Lecture 8

CSE250 – Circuits and Electronics

SUPERPOSITION PRINCIPLE

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Linearity

- *Linearity* is the property of an element describing a linear relationship between cause and effect, that is, the response in any point in a network is proportional to the stimulus which is causing it. This property is a combination of both the *homogeneity* (scaling) property and the *additivity* property.

- Homogeneity property
A system is homogeneous if scaling the input by a constant factor scales the output by the same factor. For a resistor, for example, Ohm's law relates the input i to the output v as $v = iR$. If the current is increased by a constant k, then the voltage increases correspondingly by k; that is, $kv = (ki)R$.

- Additivity or Superposition property
A system is additive if the response to the sum of inputs equals the sum of the responses to each input applied separately. For example, if applying v_1 & v_2 separately to a resistor gives rise to currents i_1 & i_2 respectively, then applying $(v_1 + v_2)$ should give rise to the current $(i_1 + i_2)$.
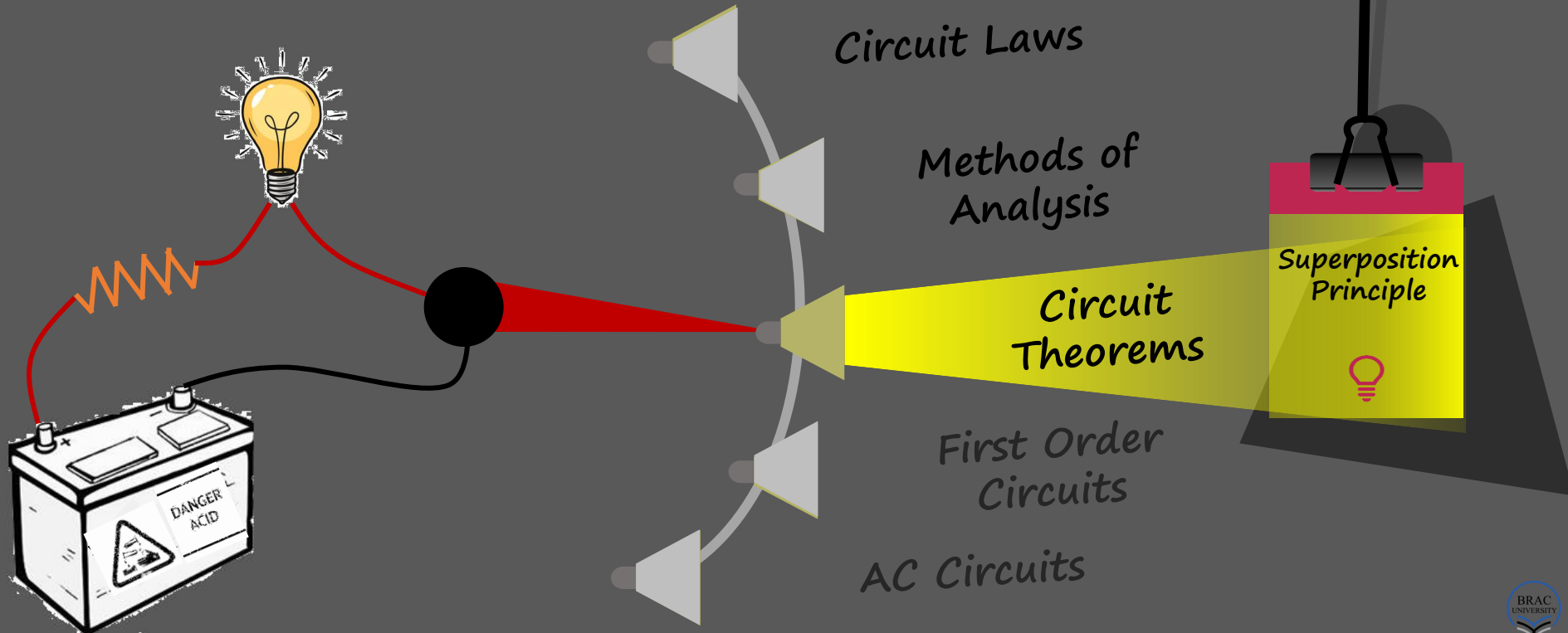
Superposition in Linear Circuits

- For a linear function,

$$f(kx) = kf(x),$$

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

- A *linear circuit* is one whose input and output are related by a straight line. A linear circuit obeys the Superposition or Additivity property.

- Superposition is an extension of the linearity principle to the case of multiple excitation. When the superposition principle applies, the output response from a number of sources acting simultaneously is simply the sum of responses which would be produced by each of the sources acting alone with all of the other sources dead.

- The superposition theorem includes the linearity theorem as a special case since increasing the value of a stimulus is equivalent to adding a second stimulus to it.

Course Outline: broad themes

Circuit Laws

Methods of Analysis

Circuit Theorems

Superposition Principle

First Order Circuits

AC Circuits

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Superposition Principle

- The *superposition principle* states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

- Keep in mind that superposition is based on linearity. For this reason, it is not applicable to the effect on power due to each source.

$$P_{Total}^2 \neq P_1^2 + P_2^2 + \ldots + P_N^2$$

- If the power value is needed, the current through (or voltage across) the element must be calculated first using superposition.

Steps to Apply Superposition Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques covered in Chapters 2 and 3.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.
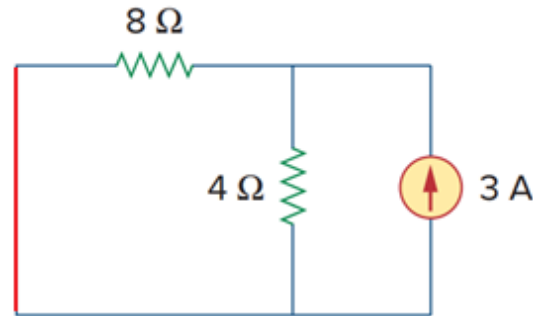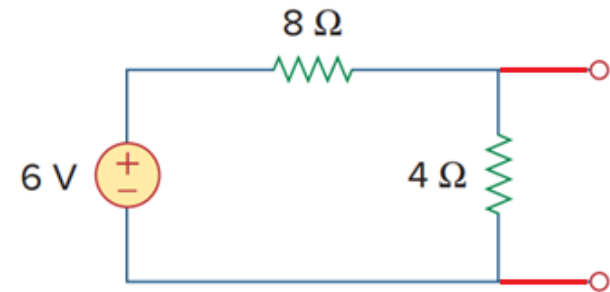
Killing independent sources

- In superposition principle, we consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by $0\,V$ (or a short circuit), and every current source by $0\,A$ (or an open circuit).

- Dependent sources are left intact because they are controlled by circuit variables.

Original circuit
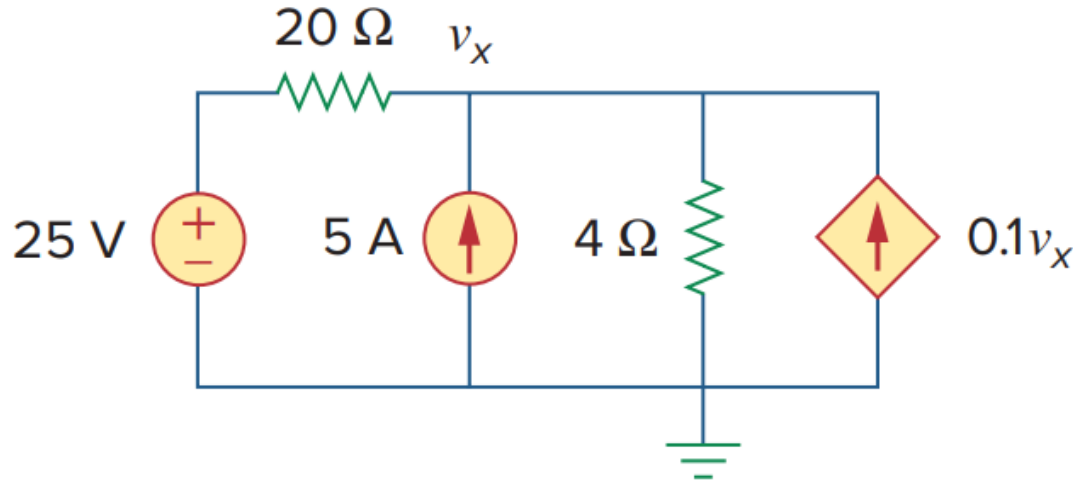
The voltage source has been replaced with a short circuit

The current source has been replaced with an open circuit
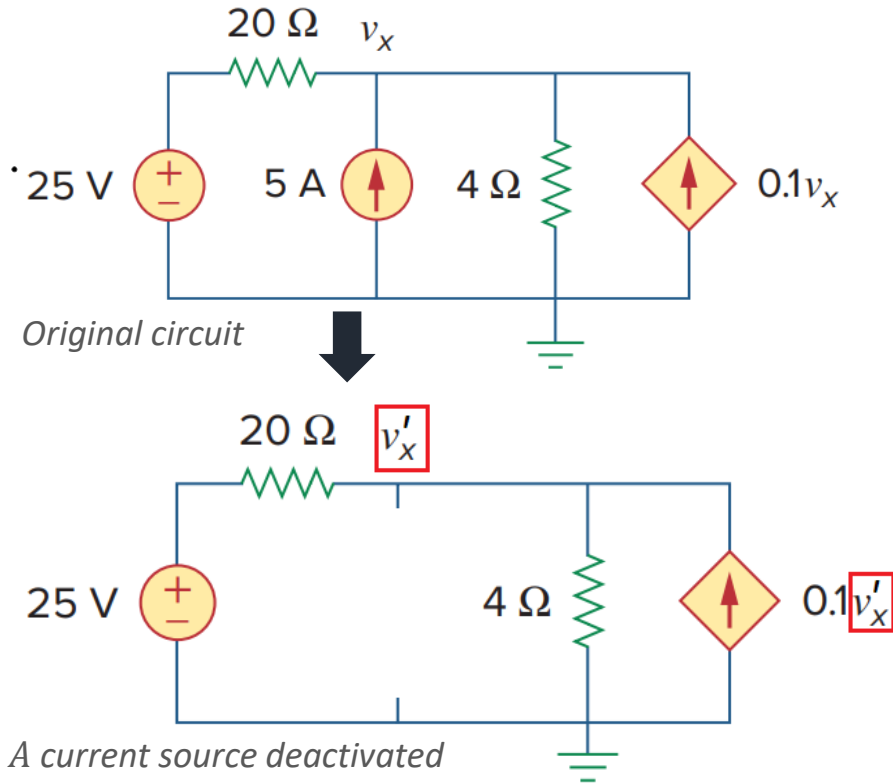
Example 1

- Use Superposition Principle to find v_x.

There are two independent and one dependent sources. The principle requires us to determine the individual contributions of the two independent sources to the node voltage v_x. If v'_x and v''_x are the contributions from the $25\,V$ voltage source and $5\,A$ current source respectively, then

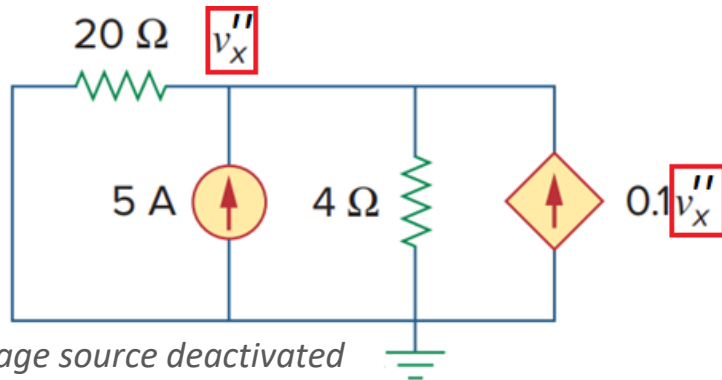$$v_x = v'_x + v''_x$$

Example 1: 25 V source is active

20 Ω v_x

25 V 5 A 4 Ω $0.1v_x$

Original circuit

20 Ω v_x'

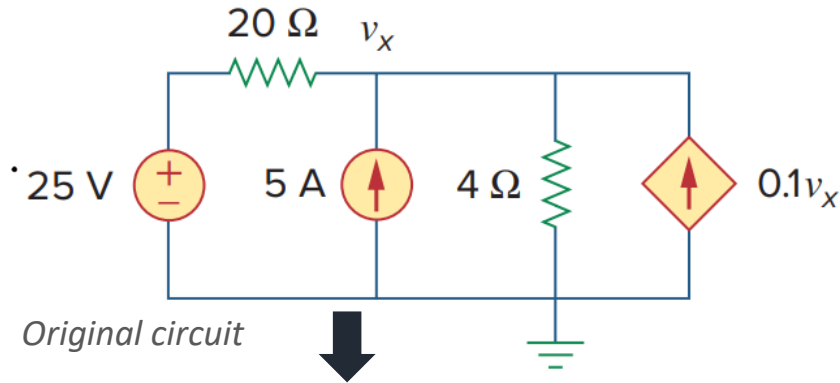25 V 4 Ω $0.1v_x'$

5 A current source deactivated

- The $5\,A$ current source has been replaced by an open circuit. The notation v_x is replaced by v_x'.

- Different circuit solving techniques (nodal analysis or mesh analysis or source transformation or voltage division) can be applied to solve for v_x'. Nodal analysis may be the easiest one.

- KCL at the node v_x',

$$\frac{v_x' - 25}{20} + \frac{v_x'}{4} = 0.1v_x'$$

Simplification yields, $v_x' = 6.25\,V$

Example 1: 5 A source is active

Original circuit

25 V voltage source deactivated

➢ The $25\,V$ voltage source has been replaced by a short circuit. The notation v_x is replaced by v_x''.

➢ KCL at the node v_x'',

$$\frac{v_x''}{20} + \frac{v_x''}{4} = 5 + 0.1v_x''$$

Simplification yields, $v_x'' = 25\,V$
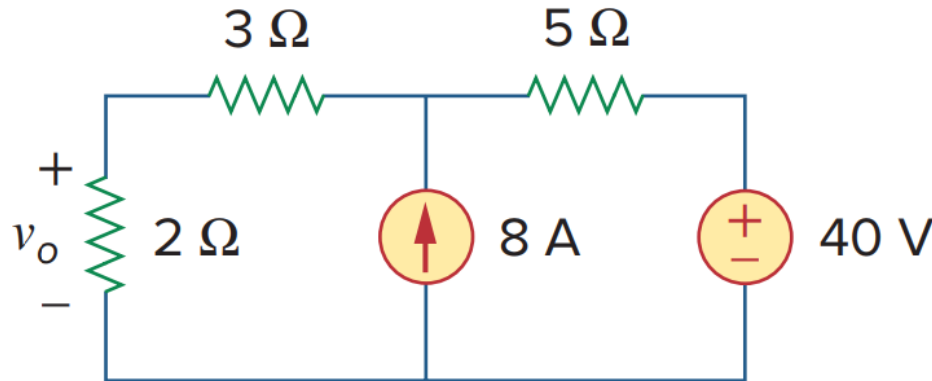
So, according to the Superposition Principle,

$$v_x = v_x' + v_x''$$

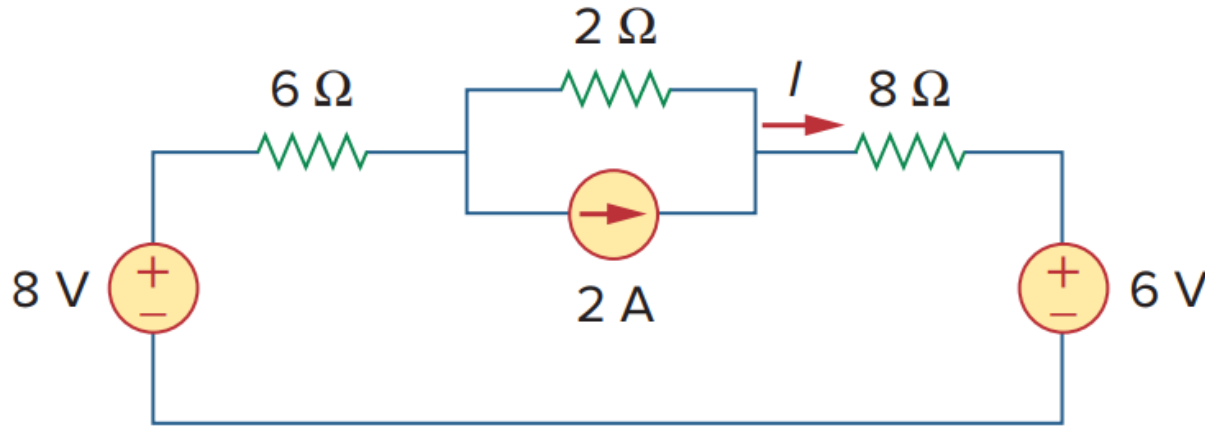$$\Rightarrow \quad v_x = 6.25 + 25 = 31.25\,V$$

Problem 1

- Using the Superposition Theorem, find v_o.

Ans: $v_0 = 16\,V$

Problem 2

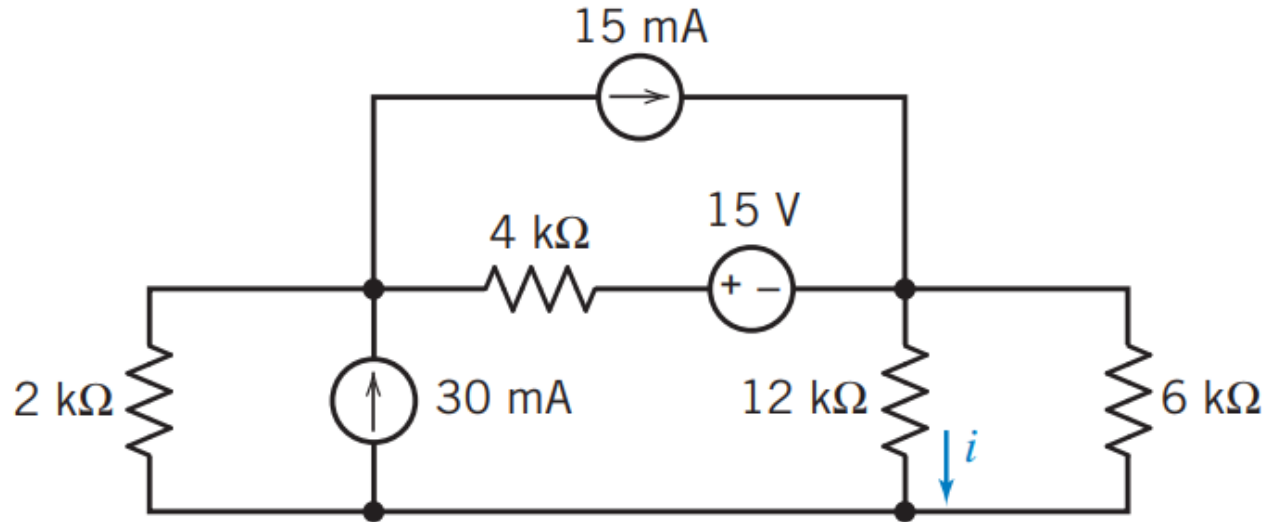- Find I in the circuit below using the Superposition Principle.

Ans: $i_0 = 0.375\,A$

Problem 3
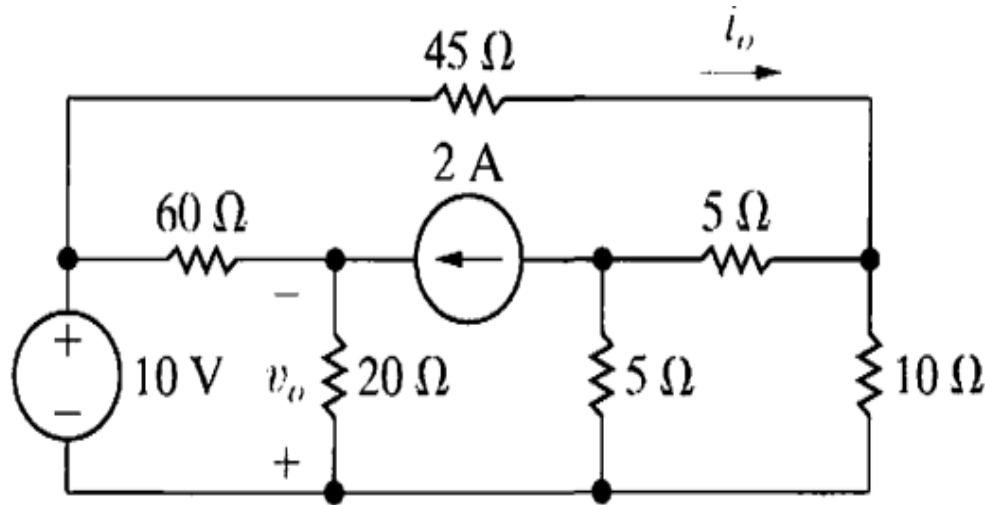
- Use Superposition Principle to find i in the circuit shown below.

Ans: $i = 2 + 2 - 0.5 = 3.5\ mA$

Problem 4
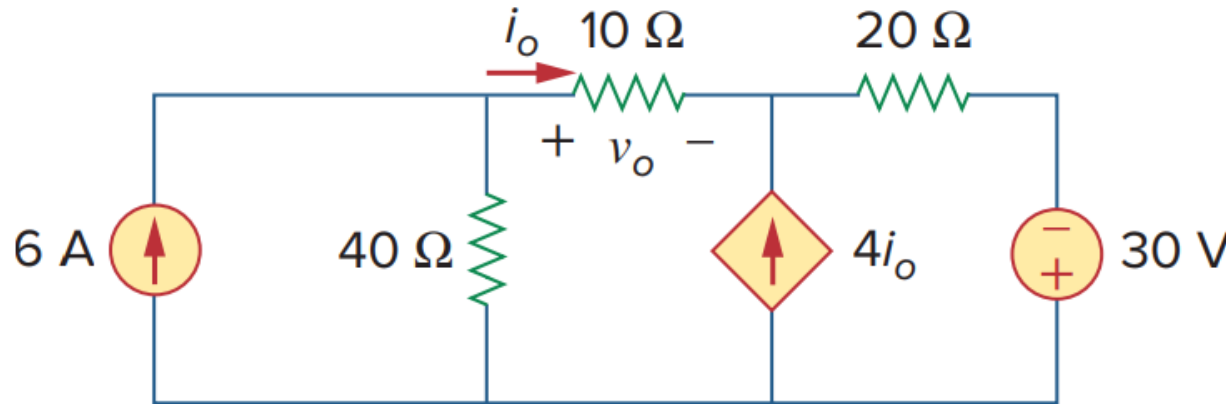
- Use Superposition Principle to solve for i_0 and v_0.

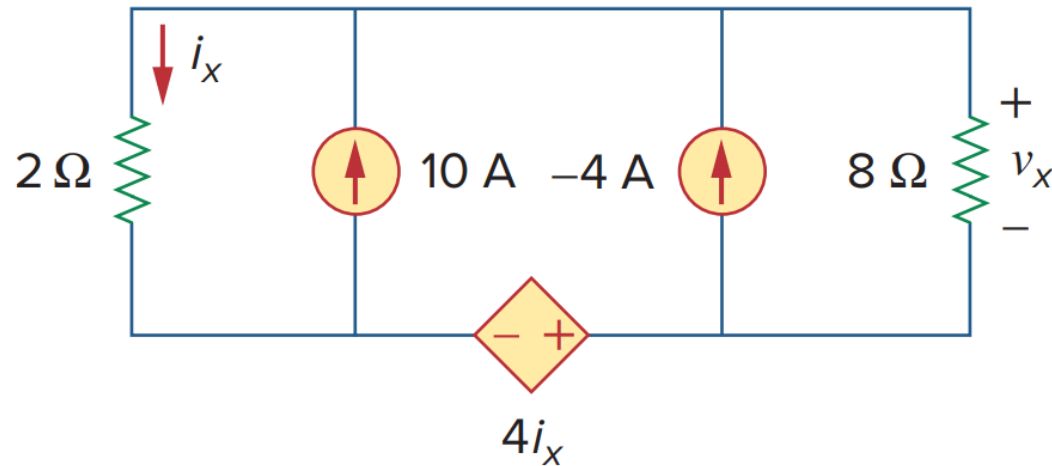Ans: $i_0 = 0.2 + 0.1 = 0.3\ A$; $v_0 = -2.5 - 30\ V = -32.5\ V$

Problem 5

- Use the Superposition Principle to find i_o and v_o.

Ans: $i_0 = 1.8\ A$; $v_0 = 18\ V$

Problem 6

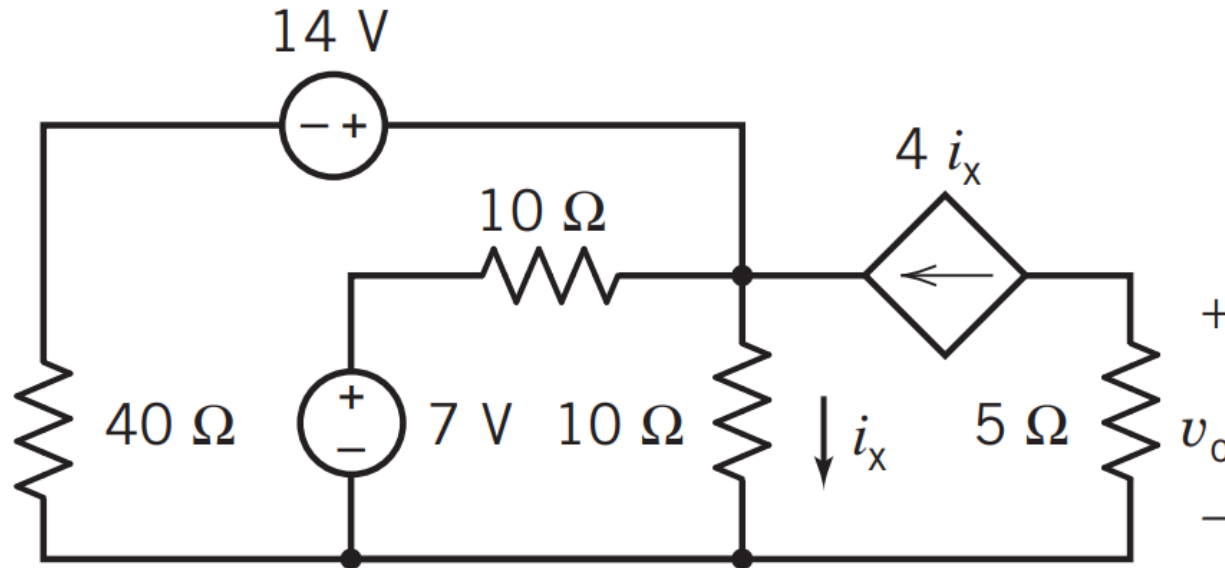- Use Superposition Principle to solve for v_x.

Ans: $\boldsymbol{v_x = -16\,V}$
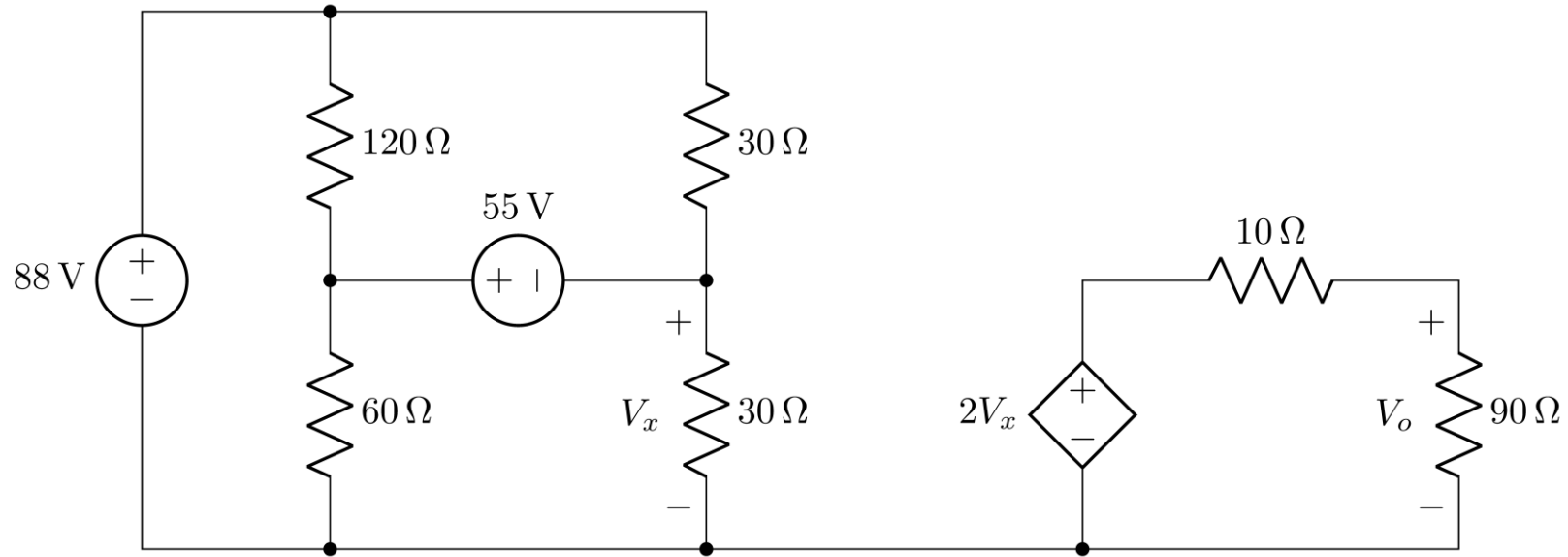
Problem 7

- Use Superposition Principle to solve for v_o.

$$\text{Ans: } v_o = 8 + 4 = 12\ V$$

Problem 8

- Use Superposition Principle to solve for V_o.
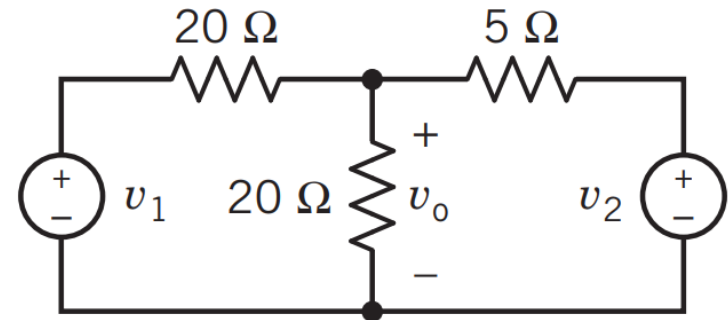

Ans: $V_o = -27V + 72V = 45\ V$

Problem 9

The inputs to the circuit shown below are the voltage source voltages v_1 and v_2. The output of the circuit is the voltage v_0. The output is related to the inputs by

$$v_0 = av_1 + bv_2$$

where a and b are the constants.

Determine the values of a and b.
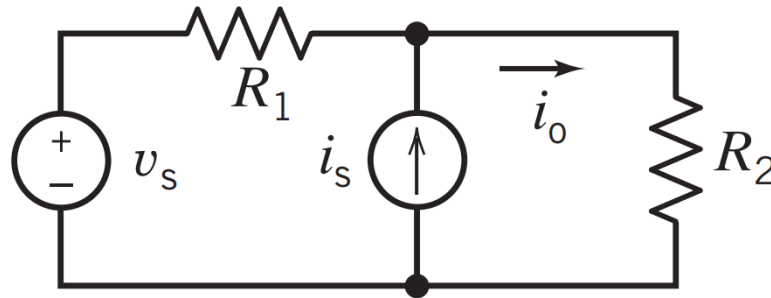

Ans: $a = {}^1/_6$, $b = {}^2/_3$

Problem 10

The inputs v_S and i_S to the following circuit and the output i_0 are related by
$$i_0 = ai_S + bv_S$$
Given the following two facts:
i. $i_0 = 0.45\,A$ when $i_S = 0.25\,A$ and $v_S = 15\,V$
ii. $i_0 = 0.3\,A$ when $i_S = 0.5\,A$ and $v_S = 0\,V$,

Determine the values of the constants a and b and the values of the resistances R_1 and R_2.

Ans: $a = 0.6\ ^A/_A$, $b = 0.02\ ^A/_V$,
$R_1 = 30\,\Omega, R_2 = 20\,\Omega$

Additional Problems

- Additional recommended practice problems: here

- Other suggested problems from the textbook: here

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Acknowledgements and References

Some of the problems, illustrations, and concepts in this lecture are taken from the following sources:

1. Sadiku, M. N. O., Fundamentals of Electric Circuits, McGraw-Hill

2. Nilsson, J. W., & Riedel, S. A., Electric Circuits, Pearson Education

3. Dorf, R. C., & Svoboda, J. A., Introduction to Electric Circuits, Wiley

Thank you for your attention