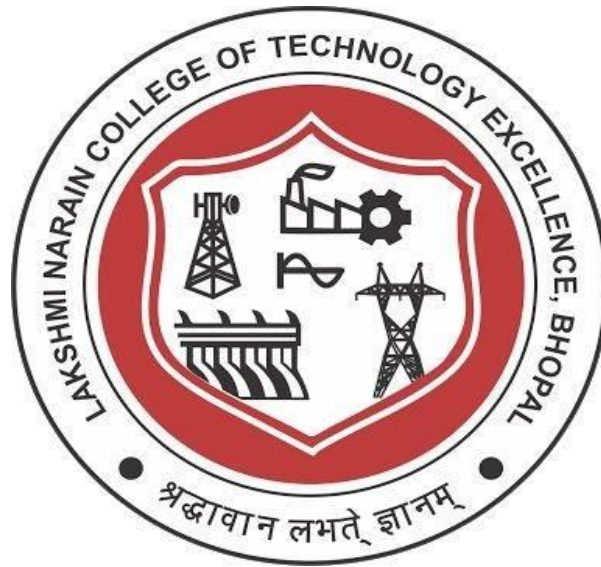


**Lakshmi Narain College Of Technology Excellence  
Bhopal**



**Computer Science  
&  
Training And Placement Department**

**Topic : Smart Health Prediction System**

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**Year : 2<sup>nd</sup> ( 3<sup>rd</sup> Semester )**

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# Introduction

Using Python and Pylab, designed and implemented a stochastic simulation of patient and virus population dynamics, and reached conclusions about treatment regimens based on the simulation results.

## Background: Viruses, Drug Treatments, and Computational Models

Viruses such as HIV and H1N1 represent a significant challenge to modern medicine. One of the reasons that they are so difficult to treat is their ability to evolve.

As you may know, the traits of an organism are determined by its genetic code. When organisms reproduce, their offspring will inherit genetic information from their parent. This genetic information will be modified, either because of mixing of the two parents' genetic information, or through mutations in the genome replication process, thus introducing diversity into a population.

Viruses are no exception. Two characteristics of viruses make them particularly difficult to treat. The first is that their replication mechanism often lacks the error checking mechanisms that are present in more complex organisms. This speeds up the rate of mutation. Secondly, viruses replicate extremely quickly (orders of magnitude faster than humans) -- thus, while we may be used to thinking of evolution as a process which occurs over long time scales, populations of viruses can undergo substantial evolutionary changes within a single patient over the course of treatment.

These two characteristics allow a virus population to acquire genetic resistance to therapy quickly. In this problem set, we will make use of simulations to explore the effect of introducing drugs on the virus population and determine how best to address these treatment challenges within a simplified model.

Computational modeling has played an important role in the study of viruses such as HIV (for example, see [this paper](#), by MIT graduate David Ho). We will implement a highly simplified stochastic model of virus population dynamics. Many details have been swept under the rug (host cells are not explicitly modeled and the size of the population is several orders of magnitude less than the size of actual virus populations). Nevertheless, our model exhibits biologically relevant characteristics and will give you a chance to analyze and interpret interesting simulation data.

## Spread of a Virus in a Person

In reality, diseases are caused by viruses and have to be treated with medicine, so we'll be looking at a detailed simulation of the spread of a virus within a person .

## Implementing a Simple Simulation (No Drug Treatment)

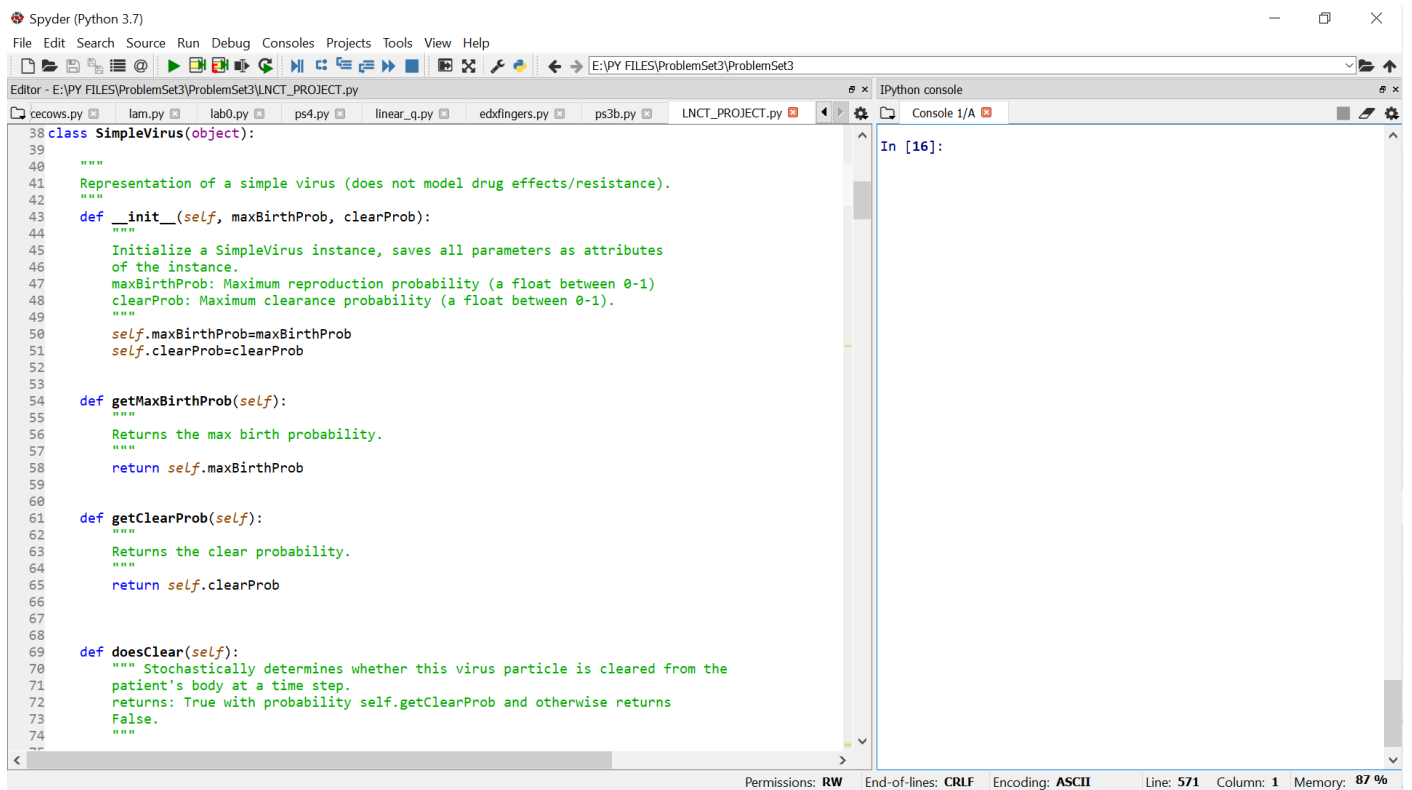
Started with a trivial model of the virus population - the patient does not take any drugs and the viruses do not acquire resistance to drugs. We simply model the virus population inside a patient as if it were left untreated.

## SimpleVirus class

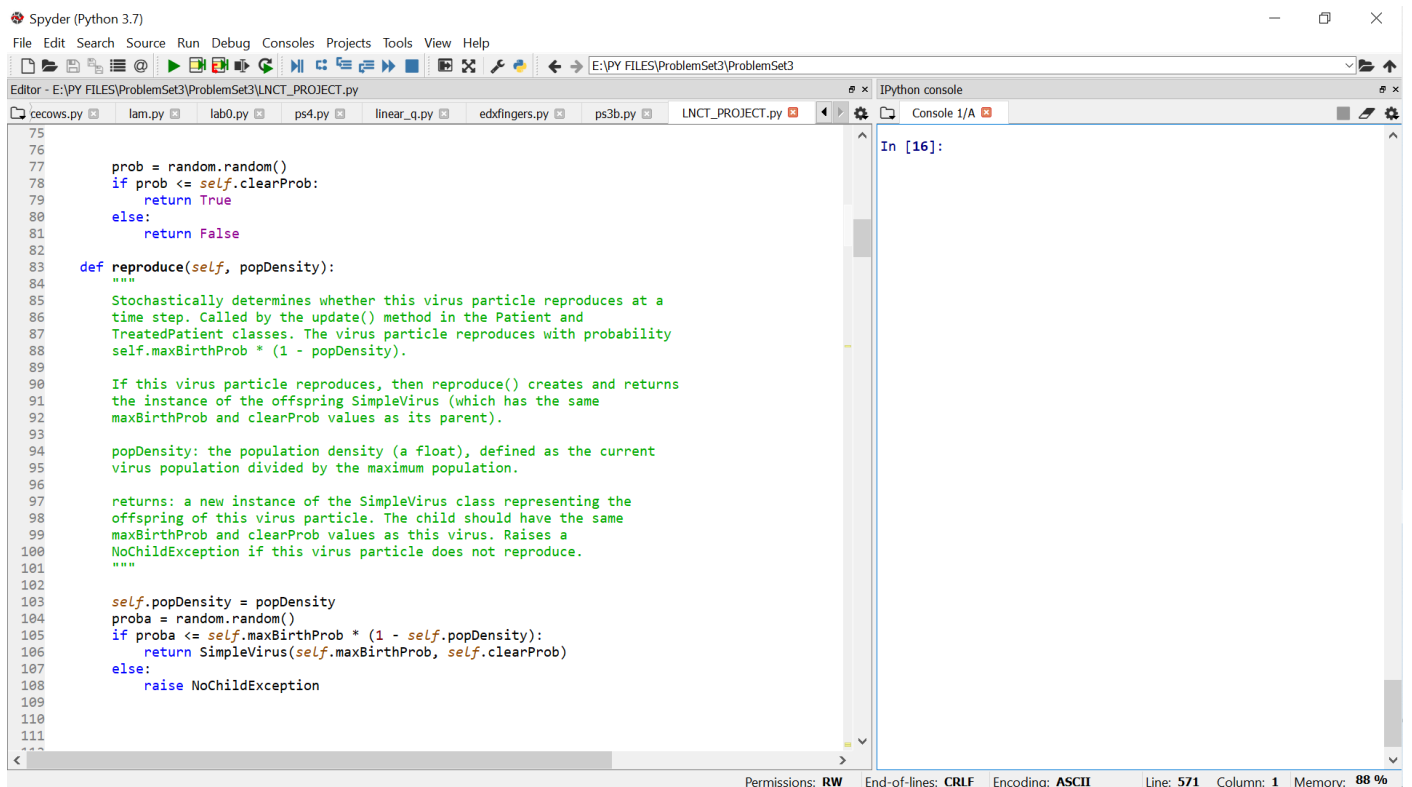
For implementing this model, I made `SimpleVirus` class, which maintains the state of a single virus particle and implemented following methods : `__init__`, `getMaxBirthProb`, `getClearProb`, `doesClear`, and `reproduce` .

The `reproduce` method in `SimpleVirus` produces an offspring by returning a new instance of `SimpleVirus` with probability: `self.maxBirthProb * (1 - popDensity)`. This method raises a `NoChildException` if the virus particle does not reproduce.

`self.maxBirthProb` is the birth rate under optimal conditions (the virus population is negligible relative to the available host cells so there is ample nourishment available). `popDensity` is defined as the ratio of the current virus population to the maximum virus population for a patient and should be calculated in the `update` method of the `Patient` class.



```
38 class SimpleVirus(object):
39
40     """
41     Representation of a simple virus (does not model drug effects/resistance).
42     """
43     def __init__(self, maxBirthProb, clearProb):
44         """
45         Initialize a SimpleVirus instance, saves all parameters as attributes
46         of the instance.
47         maxBirthProb: Maximum reproduction probability (a float between 0-1)
48         clearProb: Maximum clearance probability (a float between 0-1).
49         """
50         self.maxBirthProb = maxBirthProb
51         self.clearProb = clearProb
52
53     def getMaxBirthProb(self):
54         """
55         Returns the max birth probability.
56         """
57         return self.maxBirthProb
58
59     def getClearProb(self):
60         """
61         Returns the clear probability.
62         """
63         return self.clearProb
64
65     def doesClear(self):
66         """
67         Stochastically determines whether this virus particle is cleared from the
68         patient's body at a time step.
69         returns: True with probability self.getClearProb and otherwise returns
70         False.
71         """
```



```
75
76     prob = random.random()
77     if prob <= self.clearProb:
78         return True
79     else:
80         return False
81
82     def reproduce(self, popDensity):
83         """
84         Stochastically determines whether this virus particle reproduces at a
85         time step. Called by the update() method in the Patient and
86         TreatedPatient classes. The virus particle reproduces with probability
87         self.maxBirthProb * (1 - popDensity).
88
89         If this virus particle reproduces, then reproduce() creates and returns
90         the instance of the offspring SimpleVirus (which has the same
91         maxBirthProb and clearProb values as its parent).
92
93         popDensity: the population density (a float), defined as the current
94         virus population divided by the maximum population.
95
96         returns: a new instance of the SimpleVirus class representing the
97         offspring of this virus particle. The child should have the same
98         maxBirthProb and clearProb values as this virus. Raises a
99         NoChildException if this virus particle does not reproduce.
100         """
101
102         self.popDensity = popDensity
103         proba = random.random()
104         if proba <= self.maxBirthProb * (1 - self.popDensity):
105             return SimpleVirus(self.maxBirthProb, self.clearProb)
106         else:
107             raise NoChildException
108
109
110
111
```

## Patient class

Implemented the `Patient` class, which maintains the state of a virus population associated with a patient.

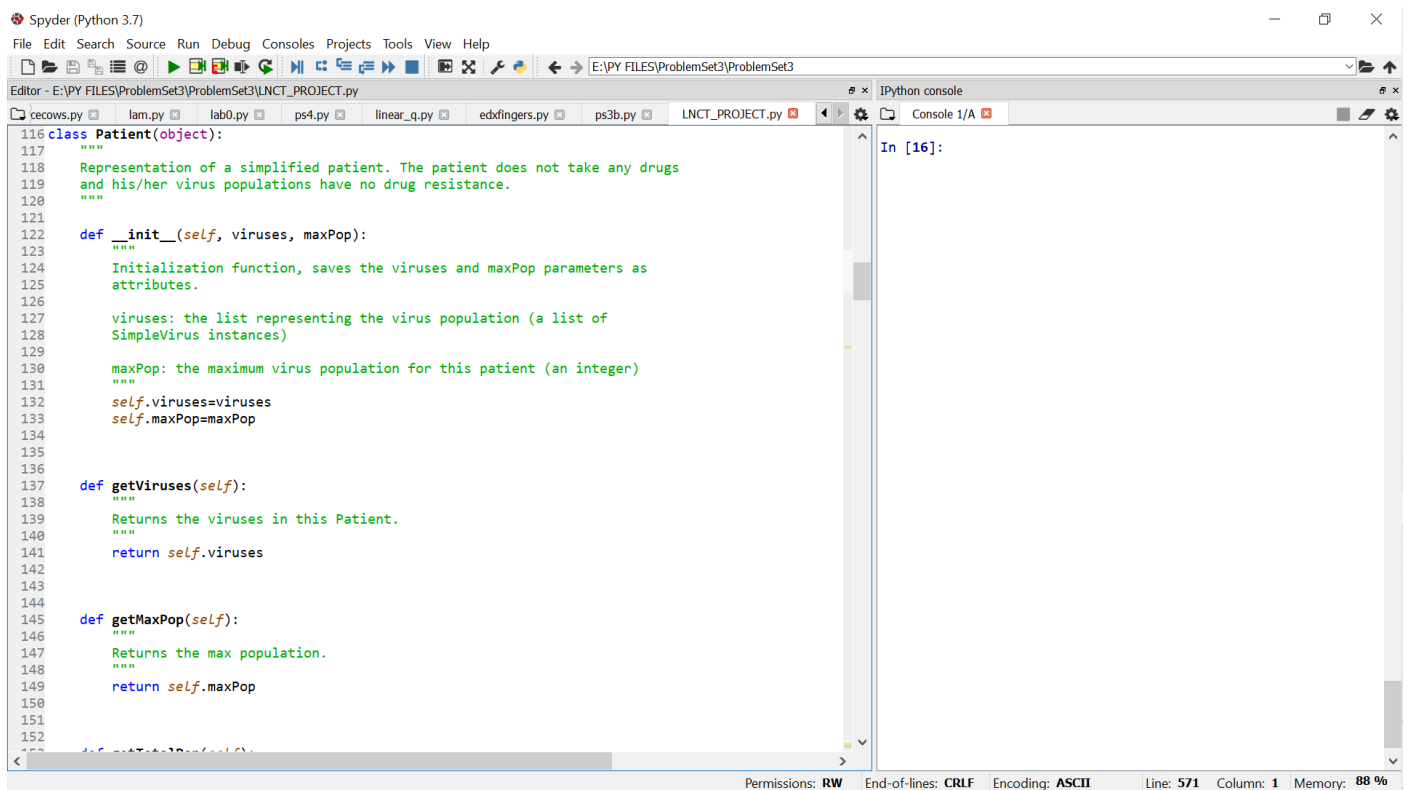
The `update` method in the `Patient` class is the inner loop of the simulation. It modifies the state of the virus population for a single time step and returns the total virus population at the end of the time step. At every time step of the simulation, each virus particle has a fixed probability of being cleared (eliminated from the patient's body). If the virus particle is not cleared, it is considered for reproduction. Utilized the population density correctly which resulted in ending up at a situation at which I shouldn't need to provide an explicit check that the virus population exceeds `maxPop` when calculating how many offspring are added to the population -- just calculated the new population density and used that for the next call to `update`.

Unlike the clearance probability, which is constant, the probability of a virus particle reproducing is a function of the virus population. With a larger virus population, there are fewer resources in the patient's body to facilitate reproduction, and the probability of reproduction will be lower. One way to think of this limitation is to consider that virus particles need to make use of a patient's cells to reproduce; they cannot reproduce on their own. As the virus population increases, there will be fewer available host cells for viruses to utilize for reproduction.

To summarize, `update` should first decide which virus particles are cleared and which survive by making use of the `doesClear` method of each `SimpleVirus` instance, then update the collection of `SimpleVirus` instances accordingly.

With the surviving `SimpleVirus` instances, `update` should then call the `reproduce` method for each virus particle.

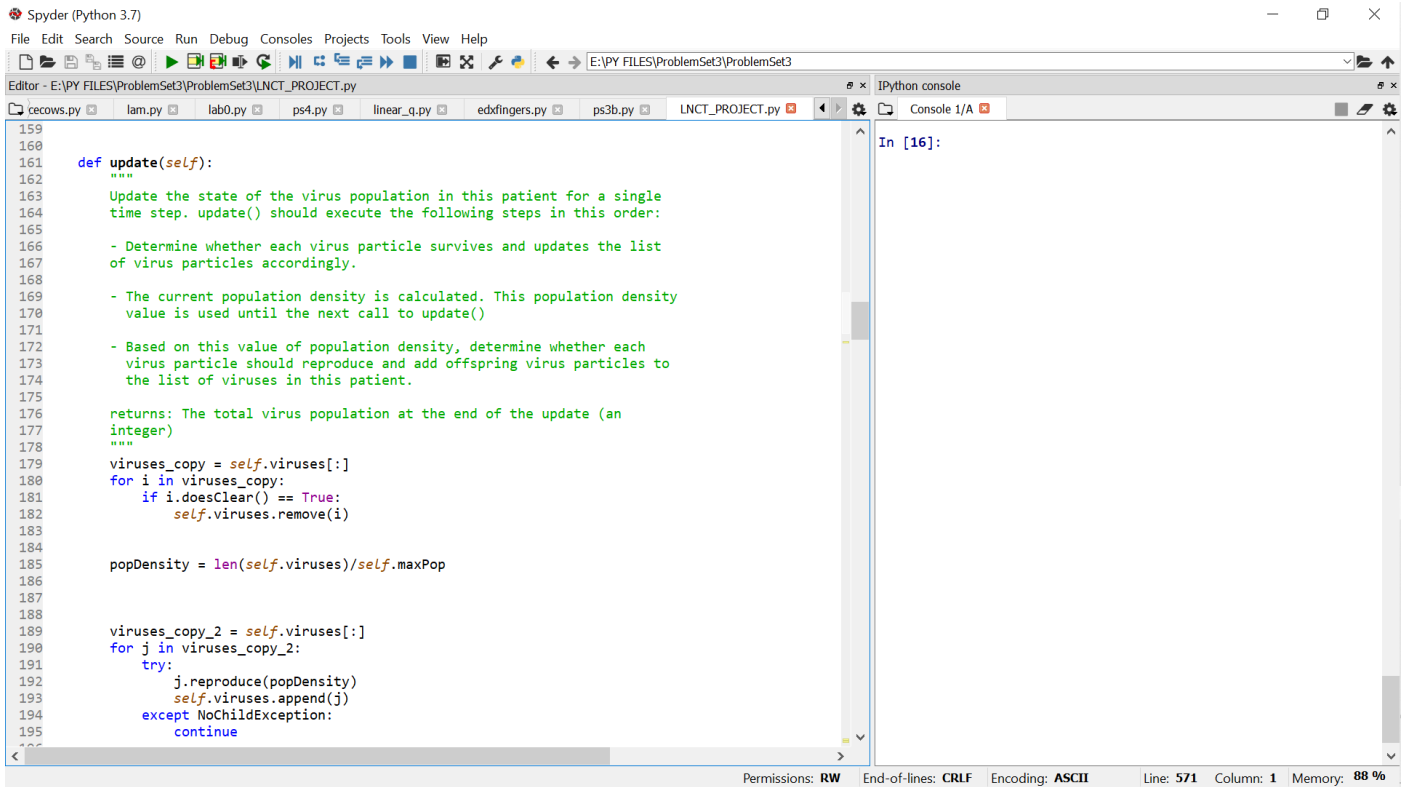
Based on the population density of the surviving `SimpleVirus` instances, `reproduce` should either return a new instance of `SimpleVirus` representing the offspring of the virus particle, or raise a `NoChildException` indicating that the virus particle does not reproduce during the current time step. The `update` method should update the attributes of the patient appropriately under either of these conditions. After iterating through all the virus particles, the `update` method returns the number of virus particles in the patient at the end of the time step.



```
116 class Patient(object):
117     """
118     Representation of a simplified patient. The patient does not take any drugs
119     and his/her virus populations have no drug resistance.
120     """
121
122     def __init__(self, viruses, maxPop):
123         """
124         Initialization function, saves the viruses and maxPop parameters as
125         attributes.
126
127         viruses: the list representing the virus population (a list of
128         SimpleVirus instances)
129
130         maxPop: the maximum virus population for this patient (an integer)
131         """
132         self.viruses=viruses
133         self.maxPop=maxPop
134
135
136
137     def getViruses(self):
138         """
139         Returns the viruses in this Patient.
140         """
141         return self.viruses
142
143
144
145     def getMaxPop(self):
146         """
147         Returns the max population.
148         """
149         return self.maxPop
150
151
152
153     def getInitialPop(self):
154         """
155         Returns the initial population.
156         """
157         return self.viruses
```

In [16]:

Permissions: RW End-of-lines: CRLF Encoding: ASCII Line: 571 Column: 1 Memory: 88 %



The screenshot shows the Spyder Python IDE interface. The main window is titled 'Spyder (Python 3.7)' and contains a menu bar (File, Edit, Search, Source, Run, Debug, Consoles, Projects, Tools, View, Help) and a toolbar. The editor pane shows a file named 'LNCT\_PROJECT.py' with the following Python code:

```
159
160
161 def update(self):
162     """
163     Update the state of the virus population in this patient for a single
164     time step. update() should execute the following steps in this order:
165
166     - Determine whether each virus particle survives and updates the list
167     of virus particles accordingly.
168
169     - The current population density is calculated. This population density
170     value is used until the next call to update()
171
172     - Based on this value of population density, determine whether each
173     virus particle should reproduce and add offspring virus particles to
174     the list of viruses in this patient.
175
176     returns: The total virus population at the end of the update (an
177     integer)
178     """
179     viruses_copy = self.viruses[:]
180     for i in viruses_copy:
181         if i.doesClear() == True:
182             self.viruses.remove(i)
183
184
185     popDensity = len(self.viruses)/self.maxPop
186
187
188     viruses_copy_2 = self.viruses[:]
189     for j in viruses_copy_2:
190         try:
191             j.reproduce(popDensity)
192             self.viruses.append(j)
193         except NoChildException:
194             continue
195
```

The IPython console on the right shows the prompt 'In [16]:'.

At the bottom of the IDE, a status bar displays: Permissions: RW, End-of-lines: CRLF, Encoding: ASCII, Line: 571, Column: 1, Memory: 88 %.

## Running and Analyzing a Simple Simulation (No Drug Treatment)

`simulationWithoutDrug(numViruses, maxPop, maxBirthProb, clearProb, numTrials)` that instantiates a `Patient`, simulates changes to the virus population for 300 time steps (i.e., 300 calls to `update`), and plots the average size of the virus population as a function of time; that is, the x-axis should correspond to the number of elapsed time steps, and the y-axis should correspond to the average size of the virus population in the patient. The population at time=0 is the population after the first call to `update`.

Run the simulation for `numTrials` trials, where `numTrials` in this case can be up to 100 trials. Use `pylab` to produce a plot (with a single curve) that displays the average result of running the simulation for many trials.

Called `simulationWithoutDrug` with the following parameters:

- `numViruses` = 100
- `maxPop` (maximum sustainable virus population) = 1000
- `maxBirthProb` (maximum reproduction probability for a virus particle) = 0.1
- `clearProb` (maximum clearance probability for a virus particle) = 0.05

Thus, your simulation should be instantiating one `Patient` with a list of 100 `SimpleVirus` instances.

Each `SimpleVirus` instance in the `viruses` list should be initialized with the proper values for `maxBirthProb` and `clearProb`.

Spyder (Python 3.7)

File Edit Search Source Run Debug Consoles Projects Tools View Help

E:\PY FILES\ProblemSet3\ProblemSet3

Editor - E:\PY FILES\ProblemSet3\ProblemSet3\LNCT\_PROJECT.py

```

204 def simulationWithoutDrug(numViruses, maxPop, maxBirthProb, clearProb, numTrials):
205     """
206     Run the simulation and plot the graph for problem 3 (no drugs are used,
207     viruses do not have any drug resistance).
208     For each of numTrials trial, instantiates a patient, runs a simulation
209     for 300 timesteps, and plots the average virus population size as a
210     function of time.
211
212     numViruses: number of SimpleVirus to create for patient (an integer)
213     maxPop: maximum virus population for patient (an integer)
214     maxBirthProb: Maximum reproduction probability (a float between 0-1)
215     clearProb: Maximum clearance probability (a float between 0-1)
216     numTrials: number of simulation runs to execute (an integer)
217
218     """
219
220
221     viruses=[SimpleVirus(maxBirthProb,clearProb) for i in range(numViruses)]
222
223     list_step=[0 for i in range(300)]
224
225     for trial in range(numTrials):
226         viruses_copy=viruses[:]
227
228         a=Patient(viruses_copy,maxPop)
229
230         for step in range(300):
231             a.update()
232             list_step[step]=list_step[step]+a.getTotalPop()
233
234     averages=[pop/numTrials for pop in list_step ]
235
236     pylab.plot(averages, label = "SimpleVirus")
237     pylab.title("SimpleVirus simulation")
238     pylab.xlabel("Time Steps")
239     pylab.ylabel("Average Virus Population")
240     pylab.legend(loc = "best")

```

IPython console

In [16]:

Permissions: RW End-of-lines: CRLF Encoding: ASCII Line: 204 Column: 71 Memory: 82 %

## Results obtained :

Spyder (Python 3.7)

File Edit Search Source Run Debug Consoles Projects Tools View Help

E:\PY FILES\ProblemSet3\ProblemSet3

Editor - E:\PY FILES\ProblemSet3\ProblemSet3\LNCT\_PROJECT.py

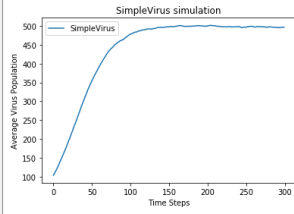
```

537
538
539     patient = TreatedPatient(virus_copy, maxPop)
540
541     for step in range(150):
542         patient.update()
543         list_step_without_prescription[step]=list_step_without_prescription[step]+patient.getTot
544         list_step_with_prescription[step]=list_step_with_prescription[step]+patient.getResistPop
545
546     patient.addPrescription('guttagonol')
547
548     for step in range(150,300):
549         patient.update()
550         list_step_without_prescription[step]=list_step_without_prescription[step]+patient.getTot
551         list_step_with_prescription[step]=list_step_with_prescription[step]+patient.getResistPop
552
553
554     average_without_prescription=[pop/numTrials for pop in list_step_without_prescription]
555     average_with_prescription=[pop/numTrials for pop in list_step_with_prescription]
556     pylab.plot(average_without_prescription, 'r-', label="Without Prescription Trend")
557     pylab.plot(average_with_prescription, 'g-', label="With Prescription Trend")
558
559     pylab.title(" Virus Simulation")
560     pylab.xlabel("Time Steps")
561     pylab.ylabel("Average Non-Resistant Virus Population and Resistant Virus Population ")
562     pylab.legend(loc = "best")
563     pylab.show()
564
565
566
567
568 #doing my test
569 #simulationWithoutDrug(numViruses, maxPop, maxBirthProb, clearProb, numTrials)
570 #simulationWithDrug(numViruses, maxPop, maxBirthProb, clearProb, resistances,mutProb, numTrials)
571 simulationWithoutDrug(100, 1000, 0.1,0.05,200)
572 #simulationWithDrug(100, 1000, 0.1, 0.05, {'guttagonol': False}, 0.005, 100)
573

```

IPython console

In [15]: runfile('E:/PY FILES/ProblemSet3/ProblemSet3/LNCT\_PROJECT.py', wdir='E:/PY FILES/ProblemSet3/ProblemSet3')



In [16]:

Permissions: RW End-of-lines: CRLF Encoding: ASCII Line: 571 Column: 1 Memory: 85 %

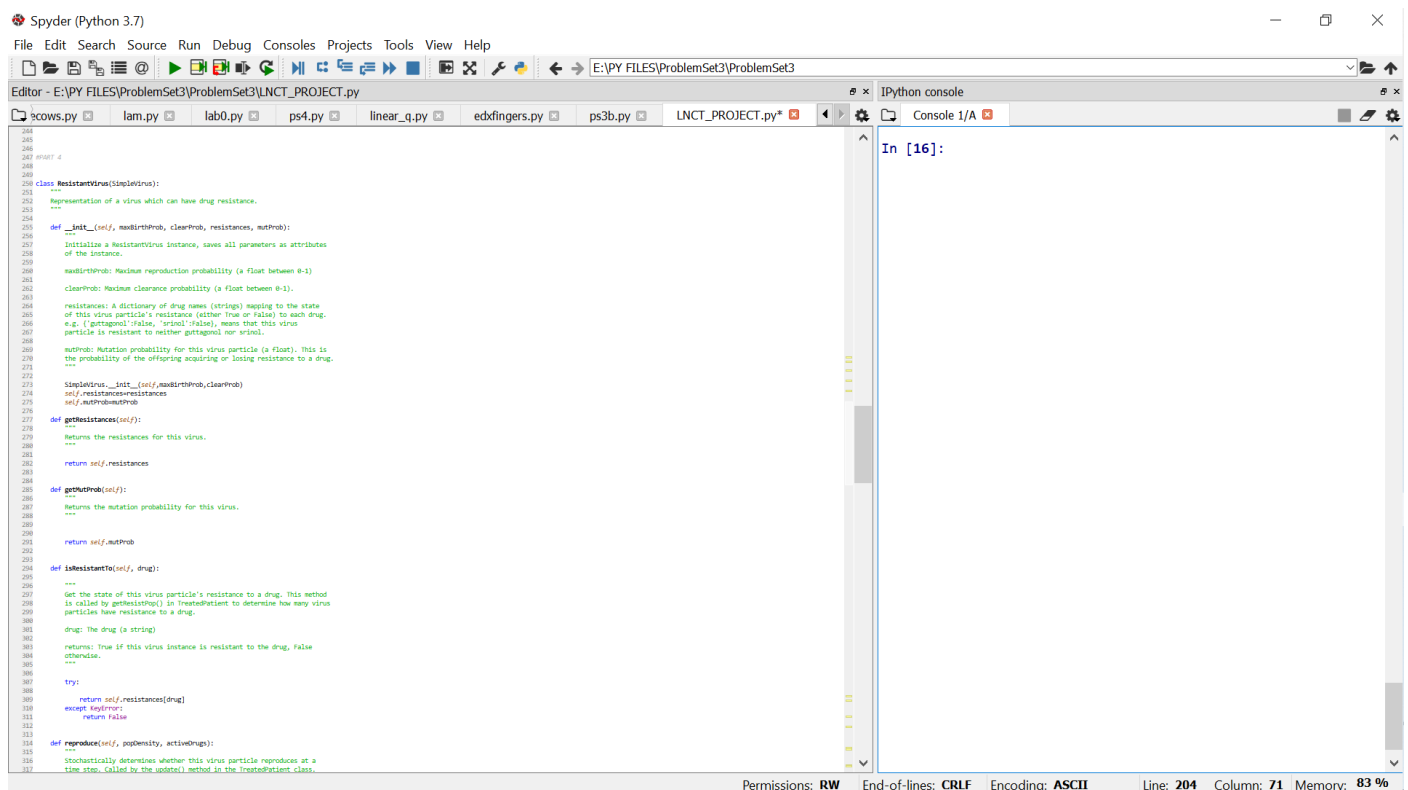
# Implementing a Simulation With Drugs

In this problem, we consider the effects of both administering drugs to the patient and the ability of virus particle offsprings to inherit or mutate genetic traits that confer drug resistance. As the virus population reproduces, mutations will occur in the virus offspring, adding genetic diversity to the virus population. Some virus particles gain favorable mutations that confer resistance to drugs.

## ResistantVirus class

In order to model this effect, we introduce a subclass of `SimpleVirus` called `ResistantVirus`. `ResistantVirus` maintains the state of a virus particle's drug resistances, and accounts for the inheritance of drug resistance traits to offspring. Implement the `ResistantVirus` class.

(zoom\_in to see clearly)

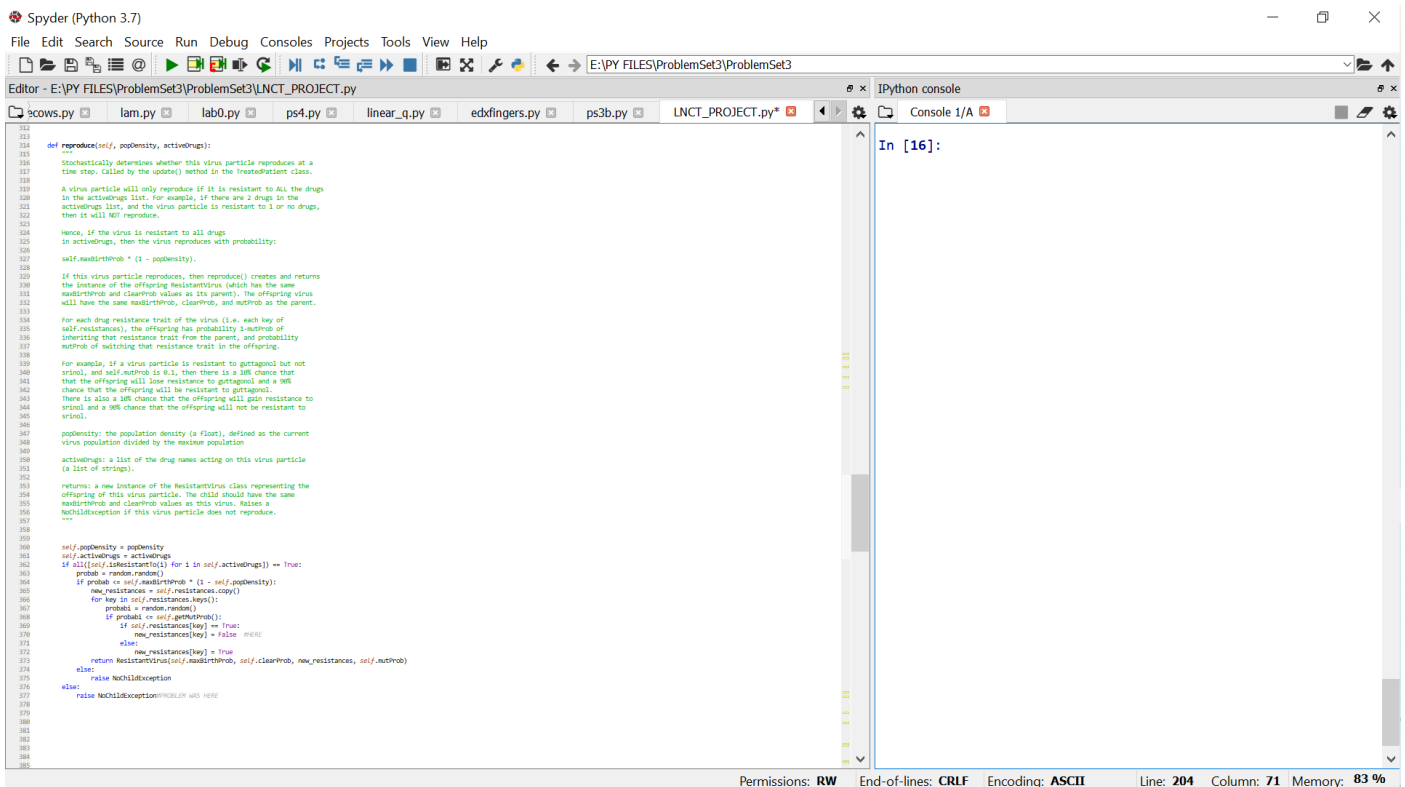


The screenshot shows the Spyder Python IDE with the `ResistantVirus` class implemented as a subclass of `SimpleVirus`. The code is as follows:

```
204
205
206 @pytest
207
208
209 class ResistantVirus(SimpleVirus):
210     """
211     Representation of a virus which can have drug resistance.
212     """
213
214     def __init__(self, maxBirthProb, clearProb, resistances, mutProb):
215         """
216         Initialize a ResistantVirus instance, saves all parameters as attributes
217         of the instance.
218
219         maxBirthProb: Maximum reproduction probability (a float between 0-1)
220
221         clearProb: Maximum clearance probability (a float between 0-1)
222
223         resistances: A dictionary of drug names (strings) mapping to the state
224         of this virus particle's resistance (either True or False) to each drug.
225         e.g. {'guttagonol':False, 'srinol':False}, means that this virus
226         particle is resistant to neither guttagonol nor srinol.
227
228         mutProb: Mutation probability for this virus particle (a float), this is
229         the probability of the offspring acquiring or losing resistance to a drug.
230         """
231
232         SimpleVirus.__init__(self, maxBirthProb, clearProb)
233         self.resistances = resistances
234         self.mutProb = mutProb
235
236     def getResistances(self):
237         """
238         Returns the resistances for this virus.
239         """
240
241         return self.resistances
242
243     def getMutProb(self):
244         """
245         Returns the mutation probability for this virus.
246         """
247
248         return self.mutProb
249
250     def isResistantTo(self, drug):
251         """
252         Get the state of this virus particle's resistance to a drug. This method
253         is called by getResistances() in TreatedPatient to determine how many virus
254         particles have resistance to a drug.
255
256         drug: the drug (a string)
257
258         returns: True if this virus instance is resistant to the drug, False
259         otherwise.
260         """
261
262         try:
263             return self.resistances[drug]
264         except KeyError:
265             return False
266
267     def reproduce(self, popDensity, activeDrugs):
268         """
269         Stochastically determines whether this virus particle reproduces at a
270         time step, called by the update() method in the TreatedPatient class.
271         """
```

The IPython console on the right shows the prompt `In [16]:`.

At the bottom of the window, the status bar displays: Permissions: RW, End-of-lines: CRLF, Encoding: ASCII, Line: 204, Column: 71, Memory: 83 %.



The screenshot shows the Spyder Python IDE interface. The top menu bar includes File, Edit, Search, Source, Run, Debug, Consoles, Projects, Tools, View, and Help. The toolbar contains icons for file operations, running, and debugging. The editor window displays a file named `LNCT_PROJECT.py` with the following Python code:

```
112 def reproduce(self, popDensity, activeDrugs):
113     """
114     Stochastically determines whether this virus particle reproduces at a
115     time step. Called by the update() method in the TreatedPatient class.
116
117     A virus particle will only reproduce if it is resistant to ALL the drugs
118     in the activeDrugs list. For example, if there are 2 drugs in the
119     activeDrugs list, and the virus particle is resistant to 1 or no drugs,
120     then it will NOT reproduce.
121
122     Hence, if the virus is resistant to all drugs
123     in activeDrugs, then the virus reproduces with probability:
124
125     self.maxBirthProb * (1 - popDensity).
126
127     If this virus particle reproduces, then reproduce() creates and returns
128     the instance of the offspring ResistantVirus (which has the same
129     maxBirthProb and clearProb values as its parent). The offspring virus
130     will have the same maxBirthProb, clearProb, and numProb as the parent.
131
132     For each drug resistance trait of the virus (i.e. each key of
133     self.resistances), the offspring has probability 1-numProb of
134     inheriting that resistance trait from the parent, and probability
135     numProb of switching that resistance trait in the offspring.
136
137     For example, if a virus particle is resistant to gattagool but not
138     vrinol, and self.numProb is 0.3, then there is a 30% chance that
139     that the offspring will lose resistance to gattagool and a 90%
140     chance that the offspring will be resistant to gattagool.
141     There is also a 30% chance that the offspring will gain resistance to
142     vrinol and a 90% chance that the offspring will not be resistant to
143     vrinol.
144
145     popDensity: the population density (a float), defined as the current
146     virus population divided by the maximum population
147
148     activeDrugs: a list of the drug names acting on this virus particle
149     (a list of strings).
150
151     returns: a new instance of the ResistantVirus class representing the
152     offspring of this virus particle. The child should have the same
153     maxBirthProb and clearProb values as this virus. Raises a
154     NoChildException if this virus particle does not reproduce.
155     """
156
157     self.popDensity = popDensity
158     self.activeDrugs = activeDrugs
159     if all([self.isResistantTo(i) for i in self.activeDrugs]) == True:
160         probab = random.random()
161         if probab < self.maxBirthProb * (1 - self.popDensity):
162             new_resistances = self.resistances.copy()
163             for key in self.resistances.keys():
164                 probab1 = random.random()
165                 if probab1 < self.getMutationProb():
166                     if self.resistances[key] == True:
167                         new_resistances[key] = False # HERE
168                     else:
169                         new_resistances[key] = True
170             return ResistantVirus(self.maxBirthProb, self.clearProb, new_resistances, self.numProb)
171         else:
172             raise NoChildException
173     else:
174         raise NoChildException # HERE HERE HERE
175
176
177
178
179
180
181
182
183
184
185
```

The IPython console on the right shows the prompt `In [16]:`.

At the bottom of the IDE, the status bar displays: Permissions: RW, End-of-lines: CRLF, Encoding: ASCII, Line: 204, Column: 71, Memory: 83 %.

## TreatedPatient Class

Needed a representation for a patient that accounts for the use of drug treatments and manages a collection of `ResistantVirus` instances. For that introduced the `TreatedPatient` class, which is a subclass of `Patient`. `TreatedPatient` must make use of the new methods in `ResistantVirus` and maintain the list of drugs that are administered to the patient.

Drugs are given to the patient using the `TreatedPatient` class's `addPrescription()` method. What happens when a drug is introduced? The drugs we consider do not directly kill virus particles lacking resistance to the drug, but prevent those virus particles from reproducing (much like actual drugs used to treat HIV). Virus particles with resistance to the drug continue to reproduce normally. Implement the `TreatedPatient` class.

(zoom\_in\_to\_view)





- `maxPop`, maximum sustainable virus population = 1000

Each `ResistantVirus` instance in the viruses list should be initialized with the following parameters:

- `maxBirthProb`, maximum reproduction probability for a virus particle = 0.1
- `clearProb`, maximum clearance probability for a virus particle = 0.05
- `resistances`, The virus's genetic resistance to drugs in the experiment = {'guttagonol': False}
- `mutProb`, probability of a mutation in a virus particle's offspring = 0.005

Ran simulation that consists of 150 time steps, followed by the addition of the drug, guttagonol, followed by another 150 time steps.

Used of the function `simulationWithDrug(numViruses, maxPop, maxBirthProb, clearProb, resistances, mutProb, numTrials)` and performed up to 100 trials and made sure that results were repeatable and representative.

Created one plot that recorded both the average total virus population and the average population of guttagonol-resistant virus particles over time.

And considered some aspects like : What trends do you observe? Are the trends consistent with your intuition?

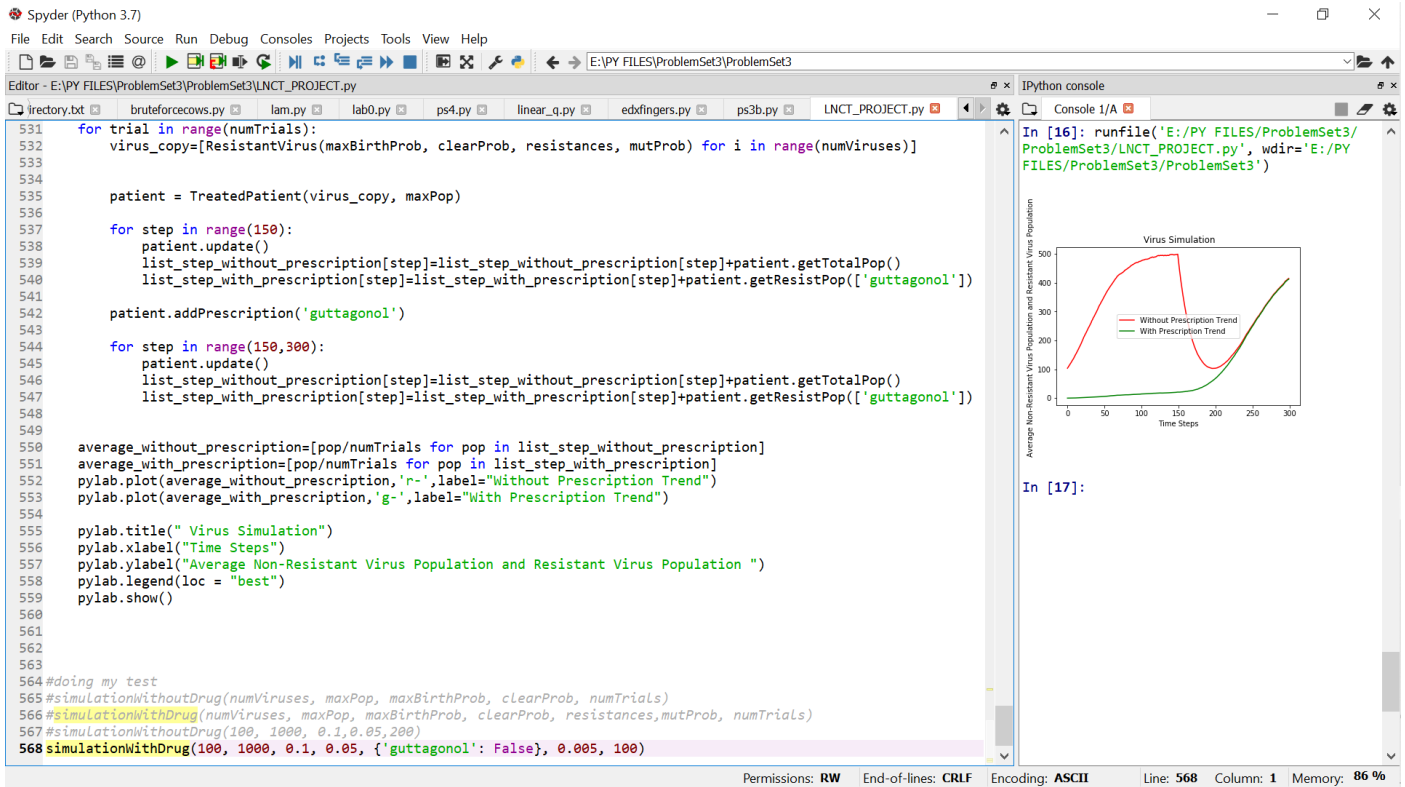
The screenshot shows the Spyder Python IDE interface. The editor window displays the `simulationWithDrug` function, which is a docstring and a function definition. The docstring describes the function's purpose and parameters. The function definition includes a loop over `numTrials` and a loop over `step` (0 to 150) to simulate the virus population over time. The IPython console on the right shows the prompt `In [16]:`.

```

504 def simulationWithDrug(numViruses, maxPop, maxBirthProb, clearProb, resistances, mutProb, numTrials):
505     """
506     Runs simulations and plots graphs for problem 5.
507
508     For each of numTrials trials, instantiates a patient, runs a simulation for
509     150 timesteps, adds guttagonol, and runs the simulation for an additional
510     150 timesteps. At the end plots the average virus population size
511     (for both the total virus population and the guttagonol-resistant virus
512     population) as a function of time.
513
514     numViruses: number of ResistantVirus to create for patient (an integer)
515     maxPop: maximum virus population for patient (an integer)
516     maxBirthProb: Maximum reproduction probability (a float between 0-1)
517     clearProb: maximum clearance probability (a float between 0-1)
518     resistances: a dictionary of drugs that each ResistantVirus is resistant to
519     (e.g., {'guttagonol': False})
520     mutProb: mutation probability for each ResistantVirus particle
521     (a float between 0-1).
522     numTrials: number of simulation runs to execute (an integer)
523     """
524
525
526
527     list_step_without_prescription=[0 for i in range(300)]
528     list_step_with_prescription=[0 for i in range(300)]
529
530
531     for trial in range(numTrials):
532         virus_copy=[ResistantVirus(maxBirthProb, clearProb, resistances, mutProb) for i in range(numViruses)]
533
534
535         patient = TreatedPatient(virus_copy, maxPop)
536
537         for step in range(150):
538             patient.update()
539             list_step_without_prescription[step]=list_step_without_prescription[step]+patient.getTotalPop()
540             list_step_with_prescription[step]=list_step_with_prescription[step]+patient.getResistPop(['guttagonol'])
541
542         patient.addPrescription('guttagonol')

```

Permissions: RW End-of-lines: CRLF Encoding: ASCII Line: 204 Column: 71 Memory: 84 %



**CODES:** <https://drive.google.com/drive/folders/1PEbsRDEMv2yJLBxthiUHIinhmrh85bXJ?usp=sharing> (.txt and .py files )

## Bibliography And Resources :

**REFERENCES** - 1. Introduction to programming and data science with python by John V Guttag &  
. 2. Edx courses 6.00.2x (especially pset 3 ) and 6.00.1x

**Certification ( For authenticity of the work )** - <https://www.linkedin.com/in/prince-dwivedi>

OR

<https://courses.edx.org/certificates/2a92171aa82a436ea2c7a45691115567>