Call-By-Unboxed-Value

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Unboxing in Practice

The Good, the Bad, and the Ugly

- Good: Unboxed values enables high-performance
- Bad: Low-level code clashes with high-level abstractions (e.g., polymorphism)
- Representation irrelevance resolves the low-level tension (e.g., Levity Polymorphism and Kinds Are Calling Conventions)
    - Restrictions (sometimes surprising) needed for operational meaning & compilability
    - *"If I can't compile it, the type checker must reject it"*

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  - Restrictions (sometimes surprising) needed for operational meaning & compilability
  - *"If I can't compile it, the type checker must reject it"*
- Call-By-Unboxed-Value explains the high-level meaning of unboxing
  - Logical & semantic foundation ensures meaningful programs
  - *"If I can write it, I can compile & run it"*

Compiling With Call-By-Unboxed-Value

- Compiling unboxed polymorphism before:
  - Only compile well-typed source programs; need typing information to generate code
  - Generate ill-typed target programs; compilation can break precise typing
  - *"Types describe the source, kinds describe the machine"*

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- Compiling unboxed polymorphism before:
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  - *"Types describe the source, kinds describe the machine"*
- Compiling unboxed polymorphism with Call-By-Unboxed-Value:
  - Can compile untyped source programs; no typing information needed
  - Compilation preserves typing if the source was well-typed
  - Lower-level abstract machine code can be expressed in a type-safe target language
  - Still support type erasure without changing answers

Unboxed Values

Holding Numbers in Registers

To avoid creating garbage & chasing pointers

$$
\begin{aligned}
\mathit{sumTo0} &: \mathsf{Int} \to \mathsf{Int} \\
\mathit{sumTo0}\ 0 &= 0 \\
\mathit{sumTo0}\ n &= n + \mathit{sumTo0}(n - 1)
\end{aligned}
$$

Is *n* an integer register, or a pointer into the heap?

Holding Numbers in Registers

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Is n an integer register, or a pointer into the heap?

Accumulator style $\implies$ fast loop

$$\begin{aligned}
&\mathit{sumTo0}' \quad : \mathsf{Int} \to \mathsf{Int} \\
&\mathit{sumTo0}'\ n = \mathit{go}\ n\ 0 \\
&\qquad \textbf{where}\ \mathit{go}\ 0\ \mathit{acc} = \mathit{acc} \\
&\qquad\qquad \mathit{go}\ n\ \mathit{acc} = \mathit{go}\ (n - 1)\ (n + \mathit{acc})
\end{aligned}$$

Problems with Polymorphism

What does a compiler need to know to generate code?

Could the polymorphic a really be any type?

$$
\begin{aligned}
id &: a \to a \\
id\ x &= x
\end{aligned}
$$

Problems with Polymorphism

Could the polymorphic *a* really be <u>any</u> type?

$$\begin{aligned} id &: a \to a \\ id\ x &= x \end{aligned}$$

Need to know *a*'s representation to generate low-level machine code:

- Where does x live? (General or specialized register? Heap?)
- How many bits does x occupy? (32? 64? 8?)
- How to copy/move x from (incoming) parameter to (outgoing) return?

Polymorphic Ambiguity

Do we need to know a and b's representations to compile app?

$$
\begin{aligned}
&app &&: (a \to b) \to a \to b \\
&app\ f\ x &&= f\ x
\end{aligned}
$$

Polymorphic Ambiguity

Do we need to know a and b's representations to compile app?

$$\begin{aligned} app &: (a \to b) \to a \to b \\ app\ f\ x &= f\ x \end{aligned}$$

- a: Yes, to move x
- b: It depends...
  - Naïvely yes, to move f's result to $(app\ f\ x)$'s caller
  - But with tail-call optimization, app never handles any b's

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What about after η-reduction?

$$\begin{aligned} &app' &&: (a \to b) \to a \to b \\ &app'\, f = f \end{aligned}$$

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What about after η-reduction?

$$\begin{aligned} app' &: (a \to b) \to a \to b \\ app'\ f &= f \end{aligned}$$

- a and b's representations are irrelevant!
- Only move $f : a \to b$, always a pointer

Higher-Order Ambiguity

What do we need to know about a and b?

$$
\begin{aligned}
&map && : (a \to b) \to [a] \to [b] \\
&map\ f\ [\,] && = [\,] \\
&map\ f\ (x : xs) && = (f\ x) : (map\ f\ xs)
\end{aligned}
$$

Higher-Order Ambiguity

When calling unknown functions

What do we need to know about a and b?

$$
\begin{aligned}
&\mathit{map} && : (a \to b) \to [a] \to [b] \\
&\mathit{map}\ f\ [] && = [] \\
&\mathit{map}\ f\ (x : xs) && = (f\ x) : (\mathit{map}\ f\ xs)
\end{aligned}
$$

- Representations of both a and b
  - To move $x : a$ around
  - To store $(f\ x) : b$ in a list
- Calling convention of b
  - What if $f : \mathsf{Int} \to \mathsf{Int} \to \mathsf{Int}$?
  - $b = \mathsf{Int} \to \mathsf{Int}$ is a function, needs 1 more argument
  - $(f\ x) : \mathsf{Int} \to \mathsf{Int}$ might be a partial application, can't jump to f's body
  - To generate code, need to distinguish partial applications from real calls

A First Taste of Call-By-Unboxed-Value

The Two Axes of Unboxing

- Familiar: Values versus computations
  - Values = being
  - Computations = doing
- New: Complexity versus Atomicity
  - Atomic = one
  - Complex = many (parts, choices, ...)

Call-By-Unboxed-Value

Having a name is a privilege, not a right

- Functions are called with complex unboxed values
  - Only atomic values are first class, can be named
  - Complex values are second class, <u>must</u> be matched
- Functions themselves are complex computations
  - Only atomic computations can be run directly
  - Complex computations are inert on their own, <u>must</u> match their context (η-long)

Elaborating Functional Code to Call-By-Unboxed-Value

Source (CBV)

$$\begin{aligned} &\mathit{sumTo0} \quad : \mathsf{Nat} \to \mathsf{Nat} \\ &\mathit{sumTo0}\ 0 = 0 \\ &\mathit{sumTo0}\ n = n + \mathit{sumTo0}(n-1) \end{aligned}$$

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CBPV

$$\begin{aligned} &\mathit{sumTo0} : \mathsf{Nat} \to \mathsf{F}\,\mathsf{Nat} \\ &\mathit{sumTo0} = \lambda n.\ \textbf{if}\ n == 0\ \textbf{then return}\ 0 \\ &\qquad\qquad\qquad\qquad\quad \textbf{else do}\ x \leftarrow n - 1; \\ &\qquad\qquad\qquad\qquad\qquad\quad \textbf{do}\ y \leftarrow \mathit{sumTo0}\ x; \\ &\qquad\qquad\qquad\qquad\qquad\quad n + y \end{aligned}$$

$(-) : \mathsf{Nat} \to \mathsf{Nat} \to \mathsf{F}\,\mathsf{Nat}$

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&\mathit{sumTo0} : \mathsf{Val\,Nat} \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val\,Nat})) \\
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ELABORATING FUNCTIONAL CODE TO CALL-BY-UNBOXED-VALUE

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Passing & Returning Multiple Arguments

$$quotRem : \mathsf{Val\,Nat} \to \mathsf{Val\,Nat} \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val\,Nat} \times \mathsf{Val\,Nat}))$$

Complex answers must be immediately destructed in place at the call site

OK	$\mathbf{do}\,(\mathsf{val\,int}\; q, \mathsf{val\,int}\; r) \leftarrow quotRem\;(\mathsf{val}\;12)\;(\mathsf{val}\;5)\;.\,\mathsf{eval}$
Illegal	$\mathbf{do}\;\mathsf{val}\;?\;qr \leftarrow quotRem\;(\mathsf{val}\;12)\;(\mathsf{val}\;5)\;.\,\mathsf{eval}$

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Illegal	$\mathbf{do}\,\mathsf{val}\ ?\ qr \leftarrow quotRem\,(\mathsf{val}\ 12)\,(\mathsf{val}\ 5)\ .\ \mathsf{eval}$

$$distance : (\mathsf{Val\,Float} \times \mathsf{Val\,Float}) \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val\,Float}))$$

Complex arguments must be immediately constructed in place at the call site

OK	$distance\,(\mathsf{val}\ 3.14, \mathsf{val}\ 2.71)$
OK	$distance\,(\mathsf{val}\ x, \mathsf{val}\ y)$
Illegal	$distance\ xy$
Illegal	$distance\,(f\ x)$

Polymorphic Code

With type annotations...

Source	$id\ : \forall a.\ a \to a$
(System F)	$id\ = \Lambda a. \lambda(x : a).\ x$
CBUV_1	$id_1 : \forall a : \mathsf{Type}\,\mathsf{ref}\,\mathbf{val}\,.\ \ \mathsf{Val}\,a \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val}\,a))$
	$id_1 = \{\, \mathsf{ty}\,a \cdot \mathsf{val}\,\mathsf{ref}(x : a) \cdot \mathsf{eval} \to \mathbf{ret}\,\mathsf{val}\,x \,\}$

Polymorphic Code

With type annotations…

Source (System F)

$$id : \forall a.\ a \to a$$
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CBUV$_1$

$$id_1 : \forall a : \mathsf{Type\,ref}\ \mathbf{val}\,.\ \ \mathsf{Val}\,a \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val}\,a))$$
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CBUV$_2$

$$id_2 : \forall a : \mathsf{Type\,int}\ \mathbf{val}\,.\ (\mathsf{Val}\,a \times \mathsf{Val\,Float}) \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val}\,a \times \mathsf{Val\,Float}))$$
$$id_2 = \{\ \mathsf{ty}\,a \cdot (\mathsf{val\,int}(x : a), \mathsf{val\,flt}(y : \mathsf{Float})) \cdot \mathsf{eval} \to \mathbf{ret}\,(\mathsf{val}\,x, \mathsf{val}\,y)\ \}$$

Polymorphic Code

With type annotations...

$$
\begin{array}{rl}
\text{Source} & id \;: \forall a.\ a \to a \\
\text{(System F)} & id \;= \Lambda a.\lambda(x : a).\ x \\[1ex]
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\text{CBUV}_2 & id_2 : \forall a : \mathsf{Type}\,\mathsf{int}\,\mathbf{val}\,.\ (\mathsf{Val}\,a \times \mathsf{Val}\,\mathsf{Float}) \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val}\,a \times \mathsf{Val}\,\mathsf{Float})) \\
& id_2 = \{\,\mathsf{ty}\,a \cdot (\mathsf{val}\,\mathsf{int}(x : a), \mathsf{val}\,\mathsf{flt}(y : \mathsf{Float})) \cdot \mathsf{eval} \to \mathbf{ret}\,(\mathsf{val}\,x, \mathsf{val}\,y)\,\}
\end{array}
$$

id_1 (Val Int $\times$ Val Float) ill-kinded, but id_1 (Box(Val Int $\times$ Val Float)) is OK because

$$\mathsf{Box} : \mathbf{cplx}\,\mathbf{val} \to \mathsf{ref}\,\mathbf{val}$$

Polymorphic Code

With type annotations...and without

$$\begin{array}{rl} \text{Source} & id \;: \forall a.\, a \to a \\ \text{(System F)} & id \;= \Lambda a.\lambda(x : a).\, x \end{array}$$

$$\begin{array}{rl} \text{CBUV}_1 & id_1 : \forall a : \mathsf{Type\,ref}\,\mathbf{val}\,.\;\; \mathsf{Val}\,a \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val}\,a)) \\ & id_1 = \{\, \mathsf{ty}\,a \cdot \mathsf{val\,ref}(x : a) \cdot \mathsf{eval} \to \mathbf{ret}\,\mathsf{val}\,x \,\} \end{array}$$

$$\begin{array}{rl} \text{CBUV}_2 & id_2 : \forall a : \mathsf{Type\,int}\,\mathbf{val}\,.\; (\mathsf{Val}\,a \times \mathsf{Val\,Float}) \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val}\,a \times \mathsf{Val\,Float})) \\ & id_2 = \{\, \mathsf{ty}\,a \cdot (\mathsf{val\,int}(x : a), \mathsf{val\,flt}(y : \mathsf{Float})) \cdot \mathsf{eval} \to \mathbf{ret}\,(\mathsf{val}\,x, \mathsf{val}\,y) \,\} \end{array}$$

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$$\mathsf{Box} : \mathbf{cplx\,val} \to \mathsf{ref}\,\mathbf{val}$$

Unboxed code still has well-defined operational meaning after type erasure!

$$id_1 = \{\, \mathsf{ty}\,a \cdot\; \mathsf{val\,ref}\,x \cdot \mathsf{eval} \to \mathbf{ret}\,\mathsf{val}\,x \,\}$$

$$id_2 = \{\, \mathsf{ty}\,a \cdot (\mathsf{val\,int}\,x, \mathsf{val\,flt}\,y) \cdot \mathsf{eval} \to \mathbf{ret}\,(\mathsf{val}\,x, \mathsf{val}\,y) \,\}$$

Fusing Values and Calling Conventions

Nested tuples & call stacks

Unboxed tuples are flattened at compile time (a, b, c : ref **val**; $x : a, y : b, z : c$):

$$
\begin{array}{rcccr}
(\mathsf{Val}\, a \times \mathsf{Val}\, b) \times \mathsf{Val}\, c & \approx & \mathsf{Val}\, a \times (\mathsf{Val}\, b \times \mathsf{Val}\, c) & \approx & \mathsf{Val}\, a \times \mathsf{Val}\, b \times \mathsf{Val}\, c \\
((\mathsf{val}\, x, \mathsf{val}\, y), \mathsf{val}\, z) & \approx & (\mathsf{val}\, x, (\mathsf{val}\, y, \mathsf{val}\, z)) & \approx & \mathsf{val}\, x, \mathsf{val}\, y, \mathsf{val}\, z
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Curried & Uncurried Functions

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$$

(Un)Curried functions are compiled to the same code (a, b : ref **val**; c : sub **comp**):

$$
\begin{array}{lcl}
f : (\mathsf{Val}\,a \times \mathsf{Val}\,b) \to \mathsf{Eval}\,c & \approx & g : \mathsf{Val}\,a \to (\mathsf{Val}\,b \to \mathsf{Eval}\,c) \\
f = \{\, (\mathsf{val\,ref}\,x, \mathsf{val\,ref}\,y) \cdot \mathsf{eval} \to \ldots \} & \approx & g = \{\, \mathsf{val\,ref}\,x \cdot (\mathsf{val\,ref}\,y \cdot \mathsf{eval}) \to \ldots \}
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\end{array}$$

Safe due to second-class status of complex values & computations

$$\begin{array}{lrcrl}
\text{OK} & f\ (\mathsf{val}\,x, \mathsf{val}\,y)\ .\,\mathsf{eval} & \approx & g\ (\mathsf{val}\,x)\ (\mathsf{val}\,y)\ .\,\mathsf{eval} & \text{OK} \\
\text{Illegal} & f\ xy\ .\,\mathsf{eval} & \not\approx & h\ (g\ (\mathsf{val}\,x)) & \text{Illegal}
\end{array}$$

Curried & Uncurried Functions

Unboxed tuples are flattened at compile time (a, b, c : ref **val**; $x : a$, $y : b$, $z : c$):

$$\begin{array}{ccccc} (\mathsf{Val}\,a \times \mathsf{Val}\,b) \times \mathsf{Val}\,c & \approx & \mathsf{Val}\,a \times (\mathsf{Val}\,b \times \mathsf{Val}\,c) & \approx & \mathsf{Val}\,a \times \mathsf{Val}\,b \times \mathsf{Val}\,c \\ ((\mathsf{val}\,x, \mathsf{val}\,y), \mathsf{val}\,z) & \approx & (\mathsf{val}\,x, (\mathsf{val}\,y, \mathsf{val}\,z)) & \approx & \mathsf{val}\,x, \mathsf{val}\,y, \mathsf{val}\,z \end{array}$$

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$$\begin{array}{lcl} f : (\mathsf{Val}\,a \times \mathsf{Val}\,b) \to \mathsf{Eval}\,c & \approx & g : \mathsf{Val}\,a \to (\mathsf{Val}\,b \to \mathsf{Eval}\,c) \\ f = \{\, (\mathsf{val\,ref}\,x, \mathsf{val\,ref}\,y) \cdot \mathsf{eval} \to \ldots \} & \approx & g = \{\, \mathsf{val\,ref}\,x \cdot (\mathsf{val\,ref}\,y \cdot \mathsf{eval}) \to \ldots \} \end{array}$$

Safe due to second-class status of complex values & computations

$$\begin{array}{lrcll} \text{OK} & f\ (\mathsf{val}\,x, \mathsf{val}\,y)\ .\,\mathsf{eval} & \approx & g\ (\mathsf{val}\,x)\ (\mathsf{val}\,y)\ .\,\mathsf{eval} & \text{OK} \\ \text{Illegal} & f\ xy\ .\,\mathsf{eval} & \not\approx & h\,(g\,(\mathsf{val}\,x)) & \text{Illegal} \\ \text{OK} & \begin{array}{l}\mathbf{unbox}\,(\mathsf{val\,ref}\,x, \mathsf{val\,ref}\,y) \leftarrow xy; \\ f\ (\mathsf{val}\,x, \mathsf{val}\,y)\ .\,\mathsf{eval}\end{array} & \approx & \begin{array}{l} h\,(\mathsf{clos}\{\mathsf{val\,ref}\,y \cdot \mathsf{eval} \to \\ \quad g\ (\mathsf{val}\,x)\ (\mathsf{val}\,y)\ .\,\mathsf{eval}\}) \end{array} & \text{OK} \end{array}$$

Unboxed Sums

Invariant: all complex patterns can be fully enumerated at compile time

Unboxed sums are also flattened at compile time (a, b, c : ref **val**; $x : a, y : b, z : c$):

$$
\begin{array}{rcrl}
(\mathsf{Val}\ a + \mathsf{Val}\ b) + \mathsf{Val}\ c & \approx & \mathsf{Val}\ a + (\mathsf{Val}\ b + \mathsf{Val}\ c) & \\
(0, (0, \mathsf{val}\ x)) & \approx & (0, \mathsf{val}\ x) & \text{Choice \#0} \\
(0, (1, \mathsf{val}\ y)) & \approx & (1, (0, \mathsf{val}\ y)) & \text{Choice \#1} \\
(1, \mathsf{val}\ z) & \approx & (1, (1, \mathsf{val}\ z)) & \text{Choice \#2}
\end{array}
$$

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(1, \mathsf{val}\ z) & \approx & (1, (1, \mathsf{val}\ z)) & \text{Choice \#2}
\end{array}
$$

Unboxed tuples distribute over unboxed sums (a, b, c : ref **val**; $x : a, y : b, z : c$):

$$
\begin{array}{rclr}
(\mathsf{Val}\ a + \mathsf{Val}\ b) \times \mathsf{Val}\ c & \approx & (\mathsf{Val}\ a \times \mathsf{Val}\ c) + (\mathsf{Val}\ b \times \mathsf{Val}\ c) & \\
((0, \mathsf{val}\ x), \mathsf{val}\ z) & \approx & (0, (\mathsf{val}\ x, \mathsf{val}\ z)) & \text{Choice \#0} \\
((1, \mathsf{val}\ y), \mathsf{val}\ z) & \approx & (1, (\mathsf{val}\ y, \mathsf{val}\ z)) & \text{Choice \#1}
\end{array}
$$

Choice Fusion

$$\begin{aligned} \mathit{maybeAdd}\ \mathrm{Nothing}\ y &= y \\ \mathit{maybeAdd}\ (\mathrm{Just}\ x)\ y &= x + y \end{aligned}$$

Invariant: mandatory pattern-matching on complex values

Choice Fusion

$$maybeAdd\ \mathsf{Nothing}\ y = y$$
$$maybeAdd\ (\mathsf{Just}\ x)\ y = x + y$$

Invariant: mandatory pattern-matching on complex values

Two equivalent versions (Maybe $a = 1 + a$; Nothing $= (0, ())$; Just $x = (1, x)$):

$$maybeAdd_1 : (1 + \mathsf{Val\,Int}) \to \mathsf{Val\,Int} \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val\,Int}))$$
$$\begin{aligned} maybeAdd_1 = \{&(0, ()) \cdot (\mathsf{val\,int}\ y) \cdot \mathsf{eval} \to \mathbf{ret}\ \mathsf{val}\ y && \text{(Choice \#0)}\\ &(1, \mathsf{val\,int}\ x) \cdot (\mathsf{val\,int}\ y) \cdot \mathsf{eval} \to x + y && \text{(Choice \#1)}\} \end{aligned}$$

$$maybeAdd_2 : (\mathsf{Val\,Int} \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val\,Int}))) \,\&\, (\mathsf{Val\,Int} \to \mathsf{Val\,Int} \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val\,Int})))$$
$$\begin{aligned} maybeAdd_2 = \{&0 \cdot (\mathsf{val\,int}\ y) \cdot \mathsf{eval} \to \mathbf{ret}\ \mathsf{val}\ y && \text{(Choice \#0)}\\ &1 \cdot (\mathsf{val\,int}\ x) \cdot (\mathsf{val\,int}\ y) \cdot \mathsf{eval} \to x + y && \text{(Choice \#1)}\} \end{aligned}$$

Choice Fusion

Unboxed sum parameters ≈ higher-order products

$$maybeAdd\ \mathsf{Nothing}\ y = y$$
$$maybeAdd\ (\mathsf{Just}\ x)\ y = x + y$$

Invariant: mandatory pattern-matching on complex values

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$$maybeAdd_2 : (\mathsf{Val\,Int} \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val\,Int}))) \mathbin{\&} (\mathsf{Val\,Int} \to \mathsf{Val\,Int} \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val\,Int})))$$
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$maybeAdd_1$ takes a Maybe argument; $maybeAdd_2$ gives a product of 2 functions

Stopping Fusion

Putting complex values in a Box pauses pattern-matching.

$$
\begin{aligned}
&\mathit{maybeAdd}_3 : \mathsf{Val}(\mathsf{Box}(1 + \mathsf{Val\,Int})) \to \mathsf{Val\,Int} \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val\,Int}))\\
&\mathit{maybeAdd}_3 = \{\, \mathsf{val\,ref}\ x \cdot \mathsf{val\,int}\ y \cdot \mathsf{eval} \to\\
&\qquad\qquad \mathbf{unbox}\ x\ \mathbf{as}\ \{\\
&\qquad\qquad\qquad (0, ()) \qquad\quad \to \mathbf{ret}\ \mathsf{val}\ y\\
&\qquad\qquad\qquad (1, \mathsf{val\,int}\ x) \to x + y\\
&\qquad\qquad \}\\
&\qquad \}
\end{aligned}
$$

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&\mathit{maybeAdd}_3 = \{\, \mathsf{val\,ref}\; x \cdot \mathsf{val\,int}\; y \cdot \mathsf{eval} \to\\
&\qquad\qquad \mathbf{unbox}\; x\; \mathbf{as}\; \{\\
&\qquad\qquad\qquad (0, ()) \qquad\quad \to \mathbf{ret}\; \mathsf{val}\; y\\
&\qquad\qquad\qquad (1, \mathsf{val\,int}\; x) \to x + y\\
&\qquad\qquad \}\\
&\qquad \}
\end{aligned}
$$

$$\mathit{maybeAdd}_3 \not\approx \mathit{maybeAdd}_1 \qquad\qquad \mathit{maybeAdd}_3 \not\approx \mathit{maybeAdd}_2$$

Foundations for Unboxing

The Usual Division of Types

A commonly-repeated refrain

Call-By-Push-Value

$$
\begin{aligned}
\mathit{ValueType} \ni A &::= A_0 \times A_1 \mid A_0 + A_1 \mid \mathsf{U}\,\underline{B} \\
\mathit{ComputationType} \ni \underline{B} &::= A \to \underline{B} \mid \underline{B}_0 \,\&\, \underline{B}_1 \mid \mathsf{F}\,A
\end{aligned}
$$

Focusing & Polarity

$$
\begin{aligned}
\mathit{PositiveType} \ni P^+ &::= P_0^+ \otimes P_1^+ \mid P_0^+ \oplus P_1^+ \mid {\downarrow} Q^- \\
\mathit{NegativeType} \ni Q^- &::= P^+ \to Q^- \mid Q_0^- \,\&\, Q_1^- \mid {\uparrow} P^+
\end{aligned}
$$

- $\mathit{Value} = \mathit{Positive}$
- $\mathit{Computation} = \mathit{Negative}$

The Usual Division of Types

A commonly-repeated refrain

Call-By-Push-Value

$$\mathit{ValueType} \ni A ::= A_0 \times A_1 \mid A_0 + A_1 \mid \mathsf{U}\,\underline{B}$$
$$\mathit{ComputationType} \ni \underline{B} ::= A \to \underline{B} \mid \underline{B}_0 \mathbin{\&} \underline{B}_1 \mid \mathsf{F}\,A$$

Focusing & Polarity

$$\mathit{PositiveType} \ni P^+ ::= P_0^+ \otimes P_1^+ \mid P_0^+ \oplus P_1^+ \mid {\downarrow} Q^-$$
$$\mathit{NegativeType} \ni Q^- ::= P^+ \to Q^- \mid Q_0^- \mathbin{\&} Q_1^- \mid {\uparrow} P^+$$

- $\mathit{Value} = \mathit{Positive}$?
- $\mathit{Computation} = \mathit{Negative}$?
- Right?

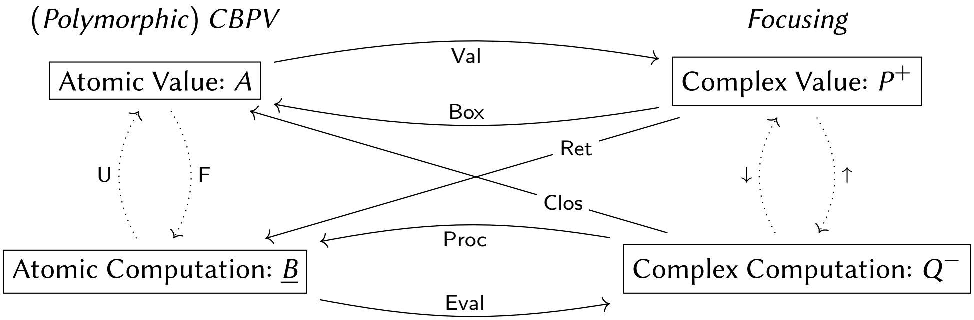
A Distinction between the Distinctions

A subtle disagreement

- In Call-By-Push-Value, "value types" are the denotable values
  - Only value types are first class, can be named
  - Computation types are second class, cannot be named unless "thunked"
- With strict focusing, pattern matching is mandatory
  - Positive types are second class, must be matched instead of named
  - Negative types are first class, cannot be matched so they are named
- Opposite sides of the complex vs atomic divide:
  - Call-By-Push-Value talks about atomic values and computations
  - Focusing talks about complex values and computations

Shifting Between Quadrants

Complexity vs Atomicity, Values vs Computations



$$\mathsf{F}\,A = \mathsf{Ret}(\mathsf{Val}\,A) \qquad {\uparrow}P^+ = \mathsf{Eval}(\mathsf{Ret}\,P^+)$$

$$\mathsf{U}\,\underline{B} = \mathsf{Clos}(\mathsf{Eval}\,\underline{B}) \qquad {\downarrow}Q^- = \mathsf{Val}(\mathsf{Clos}\,Q^-)$$

Equational theory: Sound & Complete w.r.t. Call-By-Push-Value!

$$\text{Values} = \text{Are} \quad \text{Computations} = \text{Do}$$

$$\text{Atomic} = \text{One} \qquad\qquad \text{Complex} = \text{Many}$$

Higher-Order Calling Conventions

- Default "uniform" atomic representations / calling conventions:
  - Atomic value: ref = "reference" (i.e., pointer to value)
  - Atomic computation: sub = "subroutine" (i.e., return pointer)

- First-class closure values built by Clos : **cplx comp** → ref **val**
  - Closure introduced by clos $\{ \dots \}$ around copattern-matching code
  - Closure f : Clos a eliminated with f. call operation

$$app = \lambda f\ x.\ (f\ x)$$

$$app : \forall a : \mathsf{Type\ ref}\ \mathbf{val}.\forall b : \mathsf{Type\ sub}\ \mathbf{comp}.\ {\downarrow}(\mathsf{Val}\ a \to \mathsf{Eval}\ b) \to \mathsf{Val}\ a \to \mathsf{Eval}\ b$$
$$app = \{\mathsf{ty}\ a \cdot \mathsf{ty}\ b \cdot \mathsf{val\ ref}\ f \cdot \mathsf{val\ ref}\ x \cdot \mathsf{eval\ sub} \to f.\mathsf{call}(\mathsf{val}\ x).\,\mathsf{eval\ sub}\}$$

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$$app' = \{\mathrm{ty}\,a \cdot \mathrm{ty}\,b \cdot \mathrm{val}\,\mathrm{ref}\,f : \mathrm{Clos}(a \to b) \cdot \mathrm{eval}\,\mathrm{sub} \to \mathbf{ret}\,\mathrm{val}\,f\}$$

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Stress Test: Representation-Polymorphic Overloading

Important Application: representation-polymorphic (type class) operator overloading

$$\begin{aligned} \textbf{class}\ \mathsf{Num}\ a\ \textbf{where}\ (+) \quad &:: a \to a \to a \\ \mathit{negate}\ &:: a \to a \end{aligned}$$

What can we do without explicit representation polymorphism?

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What can we do without explicit representation polymorphism?

$$\textbf{type}\ \mathsf{Num}(a : \textbf{cplx val}) : \textbf{cplx val} = \mathsf{Clos}(a \to a \to {\uparrow}a) \times \mathsf{Clos}(a \to {\uparrow}a)$$

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$$\begin{aligned}(+) &: \forall a : \mathsf{Type}\,\textbf{cplx val}\,.\ \mathsf{Num}\ a \to {\uparrow\downarrow}(a \to a \to {\uparrow}a) \\ (+) &= \{\ \mathsf{ty}\ a \cdot (\mathsf{val\,ref}\ f, \mathsf{val\,ref}\ g) \cdot \mathsf{eval} \to \textbf{ret}\ \mathsf{val}\ f\ \}\end{aligned}$$

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What can we do without explicit representation polymorphism?

After type erasure, still get well-defined, operational code

$$(+) \quad = \{\, \mathsf{ty}\ a \cdot (\mathsf{val}\ \mathsf{ref}\ f, \mathsf{val}\ \mathsf{ref}\ g) \cdot \mathsf{eval} \to \mathbf{ret}\ \mathsf{val}\ f \,\}$$

$$\mathit{negate} = \{\, \mathsf{ty}\ a \cdot (\mathsf{val}\ \mathsf{ref}\ f, \mathsf{val}\ \mathsf{ref}\ g) \cdot \mathsf{eval} \to \mathbf{ret}\ \mathsf{val}\ g \,\}$$

Compiling to the Machine

Complex patterns $\implies$ 1 simple switch

$$x : \mathsf{Box}((\mathsf{Val\,Int} + \mathsf{Val\,Float} \times \mathsf{Val\,Int}) + 1)$$

$$\begin{aligned}\textbf{unbox}\, x\, \textbf{as} \{ \; & 0, 0, \mathsf{val\,int}\, y && \to M_1; \\ & 0, 1, \mathsf{val\,flt}\, y, \mathsf{val\,int}\, z && \to M_2; \\ & 1, () && \to M_3 \}\end{aligned}$$

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```
struct {
  char tag;
  union { // case 0 = 0, 0, val int
    int zero;
           // case 1 = 0, 1, val flt, val int
    struct { float fst; int snd; } one;
           // empty case 2 = 1, ()
  } body;
} *x;
switch (x->tag) {
  case 0:
    int y = x->body.zero; M1...; break;
  case 1:
    float y = x->body.one.fst;
    int z = x->body.one.snd;
    M2...; break;
  case 2:
    M3...
}
```

Complex variables

$$\begin{aligned} \mathit{and}\ \mathsf{True}\ x &= x \\ \mathit{and}\ \mathsf{False}\ x &= \mathsf{False} \end{aligned}$$

Complex variables $x \in \{\ \mathit{pattern} \ldots \}$ match multiple patterns

$$\mathsf{Bool} = 1 + 1 \qquad \mathsf{True} = 1, () \qquad \mathsf{False} = 0, ()$$

$$\begin{aligned} &\mathit{and} : \mathsf{Bool} \to \mathsf{Bool} \to \mathsf{Eval}(\mathsf{Ret}\ \mathsf{Bool}) \\ &\mathit{and} = \{\mathsf{True} \cdot x \in \{\ \mathsf{True}; \mathsf{False}\ \} \cdot \mathsf{eval} \to \mathbf{ret}\ x \in \{\ \mathsf{True}; \mathsf{False}\ \} \\ &\qquad\quad \mathsf{False} \cdot x \in \{\ \mathsf{True}; \mathsf{False}\ \} \cdot \mathsf{eval} \to \mathbf{ret}\ \mathsf{False}\ \} \end{aligned}$$

is syntactic shorthand for

$$\begin{aligned} &\mathit{and} : \mathsf{Bool} \to \mathsf{Bool} \to \mathsf{Eval}(\mathsf{Ret}\ \mathsf{Bool}) \\ &\mathit{and} = \{\mathsf{True} \cdot \mathsf{True} \cdot \mathsf{eval} \to \mathbf{ret}\ \mathsf{True}; \qquad \mathsf{True} \cdot \mathsf{False} \cdot \mathsf{eval} \to \mathbf{ret}\ \mathsf{False}; \\ &\qquad\quad \mathsf{False} \cdot \mathsf{True} \cdot \mathsf{eval} \to \mathbf{ret}\ \mathsf{False}; \qquad \mathsf{False} \cdot \mathsf{False} \cdot \mathsf{eval} \to \mathbf{ret}\ \mathsf{False};\ \} \end{aligned}$$

Complex answers

Complex continuations more $\in \{\, \textit{copattern} \ldots \}$ match multiple calling conventions

$$
\begin{aligned}
\mathit{app} &: \forall a : \mathsf{Type\,ref\,}\mathbf{val}\,.\forall b : \mathsf{Type\,sub\,}\mathbf{comp}\,.\,{\downarrow}(\mathsf{Val}\,a \to \mathsf{Eval}\,b) \to \mathsf{Val}\,a \to \mathsf{Eval}\,b \\
\mathit{app} &= \{\mathsf{ty}\,a \cdot \mathsf{ty}\,b \cdot \mathsf{val\,ref}\,f \cdot \mathsf{more} \in \{\,\mathsf{val\,ref}\,x \cdot \mathsf{eval\,sub}\,\} \to f\,.\,\mathsf{call}\}
\end{aligned}
$$

$$
\begin{aligned}
\mathit{app2} &: \forall a, b : \mathsf{Type\,ref\,}\mathbf{val}\,.\forall c : \mathsf{Type\,sub\,}\mathbf{comp}\,. \\
&\qquad {\downarrow}(\mathsf{Val}\,a \to \mathsf{Val}\,b \to \mathsf{Eval}\,c) \to \mathsf{Val}\,a \to \mathsf{Val}\,b \to \mathsf{Eval}\,c \\
\mathit{app2} &= \{\mathsf{ty}\,a \cdot \mathsf{ty}\,b \cdot \mathsf{ty}\,c \cdot \mathsf{val\,ref}\,f \cdot \mathsf{more} \in \{\,\mathsf{val\,ref}\,x \cdot \mathsf{val\,ref}\,y \cdot \mathsf{eval\,sub}\,\} \to f\,.\,\mathsf{call}\}
\end{aligned}
$$

is syntactic shorthand for

$$
\begin{aligned}
\mathit{app} &= \{\mathsf{ty}\,a \cdot \mathsf{ty}\,b \cdot \mathsf{val\,ref}\,f \cdot \mathsf{val\,ref}\,x \cdot \mathsf{eval\,sub} \to f\,.\,\mathsf{call}(\mathsf{val}\,x)\,.\,\mathsf{eval\,sub}\} \\
\mathit{app2} &= \{\mathsf{ty}\,a \cdot \mathsf{ty}\,b \cdot \mathsf{ty}\,c \cdot \mathsf{val\,ref}\,f \cdot \mathsf{val\,ref}\,x \cdot \mathsf{val\,ref}\,y \cdot \mathsf{eval\,sub} \\
&\qquad \to f\,.\,\mathsf{call}(\mathsf{val}\,x)\;(\mathsf{val}\,y)\;.\,\mathsf{eval\,sub}\}
\end{aligned}
$$