Q1:

(a) From the given circuit, $V_L = \frac{R_L}{R_{Th}+R_L} V_{Th}$. When $V_L = 12$ V, $R_L = 1\,k\Omega$. So,

$$12 = \frac{1}{1+R_{Th}} V_{Th} \tag{1}$$

When $V_L = 20$ V, $R_L = 2\,k\Omega$. So,

$$20 = \frac{2}{2+R_{Th}} V_{Th} \tag{2}$$

Dividing the two equations 1 & 2,

$$\frac{6}{5} = \frac{2+R_{Th}}{1+R_{Th}} \Rightarrow R_{Th} = 4\,k\Omega$$

Substituting this to 1, $V_{Th} = 60$ V

Ans: 60 V, $4\,k\Omega$

Alternate Solution using Max Power Transfer:

R_L	V_L	P_L
$1\,k\Omega$	12 V	144 mW
$2\,k\Omega$	20 V	200 mW
$4\,k\Omega$	30 V	225 mW
$6\,k\Omega$	36 V	216 mW
$16\,k\Omega$	48 V	144 mW
$56\,k\Omega$	56 V	56 mW

Thus, $P_{max} = 225$ mW, when $R_L = 4\,k\Omega$. $\therefore R_{Th} = 4\,k\Omega$. And since voltage across both the resistors will be same, $V_{Th} = 2 \times V_L = 60\,V$.

(b) **Thevenin's theorem:** W.r.t. the ground given in the question, $V_b = 20$ V. Voltage of the +ve node of 10 V source = 20 - V_x. Applying KCL to this node:

$$\frac{(20-V_x)-20}{5} + \frac{20-V_x}{10} + \frac{20-V_x-V_a}{10} + \frac{20-V_x-V_x-V_a}{10} - 4 = 0$$

KCL at V_a,

$$\frac{V_a-(20-V_x)}{10} + \frac{V_a-(20-V_x-V_x)}{10} = 0$$

Solving, $V_a = 30$ V. $\therefore V_{Th} = V_a - V_b = 30 - 20 = 10\,V$.

Norton's theorem: Shorting a & b and w.r.t. the ground given, $V_a = V_b =$ 20 V. Again KCL at 20 - V_x node,

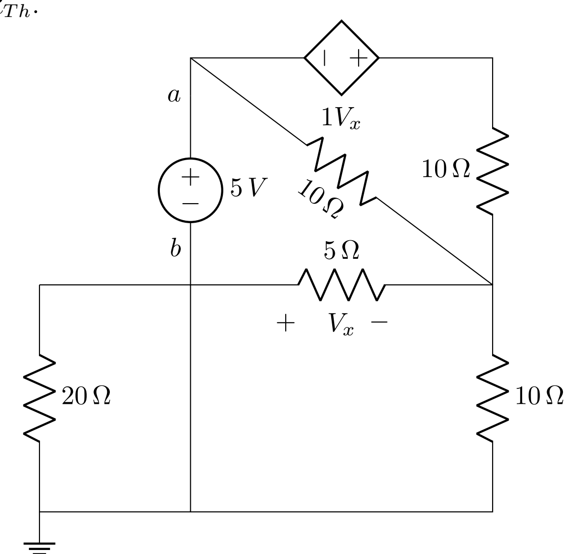
$$\frac{20-V_x-20}{5} + \frac{20-V_x}{10} + \frac{20-V_x-20}{10} + \frac{20-V_x-V_x-20}{10} - 4 = 0$$

Solving, $V_x = \frac{-10}{3}$ V. KCL at V_a,

$$\frac{V_a - (20 - V_x)}{10} + \frac{V_a + V_x - (20 - V_x)}{10} + I_N = 0$$

Substituting values of V_a & V_x, $I_N = 1$ A

R_{Th}**:** Turning off all independent sources, the reduced circuit looks like the following, with a dummy source of 5 V inserted between a-b terminals for finding R_{Th}.



20 Ω is shorted. $V_b = 0$ V, $V_a = 5$ V. Applying KCL at -V_x node,

$$\frac{-V_x}{5} + \frac{-V_x}{10} + \frac{-V_x - 5}{10} + \frac{-V_x - V_x - 5}{10} = 0$$

Solving, $V_x = -\frac{5}{3}$ V. KCL at V_a,

$$\frac{V_a - (-V_x)}{10} + \frac{V_a + V_x - (-V_x)}{10} - I_{5\,V} = 0$$

Substituting values of V_a & V_x, $I_{5\,V} = \frac{1}{2}$ A. Now, $R_{Th} = \frac{5}{I_{5\,V}} = \frac{5}{\frac{1}{2}} = 10\,\Omega$.

$\therefore P_{max} = \frac{V_{Th}^2}{4 \times R_{Th}} = \frac{I_N^2 \times R_N}{4} = \frac{V_{Th} \times I_N}{4} = 2.5\,W$

Ans: 2.5 W

Q2: (a)

i. $5\tau = 5 \Rightarrow \tau = 1\,\text{ms}$

ii. $V_c(t) = V_f + [V_0 - V_f]e^{-\frac{t - t_i}{\tau}} = 6 + [2 - 6]e^{-\frac{t-0}{1 \times 10^{-3}}}$

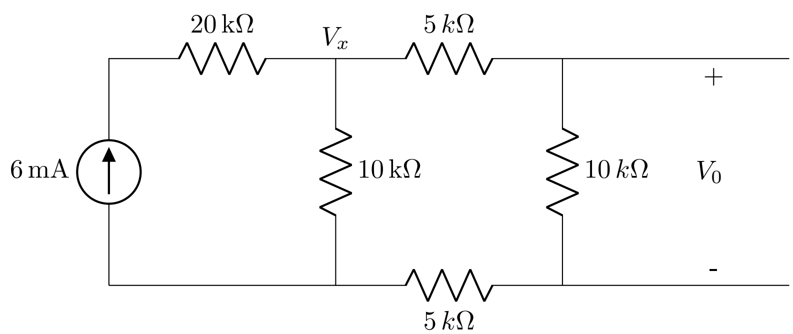
iii. $R_{Th} = \frac{\tau}{C} = \frac{1\times10^-3}{1\times10^{-6}} = 1\,k\Omega$

$R||2\,k\Omega = R_{Th} \Rightarrow R = 2\,k\Omega$

Ans: $\tau = 1\,ms$, $V_c(t) = 6 - 4e^{\frac{-t}{1\times10^{-3}}}$, $R = 2\,k\Omega$

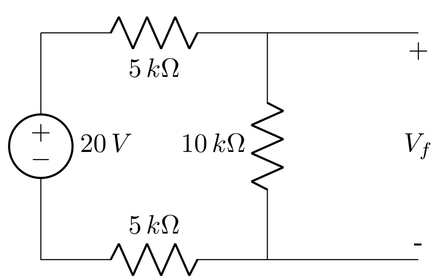
(b)

i. Open circuit voltage at the terminals of the capacitor for $t < 0$, V_0:



Using current divider, $I_{(5+10+5)\,k\Omega} = \frac{10}{10+20}\times6 = 2\,mA$. $\therefore\ V_0 = 2\times10 = 20\,V$.

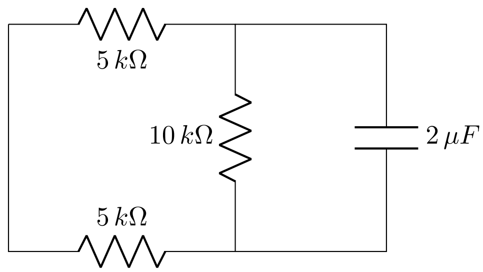
Open circuit voltage at the terminals of the capacitor for $t > 0$, V_f:



Using voltage divider, $V_f = \frac{10}{5+10+5}\times20 = 10\,V$

Ans: $V_0 = 20\,V$, $V_f = 10\,V$

ii. For $t > 0$, killing all the independent sources:



$\therefore R_{Th} = (5+5)||10 = 5\,k\Omega.$

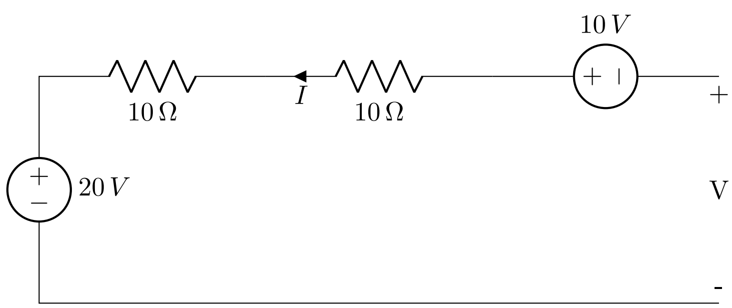
$\therefore \tau = R_{Th} \times C = 5 \times 2 = 10\,ms.$

iii. $V(t) = 10 + [20-10]e^{-\frac{t}{10\times 10^{-3}}}$

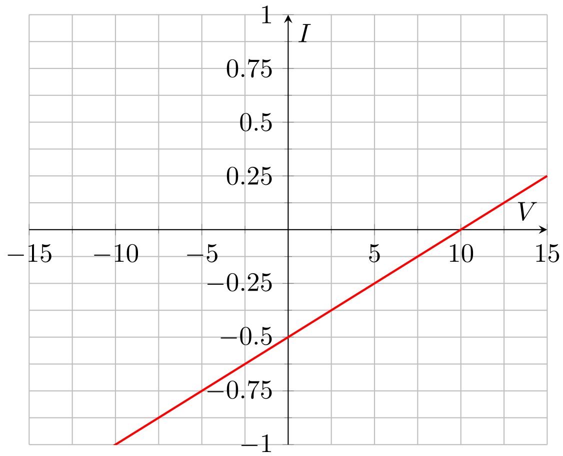
iv. $i(t) = C\frac{dV_c}{dt} = 2\times 10^{-6} \times \frac{-1}{10\times 10^{-3}} \times (10e^{-\frac{t}{10\times 10^{-3}}}) = -2e^{-\frac{1}{10\times 10^{-3}}}\,mA$

v. Using voltage divider, $V_R(t) = \frac{6}{4+6} \times V(t) = 6 + 6e^{-\frac{1}{10\times 10^{-3}}}$

Q3: Using source transfromation:



Applying KVL in the loop, $-20 - 10I - 10I + 10 + V = 0 \Rightarrow I = \frac{V}{20} - \frac{1}{2}$



Q4: (a)

$$-16sin(150\pi t + 45°) = 16cos(150\pi t + 45° + 90°) = 16cos(150\pi t + 135°)$$

$\therefore \Delta\phi = 135° \equiv \frac{T}{360°} \times 135° = \frac{2\pi}{150\pi} \times \frac{3}{8} = \frac{1}{200}\, s = 5\, ms$

Ans: $5\,ms$ shifted to the left

(b) $T = 72 - 8 = 64\,\text{ms}$. Let the initial phases be ϕ_1 and ϕ_2, respectively.

$$\phi_1 = (16 - 8) \times 10^{-3} \times \frac{360000°}{64} = 45°$$

$$\phi_2 = (0 - 16) \times 10^{-3} \times \frac{360000°}{64} = -90°$$

$$\therefore v_1(t) = 32cos(\frac{360000}{T}t + \phi_1) = 32cos(\frac{360000}{64}t + 45°)$$

$$\therefore v_2(t) = 16sin(\frac{360000}{64}t - 90°) = -16cos(\frac{360000}{64}t) = 16cos(\frac{360000}{64}t + 180°)$$

i. $\Delta\phi = 180° - 45° = 135°$. $v_2(t)$ is leading.

ii. $v_1(t) = 32cos(\frac{360000}{64}t + 45°)$

$v_2(t) = 16cos(\frac{360000}{64}t + 180°)$