

Kinds Are Calling Conventions

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Efficient Function Calls

Parameter Passing Techniques

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- Representation — What & Where?

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- Arity — How many?

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- **Levity (aka Evaluation Strategy) — When to compute?**

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- Representation — What & Where?
- **Arity** — How many?
- Levity (aka Evaluation Strategy) — When to compute?

Determining Function Arity

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Type suggests arity 2

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f1 = \x -> \y ->  
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Arity 2

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Hint: ‘`expensive x`’ may be costly, or even cause side effects

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$\text{let } z = \text{expensive } x$ **Arity 1** $f4 = \backslash x \rightarrow f3 \ x$
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$$\begin{array}{l} f3 = \lambda x \rightarrow \\ \quad \text{let } z = \text{expensive } x \quad \textbf{Arity 1} \quad f4 = \lambda x \rightarrow f3 \ x \\ \quad \text{in } \lambda y \rightarrow y + z \quad \neq \lambda x \rightarrow \lambda y \rightarrow f3 \ x \ y \end{array}$$

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What Is Arity?

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Goal: An IL with *unrestricted* η
for functions, along with
restricted β for other types

Static Arity

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- Application may still be *restricted* for efficiency, like source functions
 - $(\lambda x . x + x)$ (*expensive* 10^6) does not recompute *expensive* 10^6
- With full η , types express arity — just count the arrows
 - $f : Int \rightsquigarrow Bool \rightsquigarrow String$ has arity 2, no matter f ’s definition

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f3 :: Int ~> Int ~> Int
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 - `map (App (f3' 100)) [1..106]` computes ‘expensive 100’ only once ☺

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Clos :: (Int ~> Int) ~> {Int ~> Int}   App  :: {Int ~> Int} ~> Int ~> Int
```

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Not *Evaluated*

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- $x = x'$ by η , and x' always follows call-by-name order!
- Primitive functions are never just *evaluated*; they are always *called*

The Problem With Polymorphism

And Static Compilation

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poly :: forall a. (Int ~> Int ~> a) ~> (a, a)
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 - $f :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Bool} \rightarrow \text{Bool}$ has arity 3...
 - $g :: \text{Int} \rightarrow \text{Bool} \rightarrow \text{Bool}$ has arity 2... oops...
- How to statically compile? Is 'g 5' a call? A partial application?

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revapp :: forall a b. a -> (a -> b) -> b
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revapp [0..3] (++) [4..9]) vs revapp 2.5 (plusFloat# 1.5)

A Stop-Gap Solution

Uniform Polymorphism in a Nonuniform Language

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- Draconian restriction is unsatisfactory
 - **Too restrictive:** Identical definitions/code repeated for different types (like `error :: String -> a`)
 - **Incompatible with kind polymorphism:** `forall k::Kind. forall a::k. ???`

Representation Polymorphism

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 - $r :: \text{Rep}$ is the *representation* of a
 - $\star = \text{TYPE Ptr}$

$\text{revapp } x \ f = f \ x$

$\text{revapp} :: \text{forall } (r1, r2 :: \text{Rep}) \ (a :: \text{TYPE } r1) \ (b :: \text{Type } r2).$
 $\quad a \rightarrow (a \rightarrow b) \rightarrow b$

Representation Polymorphism

Kinds As Representations

- Generalize $a :: \star$ to $a :: \text{TYPE } r$
 - $r :: \text{Rep}$ is the *representation* of a
 - $\star = \text{TYPE Ptr}$

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revapp :: forall (r1,r2::Rep) ($a :: \text{TYPE } r1$) ($b :: \text{Type } r2$).
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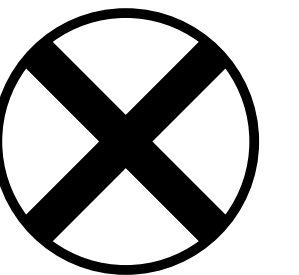
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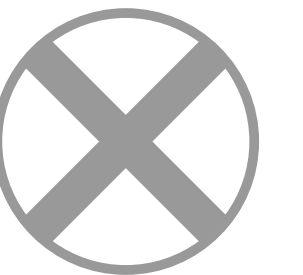
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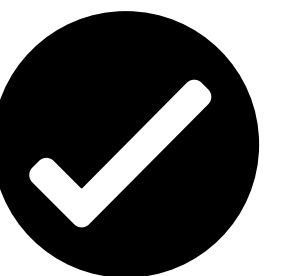
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- $\text{poly} :: \text{forall } a :: \text{TYPE } \text{Ptr } \text{Call}[2]. (\text{Int} \sim> \text{Int} \sim> a) \sim> (a, a)$
 $\text{poly } f = \text{let } g :: \text{Int} \sim> a = f \ 3 \text{ in } (g \ 4, g \ 5)$

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- $f :: \text{Int } \sim> \text{Int } \sim> a :: \text{TYPE } \text{Ptr } \text{Call}[4]$ has arity 4 ($2 + 1 + 1$)

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- $a :: \text{TYPE } \text{Ptr } \text{Call}[n]$ says values of a are pointers with arity n (simplified)

```
poly :: forall a :: TYPE Ptr Call[2]. (Int ~> Int ~> a) ~> (a, a)
poly f = let g :: Int ~> a = f 3 in (g 4, g 5)
```

- $f :: \text{Int } \sim\> \text{Int } \sim\> a :: \text{TYPE } \text{Ptr } \text{Call}[4]$ has arity 4 ($2 + 1 + 1$)
- $g :: \text{Int } \sim\> a :: \text{TYPE } \text{Ptr } \text{Call}[3]$ has arity 3 ($2 + 1$)

```
revapp :: forall (c :: Conv) (r :: Rep)
         (a :: TYPE Ptr c) (b :: TYPE r Call[1]).
         a ~> (a ~> b) ~> b
revapp x f = f x
```

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- $f :: a \sim\!> b :: \text{TYPE } \text{Ptr } \text{Call}[2]$ has arity 2

Arity Polymorphism

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- $a :: \text{TYPE } \text{Ptr } \text{Call}[n]$ says values of a are pointers with arity n (simplified)

```
poly :: forall a :: TYPE Ptr Call[2]. (Int ~> Int ~> a) ~> (a, a)
poly f = let g :: Int ~> a = f 3 in (g 4, g 5)
```

- $f :: \text{Int } \sim\!> \text{Int } \sim\!> a :: \text{TYPE } \text{Ptr } \text{Call}[4]$ has arity 4 ($2 + 1 + 1$)
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```
revapp :: forall (c :: Conv) (r :: Rep)
         (a :: TYPE Ptr c) (b :: TYPE r Call[1]).
         a ~> (a ~> b) ~> b
```

```
revapp x f = f x
```

- $f :: a \sim\!> b :: \text{TYPE } \text{Ptr } \text{Call}[2]$ has arity 2
- $x :: a :: \text{TYPE } \text{Ptr } c$ is represented as a pointer

Static Compilation

To the Machine

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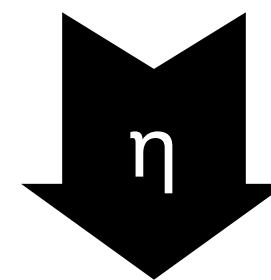
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poly :: forall a :: TYPE Ptr Call[2].  
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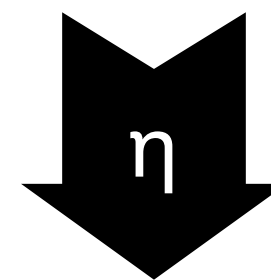


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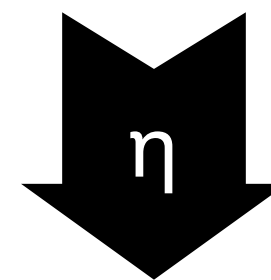
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poly = \(f::Ptr) ->
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```



```
poly = \(f::Ptr) ->  
      let g::Ptr = \(x::I32, y::?, z::?) -> f(3, x, y, z)
```

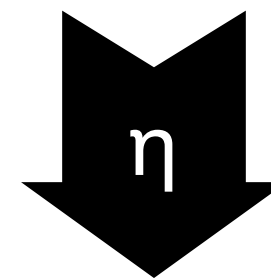
Static Compilation

With Polymorphic η -Expansion

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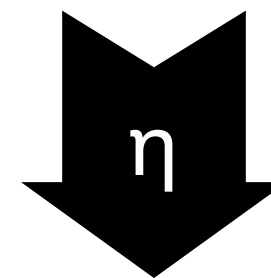
```
poly :: forall a::TYPE Ptr Call[Ptr, F64].  
      (Int# ~> Int# ~> a) ~> (a, a)  
poly f = let g :: Int# ~> a = f 3  
          in (g 4, g 5)
```



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        in (g 4, g 5)
```

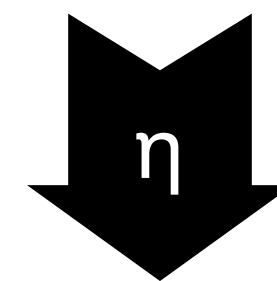


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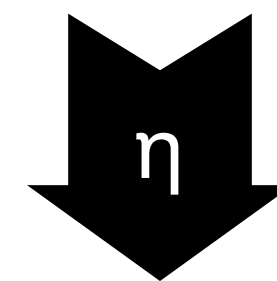


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poly = \ (f::Ptr) ->  
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        in (g 4, g 5)
```



```
poly = \ (f::Ptr) ->  
      let g::Ptr = \ (x::I32, y::Ptr, z::F64) -> f(3,x,y,z)  
      in (\ (y::Ptr, z::F64) -> g(4, y, z),  
         \ (y::Ptr, z::F64) -> g(5, y, z))
```

Even More

In the Paper

- Levity Polymorphism
 - For when evaluation strategy doesn't matter
- Compiling Source \rightarrow Intermediate \rightarrow Target
 - Via kind-directed η -expansion and register assignment
- Type system for ensuring static compilation
 - Of definitions with arity, levity, and representation polymorphism

Kinds capture the details of
efficient calling conventions in
low-level machine code