivity equal $\hat{\theta}$.[23] Conditional on the initial productivity and the initial report, (Ψ, ρ) and (Ψ', ρ') lead to (i) the same expected lifetime utility from consumption (by construction of c') and (ii) the same expected lifetime disutility from labor (by Lemma 6, since the expected lifetime income is the same in the two cases). Since the distribution of initial reports is identical in ρ and ρ', the unconditional expected utility given (Ψ, ρ) and (Ψ', ρ') coincide as well: $\mathbb{E}_{\rho}\{U(c, n')\} = \mathbb{E}_{\rho'}\{U(c', n')\}$. Since $\mathbb{E}_{\rho}\{U(c, n')\} > \mathbb{E}_{\rho^*}\{U(c', n)\} = \mathbb{E}_{\rho^*}\{U(c, n)\}$, the original mechanism is not incentive-compatible which is a contradiction. ∎

Proposition B.1. *Suppose $\kappa = \phi = \infty$. There is an incentive-efficient mechanism (c, y, n) corresponding to the probability measure $\tilde{\mu}_\Theta$ such that $c(\theta_t^T) = \bar{c}(\theta_1^T)$ and $y(\theta_t^T) = \bar{y}(\theta_1^T)$ for all $t \leq T$, where $\bar{c}(\cdot)$ and $\bar{y}(\cdot)$ solve*

$$\max_{\substack{c:\Theta \to \mathbb{R}_+ \\ y:\Theta \to \mathbb{R}_+}} \tilde{\mathbb{E}}\{u(c(\theta_1^T) - \tilde{v}(y(\theta_1^T), \theta_1^T)\} \tag{32}$$

subject to the resource constraint

$$\mathbb{E}\{y(\theta_1^T) - c(\theta_1^T)\} \geq 0 \tag{33}$$

[23] Formal definition is as follows. Suppose that $\rho' = ((R_{m'}, \mathcal{B}_{R_{m'}}, \mu_{R_{m'}}), m')$. Fix initial type θ. Define σ_θ as a push-forward measure on $(\Theta, \mathcal{B}_\Theta)$ defined by a map $m'_1(\theta, \cdot)$. Define $\mu_{\Theta^T|\theta}$ as a probability measure over productivity histories conditional on initial productivity θ. Now define a probability measure $\tilde{\mu}_\theta$ over $(\Theta^T, \mathcal{B}_{\Theta^T})$ as $\tilde{\mu}_\theta(A \times B) = \sigma_\theta(A) \int_A \mu_{\Theta^{T-1}|a}(B) da$, $A \in \mathcal{B}_\Theta$ and $B \in \mathcal{B}_{\Theta^{T-1}}$. For each θ we have a probability space $(\Theta^T, \mathcal{B}_{\Theta^T}, \tilde{\mu}_\theta)$. Denote the Cartesian product of all such probability spaces as $(R_m, \mathcal{B}_{R_m}, \mu_{R_m})$. Finally, define ρ as $((R_m, \mathcal{B}_{R_m}, \mu_{R_m}), m)$ where $m(\theta^T, r_m^T) = r_{m1}^T(\theta_1^T)$.

and the incentive-compatibility constraints in mixed reporting strategies

$$u(c(\theta')) - u\left(c\left(\theta\right)\right) - \big(y(\theta') - y\left(\theta\right)\big)\, \tilde{v}_y(y(\theta),\theta) \geq 0 \textit{ for all } \theta \textit{ and } \theta'. \tag{34}$$

Proof of Proposition B.1. Define the average disutility from labor of an initial type θ with average income $\bar{y}$ as

$$\tilde{v}(\bar{y},\theta) \equiv \min_{n:\Theta^T \to \mathbb{R}_+^T} \mathbb{E}\left\{\sum_{t=1}^T \beta^{t-1} v\left(n_t\right) \mid \theta\right\} \Big/ \sum_{t=1}^T \beta^{t-1} \text{ s.t. } \mathbb{E}\left\{\sum_{t=1}^T \beta^{t-1}\theta_t n_t \mid \theta\right\} = \sum_{t=1}^T \beta^{t-1}\bar{y}. \tag{35}$$

First I'll prove the postulated properties of function $\tilde{v}(\cdot,\theta)$. Clearly, it is strictly increasing. To see that it is strictly convex, consider two average income levels y_1 and y_2, $y_1 \neq y_2$. Denote by n_1 and n_2 the labor strategies consistent with Lemma 6 which generate these an average incomes. For some $\sigma \in (0,1)$ and all θ^T define $\overline{n}(\theta^T) = \sigma n_1(\theta^T) + (1-\sigma)n_2(\theta^T)$, which generates average income $\sigma y_1 + (1-\sigma)y_2$. Note that due to strict convexity of v the disutility from following $\overline{n}$ is strictly lower than $\sigma\tilde{v}\left(y_1,\theta\right) + (1-\sigma)\tilde{v}\left(y_2,\theta\right)$ for any θ, which proves the strict convexity of $v(\tilde{\cdot},\theta)$. By Corollary 5 from Milgrom and Segal (2002), directional derivatives of $\tilde{v}(\cdot,\theta)$ are equal to the maximal and minimal Lagrange multiplier of the zero profit constraint. In the proof of Lemma 6 I show that this multiplier is unique, which implies that $\tilde{v}(\cdot,\theta)$ is differentiable.

Consider some incentive-efficient mechanism $(c,\tilde{y},n)$. By Theorem 2, this mechanism prescribes constant consumption and a deterministic labor income which, conditional on initial productivity, is independent of subsequent productivity realizations. By Lemma 6 we can replace the labor income function $\tilde{y}$ with y which has the same average income of all initial types, yet, given an initial type report, prescribes a constant labor income: $y_t(\theta^T) = \sum_{t=1}^T \beta^{t-1}\tilde{y}(\theta^T)/\sum_{t=1}^T$. Then the expected lifetime utility of initial type θ is

$$\mathbb{E}\{U(c,n) \mid \theta_1^T = \theta\} = \sum_{t=1}^T \beta^{t-1}(u(c(\theta)) - \tilde{v}(y(\theta),\theta)). \tag{36}$$

The resource constraint combined with ZPC can be expressed as

$$\mathbb{E}\left\{\sum_{t=1}^T \beta^{t-1}\left(\theta_t^T n_t - c_t\right)\right\} = \sum_{t=1}^T \beta^{t-1}\mathbb{E}\{y(\theta_1^T) - c(\theta_1^T)\} \geq 0. \tag{37}$$

Divide the expected utility and the resource constraint by a positive constant $\sum_{t=1}^T \beta^{t-1}$ to express them as in the statement of the lemma.

What remains to be shown is that the incentive-compatibility constraint can be expressed as claimed. By Theorem 2 only initial type report matters and, as is shown in the more general case in Lemma 7, only mixed reporting strategies are relevant. The form of the incentive-compatibility constraint then follows from the reasoning analogous to that in

Lemma 4. ■

Proof of Claim 2. By Lemma 6 and Proposition B.1 we know that $\tilde{v}_y(y,\theta)=\lambda(\theta)$, where $\lambda(\theta)$ is pinned down by

$$\sum_{t=1}^{T}\beta^{t-1}\mathbb{E}\left\{\theta_t^T v'^{-1}\left(\theta_t^T\lambda(\theta_1^T)\right)\mid\theta_1^T=\theta\right\}=\sum_{t=1}^{T}\beta^{t-1}y. \tag{38}$$

Increase initial productivity, while keeping $\lambda(\theta_1^T)$ constant. Since the function $\theta_t^T v'^{-1}\left(\theta_t^T\lambda(\theta_1^T)\right)$ is increasing in θ_t^T and the distribution of productivities is shifted upwards according to FOSD, the left-hand side weakly rises. In order for equality to hold, the multiplier needs to be weakly reduced. It means that $\tilde{v}_y(y,\theta)$ is nonincreasing in θ. ■

Proof of Theorem 3. The proof is a generalization of the proof of Theorem 1: simply substitute $\tilde{v}(y,\theta)$ for $v(y/\theta)$, $\tilde{v}_y(y,\theta)$ for $v'(y/\theta)/\theta$ and $\tilde{v}_\theta(y,\theta)$ for $-v(y/\theta)y/\theta^2$. The assumption of absolute continuity is required to obtain the integral representation of $\tilde{v}(y(\theta'),\theta')-\tilde{v}(y(\theta),\theta))$. The single-crossing assumption is required for the necessity of income monotonicity and for the implication $\theta'>\theta\wedge v_y(y(\theta),\theta)>v_y(y(\theta'),\theta')\implies y(\theta')-y(\theta)>0$. ■

Proof of Proposition 1. Set $\Delta=\kappa-\kappa'$ if $\kappa-\kappa'$ is finite or $\Delta=\phi-\phi'$ otherwise. Construct a new income function $\tilde{y}$ in the following way: $\tilde{y}_T(\theta^T)=y_T(\theta^T)+\Delta$, $\tilde{y}_t(\theta^T)=y_t(\theta^T)+(1-\beta)\Delta$ for $T>t>1$ and $\tilde{y}_1(\theta^T)=y_1(\theta^T)-\beta\Delta$ for all θ^T. Then we have $\pi_t(y,n,\theta^T)=\pi_t(\tilde{y},n,\theta^T)+\Delta$ for $t\geq 2$ and $\pi_1(y,n,\theta^T)=\pi_1(\tilde{y},n,\theta^T)$ for all θ^T.

Consider a new mechanism $(c,\tilde{y},n)$. I will show that any strategies $(\hat{\rho},\hat{n})$ satisfy ZPC_d and LCC_d with mechanism (c,y,n) and costs (κ,ϕ) if and only if they satisfy ZPC_d and LCC_d with mechanism $(c,\tilde{y},n)$ and cost (κ',ϕ'). Regarding ZPC_d, it follows from the equality of the continuation profits in the initial period. Regarding LCC_d, for any histories θ^t and $\hat{r}_m^t$ of length at least 2 ($t\geq 2$) we have

$$\kappa\geq\mathbb{E}_{\hat{\rho}}\{\pi_t(y,\hat{n},\theta^T)\mid\theta^t,\hat{r}_m^t\}\geq-\phi \tag{39}$$

which can also be expressed as

$$\kappa'+\Delta\geq\mathbb{E}_{\hat{\rho}}\{\pi_t(\tilde{y},\hat{n},\theta^T)\mid\theta^t,\hat{r}_m^t\}+\Delta\geq=-\phi'+\Delta. \tag{40}$$

Subtract Δ from all sides to recover LCC_d under the new mechanism. Under Assumption 1 we can focus on reporting strategies that are truthful in the initial period. Then ZPC_d implies that LCC_d hold with κ' and ϕ' also in the initial period.

Note that any $(\hat{\rho},\hat{n})$ implies exactly the same expected utility with (c,y,n) as with $(c,\tilde{y},n)$. Since the original mechanism is incentive-compatible, so is $(c,\tilde{y},n)$. The feasibility of $(c,\tilde{y},n)$ follows from the feasibility of (c,y,n) and ZPC_d. ■

Proof of Corollary 2. First I will show that (c, y, n) is incentive-efficient for some quitting and firing costs (κ, ϕ) where $\max\{\kappa, \phi\} = \infty$ and $\min\{\kappa, \phi\} < \infty$. By Lemma 6 we can focus on labor income functions prescribing a constant labor income which depends only on the initial report: $y_t(\theta^T) = y_1(\theta^T)$ for all $t \leq T$ and all θ^T. The boundedness of Θ and the Inada condition $v'(\infty) = \infty$ ensure that output $\theta_t^T n_t(\theta^T)$ is bounded for all $t \leq T$ and all θ^T. Then the labor income $y_t(\theta^T)$, equal to the average output, is bounded as well. Thus, for any reporting strategy $\hat{\rho}$ and the corresponding labor strategy $\hat{n}$ consistent with Lemma 6 the expected continuation profits $\mathbb{E}_{\hat{\rho}}\{\pi_t \mid \theta^t, \hat{r}_m^t\}$ are bounded for all $t \leq T$ and all $(\theta^T, \hat{r}_m^T)$ - denote the upper bound by $\bar{\pi}$ and the lower bound by $\underline{\pi}$. Note that all strategies that are consistent with LCC_d under full two-sided commitment are consistent with LCC_d also when either $\kappa = \infty$ and $\phi \geq -\underline{\pi}$ or $\kappa \geq \bar{\pi}$ and $\phi = \infty$. Therefore, the mechanism (c, y, n) is incentive-efficient also under such (κ, ϕ).

Then by Proposition 1, there exists a labor income function $\tilde{y}$ such that $(c, \tilde{y}, n)$ is incentive-efficient for any quitting cost κ' and any firing cost ϕ' such that $\max\{\kappa', \phi'\} = \infty$ and $\min\{\kappa', \phi'\} < \infty$. ■

Proof of Lemma 7. It follows easily from Lemma 3. To prove part 1, we can write the indifference between (ρ^*, n) and $(\hat{\rho}, \hat{n})$ as $u(c(\theta)) - v(n_1(\theta)) + U_{\theta,(\rho^*,n)} = u(c(\hat{m}_1(\theta))) - v(\hat{n}_1(\theta)) + U_{\theta,(\hat{\rho},\hat{n})}$, where $U_{\theta,(\rho',n')}$ stands for the continuation value of the initial type θ following strategies (ρ', n'). The inequality analogous to (7) holds when we incorporate the continuation values and average labor supply only initially. Regarding part 2, when $\min\{\kappa, \phi\} = 0$ then any labor averaging in the initial period violates the limited commitment constraints. Hence, the contracts which randomize in the initial period give no advantage over the contracts which do not randomize in the initial period. ■

Proof of Proposition 2. Construct $\tilde{y}$ as in Proposition 1. By Proposition 1 we only need to verify that the mechanism $(c, \tilde{y}, n)$ under (κ', ϕ') does not allow agents to gain by misreporting in the initial period. Suppose that $(c, \tilde{y}, n)$ is not incentive-compatible. If the tempting deviation does not involve randomization in the initial period, naturally it would be tempting also with (c, y, n), which is a contradiction. If the tempting deviation involves randomization in the initial period and labor averaging, note that, since $\min\{\kappa, \phi\} > \min\{\kappa', \phi'\}$, the original mechanism under the cost structure (κ, ϕ) allows for no less labor averaging. Therefore, the same deviation is tempting and the original mechanism is not incentive-compatible, which is a contradiction. ■