

Homework-projects

Consider following eight systems of linear algebraic equations $Ax=b$

$$\begin{aligned} -2x_1 + 3x_2 + x_3 &= 9 \\ 3x_1 + 4x_2 - 5x_3 &= 0 \\ x_1 - 2x_2 + x_3 &= -4 \end{aligned} \quad (A) \quad \begin{bmatrix} 1 & 1 & 3 \\ 5 & 3 & 1 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \quad (B)$$

$$\begin{aligned} x_1 + 3x_2 + 2x_3 - x_4 &= 9 \\ 4x_1 + 2x_2 + 5x_3 + x_4 &= 27 \\ 3x_1 - 3x_2 + 2x_3 + 4x_4 &= 19 \\ -x_1 + 2x_2 - 3x_3 + 5x_4 &= 14 \end{aligned} \quad (C) \quad \begin{bmatrix} 3 & 1 & -1 & 3 \\ 2 & 1 & -2 & 0 \\ 0 & 3 & 2 & -2 \\ 1 & 1 & 1 & 5 \end{bmatrix} [x_i] = \begin{bmatrix} 4 \\ -1 \\ 4 \\ -2 \end{bmatrix} \quad (D)$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 2 \\ -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 8 \end{bmatrix} \quad (E) \quad \begin{bmatrix} 2 & 3 & 5 \\ 3 & 1 & -2 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ -3 \end{bmatrix} \quad (F)$$

$$\begin{bmatrix} 2 & -2 & 2 & 1 \\ 2 & -4 & 1 & 3 \\ -1 & 3 & -4 & 2 \\ 2 & 4 & 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7 \\ 10 \\ -14 \\ 1 \end{bmatrix} \quad (G) \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 3 & 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \\ 1 \end{bmatrix} \quad (H)$$

Number on the list of exercise corresponds to the number which was fixed to each the student during laboratory exercises

1. Solve numerically Eq.(E) by Gauss elimination without pivoting
2. Solve numerically Eq.(F) by Gauss elimination without pivoting
3. Solve numerically Eq.(G) by Gauss elimination without pivoting
4. Solve numerically Eq.(H) by Gauss elimination without pivoting
5. Solve numerically Eq.(A) by Gauss-Jordan elimination
6. Solve numerically Eq.(B) by Gauss-Jordan elimination
7. Solve numerically Eq.(C) by Gauss-Jordan elimination
8. Solve numerically Eq.(D) by Gauss-Jordan elimination
9. Solve Eq.(A) by Doolittle LU factorization method
10. Solve Eq.(B) by Doolittle LU factorization method
11. Solve Eq.(C) by Doolittle LU factorization method
12. Solve Eq.(A) by Doolittle LU factorization method

Iterative Methods

Solve the following problems by iterative methods. Let $x^{(0)T} = [0.0, 0.0, 0.0, 0.0]$.

For computer solutions, iterate until six digits after the decimal place converges.

Tridiagonal Systems of Equations

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 12 \\ 11 \end{bmatrix} \quad (\text{I}) \quad \begin{bmatrix} 3 & 2 & 0 & 0 \\ 2 & 3 & 2 & 0 \\ 0 & 2 & 3 & 2 \\ 0 & 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 12 \\ 17 \\ 14 \\ 7 \end{bmatrix} \quad (\text{J})$$
$$\begin{bmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -7 \\ -1 \end{bmatrix} \quad (\text{K}) \quad \begin{bmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 0 \\ 8 \end{bmatrix} \quad (\text{L})$$
$$\begin{bmatrix} 4 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{bmatrix} [x_i] = \begin{bmatrix} 150 \\ 200 \\ 150 \\ 100 \end{bmatrix} \quad (\text{M}) \quad \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 0 \\ 8 \end{bmatrix} \quad (\text{N})$$

13. Solve Eq. (I) by the SOR method for $1.25 \leq \omega \leq 1.35$ with $\Delta\omega = 0.01$.

14. Solve Eq. (K) by the SOR method for $1.25 \leq \omega \leq 1.35$ with $\Delta\omega = 0.01$.

15. Solve Eq. (L) by the SOR method for $1.25 \leq \omega \leq 1.35$ with $\Delta\omega = 0.01$.

16. Solve Eq. (I) by the Thomas algorithm.

17. Solve Eq. (J) by the Thomas algorithm.

18. Solve Eq. (K) by the Thomas algorithm.