Lecture 7

CSE250 - Circuits and Electronics

MESH ANALYSIS

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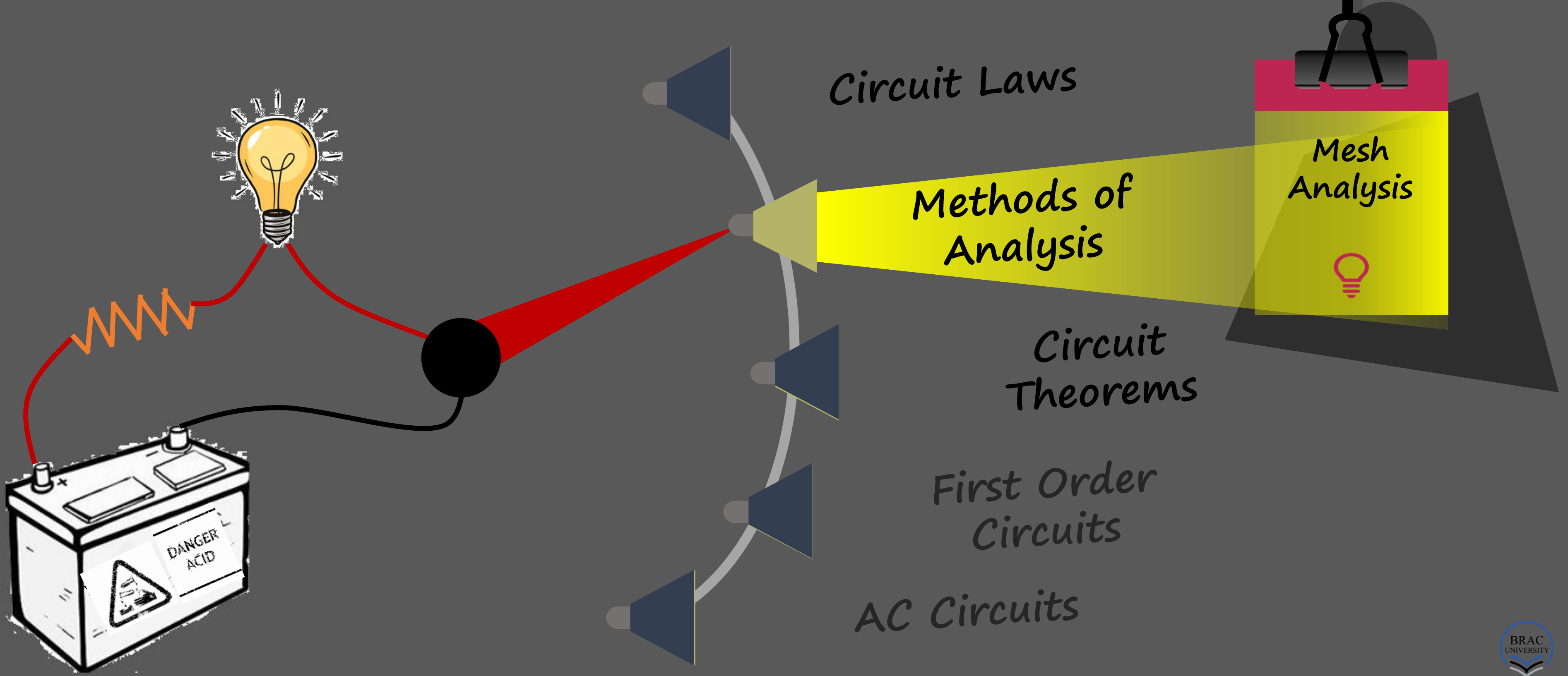
Course Outline: broad themes



Mesh Analysis

- *Mesh analysis* provides another general procedure for analysing circuits, using mesh currents as the circuit variables. Mesh analysis applies KVL to find unknown currents in a given circuit.
- A *mesh* is a loop that does not contain any other loops within it.
- *Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is planar. Nonplanar circuits cannot be handled with mesh analysis.*
- *A nonplanar circuit is one that has branches that cross each other and cannot be redrew without doing so.*

Steps to Determine Mesh Currents:

1. Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

Mesh Analysis: steps

Start

<u>Step 1</u>: Identify all the meshes and assign mesh variables

Recall that, 'Mesh' is a loop which does not contain any other loops within it. The mesh currents can be taken CW or CCW or a mix of both. It is better to follow any one.

<u>Step 2</u>: Write equations for all the current sources

If a current source exists in a mesh which is not part of any other mesh, the current of that mesh is equal to the value of the current source. If a current exists between two loops, it will form a Supermesh, however, we can also write the equations for such current sources.

<u>Step 3</u>: Apply KVL

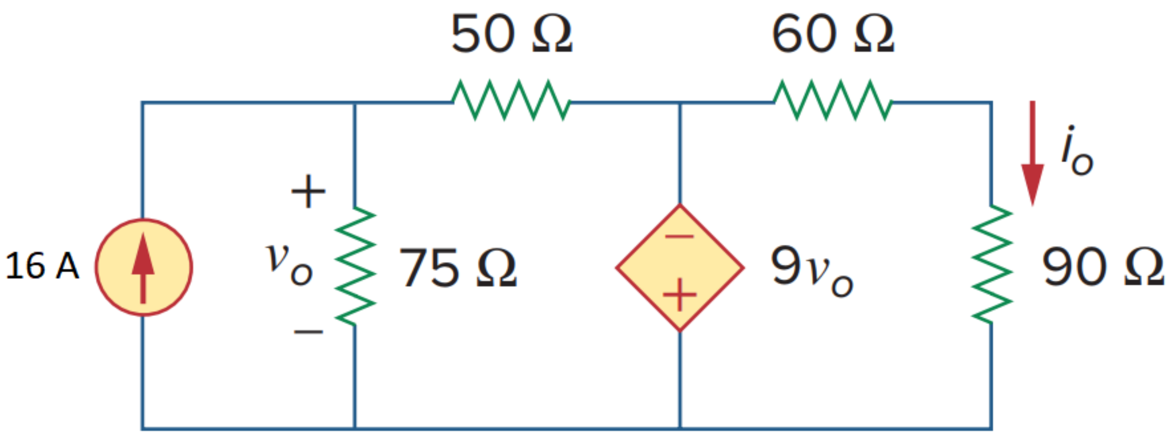
No need to apply KVL to those meshes where mesh currents are already known from Step 3.

<u>Step 4</u>: Solve the simultaneous equations

You should have a number of independent equations equal to the number of meshes.

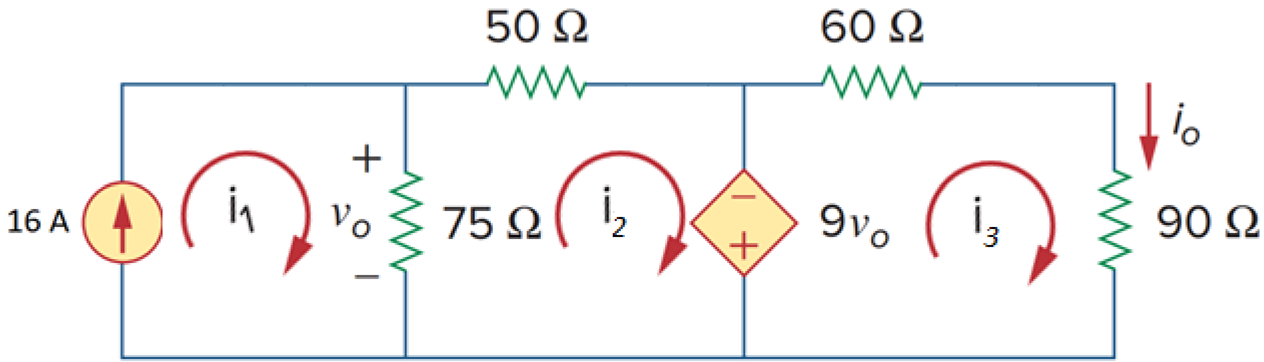
Example 1 - 1/7

Use mesh analysis, determine v_0. What is the current supplied by the dependent voltage source? What is the power of it? Is it absorbing or supplying?



Before solving the circuit using mesh analysis, recall that, "*For passive elements, current enters through the positive terminal of the voltage drop across it*." This is according to the *passive sign convention*, current must always flow from a higher potential to a lower potential through a passive element that is absorbing power.

Example 1 - 2/7

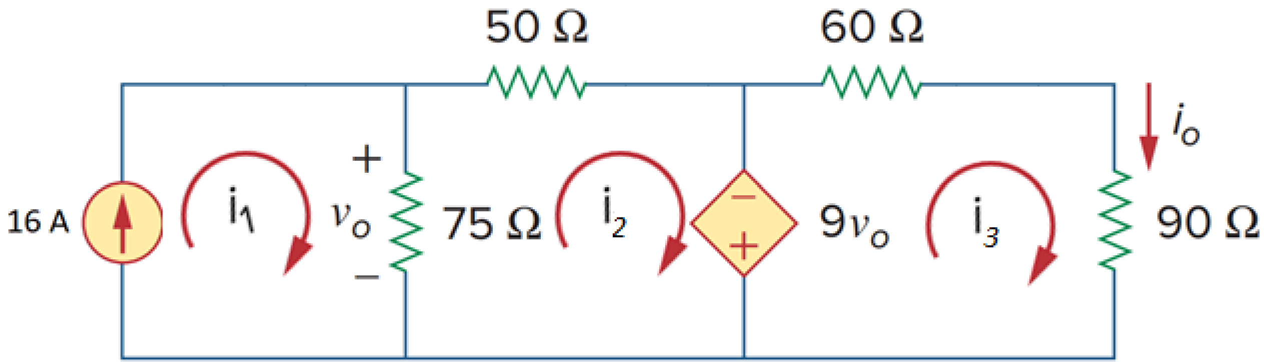


First identify all the meshes (independent loops) in this circuit.

There are 3 meshes as identified in the circuit.

Assign mesh currents $(i_1, i_2, and\ i_3)$ to all the meshes. The assigned currents can be clockwise, anti-clockwise, or a combination of the two.

Example 1 - 3/7



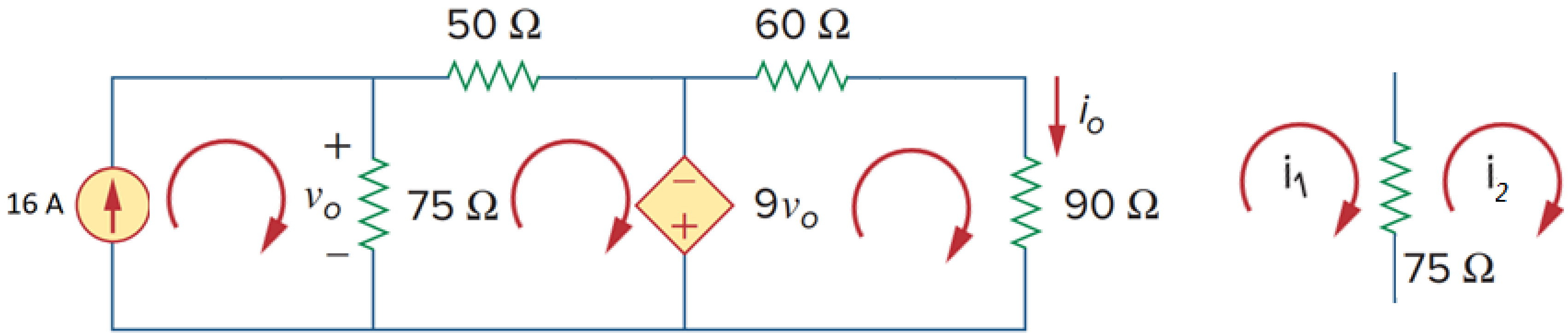
The 2nd step is to apply KVL to each mesh.

Note that, we already know the mesh 1 current. i_1 and the 16 A current flow through the same wire in the same direction. We can write directly,

$$i_1 = 16\,A ---- (i)$$

For meshes whose mesh currents are already known, we don't need to apply KVL.

Example 1 - 4/7

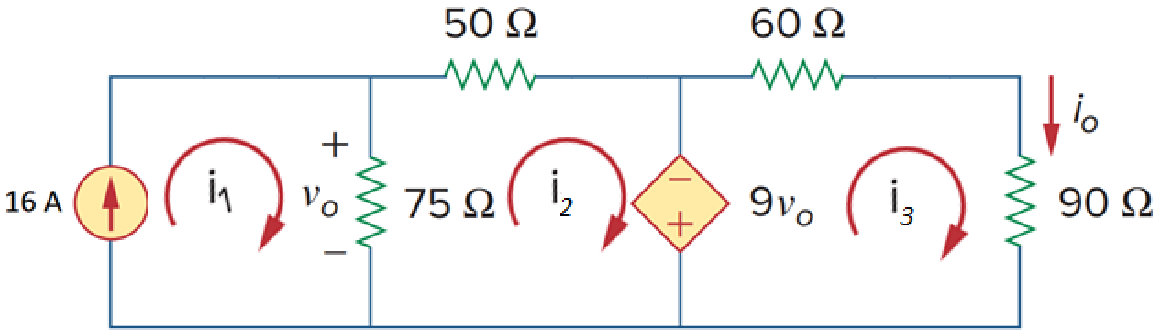


Next, apply KVL to mesh 2.

$$75\,(i_2 - i_1)\; + 50i_2 - 9v_0 = 0$$

Notice that, the two mesh currents ($i_1$and i_2) overlap through the 75 Ω. As there can be no more than a current in a wire, the resulting current through the 75 Ω will be either $i_1 - i_2$ or $i_2 - i_1$. But we won't know exactly before solving. As we are moving in the direction of i_2, we take $i_2 - i_1$ as the resulting current and the KVL equation is written accordingly.

Example 1 - 5/7



$$75\,(i_2 - i_1)\ + 50i_2 - 9v_0 = 0\ [from\ the\ previous\ slide]$$

Now we have to replace v_0 in terms of the mesh currents as the mesh equations should not contain unknowns other than the mesh currents.

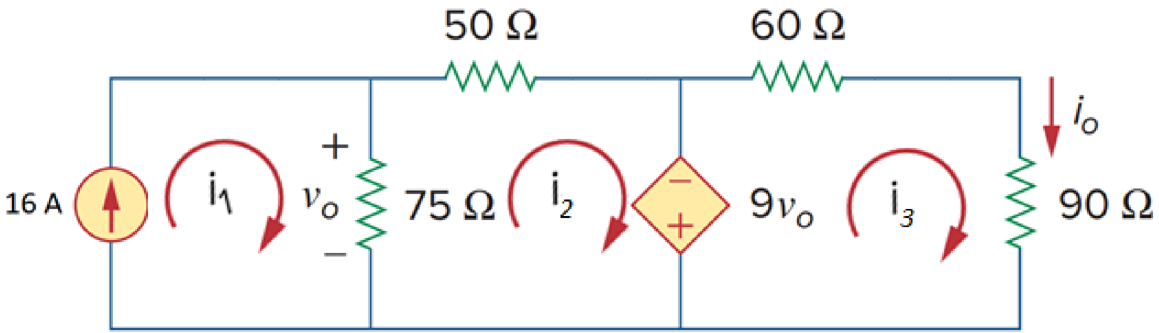
v_0 is the voltage drop across the 75 Ω resistor. With the polarity of v_0 given,

$$v_0 = 75\,(i_1 - i_2)$$

Substituting, $$75\,(i_2 - i_1)\ + 50i_2 - 9 \times 75\,(i_1 - i_2) = 0$$

$$750\, i_1 - 800i_2 = 0 ---- -(ii)$$

Example 1 - 6/7



Next, apply KVL to mesh 3.

$$9v_0 + 60i_3 + 90i_3 = 0$$

Substituting $v_0 = 75\ (i_1 - i_2)$ for v_0,

$$9 \times 75\ (i_1 - i_2) + 60i_3 + 90i_3 = 0$$

After simplifying,

$$675\ i_1 - 675i_2 + 150i_3 = 0 ---- -(iii)$$

Example 1 - 7/7

We have derived the three mesh equations,

$$i_1 = 16\ A$$

$$750\ i_1 - 800 i_2 = 0$$

$$675\ i_1 - 675 i_2 - 150 i_3 = 0$$

Solving … …,

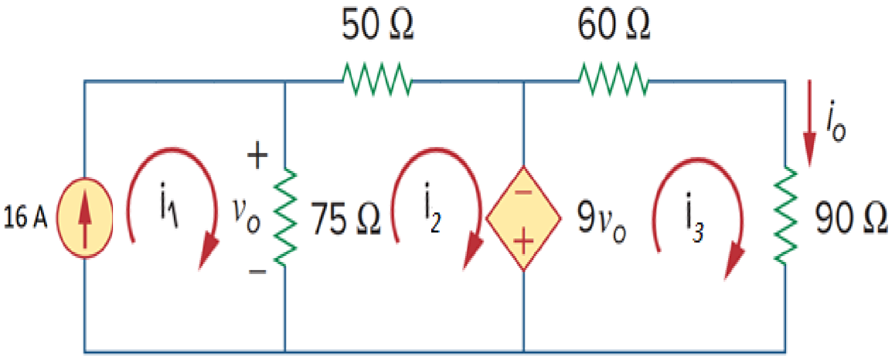
$$i_1 = 16\ A; \qquad i_2 = 15\ A; \qquad i_3 = -\ 4.5\ A;$$

So,

$$v_0 = 75(i_1 - i_2) = 75\ (16 - 15) = 75\ V$$

Current supplied (entering into the –ve terminal) by the dependent source is,

$$i_2 - i_3 = 15 - (-4.5) = 19.5\ A$$



Power supplied by the dependent source is thus,

$$p = -\ vi = 9v_0 \times 19.5$$

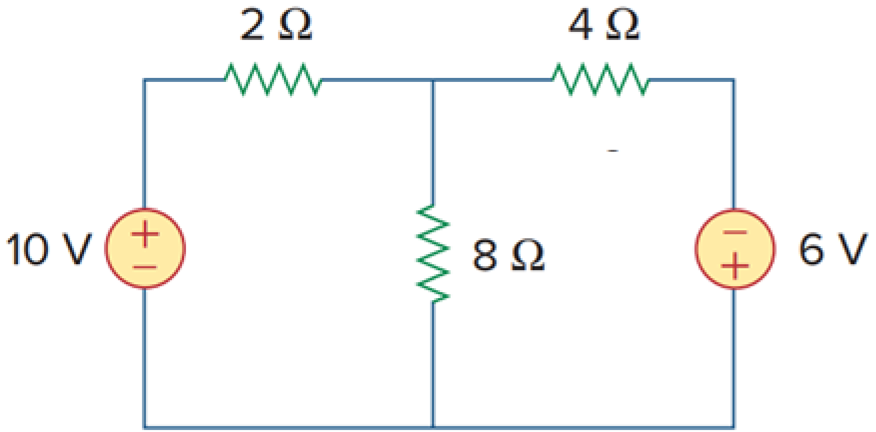
$$= 9 \times 75 \times 19.5$$

$$= 13162.5\ W$$

Problem 1

i. Perform *branch current analysis* to determine the current absorbed by the $6\ V$ source in the following circuit.

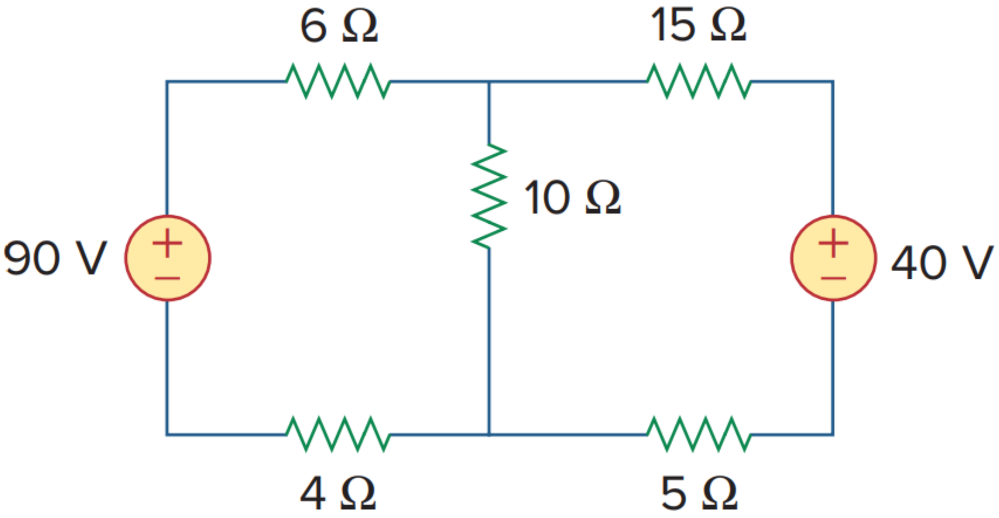
ii. Perform *mesh analysis* to determine the current absorbed by the $6\ V$ source in the following circuit.



Ans: $-\mathbf{2.5}\ \boldsymbol{A}$

Problem 2

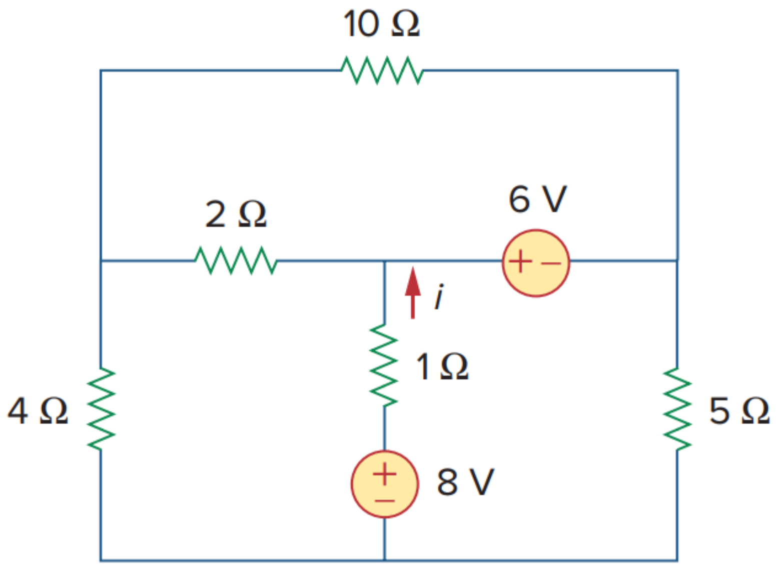
- Calculate the current through the $10\ \Omega$ resistor using mesh analysis.



Ans: $\boldsymbol{I_{10\Omega} = 4.4\ A}$

Problem 3

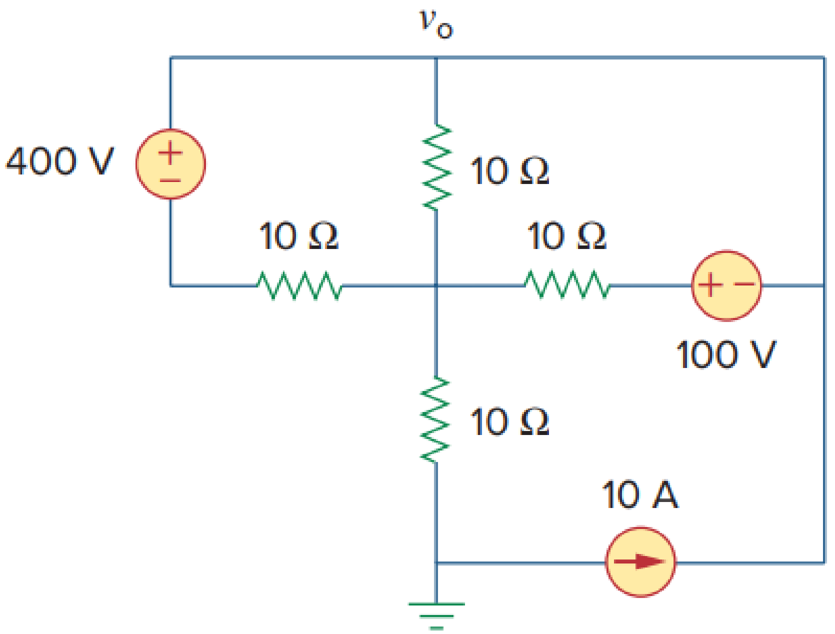
- Calculate the current i using mesh analysis.



Ans: $\boldsymbol{i = 1.188\ A}$

Problem 4

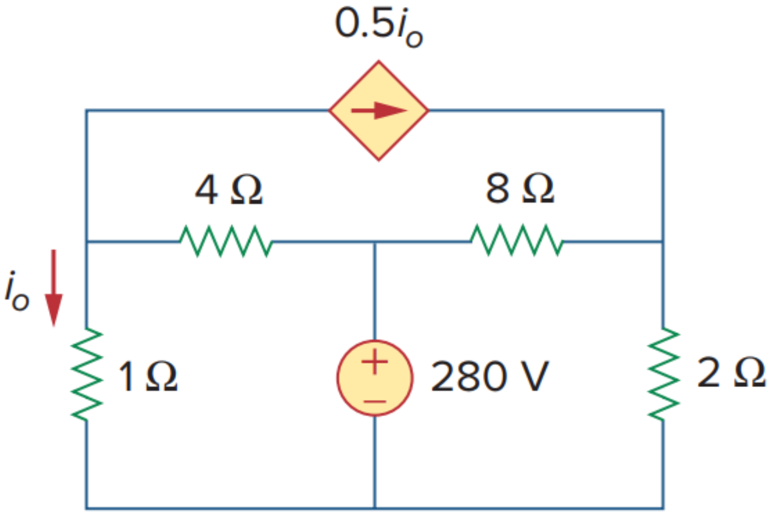
- Apply mesh analysis to find v_o in the following circuit.



Ans: $v_0 = 233.3\ V$

Problem 5

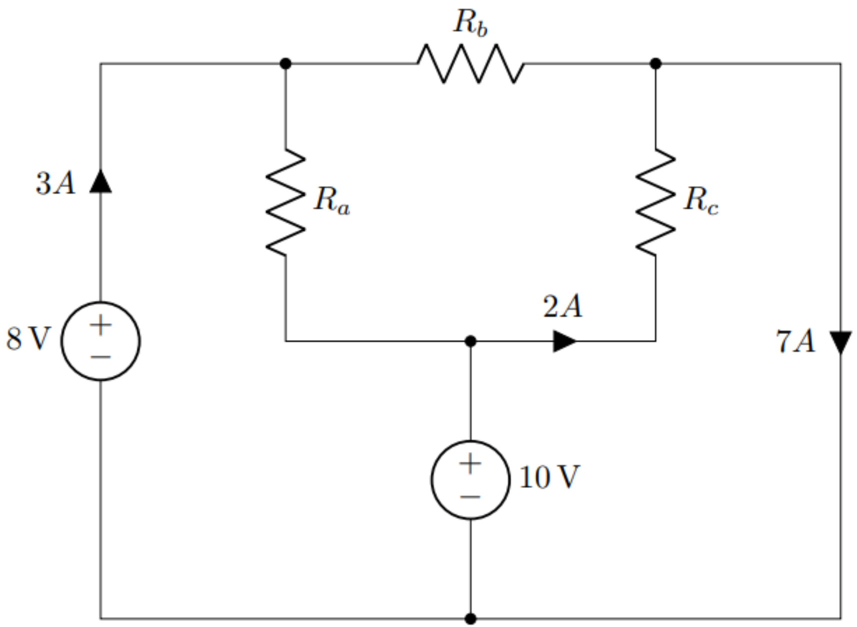
- Find i_0 using mesh analysis. What is the voltage across the $0.5i_0$ source?



Ans: $\boldsymbol{i_0 = 40\ A;\ \pm 48\ V}$

Problem 6

- Determine the values of R_a, R_b, and R_c for the circuit shown below.



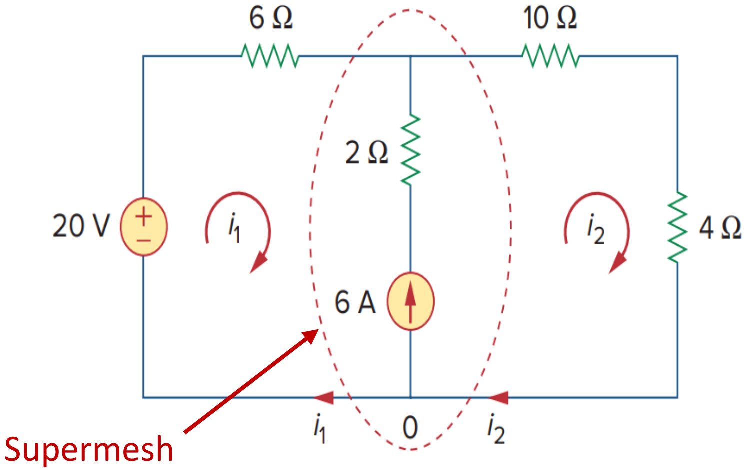
Ans: $R_a = 1\,\Omega, R_b = 1.6\,\Omega, R_c = 5\,\Omega$

Current Source Bet^n Loops

■ **CASE 1** When a current source (dependent or independent) exists only in one mesh, we simply set the current at that mesh equal to the current of the current source. (We have already seen this in example 1).

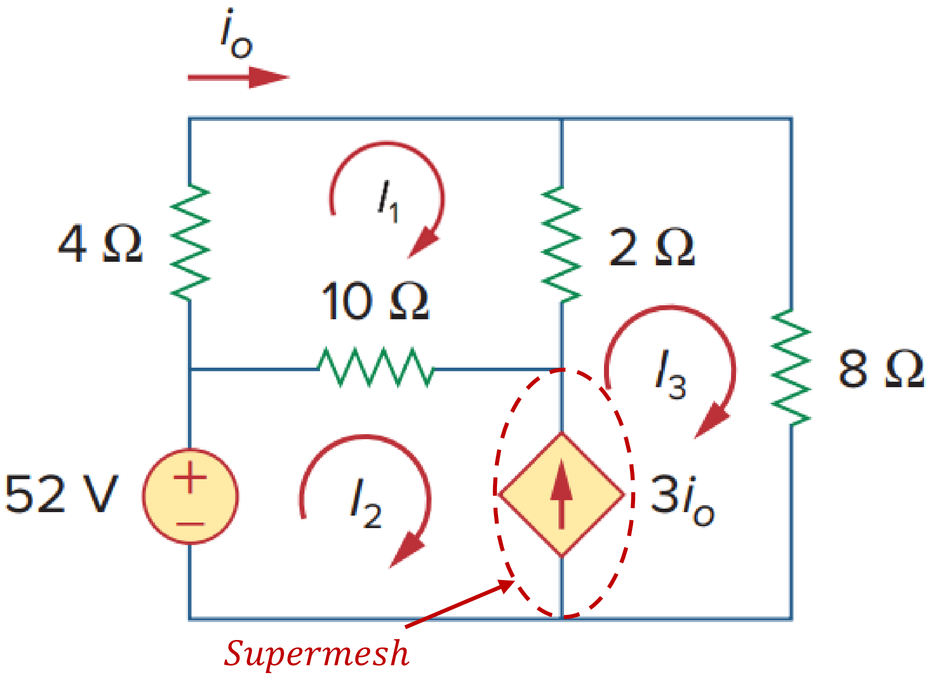
■ **CASE 2** When a current source (dependent or independent) exists between two meshes, the two meshes form a generalized mesh or supermesh.

In other words, a *supermesh* results when two meshes have a (dependent or independent) current source in common.



Example 2 - 1/5

- Find i_0 using mesh analysis. Also, calculate the voltage across the $3i_0$ source.



Step 1: Identify all the meshes and assign mesh variables to each of the meshes.

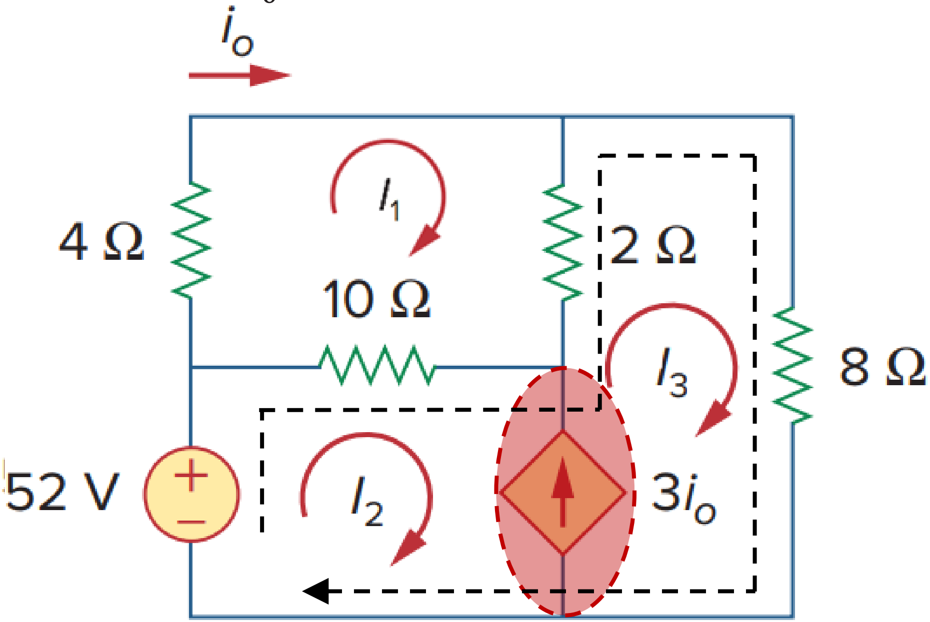
Check for supermeshes. Check if a current source (dependent or independent) is connected between two meshes. There can be multiple supermeshes in a circuit.

In this circuit, the $3i_0$ current source forms a supermesh between meshes 2 and 3.

We need to handle such conditions differently because there is no way to know the voltage across a current source in advance.

Example 2 - 2/5

- Find i_0 using mesh analysis. Also, calculate the voltage across the $3i_0$ source.



Step 2: Apply KVL to each of the meshes.

KVL to the mesh 1,

$$4i_1 + 2(i_1 - i_3) + 10(i_1 - i_2) = 0$$

$$\Rightarrow \quad 16i_1 - 10i_2 - 2i_3 = 0 ---- -(i)$$

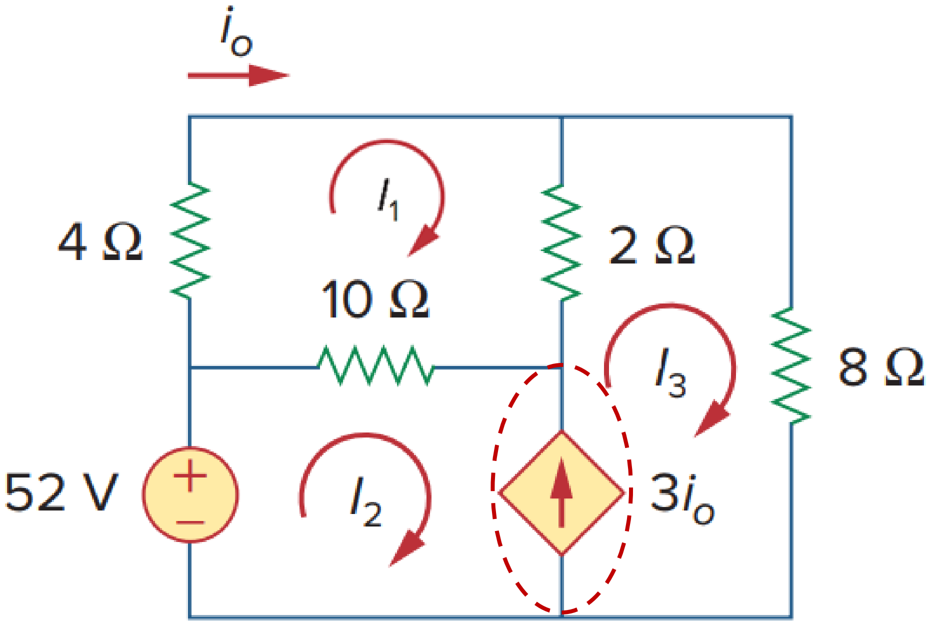
Next, ignore the current source that forms the supermesh and apply KVL to the corresponding meshes together. Careful with the current notations. Applying KVL to the supermesh along the black dotted line shown in the figure,

$$-52 + 10(i_2 - i_1) + 2(i_3 - i_1) + 8i_3 = 0$$

$$\Rightarrow \quad 12i_1 - 10i_2 - 10i_3 = -52 ---- -(ii)$$

Example 2 - 3/5

- Find i_0 using mesh analysis. Also, calculate the voltage across the $3i_0$ source.



We have 2 equations, 3 variables, and no remaining mesh for KVL.

The 3rd equation required, can be found by applying KCL to the supermesh.

$$i_3 - i_2 = 3i_0$$

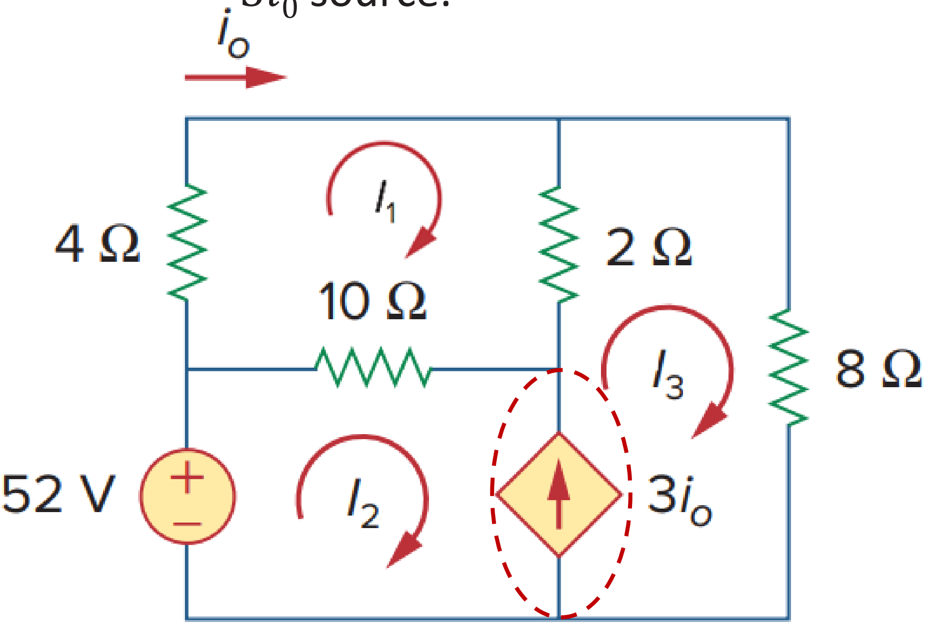
Now replace i_0 in terms of the mesh currents. It can be seen from the figure that, $i_0 = i_1$. Substituting,

$$i_3 - i_2 = 3i_1$$

$$\Rightarrow \quad 3i_1 + i_2 - i_3 = 0 ---- -(iii)$$

Example 2 - 4/5

- Find i_0 using mesh analysis. Also, calculate the voltage across the $3i_0$ source.



We have derived the three equations,

$$16i_1 - 10i_2 - 2i_3 = 0$$

$$12i_1 - 10i_2 - 10i_3 = -52$$

$$3i_1 + i_2 - i_3 = 0$$

Solving … …,

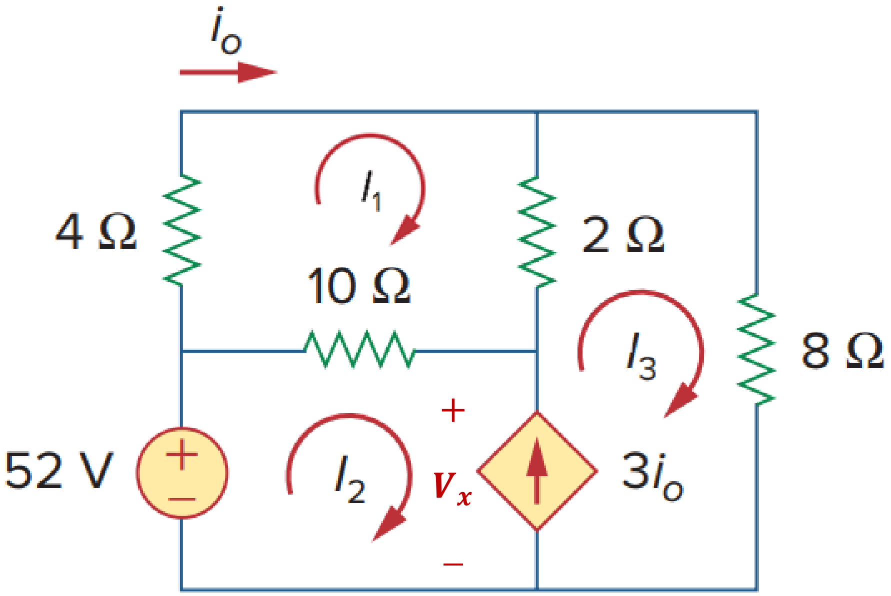
$$i_1 = 1.5\ A; \quad i_2 = 1.25\ A; \quad i_3 = 5.75\ A$$

So, $\quad i_0 = i_1 = 1.5\ A$

To calculate the voltage across the $3i_0$ dependent source, we have to apply KVL to either loop 2 or loop 3.

Example 2 - 5/5

- Find i_0 using mesh analysis. Also, calculate the voltage across the $3i_0$ source.



Let the voltage across the $3i_0$ source is V_x as indicated in the figure.

Applying KVL to the loop 2,

$$-52 + 10(i_2 - i_1) + V_x = 0$$

$$\Rightarrow \quad V_x = 52 - 10(1.25 - 1.5) = 54.5\ V$$

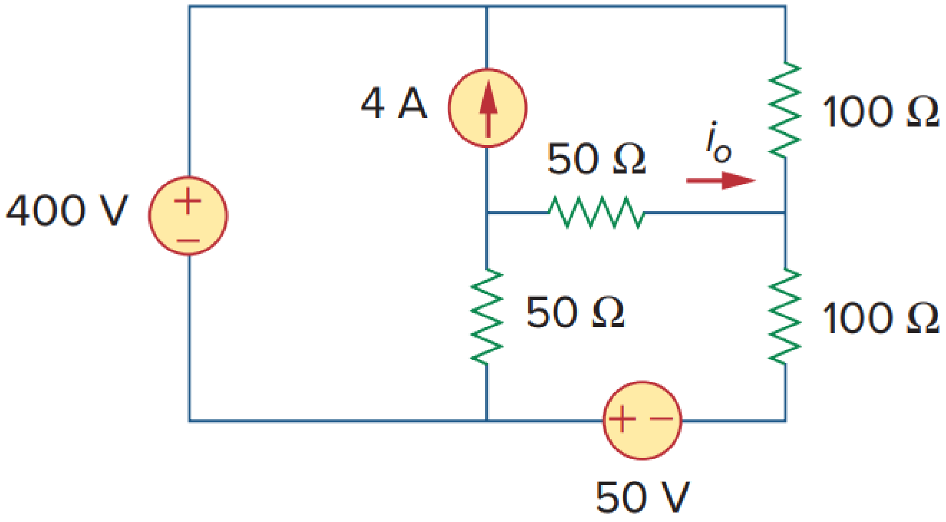
As observed by the polarities of voltage and current, the dependent source is supplying power.

$$p = +vi = 54.5 \times 3i_0 = 54.5 \times 3i_1$$

$$\Rightarrow \quad p = 54.5 \times 3 \times 1.5 = 245.25\ W$$

Problem 7

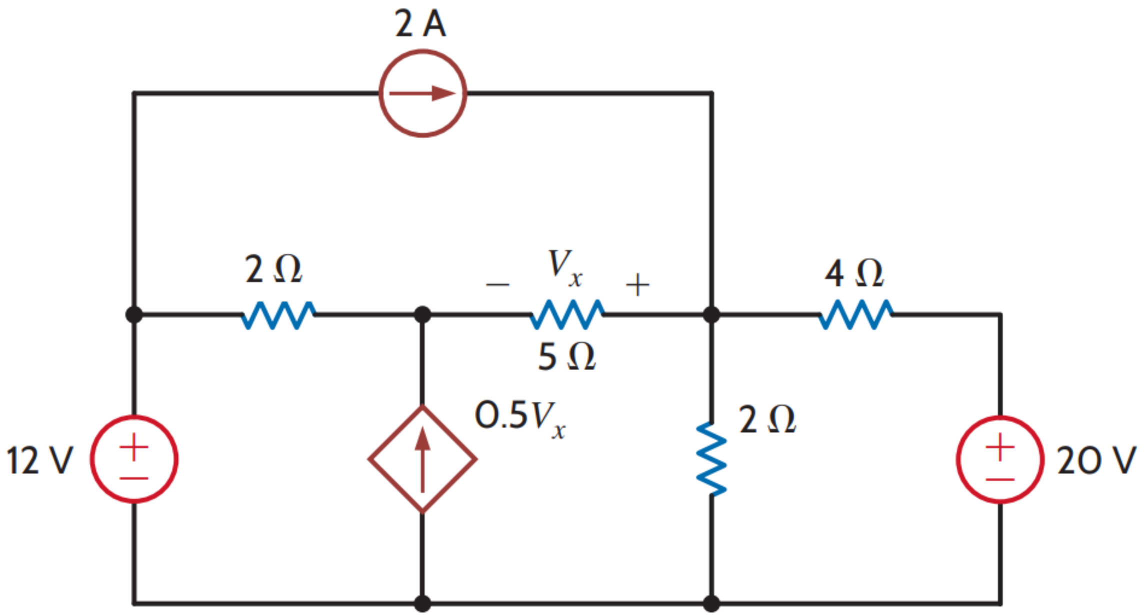
- Find i_0 using mesh analysis.



Ans: $\boldsymbol{i_0 = -2.5\,A}$

Problem 8

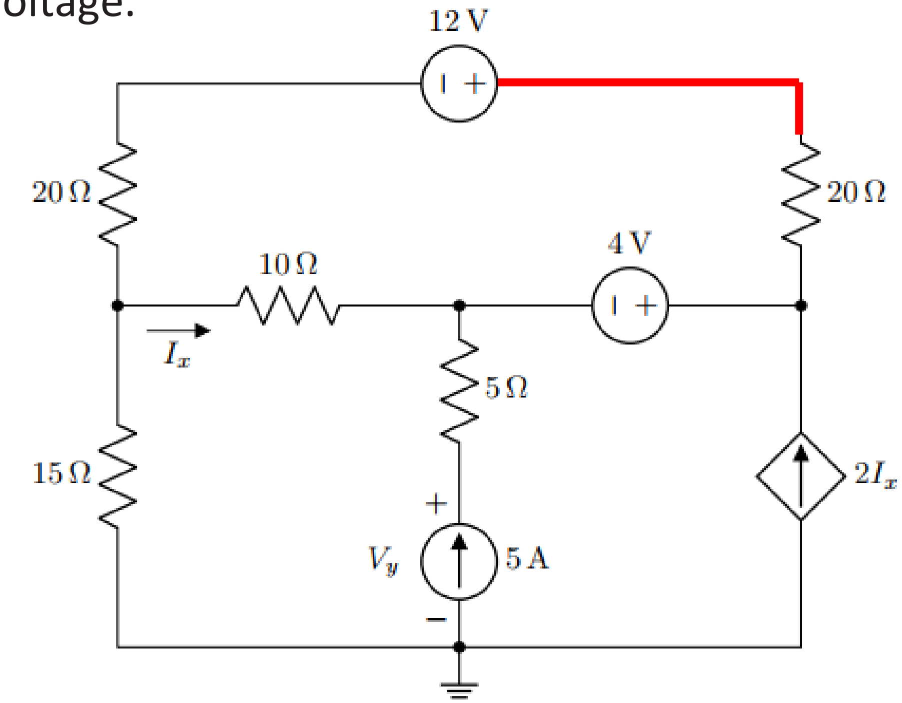
- Find V_x using mesh analysis.



Ans: $\boldsymbol{V_x = -1\ V}$

Problem 9

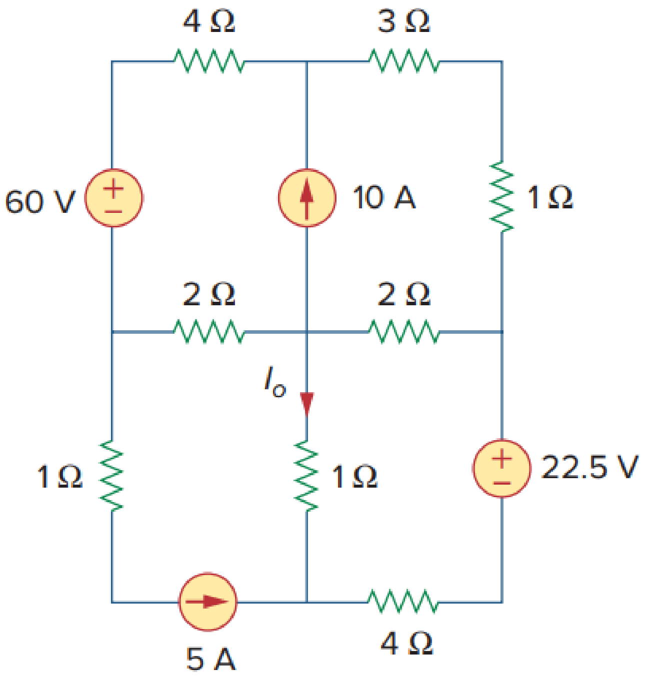
- Use Mesh Analysis to analyze the circuit. Find V_y. Determine the red colored node voltage.



Ans: $V_y = 68\ V; V_{red} = 43\ V$

Problem 10

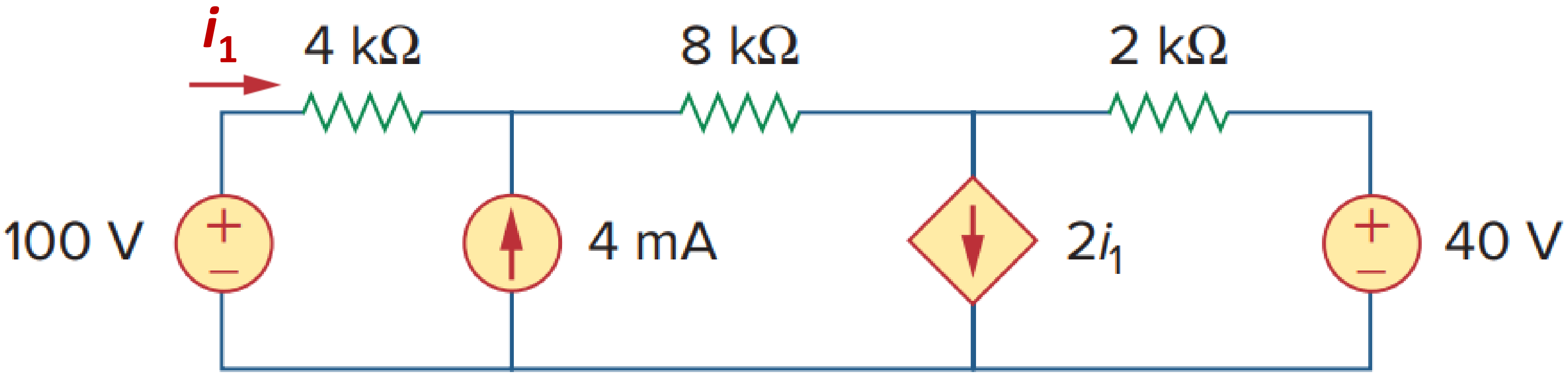
- Derive the mesh equations for the following circuit. Determine i_0.



Ans: $\boldsymbol{i_0 = -3.62\ A}$

Problem 11

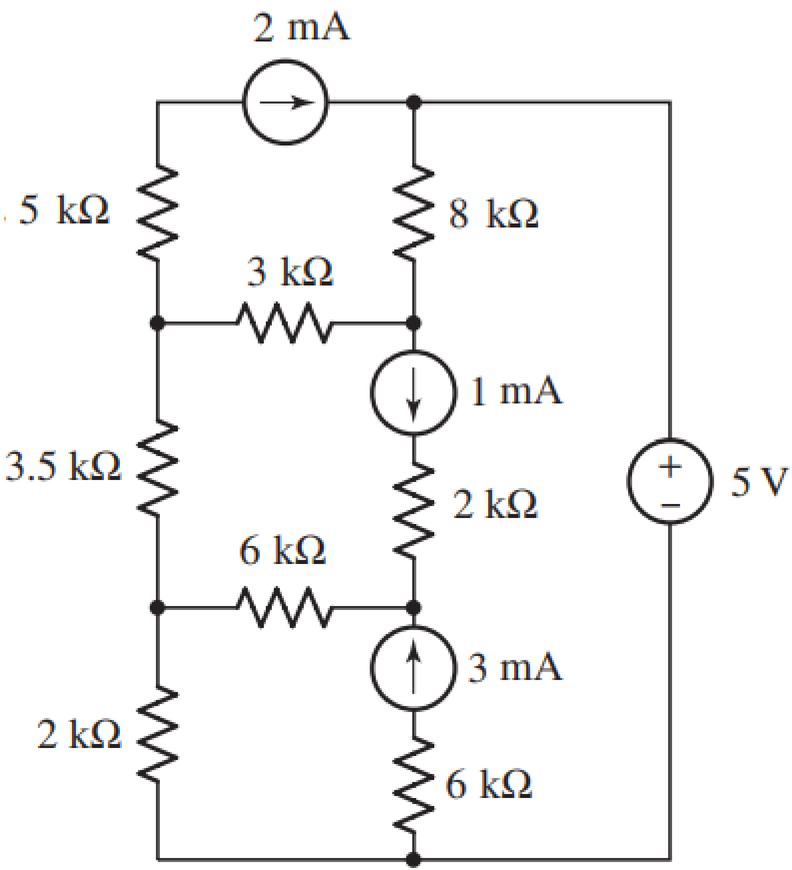
- Find the mesh currents.



Ans: $\pm 2\ mA;\ \pm 6\ mA;\ \pm 2\ mA$

Problem 12

- Find the mesh currents.

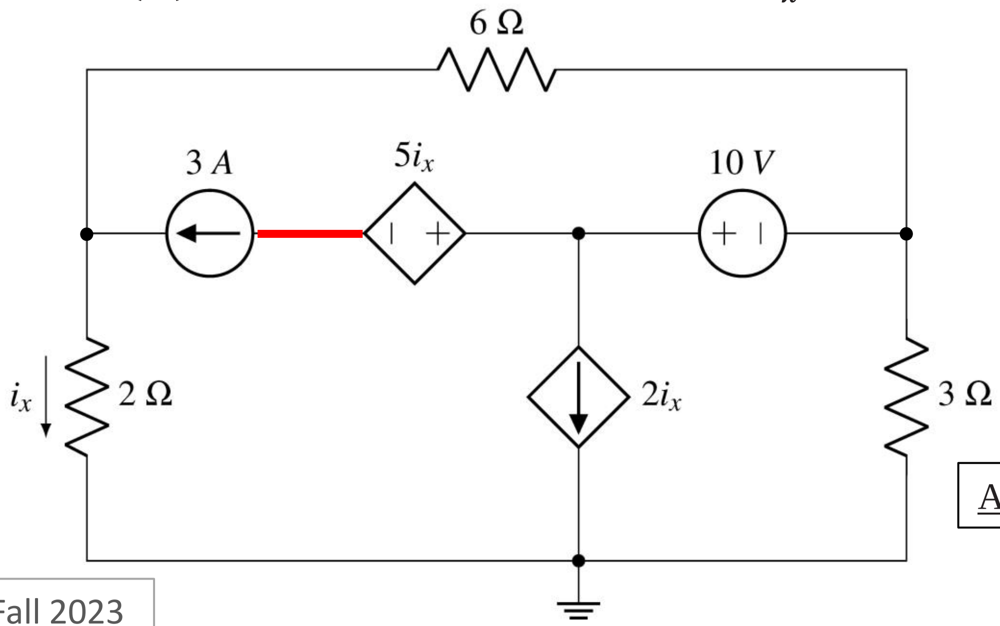


Ans: $\pm \mathbf{2}\ \boldsymbol{mA}$; $\pm \mathbf{2}\ \boldsymbol{mA}$; $\mp \mathbf{2}\ \boldsymbol{mA}$; $\pm \mathbf{1}\ \boldsymbol{mA}$

Problem 13

(i) Use nodal analysis to find i_x. Determine the voltage across the $3\ A$ source.

(ii) Use mesh analysis to find i_x. Determine the voltage of the red colored node.

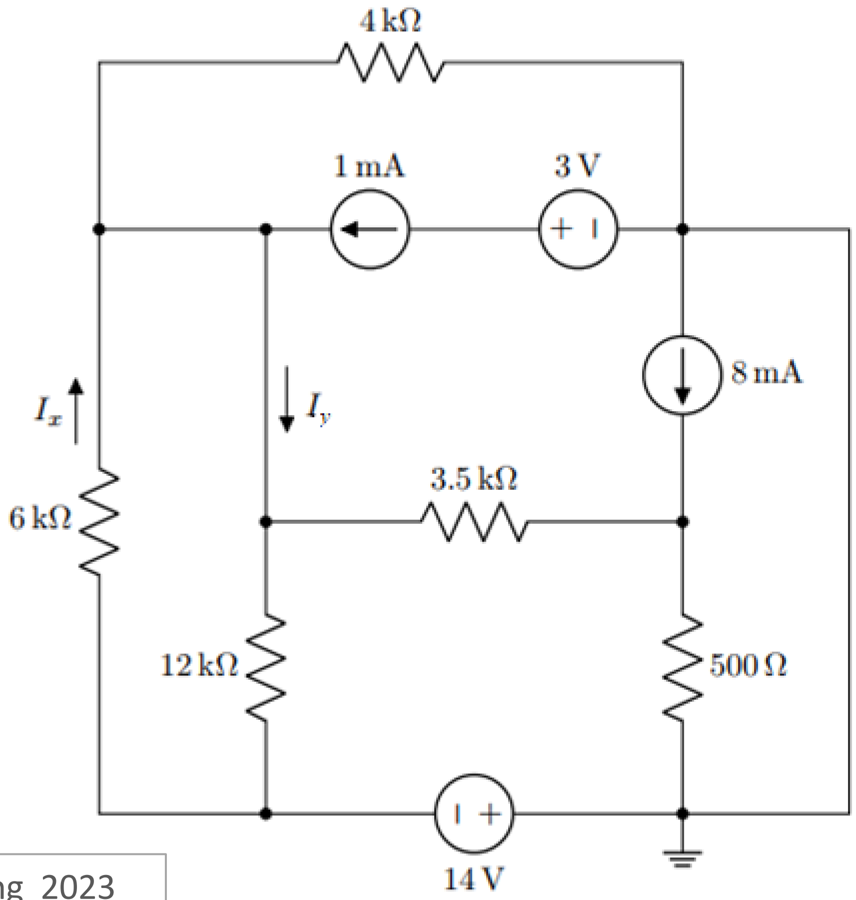


Ans: $\boldsymbol{i_x = 1.059\ A;\ v_{3A} = \pm 6.94\ V;\ v_{red} = -4.82\ V}$

Fall 2023

Problem 14

- Use mesh analysis to analyze the circuit. Find I_x.
- Determine the current I_y.
- Now repeat using Nodal analysis.
- Compare the two methods in solving this circuit.

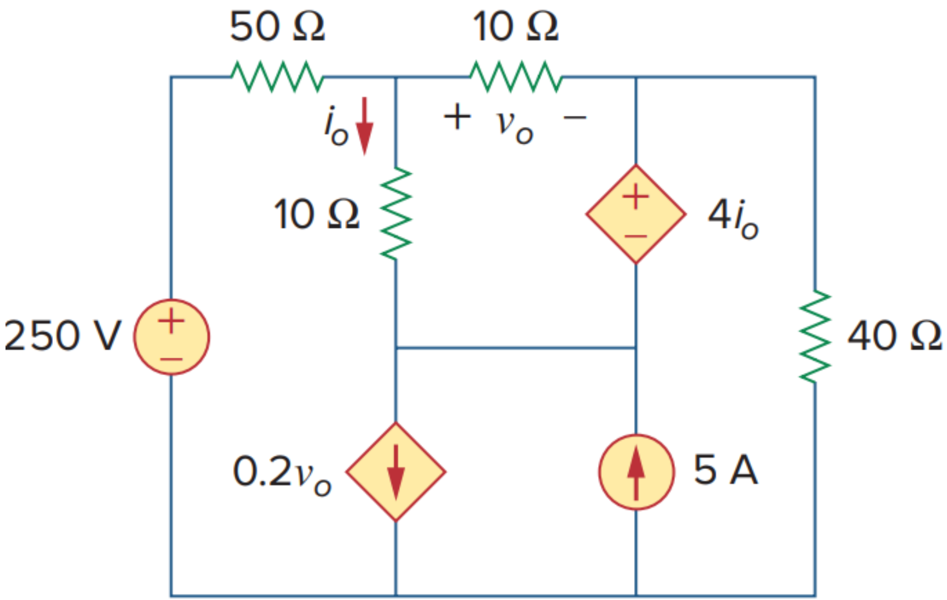


Ans: $\boldsymbol{I_x = -2\ mA;\ I_y = -0.5\ mA}$

Spring 2023

Problem 15

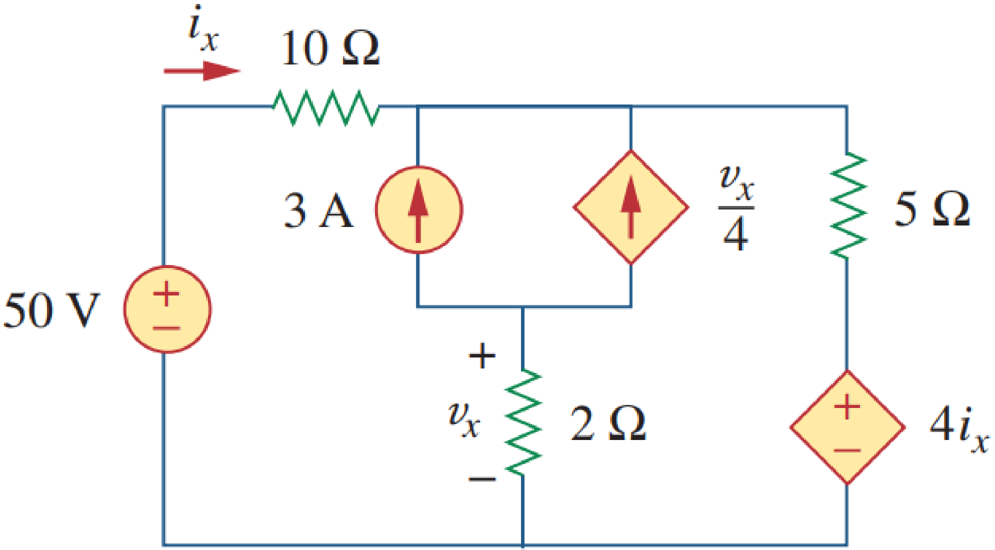
- Use mesh analysis to determine v_0 and i_0. What is the voltage across the $5\ A$ source?



Ans: $\boldsymbol{v_0 = 2.941\ V;\ i_0 = 0.49\ A}$

Problem 16

- Use mesh analysis to determine v_x and i_x. What is the voltage across the 3 A source?



Ans: $\boldsymbol{v_x = -4\ V;\ i_x = 2.105\ A}$

Nodal vs Mesh Analysis

- Given a network to be analysed, how do we know which method is better or more efficient? *The choice of the better method is dictated by two factors:*

■ Nature of the network

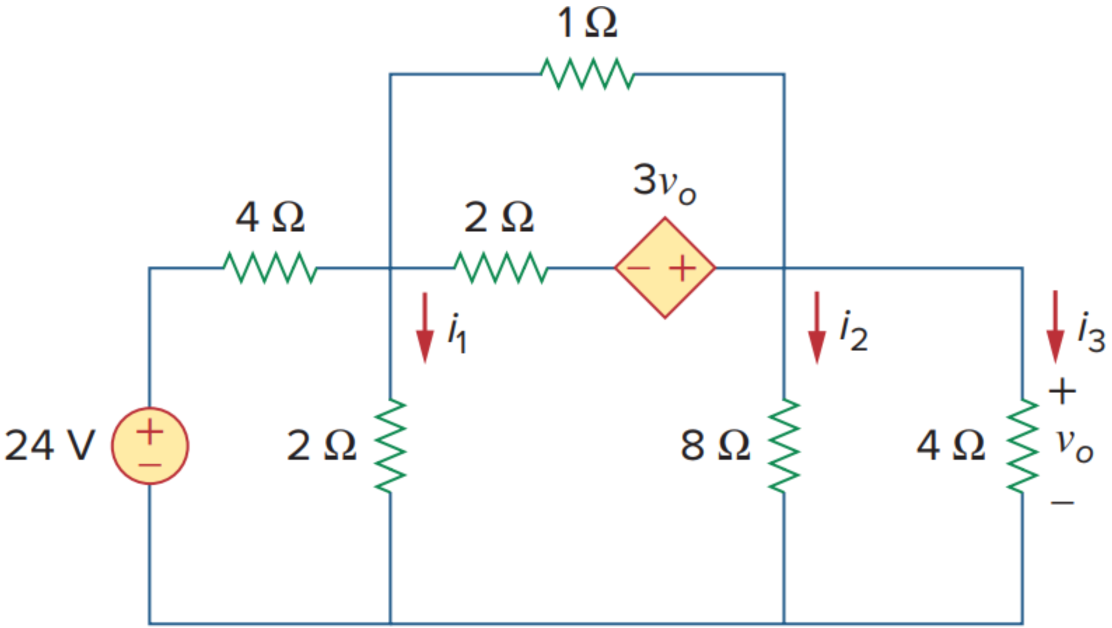
Mesh analysis is easier for networks that contain many series-connected elements, voltage sources, or supermeshes	Nodal analysis is easier for networks with parallel connected elements, current sources, or supernodes.
A circuit with fewer nodes than meshes is better analysed using nodal analysis, and vice versa. The key is to select the method that results in the smaller number of equations.	

■ Information required

Mesh analysis is easier if branch or mesh currents are required	Nodal analysis is easier if node voltages are required
As we shall see in CSE251, mesh analysis is the only method to use in analysing transistor circuits. But mesh analysis cannot easily be used to solve an op amp circuit, because there is no direct way to obtain the voltage across the op amp itself. For nonplanar networks, nodal analysis is the only option.	

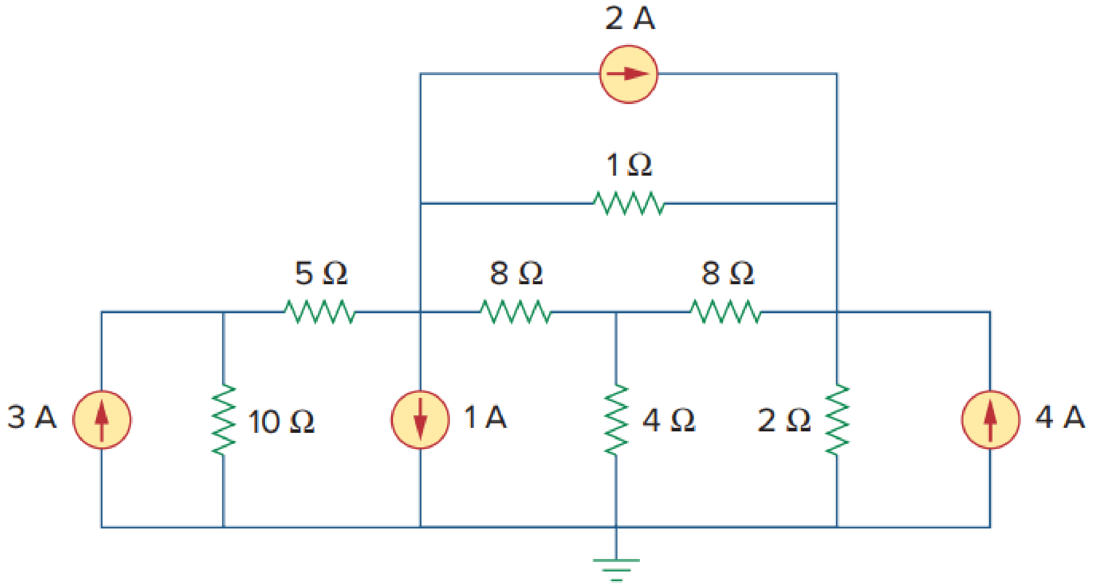
Problem 17

- Which method, nodal or mesh, is more convenient for solving the circuit? Derive the equations that correspond to the convenient one.



Problem 18

- Count how many nodes and meshes there are in this circuit. What is the bare minimum of variables that need to be considered for both nodal and mesh analysis? Which of these methods is the most convenient for solving the circuit? Determine the equations that correspond to the convenient one.

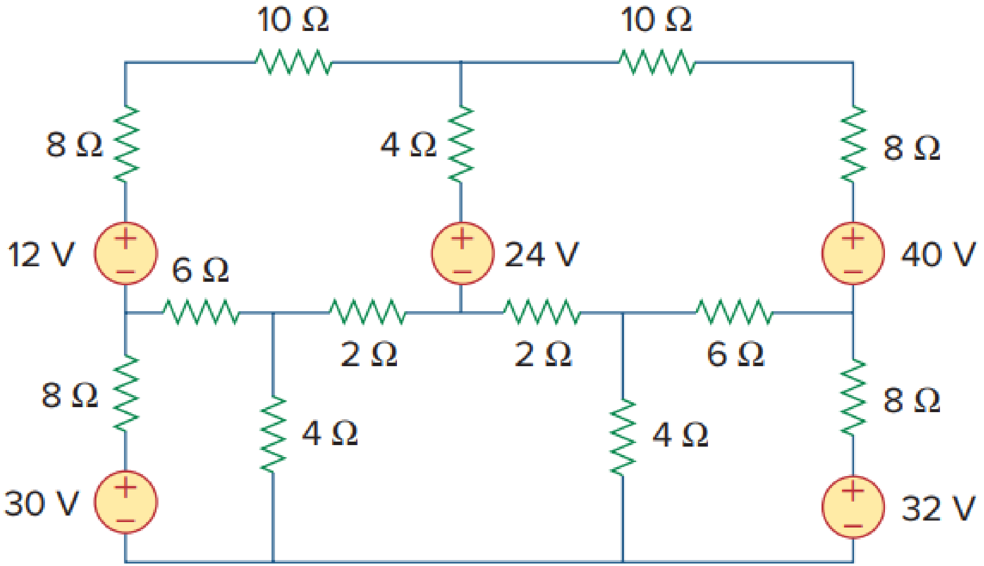


Ans:

- # of nodes = 4;
- # of meshes = 7;
- minimum # of variables for nodal analysis = 4;
- minimum # of variables for mesh analysis = 7.

Problem 19

- Count how many nodes and meshes there are in this circuit. What is the bare minimum of variables that need to be considered for both nodal and mesh analysis? Which of these methods is the most convenient for solving the circuit? Determine the equations that correspond to the convenient one.



Ans:
- # of nodes = 14;
- # of meshes = 5;
- minimum # of variables for nodal analysis = 6;
- minimum # of variables for mesh analysis = 5.

Additional Problems

- Additional recommended practice problems: here
- Other suggested problems from the textbook: here

Acknowledgements and References

Some of the problems, illustrations, and concepts in this lecture are taken from the following sources:

1. Sadiku, M. N. O., Fundamentals of Electric Circuits, McGraw-Hill
2. Irwin, J. D., & Nelms, R. M., Basic Engineering Circuit Analysis, Wiley

Thank you for your attention