Université Fédérale
Toulouse Midi-Pyrénées
École Doctorale Mathématiques, Informatique et Télécommunications de Toulouse

Institut Supérieur de l'Aéronautique et de l'Espace
SUPAERO

Synchronization Performance Limits of GNSS Receivers

Priyanka DAS

PhD Thesis Defense | 30th March, 2021

Institut Supérieur de l'Aéronautique et de l'Espace (ISAE-Supaéro)
ISAE-ONERA SCANR (Signal, Communication, Antennae, Navigation & Radar)

Advisor: Eric CHAUMETTE — ISAE-Supaéro
Co-advisors: Silvère BONNABEL — Mines ParisTech
Jordi VILA-VALLS — ISAE-Supaéro

SAFRAN
Tutor: Loïc DAVAIN
Manager: François GUILLOT

Speaker background

2013
Bachelor of Science
Physics (Hons)

2016
Ingénieur polytechnicien
Data Science

2017
Master of Science
Autonomous Systems

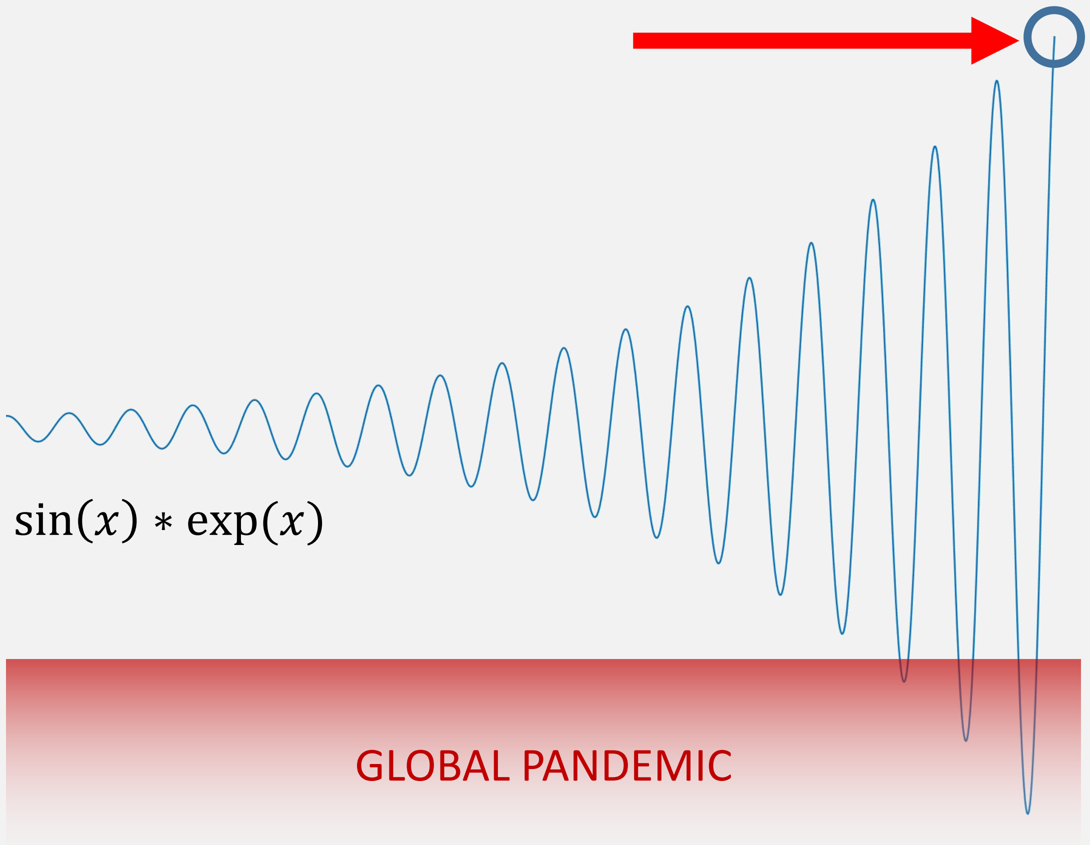
Why do a PhD?

Caltech Space Challenge 2015

What is a PhD?

Doctorat:
(*Latin*) *doctorem*,
enseigner (*to teach*)

PhD:
(Latin) Philosophiae Doctor,
“Doctor of Philosophy”



In the next 42 minutes

SAFRAN ISAE SUPAERO

For beginners

- What is GNSS?
- Positioning
- GNSS signals
- Cramér-Rao bound (CRB)?

For the experts

- Easy-to-use, compact CRB for a **generic band-limited signal** based on **signal samples**
- Asymptotic performance characterization of GNSS **time-delay**, **Doppler** and **phase** estimation
- Is GNSS **code-only precise positioning** solution?

Talk structure

- **Introduction** to basic theoretical concepts
- **Context** in detail and problem statement
- **Approach** and methodology adopted
- **Results** obtained and validation with examples
- **Conclusion** and perspectives

INTRODUCTION

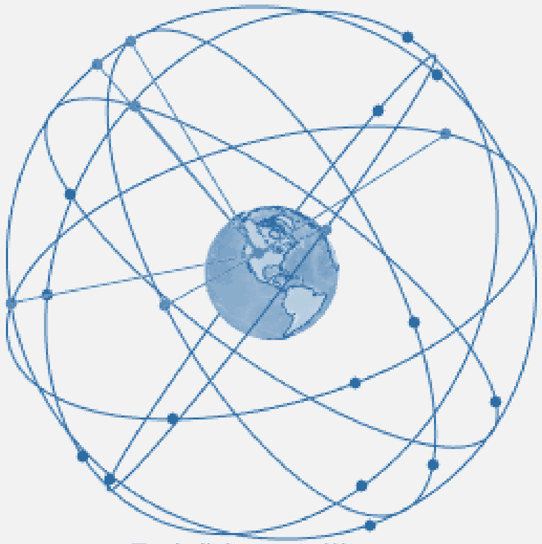
What is GNSS?

SAFRAN ISAE SUPAERO

GLOBAL NAVIGATION SATELLITE SYSTEM	
Global-level	
GPS	USA
Galileo	European
GLONASS	Russian
BeiDou	Chinese
Regional-level	
IRNSS/NavIC	Indian
QZSS	Japanese
Augmentation Systems	
EGNOS	European
WAAS	USA
MSAS	Japan

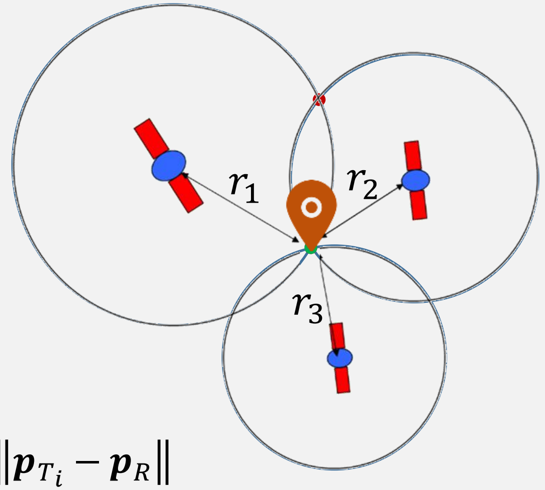
Constellation:
18-30 MEO satellites

Orbital altitude: ~20,000 km
R_{Earth} = 6,371 km



(Source: Paulsava)

Basic principle:
Trilateration



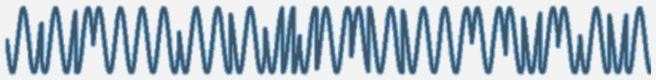
$$r_i = \|\boldsymbol{p}_{T_i} - \boldsymbol{p}_R\| = \sqrt{(x_i - x_R)^2 + (y_i - y_R)^2 + (z_i - z_R)^2} = c\tau_i$$

In reality:
r_i : **Pseudorange**

- Error sources (clock, iono, …)
- τ to be estimated (not measured)
- **Multilateration** $n_{sat} \geq 4$

GNSS: Signal Structure

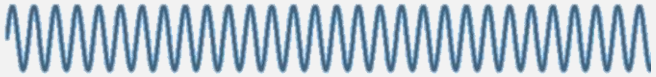
GPS L1 C/A signal



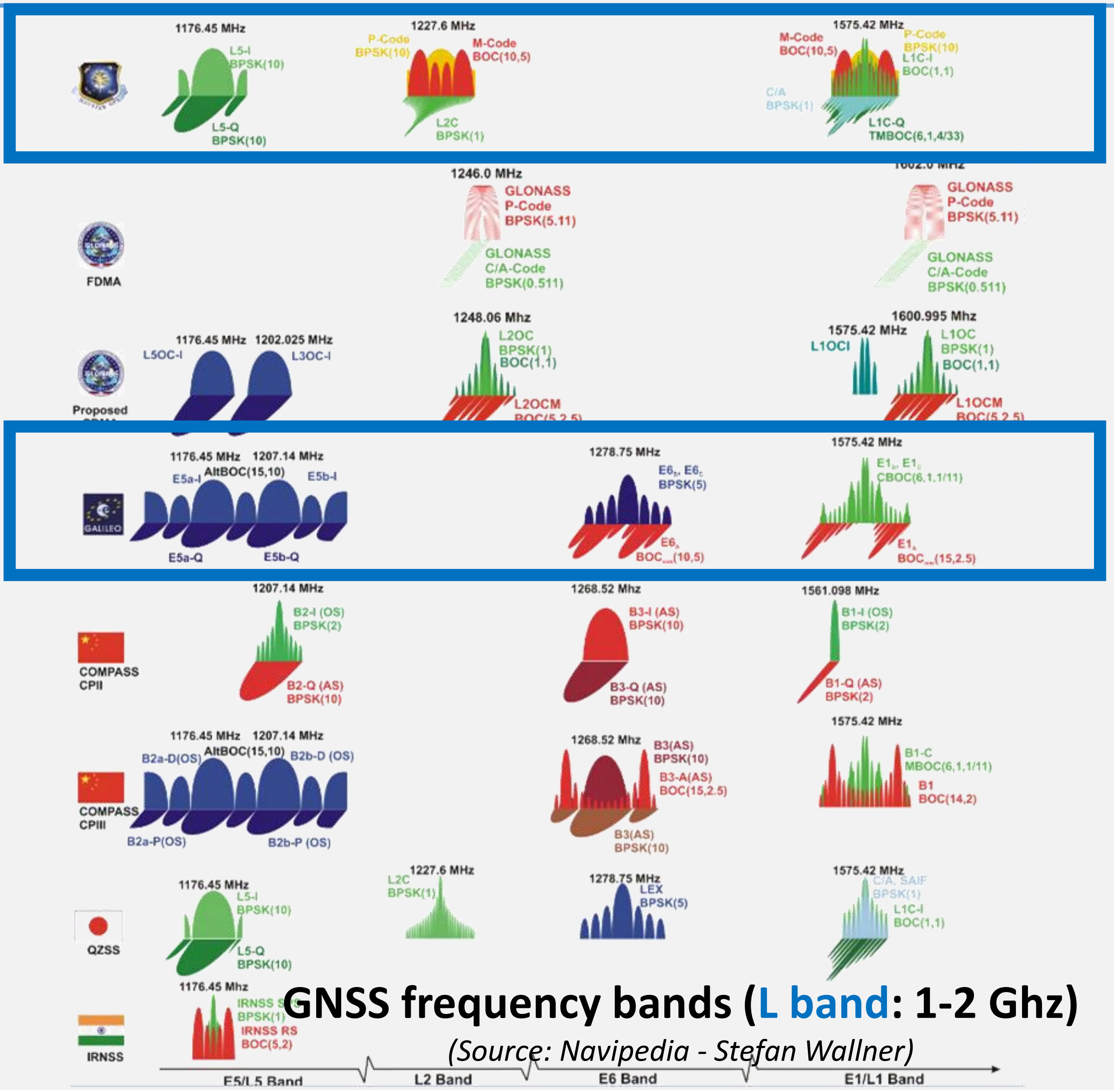
Transmitted signal

Navigation message: 50 Hz

PRN Code: 1.023 MHz



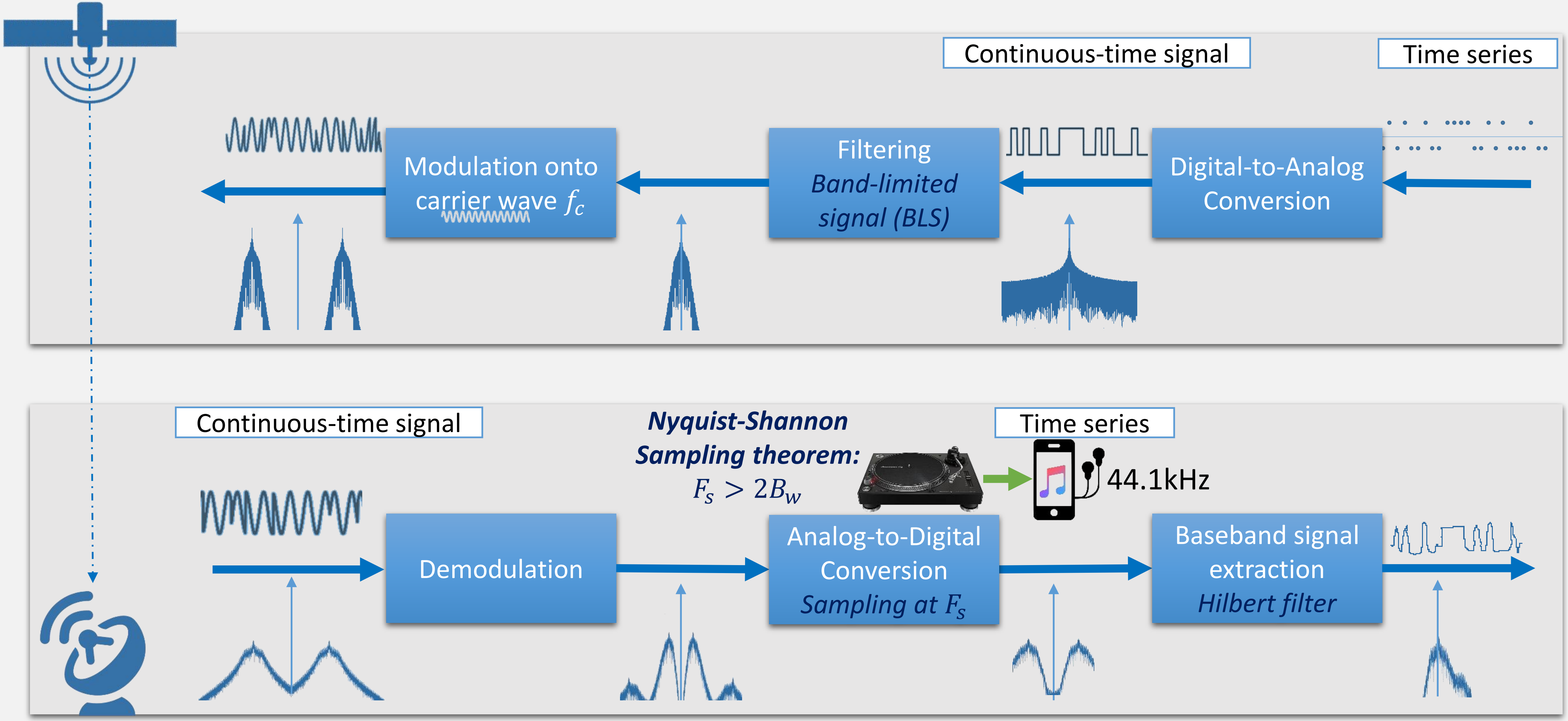
Carrier: 1575.42 MHz



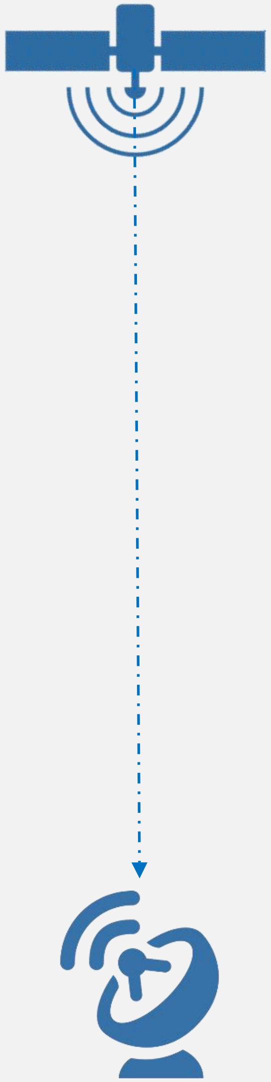
GNSS frequency bands (L band: 1-2 Ghz)

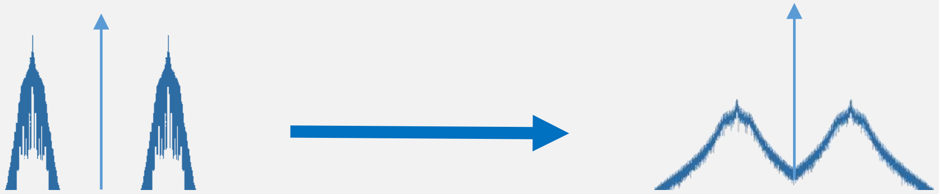
(Source: Navipedia - Stefan Wallner)

GNSS: Transmission and Reception



GNSS: Sources of Error





“Pseudorange” error sources:

• Satellite clock offset	~ few hundred km
Relativistic clock corrections	< 13 m
Satellite instrument errors	~ m
• Ionospheric delay	2-50 m
Tropospheric delay	2-50 m
• Receiver clock offset	< 300 km
Receiver instrument error	~ m

$$1\ \mu s \times c = 10^{-6} \times 3 \cdot 10^{8}\ ms^{-1}$$
$$= \color{red}{300\ m}$$

GNSS: Time-Delay and Doppler Estimation

Sampled signal at Hilbert filter output:

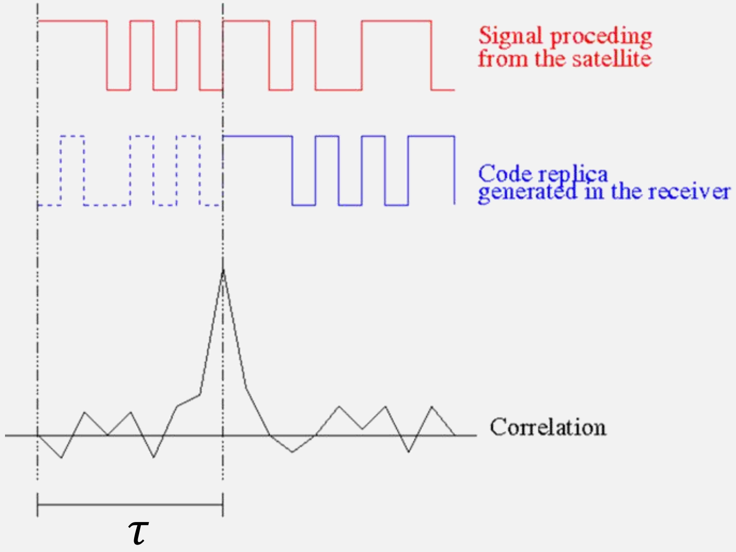
- Sum of signals from each visible satellite

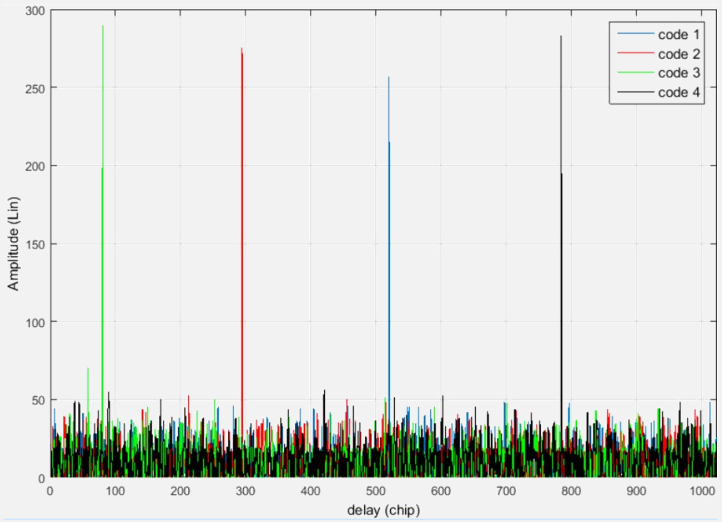
Problem:

- How to extract each satellite signal information
  - To find **pseudoranges** and estimate **position**

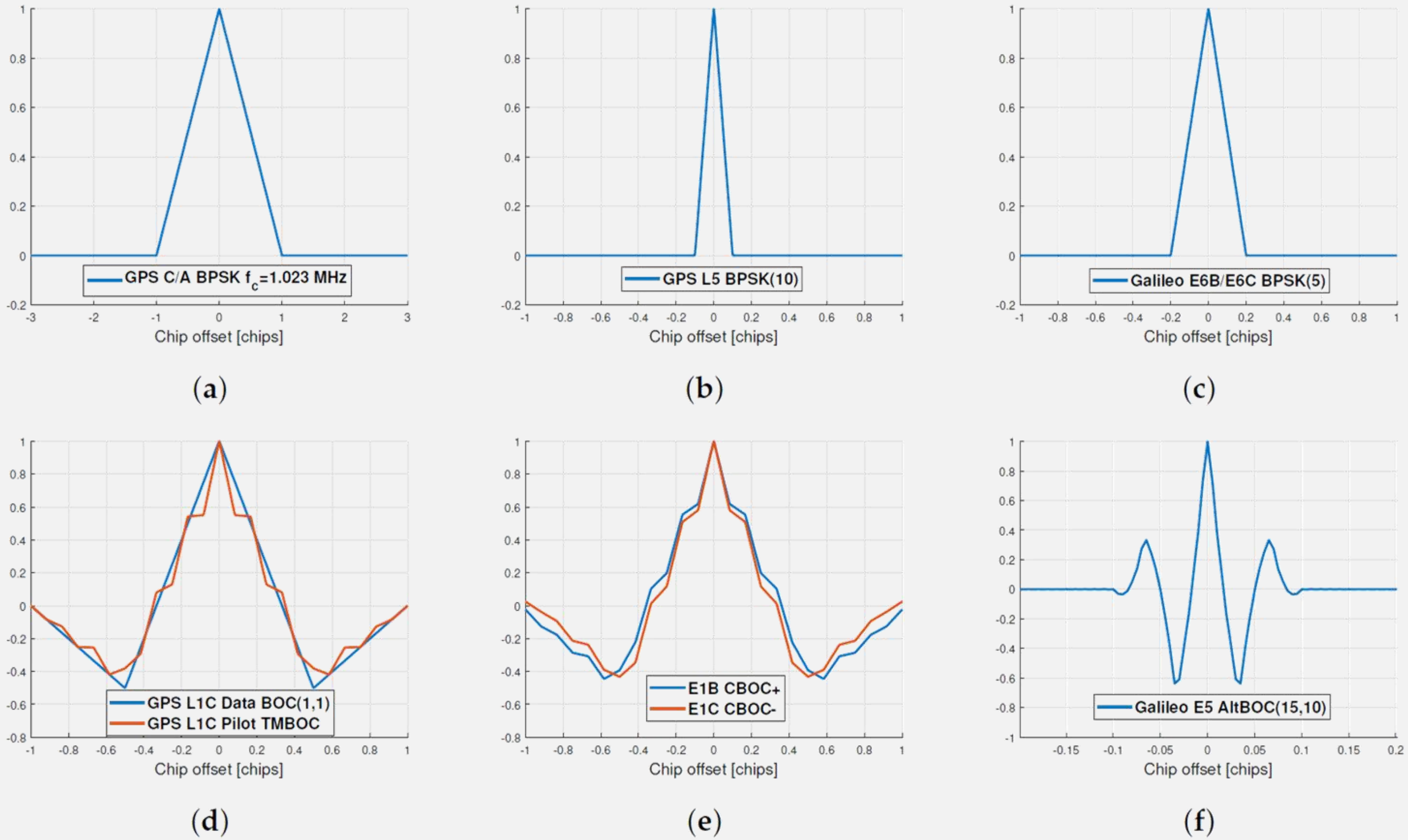
Solution:

- Signal code: pseudo-orthogonal
  - e.g. **Gold codes** for L1 C/A
- **Correlation** with code replica
  - Low cross-correlation, high autocorrelation
  - Performed by a **matched filter**





Some GNSS Autocorrelation Functions (ACF)



CONTEXT

Synchronization problem: Time-Delay and Doppler Estimation

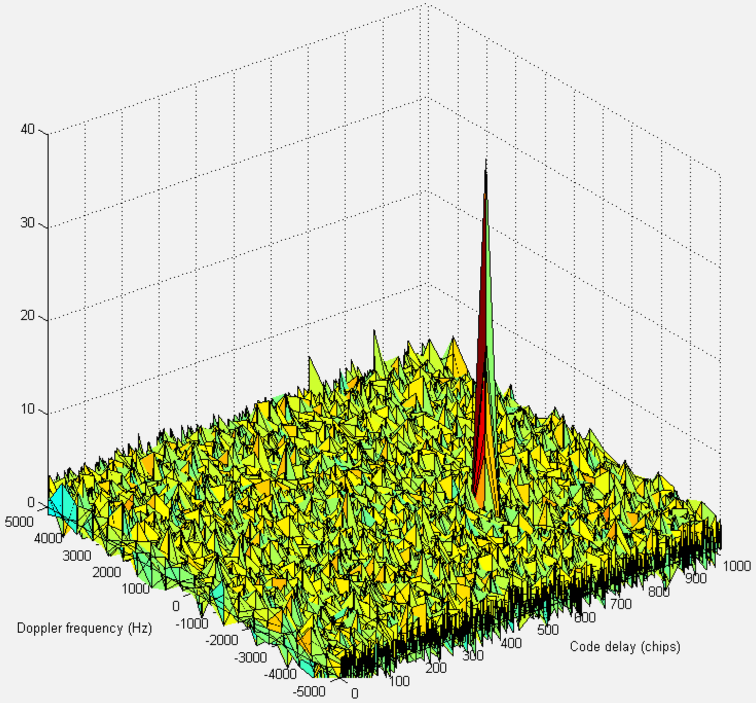
Synchronization:

- **Estimation** of propagation delay and Doppler shift
  - Objective: identify, localize and track radiating sources
  - Key first stage in many applications (Radar, Sonar, Speech,…)
- Instance of **Maximum Likelihood Estimator (MLE)**
  - Matched filter – optimal linear filter for maximising SNR in presence of additive noise

Fundamental question: what is the ultimate achievable performance

- **Mean Square Error (MSE)** sense
  - Handy (continuous, differentiable, ...) measure of MLE performance

Solution: lower bounds on the MSE



Ambiguity function

Cramér-Rao Lower Bounds

SAFRAN | ISAE SUPAERO

Signal containing θ

Parameter $\boldsymbol{\theta}$ to be estimated, contained in signal **s**

$\hat{\theta}$

Unbiased estimator of $\boldsymbol{\theta}$

Does $\hat{\boldsymbol{\theta}}$ attain CRB?

Cramér-Rao Bound (CRB)

Minimum covariance that can be reached by estimating $\boldsymbol{\theta}$

Fisher Information Matrix $\mathrm{F}(\theta)$

Quantifies the information in the distribution

inverse

Theorem: Cramér-Rao Lower Bound

If $p(\boldsymbol{x};\boldsymbol{\theta})$ verifies the regularity condition:

$$E\left[\frac{\partial \ln p(\boldsymbol{x};\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right] = \mathbf{0} \;\forall \boldsymbol{\theta}$$

Then, for any unbiased estimator $\hat{\boldsymbol{\theta}}$ of $\boldsymbol{\theta}$:

Covariance matrix: $cov(\hat{\boldsymbol{\theta}}) \geq \mathbf{F}^{-1}(\boldsymbol{\theta})$

Fisher Information: $\mathbf{F}(\boldsymbol{\theta}) = -E\left[\frac{\partial^2 \ln p(\boldsymbol{x};\boldsymbol{\theta})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}^T}\right]$

An unbiased estimator $\hat{\boldsymbol{\theta}}$, which attains this bound $\forall \boldsymbol{\theta}$, exists if and only :

$$\frac{\partial \ln p(\boldsymbol{x};\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \mathbf{F}(\boldsymbol{\theta})(\boldsymbol{g}(\boldsymbol{x}) - \boldsymbol{\theta})$$

for some functions $\mathbf{F}(\boldsymbol{\theta})$ and $\boldsymbol{g}(\boldsymbol{x})$

Minimum Variance Unbiased Estimator: $\hat{\boldsymbol{\theta}} = \boldsymbol{g}(\boldsymbol{x})$

Cramér-Rao Lower Bounds

Signal containing θ
Parameter $\boldsymbol{\theta}$ to be estimated, contained in signal **s**

$\hat{\boldsymbol{\theta}}$
Unbiased estimator of $\boldsymbol{\theta}$

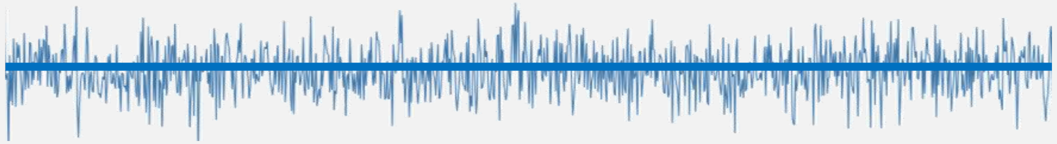
Does $\hat{\boldsymbol{\theta}}$ attain CRB?

Cramér-Rao Bound (CRB)
Minimum covariance that can be reached by estimating $\boldsymbol{\theta}$

Fisher Information Matrix $\mathrm{F}(\boldsymbol{\theta})$
Quantifies the information in the distribution

inverse

Example: Estimating DC voltage (with Gaussian noise)



$$s(n) = \theta + w(n)$$

(N observations, w i.i.d.)

$$p(\boldsymbol{s};\theta) = \prod_{n=0}^{N-1}\left(\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{1}{2\sigma^2}(s(n)-\theta)^2\right]\right)$$

Average over N observations

$$\frac{\partial \ln p(\boldsymbol{s};\theta)}{\partial\theta} = \frac{N}{\sigma^2}\left(\sum_{n=0}^{\mathrm{N}} s(n) - \theta\right)$$

$$var(\hat{\theta}) = \frac{\sigma^2}{N} = \mathrm{F}(\theta)^{-1}$$

Cramér-Rao Lower Bounds

Interest: [Kay93]

✓ **Simplicity** of calculation

✓ **Accurate estimation** of the MSE of the MLE

- In the asymptotic region of operation (large sample regime)
- Under certain conditions
  - High SNR for a Conditional Signal Model [SN90] [Ren+06]

Background:

Several delay-Doppler CRB expressions for finite or infinite bandwidth signals exist, but:

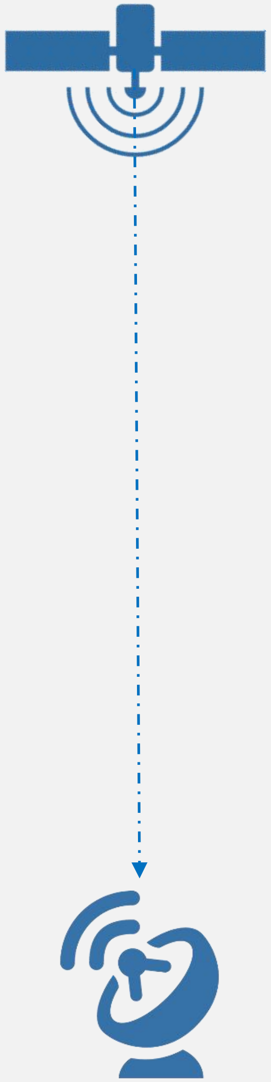
- Lack of easy-to-use **compact closed-form CRB** for generic BLS
- Most CRB expressions only address **standard narrowband signal model**
- Ignore **compression or stretch** due to range rate on the envelope

Existing delay-Doppler CRB <u>do not</u> match our needs:

- CRBs **delay, Doppler** and **phase** depending on **baseband signal samples**
- Incorporate **baseband signal dilation** due to Doppler effect into MLE
- Closed-form expressions for **amplitude** and **phase**

Extension to the GNSS case

SAFRAN | isae SUPAERO



Key assumption: reception of band-limited GNSS signals

Single Point Positioning:

Performance bounds of GNSS signals?

Precise positioning:

Standard method: Exploiting signal phase information
- e.g. GPS L1/CA: 19cm, while baseband: 300m

RTK – differential technique
- Requires reference station

PPP – precise phase measurements
- **Ambiguity resolution**, not practical in real-time

Hybrid architectures (e.g. fusion with inertial unit)
- **Kalman filter** – estimation of covariance

Can we obtain a GNSS code-only precise (< 20cm) positioning solution and under which conditions?

Existing delay-Doppler CRB do not match our needs:

- CRBs **delay, Doppler** and **phase** depending on **baseband signal samples**
- Incorporate **baseband signal dilation** due to Doppler effect into MLE
- Closed-form expressions for **amplitude** and **phase**

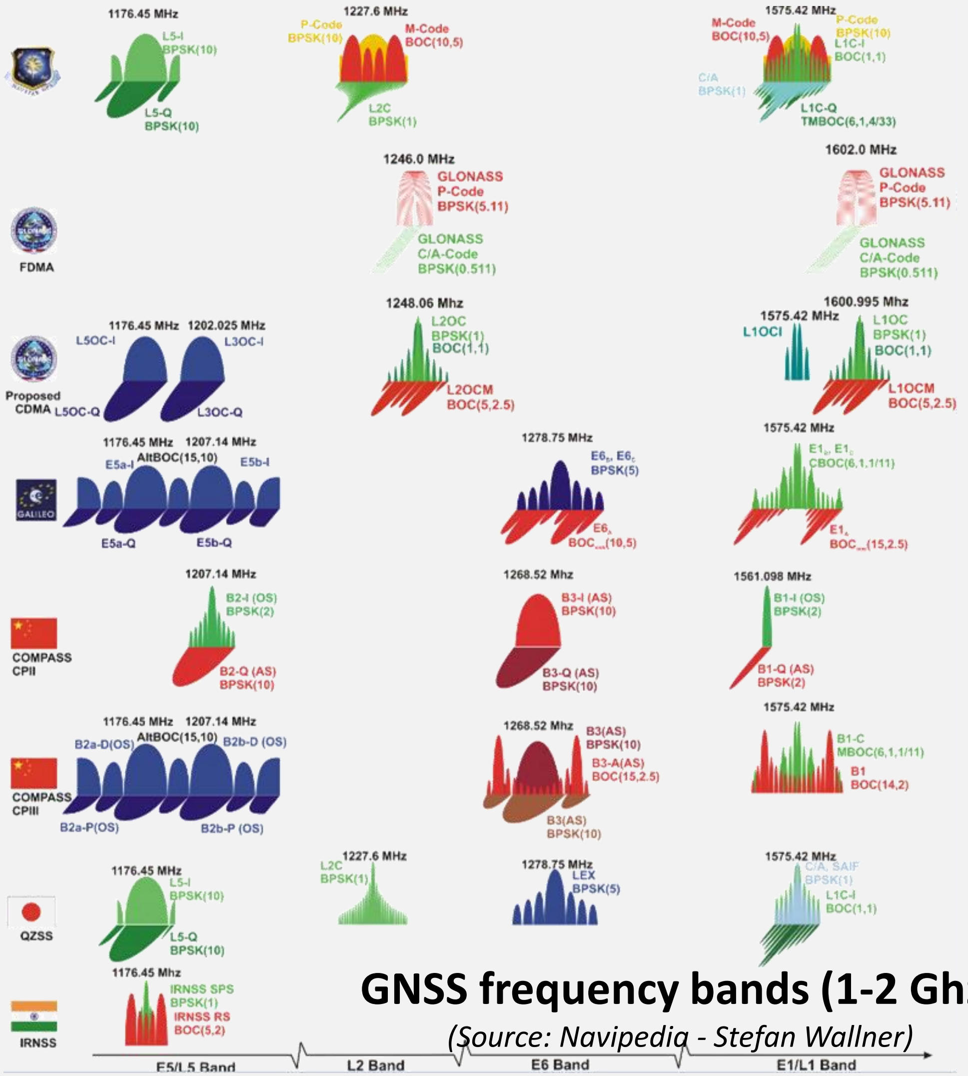
Problem Statement: Summary

1. **Lack of time delay, Doppler and phase CRB which are:**
   - Sufficiently generalized and easy-to-use
   - (Magic words) compact and closed-form
   - Depending on baseband signal samples
   - For generic BLS

2. **Existing delay-Doppler CRB do not match our needs**
   - Mostly standard narrowband signal model CRB
   - Baseband signal dilation due to the Doppler effect into MLE

3. **Lack of comprehensive performance analysis of GNSS signals**

4. **GNSS code-only precise positioning solution:**
   - With which signal(s)?
   - Under which conditions?



GNSS frequency bands (1-2 Ghz)

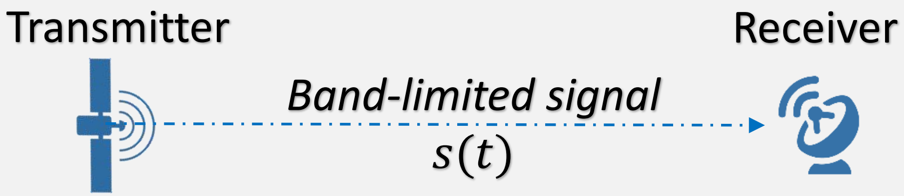
(Source: Navipedia - Stefan Wallner)

APPROACH

Estimation in the GNSS context: PVT

Key assumption:

- GNSS: reception of **band-limited signals**



Optimal positioning solution:

- Direct Position Estimation (DPE) [CFF07]
  - ML position estimation from sum of all signals

❖ Exploits links in different channels: **Accurate** and **robust**

! Real time applications: **Heavy computational burden**

DPE

- Receiver position
- Receiver velocity
- Clock offset and drift from each satellite

Standard solution – Two-step procedure:

- Pseudo-orthogonal codes
- Two steps: synchronization + multilateration

❖ **Asymptotically optimal at high SNR** [Vin+17]

Synchronization

- Clock offset and drift from each satellite

→

Multilateration

- Receiver position
- Receiver speed

Least Square (LS)

Signal Model

$$\boldsymbol{x}_A = \alpha_A \, \boldsymbol{s}_A(\tau) + \boldsymbol{n}_A$$

Gaussian noise

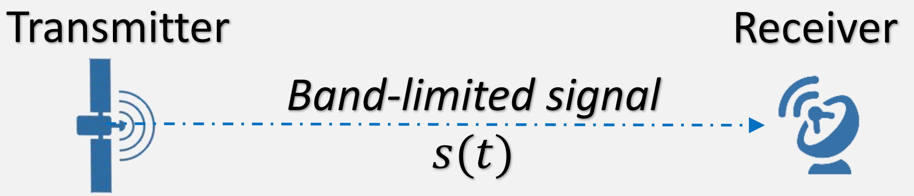
Transmitted signal

Amplitude factor

Complex analytic signal at receiver antenna output

Time-series: $s(t) = \sum_{n=N_1}^{N_2} s\left(\frac{n}{F_s}\right) sinc\left(\pi F_s\left(t - \frac{n}{F_s}\right)\right)$

Fourier series: $s(f) = \left(\frac{1}{F_s}\sum_{n=N_1}^{N_2} s\left(\frac{n}{F_s}\right) e^{-j2\pi n\frac{f}{F_s}}\right) 1_{\left[-\frac{B}{2},\frac{B}{2}\right]}(f)$



$$\|\boldsymbol{p}_{TR}(t)\| \triangleq \|\boldsymbol{p}_R(t) - \boldsymbol{p}_T(t - \tau(t))\| = c\tau(t)$$

- **Line-of-sight** transmission
- **Band-limited** transverse electromagnetic (TEM) signal $s(t)$ with **bandwidth** B
- Transmitter and receiver in **Uniform Rectilinear Motion (URM)**
- **Carrier wave**: f_c and $\lambda_c = \frac{c}{f_c}$

Two cases: Static and dynamic

SAFRAN isae SUPAERO

$\boldsymbol{x}_A = \alpha_A\, \boldsymbol{s}_A(\tau) + \boldsymbol{n}_A$	Static case	Dynamic case
Propagation delay	$\tau(t) = \tau$	$\tau(t) \simeq \tau + bt,$ $\lVert \boldsymbol{p}_{TR}(t) \rVert = d + vt,$ $\tau = \frac{d}{c}, b = \frac{v}{c}$
Signal model (N samples, F_s)	Code samples $\boldsymbol{x} = \rho e^{j\varphi}\, \boldsymbol{s}(\tau) + \boldsymbol{n}$ $\alpha = \rho e^{j\varphi}, \boldsymbol{\theta} = [\varphi, \tau]^T$ Signal samples Noise samples	$\boldsymbol{x} = \rho e^{j\varphi}\, \boldsymbol{a}(\boldsymbol{\eta}) + \boldsymbol{n}$ $\boldsymbol{\eta} = [\tau, b]^T$
Unknown deterministic parameters [Men+12]	$\boldsymbol{\epsilon} = [\sigma_n^2, \rho, \boldsymbol{\theta}^T]^T$	$\boldsymbol{\epsilon} = [\sigma_n^2, \rho, \varphi, \boldsymbol{\eta}^T]^T$

CRB, MLE and Ambiguity Function

SAFRAN isae SUPAERO

$\boldsymbol{x}_A = \alpha_A\, \boldsymbol{s}_A(\tau) + \boldsymbol{n}_A$	Static case	Dynamic case
Fisher Information $\mathrm{CRB} = (\mathrm{F})^{-1}$ [YB92]	$\mathrm{F}_{\tau\mid\epsilon}(\boldsymbol{\epsilon}) = \frac{2\lvert\alpha\rvert^2}{\sigma_n^2}\Re\{\boldsymbol{\Phi}(\tau)\}$ $\boldsymbol{\Phi}(\tau) = \frac{\partial \boldsymbol{s}(\tau)^H}{\partial\tau}\boldsymbol{\Pi}^{\perp}_{s(\tau)}\frac{\partial \boldsymbol{s}(\tau)}{\partial\tau}$	$\mathbf{F}_{\eta\mid\epsilon}(\boldsymbol{\epsilon}) = \frac{2\lvert\alpha\rvert^2}{\sigma_n^2}\Re\{\boldsymbol{\Phi}(\tau)\}$ $\boldsymbol{\Phi}(\boldsymbol{\eta}) = \frac{\partial \boldsymbol{a}(\boldsymbol{\eta})^H}{\partial\boldsymbol{\eta}^T}\boldsymbol{\Pi}^{\perp}_{a(\eta)}\frac{\partial \boldsymbol{a}(\tau)}{\partial\boldsymbol{\eta}^T}$
MLE [Ott+93]	$\hat{\tau} = \arg\max_{\tau}\left\{\frac{\lvert\boldsymbol{s}(\tau)^H\boldsymbol{x}\rvert^{2}}{\boldsymbol{s}(\tau)^H\boldsymbol{s}(\tau)}\right\}$	$\hat{\boldsymbol{\eta}} = \arg\max_{\eta}\left\{\frac{\lvert\boldsymbol{a}(\boldsymbol{\eta})^H\boldsymbol{x}\rvert^{2}}{\boldsymbol{a}(\boldsymbol{\eta})^H\boldsymbol{a}(\boldsymbol{\eta})}\right\}$
Ambiguity Function [Sko90]	$\Xi(\tau;\tau^0) = \left\lvert\frac{\boldsymbol{s}(\tau)^H\boldsymbol{s}(\tau^0)}{\lVert\boldsymbol{s}(\tau)\rVert\lVert\boldsymbol{s}(\tau^0)\rVert}\right\rvert^2$ $\Xi(\tau^0+d\tau;\tau^0) \approx 1 - \frac{1}{2}\left(\frac{2\,\Re\{\boldsymbol{\Phi}(\tau^0)\}}{\lVert\boldsymbol{s}(\tau^0)\rVert^2}\right)d\tau^2$	$\Xi(\boldsymbol{\eta};\boldsymbol{\eta}^0) = \frac{\lvert\alpha\rvert^2}{N}\lVert\boldsymbol{a}(\boldsymbol{\eta}^0)\rVert^2\left\lvert\frac{\boldsymbol{a}(\boldsymbol{\eta})^H\boldsymbol{a}(\boldsymbol{\eta}^0)}{\lVert\boldsymbol{a}(\boldsymbol{\eta})\rVert\lVert\boldsymbol{a}(\boldsymbol{\eta}^0)\rVert}\right\rvert^2$ $\Xi(\boldsymbol{\eta}^0+d\boldsymbol{\eta};\boldsymbol{\eta}^0)$ [Ric03] $\approx \frac{\lvert\alpha\rvert^2}{N}\lVert\boldsymbol{a}(\boldsymbol{\eta}^0)\rVert^2\left(1-\frac{1}{2}d\boldsymbol{\eta}^T\left(\frac{2\,\Re\{\boldsymbol{\Phi}(\boldsymbol{\eta}^0)\}}{\lVert\boldsymbol{a}(\boldsymbol{\eta}^0)\rVert^2}\right)d\boldsymbol{\eta}\right)$

RESULTS

Results Overview

SAFRAN isae SUPAERO

Three principle results:

1. Compact Closed-form CRB Expression for Time-Delay Estimation
2. Joint Time-Delay and Phase Estimation CRB
3. Joint Time-Delay and Doppler-Stretch Estimation

Presentation structure:

- Main contributions
- Validation of results
- Conclusion + publication

Compact Closed-form CRB Expression for Time-Delay Estimation

Case1: Constant propagation delay $\tau(t) = \tau$

Compact Closed-form CRB expression of Time-Delay Estimation:

$$\mathrm{F}_{\tau|\epsilon}(\epsilon) = \frac{2|\alpha|^2}{\left(\frac{\sigma_n^2}{F_s}\right)} F_s(\boldsymbol{s}^H\boldsymbol{s})\left(\frac{\boldsymbol{s}^H\boldsymbol{V}\boldsymbol{s}}{\boldsymbol{s}^H\boldsymbol{s}} - \left|\frac{\boldsymbol{s}^H\boldsymbol{\Lambda}\boldsymbol{s}}{\boldsymbol{s}^H\boldsymbol{s}}\right|^2\right)$$

- For generic BLS
- In terms of $\boldsymbol{s}$
- $\boldsymbol{V}, \boldsymbol{\Lambda}$: square matrices
- Does not depend on τ

Relationship with Signal to Noise Ratio (SNR):

$$\mathrm{SNR}_{out} = \frac{|\alpha|^2}{\left(\frac{\sigma_n^2}{F_s}\right)}\mathbb{E} \quad \text{(from MLE)}, \; \text{Signal energy: } \mathbb{E} = \frac{\boldsymbol{s}^H\boldsymbol{s}}{F_s} = \int_{-\infty}^{+\infty}|s(t)|^2$$

$$\mathrm{F}_{\tau|\epsilon}(\epsilon) = \mathrm{SNR}_{out} \times 2F_s^2\left(\frac{\boldsymbol{s}^H\boldsymbol{V}\boldsymbol{s}}{\boldsymbol{s}^H\boldsymbol{s}} - \left|\frac{\boldsymbol{s}^H\boldsymbol{\Lambda}\boldsymbol{s}}{\boldsymbol{s}^H\boldsymbol{s}}\right|^2\right) \xrightarrow{Real\ BLS} \mathrm{F}_{\tau|\epsilon}(\epsilon) = \mathrm{SNR}_{out} \times 2F_s^2\left(\frac{\boldsymbol{s}^H\boldsymbol{V}\boldsymbol{s}}{\boldsymbol{s}^H\boldsymbol{s}}\right)$$

Compact Closed-form CRB Expression for Time-Delay Estimation

CRB used for optimal signal design

Minimum $\mathrm{CRB}_{opt} = \mathrm{F}_{\tau|\epsilon}(\epsilon)^{-1}$ **for a real code (for given length** N **and energy** $\mathbb{E}$**):**

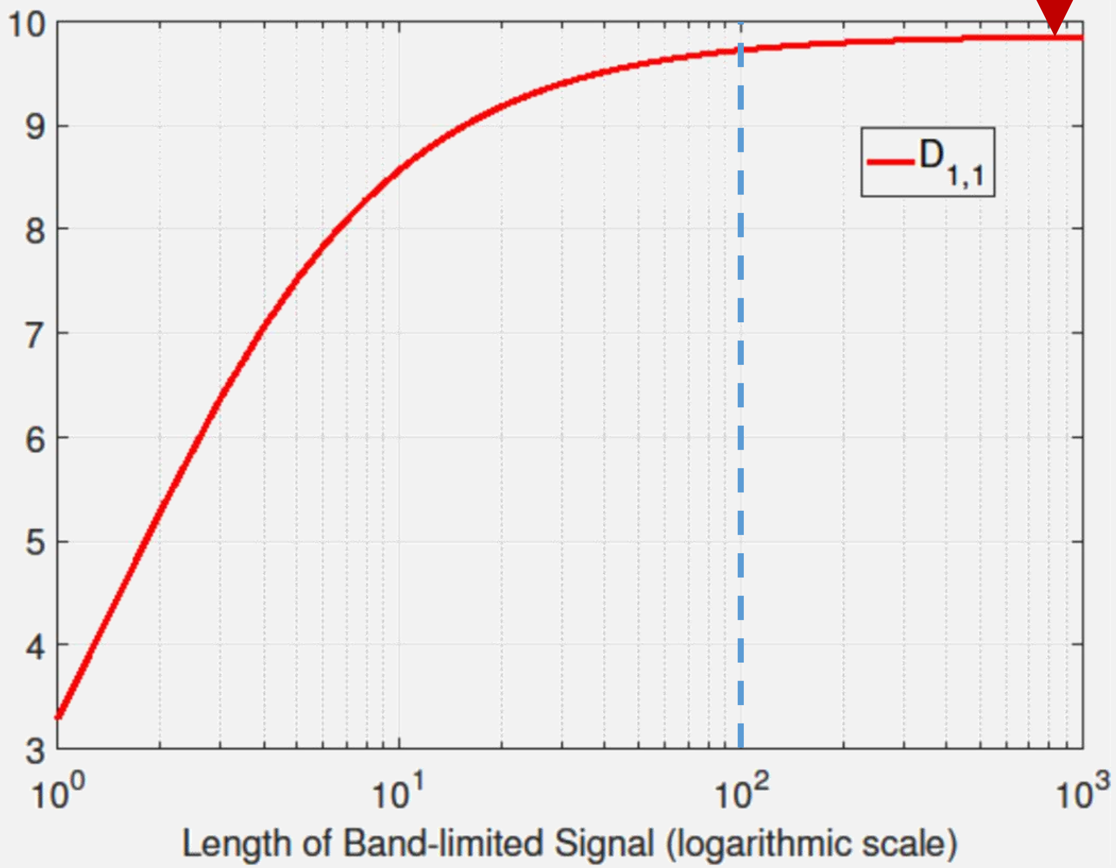
$$\mathrm{F}_{\tau|\epsilon}(\epsilon) \leq \frac{2|\alpha|^2\mathbb{E}}{\left(\frac{\sigma_n^2}{F_s}\right)} F_s^2 D_{1,1} = \mathrm{SNR}_{out} \times 2F_s^2 D_{1,1}$$

Performance Loss Metric:

$$\rho = \frac{\mathrm{CRB}_{opt}}{\mathrm{CRB}_s} = \frac{\frac{\boldsymbol{s}^H \boldsymbol{V} \boldsymbol{s}}{\boldsymbol{s}^H \boldsymbol{s}} - \left|\frac{\boldsymbol{s}^H \boldsymbol{\Lambda} \boldsymbol{s}}{\boldsymbol{s}^H \boldsymbol{s}}\right|^2}{D_{1,1}}$$

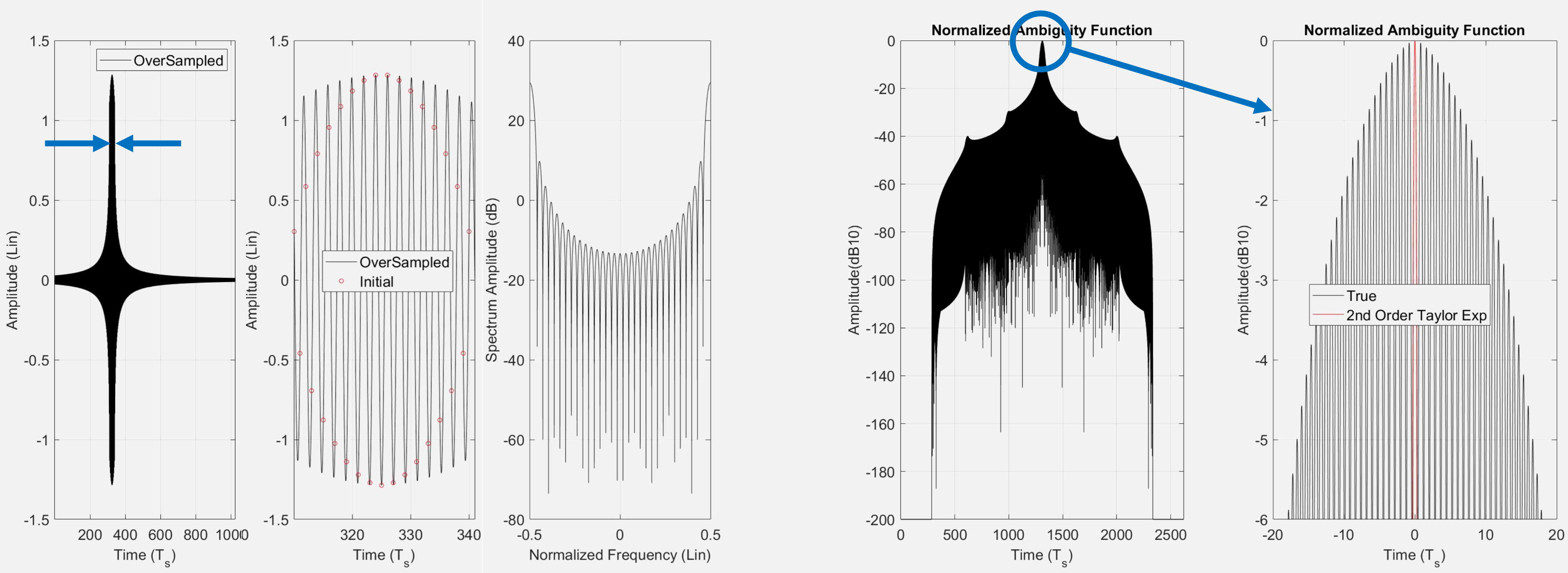
GPS L1 C/A: N = 1023

$$\lim_{N\to\infty} D_{1,1} = \pi^2$$



Compact Closed-form CRB Expression for Time-Delay Estimation

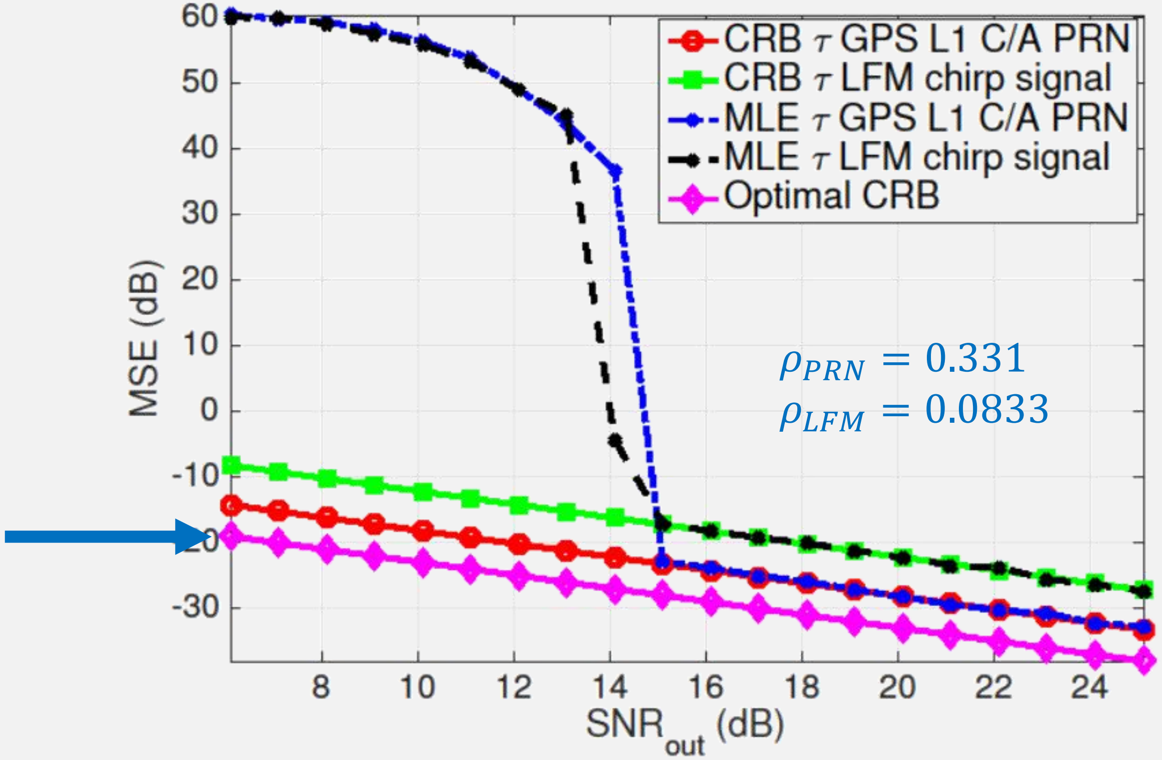
Optimal-bound signal (Bound of the bound)



N = 31

Compact Closed-form CRB Expression for Time-Delay Estimation

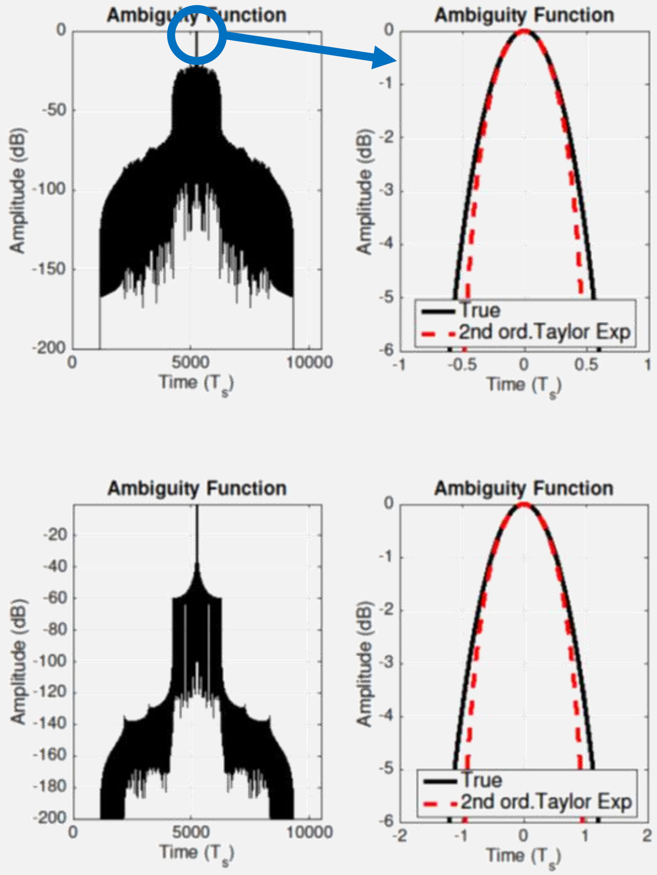
CRB and MLE for two band-limited signals, N=1023



Ambiguity function and its 2nd order Taylor expansion

GPS L1 C/A code

LFM chirp signal



Compact Closed-form CRB Expression for Time-Delay Estimation

✓ **Derivation of a compact closed-form CRB expression for time-delay estimation**

- Considering generic band-limited transmitted signal
- Constant transmitter to receiver propagation delay.

✓ **Optimal CRB**

- Optimal signal design
- Performance loss metric

✓ **Publication:**

❖ P. Das, J. Vilà-Valls, E. Chaumette, F. Vincent, L. Davain, and S. Bonnabel.
"On the Accuracy Limit of Time-delay Estimation with a Band-limited Signal".
In: Proc. of the IEEE Intl. Conf. on Acoustics, Speech and Signal Processing (ICASSP). Brighton, UK, May 2019

Joint Time-Delay and Phase Estimation CRB

SAFRAN isae SUPAERO

Contribution | Validation | Conclusion

- For generic BLS
- In terms of $\boldsymbol{s}$
- Does not depend on τ, φ

Time-delay and phase CRB

$$\mathbf{CRB}_{\boldsymbol{\theta}|\epsilon} = \begin{bmatrix} \mathrm{CRB}_{\varphi|\epsilon} & \mathrm{CRB}_{\tau,\varphi|\epsilon} \\ \mathrm{CRB}_{\tau,\varphi|\epsilon} & \mathrm{CRB}_{\tau|\epsilon} \end{bmatrix}$$

$$\mathrm{CRB}_{\tau|\epsilon} = \frac{1}{2\mathrm{SNR_{out}}} \frac{1}{F_s^2\left(\frac{\boldsymbol{s}^H\boldsymbol{V}\boldsymbol{s}}{\boldsymbol{s}^H\boldsymbol{s}} - \mathrm{Im}\left\{\frac{\boldsymbol{s}^H\boldsymbol{\Lambda}\boldsymbol{s}}{\boldsymbol{s}^H\boldsymbol{s}}\right\}^2\right)} \underset{Real\ signal}{=} \frac{1}{2\mathrm{SNR_{out}}} \frac{1}{F_s^2\left(\frac{\boldsymbol{s}^H\boldsymbol{V}\boldsymbol{s}}{\boldsymbol{s}^H\boldsymbol{s}}\right)}$$

Time-delay CRB same as in the previous case (without phase)

$$\mathrm{CRB}_{\varphi|\epsilon} = \frac{1}{2\mathrm{SNR_{out}}}\left(1 + \frac{\mathrm{Im}\left\{\frac{\boldsymbol{s}^H\boldsymbol{\Lambda}\boldsymbol{s}}{\boldsymbol{s}^H\boldsymbol{s}}\right\}^2}{\frac{\boldsymbol{s}^H\boldsymbol{V}\boldsymbol{s}}{\boldsymbol{s}^H\boldsymbol{s}} - \mathrm{Im}\left\{\frac{\boldsymbol{s}^H\boldsymbol{\Lambda}\boldsymbol{s}}{\boldsymbol{s}^H\boldsymbol{s}}\right\}^2}\right) \underset{Real\ signal}{=} \frac{1}{2\mathrm{SNR_{out}}}$$

Performance of phase estimation independent of $\boldsymbol{s}(t)$
Depends on $\mathrm{SNR_{out}}$ of matched filter

$$\mathrm{CRB}_{\tau,\varphi|\epsilon} = \frac{1}{2\mathrm{SNR_{out}}} \frac{\mathrm{Im}\left\{\frac{\boldsymbol{s}^H\boldsymbol{\Lambda}\boldsymbol{s}}{\boldsymbol{s}^H\boldsymbol{s}}\right\}}{F_s^2\left(\frac{\boldsymbol{s}^H\boldsymbol{V}\boldsymbol{s}}{\boldsymbol{s}^H\boldsymbol{s}} - \mathrm{Im}\left\{\frac{\boldsymbol{s}^H\boldsymbol{\Lambda}\boldsymbol{s}}{\boldsymbol{s}^H\boldsymbol{s}}\right\}^2\right)} \underset{Real\ signal}{=} 0$$

No impact of signal phase on time-delay estimation

Joint Time-Delay and Phase Estimation CRB

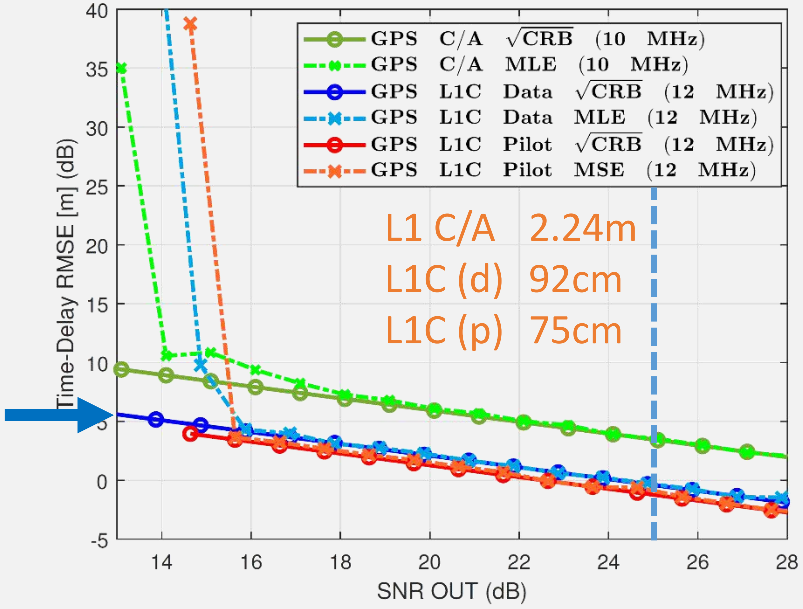
SAFRAN

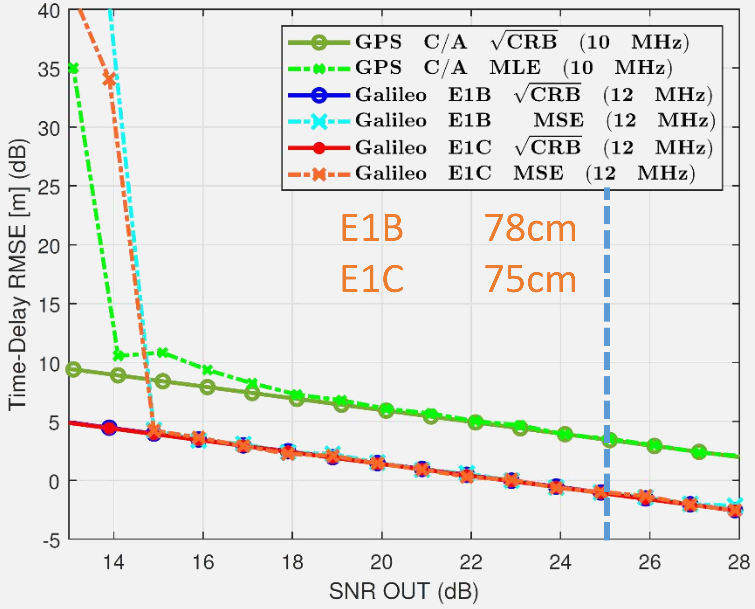
isae SUPAERO

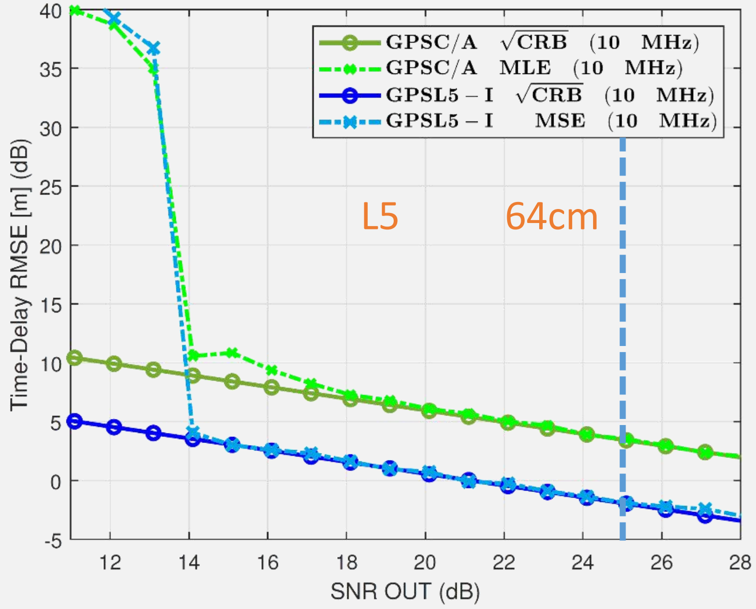
Contribution | Validation | Conclusion

CRB/MLE for various representative GNSS signals

Standard deviations at SNRout = 25dB







GPS L1 C/A vs GPS L1C (data & pilot) | GPS L1 C/A vs Gal E1B/E1C | GPS L1 C/A vs GPS L5

LIC and E1 OS: BOC modulation
GPS L1 C/A: BPSK(1)

L5: BPSK(10)

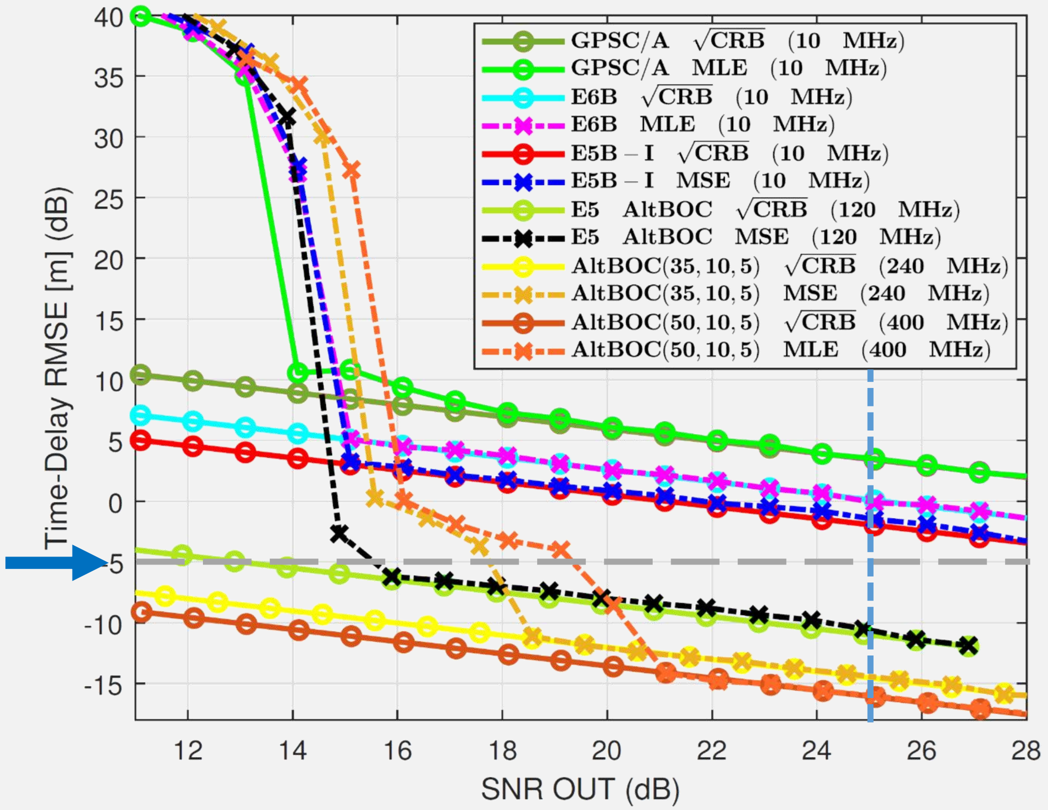
BOC : Binary Offset Carrier
BPSK: Binary Phase Shift Keying

Joint Time-Delay and Phase Estimation CRB

SAFRAN isae SUPAERO

Contribution | Validation | Conclusion

CRB/MLE for various representative GNSS signals



Theoretical performance limits of Alt-BOC type signals

Time-delay Estimation Standard Deviation [cm]

SNRout (dB) $= {}^{C}/_{N_0}\, T_I$	$T_I = TP_{RN}\, L_c$	AltBOC (15,10)	AltBOC (35,10,5)	AltBOC (50,10,5)
25	10 ms	8	3.55	2.51
30	30 ms	4.47	2	1.41
35	100 ms	2.5	1.12	0.79
40	300 ms	1.41	0.63	0.45
45	1 s	0.8	0.35	0.25
50	3 s	0.45	0.2	0.14

		2.24m (L1 C/A)
Galileo E6	BPSK(5)	1m
Galileo E5b-I	BPSK(10)	64cm
Galileo E5	AltBOC(15,10)	**8cm**
and meta signals	AltBOC(35,10,5)	**3.5cm**
	AltBOC(50,10,5)	**2.5cm**

BOC : Binary Offset Carrier
BPSK: Binary Phase Shift Keying

Joint Time-Delay and Phase Estimation CRB

✓ Derivation of joint time-delay and phase estimation CRB

- Extension of previous **compact closed-form time-delay estimation CRB**
- Directly computed from the **signal samples** (fits GNSS problem)

✓ Comprehensive asymptotic performance analysis of GNSS signals

- Time-delay CRB: **achievable synchronization performance** with different GPS and Galileo signals
- MSE threshold after which carrier phase is required for positioning

❖ L. Ortega et al. ”Positioning Performance Limits of GNSS Meta-Signals and HO-BOC Signals”, *in Sensors 2020, 20(12)*

✓ Publication:

❖ P. Das, L. Ortega, J. Vilà-Valls, F. Vincent, E. Chaumette, and L. Davain.
“Performance Limits of GNSS Code-Based Precise Positioning: GPS, Galileo & Meta-Signals”.
In: Sensors (2020), Vol. 20, Issue 8

Joint Time-Delay and Doppler-Stretch Estimation

SAFRAN isae SUPAERO

Contribution Validation Conclusion

CRB of ρ, φ and $\boldsymbol{\eta} = [\tau, b]^T$ of Conditional Signal Model

Preliminary compact CRB (for $\boldsymbol{\eta}$):

$$\mathrm{CRB}_{\boldsymbol{\eta}|\epsilon}^{-1} = \frac{2|\alpha|^2}{\sigma_n^2} \Re\{\boldsymbol{\Phi}(\boldsymbol{\eta})\} \qquad \Re\{\boldsymbol{\Phi}(\boldsymbol{\eta})\} = \begin{bmatrix} (\cdot)_{1,1} & (\cdot)_{1,2} \\ (\cdot)_{1,2} & (\cdot)_{2,2} \end{bmatrix}$$

Versatile compact CRB for delay, Doppler stretch and phase estimation with a BLS ($\beta = 1 - b$)

For real-valued $s(t)$:

Narrowband

$$(\cdot)_{1,1} = F_s \boldsymbol{s}^T \boldsymbol{s} \frac{\boldsymbol{s}^T \boldsymbol{V} \boldsymbol{s}}{\boldsymbol{s}^T \boldsymbol{s}}$$
$$(\cdot)_{1,2} = 0$$
$$(\cdot)_{2,2} = \frac{\boldsymbol{s}^T \boldsymbol{s}}{F_s \beta^3} \frac{\omega_c^2}{F_s^2} \left(\frac{\boldsymbol{s}^T \boldsymbol{D}^2 \boldsymbol{s}}{\boldsymbol{s}^T \boldsymbol{s}} - \left(\frac{\boldsymbol{s}^T \boldsymbol{D}^2 \boldsymbol{s}}{\boldsymbol{s}^T \boldsymbol{s}} \right)^2 \right)$$

- Signal samples
- No impact of τ
- Small b, $\beta \sim 1$
  -> narrowband

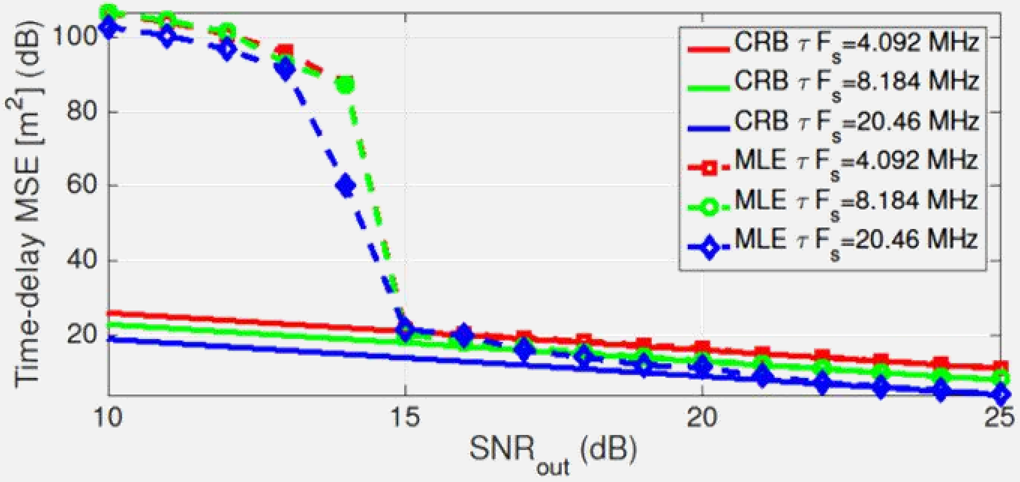
Wideband

$$(\cdot)_{1,1} = F_s \beta \boldsymbol{s}^T \boldsymbol{s} \frac{\boldsymbol{s}^T \boldsymbol{V} \boldsymbol{s}}{\boldsymbol{s}^T \boldsymbol{s}}$$
$$(\cdot)_{1,2} = \frac{\boldsymbol{s}^T \boldsymbol{s}}{\beta} \frac{\boldsymbol{s}^T \boldsymbol{V} \boldsymbol{D} \boldsymbol{s}}{\boldsymbol{s}^T \boldsymbol{s}}$$
$$(\cdot)_{2,2} = \frac{\boldsymbol{s}^T \boldsymbol{s}}{F_s \beta^3} \left(\frac{\omega_c^2}{F_s^2} \left(\frac{\boldsymbol{s}^T \boldsymbol{D}^2 \boldsymbol{s}}{\boldsymbol{s}^T \boldsymbol{s}} - \left(\frac{\boldsymbol{s}^T \boldsymbol{D}^2 \boldsymbol{s}}{\boldsymbol{s}^T \boldsymbol{s}} \right)^2 \right) + 2 + \frac{\boldsymbol{s}^T \boldsymbol{D} \boldsymbol{V} \boldsymbol{D} \boldsymbol{s}}{\boldsymbol{s}^T \boldsymbol{s}} - \left(1 + \frac{\boldsymbol{s}^T \boldsymbol{D} \boldsymbol{\Lambda} \boldsymbol{s}}{\boldsymbol{s}^T \boldsymbol{s}} \right)^2 \right)$$

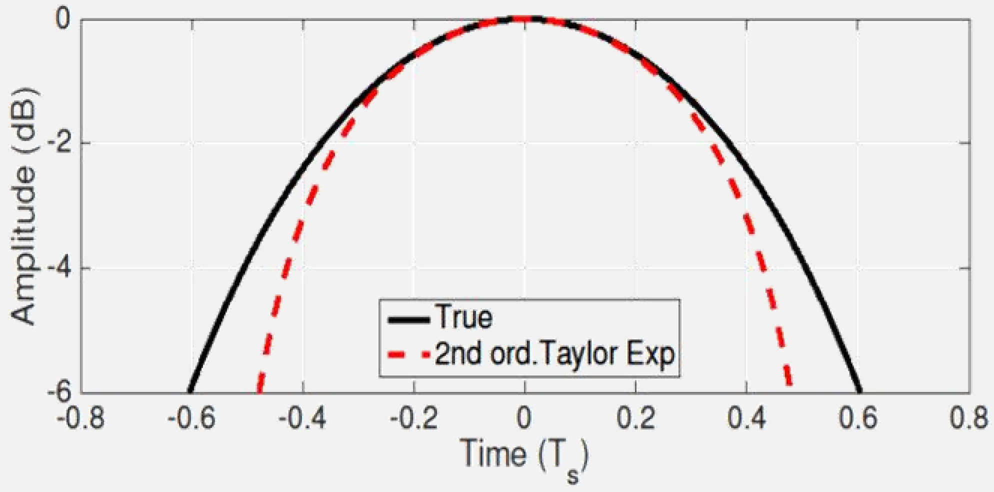
Joint Time-Delay and Doppler-Stretch Estimation

Synthetic signal experiment

GPS L1 C/A PRN code



Time-delay CRB and MLE



Ambiguity function
and corresponding 2nd order Taylor expansion

$$Fs = \frac{1}{Ts} = 1.023 \text{ MHz (chip frequency)}$$

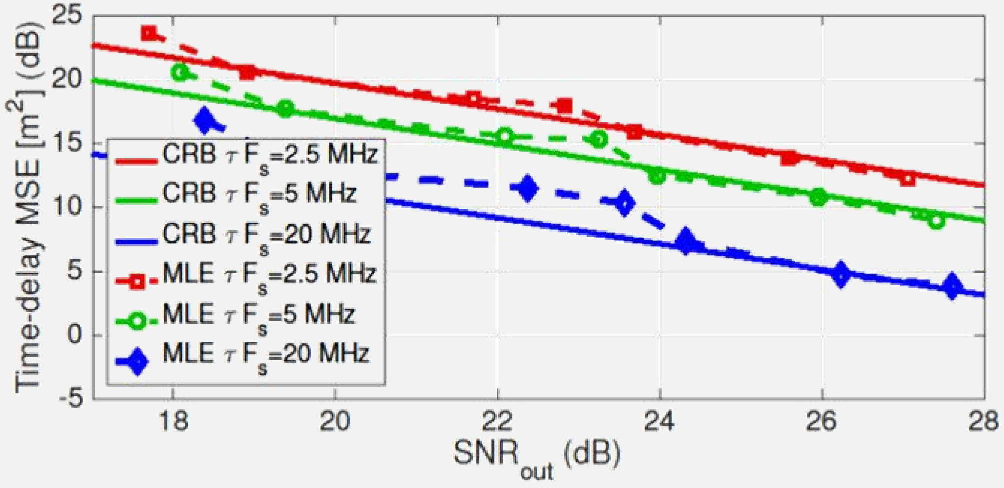
Joint Time-Delay and Doppler-Stretch Estimation

Real signal experiment: GPS L1 C/A PRN code

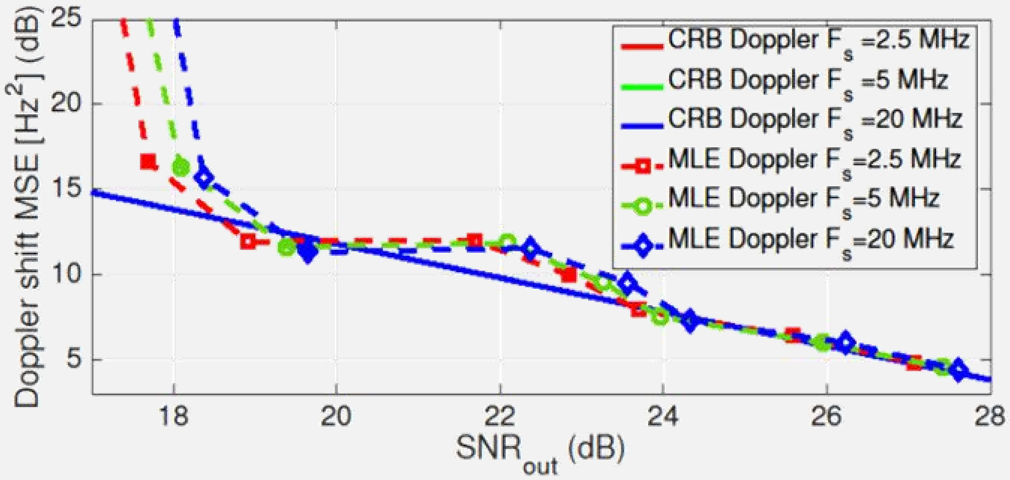
Static L1 patch
Open sky conditions

- **USRP X310**
  $F_s = 20\text{MHz}$
- *Baseband extraction*

- Length = 10 GPS L1 C/A PRN = 10ms
- 100 correlations = 1s
- Time-delay and Doppler shift MLE



Time-delay CRB and MLE



Doppler frequency shift CRB and MLE

Joint Time-Delay and Doppler-Stretch Estimation

SAFRAN isae SUPAERO

Contribution Validation Conclusion

- ✓ **Derivation of a general compact closed-form CRB expression for the delay-Doppler estimation**
- ✓ **General compact closed-form CRB expression for amplitude and phase**
- ✓ **Compact CRBs expressed in terms of signal samples**
- ✓ **Publication:**
  - ❖ P. Das, J. Vilà-Valls, F. Vincent, L. Davain, and E. Chaumette.
    "A New Compact Delay, Doppler Stretch and Phase Estimation CRB with a Band-Limited Signal for Generic Remote Sensing Applications".
    In: Remote Sensing (2020), Vol. 12, issue 18

CONCLUSION

Conclusion

SAFRAN isae SUPAERO

Main objective:

- Characterization of **asymptotic performance** of GNSS time-delay, Doppler and phase estimation
- Using **CRB** family of lower bounds

Proposed CRBs:

- ✓ Easy-to-use and compact
- ✓ In terms of **baseband signal samples**
- ✓ For generic **band-limited signal**

Results:

1. **Compact closed-form CRB for time-delay estimation**
   - ❖ Performance loss metric
2. **Joint time-delay and phase estimation CRB:**
   - ❖ Characterization of GNSS signals
   - ❖ MSE thresholds requiring carrier phase for GNSS
3. **Joint time-delay and Doppler stretch estimation**
   - ❖ Narrowband and wideband

NOVELTY!

CRBs do not depend on estimation parameters

Comparison of different signals possible over same criteria

Conclusion: List of publications

Main objective:

- Characterization of **asymptotic performance** of GNSS time-delay, Doppler and phase estimation
- Using **CRB** family of lower bounds

Results:

1. **Compact closed-form CRB for time-delay estimation**
   - Performance loss metric
2. **Joint time-delay and phase estimation CRB:**
   - Characterization of GNSS signals
   - MSE thresholds requiring carrier phase for GNSS
3. **Joint time-delay and Doppler stretch estimation**
   - Narrowband and wideband

Conferences

[1] P. Das, J. Vilà-Valls, E. Chaumette, F. Vincent, L. Davain, and S. Bonnabel. **"On the Accuracy Limit of Time-delay Estimation with a Band-limited Signal"**. *IEEE Intl. Conf. on Acoustics, Speech and Signal Processing (ICASSP). Brighton, UK, May 2019*

[2] P. Das, B. Priot, J. Vilà-Valls, and E. Chaumette. **"A Signal Processing Approach to GNSS Precise Positioning for Lunar Exploration"**. *70th International Astronautical Congress (IAC'19). Washington DC, USA, Nov. 2019 – Awarded the ESA ISEB grant*

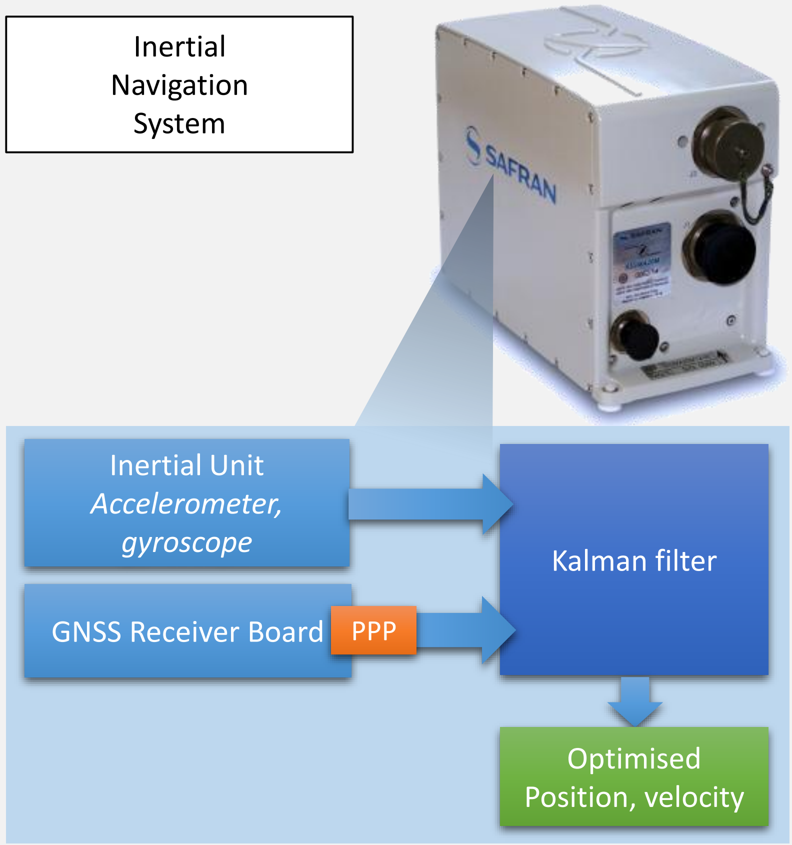
Journals

[3] P. Das, L. Ortega, J. Vilà-Valls, F. Vincent, E. Chaumette, and L. Davain. **"Performance Limits of GNSS Code-Based Precise Positioning: GPS, Galileo & Meta-Signals"**. *Sensors (2020) Vol. 20, Issue 8*

[4] P. Das, J. Vilà-Valls, F. Vincent, L. Davain, and E. Chaumette. **"A New Compact Delay, Doppler Stretch and Phase Estimation CRB with a Band-Limited Signal for Generic Remote Sensing Applications"**. *Remote Sensing (2020) Vol. 12, Issue 18*

Perspectives

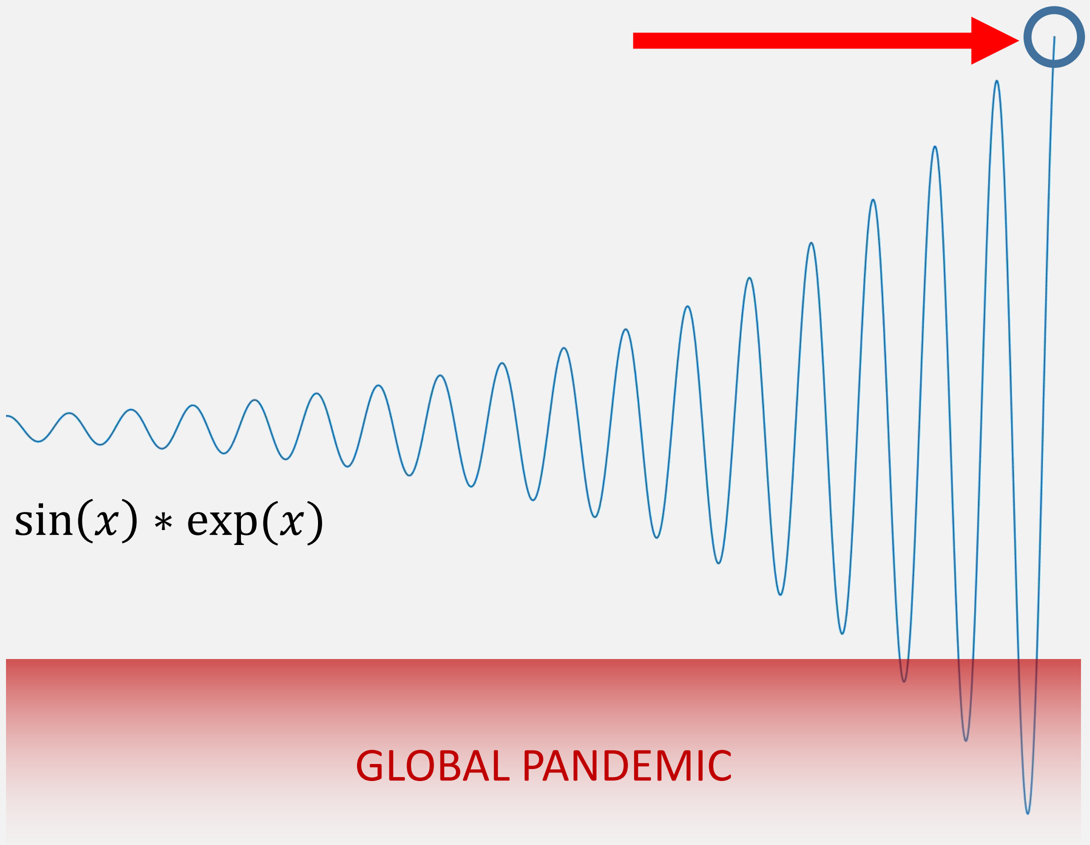
- **Closed-form CRBs:** Generalized, depending only on **baseband signal sample vector**
  - ❖ Formulation of **new cost-functions** dedicated to waveform optimization
- **Asymptotic performance analysis**
  - ❖ With **longer coherent integration** time
  - ❖ Reaching **code-based sub-cm accuracy**?
  - ❖ CRB with **acceleration** and higher-order terms?
  - ❖ Considering **external corrections** (ionosphere, clock errors,…)
- **Further performance analysis**
  - ❖ **Hybrid architectures**: fusion with other sensors, e.g. inertial navigation systems (INS)
  - ❖ **Harsh environments**: e.g. in multipath conditions



What is a PhD?

Doctorat:
(*Latin*) *doctorem*,
enseigner (*to teach*)

PhD:
(*Latin*) *Philosophiae Doctor,*
"Doctor of Philosophy"



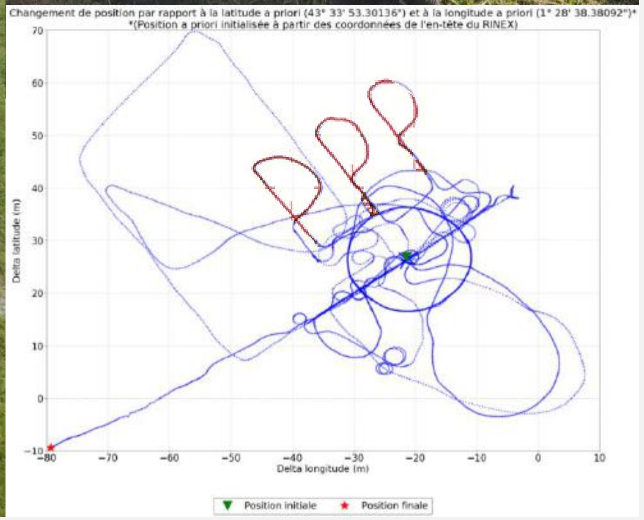
PREX: Experimental Project (1st year students, ISAE-Supaéro)

SAFRAN ISAE SUPAERO

20h project on SPP and RTK algorithms + Kalman filter (Linear and extended) + INS / GPS hybridization

Doctorat:
(Latin) doctorem,
enseigner (*to teach*)

PhD:
(Latin) Philosophiae Doctor,
“Doctor of Philosophy”



HI-SEAS Lunar Simulation Mission (January 2020)

Doctorat:
(*Latin*) *doctorem*,
enseigner (*to teach*)

PhD:
(Latin) Philosophiae Doctor,
“Doctor of Philosophy”

Université Fédérale Toulouse Midi-Pyrénées
Edmitt
Ecole Doctorale Mathématiques, Informatique et Télécommunications de Toulouse

ISAE
Institut Supérieur de l'Aéronautique et de l'Espace
SUPAERO

"If chance is the antithesis of law, then we need to discover the laws of chance."
- *Prof. C. R. Rao*

Thank you for you attention.

Special thanks!

- Advisors and Jury
- Safran ("*Humans of Éragny*")
- ISAE: SCAN/DEOS and TéSA
- EuroMoonMars and #TeamHB5
- Mentors, doctors, family (including pets) and friends

Synchronization Performance Limits of GNSS Receivers

Priyanka DAS

PhD Thesis Defense | 30th March, 2021

Annex

References

[CFF07] P. Closas, C. Fernández-Prades, and J. A. Fernández-Rubio. “Cramér-Rao bound analysis of positioning approaches in GNSS receivers”. In: *IEEE Trans. on Signal Processing* 57.10 (Nov. 2009),

[Kay93] S. M. Kay. *Fundamentals of Statistical Signal Processing: Estimation Theory* Englewood Cliffs, New Jersey, USA: Prentice-Hall, 1993

[Ott+93] B. Ottersten, M. Viberg, P. Stoica, and A. Nehorai. “Exact and Large Sample Maximum Likelihood Techniques for Parameter Estimation and Detection in Array Processing”. In: *Radar Array Processing*. Ed. by S. Haykin, J. Litva, and T. J. Shepherd. Heidelberg: Springer-Verlag, 1993

[Ren+06] A. Renaux, P. Forster, E. Chaumette, and P. Larzabal. “On the High-SNR Conditional Maximum-Likelihood Estimator Full Statistical Characterization”. In:*IEEE Trans. Signal Process.* 54.12 (Dec. 2006)

[Ric03] D. W. Ricker. *Echo Signal Processing*. Springer, New York, USA: Kluwer Academic, 2003

[Sko90] M. I. Skolnik. *Radar Handbook*. 3rd. New York, USA: McGraw-Hill, 1990

[SN90] P. Stoica and A. Nehorai. “Performances study of conditional and unconditional direction of arrival estimation”. In: *IEEE Trans. Acoust., Speech, Signal Process.* 38.10 (Oct. 1990)

[TB07] H. L. Van Trees and K. L. Bell, eds. *Bayesian Bounds for Parameter Estimation and Nonlinear Filtering/Tracking*. NY, USA: Wiley/IEEE Press, 2007

[Vin+17] F. Vincent, E. Chaumette, C. Charbonnieras, J. Israel, L.l Ries, M. Aubourg, and F. Barbiero. “Asymptotically Efficient GNSS Trilateration”. In: 133 (Apr. 2017)

[YB92] S. F. Yau and Y. Bresler. “A Compact Cramér-Rao Bound Expression for Parametric Estimation of Superimposed Signals”. In: *IEEE Trans. Signal Process.* 40.5 (May 1992)