Department of Computer Science and Engineering (CSE)
BRAC University

Lecture 2

CSE250 - Circuits and Electronics

SERIES AND PARALLEL CONFIGURATIONS

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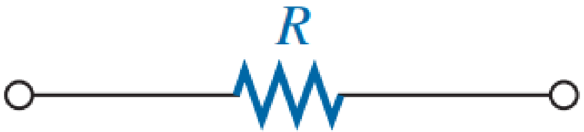
Resistance



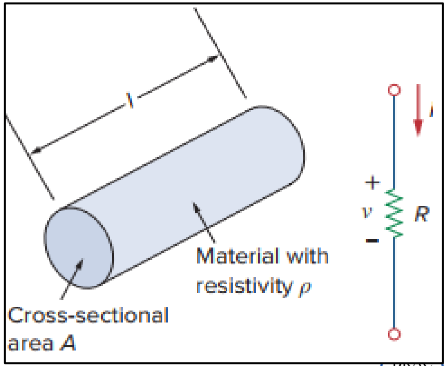
- What determines the level of current that results when a particular voltage is applied across a wire? *The answers lie in the fact that there is an opposition to the flow of charge in the system that depends on the components of the circuit. This opposition to the flow of charge through an electrical circuit, called resistance*.

- This opposition, due primarily to collisions and friction between the free electrons and other electrons, ions, and atoms in the path of motion, converts the supplied electrical energy into heat that raises the temperature of the electrical component and surrounding medium.

- So, *resistance* is a physical property of materials that refers to the ability to resist current.

- The resistance of any material with a uniform cross-sectional area A depends on A and its length ℓ. Mathematically, $R = \rho\frac{L}{A}$, where ρ is known as the resistivity of the material in ohm-meters $(\Omega - m)$.

- The measuring unit for resistance is ohm (Ω)



Conductance

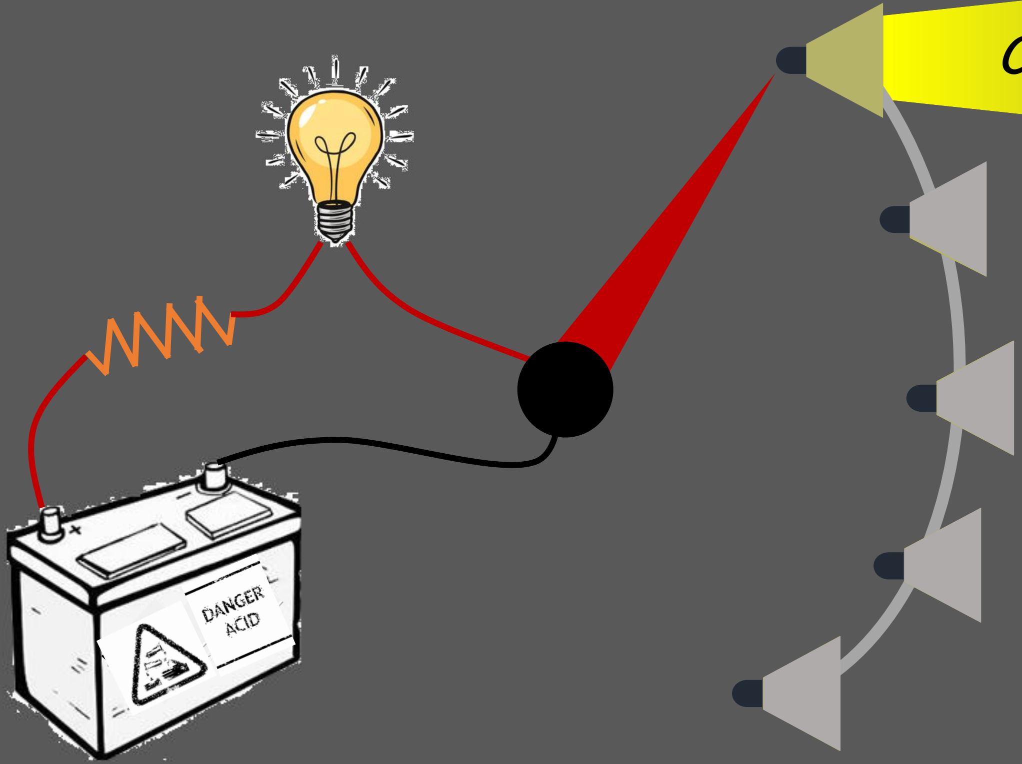
- By finding the reciprocal of the resistance of a material, we have a measure of how well the material conducts electricity. The quantity is called conductance, has the symbol G, and is measured in $siemens$ (S) or $mhos$ $(\mho)$

- So, the *conductance* is a measure of how well an element will conduct electric current.

- $G = \frac{1}{R}$ $[1\, \mho = 1\, A/V = 1\, Siemen\, (S)]$

- $G = \frac{1}{R} = \frac{A}{\rho L} = \frac{\sigma A}{L}$, where $\sigma = \frac{1}{\rho}$ is a material-specific parameter called *conductivity*, measured in *siemens per meter* (Sm^{-1})

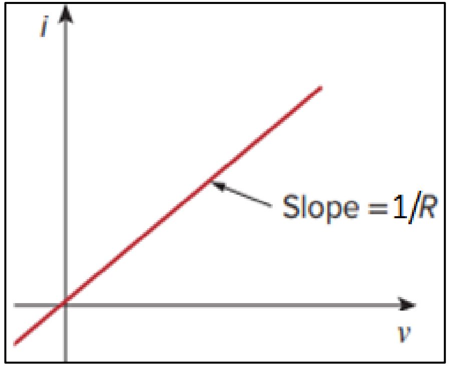
Course Outline: broad themes



- Circuit Laws
  - Ohm's Law
- Methods of Analysis
- Circuit Theorems
- First Order Circuits
- AC Circuits

Ohm's Law

- One of the basic equations for any physical system is, $Effect = \frac{Cause}{Opposition}$. Every conversion of energy from one form to another can be related to this equation.
- In electric circuits, the effect we are trying to establish is the flow of charge, or current. The potential difference, or voltage, between two points is the cause ("pressure"), and the opposition is the resistance encountered. Substituting the terms,
- $Current = \frac{Voltage}{Resistanee} \quad \Rightarrow \quad I = \frac{V}{R}$



- *Ohm's law* states that the voltage across a resistor is directly proportional to the current flowing through the resistor.
- That is, $v \propto i$ or $v = Ri$. Ohm defined the constant of proportionality for a resistor to be the resistance, R, measured in $ohm\ (\Omega)$. [1 Ω = 1 V/A]
- $p = vi = i^2R = \frac{v^2}{R}$ *(always $+ve$ for passive elements like resistor)*

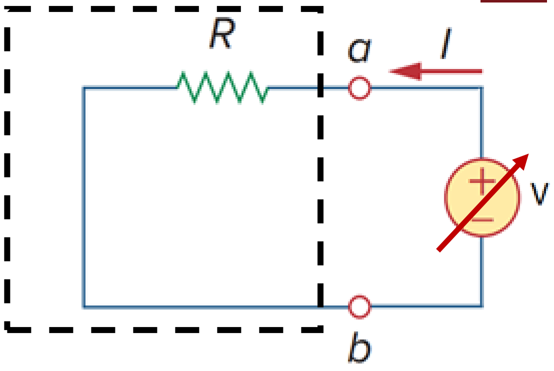
I-V of a Resistor

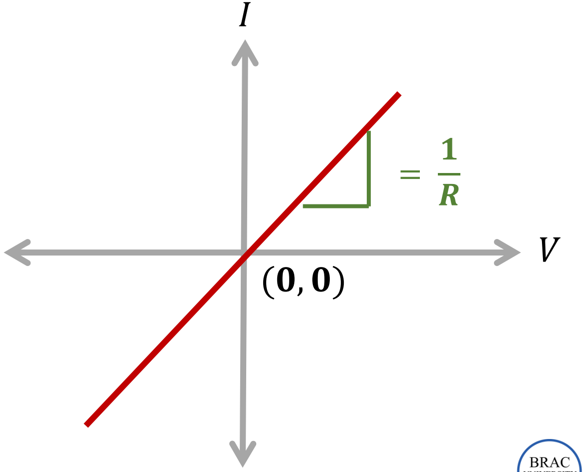
- The current voltage relationship or $I - V$ *characteristics* of a resistor follows Ohm's law.
- Let's say we have a resistor (R) between terminals a and b. To determine the $I - V$ characteristics, if applying a voltage V between $a - b$ makes the resistor to draw a current I, we can write,

$$V = IR$$

$$\Rightarrow I = \frac{1}{R}V$$

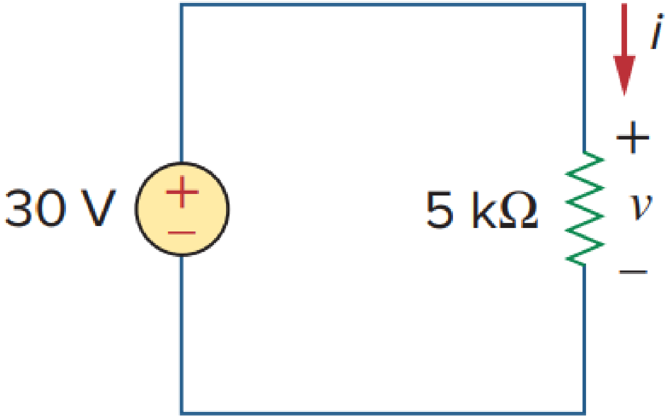
- The equation results in a linear I vs V plot that intersects the axes at the origin.
- The slope of the line depends on the value of the resistance. The greater the resistance, the less current flows through it, and the curve deviates from the current axis and flattens.





Example 1

- In the circuit shown below, calculate the current i, the conductance G, and the power p.



Solution

The voltage across the resistor is the same as the source voltage $(30\ V)$ because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is,

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 0.006\ A = 6\ mA$$

The conductance is,

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.0002\ S = 0.2\ mS$$

The power can be calculated in various ways

$$p = vi = 30 \times (6 \times 10^{-3}) = 0.18\ W = 180\ mW$$

Or,

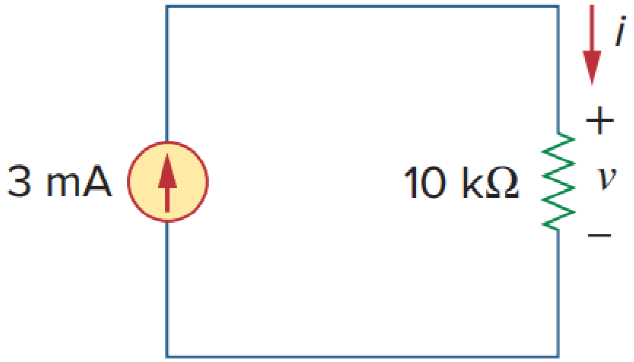
$$p = i^2 R = (6 \times 10^{-3})^2 \times (5 \times 10^3) = 180\ mW$$

Or,

$$p = \frac{v^2}{R} = \frac{30^2}{5 \times 10^3} = 180\ mW$$

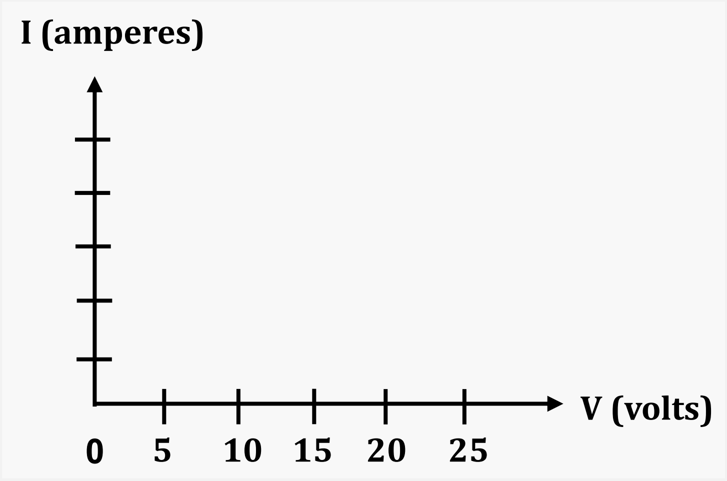
Problem 1

i. For the circuit shown below, calculate the voltage v, the conductance G, and the power p.



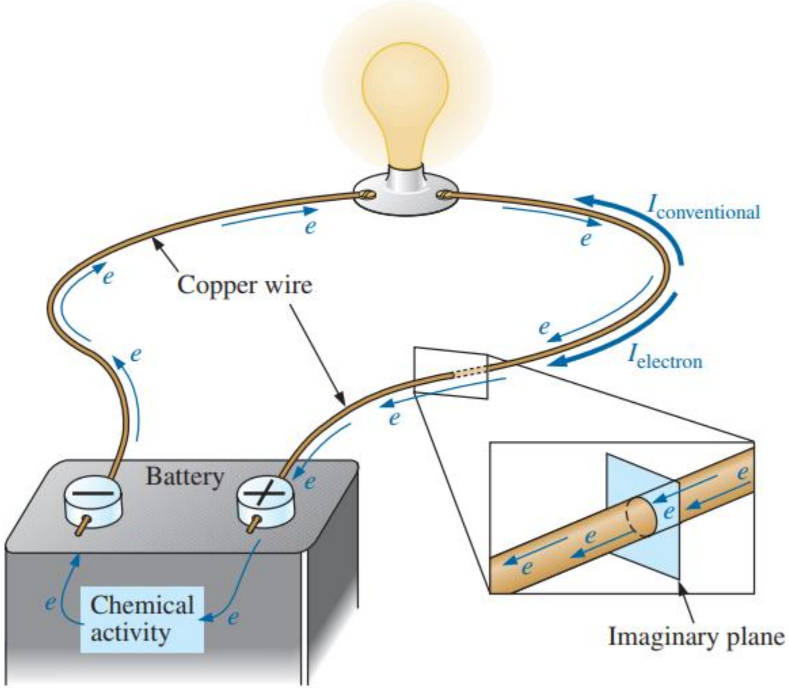
Ans: (i) $v = 30\ V$; $G = 100\ \mu S$; $p = 90\ mW$

ii. Draw the $I - V$ characteristics of a $10\ k\Omega$ resistor using the following template. Label the axes appropriately.



Problem 2

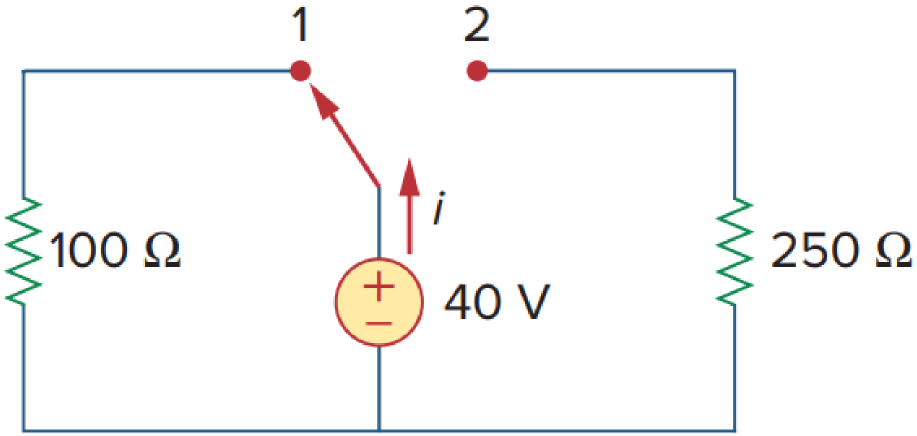
- Find the hot resistance of a light bulb rated $60\ W, 120\ V$.



Ans: $R = 240\ \Omega$

Problem 3

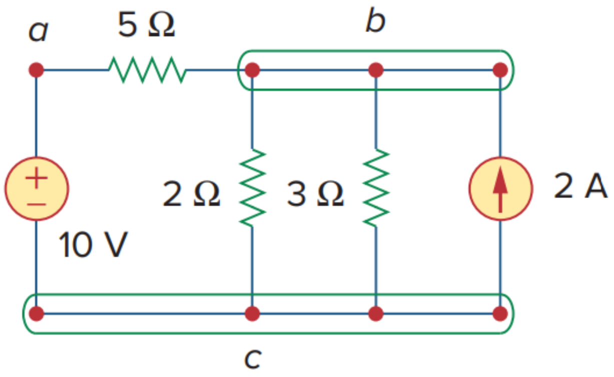
- (a) Calculate current i when the switch is in position 1.
- (b) Find the current when the switch is in position 2.



Ans: (a) $\boldsymbol{i = 0.4\ A}$; (b) $\boldsymbol{i = 0.16\ A}$;

Circuit Terminology

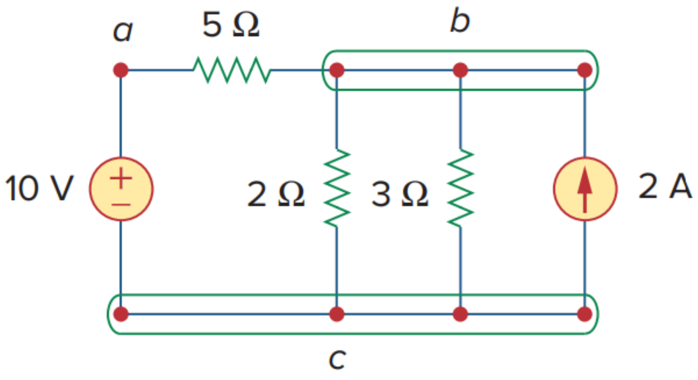
- A *branch* is single path, containing one simple element, which connects one node to any other node.
- A *node* is the point of connection between two or more branches. A node is an equipotential region in a circuit.
- A *path* is a set of elements formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.
- A *loop* is a closed path.
- A *mesh* is a loop which does not contain any other loops within it.
- In general, a circuit with n nodes and b branches will have $b - n + 1$ *independent KVL equations*.
- Therefore, while analyzing a circuit it is necessary to apply KVL only to these loops, which will in total, traverse each branch at least once in the process.



- 5 branches: $10\ V$ source, $2\ \Omega$, $3\ \Omega$, and $5\ \Omega$ resistors, $2\ A$ current source
- 3 nodes (n): a, b, c
- 6 loops
- 3 meshes
- 3 independent KVL equations

Loop vs. Mesh

- A *loop* is a <u>physically closed</u> path whereas a *mesh* is a loop which does not contain any other loops within it.
- In the circuit at the right, there are total *6* loops which can be written as follows (order doesn't matter):
- Loop *1* consists of $10\ V$, $5\ \Omega$, and $2\ \Omega$. (*Mesh*)
- Loop *2*: $10\ V$, $5\ \Omega$, and $3\ \Omega$.
- Loop *3*: $10\ V$, $5\ \Omega$, and $2\ A$.
- Loop *4*: $2\ \Omega$ and $3\ \Omega$. (*Mesh*)
- Loop *5*: $2\ \Omega$ and $2\ A$.
- Loop *6*: $3\ \Omega$ and $2\ A$. (*Mesh*)
- *Loop* is geometry independent whereas a *mesh* is not. For example, if the positions of $3\ \Omega$ and $2\ A$ are interchanged, loop *5* will now be a mesh instead of loop *4*.



- Total number of *loops* and *meshes* in a circuit are always constant.
- Number of meshes = Number of <u>independent</u> KVL equations

Problem 5

Ans:

1. $b = 6; n = 4; m = 3; l = 6$
2. $b = 5; n = 3; m = 3; l = 6$
3. $b = 6; n = 2; m = 5; l = 15$

- Determine the number of **branches**, **nodes, meshes,** and **loops** in the following circuits.

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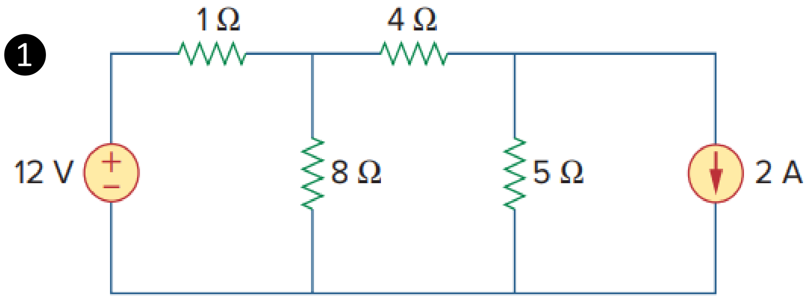

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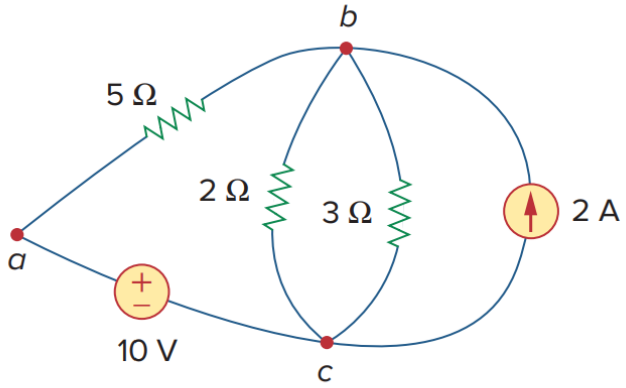

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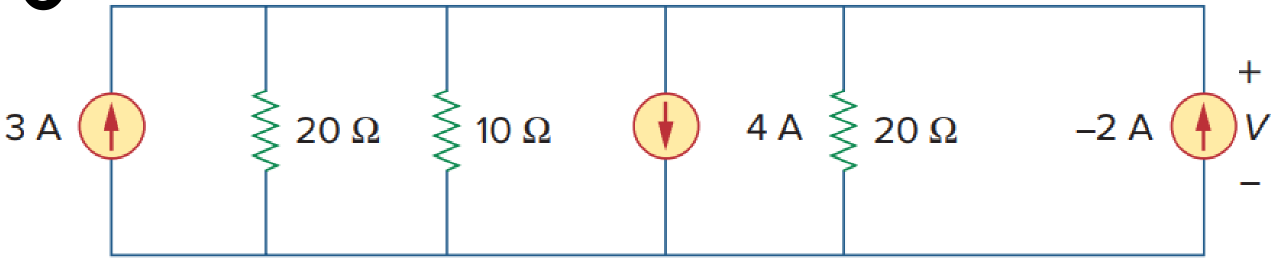
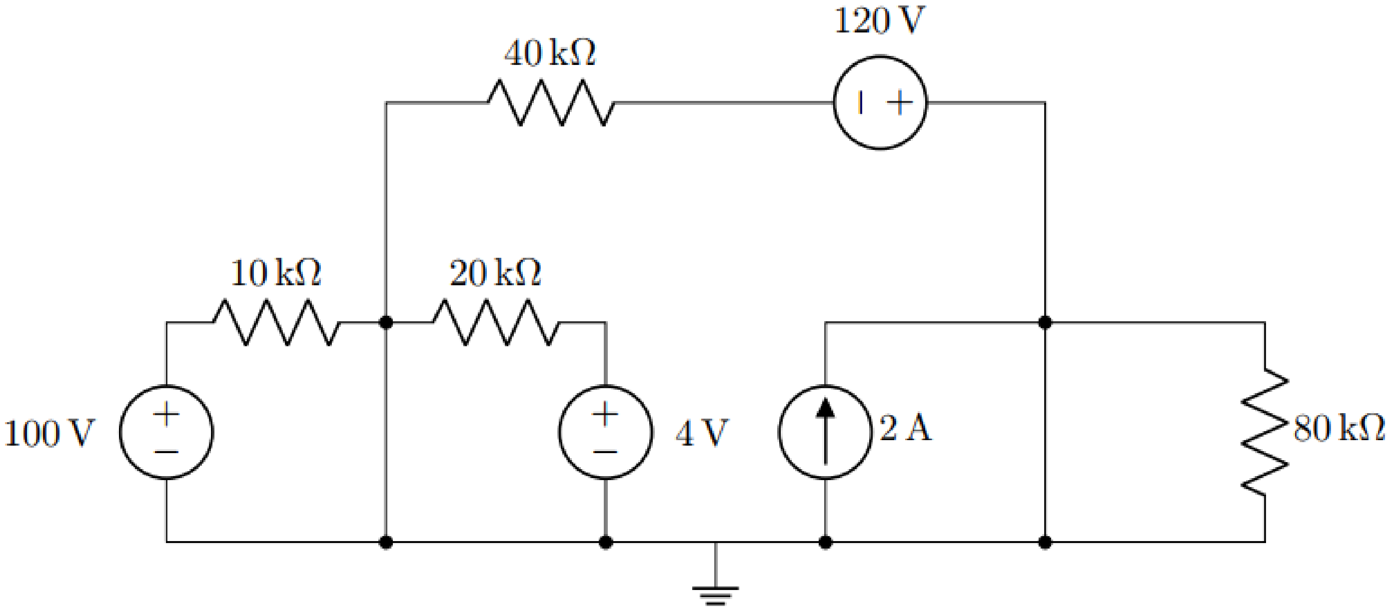


Problem 6

- Determine the number of **nodes** and **meshes** in the following circuit.

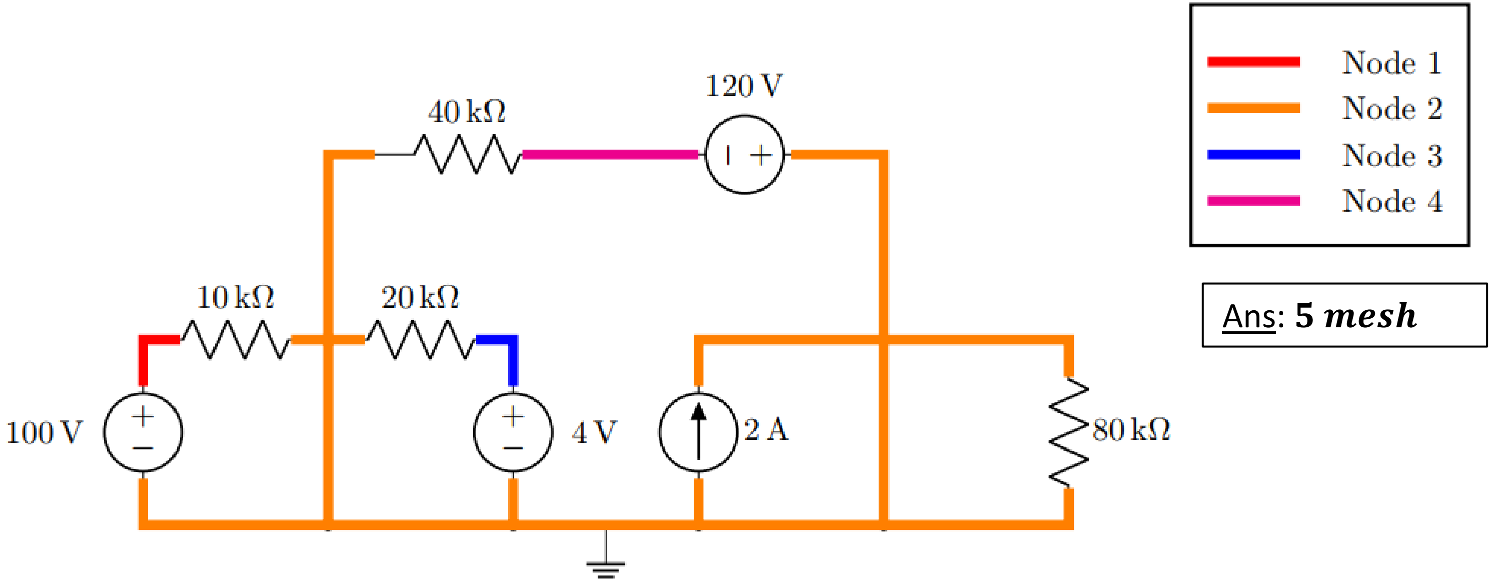


Ans: **Try yourself**

Problem 6: Solution

- Question: Determine the number of **nodes** in the following circuit.

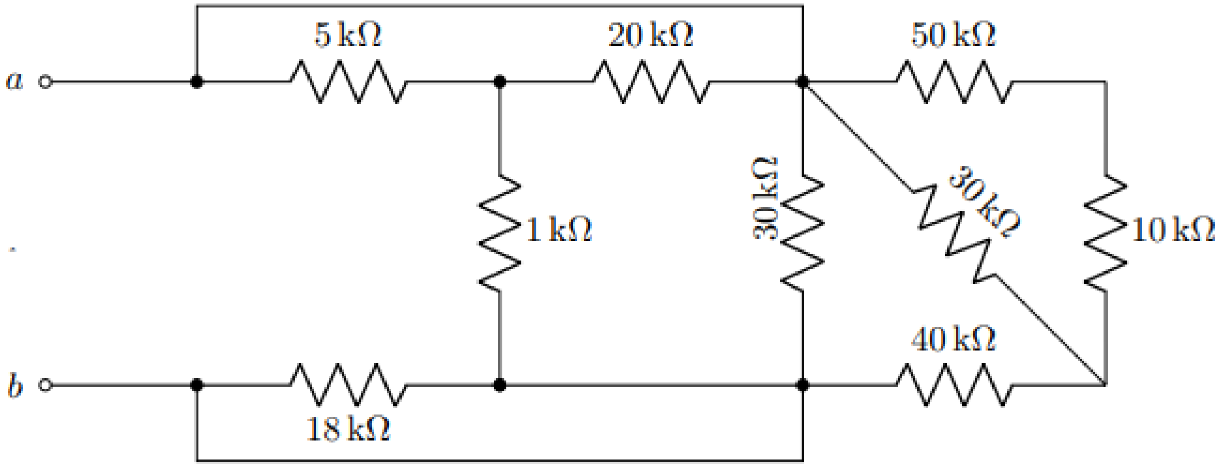
Solution:



Ans: **5** ***mesh***

Problem 7

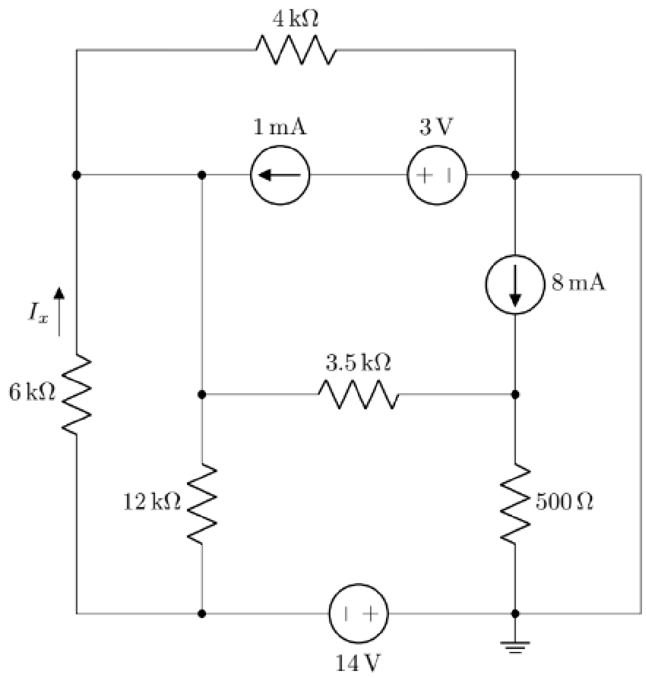
- Determine the number of **nodes** in the following circuit.



Ans: **Try yourself**

Problem 8

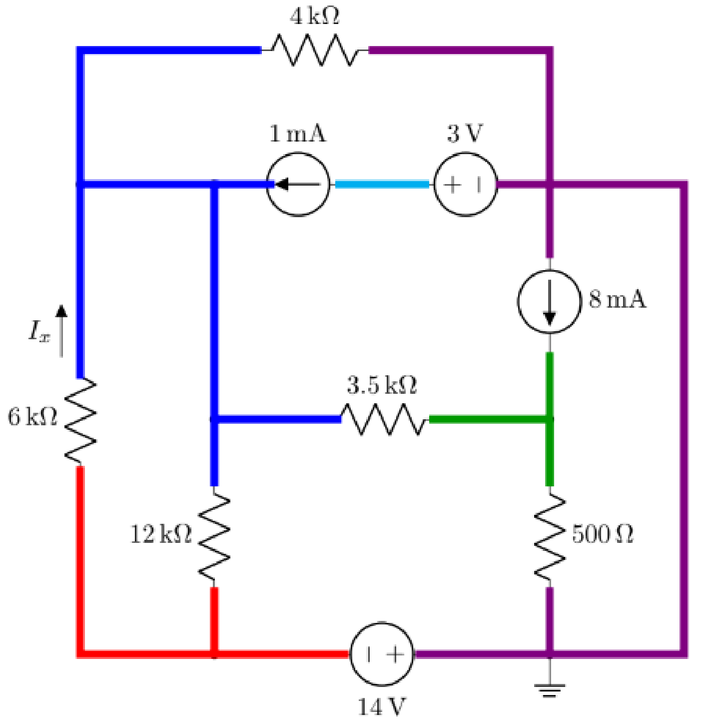
- Determine the number of **nodes** in the following circuit.



Ans: **Try yourself**

Problem 8: solution

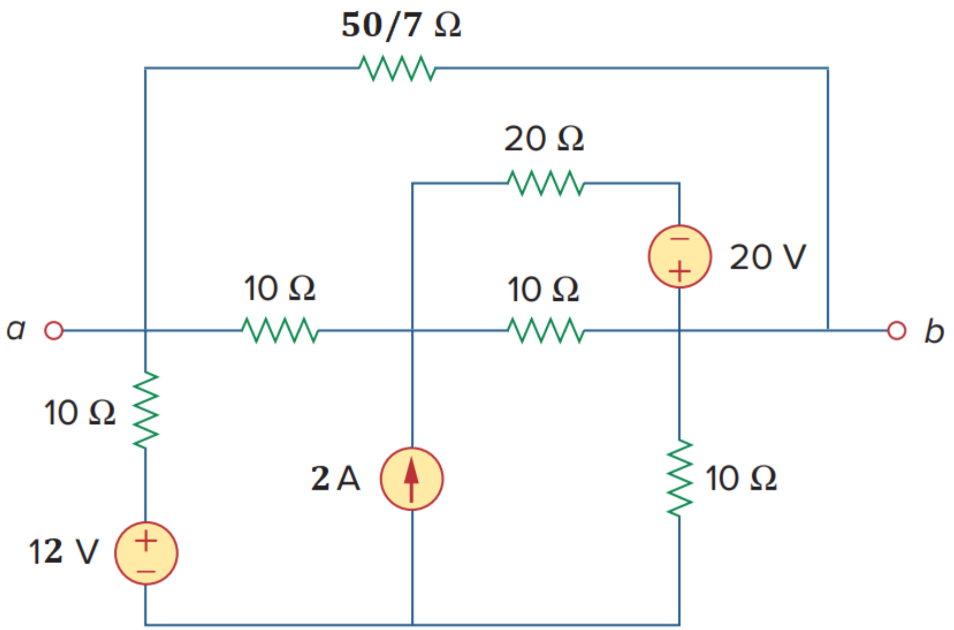
- Question: Determine the number of **nodes** and **meshes** in the following circuit.



Ans: $\mathbf{5}\ \boldsymbol{nodes}; \mathbf{5}\ \boldsymbol{meshes}$

Problem 9

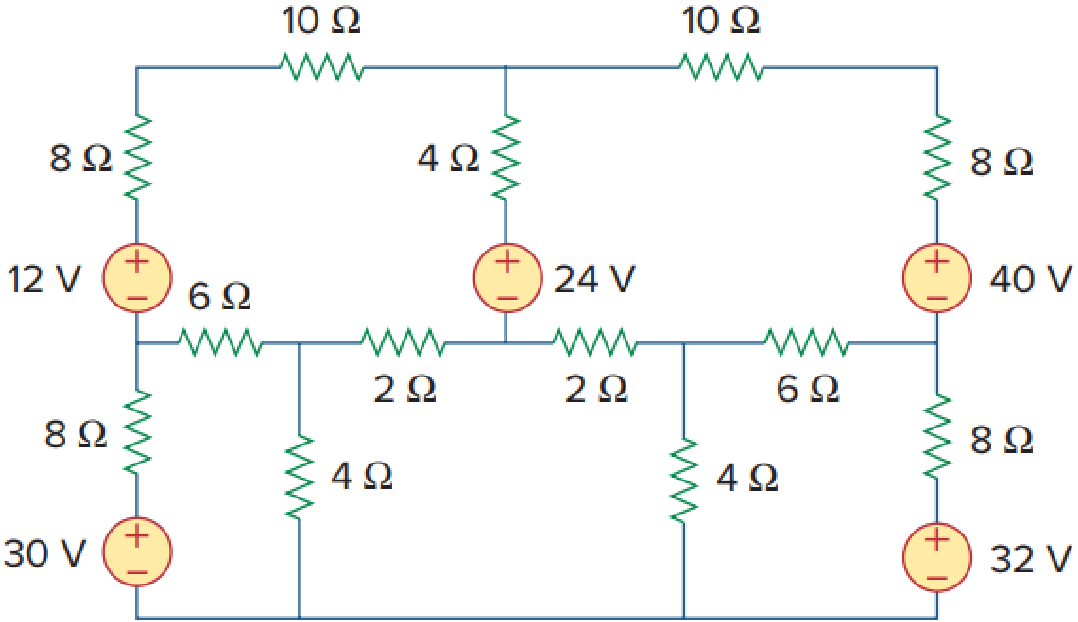
- Determine the number of **nodes** in the following circuit.



Ans: **Try yourself**

Problem 10

- How many **nodes** and **meshes** are there in the following circuit.



Ans: **14 nodes; 5 meshes**

Test Your Understanding

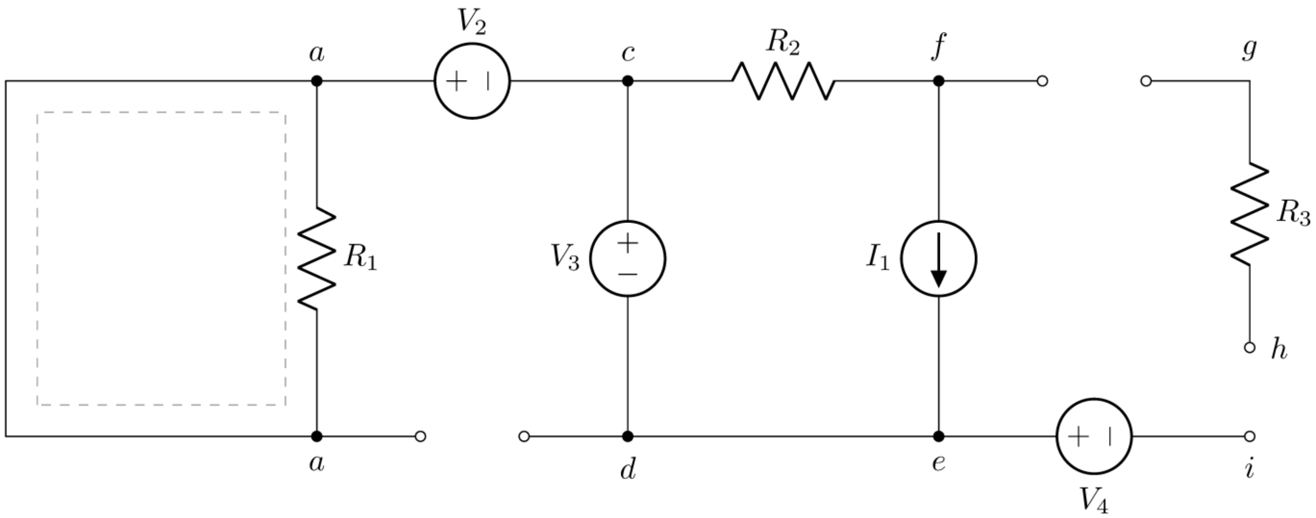
- If a loop is defined as the *closed path formed by starting at a node, passing a* <u>*set*</u> *of nodes, and returning to the same node without passing any node more than once*, for the following circuit, which of the pathways in the circuit shown above is/are loop(s)?

I. path indicated by the dashed grey line.

II. path $cdaac$.

III. path $cfedc$.

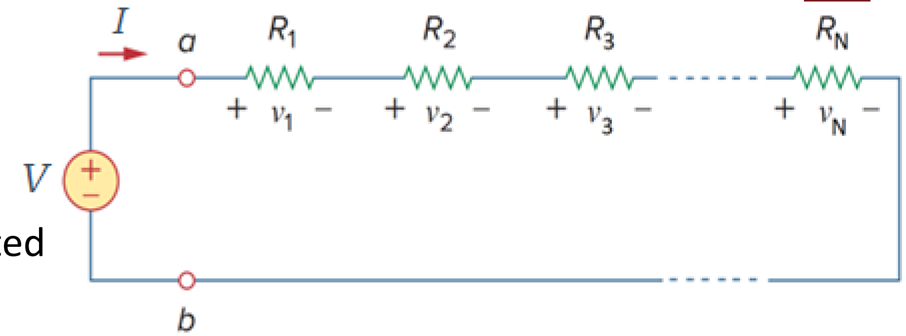
IV. path $fghief$.



Circuit Configurations

- Circuit elements can be connected to each other in two main ways: series connection and parallel connection.
- In *series configuration*, components are connected end-to-end in a line. The same current flows through all the components. If one component breaks down, the whole circuit will burn out. *So, if same current flows through two circuit elements having a common node, they are said to be in series.*
- In *parallel configuration*, all components are connected across each other leads forming exactly two sets of electrically common points. There are many paths for current flow, but only one voltage across all components. *So, if multiple circuit elements are connected between the same two nodes, they are said to be in parallel.*
- Another configuration occurs when the circuit components are not connected in series or parallel but rather in a 'Y' or 'Δ' configuration. *Wye-Delta transformation* is required to simplify such configuration.
- The majority of electric circuits use all configurations simultaneously.

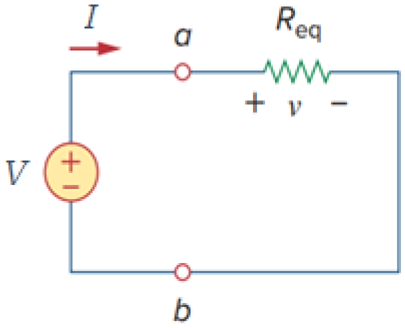
Series resistors



- Consider N number of resistors connected in series with a power supply of v volts.
- If the current flowing through the series circuit is i, then the voltage drops across the resistors can be written as,
- $v_1 = IR_1, v_2 = IR_2, v_3 = IR_3, \ldots \ldots, v_N = IR_N$
- According to the KVL, $V = v_1 + v_2 + v_3 + \ldots \ldots + v_N$

$$\Rightarrow \quad V = I(R_1 + R_2 + R_3 + \ldots \ldots + R_N)$$

$$\Rightarrow \quad I = \frac{V}{R_1 + R_2 + R_3 + \ldots \ldots + R_N}$$



- It can be written as, $v = V = IR_{eq}$, implying that the series resistors can be replaced by an equivalent resistor R_{eq}; that is,

$$\boxed{R_{eq} = R_1 + R_2 + R_3 + \ldots \ldots + R_N}$$

Graphically

- We can also graphically derive the condition for the two circuits two be equivalent to each other.
- From the previous slide, the current voltage relationship for N number of resistors connected in series is,

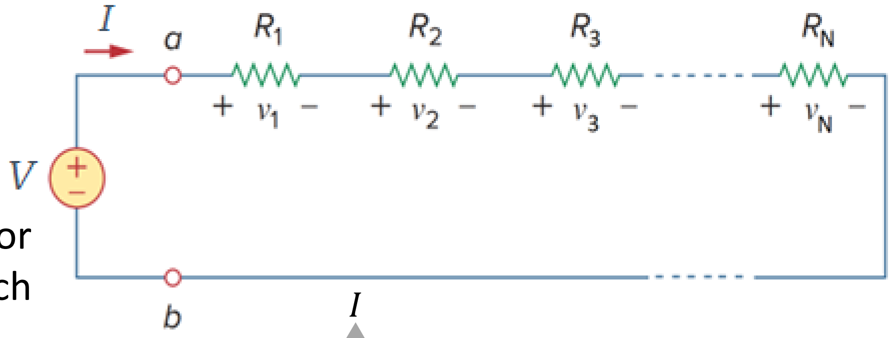
$$I = \frac{1}{R_1 + R_2 + R_3 + \dots \dots + R_N} V$$

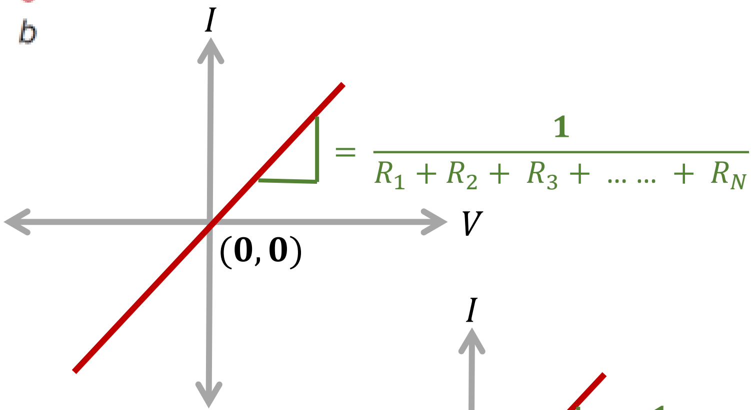
- And the $I - V$ relationship for a resistor R_{eq} is,

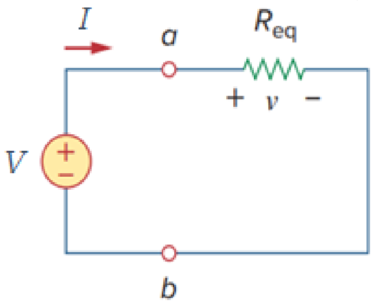
$$I = \frac{1}{R_{eq}} V$$

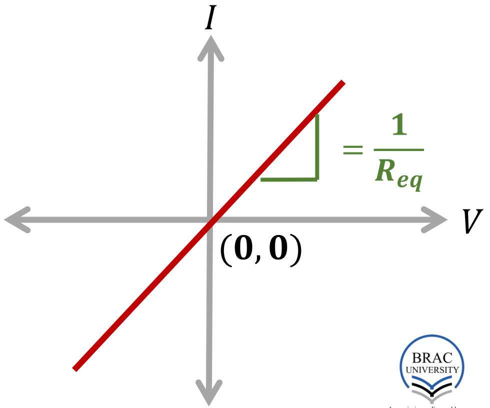
- The two circuits will be equivalent to each other if the two $I - V$ lines are identical. It requires the slopes to be equal. Thus,

$$\boxed{R_{eq} = R_1 + R_2 + R_3 + \dots \dots + R_N}$$

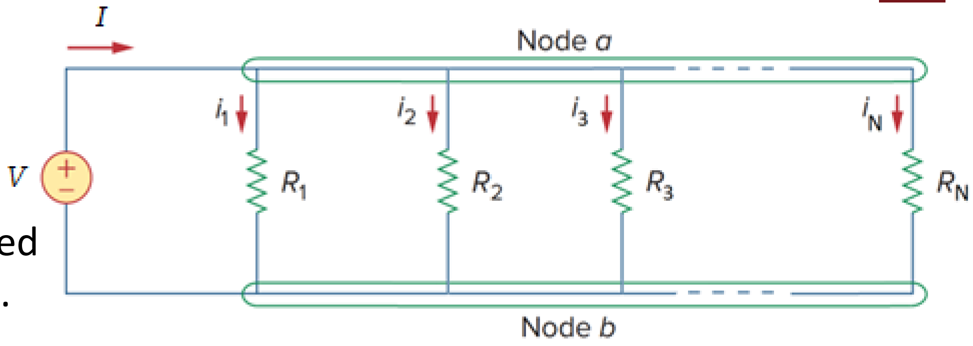






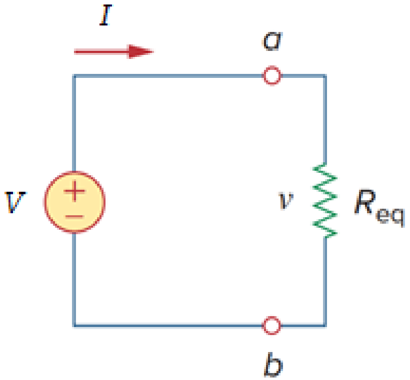


Parallel resistors



- Consider N number of resistors connected in parallel with a power supply of V volts.
- Therefore, they have the same voltage V across them. So,

$$V = i_1R_1 = i_2R_2 = i_3R_3 = \ \dots\dots = i_NR_N$$



- According to the KCL, $\quad I = i_1 + i_2 + i_3 + \ \dots\dots + \ i_N$

$$\Rightarrow \quad I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \ \dots\dots + \frac{V}{R_N}$$

$$\Rightarrow \quad I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ \dots\dots + \frac{1}{R_N}\right)$$

- It can be written as, $I = \frac{V}{R_{eq}}$, implying that the parallel resistors can be replaced by an equivalent resistor R_{eq}; that is,

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ \dots\dots + \frac{1}{R_N}}$$

Graphically

- From the previous slide, the current voltage relationship for N number of resistors connected in parallel is,

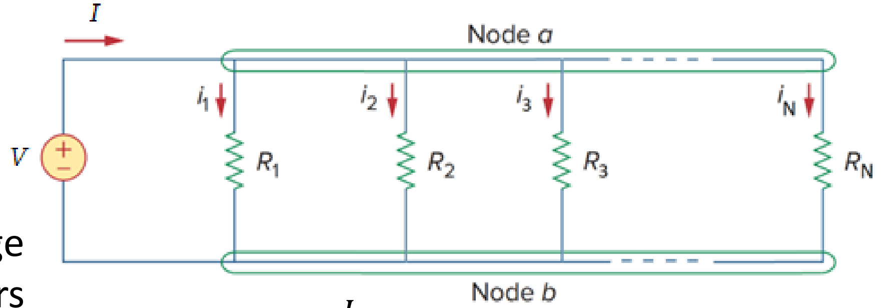
$$I = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ \dots\dots + \frac{1}{R_N}\right) V$$

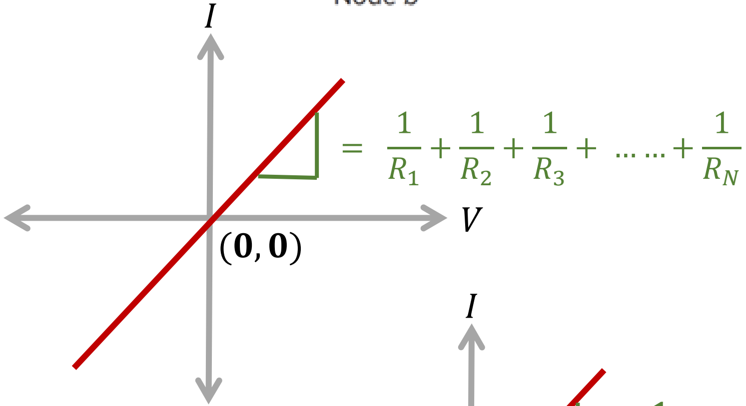
- And the $I - V$ relationship for a resistor R_{eq} is,

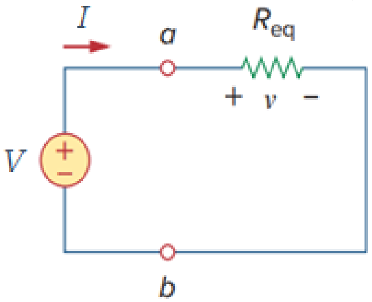
$$I = \frac{1}{R_{eq}} V$$

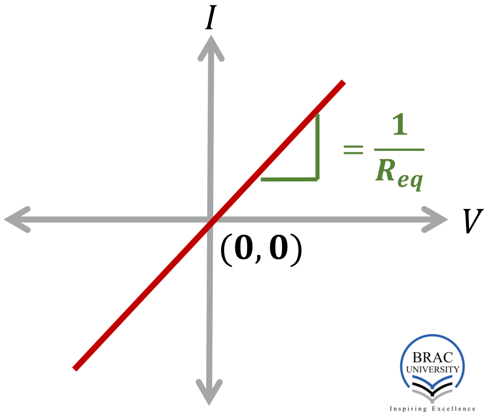
- The two circuits will be equivalent to each other if the two $I - V$ lines are identical. It requires the slopes to be equal. Thus,

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ \dots\dots + \frac{1}{R_N}}$$



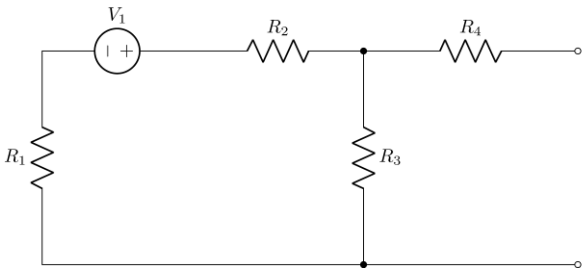




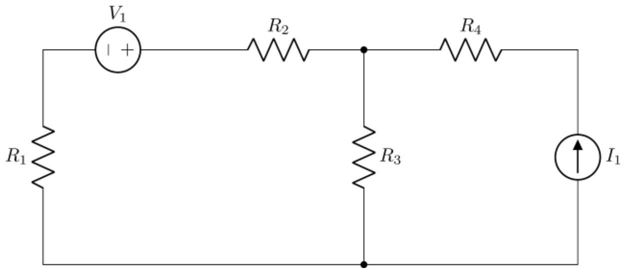


Test Your Understanding

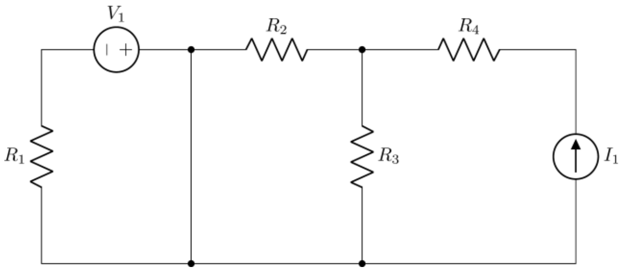
- For each of the following circuits, how are R_2 and R_3 connected?
  - i. Series
  - ii. Parallel
  - iii. Neither series nor parallel



(b)



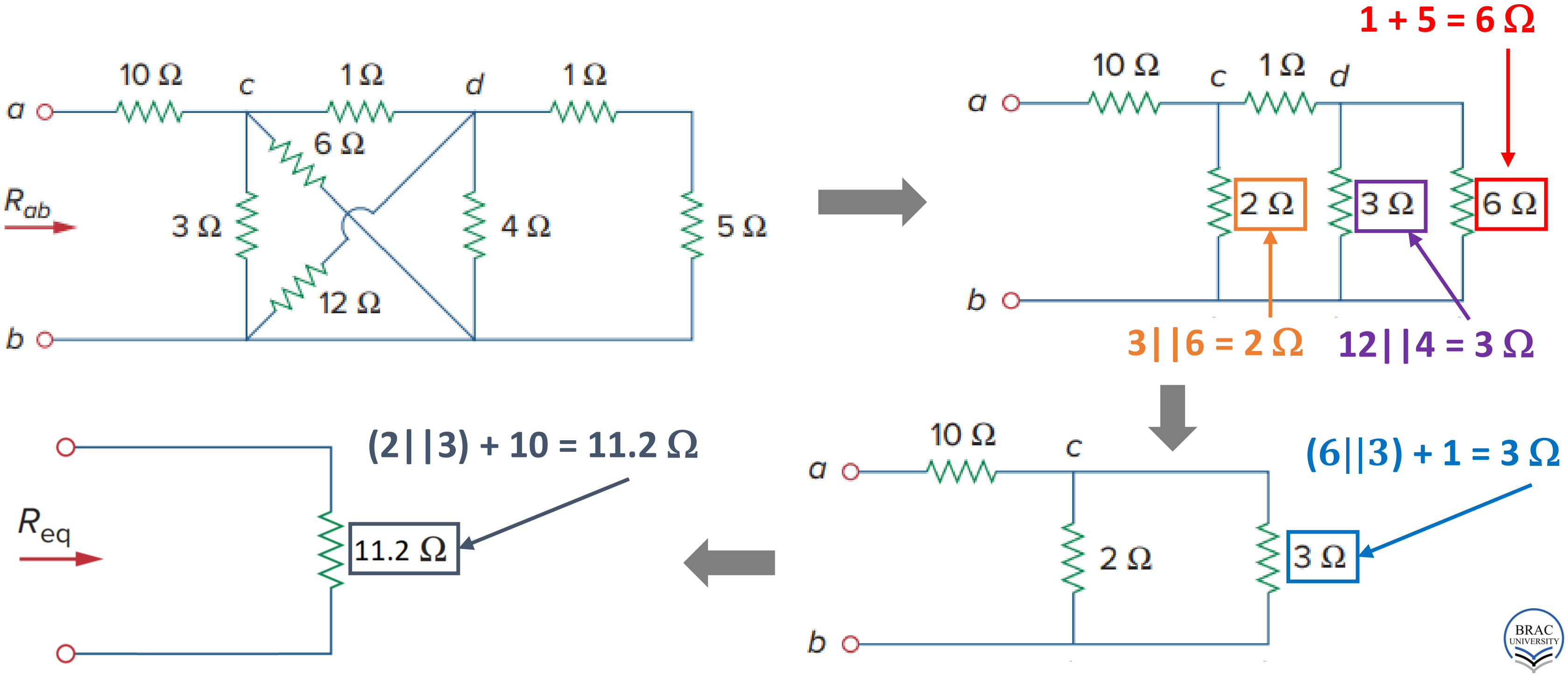
(a)



(c)

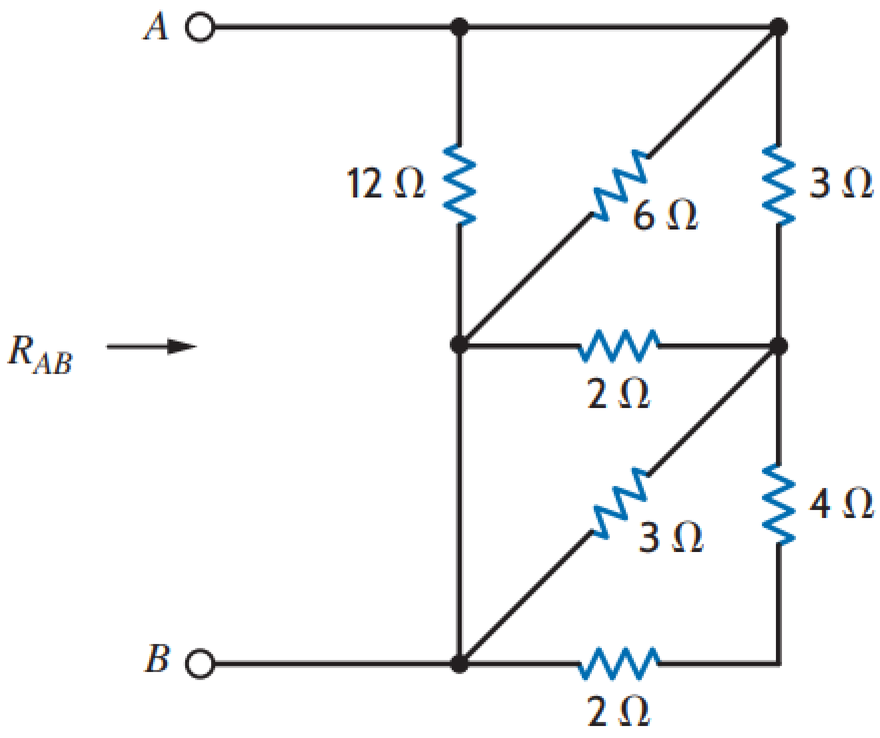
Example 2

- Using series/parallel resistance combination, find R_{ab} for the circuit shown below.



Problem 11

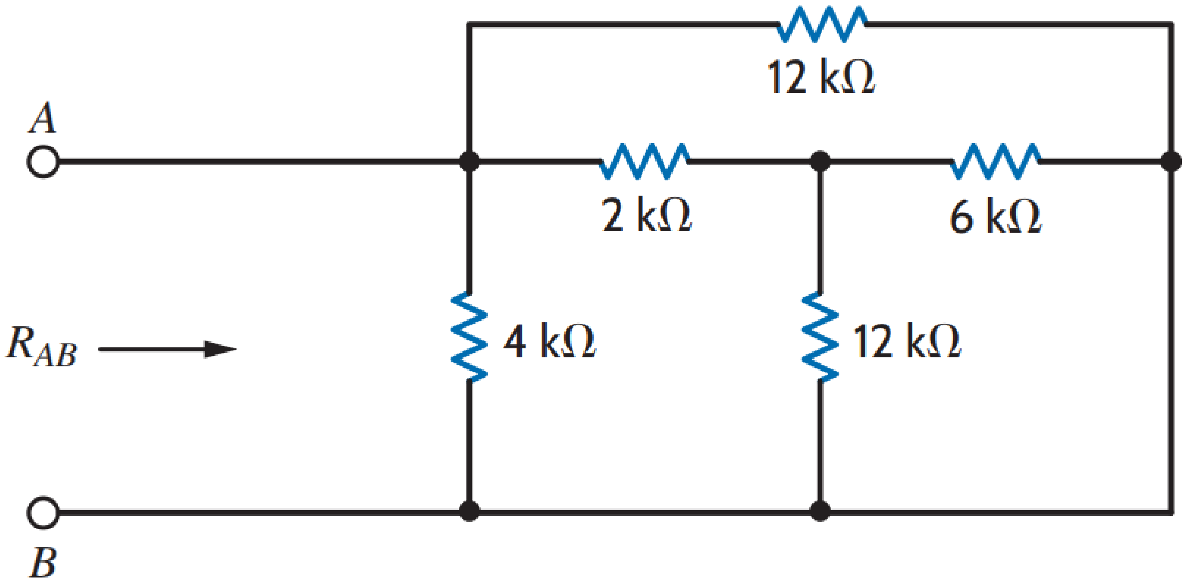
- Using series/parallel resistance combination in the adjacent circuit, find the equivalent resistance R_{AB}.



Ans: $\boldsymbol{R_{AB} = 2\ \Omega}$

Problem 12

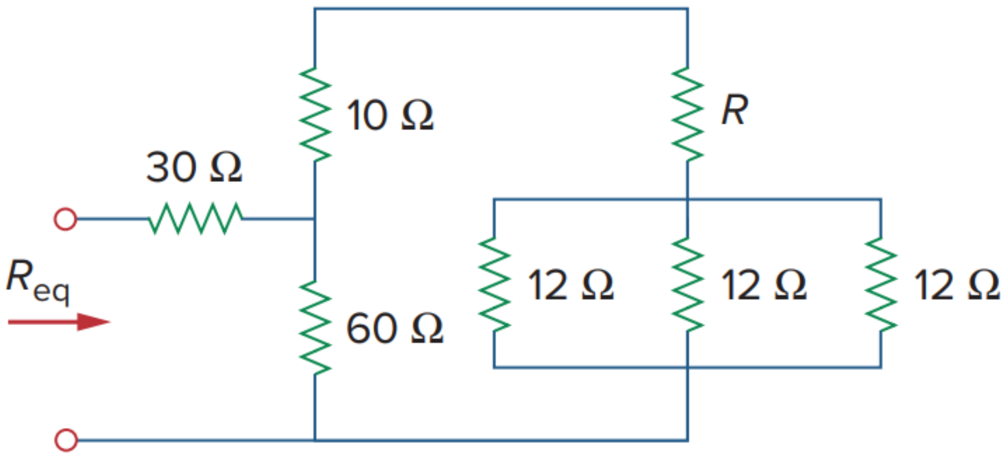
- Find the equivalent resistance between terminals A and B.



Ans: $\boldsymbol{R_{AB}} = \mathbf{2}\ \Omega$

Problem 13

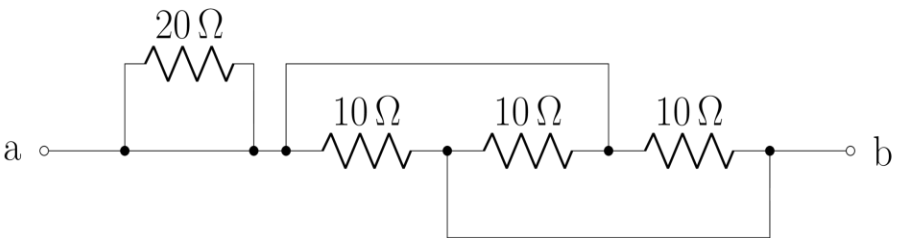
- If $R_{eq} = 50\ \Omega$ in the circuit, find R.

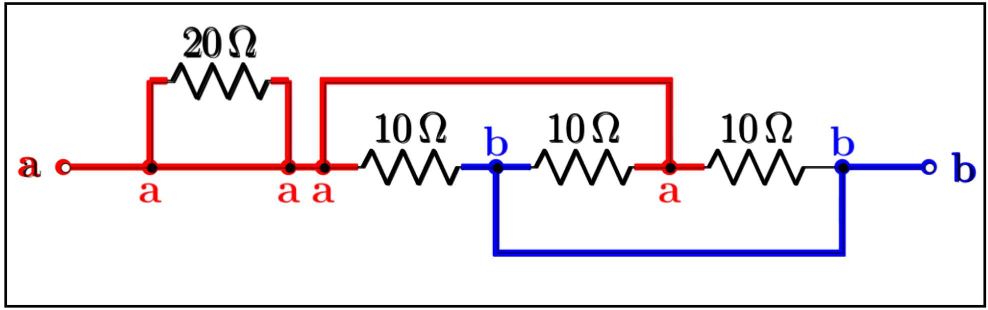


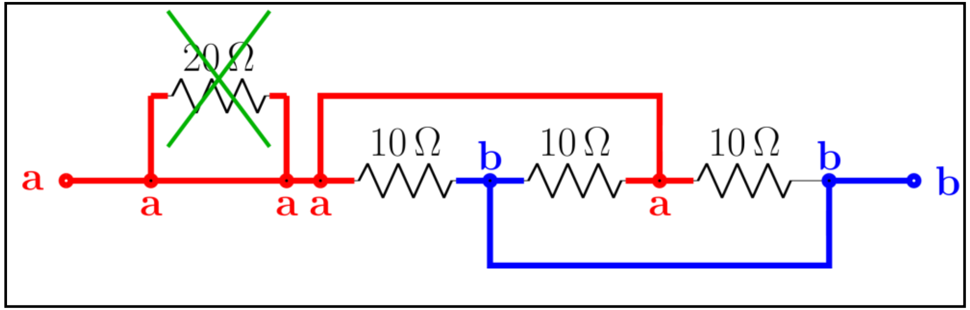
Ans: $\boldsymbol{R} = \mathbf{16}\ \Omega$

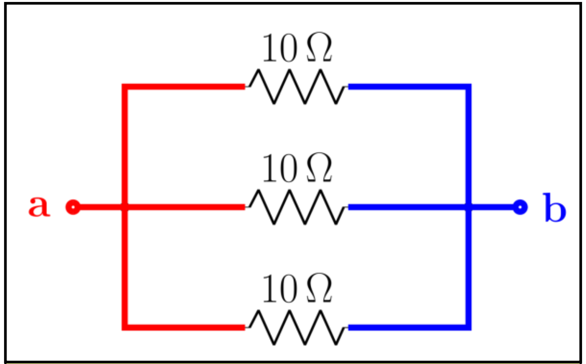
Example 3

- Find R_{ab}





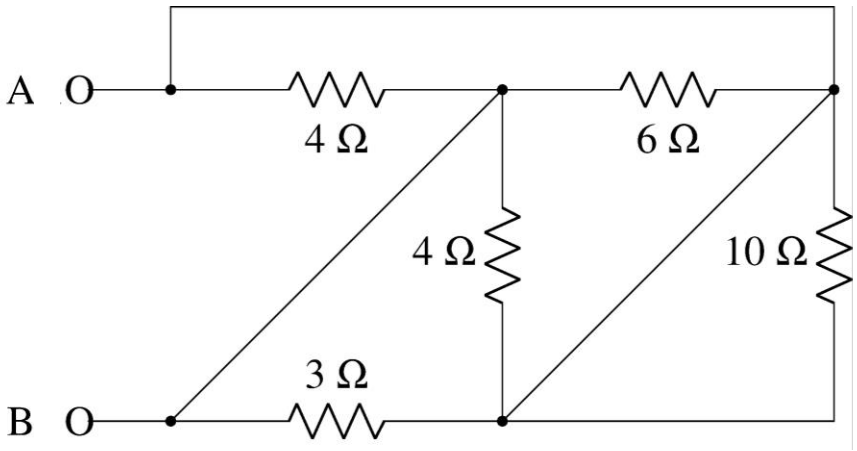




$R_{ab} = 10 \,||\, 10 \,||\, 10 = \frac{10}{3}\,\Omega$

Problem 14

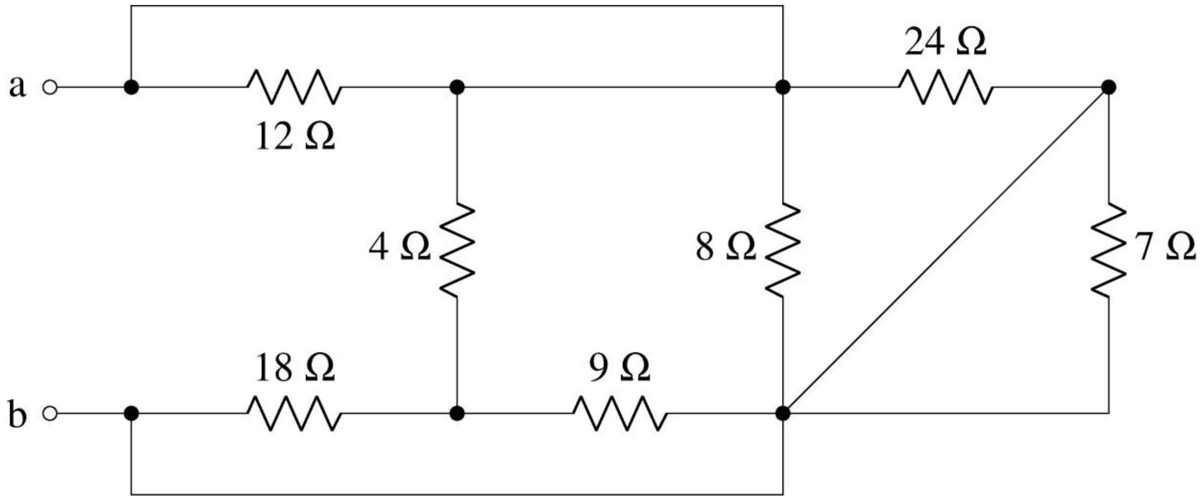
- Find the equivalent resistance between terminals A and B.



Ans: $\boldsymbol{R_{AB}} = \mathbf{1}\ \Omega$

Problem 15

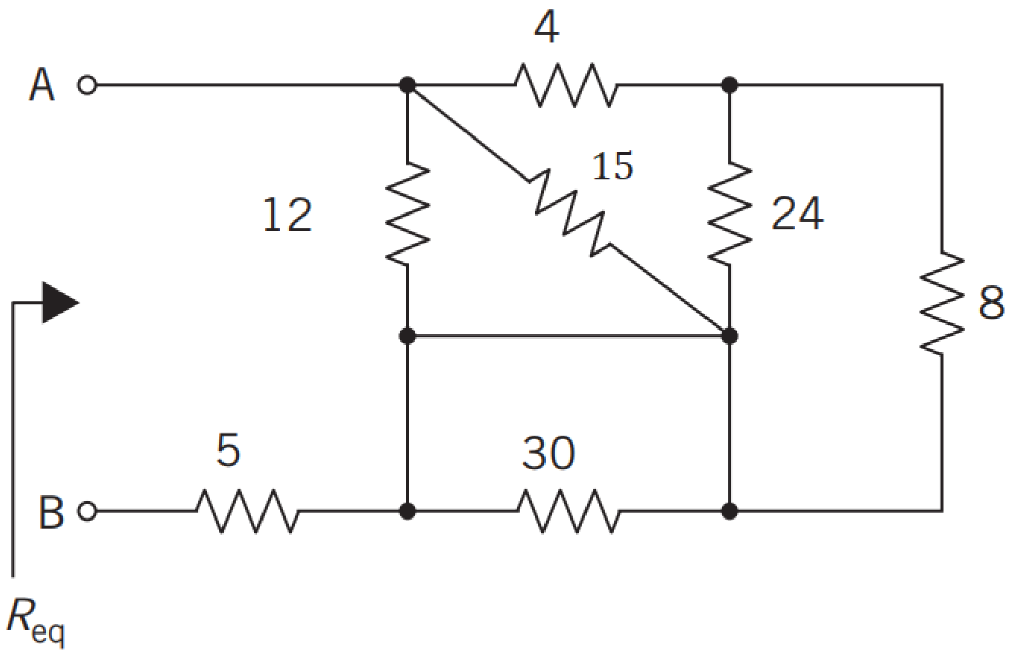
- Find the equivalent resistance between terminals a and b.



Ans: $\boldsymbol{R_{ab}} = \mathbf{3.75}\ \Omega$

Problem 16

- Determine R_{eq}.

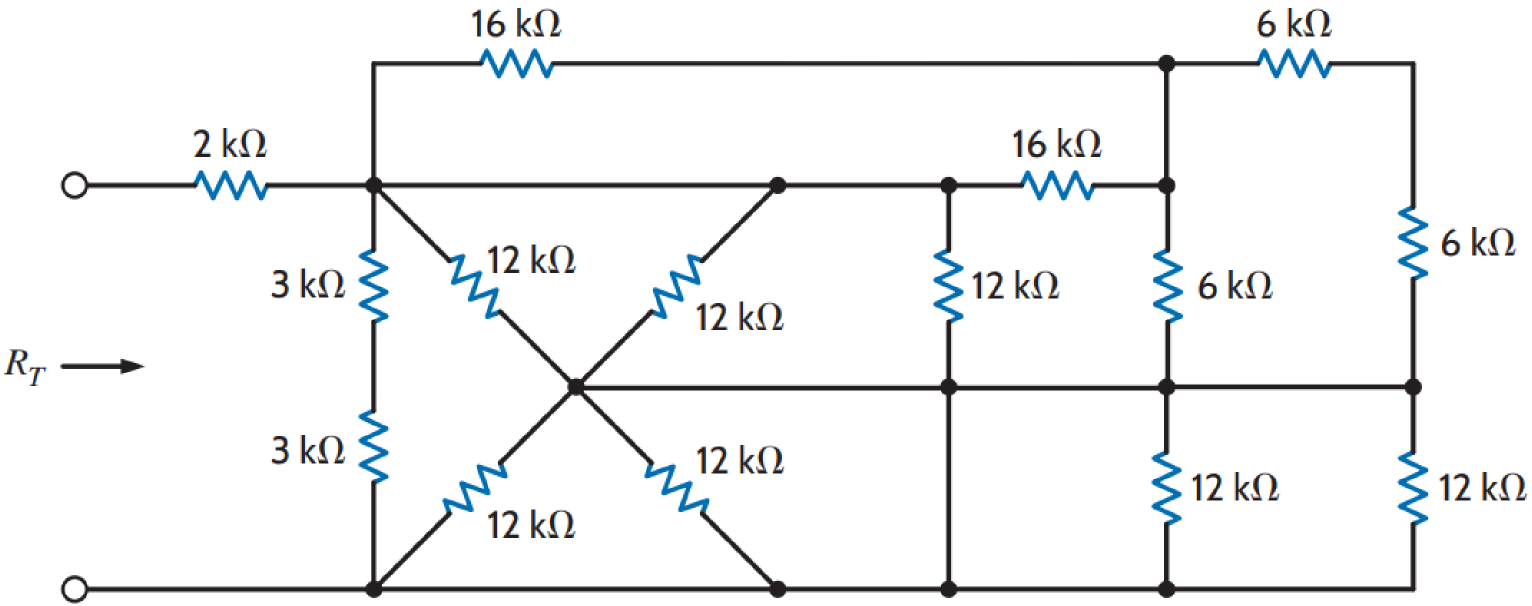


Ans: $R_{eq} = 9$

Problem 17

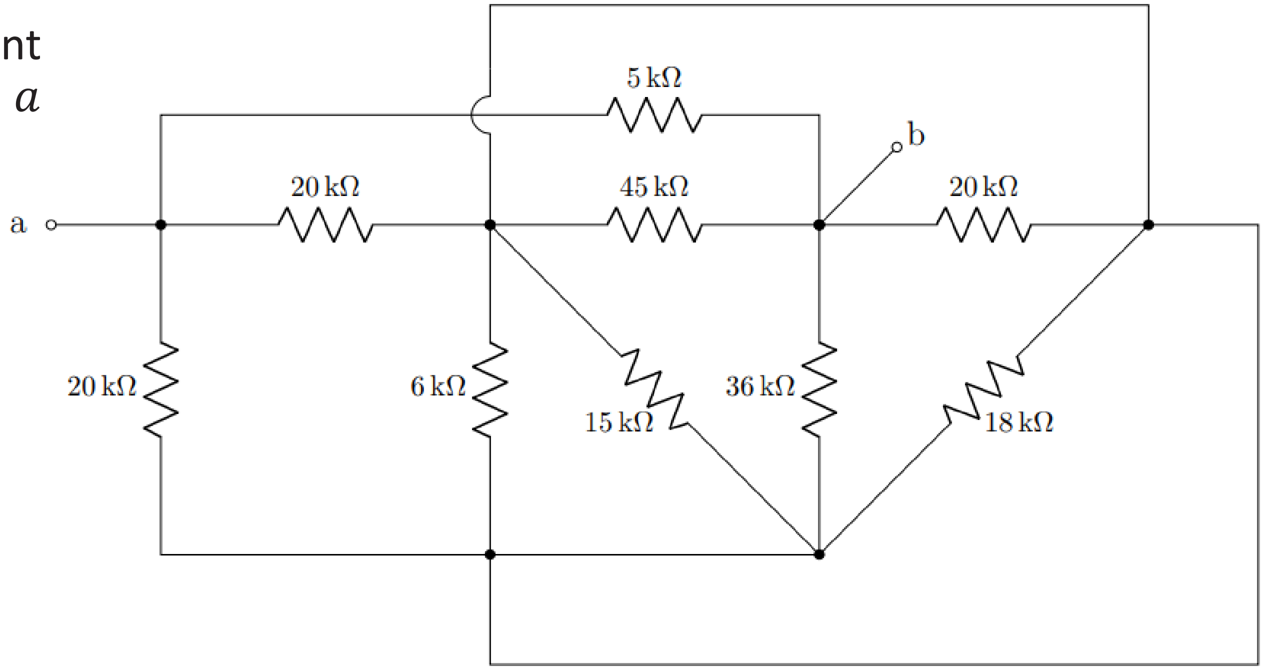
Ans: $R_{eq} = 4\ \text{k}\Omega$

- Determine R_T.



Problem 18

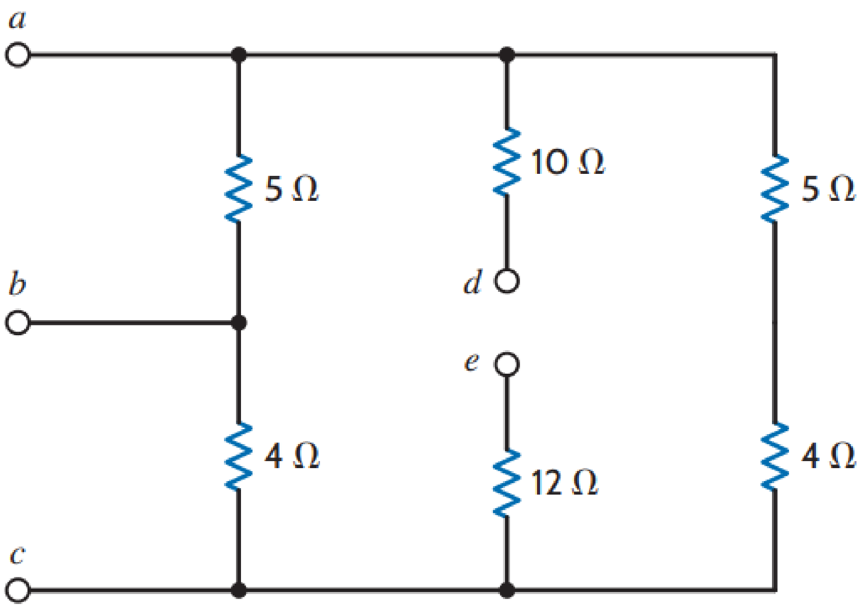
- Find the equivalent resistance between a and b.



Ans: $R_{ab} = 4\ k\Omega$

Problem 19

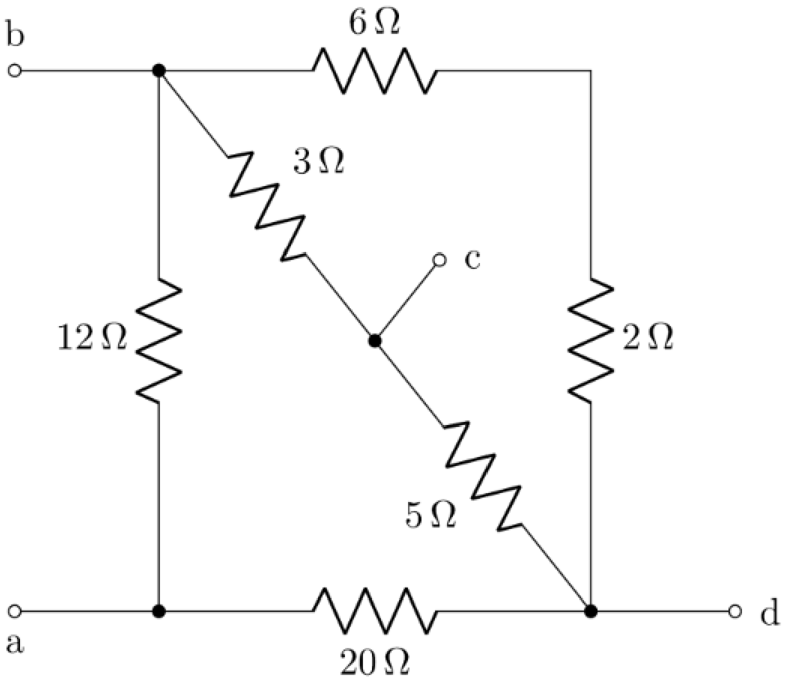
- Find the equivalent resistance between—
  - *I.* a and b,
  - *II.* b and c,
  - *III.* a and c,
  - *IV.* d and e,
  - *V.* a and e,
  - *VI.* c and d,
  - *VII.* a and d,
  - *VIII.* c and e,
  - *IX.* b and d, and
  - *X.* b and e.



Ans: $R_{ab} = 3.6\ \Omega$; $R_{bc} = 3.27\ \Omega$; $R_{ac} = 4.5\ \Omega$; $R_{de} = 26.5\ \Omega$; $R_{ae} = 16.5\ \Omega$; $R_{cd} = 14.5\ \Omega$; $R_{ad} = 10\ \Omega$; $R_{ce} = 12\ \Omega$; $R_{bd} = 13.6\ \Omega$; $R_{be} = 15.1\ \Omega$

Problem 20

- Find the equivalent resistance between—
  - *I.* a and b,
  - *II.* a and d,
  - *III.* b and d, and
  - *IV.* b and c



Ans: $\boldsymbol{R_{ab}} = \mathbf{8}\ \Omega;\ \boldsymbol{R_{ad}} = {}^{80}/_{9}\ \Omega;\ \boldsymbol{R_{bd}} = {}^{32}/_{9}\ \Omega;\ \boldsymbol{R_{bc}} = {}^{19}/_{8}\ \Omega$

Additional Problems

- Additional recommended practice problems: here
- Other suggested problems from the textbook: here

Acknowledgements and References

- Some of the problems, illustrations, and concepts in this lecture are taken from the following sources:

1. Sadiku, M. N. O., Fundamentals of Electric Circuits, McGraw-Hill
2. Nilsson, J. W., & Riedel, S. A., Electric Circuits, Pearson Education
3. Boylestad, R. L., Introductory Circuit Analysis, Pearson
4. Irwin, J. D., & Nelms, R. M., Basic Engineering Circuit Analysis, Wiley

Thank you for your attention