

Kinds Are Calling Conventions

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Efficient Function Calls

Parameter Passing Techniques

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- Representation — What & Where?

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- **Arity — How many?**

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- **Levity (aka Evaluation Strategy) — When to compute?**

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- Representation — What & Where?
- **Arity** — How many?
- Levity (aka Evaluation Strategy) — When to compute?

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Type suggests arity 2

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f1 = \x -> \y ->  
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Hint: 'expensive x' may be costly, or even cause side effects

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Goal: An IL with *unrestricted* η
for functions, along with
restricted β for other types

Static Arity

In an Intermediate Language

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- Application may still be *restricted* for efficiency, like source functions
 - $(\lambda x . x + x)\ (\text{fact } 10^6)$ does not recompute $\text{fact } 10^6$
- With full η , types express arity — just count the arrows
 - $f : \text{Int} \rightsquigarrow \text{Bool} \rightsquigarrow \text{String}$ has arity 2, no matter f ’s definition

Currying

When Partial Application Matters

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f3 :: Int ~> Int ~> Int
```

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- `f3'` is an arity 1 function; returns a closure `{Int~>Int}` of an arity 1 function
 - `map (App (f3' 100)) [1..106]` computes ‘`expensive 100`’ only once ☺

```
Clos :: (Int ~> Int) ~> {Int ~> Int}   App  :: {Int ~> Int} ~> Int ~> Int
```

Functions are *Called*

Not *Evaluated*

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`x = let f :: Int ~> Int = expensive 100 in ...f...f...`

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 - Call-by-value: first, before binding `f`

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- Primitive functions are never just *evaluated*; they are always *called*

The Problem With Polymorphism

And Static Compilation

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poly :: forall a. (Int ~> Int ~> a) ~> (a, a)
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- But what if $a = \text{Bool} \rightarrow \text{Bool}$?

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 - $g :: \text{Int} \rightarrow \text{Bool} \rightarrow \text{Bool}$ has arity 2... oops...
- How to statically compile? Is 'g 5' a call? A partial application?

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revapp [0..3] (++) [4..9]) vs revapp 2.5 (plusFloat# 1.5)

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 - **Too restrictive:** Identical definitions/code repeated for different types (like `error :: String -> a`)
 - **Incompatible with kind polymorphism:** `forall $k :: \text{Kind}$. forall $a :: k$. ???`

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 $\quad a \rightarrow (a \rightarrow b) \rightarrow b$

Representation Polymorphism

Kinds As Representations

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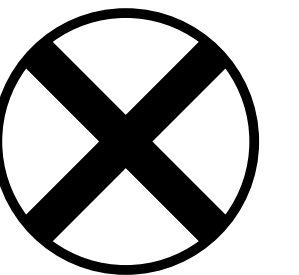
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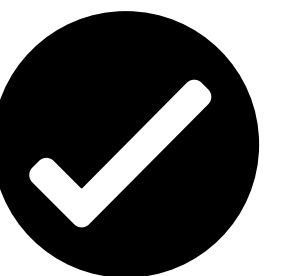
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Arity Polymorphism

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```
poly :: forall a :: TYPE Ptr Call[2]. (Int ~> Int ~> a) ~> (a, a)
poly f = let g :: Int ~> a = f 3 in (g 4, g 5)
```

- $f :: \text{Int } \sim\> \text{Int } \sim\> a :: \text{TYPE } \text{Ptr } \text{Call}[4]$ has arity 4 ($2 + 1 + 1$)
- $g :: \text{Int } \sim\> a :: \text{TYPE } \text{Ptr } \text{Call}[3]$ has arity 3 ($2 + 1$)

```
revapp :: forall (c :: Conv) (r :: Rep)
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revapp x f = f x
```

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         a ~> (a ~> b) ~> b
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```
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```

- $f :: a \sim\> b :: \text{TYPE } \text{Ptr } \text{Call}[2]$ has arity 2
- $x :: a :: \text{TYPE } \text{Ptr } c$ is represented as a pointer

Levity Polymorphism

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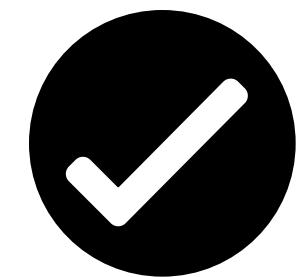
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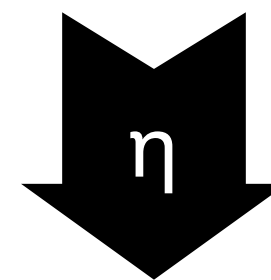
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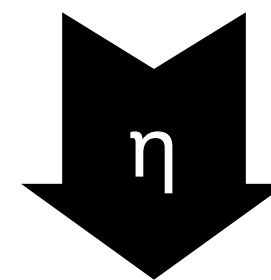


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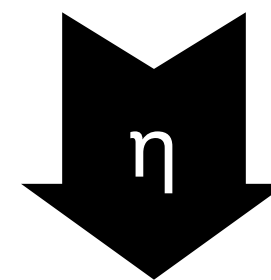
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```
poly = \(f::Ptr) ->  
      let g::Ptr = \(x::I32, y::?, z::?) -> f(3, x, y, z)
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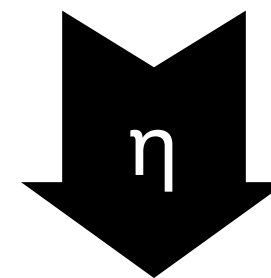

Static Compilation

With Polymorphic η -Expansion

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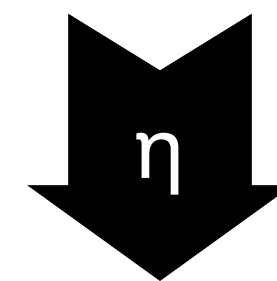
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poly :: forall a::TYPE Ptr Call[Ptr, F64].  
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Static Compilation

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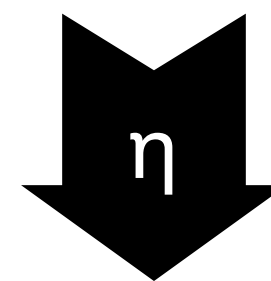


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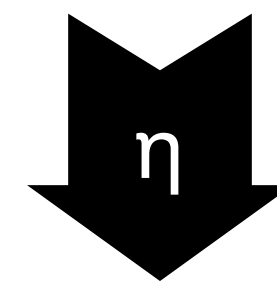


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```
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      in (\ (y::Ptr, z::F64) -> g(4, y, z),  
         \ (y::Ptr, z::F64) -> g(5, y, z))
```

Even More

In the Paper

- Levity Polymorphism
 - For when evaluation strategy doesn't matter
- Compiling Source \rightarrow Intermediate \rightarrow Target
 - Via kind-directed η -expansion and register assignment
- Type system for ensuring static compilation
 - Of definitions with arity, levity, and representation polymorphism

Kinds capture the details of
efficient calling conventions in
low-level machine code