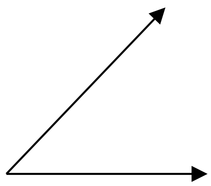
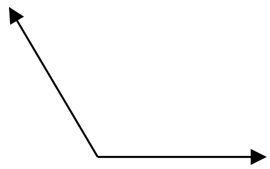
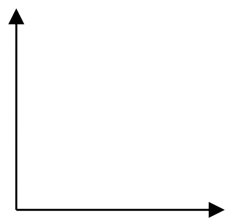
“Education is not the learning of facts, but the training of the mind to think." – Albert Einstein

Chapter 1: Trigonometric Functions

Section 1.1: Angles

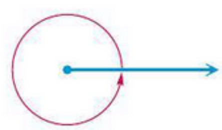
Angles:

An **angle** is formed by two rays with the same end point. The common end point is called the **vertex** of the angle, and the rays are called the **sides** of angle.

Degree Measure:

One way to measure the size of an angle is with **degree measure**. The angle formed by rotating a ray through one complete revolution has a measure of 360-degree, written 360°.



One degree (1°) is 1/360 of a full rotation. Likewise, 180°, is one half of a full rotation, and 90° is one quarter of a full rotation.

Acute Angle:

Right Angle:

Obtuse Angle:

Straight Angle:

Complementary Angles:

Supplementary Angles:

Ex 1: - If an angle measures 30 degrees, then find the measure of its…

Complement:

Supplement:

- If an angle measures x degrees, then find the measure of its…

Complement:

Supplement:

Traditionally, portions of a degree have been measured with **minutes** and **seconds**. One minute is $\frac{1}{60}$ of a degree, and one second is $\frac{1}{60}$ of a minute. We can say that one second is $\frac{1}{3600}$ of a degree

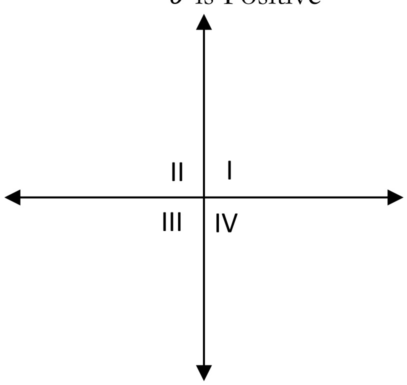
Ex 2:   Perform each following calculation

a) $12^0 19' + 27^0 45'$ =

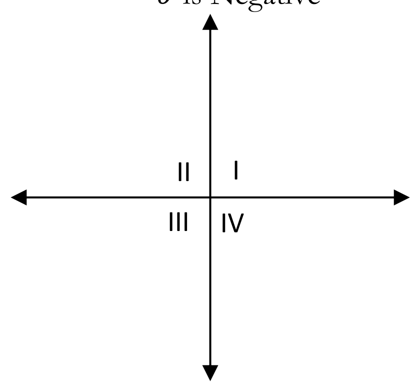
b) $67^0 - 32^0 45'$ =

Defn: An **angle** is said to be in **standard position** if its initial side is along the positive x-axis and its vertex is at the origin.

θ is Positive



θ is Negative



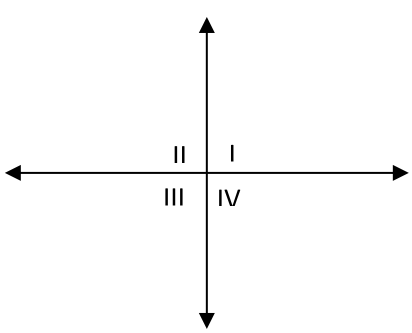
Defn: **Coterminal Angles** – share the same initial and terminal sides but have different number of rotations or direction.

Ex 3:   For each given angle…

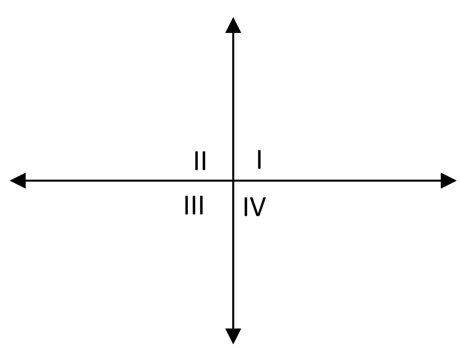
(i) Draw the angle in standard position.

(ii) Find the smallest positive coterminal angle.

$\theta = 400°$



$\theta = -270°$



Section 1.2: Angles Relationships and Similar Triangles

Vertical Angles:

Vertical angles have equal measures.

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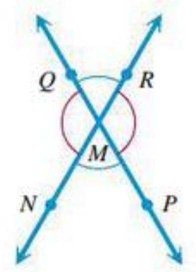
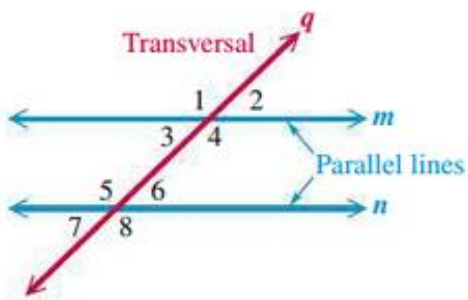
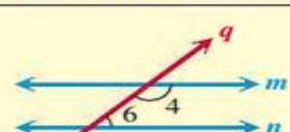
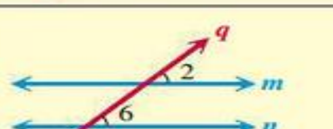


Parallel lines are lines that lie in the same plane and do not intersect. When a line q intersects two parallel lines, m, and n, q is called a ***transversal***

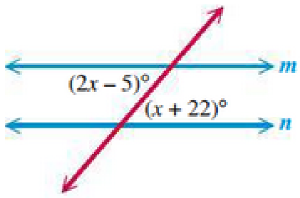


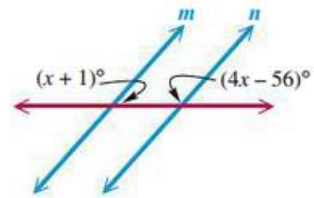
The following table gives the names of these angles and rules about their measures.

Angle Pairs of Parallel Lines Intersected by a Transversal

Name	Sketch	Rule
Alternate interior angles	 (also 3 and 6)	Angle measures are equal.
Alternate exterior angles	 (also 2 and 7)	Angle measures are equal.
Interior angles on the same side of a transversal	 (also 3 and 5)	Angle measures add to 180°.
Corresponding angles	 (also 1 and 5, 3 and 7, 4 and 8)	Angle measures are equal.

Ex 1: Find the measure of each marked angles



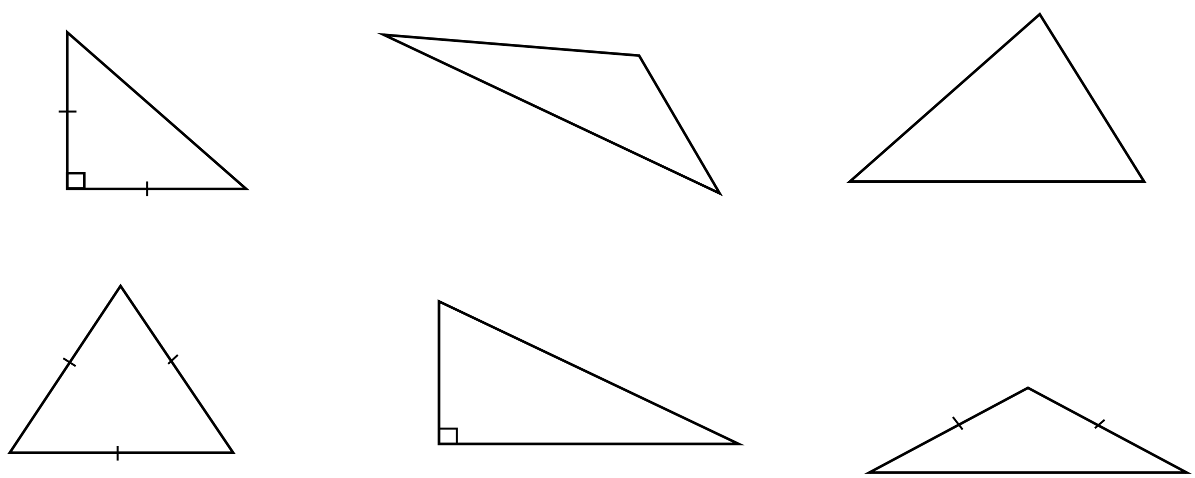


Triangles:

A **triangle** is a **three-side** polygon. Every triangle has three sides and three angles. We denote the angles (vertices) with uppercase letters and the lengths of the sides with lowercase letters.

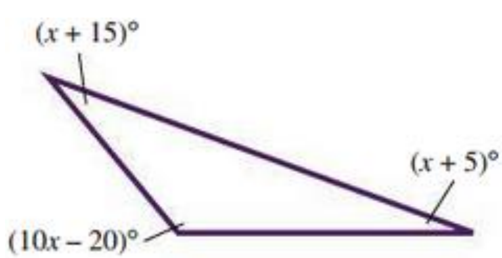
Ex 2: Classify each triangle by its sides and angles by the following names:

Acute Δ, Right Δ, Obtuse Δ, Scalene Δ, Isosceles Δ, Equilateral Δ



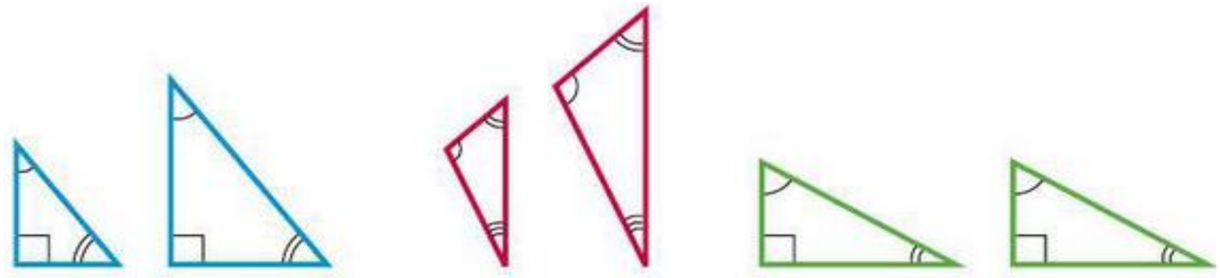
The sum of the measures of the angles of any triangle is 180 degrees.

Ex 3: Find the value of *x*.

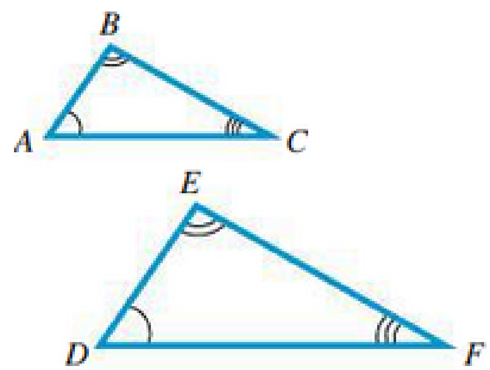


Similar Triangles:

Similar triangles are triangles of exactly the same shape but not necessarily the same size. Triangles that are both the same size and the same shape are **congruent triangles**.



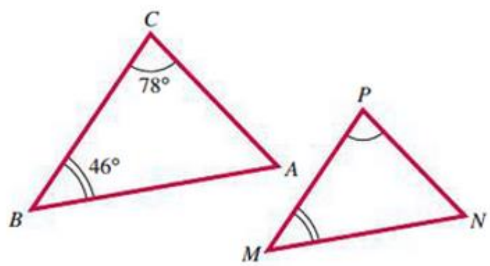
Conditions for Similar Triangles:



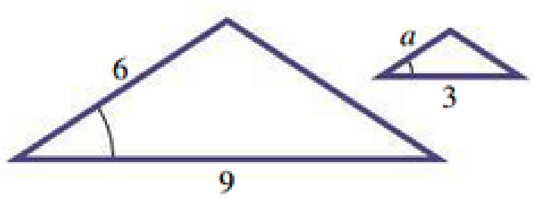
Triangle ABC is similar to triangle DEF if the following conditions hold.

1. Corresponding angles have the same measure.
2. Corresponding sides are proportional. (That is, the ratios of their corresponding sides are equal.)

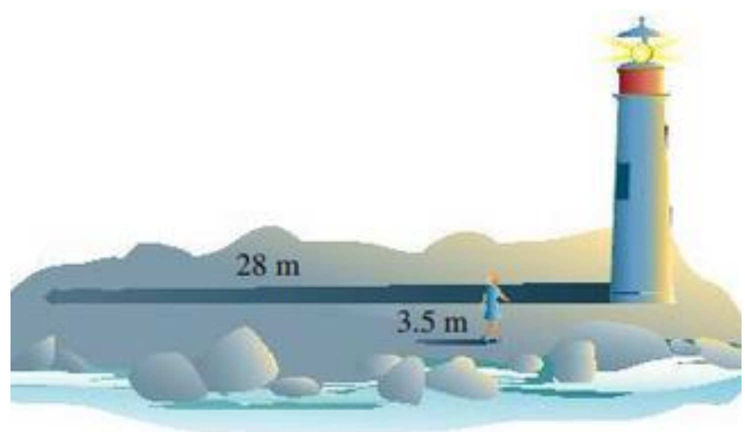
Ex 4: Find all unknown angle measures in each pair of similar triangles.



Ex 5: Find the unknown side lengths in each pair of similar triangles.



Ex 6: The Biloxi lighthouse in the figure casts a shadow 28 m long at 7 AM. At the same time, the shadow of the lighthouse keeper, who is 1.75 m tall, is 3.5 m long. How tall is the lighthouse?



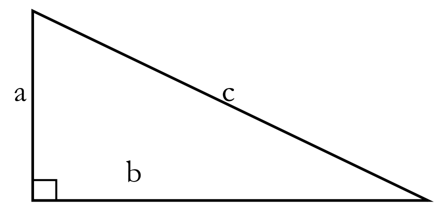
NOT TO SCALE

“Failure is success if we learn from it.” – Malcolm Forbes

Section 1.3: Trigonometric Functions

Pythagorean Theorem:

In any right triangle, the square of the length of the longest side (called hypotenuse) is equal to the sum of the square of the lengths of the other two sides (called legs)



$$c^2 = a^2 + b^2$$

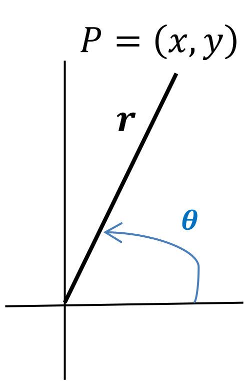
Ex 1: a) If a = 4 and b=3, find c.

b) If a = 5 and c = 8, find b.

Trigonometric Functions:

Let P be a point on the terminal side of an angle, θ, in standard position, then the six trigonometric functions of angle θ are defined as follows:

Trig Function Definitions



$The\ sine\ of\ \theta = sin\theta = \frac{y}{r} = \frac{opposite}{hypotenuse}$

$The\ cosine\ of\ \theta = cos\theta = \frac{x}{r} = \frac{adjacent}{hypotenuse}$

$The\ tangent\ of\ \theta = tan\theta = \frac{y}{x} = \frac{adjacent}{hypotenuse}$ $(x \neq 0)$

$The\ cotangent\ of\ \theta = cos\theta = \frac{x}{y}$ $(y \neq 0)$

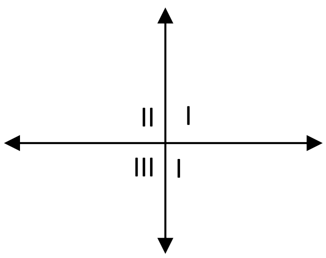
$The\ secant\ of\ \theta = sec\theta = \frac{r}{x}$ $(x \neq 0)$

$The\ cosecant\ of\ \theta = csc\theta = \frac{r}{y}$ $(y \neq 0)$

Where $x^2 + y^2 = r^2$ or $r = \sqrt{x^2 + y^2}$. That is, r is the distance from the origin to (x, y).

We use: s o h - c a h - t o a to remember sine, cosine, and tangent.

Ex 3: The terminal side of θ goes through $P = (12, -5)$. Find all six trig functions of θ.

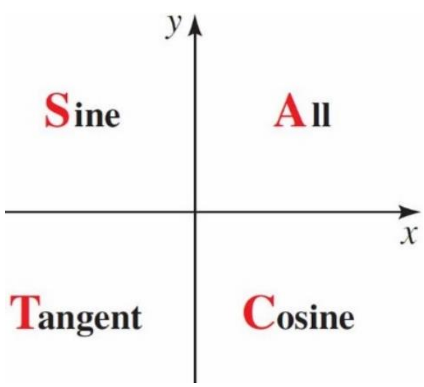


First, we want to find r

$\sin\theta =$ $\tan\theta =$ $\sec\theta =$

$\cos\theta =$ $\cot\theta =$ $\csc\theta =$

Algebraic Signs of Trigonometric Functions:



SIGNS OF THE TRIGONOMETRIC FUNCTIONS

Quadrant	Positive Functions	Negative Functions
I	all	none
II	sin, csc	cos, sec, tan, cot
III	tan, cot	sin, csc, cos, sec
IV	cos, sec	sin, csc, tan, cot

"**All Students Take Calculus**."

"The road to success is always under contruction." – Lily Tomlin

Section 1.4: Using the Definitions of the Trigonometric Functions

Recall the six trigonometric functions from section 1.3

We use: s o h - c a h - t o a to remember sine, cosine, and tangent.

$The\ sine\ of\ \theta = sin\theta = \frac{y}{r} =$

$The\ cosine\ of\ \theta = cos\theta = \frac{x}{r} =$

$The\ tangent\ of\ \theta = tan\theta = \frac{y}{x} =$ $(x \neq 0)$

$The\ cotangent\ of\ \theta = cos\theta = \frac{x}{y} =$ $(y \neq 0)$

$The\ secant\ of\ \theta = sec\theta = \frac{r}{x} =$ $(x \neq 0)$

$The\ cosecent\ of\ \theta = csc\theta = \frac{r}{y} =$ $(y \neq 0)$

Where $x^2 + y^2 = r^2$ or $r\ \sqrt{x^2 + y^2}$. That is, r is the distance from the origin to (x, y).

Then it follows directly that

Reciprocal Identities

$tan\theta =$ $cot\theta =$

$csc\theta =$ $sec\theta =$

Ex 1: a) If sinθ=3/5, then secθ =

b) If tanθ=5/8, then cotθ =

c) If cotθ=-8/3, then tanθ =

Quotient Identities $tan\theta =$ $cot\theta =$

Ex 2: If $cos\theta = -\frac{3}{5}$ and $sin\theta = \frac{4}{5}$. Find $tan\theta$ and $cot\theta$

Pythagorean Identities

Recall that for θ in standard position, $x^2 + y^2 = r^2$, and consider three cases…

Dividing by r^2	Dividing by x^2	Dividing by y^2
$x^2 + y^2 = r^2$	$x^2 + y^2 = r^2$	$x^2 + y^2 = r^2$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

Ex 3: a) If $\sin\theta = -\frac{4}{5}$ and θ terminates in QIV, find $\cos\theta$.

b) Find $\tan\theta$ if $\sin\theta = \frac{1}{3}$ and θ terminates in QI