Optimizer Performance Comparison Between ModelCenter, Isight and OASIS

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Empower Operations Corp.

Executive Summary

OASIS is in direct competition with currently market leaders, ModelCenter and Isight. This report documents comparison results with these two tools on a diverse set of benchmark problems. Thirteen problems (20 in total with different dimensionalities) are chosen from three categories: 1) problems of many local optima, 2) problems of ill-shape, and 3) constrained problems.

Tests are performed independently at Beijing Institute of Technology for ModelCenter and Isight, and at Simon Fraser University for OASIS. Thirty runs of each test are carried out and statistical performances of the algorithms are compared with box plots.

Overall, OASIS demonstrates the strongest capability in terms of quality of solution, robustness, and speed of convergence, while there is no clear performance distinction between ModelCenter and Isight.

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1. Introduction

A number of design software tools are developed to optimize engineering design problems. ModelCenter and Isight are two dominating commercial tools and both contain some optimization algorithms to tackle different optimization problems, especially engineering problems involving simulations. OASIS (Optimization Assisted Simulation Integration Software), as a new design tool provided by Empower Operations Corp. (in short, Empower), features one-button optimization capability that solves a wide range of engineering optimization problems.

This report documents the test results of the optimizers in ModelCenter, Isight, and OASIS with standard benchmark math problems in order to gain insight into the capabilities and potential shortcomings of the three tools.

2. Testing Platform

2.1 Scope of comparison

The comparison focuses on the needs of real-world engineers, who are mostly interested in finding the global optimum. In addition, most engineering problems do not have explicit equations but rather computed via finite element analysis (FEA), computational fluid dynamics (CFD), or other simulation or analysis models and codes. Last, most engineering problems are constrained. The comparison thus focuses on the following aspects:

i. Simulation-based problems with no equations and gradients are not reliable or not readily available

ii. Multimodal problems, i.e., problems with many local optima

iii. Ill-shaped problems, i.e., problem with strange shapes that make it hard to identify the optimal, and

iv. Constrained problems

Please note even though benchmarks used in this comparison are math problems, they are treated like simulation processes while assuming their mathematical expression is unknown and the gradient information is not available. The use of math benchmark problems ensures that tests can be performed in a reasonable timeframe and the tests can be repeated and validated.

2.2 Participating algorithms

ModelCenter Version 9.0.1

There are three optimization tool packages in ModelCenter Version 9.0.1, gradient optimizer, design optimization tool and Darwin Genetic Tool. Gradient optimizer and design optimization tool are gradient-based local search methods. Given the Scope of our comparison as defined in Section 2.1, the Darwin Genetic Optimization Tool is selected as the Optimizer in ModelCenter. In this test, the population size is set to be 65, while the selection scheme is multiple elitist. The maximum generation is 1000. Additionally, the crossover probability is 1 and the mutation probability is 0.1. The parameter setting is as shown in Fig. 1.

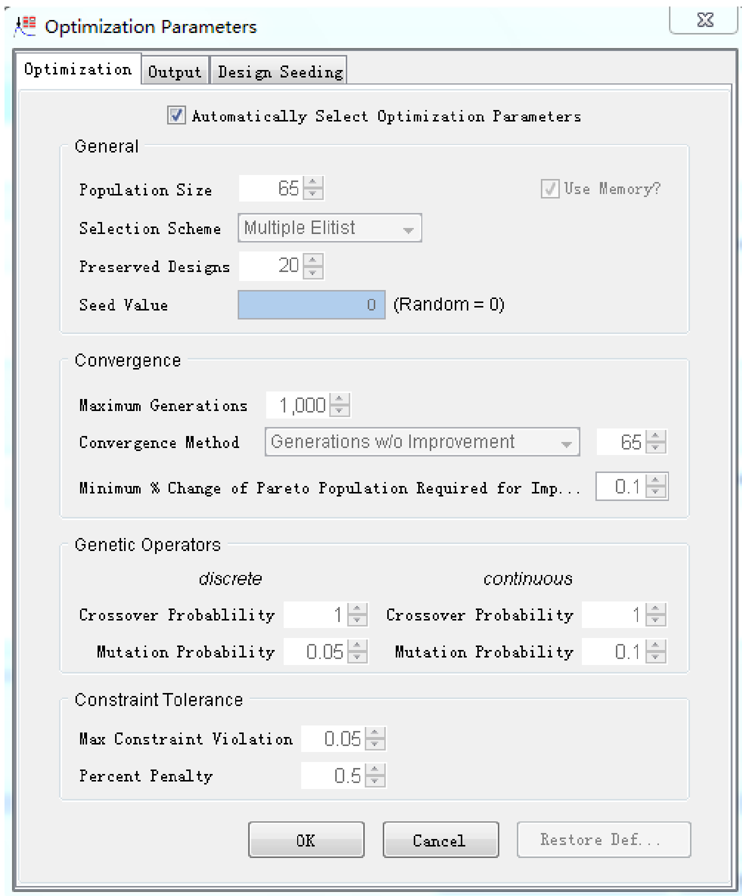


Figure 1 Parameter settings in ModelCenter

Isight Version 5.7.1

Isight Version 5.7.1 provides numerical optimizers including Nonlinear Programming by Quadratic Lagrangian (NLPQL), Modified Method of Feasible Directions (MMFD), Hooke-Jeeve Direct Search, Adaptive Simulated Annealing (AIA), Multi-Island Genetic Algorithm (MIGA), and Pointer automatic optimizer. In this test, considering the global search ability, the Pointer automatic optimizer is selected in Isight. The proprietary Pointer optimizer automatically picks the best algorithm from the existing pool of algorithms for a given problem. The parameters are all set to be the default settings. The parameter setting window of Isight is shown in Fig. 2.

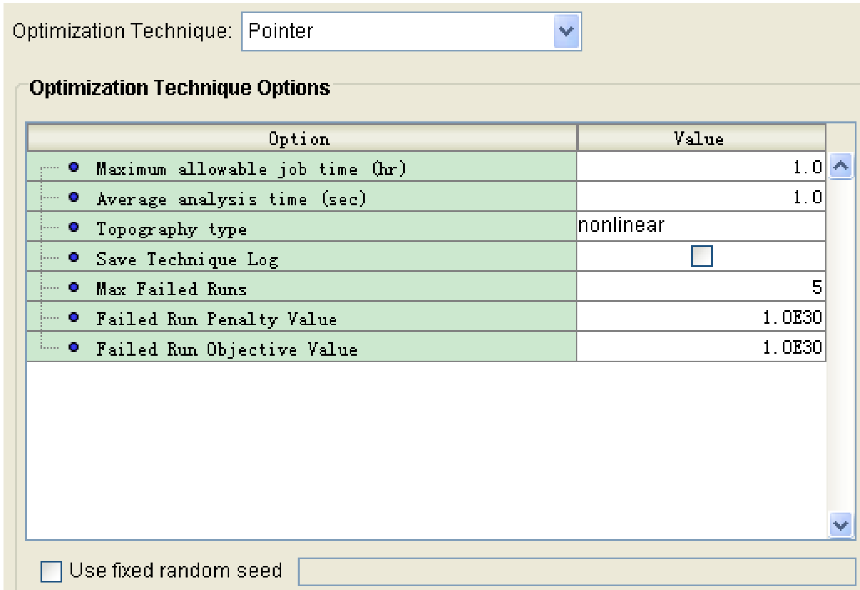


Figure 2 Parameter settings in Isight

It is to be noted that Isight Pointer requires the user give an initial point for each optimization run. In the test, to ensure fairness, a random initial point is generated for each independent run for each test problem.

OASIS Version 1.1

OASIS Version 1.1 automatically picks its algorithm for the user. The algorithm for global optimization is metamodel-based and is called SOGO (Single Objective Global Optimization). No parameter setting, nor an initial point, is needed by SOGO.

2.3 Testers of the algorithms

All the test results in this document on ModelCenter and Isight were independently obtained by Beijing Institute of Technology (BIT), China. The test of OASIS was conducted by Product Design and Optimization Laboratory at Simon Fraser University, Canada.

2.4 Test Problems

Test problems are obtained from the open literature and websites; they are commonly used as benchmark problems for testing simulation-based optimization algorithms[1]. These problems are divided into three categories:

[1] http://www.sfu.ca/~ssurjano/optimization.html, last access July 6, 2016.
http://www-optima.amp.i.kyoto-u.ac.jp/member/student/hedar/Hedar_files/TestGO.htm, last access July 6, 2016.

i) Problems with multiple local optima

(1) Levy function

$$f(x) = sin^2(\pi y_1) + \sum_{i=1}^{3} \left((y_i - 1)^2\left(1 + 10sin^2(\pi y_i + 1)\right)\right)$$
$$+(y_3 - 1)^2\left(1 + sin^2(2\pi x_3)\right)$$
$$y_i = 1 + \frac{x_i - 1}{4}, i = 1,2,3,4$$
$$-10 \le x_i \le 10$$

(2) Ackley function

$$f(x) = 20 + e - 20e^{-\frac{1}{5}\sqrt{\frac{1}{20}\sum_{i=1}^{20} x_i^2}} - e^{\frac{1}{20}\sum_{i=1}^{20}\cos(2\pi x_i)}$$
$$-30 \le x_i \le 30, i = 1,2, \ldots ,20$$

(3) Griewank function

$$f(x) = \sum_{i=1}^{n} \frac{x_i^2}{4000} - \prod_{i=1}^{n} \cos\left(x_i/\sqrt{i}\right) + 1$$
$$-600 \le x_i \le 600, i = 1,2, \ldots , n$$

(4) Rastrigin function

$$f(x) = 10 \times 80 + \sum_{i=1}^{80} (x_i^2 - 10\cos(2\pi x_i))$$
$$-5.12 \le x_i \le 5.12, i = 1,2, \ldots ,80$$

(5) Shubert Function

$$f(\boldsymbol{x}) = \prod_{i=1}^{d} \sum_{j=1}^{5} j\cos\left((j+1)x_i + j\right)$$

$$x \in [-5.12, 5.12]; d = \#\, variables$$

ii) Ill-shaped Problems

(1) Trid function (Bowl-shaped)

$$f(x) = \sum_{i=1}^{10} (x_i - 1)^2 - \sum_{i=2}^{10} x_i x_{i-1}$$
$$-100 \le x_i \le 100, i = 1,2, \ldots , 10$$

(2) Perm function D, Beta=0.5

$$f(x) = \sum_{k=1}^{30} \left[\sum_{i=1}^{30} (i^k + \beta)((x_i/i)^k - 1)\right]^2$$
$$-30 \leq x_i \leq 30, i = 1,2, \dots, 30$$

(3) Rosenbrock function (Valley-shaped)

$$f(x) = \sum_{i=1}^{49} [100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2]$$
$$-10 \leq x_i \leq 10, i = 1,2, \dots, 50$$

(4) Michalewicz Function (Steep Ridges/Drops)

$$f(x) = -\sum_{i=1}^{d} \sin(x_i)\, sin^{2m}(\frac{ix_i^2}{\pi})$$

$$m = 10; x \in [0, \pi]; d = \#\ of\ variables$$

(5) Powell Function

$$f(x) = \sum_{i=1}^{n/4} (x_{4i-3} + 10x_{4i-2})^2 + 5(x_{4i-1} - x_{4i})^2 + (x_{4i-2} - 2x_{4i-1})^4 + 10(x_{4i-3} - x_{4i})^2$$
$$-4 \leq x_j \leq 5, j = 1,2, \dots n$$

iii) Constrained problems

(1) G2

$$f(x) = -\left|\frac{\sum_{i=1}^{20} cos^4(x_i) - 2\prod_{i=1}^{20} cos^2(x_i)}{\sqrt{\sum_{i=1}^{20} ix_i^2}}\right|$$
$$s.t.\ \ g_1(x) = -\prod_{i=1}^{20} x_i + 0.75 \leq 0$$
$$g_2(x) = \sum_{i=1}^{20} x_i - 7.5 \times 20 \leq 0$$
$$0 \leq x_i \leq 10, i = 1,2, \dots, 20$$

(2) G7

$$f(x) = x_1^2 + x_2^2 + x_1x_2 - 14x_1 - 16x_2 + (x_3 - 10)^2 + 4(x_4 - 5)^2 + (x_5 - 3)^2$$
$$+2(x_6 - 1)^2 + 5x_7^2 + 7(x_8 - 11)^2 + 2(x_9 - 10)^2 + (x_{10} - 7)^2 + 45$$

$$
\begin{aligned}
s.t.\ g_1(x) &= 4x_1 + 5x_2 - 3x_7 + 9x_9 - 105 \le 0 \\
g_2(x) &= 10x_1 - 8x_2 - 17x_7 + 2x_8 \le 0 \\
g_3(x) &= -8x_1 + 2x_2 + 5x_9 - 2x_{10} - 12 \le 0 \\
g_4(x) &= 3(x_1 - 2)^2 + 4(x_2 - 3)^2 + 2x_3^2 - 7x_4 - 120 \le 0 \\
g_5(x) &= 5x_1^2 + 8x_2 + (x_3 - 6)^2 - 2x_4 - 40 \le 0 \\
g_6(x) &= 0.5(x_1 - 8)^2 + 2(x_2 - 4)^2 + 3x_5^2 - x_6 - 30 \le 0 \\
g_7(x) &= x_1^2 + 2(x_2 - 2)^2 - 2x_1x_2 + 14x_5 - 6x_6 \le 0 \\
g_8(x) &= -3x_1 + 6x_2 + 12(x_9 - 8)^2 - 7x_{10} \le 0
\end{aligned}
$$

$$-10 \le x_i \le 10, i = 1,2,\dots,10$$

(3) Stepped cantilever beam

$$V = D(b_1h_1l_1 + b_2h_2l_2 + b_3h_3l_3 + b_4h_4l_4 + b_5h_5l_5)$$

$$
\begin{aligned}
s.t.\, g_1(x) &= \frac{6Pl_5}{b_5h_5^2} - 14000 \le 0 \\
g_2(x) &= \frac{6P(l_5 + l_4)}{b_4h_4^2} - 14000 \le 0 \\
g_3(x) &= \frac{6P(l_5 + l_4 + l_3)}{b_3h_3^2} - 14000 \le 0 \\
g_4(x) &= \frac{6P(l_5 + l_4 + l_3 + l_2)}{b_2h_2^2} - 14000 \le 0 \\
g_5(x) &= \frac{6P(l_5 + l_4 + l_3 + l_2 + l_1)}{b_1h_1^2} - 14000 \le 0 \\
g_6(x) &= \frac{Pl^3}{3E}\left(\frac{1}{I_1} + \frac{7}{I_2} + \frac{19}{I_3} + \frac{37}{I_4} + \frac{61}{I_5}\right) - 2.7 \le 0 \\
g_7(x) &= \frac{h_5}{b_5} - 20 \le 0 \\
g_8(x) &= \frac{h_4}{b_4} - 20 \le 0 \\
g_9(x) &= \frac{h_3}{b_3} - 20 \le 0 \\
g_{10}(x) &= \frac{h_2}{b_2} - 20 \le 0 \\
g_{11}(x) &= \frac{h_1}{b_1} - 20 \le 0
\end{aligned}
$$

All the test problems are tabulated below with their dimensionalities and analytical global optimal values. In total there are 13 problems and 20 test cases considering some functions have variations of multiple dimensionalities.

Table 1 List of benchmark functions

Category	Problem name	# of dimension	Optimal value
Multimodal Functions	Levy	4	0
	Ackley	20	0
	Griewank	20, 50, 100	0
	Rastrigin	80	0
	Shubert	10, 30, 60	Unknown
Ill-shaped Functions	Trid	10	-210
	Perm D, Beta	30	0
	Rosenbrock	50	0
	Michalewicz	10, 30, 60	Unknown
	Powell	100, 120	0
Constrained Problems	G2	20	-0.8036
	G7	10	24.3062
	Stepped Beam	10	Unknown

2.5 Comparison Criteria

The optimization algorithms are compared based on 1) effectiveness, 2) robustness, and 3) efficiency. For effectiveness and robustness, we measure the average optimal function value, f^*, **over 30 independent runs of the algorithm on the same problem**, as well as the range of variations of the objective function value. We can also observe from the box plot for each test case to see the variations of results over the 30 runs. For efficiency, the best optimal values are recorded at certain stages of expenses, i.e., the number of function calls at 200, 500, 1000, etc. Note the CPU time was not used as an indicator because the assumption for such a comparison is on expensive engineering simulations. Unless there is a significant difference in the time for optimization, the number of function evaluation is often considered a better indication of algorithm efficiency since the optimization time is assumed comparatively negligible than the engineering simulation time.

3. Testing Results

3.1 Problems with many local minima (multimodal)

As one can see from Table 2, "dim" indicates the dimensionality of the problem; "Max NFE" sets the maximum number of function evaluations (NFE). The table compares the f^* and its range at each NFE setting for the three tools. The same table format is used for other tests as well. The green cell indicates the lowest average value for the row.

Table 2 Optimization results of multimodal functions

Problem	dim	Max NFE	ModelCenter		Isight		OASIS	
			*f**	range	*f**	range	*f**	range
Levy	4	200	1.151	[0.239,3.992]	4.7622	[0.0002,11.7873]	0.05149	[0.0002, 0.1511]
Ackley	20	1000	11.326	[9.571,13.637]	12.551	[2.954,19.254]	1.468	[0.406,2.758]
		2000	6.756	[5.317,8.910]	5.295	[1.142,19.110]	0.533	[0.324,2.470]
		5000	1.048	[0.340,2.391]	2.561	[0.062,5.013]	0.344	[0.3240.398]
		10000	0.000	[0.000,0.000]	2.117	[0.018,4.268]	0.326	[0.300,0.380]
Griewank	20	1000	25.44	[14.861,41.521]	1.079	[0.935,1.559]	1.044	[0.930,1.200]
		2000	5.897	[2.819,12.623]	0.992	[0.746,1.086]	0.956	[0.853,1.004]
		5000	0.687	[0.059,1.041]	0.812	[0.245,1.057]	0.948	[0.798,0.958]
		10000	0.001	[0,0.1298]	0.653	[0.005,1.057]	0.883	[0.553,0.944]
	50	1000	193.31	[151.90,297.37]	9.460	[3.350,20.042]	4.186	[2.791,6.650]
		2000	66.85	[43.92,98.42]	1.317	[1.101,2.816]	1.056	[1.043,1.070]
		5000	6.985	[3.99,15.07]	0.911	[0.456,1.204]	1.040	[1.022,1.052]
		10000	0.903	[0.145,1.102]	0.777	[0.031,1.193]	1.036	[1.022,1.044]
	100	1000	648.42	[528.30,760.62]	2026.0	[1.64e3,2.69e3]	614.30	[363.45,943.08]
		2000	294.87	[219.70,404.90]	30.767	[15.26,49.61]	89.561	[35.86,117.33]
		5000	84.20	[55.03,149.73]	2.966	[1.440,5.733]	3.736	[2.275,7.275]
		10000	8.162	[4.659,16.91]	0.907	[0.272,1.250]	2.935	[2.275,3.661]
Rastrigin	80	1000	842.31	[745.69,906.41]	1113.5	[911.08,1.24e3]	685.53	[471.13,811.39]
		2000	682.84	[599.25,754.12]	951.70	[677.80,1.13e3]	619.88	[385.23,775.91]
		5000	363.50	[283.28,457.45]	642.64	[427.75,803.41]	557.58	[379.14,725.45]
		10000	277.69	[167.50,445.52]	490.35	[245.09,680.80]	508.10	[374.64,616.58]
Shubert	10	500	-7.6e7	[-4.0e8,-1.17e7]	-7.5e8	[-7.8e9,-7.0e5]	-6.0e9	[-4.8e10,-3.4e7]
		1000	-1.0e9	[-1.2e10,-1.8e7]	-2.3e9	[-1.9e10,-2.5e7]	-1.4e10	[-1.4e11,-1.9e8]
	30	1000	-2.6e21	[-3.4e22,-1.2e18]	-5.4e20	[-1.8e21,-1.6e20]	-1.3e26	[-2.9e27,-4.8e20]
		2000	-4.0e23	[-4.9e24,-2.9e19]	-5.6e20	[-2.0,-1.7e20]	-3.5e26	[-4.2e27,-2.4e22]
		5000	-7.7e26	[-9.9e27,-2.4e22]	-1.4e21	[-1.7e22,-2.3e20]	-3.0e27	[-2.7e28,-5.9e23]
		10000	-4.8e28	[-1.0e30,-7.2e22]	-3.7e21	[-6.6e22,-2.3e20]	-7.5e27	[-4.9e28,-1.4e25]
	60	1000	-2.9e37	[-5.6e38,-4.6e+32]	-4.5e31	[-1.3e33,-4.9e25]	-2.7e48	[-5.4e49,-2.0e41]
		2000	-1.8e40	[-2.2e41,-1.6e36]	-4.7e31	[-1.3e33,-7.1e25]	-1.1e51	[-2.6e52,-1.3e44]
		5000	-3.5e45	[-3.5e46,-2.3e40]	-1.2e32	[-1.3e33,-5.2e27]	-2.0e51	[-4.1e52,-3.0e45]
		10000	-1.1e52	[-3.1e53,-8.7e43]	-1.3e32	[-1.3e33,-1.4e29]	-3.9e51	[-7.9e52,-5.9e48]

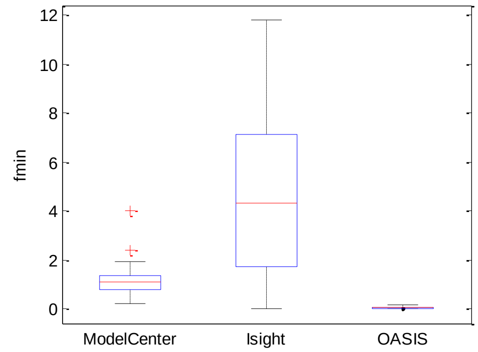
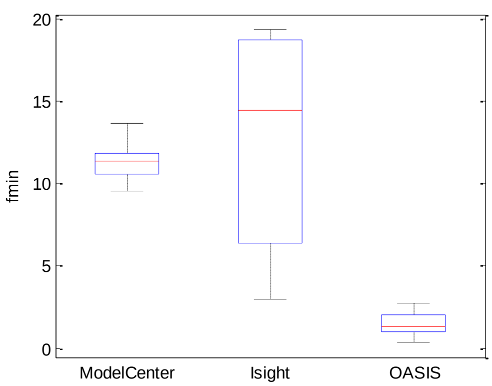
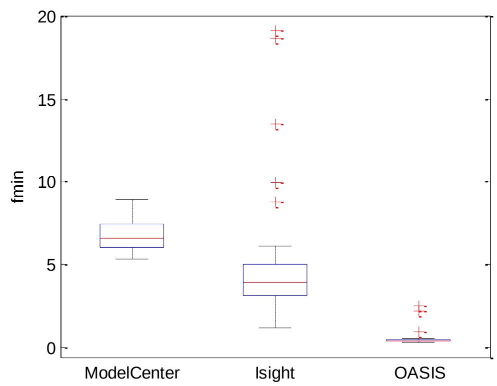


Figure 3 Box plot of Levy function

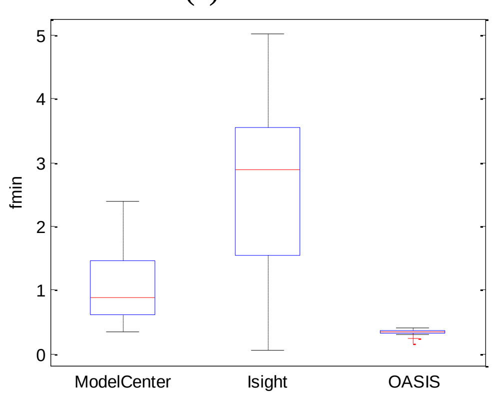
Figure 3 shows the box plot for the Levy function of four dimensions. The red line shows the median of the optimal function value, the blue box shows the value range between the first quartile and third quartile, the top and bottom bar show the maximum and minimum values respectively, and points beyond the two bars are outliers. In general the smaller is the size of the blue box the more robust is the solution. This shows OASIS has the best performance and ModelCenter performs better than Isight for both the quality of solution and robustness.



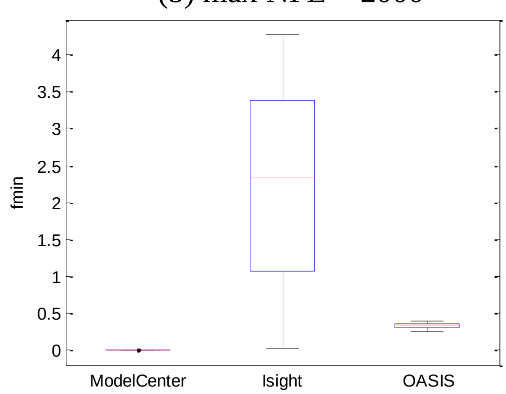
(a) max NFE = 1000



(b) max NFE = 2000



(c) max NFE = 5000

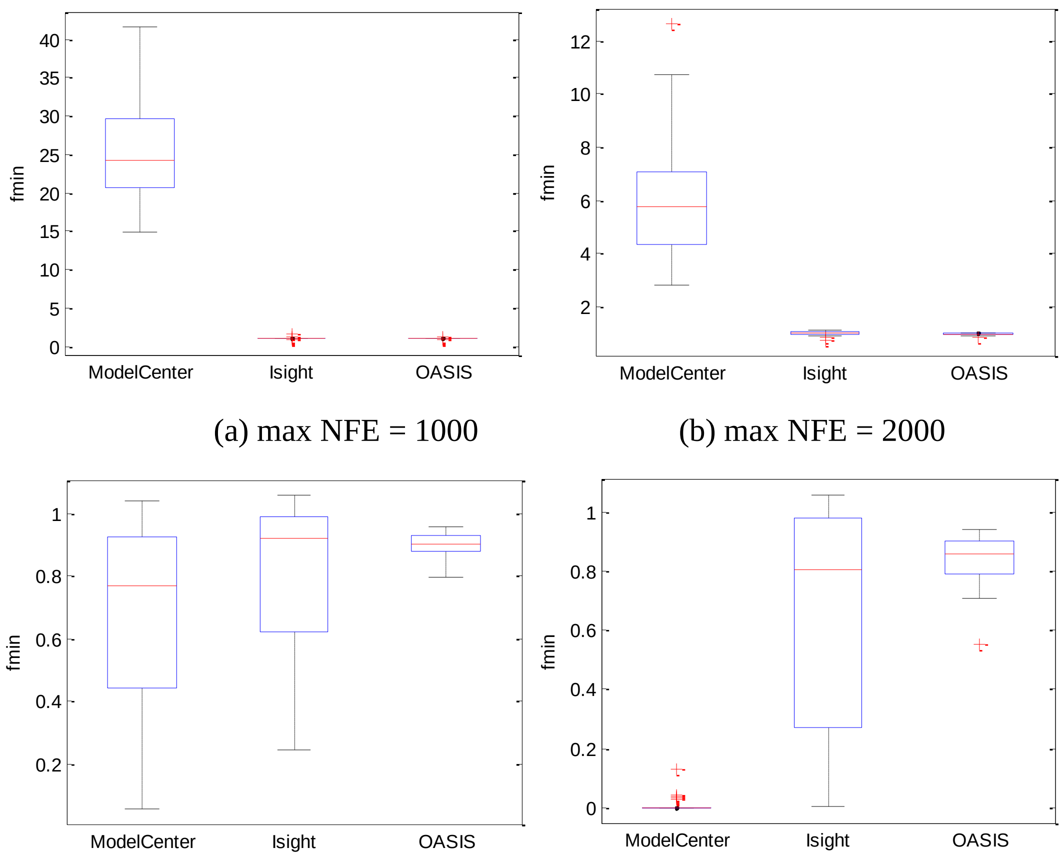


(d) max NFE = 10000

Figure 4 Box plot of Ackley function

For the Ackley function of 20 dimensions, OASIS reaches better solutions much faster than the other two. ModelCenter catches up when NFE reaches 10,000. The box plot of this function is shown in Figure 4. From the box plots, one can see OASIS not only achieved the best values among the three, it also shows high robustness after 2000 NFEs. Isight shows higher variation and overall worse results than ModelCenter.

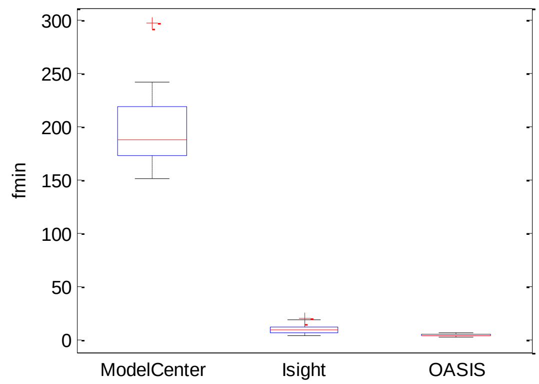
For the 20-D Griewank function, shown in Figure 5, OASIS shows faster convergence with low number of NFEs and high robustness throughout. ModelCenter catches up with the other two when NFE reaches 5,000.



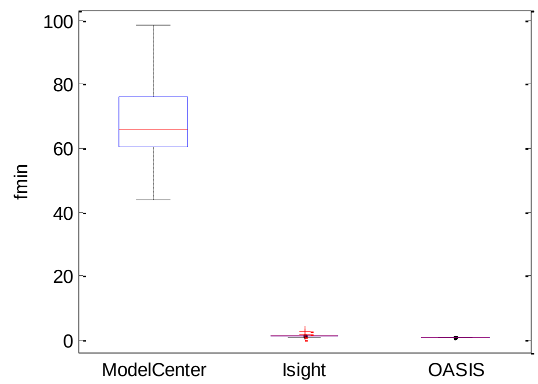
(a) max NFE = 1000 (b) max NFE = 2000

(c) max NFE = 5000 (d) max NFE = 10000

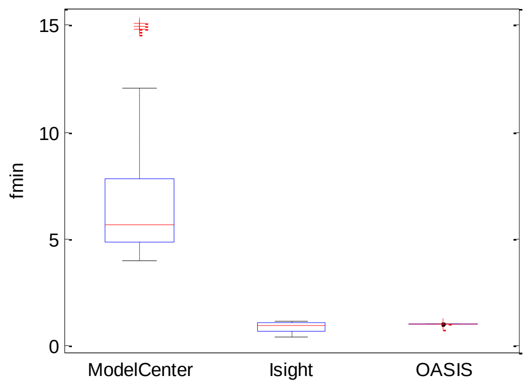
Figure 5 Box plot of Griewank function (20-D)



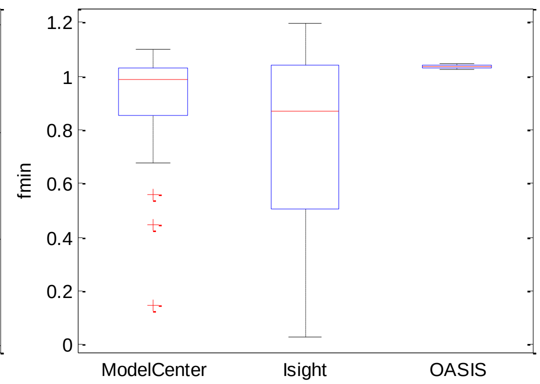
(a) max NFE = 1000



(b) max NFE = 2000



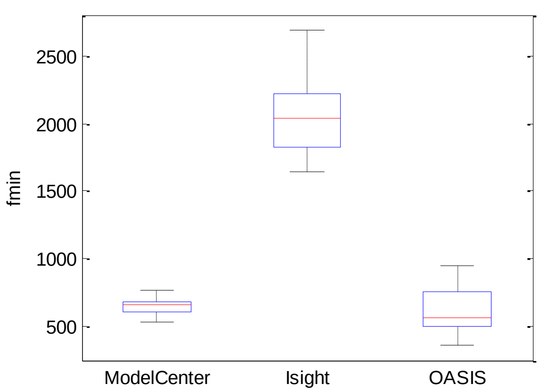
(c) max NFE = 5000



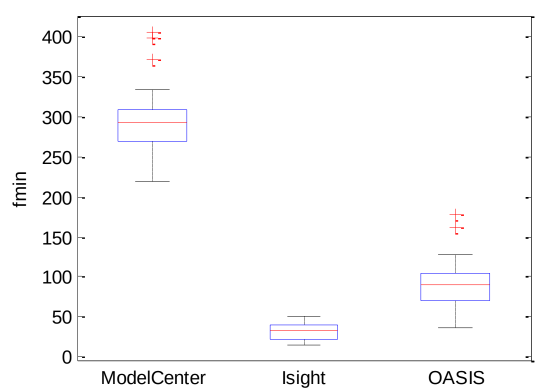
(d) max NFE = 10000

Figure 6 Box plot of Griewank function (50-D)

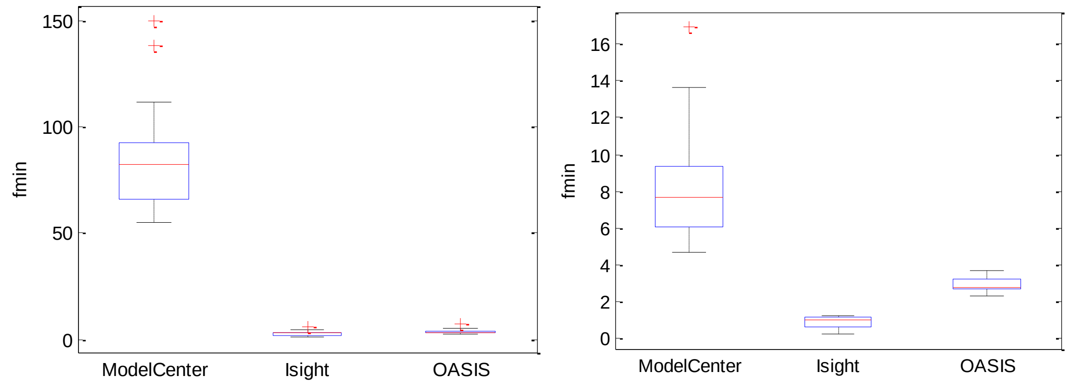
For 50-D Griewank, OASIS shows overall best performance with similar optimal solution and highest robustness.



(a) max NFE = 1000



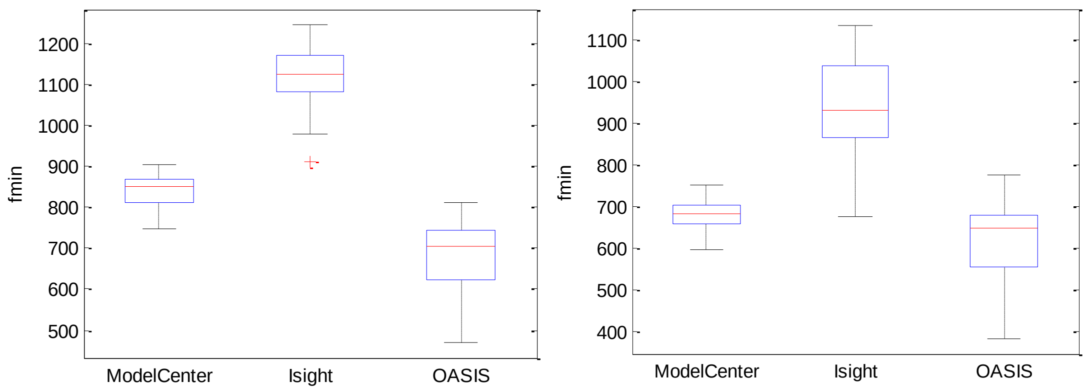
(b) max NFE = 2000



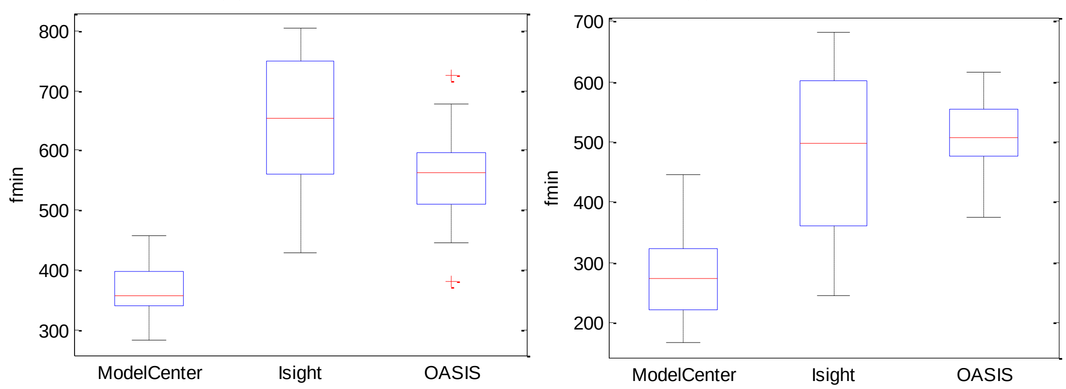
(c) max NFE = 5000 (d) max NFE = 10000

Figure 7 Box plot of Griewank function (100-D)

For 100-D Griewank, OASIS gets the best solution with 1000 NFEs, and shows a similar performance with Isight at higher NFEs. ModelCenter works well at low NFEs but then falls behind.



(a) max NFE = 1000 (b) max NFE = 2000

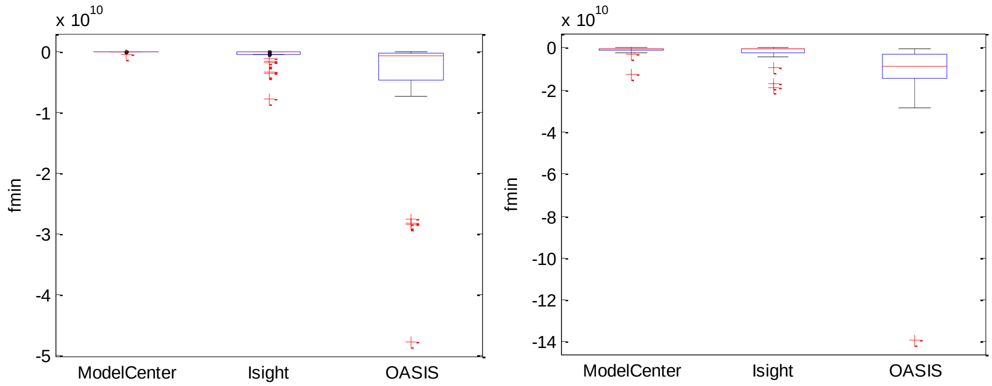


(c) max NFE = 5000 (d) max NFE = 10000

Figure 8 Box plot of Rastrigin function

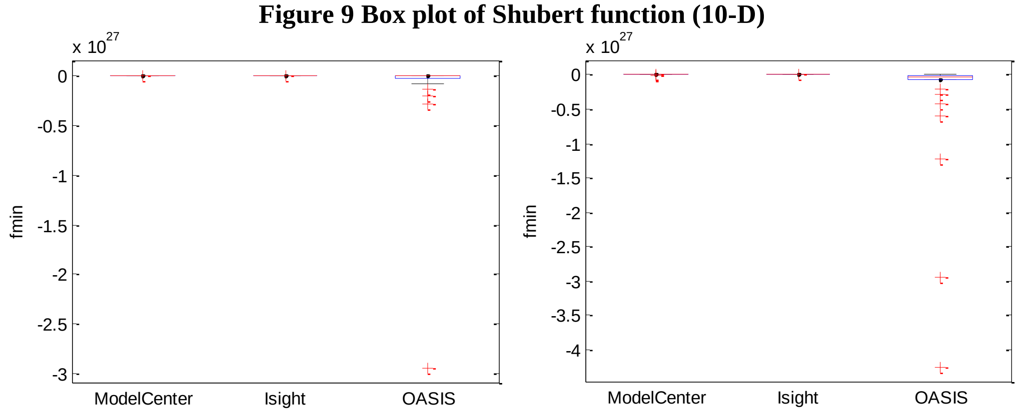
For Rastrigin, OASIS shows the best performance with low NFEs, then ModelCenter catches up for high NFEs.

For the Shubert function, we tested d=10, 30 and 60. Since the numbers are large, it is best to check Table 2 to compare them rather than the plots shown in Figures 9-11. In all cases, OASIS is a clear winner.

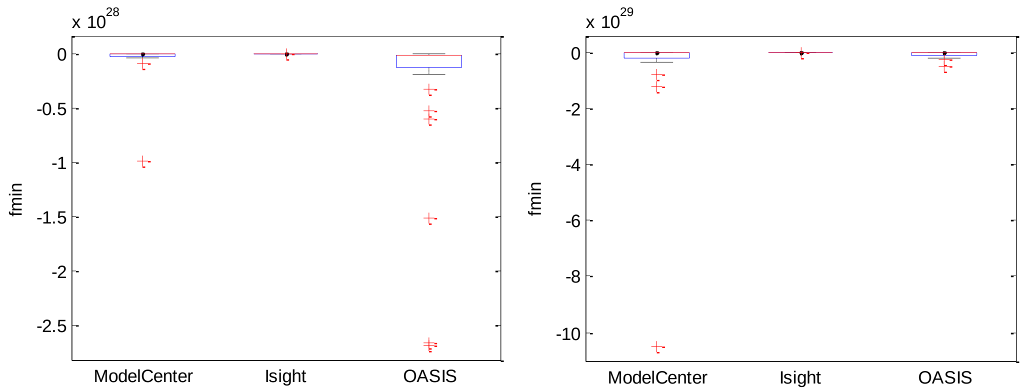


(a) max NFE = 500 (b) max NFE = 1000

Figure 9 Box plot of Shubert function (10-D)

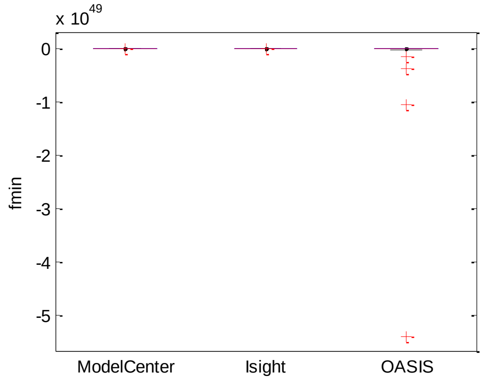


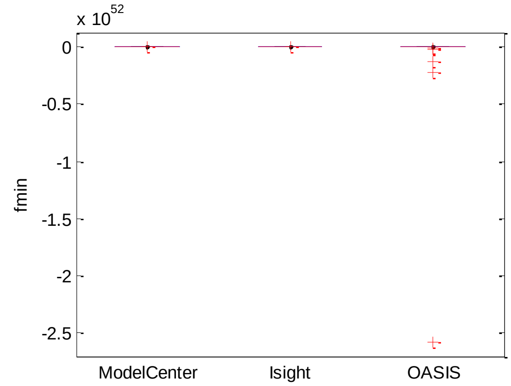
(a) max NFE = 1000 (b) max NFE = 2000



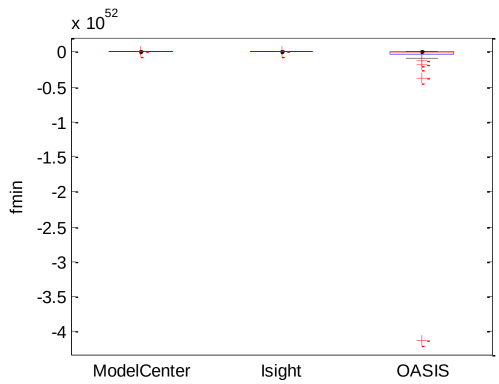
(c) max NFE = 5000 (d) max NFE = 10000

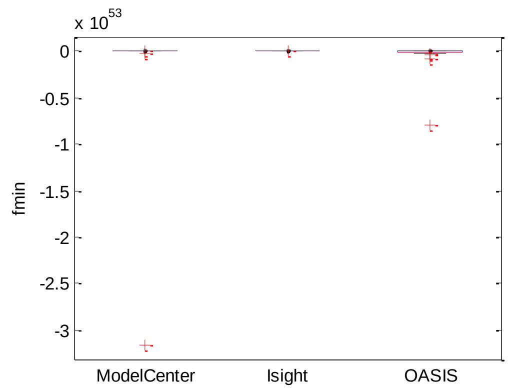
Figure 10 Box plot of Shubert function (30-D)





(a) max NFE = 1000 (b) max NFE = 2000





(c) max NFE = 5000 (d) max NFE = 10000

Figure 11 Box plot of Shubert function (60-D)

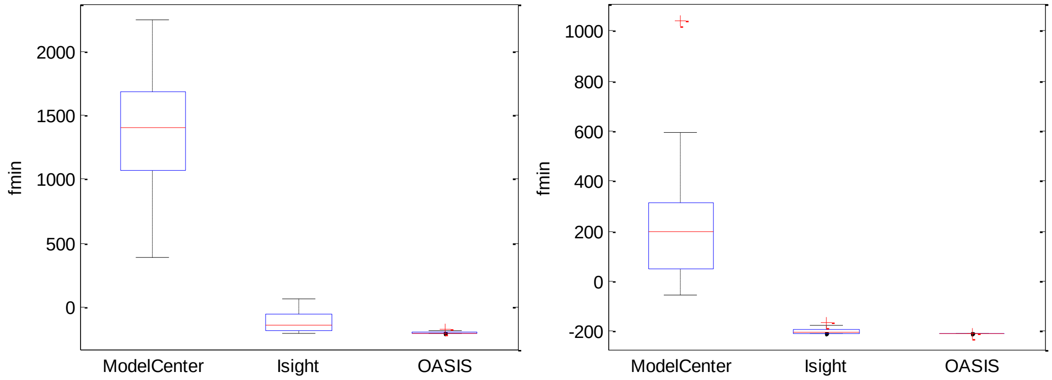
From the first group of problems with dimensionality spanning from 4 to 100, OASIS shows better performance with low number of function calls (NFEs<=2000) in all cases. With high NFEs, OASIS got better optimal than the other two in most of cases. From the box plots, OASIS shows overall higher robustness than the other two. In practical engineering when the budget and time are limited, OASIS has clear advantage. There is no clear differentiation between ModelCenter and Isight.

3.2 Ill-Shaped Problems

The second category of problems may not have many local optima but are of strange shapes that make it hard to identify the global optimum. The test results for the three problems are shown in Table 3. From the table, one can see that OASIS gets the best results for almost all the problems except for Perm. Even for the few exceptions, OASIS' solution is close to the best or when NFEs are at the high end. From Figures 12-19, OASIS consistently shows the best performances at low NFE settings and better or similar performances at high NFE settings.

Table 3 Optimization results of ill-shaped functions

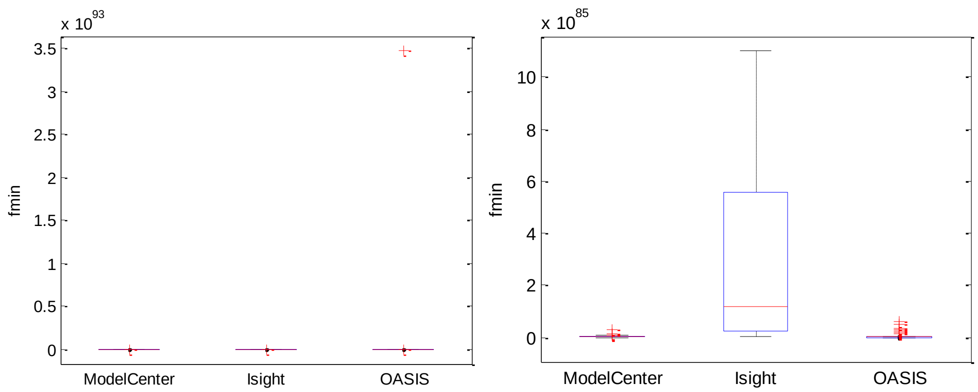
Problem	dim	Max NFE	ModelCenter		Isight		OASIS	
			*f**	range	*f**	range	*f**	range
Trid	10	500	1409.8	[390.62,2247.97]	-115.89	[-204.18,63.74]	-200.78	[-209.30, -180.60]
		1000	235.36	[-57.57,1041.79]	-199.52	[-209.17,-166.07]	-209.70	[-209.98, -207.75]
Perm	30	1000	2.67e84	[3.4e82,2.5e85]	5.58e85	[2.6e83,2.3e86]	1.20e92	[3.7e81,3.4e93]
		2000	3.40e83	[5.7e80,2.9e84]	2.83e85	[1.7e83,1.1e86]	8.78e83	[9.2e80,6.0e84]
		5000	4.24e82	[5.7e80,1.8e83]	9.53e84	[1.8e82,6.3e85]	1.41e83	[4.4e80,8.6e83]
		10000	2.34e82	[2.9e80,4.9e82]	3.84e84	[1.5e82,2.6e85]	4.17e82	[4.4e80,4.1e83]
Rosenbrock	50	1000	3.33e5	[1.52e5,5.42e5]	1.99e4	[6.27e3,7.68e4]	1.04e3	[497.47,2.11e3]
		2000	6.82e4	[2.79e4,1.61e4]	3.62e3	[240.95,7.83e3]	81.26	[51.14,257.66]
		5000	3.82e3	[748.44,1.03e4]	139.90	[53.59,339.82]	63.24	[49.64,157.20]
		10000	325.42	[109.62,793.97]	100.26	[43.43,284.20]	57.74	[49.64,150.36]
Michalewicz	10	500	-4.629	[-6.44,-3.57]	-6.156	[-8.88,-4.10]	-6.374	[-7.44，-4.89]
		1000	-5.958	[-6.85,-5.01]	-6.645	[-9.05,-4.45]	-7.082	[-8.22，-5.07]
	30	1000	-9.580	[-11.71,-7.53]	-11.756	[-14.97,-9.46]	-15.083	[-18.61，-12.57]
		2000	-12.337	[-14.79,-9.95]	-14.925	[-17.84,-11.72]	-16.301	[-18.72，-14.15]
		5000	-18.249	[-20.60,-15.21]	-18.346	[-21.41,-14.99]	-17.280	[-20.14，-15.61]
		10000	-19.931	[-22.18,-17.30]	-19.723	[-23.90,-15.20]	-18.122	[-20.86，-15.75]
	60	1000	-13.774	[-15.97,-12.31]	-16.255	[-20.09,-13.25]	-22.006	[-24.96,-17.31]
		2000	-15.657	[-17.76,-14.26]	-18.205	[-22.17,-14.79]	-23.816	[-27.02,-20.42]
		5000	-21.745	[-27.79,-17.82]	-24.697	[-29.84,-20.00]	-26.911	[-31.16,-23.65]
		10000	-29.672	[-35.81,-24.13]	-30.882	[-35.60,-26.29]	-28.579	[-31.16,-25.51]
Powell	100	1000	1.51e4	[8.59e3,2.36e4]	4.19e4	[1.97e4,7.23e4]	7.31e3	[3.96e3,1.36e4]
		2000	5.64e3	[3.35e3,8.16e3]	1.14e3	[465.90,2.03e3]	865.01	[529.73,1.78e3]
		5000	1.45e3	[776.23,2.93e3]	191.82	[42.85,338.80]	182.24	[101.14,241.10]
		10000	238.73	[87.96,592.28]	7.47	[2.12,20.84]	155.33	[100.83,221.06]
	120	1000	2.04e4	[1.24e4,2.72e4]	6.80e4	[4.26e4,9.19e4]	1.11e4	[4.32e3,2.18e4]
		2000	7.82e3	[5.76e3,1.32e4]	5.66e3	[3.26e3,8.13e3]	1.76e3	[880.31,3.78e3]
		5000	2.34e3	[1.29e3,5.04e3]	437.31	[217.07,627.58]	273.10	[182.79,493.37]
		10000	446.04	[209.45,1.10e3]	35.95	[3.79,250.92]	221.91	[144.21,294.98]



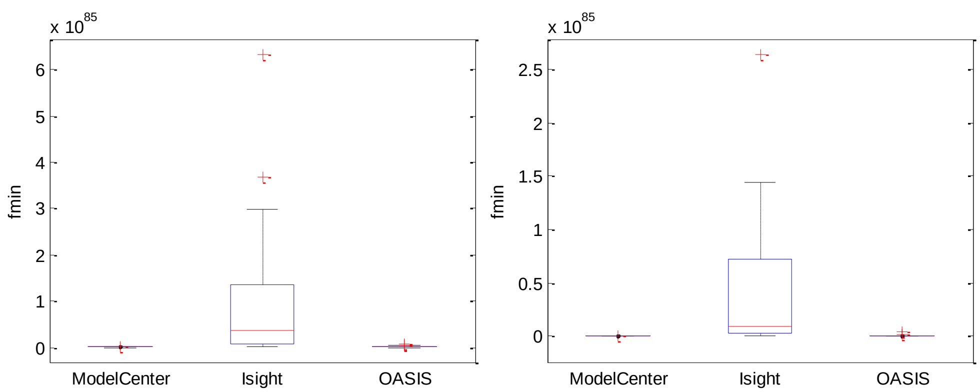
(a) Trid-500 (b) Trid-1000

Figure 12 Box Plot of the Trid Function

Clearly OASIS performs the best for Trid in terms of solution and robustness with Isight being the second best.



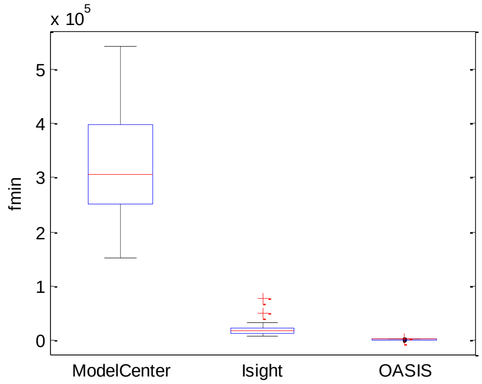
(a) max NFE = 1000 (b) max NFE = 2000



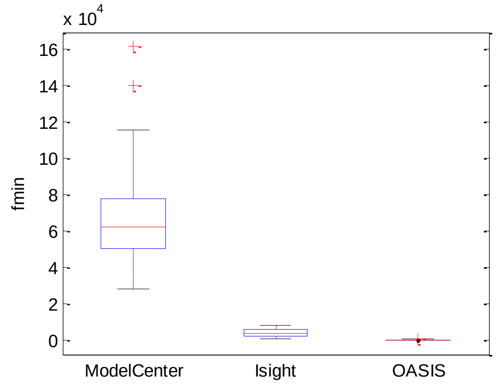
(c) max NFE = 5000 (d) max NFE = 10000

Figure 13 Box plot of Perm function

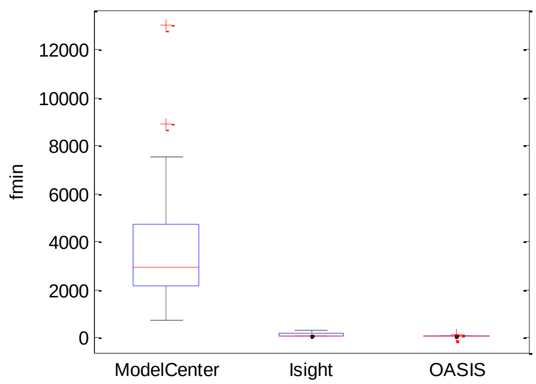
The box plot of Perm results shows that OASIS and ModelCenter have high robustness and quality of solution while Isight shows greater variation. The plot for Rosenbrock shows OASIS has best the solution quality and robustness for all NFE settings.



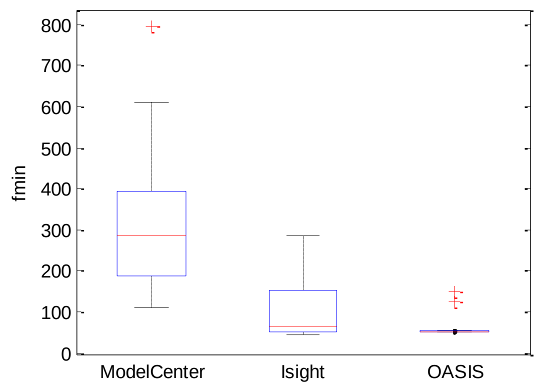
(a) max NFE = 1000



(b) max NFE = 2000

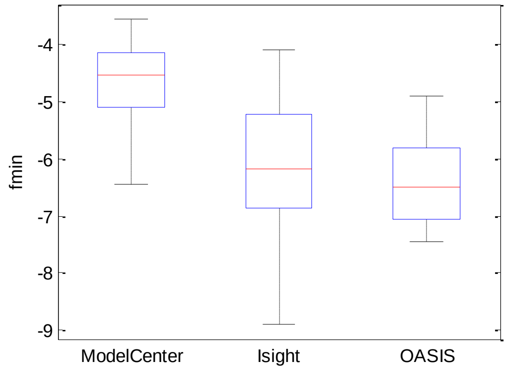


(c) max NFE = 5000

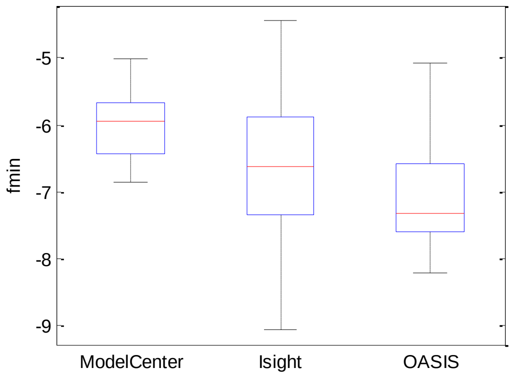


(d) max NFE = 10000

Figure 14 Box plot of Rosenbrock function

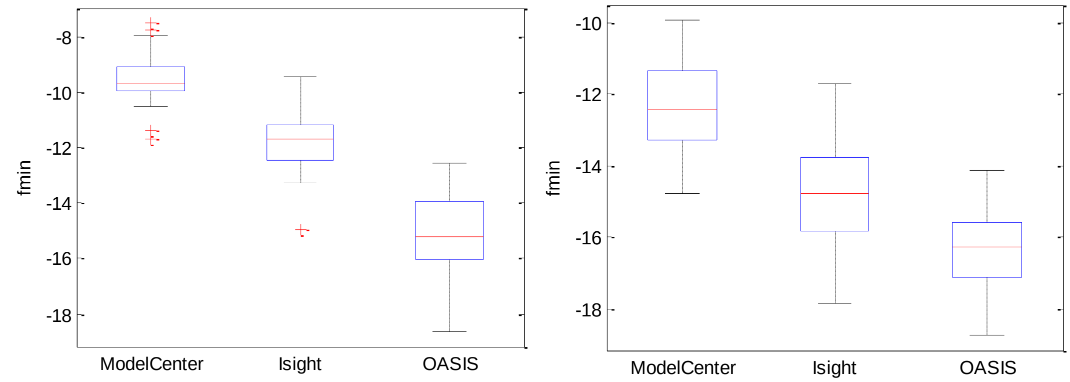


(a) max NFE = 500

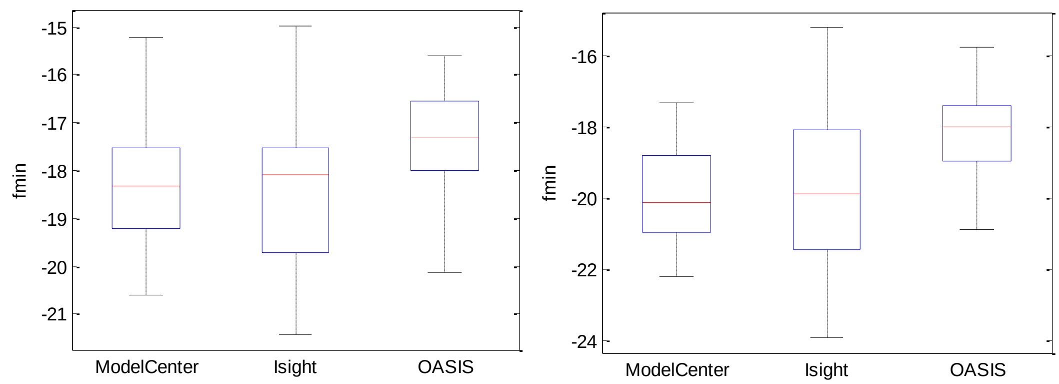


(b) max NFE = 1000

Figure 15 Box plot of Michalewicz function (10-D)

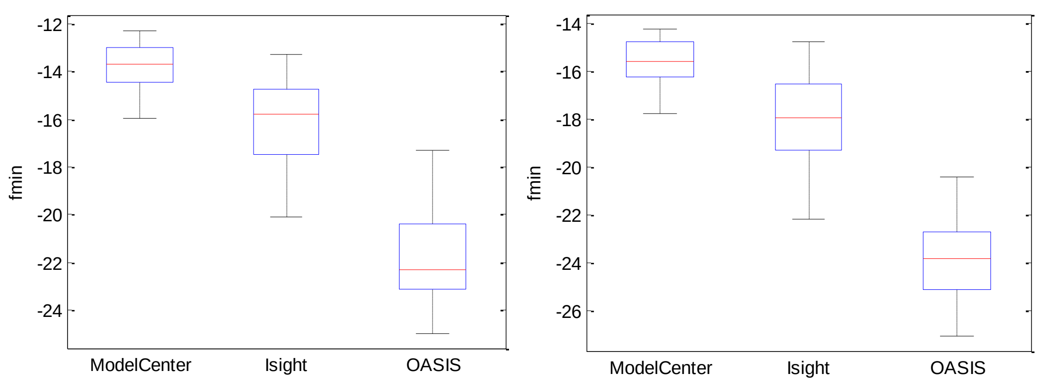


(a) max NFE = 1000 (b) max NFE = 2000

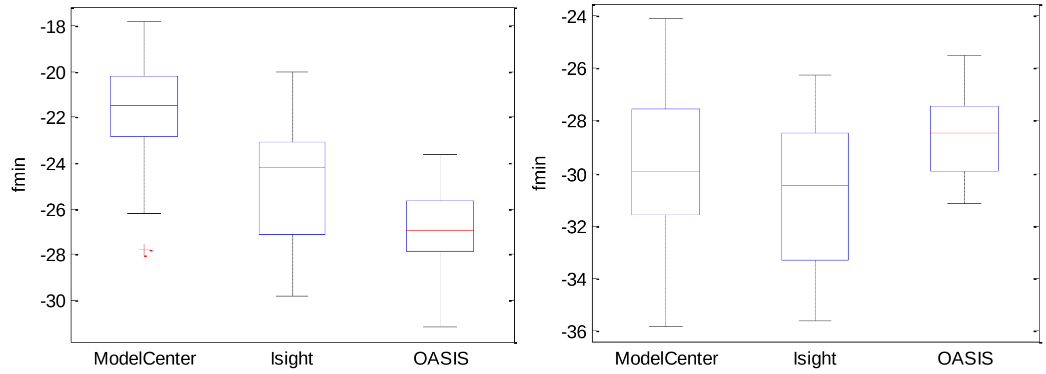


(a) max NFE = 5000 (b) max NFE = 10000

Figure 16 Box plot of Michalewicz function (30-D)



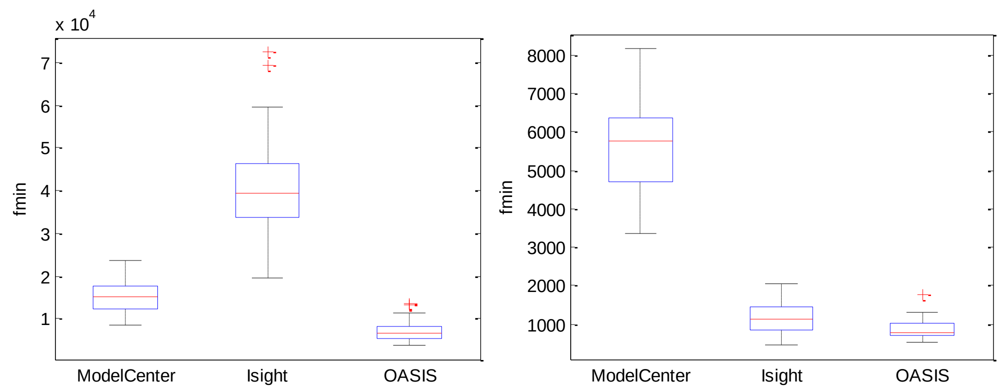
(a) max NFE = 1000 (b) max NFE = 2000



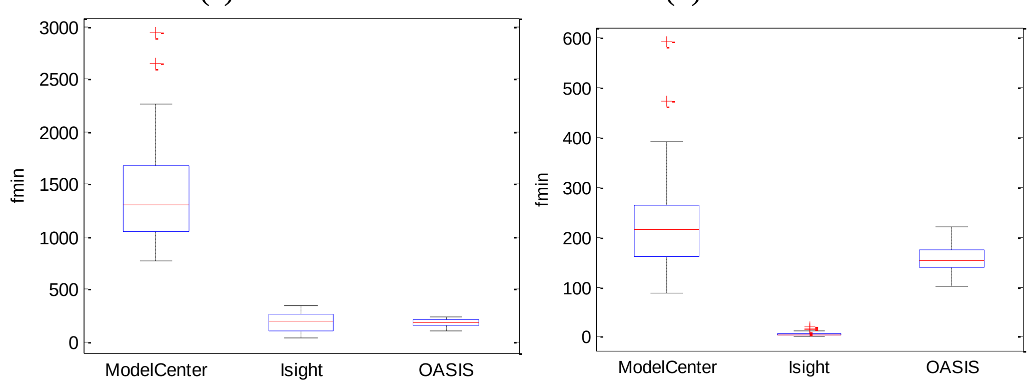
(a) max NFE = 5000 (b) max NFE = 10000

Figure 17 Box plot of Michalewicz function (60-D)

For Michalewicz function 10-D, OASIS wins in all NFEs settings; for the 30-D and 60-D variations, OASIS shows quicker convergences with low NFEs and similar performance at high NFEs with the other two tools.

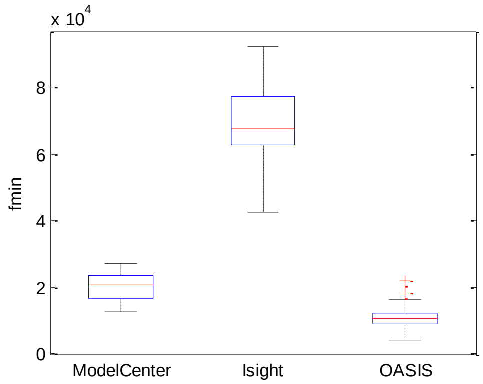


(a) max NFE = 1000 (b) max NFE = 2000

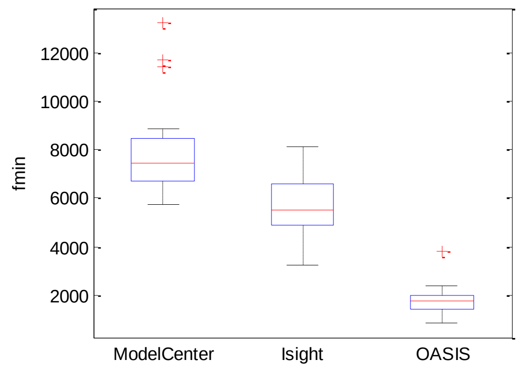


(c) max NFE = 5000 (d) max NFE = 10000

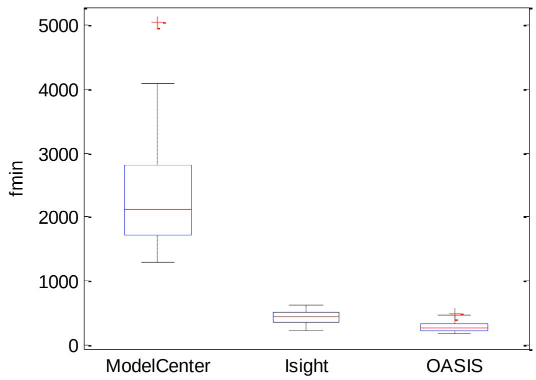
Figure 18 Box plot of Powell function (100-D)



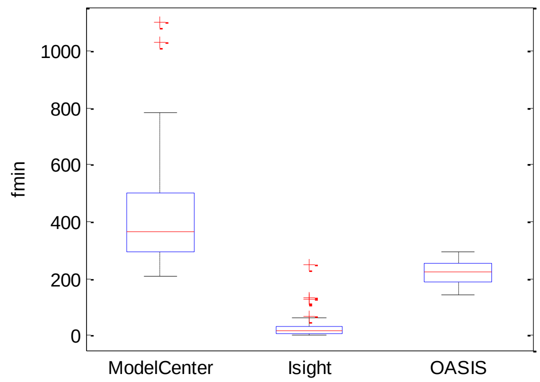
(a) max NFE = 1000



(b) max NFE = 2000



(c) max NFE = 5000



(d) max NFE = 10000

Figure 19 Box plot of Powell function (120-D)

For Powell function 100-D and 120-D, OASIS performs best when NFEs lower than 5000, and shows similar performance with the other two at NFE=10000.

From the second group, OASIS again shows overall the best performance in terms of solution quality and robustness. Especially when NFEs are low, i.e., the time and budget are limited, OASIS performs the best. Once NFEs reach high settings, OASIS shows better or similar performances with the other two.

3.3 Constrained Problems

The last groups are the constrained problems. The results are shown in Table 4.

Table 4 Optimization results of constrained functions

Problem	dim	Max NFE	ModelCenter		Isight[2]		OASIS	
			*f**	range	*f**	range	*f**	range
G2	20	1000	-0.035	[-0.044,-0.021]	-0.244	[-0.310,-0.183]	-0.579	[-0.745,-0.284]
		2000	-0.038	[-0.046,-0.018]	-0.250	[-0.354,-0.183]	-0.631	[-0.780,-0.396]
		5000	-0.042	[-0.052,-0.031]	-0.264	[-0.395,-0.193]	-0.721	[-0.797,-0.588]
		10000	-0.044	[-0.054,-0.031]	-0.281	[-0.432,-0.217]	-0.759	[-0.803,-0.670]
G7	10	500	-	-	57.396	[25.89,216.16]	-	-
		1000	572.50	[151.08,1373.99]	26.962	[25.04,31.96]	-	-
Beam	10	500	9.76e4	[8.98e4,1.10e5]	8.71e4	[7.74e4,1.11e5]	7.82e4	[7.76e4,7.86e4]
		1000	9.22e4	[8.73e4,9.92e4]	8.71e4	[7.74e4,1.11e5]	7.80e4	[7.75e4,7.86e4]

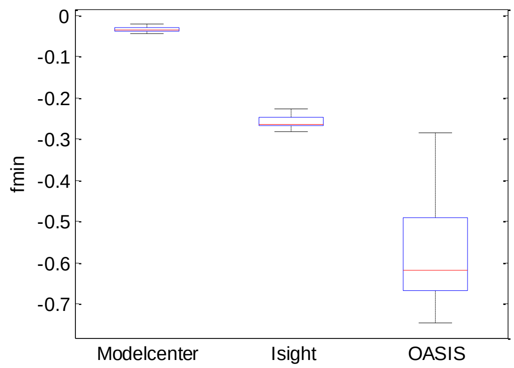
The blanks in Table 4 indicate the algorithm did not find a feasible solution with the designated NFEs. That is, ModelCenter could not find a feasible solution for G7 with 500 NFEs, while Isight only yielded six feasible points over 30 runs for the Beam problem, and OASIS failed for G7. However, a new algorithm developed by Empower for handling constrained problems yields the following results with 10 independent runs. From Table 5, one can see the new algorithm found the best solution among all with only 132 NFEs for G7 and 200 NFEs for the beam problem. This algorithm is currently not available in OASIS Version 1.1.

Table 5 Test results of a new OASIS algorithm

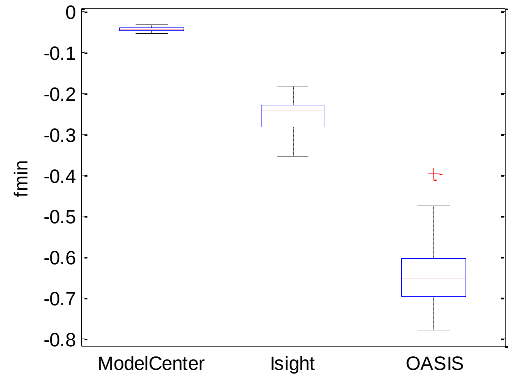
Problem	NFE	*f**	range
G7	132	24.98	[24.31 27.03]
Beam	200	78790.04	[77441 83125]

From Table 4, Table 5 and all the box plots below, it is clear that OASIS comes out as the best in the group, and the other two tools have difficulty solving all of the problems. For both G2 and G7, ModelCenter comes the last. For the Beam problem, OASIS shows significantly better performance than ModelCenter while Isight only got six feasible points over 30 runs.

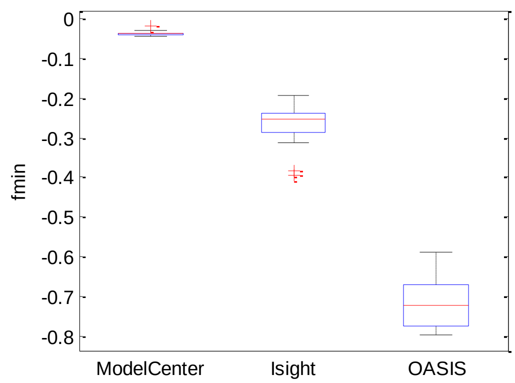
[2] *For the stepped beam problem, Isight only got six feasible solutions in 30 runs.



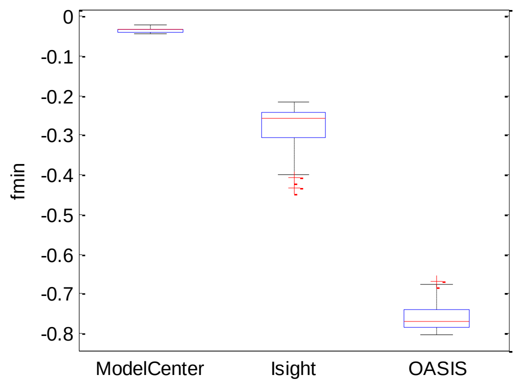
(a) max NFE = 1000



(b) max NFE = 2000

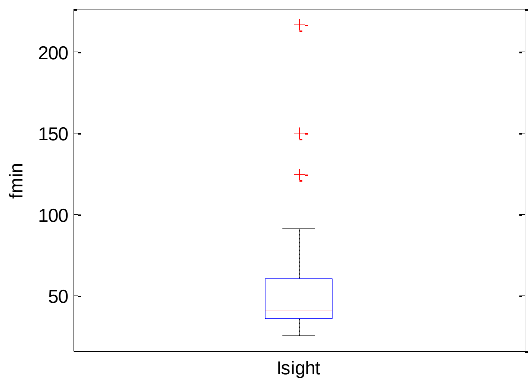


(c) max NFE = 5000

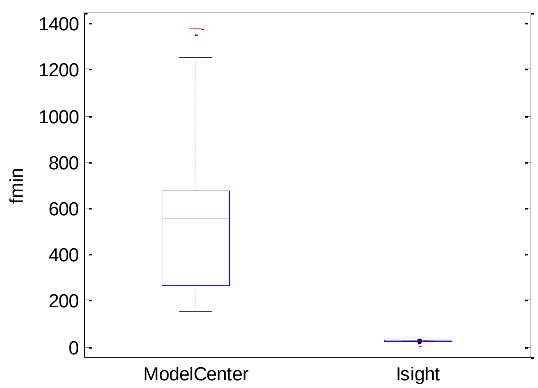


(d) max NFE = 10000

Figure 20 Box plot of G2 function

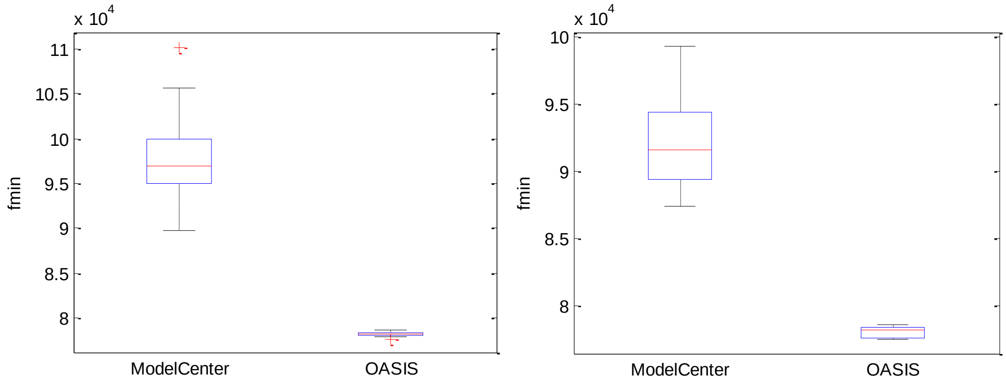


(a) max NFE = 500



(b) max NFE = 1000

Figure 21 Box plot of G7 function



(a) max NFE = 500 (b) max NFE = 1000

Figure 22 Box plot of stepped cantilever beam problem

For the beam problem since Isight only got six feasible points in 30 runs, it is not drawn in the boxplot. For this group of problems, OASIS shows the best performance, especially considering the new algorithm that is recently developed to handle constraints.

4. Summary

The optimization capability of ModerCenter, Isight and OASIS are tested with 13 numerical benchmark functions (20 in total if considering dimension variations) divided into three categories. These functions are chosen to represent problems with multiple local optima, ill-shape, and constraints. Combing all the test results, the following can be observed that:

1. OASIS consistently performs well for problems with many local optima, difficult terrains, and many constraints. Overall, OASIS yields better results with higher robustness than the other two tools.

2. OASIS is the mostly effective algorithm when NFEs are low, i.e., the time and budget is limited. In practice, it is generally not practical to reach to 5000 or more NFEs.

3. Even when NFEs are high (i.e., 5000 or more), OASIS shows similar or sometimes higher performance than the other two tools.

4. OASIS performs consistently well for both low dimensional and high dimensional problems, and

5. There is no clear performance distinction between ModeCenter and Isight; both work well for some problems but left behind for others.