Kinds Are Calling Conventions

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Efficient Function Calls

Parameter Passing Techniques

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- Representation — What & Where?

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- Representation — What & Where?
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- Levity (aka Evaluation Strategy) — When to compute?

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Goal: An IL with ***unrestricted*** η for functions, along with *restricted* β for other types

Static Arity

In an Intermediate Language

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- Application may still be *restricted* for efficiency, like source functions
  - $(\lambda x \,.\, x + x)\ (\mathit{fact}\ 10^6)$ does not recompute $\mathit{fact}\ 10^6$
- With full η, types express arity — just count the arrows
  - $f : \mathit{Int} \rightsquigarrow \mathit{Bool} \rightsquigarrow \mathit{String}$ has arity 2, no matter f’s definition

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  - map (App (f3' 100)) [1..10^6] computes 'expensive 100' only once ☺

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Clos :: (Int ~> Int) ~> {Int ~> Int}   App  :: {Int ~> Int} ~> Int ~> Int
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Not ***Evaluated***

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- Primitive functions are never just *evaluated*; they are always *called*

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And Static Compilation

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poly :: forall a. (Int ~> Int ~> a) ~> (a, a)
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- How to statically compile? Is '`g 5`' a call? A partial application?

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`revapp [0..3] (++ [4..9])` vs `revapp 2.5 (plusFloat# 1.5)`

A Stop-Gap Solution

Uniform Polymorphism in a Nonuniform Language

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  - **Too restrictive**: Identical definitions/code repeated for different types (like `error :: String -> a`)
  - **Incompatible with kind polymorphism**: `forall k::Kind. forall a::k.  ???`

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Representation Polymorphism

Kinds As Representations

- Generalize `a::⋆` to `a::TYPE r`
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  - `f :: a ~> b :: TYPE Ptr Call[2]` has arity 2
  - `x :: a :: TYPE Ptr c` is represented as a pointer

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poly = \(f::Ptr) ->
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With Polymorphic η-Expansion

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poly :: forall a::TYPE Ptr Call[Ptr, F64].
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       in (\(y::Ptr, z::F64) -> g(4, y, z),
           \(y::Ptr, z::F64) -> g(5, y, z))
```

Even More

In the Paper

- Levity Polymorphism
  - For when evaluation strategy doesn't matter
- Compiling Source → Intermediate → Target
  - Via kind-directed η-expansion and register assignment
- Type system for ensuring static compilation
  - Of definitions with arity, levity, and representation polymorphism

Kinds capture the details of efficient calling conventions in low-level machine code