Department of Computer Science and Engineering (CSE)
BRAC University

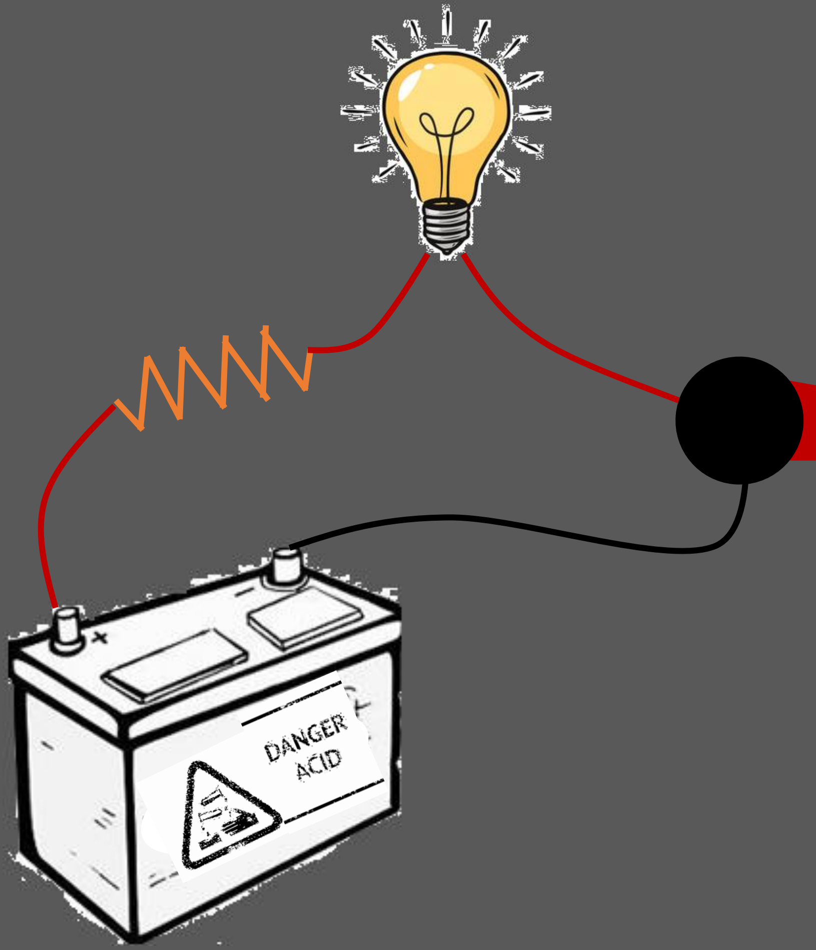
Lecture 10

CSE250 - Circuits and Electronics

# Maximum Power Transfer Theorem

Purbayan Das, Lecturer
*Department of Computer Science and Engineering (CSE)*
*BRAC University*

# Course Outline: broad themes




- Circuit Laws
- Methods of Analysis
- Circuit Theorems
  - Maximum Power Transfer Theorem
- First Order Circuits
- AC Circuits

# Maximum Power Transfer

- In many practical situations, a circuit is designed to provide power to a load. There are applications in areas such as communications and amplification where it is desirable to maximize the power delivered to an antenna (load) and a speaker respectively.

- When designing a circuit, it is often important to be able to answer the question, *"What load should be applied to a system or what driving circuitry for a particular load should be designed to ensure that the load is receiving maximum power from the system or from the circuit respectively?"*

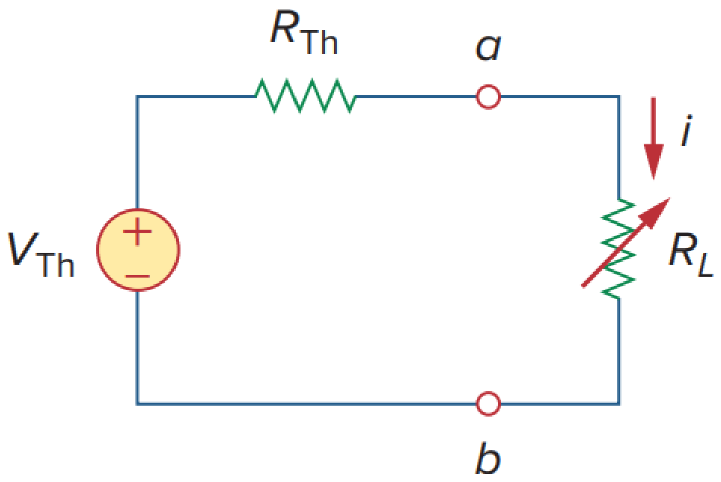
- Given a system with known internal losses the Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the load resistance $R_L$.

- Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$). This is known as the *Maximum Power Transfer Theorem.*

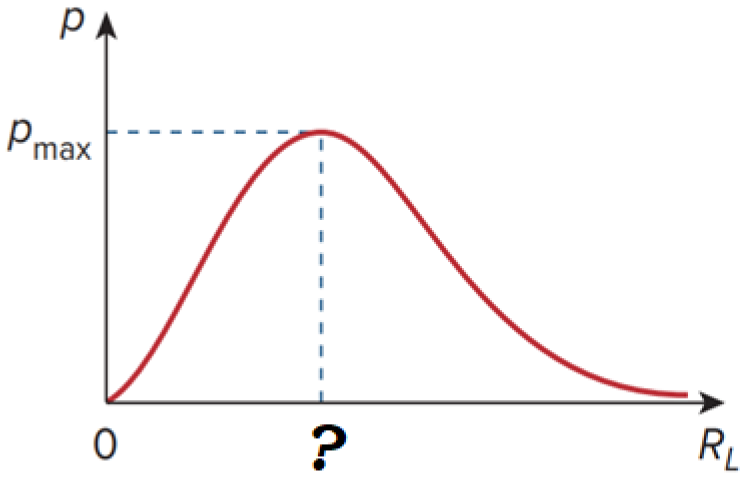
# Graphically

- Given any linear two terminal circuit, it can be reduced to a Thevenin equivalent as shown. Power delivered to the load by the Thevenin equivalent circuit is then,

$$p = i^2 R = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$




- For a given circuit, $V_{Th}$ and $R_{Th}$ are fixed. By varying the load resistance $R_L$, the power delivered to the load varies as sketched in the figure.

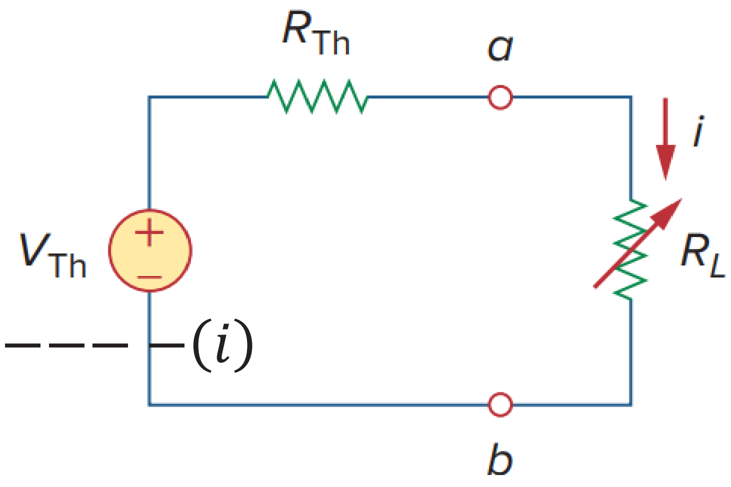



- Notice that the power is small for small or large values of $R_L$ but maximum for some value of $R_L$ between 0 and ∞.

- Let's now see mathematically that this maximum power occurs when $R_L$ is equal to $R_{Th}$.

# Mathematically

- The Thevenin equivalent circuit for a load $R_L$ is shown below. The load current is, $i = \frac{V_{Th}}{R_{Th}+ R_L}$.




- Power delivered to the load is,

$$p = i^2 R = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L = V^2{}_{Th}\left[\frac{R_L}{(R_{Th} + R_L)^2}\right] \text{-----} (i)$$

- Differentiating with respect to $R_L$,

$$\frac{dp}{dR_L} = V^2{}_{Th}\left[\frac{(R_{Th} + R_L)^2 \frac{d}{dR_L}(R_L) - R_L \frac{d}{dR_L}\{(R_{Th} + R_L)^2\}}{(R_{Th} + R_L)^4}\right] = V^2{}_{Th}\left[\frac{(R_{Th} + R_L)^2 - 2R_L (R_{Th} + R_L)^2}{(R_{Th} + R_L)^4}\right]$$

- Setting $\frac{dp}{dR_L}$ to zero will lead to the condition for maximum power transfer to the load.

# Condition to $P_{max}$ transfer & $P_{max}$

- For maxima/minima,

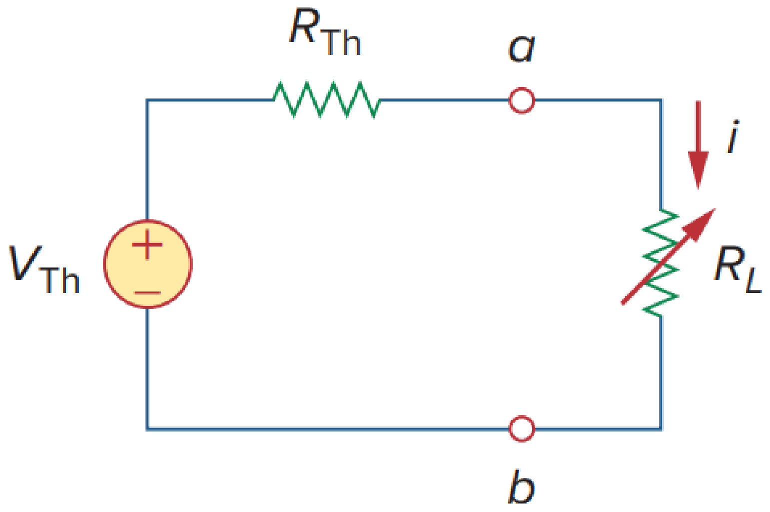
$$\frac{dp}{dR_L} = 0 = V^2{}_{Th}\left[\frac{(R_{Th} + R_L - 2R_L)}{(R_{Th} + R_L)^3}\right]$$

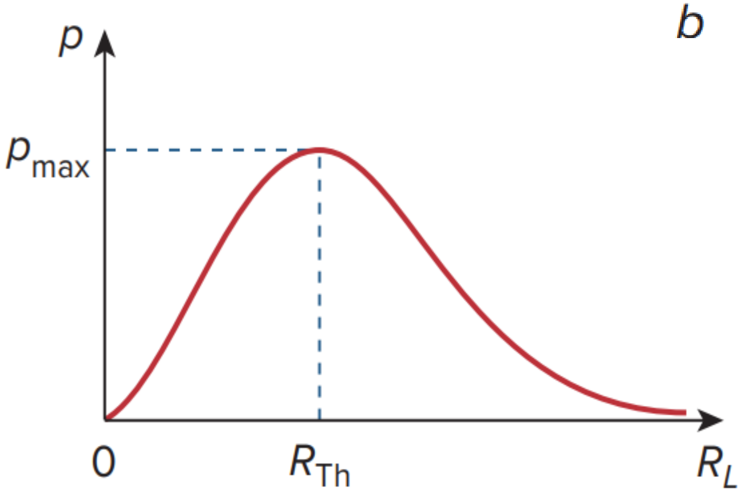
$$\Rightarrow R_{Th} + R_L - 2R_L = 0$$

$$\Rightarrow R_L = R_{Th}$$

Substituting in $(i)$ in the previous slide,

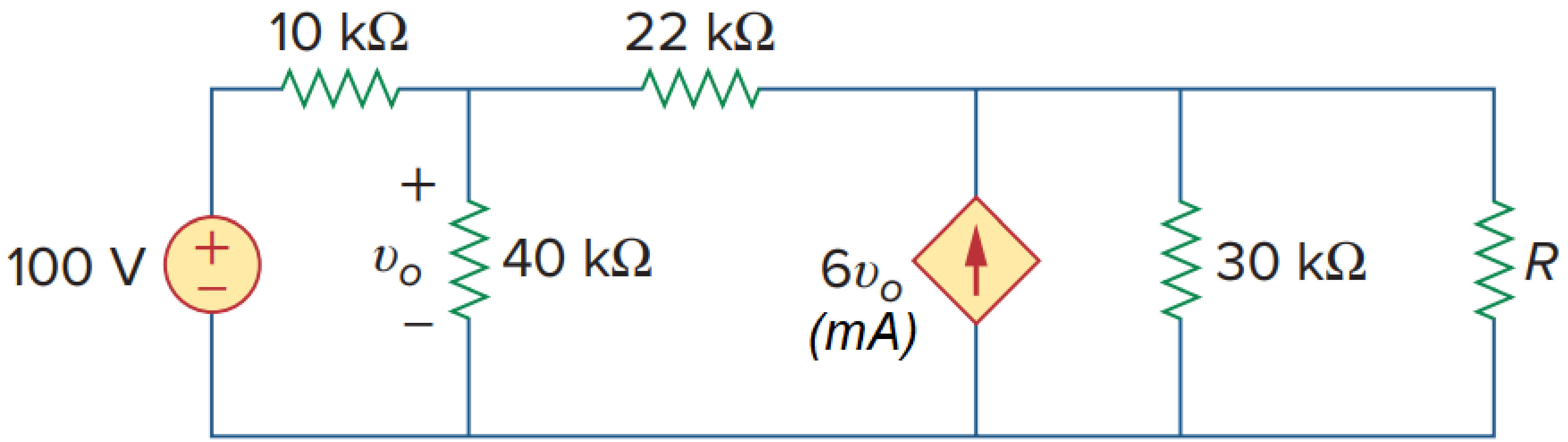
$$p_{max} = \frac{V_{Th}^2}{4R_{Th}}$$



# Example 1

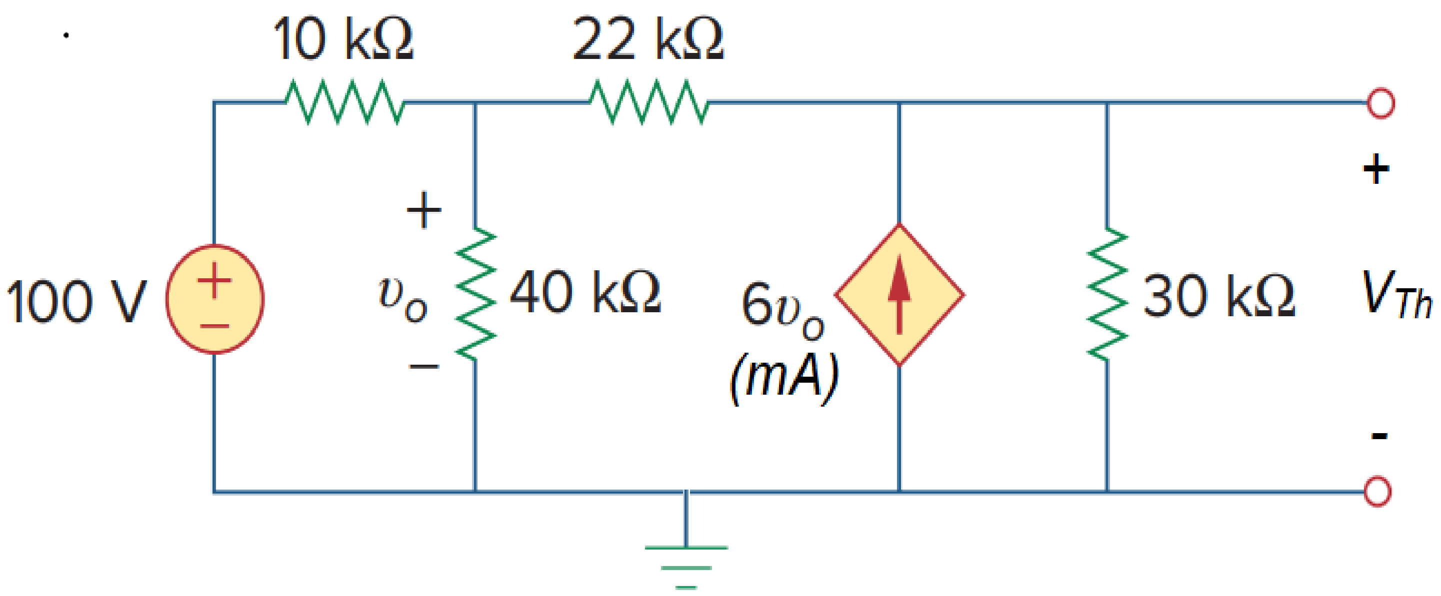
- Find the maximum power that can be delivered to the resistor *R*.




Ans: $\boldsymbol{V_{Th} = -231.304\ V;\ R_{Th} = -650\ \Omega; p_{max} = \infty}\ (Theoretically)$

* *See solution in the next slide if necessary*

# Example 1: finding $V_{Th}$




To find $P_{max}$, we have to first find $V_{Th}$ and $R_{Th}$.

Let's use nodal analysis to find the $V_{Th}$.

KCL at node $v_0$,

$$\frac{v_0 - 100}{10} + \frac{v_0}{40} + \frac{v_0 - V_{Th}}{22} = 0$$

$$\Rightarrow \quad 75v_0 - 20V_{Th} = 4400 \ ----(i)$$

KCL at node $V_{Th}$,

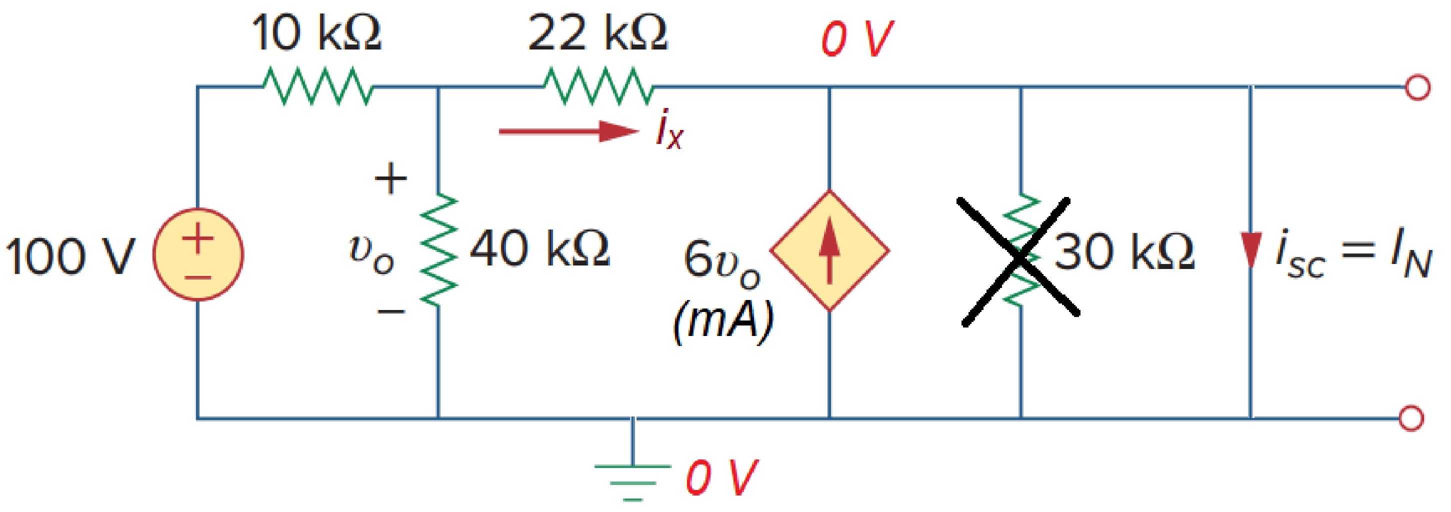
$$\frac{V_{Th} - v_0}{22} + \frac{V_{Th}}{30} = 6v_0$$

$$\Rightarrow \quad 1995v_0 - 26V_{Th} = 0 \ -----(ii)$$

Solving $(i)$ and $(ii)$,

$$V_{Th} = -231.304\,V$$

# Example 1: finding $R_{Th}$




As $V_{Th} \neq 0$, let's use $R_{Th} = \frac{V_{Th}}{I_N}$ to determine the Thevenin equivalent resistance. The load terminals have been short circuited as shown in the figure.
Upon short circuiting the terminals $a - b$, the 10 Ω is shorted out. The whole circuit to the left of the dependent source is shorted with respect to it. As a result, the $6v_0$ current supplied by the dependent source will only flow through the short circuit.

Let's use nodal analysis to solve for the current $i_x$ going towards the short circuit though the 22 $k\Omega$ resistor.

KCL at node $v_0$,

$$\frac{v_0 - 100}{10} + \frac{v_0}{40} + \frac{v_0 - 0}{22} = 0$$

$$\Rightarrow \quad 75v_0 = 4400$$

$$\Rightarrow \quad v_0 = 58.667\,V$$

$$\Rightarrow \quad i_x = \frac{v_0 - 0}{22} = 2.667\,mA$$

So,

$$I_N = i_x + 6v_0 = 354.669\,mA$$

$$R_{Th} = \frac{V_{Th}}{I_N} = -650\,\Omega$$

# Example 1: finding $P_{max}$

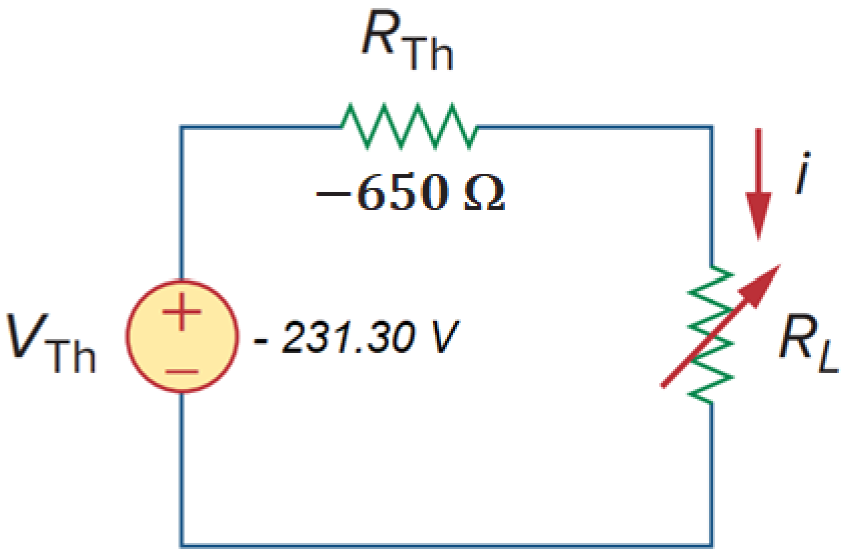
From the previous slides,

$$V_{Th} = -231.30\,V; \qquad R_{Th} = -650\,\Omega$$

What does a negative Thevenin resistance mean!

*Negative Thevenin resistance is a part of the circuit model. The conversion of an actual circuit to a Thevenin equivalent is a mechanism for solving circuit problems and does not mean that the Thevenin equivalent circuit replaces the real circuit in all aspects.*

Again, negative resistance means an active circuit. This means the circuit is trying to deliver infinite power to the load (assuming the load is practical, that is, $R_L > 0$).



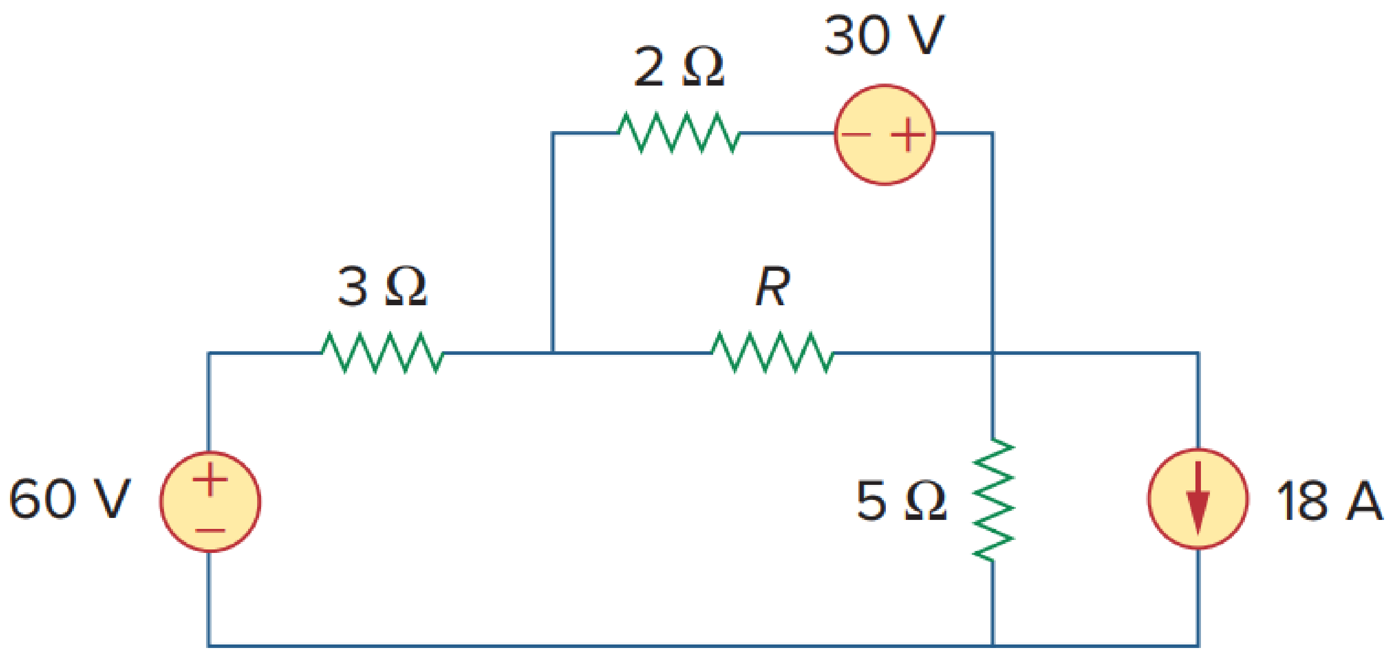

So, the correct answer is,

$$i = \frac{V_{Th}}{R_{Th} + R_L} = \frac{-231.30}{-650 + 650} = \infty$$

$$p_{max} = i^2 R_L = \infty\ (theoretically)$$

# Problem 1

- Find the maximum power that can be delivered to the resistor $R$. Calculate the power efficiency at the maximum power point.




Ans: $V_{Th} = 6\ V$; $R_{Th} = 1.6\ \Omega$; $p_{max} = 5.625\ W$; $\eta = 50\ \%$

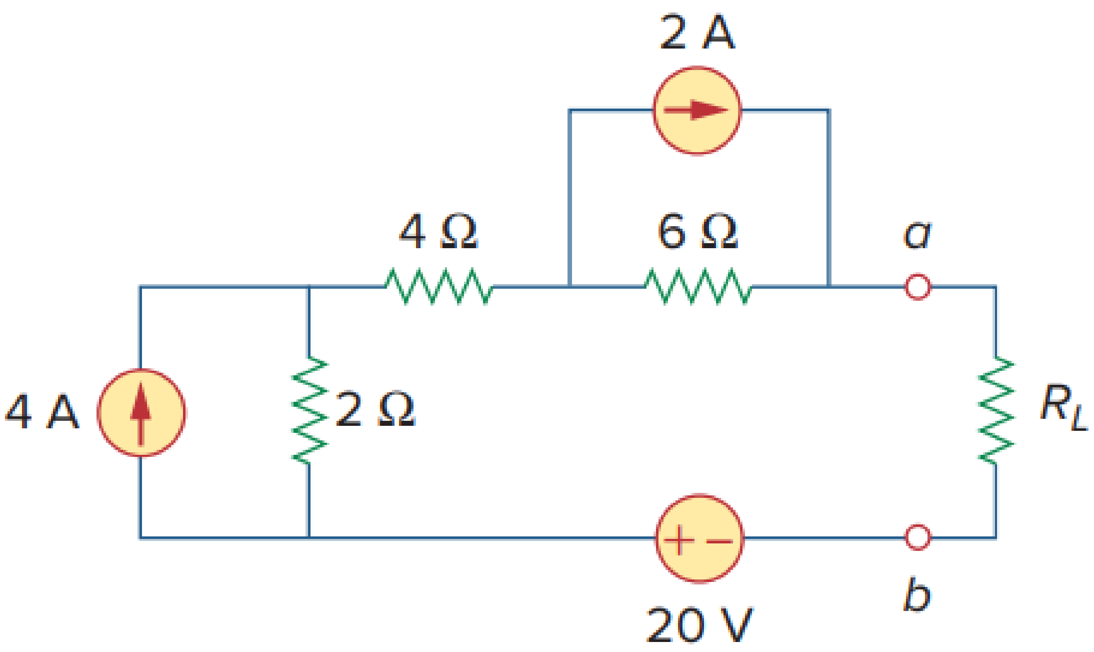
# Problem 2

a) For the circuit in the figure below, obtain the Thevenin equivalent at terminals $a - b$.

b) Calculate the current if $R_L = 8\ \Omega$.

c) Find $R_L$ for maximum power deliverable to $R_L$.

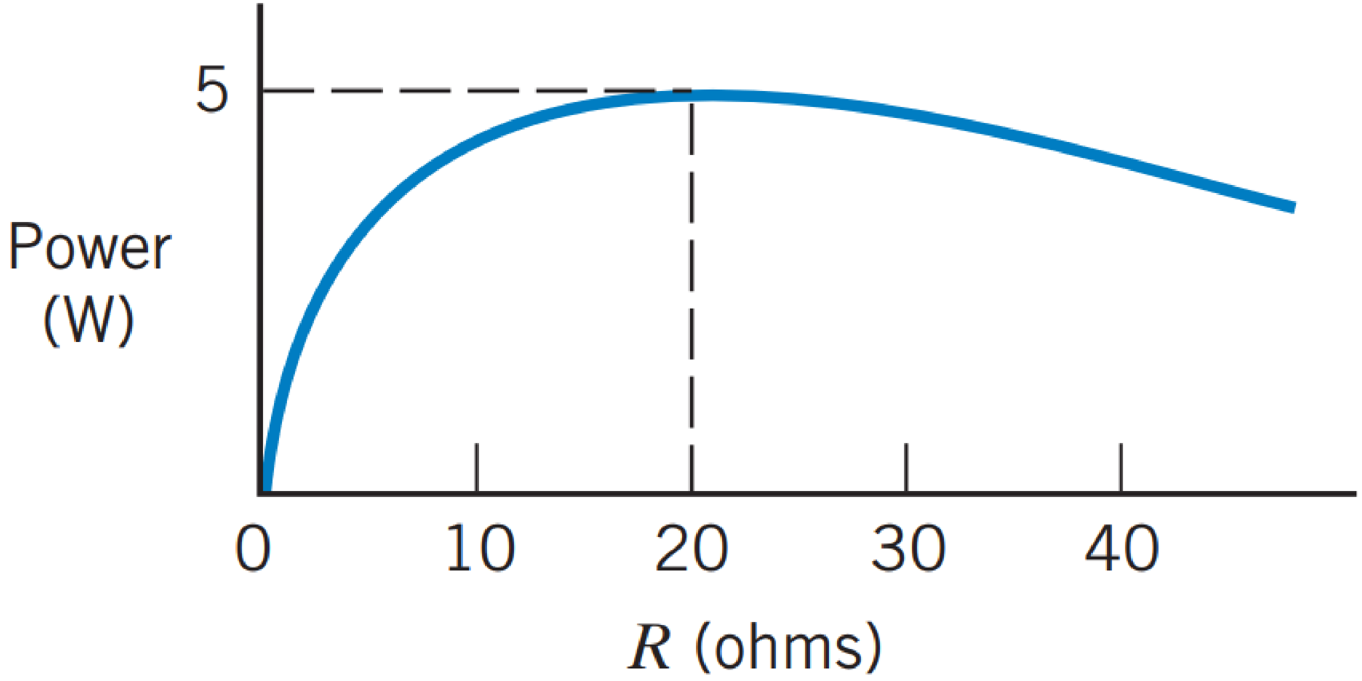
d) Determine that maximum power.




Ans: $\boldsymbol{V_{Th} = 40\ V;\ R_{Th} = 12\ \Omega;\ p_{max} = 33.33\ W}$

# Problem 3

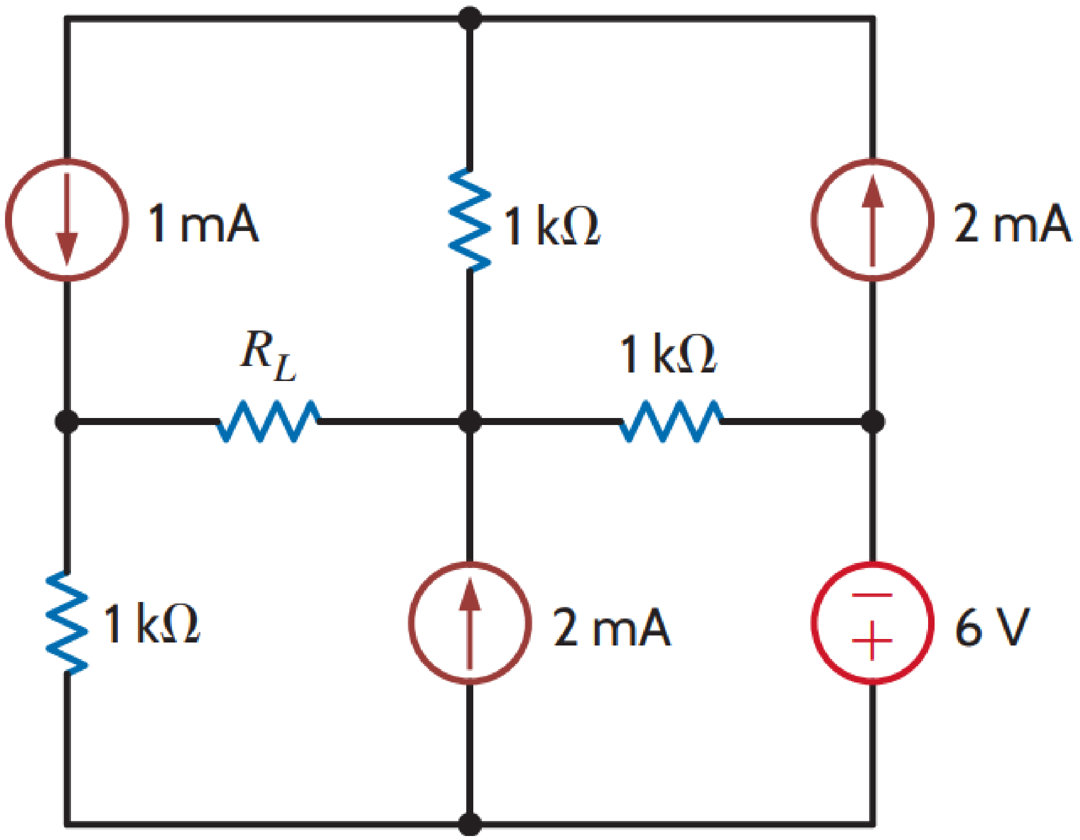
- A linear circuit was connected to a variable resistor $R$, and the power delivered to the resistor was measured as shown in the plot below. Determine the Thevenin equivalent circuit




Ans: $V_{Th} = 20\ V$; $R_{Th} = 20\ \Omega$

# Problem 4

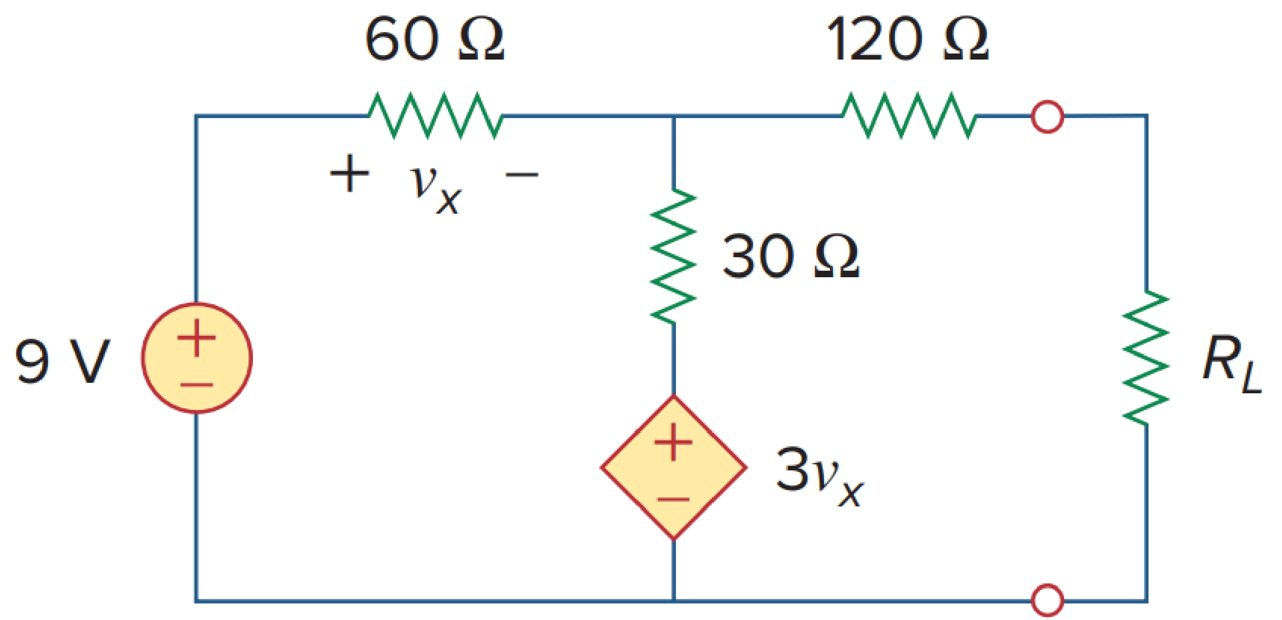
- Find the maximum power that can be delivered to the resistor $R_L$.




Ans: $V_{Th} = \pm 4\ V; I_N = \pm 2\ mA;\ R_{Th} = R_N = 2\ k\Omega;\ p_{max} = 2\ mW$

# Problem 5

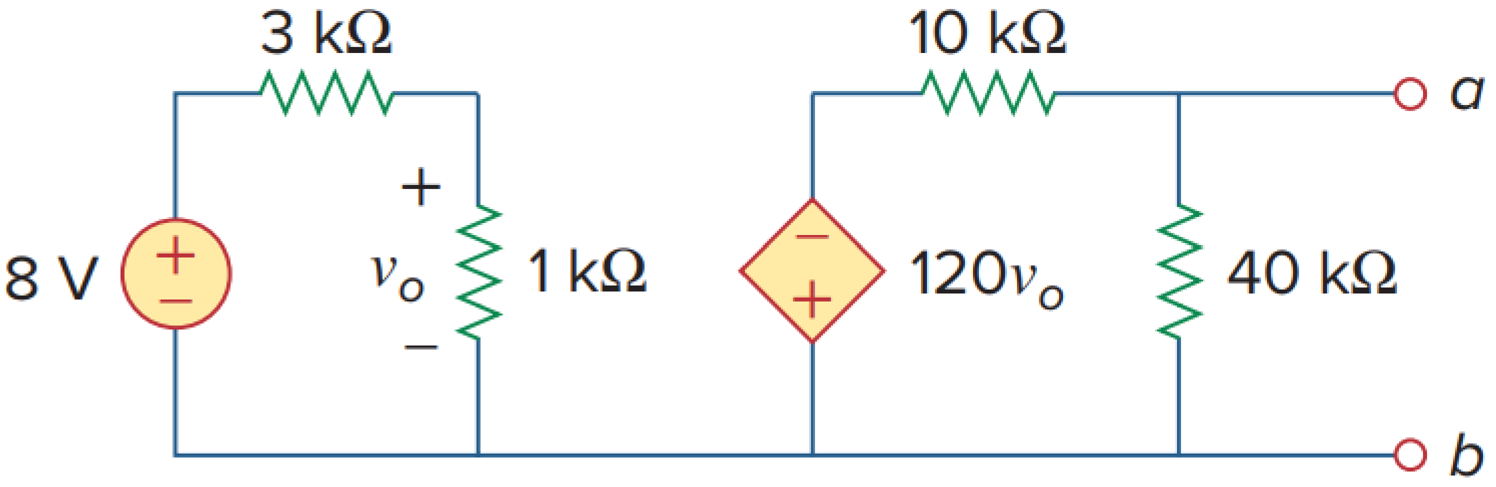
- Determine the value of $R_L$ that will draw the maximum power from the rest of the circuit. Calculate the maximum power.




Ans: $\boldsymbol{R_L} = \mathbf{126.67}\ \Omega$; $\boldsymbol{p_{max}} = \mathbf{96.71}\ \boldsymbol{mW}$

# Problem 6

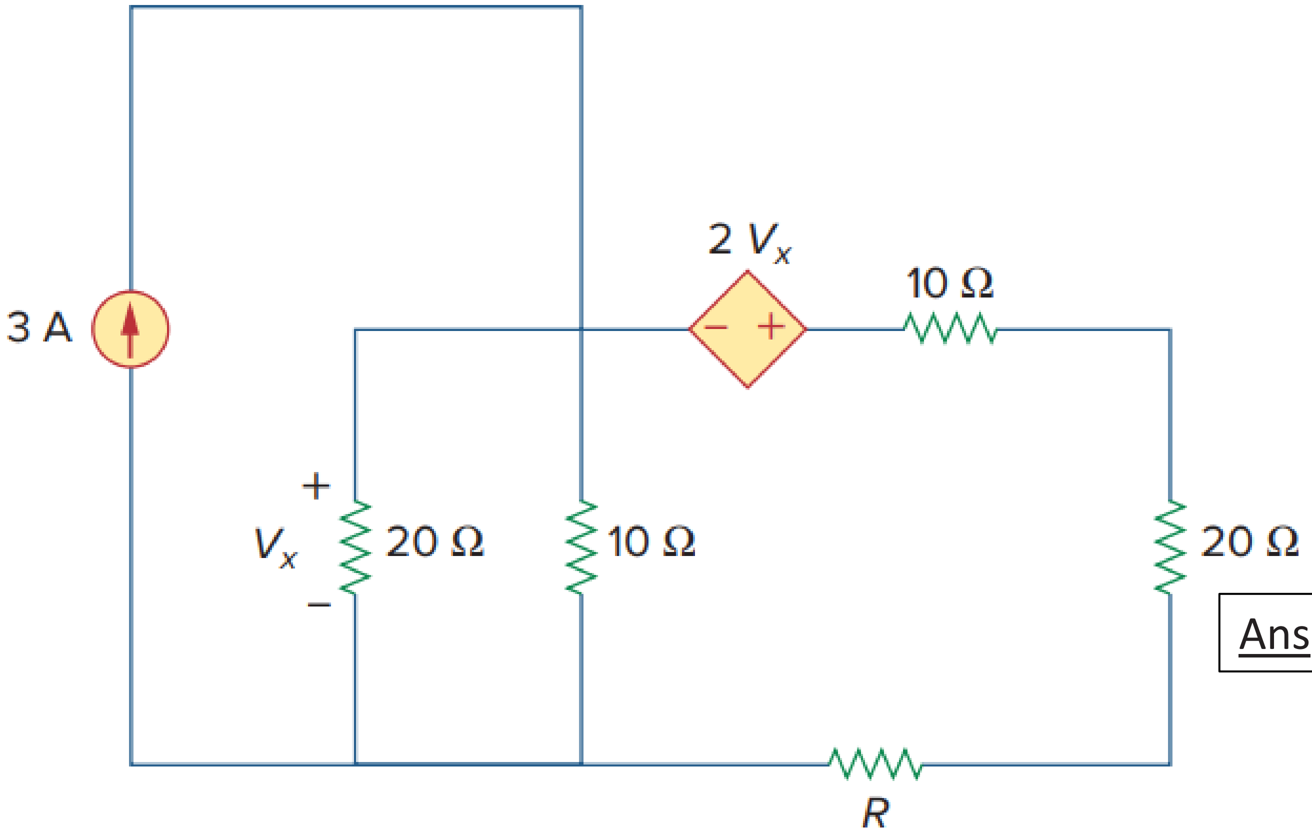
- What resistor connected across terminals will absorb maximum power from the circuit? What is that power?




<u>Ans</u>: $\boldsymbol{V_{Th} = -192\ V;\ R_{Th} = 8\ k\Omega;\ p_{max} = 1.152\ W}$

# Problem 7

- Determine the maximum power delivered to the variable resistor $R$ shown.

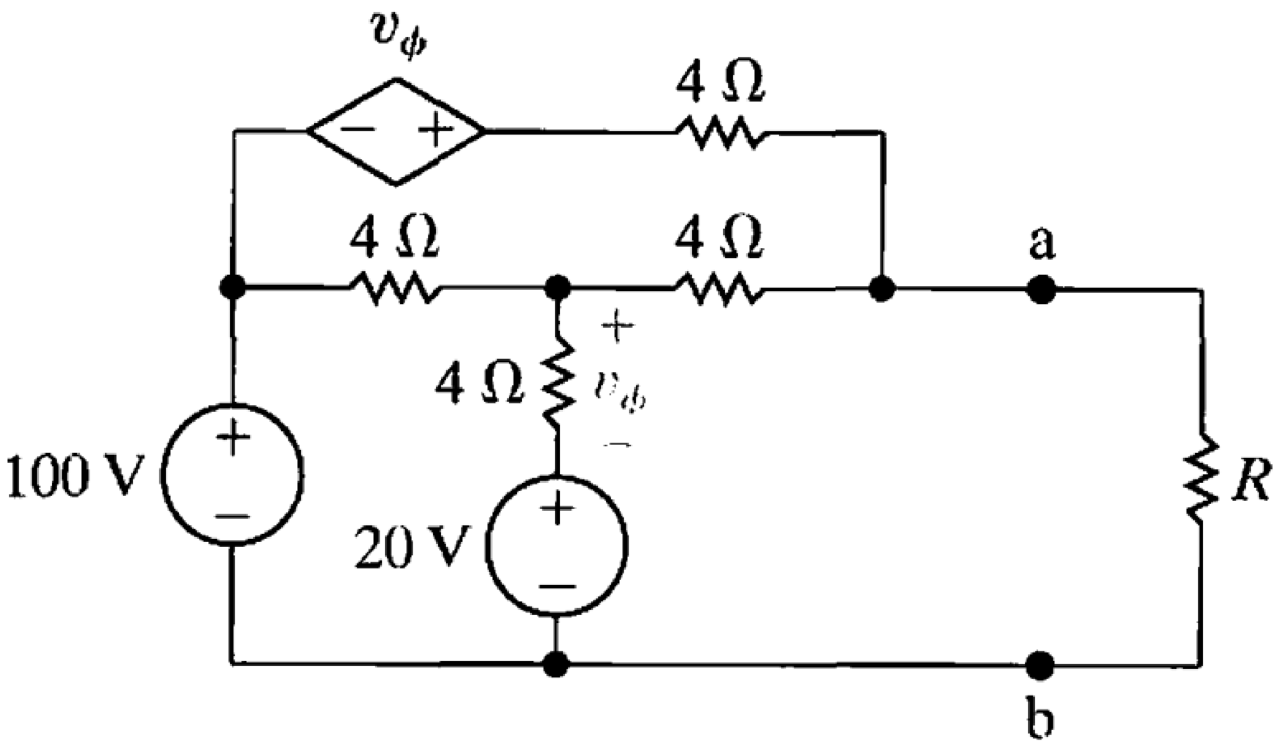



Ans: $\boldsymbol{V_{Th} = -60\ V;\ R_{Th} = 50\ \Omega; p_{max} = 18\ W}$

# Problem 8

i. Find the value of $R$ that enables the circuit shown to deliver maximum power to the terminals $a - b$.

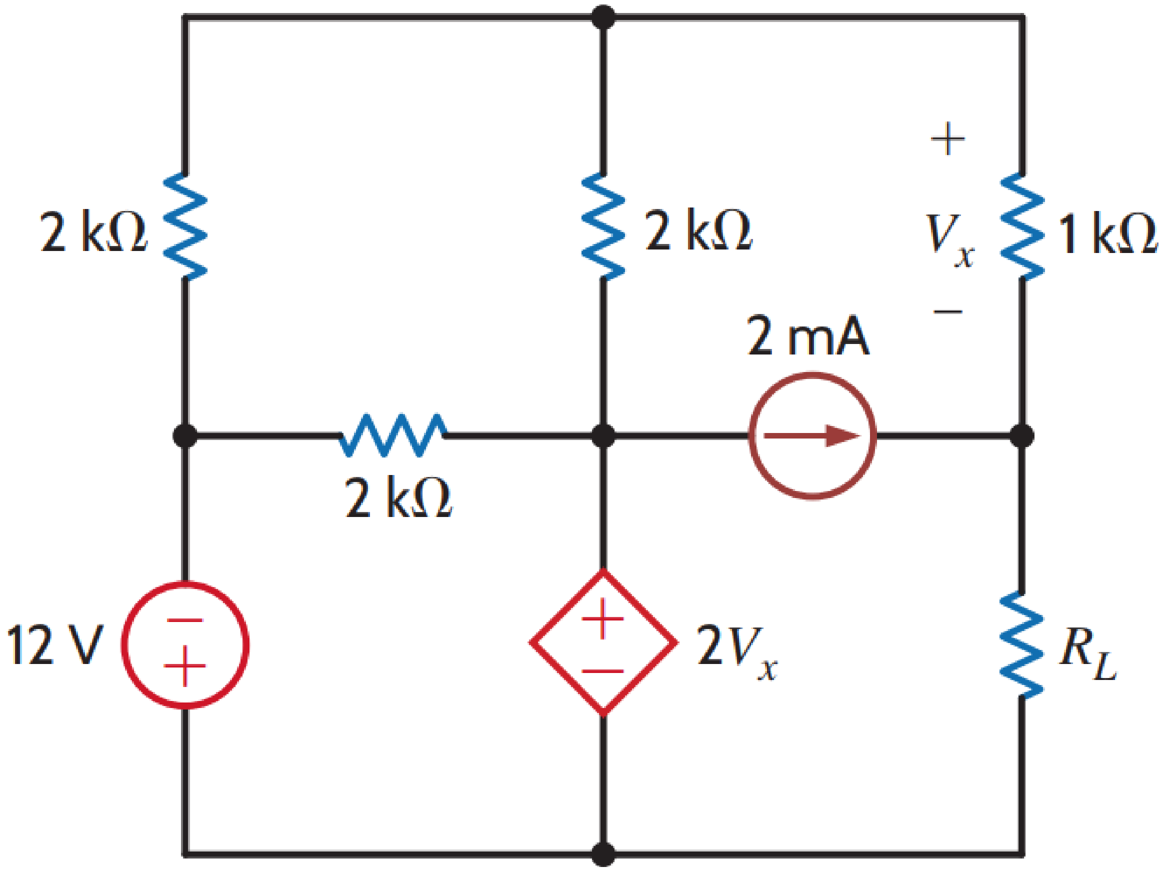
ii. Find the maximum power delivered to $R$.




Ans: $(i)\ 3\ \Omega;\ (ii)\ V_{Th} = 120\ V;\ P_{max} = 1.2\ kW$

# Problem 9

- Find the maximum power that can be delivered to the resistor $R_L$.

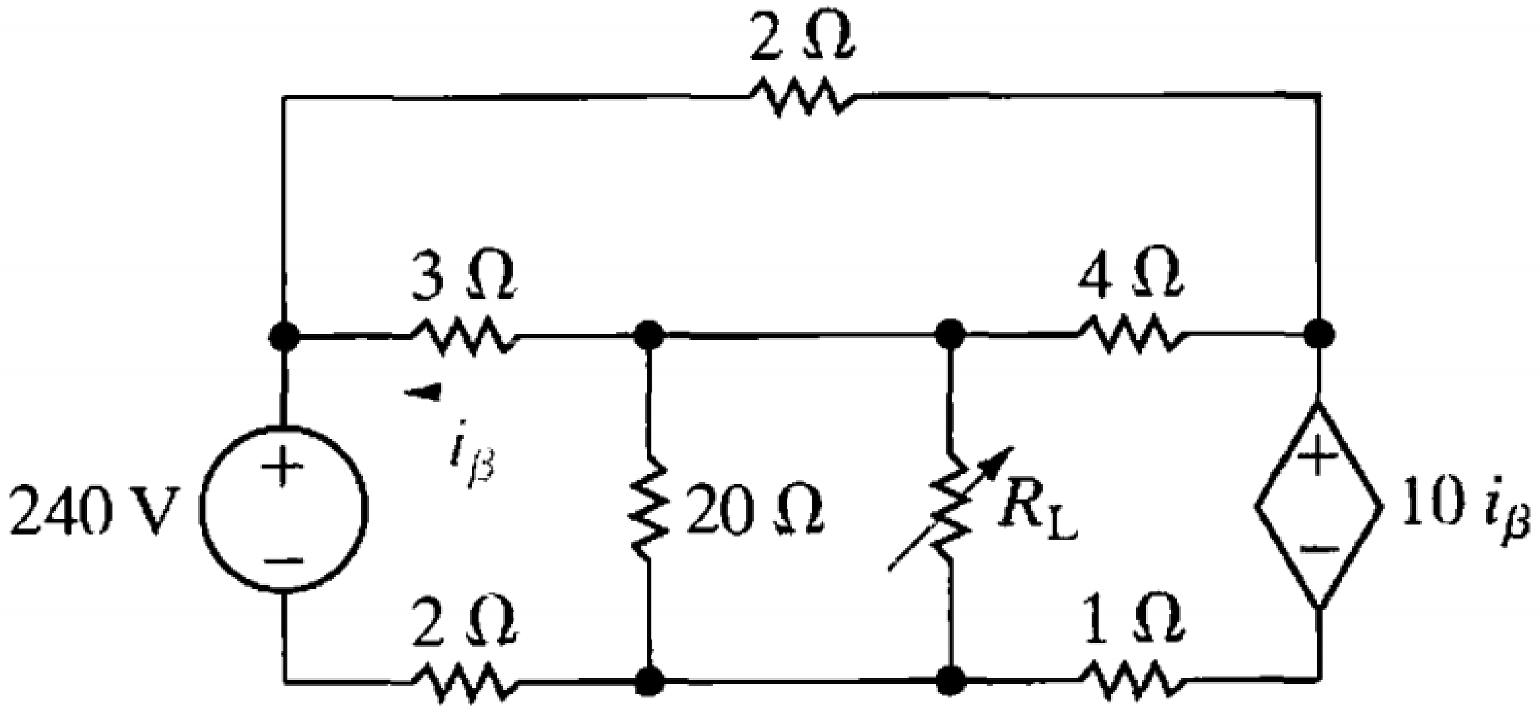



Ans: $V_{Th} = \mp 4\ V; I_N = \mp 4\ mA;\ R_{Th} = R_N = 1\ k\Omega;\ p_{max} = 4\ mW$

# Problem 10

i. Find the value of $R_L$ that enables the circuit shown to deliver maximum power to the load $(R_L)$.

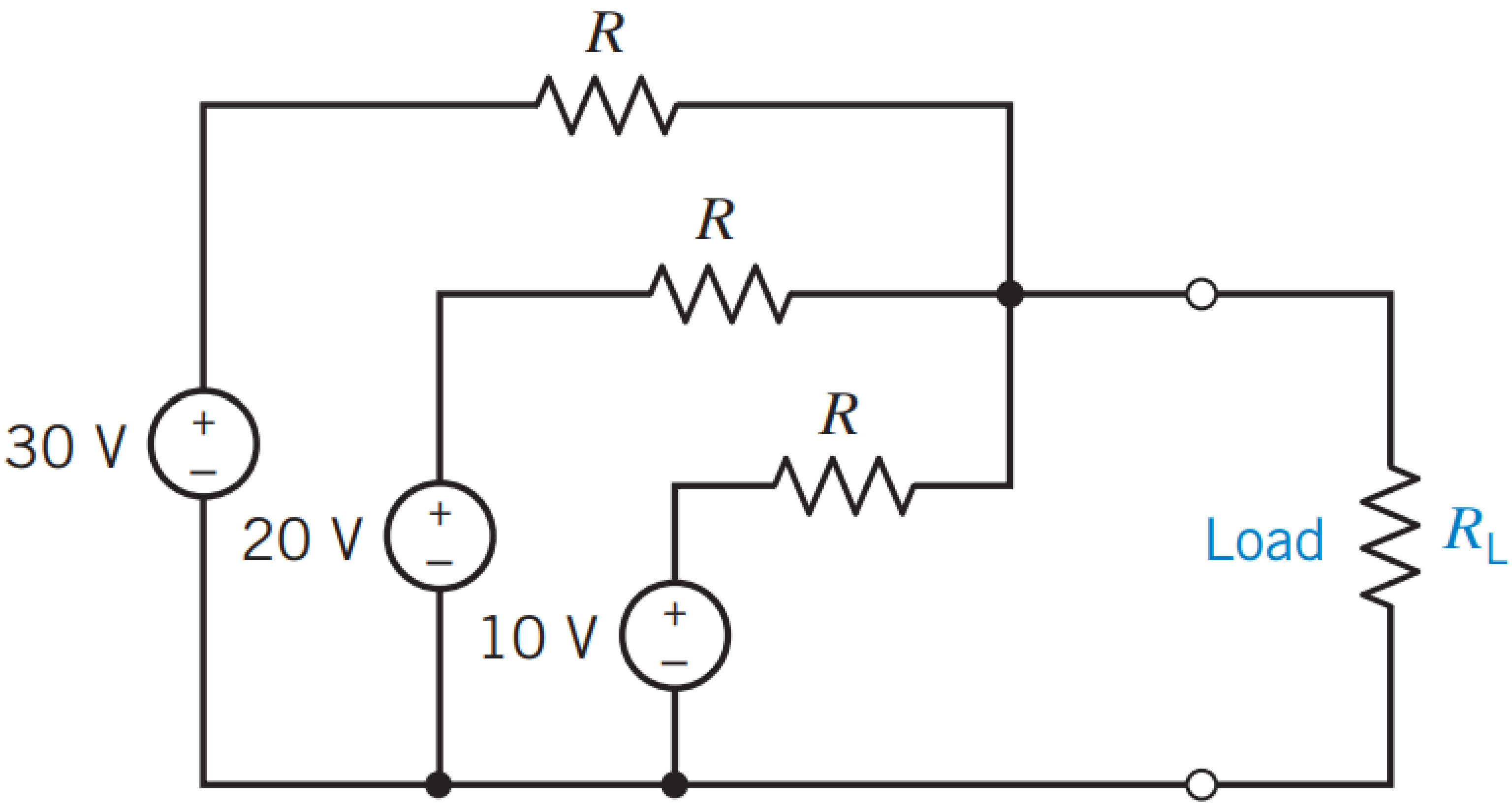
ii. Find the maximum power delivered to $R_L$.




Ans: $(i)\ \mathbf{6}\ \Omega\ (ii)\ \boldsymbol{P_{max}} = \mathbf{24\ W}$

# Problem 11

- If the following circuit delivers a maximum of $3\ W$ power to the load, what is the value of $R$?




Ans: $\mathrm{R} = \mathbf{100}\ \Omega$

# Additional Problems

- Additional recommended practice problems: here
- Other suggested problems from the textbook: here

# Acknowledgements and References

- Some of the problems, illustrations, and concepts in this lecture are taken from the following sources:


1. Sadiku, M. N. O., Fundamentals of Electric Circuits, McGraw-Hill
2. Nilsson, J. W., & Riedel, S. A., Electric Circuits, Pearson Education
3. Irwin, J. D., & Nelms, R. M., Basic Engineering Circuit Analysis, Wiley

# Thank you for your attention