# Major Cooperative Transitions and Management

## Theory in the Game of Life

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### **Abstract**

Biological and cultural evolution show a trend towards increasing hierarchical organization, in which entities at one level combine cooperatively to form a new entity at a higher level of organization. In each case where such a cooperative transition has been studied, we have some understanding of how the transition came about, but it is difficult to formulate a unified theory that covers all of these transitions. John Stewart has proposed a theoretical framework called *Management Theory*, which attempts to explain all of the major cooperative transitions in biological and cultural evolution. The idea is that successful transitions require the integration of managers and workers into a cooperative organization. This theory seems appropriate when we consider the cultural evolution of corporations, where managers and workers are clearly essential, but it seems less plausible when we consider the biological evolution of entities that do not invite anthropomorphic projection. However, in the following article, we define managers and workers in an abstract way that enables us to apply these terms over a broad range of cases, including cultural evolution, biological evolution, and computational simulations of evolution. The core idea is that a worker is an entity that takes the main role in the production of something and a manager is an entity that plays a supporting role in the production of something. We apply this abstract view of managers and workers to a computational simulation of evolving cooperative transitions in John Conway's Game of Life. The simulation confirms the expectations of Management Theory: Manager-worker relations result in robust and productive cooperation, whereas workers without managers tend to lack robustness, and managers without workers tend to lack productivity.

**Keywords:** Major cooperative transitions, management theory, symbiosis, evolution, cellular automata, hierarchical organization.

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### 1 Introduction

In past work, we presented *Model-S* (Turney, 2020), a computational simulation of the evolution of symbiosis in John Conway's cellular automaton, the *Game of Life* (Gardner, 1970). Model-S was created to provide some insight into the major cooperative transitions in biological and cultural evolution (Maynard Smith & Szathmáry, 1995; Nolan & Lenski, 2010; Turney, 2018). John Stewart's work on *Management Theory* (Stewart, 2020) was also intended to provide insight into the major cooperative transitions in evolution. The current article attempts to apply Stewart's Management Theory to the symbiotic entities that evolve in Model-S.

Management Theory aspires to account for all of the major cooperative transitions in biological and cultural evolution, yet the examples and arguments for the theory mostly depend on cultural evolution, especially human cultural evolution (Stewart, 1995; 1997; 2014; 2020). On the other hand, Model-S is a highly abstract and simplified mathematical model. As such, it seems most suited to the earliest instances of biological evolution. This suggests that applying Stewart's Management Theory to entities in Model-S is a challenging test for both Management Theory and Model-S.

The first step in applying Management Theory to Model-S is to define *managers* and *workers* in a way that is precise yet general enough to apply to biology, culture, and cellular automata. Our proposal is that a *worker* is an entity that takes the main role in the production of something and a *manager* is an entity that plays a supporting role in the production something. In general, an entity may sometimes act as a worker and other times act as a manager. We will classify entities at a specific time as workers or managers based on whether they mostly act as managers or mostly act as workers at that point in time.

In this article, we will focus on the simplest hierarchical structure, in which there are only two entities. We examine the case of two entities that are joined to form a *symbiotic* entity. In general, *symbiosis* can mean a relation in which both parts benefit (*mutualism*) or a relation in which it is not specified whether any of the parts benefit (Douglas, 2010). We will use the term *mutualism* when both parts benefit and the term *symbiosis* to cover all cases, with and without benefits.

Each part of the symbiotic entity may be either a worker or a manager. Given two entities, there are three possibilities: (1) The symbiotic entity may consist of two workers. We find that two workers can be highly productive, but their relation tends to decay over time and they then switch to a manager-worker relation or (less often) a manager-manager relation. In other words, two workers are productive but lack robustness. (2) The symbiotic entity may consist of two managers. Two managers tend to have a stable, robust relation, but they lack productivity. (3) The symbiotic entity may consist of one manager and one worker. One manager and one worker tend to be less productive than two workers, but their relation is more

stable. Their robustness compensates for their slightly lower productivity than two workers. Over time, a large majority of successful relations are manager-worker relations.

The experiments we present here provide a simple, abstract, mechanistic account of how manager-worker relations come to dominate the successful symbiotic unions, including the major cooperative transitions in biological and cultural evolution. The results lend support to Stewart's Management Theory. Stewart's arguments are largely based on human societies, human governance mechanisms, and the structure of human corporations, whereas the results presented in this article rely on a mathematical, computational model. It is encouraging that two, quite different approaches to understanding major cooperative transitions agree on the importance of manager-worker relations.

In Section 2, we briefly discuss past work on symbiosis and major cooperative transitions. Section 3 introduces the background that is necessary to understand the experiments. Section 3.1 summarizes the design of Model-S and Section 3.2 explains how we apply Management Theory to the symbiotic entities that evolve in Model-S. Section 4 presents the experiments, the core of the paper. Section 4.1 examines the robustness and productivity of the three kinds of symbiotic unions (worker-worker, manager-manager, and manager-worker). Section 4.2 presents two different definitions of a *successful* symbiotic union and then explores how success is related to the three kinds of symbiotic unions. Section 5 discusses future work and limitations of the current work. We conclude in Section 6.

### 2 Related Work

Douglas (2010) provides an excellent survey of research on symbiosis. Martin and Schwab (2013) give a detailed explanation of the various terms that are used to classify the many different types of symbioses.

The major cooperative transitions in biological and cultural evolution may be seen as cases of particularly successful symbioses (mutualisms). Attempts to explain the processes that drive the major cooperative transitions include the work of Buss (1987), Maynard Smith (1988), Maynard Smith and Szathmáry (1995), Stewart (1995; 2014; 2020), Michod (1999), Wilson and Wilson (2007), Wilson (2015), West et al. (2015), Szathmáry (2015), and Wilson (2019).

Model-S is a computational model of symbiosis with shifting levels of selection (Turney, 2020). In Model-S, when two entities enter a symbiotic relation, they are fused together, hence they live or die as a unit. This kind of fusion, in which selection shifts from the two components to the fused whole, is called *endosymbiosis*. The paradigmatic example of endosymbiosis is the major transition in which two prokaryotic cells merged to form a eukaryote (Margulis, 1970; 1981).

Model-S uses the *Immigration Game* (Wainwright, 1971), created by Don Woods, as contest for measuring the fitness of evolved entities. The Immigration Game is a two-player variation of John Conway's Game of Life (Gardner, 1970). We chose the Immigration Game for our computational simulation of the evolution of symbiosis because there is a natural way to model symbiosis: two seed patterns can be merged into one symbiote by simply joining them side-by-side and treating them as a unit. We describe Model-S and the Immigration Game in detail in Section 3.

Ikegami (1994) also chose to use game theory in his pioneering computational simulation of the evolution of symbiosis. The fitness of the entities in the population was measured by playing the *Erroneous Iterated Prisoner's Dilemma* game. An entity's game strategy was represented by a tree. Two entities can be merged into one symbiote by fusing one tree onto a branch of the other tree.

### 3 Background: Model-S and Management Theory

In Section 3.1, as background information for the experiments in Section 4, we describe the Game of Life, the Immigration Game, and Model-S. We then explain our model of Stewart's (2020) Management Theory in Section 3.2.

#### 3.1 Model-S

This section is a summary of work that is described in detail in our previous article on symbiosis in the Immigration Game (Turney, 2020). Fortunately, many of the details in that work are not required in order to understand the current article.

The Game of Life (Gardner, 1970) is played on an infinite, two-dimensional grid of square cells, where each cell is either *dead* (state 0) or *alive* (state 1). Often the dead state (the background) is coloured white and the live state (the foreground) is coloured black. The state of a cell changes with time, based on the state of its eight nearest neighbours (the *Moore neighbourhood*). Time passes in discrete intervals, and the states of the cells at time t uniquely determine the states of the cells at time t 1. There is only one player in the game and the player's only actions are to choose the initial states of the cells at time t = 0 and the time limit for the game, given by a maximum value for t. The states for t > 0 are calculated by a computer. The initial states form a *seed pattern* that determines the course of the game.

The rules for updating states are compactly expressed as B3/S23: A cell is *born* (it switches from state 0 to state 1) if it has exactly three living neighbours (B3). A cell *survives* (it remains in state 1) if it has two or three living neighbours (S23). Otherwise, the cell *dies* (it switches from 1 to 0) or remains *unborn* (it remains in state 0).

The Immigration Game (Wainwright, 1971) is almost the same as the Game of Life, except there are two different *live* states (states 1 and 2, usually represented by red and blue colours). The rules for updating remain B3/S23, but there are two new rules for determining colour: (1) Live cells do not change colour unless they die (they become white). (2) When a new cell is born, it takes the colour of the majority of its neighbours. Since birth requires three live neighbours, there is always a clear majority. The initial states at time t = 0 are chosen by the two players of the game; one player makes a red seed pattern and the other player makes a blue seed pattern. The players agree on a time limit, given by a maximum value for t.

In the Immigration Game, if states 1 and 2 were coloured black, instead of red and blue, the game would appear to be exactly the same as the Game of Life. The purpose of the two colours is to score the two players, to convert the Game of Life from a solitaire game into a two-player competitive game. In Model-S, we score each player by the *growth* of their initial seed, defined as the final number of cells of their colour minus the initial number of cells of their colour. Growth will be negative if the final number is less than the initial number. The motivation for this method of scoring is to avoid biasing the game in favour of the seed that has the most living cells at the beginning of the game. The winner is the player whose colour has grown the most over the course of the game.

Model-S (Turney, 2020) is a genetic algorithm for evolving seed patterns that are good at playing the Immigration Game. It is based on a GENITOR-style genetic algorithm (Whitley, 1989), with one-at-a-time reproduction, a constant population size, and rank-based tournament selection.

Model-S uses *Golly* (Trevorrow, et al., 2021), an open source, cross-platform application for exploring cellular automata. Golly can be controlled with Python code. Model-S consists of Python routines that manage a population of entities (seed patterns). A seed pattern is stored in Model-S as a matrix. Whenever it is necessary to measure the relative fitness of two entities by running a contest in the Immigration Game, Model-S passes the seed patterns (red and blue) to Golly and receives a score from Golly when the contest is over (the score is based on the growth of red and blue). In addition to providing the initial seed patterns for time t = 0, Model-S also sets the duration of the contest by giving a maximum value for t. Larger seeds are allowed more time to develop.

Model-S is constructed with four layers, each subsequent layer building on the previous layers. The purpose of having four layers is to measure each layer's contribution to the fitness of the evolving population, by selectively enabling or disabling layers. In the current article, we use all four layers.

Layer 1 implements a simple form of asexual reproduction, with a fixed genome size (that is, a fixed matrix size). A member of the population is selected for reproduction using tournament selection (Whitley, 1989). The chosen seed pattern is mutated by randomly flipping some of the bits in the seed matrix and it

then competes in a series of one-on-one Immigration Games with the other members of the population. Its fitness is the average fraction of games it wins.

Layer 2 implements a slightly more sophisticated form of asexual reproduction. A member of the population is selected for reproduction using tournament selection. Layer 2 allows the seed matrix to grow or shrink by appending or removing a row or column to or from the seed matrix. Layer 2 then passes the seed on to Layer 1 for mutation by flipping bits.

Layer 3 selects two seeds from the population using tournament selection and then combines them with genetic crossover (sexual reproduction). Layer 3 requires the two seeds to be somewhat similar for crossover to proceed. If a suitable match is found, then Layer 3 combines the two seeds with crossover and passes the new seed on to Layer 2 for row or column adjustments. Otherwise, if no suitable match is found, Layer 3 passes only one of the two seeds on to Layer 2, without making any changes to the chosen seed.

Layer 4 adds symbiosis to Model-S. Two seeds are selected from the population and they are fused together, side-by-side, creating a new symbiotic genome. This new fused seed is treated as a whole; that is, selection shifts from the level of the two parts to the level of the whole. The parameters in Model-S are set so that fusion is rare. Most of the time, Layer 4 makes no changes and simply passes control to Layer 3. The main result of our past work (Turney, 2020) is evidence that symbiosis (Layer 4) promotes fitness improvements in the Immigration Game.

### 3.2 Management Theory

Each symbiote created by Layer 4 is composed of two parts (two seed patterns) that have been fused together to make a new whole (one new, fused seed pattern). If Model-S has been running for many generations, then the two parts may also be symbiotes, which will also be composed of two parts. To keep our analysis simple, we will not recursively consider parts within parts; we will only consider the two parts at the top level, the two parts that were joined in the most recent symbiotic fusion.

To test Management Theory, we need a way to analyze the interaction between the two top-level parts of the symbiote, in order to label the parts as workers or managers. We could colour one part red and the other part blue and then run the Immigration Game to see what happens, but this will only tell us which part grows the most, the red seed pattern or the blue seed pattern. The Immigration Game is not sufficient for distinguishing workers from managers. Our solution is to introduce a new game, the *Management Game*.

The Management Game extends the Immigration Game by adding two more *live* states, orange and green, bringing the total number of states to five. Table 1 lists the five states and introduces some terminology.

Insert Table 1 here.

Table 2 presents the rules of the Management Game. Like the Game of Life and the Immigration Game, the basic rule is still B3/S23. The only change is how colours are handled. We colour one part of the symbiote red and the other part blue and then play the Management Game. If red and blue do not interact, then we will only see changing red and blue patterns. If red and blue do interact, then we will start to see orange and green colours appearing.

Insert Table 2 here.

In Section 3.1, we noted that, if states 1 and 2 in the Immigration Game were displayed as black, then the Immigration Game would look exactly like the Game of Life. Likewise, in the Management Game, if state 3 (orange) were displayed as red and state 4 (green) were displayed as blue, then the Management Game would look exactly like the Immigration Game. We may think of orange as a variation of red and we may think of green as a variation of blue.

The four rows of Table 2 tell us four things: (1) A newly born red cell is always the work of three other red cells. (2) A newly born blue cell is always the work of three other blue cells. (3) When we see an orange cell, we know that, at some time in the past or in the current birth, a blue or green cell played a *supporting* role (one of three cells for B3) in forming the orange cell (since the initial seed contains no orange). The *main* role (two of three cells) was played by orange or red. (4) When we see a green cell, we know that, at some time in the past or in the current birth, a red or orange cell played a *supporting* role (one of three cells for B3) in forming the green cell (since the initial seed contains no green). The *main* role (two of three cells) was played by green or blue.

Table 3 defines workers and managers, based on the growth of the four *live* colours. Green growth indicates cases where red/orange was acting as a *manager* and red/orange growth indicates cases where red/orange was acting as a *worker*. Orange growth indicates cases where blue/green was acting as a *manager* and blue/green growth indicates cases where blue/green was acting as a *worker*.

Insert Table 3 here.

Figure 1 gives an example of a symbiote and its two parts, growing separately in the first two rows of the table and growing together as a symbiote in the final row of the table. The Management Game is used

for all three rows. Where red and blue come into contact with each other, we see green and orange cells (in the bottom row, on the right). Where they are isolated from each other, they maintain their original colours.

Insert Figure 1 here.

Figure 2 shows a symbiotic seed pattern that, over time, becomes a manager-manager relation. The top row shows the initial pattern and the final pattern. The bottom graph shows the growth of the four colours over the course of the run of the Management Game, from t = 0 to t = 1000. Red and blue have negative growth (they shrink). Orange and green have relatively sparse growth, which is typical for a manager-manager relation.

Insert Figure 2 here.

Figure 3 is an example of a manager-worker pair. Red/orange is the worker and blue/green is the manager. Manager-worker pairs tend to be highly productive. Note that the growth of orange in the manager-worker pair passes 500 (see Figure 3), whereas the growth of green in the manager-manager pair only reaches 100 for a short time (see Figure 2).

Insert Figure 3 here.

Figure 4 illustrates a worker-worker pair. Pairs of this type can be highly productive, but the worker-worker relation needs two workers that are well-balanced. The worker-worker relation tends to decay if the two workers are not similar in their productivity. In this example, although the initial seed patterns are not symmetrical, the final seed patterns are perfectly symmetrical, which is ideal for a worker-worker pair.

Insert Figure 4 here.

### 4 Experiments: Manager-Worker Relations

In this section, we analyse 18 runs of Model-S. Each run evolves a fixed-size population of 200 entities for 100 generations. A generation is defined as 200 births (including new symbiotes as *births*, although they are not *born* in the usual sense), so a full run yields 20,000 births of evolved entities  $(200 \times 100)$ . One run takes about two weeks with a typical desktop computer. We executed 18 runs in parallel on three computers.

Most of the births in Model-S are asexual (Layer 2) or sexual (Layer 3). Our assumption is that symbioses need a variety of non-symbiotic entities as components for creating fit symbiotic entities. Therefore we chose the parameters in Model-S so that new symbiotic entities (Layer 4) would be relatively rare, just as they are relatively rare in biological evolution. Over the 18 runs of Model-S, 844 symbiotic fusions (Layer 4) were produced, an average of 46.9 symbioses per run (844 symbioses / 18 runs = 46.9), which corresponds to 0.002345 symbioses per entity (46.9 symbioses / 20,000 evolved entities = 0.002345). The majority of the symbioses are not mutualisms: usually at least one of the parts in a fused symbiotic whole would be better off on its own. (We discuss this in Section 4.2.)

In Section 4.1, we plot the probability and productivity of manager-worker relations, based on our sample of 844 symbiotic fusions, and we discuss the implications of the plots. In Section 4.2, we examine the relation between the success (mutualism) or failure of symbiotes and the type of relationship of the parts (manager-manager, manager-worker, or worker-worker).

Software for reproducing the results presented here is freely available for downloading (Turney, 2021).

### 4.1 Probability and Productivity of Manager-Worker Relations

In Section 3.2, we saw individual examples of each of the three types of manager-worker relations (Figures 2 to 4). Now we take a more general look at the whole collection of 844 symbiotic fusions. From this collection, we can calculate the probability and productivity of each of the three types. We estimate the *probability* of a type at time t by the number of instances of the type at time t divided by 844, the size of the collection. The *productivity* of a type at time t is given by the sum of the growths of all four colours for the given type at time t divided by the number of instances of the type at time t. In all experiments, t ranges from 0 to 1000.

Figure 5 shows the probability and productivity of the three types. The graph for probability tells us that all relations begin at time t=0 as worker-worker relations. However, worker-worker relations tend to be unstable: they typically decay rapidly to manager-worker or manager-manager relations. At approximately t=300, the probability of a worker-worker relation stabilizes at about 0.1; that is, in the long term, only about 10% of relations are worker-worker relations. At roughly the same time (t=300), manager-worker relations have stabilized around 60% (p=0.6) and manager-manager relations have stabilized around 30% (p=0.3).

Insert Figure 5 here.

The graph for productivity tells a different story. At all times (t = 0 to t = 1000), worker-worker relations are the most productive relations. They stabilize with productivity (average cell growth per symbiote) ranging from about 100 to 120. Next in productivity are manager-worker relations, with productivity stabilizing around 80 to 100. Last are manager-manager relations, with productivity stabilizing at about 20 to 30.

The lesson from Figure 5 is that manager-worker relations are both *robust* (they have a high probability of persisting over time) and *productive*. They are not as productive as worker-worker relations (although they are close), but they make up for that with their stability. Manager-manager relations are more robust (higher long-term probability) than worker-worker relations, but manager-manager relations are not productive. Worker-worker relations tend to be highly productive, but they are unstable. Manager-worker relations have the best balance of productivity and stability.

Note that the curve for the productivity of worker-worker relations seems to be considerably noisier (it has more spikes) than the productivity curves for manager-worker and manager-manager relations, but we should not assume this has any special significance. The increased noise is likely due to the smaller sample size for worker-worker relations, since only about 10% of the 844 symbioses are worker-worker relations when t > 300. The curve for worker-worker relations is relatively smooth for t < 50, when the sample size is larger.

Figure 5 shows us how the probability and productivity of manager-worker relations vary over the course of a run of the Management Game, from t = 0 to t = 1000, analogous to the lifetime of a biological creature. In contrast to *lifetime* change, Table 4 shows us how the probability and productivity of manager-worker relations vary over *evolutionary* time, as Model-S runs, evolving a sequence of 20,000 seed patterns.

Insert Table 4 here.

The four groups in Table 4 represent successive eras of evolution in Model-S. The bottom line of the table tells us that the expected productivity increases with each subsequent era. As we saw with Figure 5, worker-worker relations can be highly productive, but they have lower probabilities than manager-worker relations. The expected productivity (average productivity multiplied by probability) is highest for manager-worker relations (the third row from the bottom in Table 4).

Interestingly, the expected productivity for worker-worker relations grows steadily across the four groups (see the second row from the bottom in Table 4), although it is still considerably below the expected

productivity of manager-worker relations (65.22 for manager-workers versus 31.23 for worker-workers). This suggests that, as evolution proceeds, workers are gradually getting better at working without managers.

#### 4.2 Successful Manager-Worker Relations

We mentioned at the start of Section 4 that the majority of the 844 symbioses are not mutualisms. In mutualist symbioses, both partners benefit from being together. In this section, we examine how the three types (worker-worker, manager-manager, and manager-worker) are related to mutualisms.

In biology, it is often difficult to determine whether a symbiosis is mutualist. The answer often varies over the lifetime of a symbiote, depending on its environment (Douglas, 2010). When food is plentiful and conditions are favourable, a relation may be mutualist, but it could cease to be mutualist if food becomes scarce and conditions become unfavourable.

In past work with Model-S, we determined whether a symbiote was mutualist by measuring the fitness of the symbiote as a whole and comparing it with the fitness of each of the separated parts (Turney, 2020). Fitness was measured by a series of one-on-one competitions in the Immigration Game. Fitness in Model-S is a noisy signal, involving pseudo-random sampling, which is not a problem in the context of running an evolutionary algorithm, since they are designed to work well with noise, but we decided to look for a cleaner signal for the analysis of manager-worker relations.

In the current article, we apply two different criteria for *success* (mutualism) to the 844 symbioses:

**Sum of parts success:** A symbiote is *successful* if the growth of the whole symbiote is greater than the *sum* of the growth of its two parts, when the two parts grow separately.

**Maximum of parts success:** A symbiote is *successful* if the growth of the whole symbiote is greater than the *maximum* of the growth of its two parts, when the two parts grow separately.

In general, *sum of parts success* is a more demanding criterion for success than *maximum of parts success*. A rare exception to this is cases when growth is negative.

Unlike the fitness measure in the evolutionary algorithm of Model-S, there is no noise in either of these measures of success. The result will be the same every time we run a game. The game we run could be the Game of Life, the Immigration Game, or the Management Game. Since we only care about the total growth, including all live colours, the total growth is the same for all three games.

Consider Figure 1 in Section 3.2 as an example of how success is calculated. In the top row of the figure, the left part grew by 420 live cells from t = 0 to t = 1000. In the second row, the right part grew by 933 live cells. In the third row, the whole seed grew by 438 live cells. (Note that the magnification varies in the

images, so the size of an image does not correspond to the number of live cells.) The sum of the left and right parts is 1,353 and the maximum of the left and right parts is 933. By both measures of success, the symbiote in Figure 1 is a failure, since the growth of the whole seed is much less than the maximum and the sum of the parts.

Table 5 uses the sum of parts criterion for success. The probability of success for each relation ranges from 3% for manager-manager relations to 16% for manager-worker relations. Manager-worker relations are successful in 84 cases, whereas manager-manager relations have only 7 successes and worker-worker relations have only 8 successes. When worker-worker relations succeed, they are highly productive, with an average growth of 532.9. Successful manager-worker relations, with an average growth of 285.3, are considerably less productive than successful worker-worker relations, but they are much more reliable.

Insert Table 5 here.

Table 6 uses the maximum of parts criterion for success. With this more relaxed criterion for success, we naturally have more successes, but the general pattern is the same. The probability of success ranges from 9% for manager-manager relations to 25% for manager-worker relations. Manager-worker relations are successful in 129 cases, whereas manager-manager relations have only 20 successes and worker-worker relations have only 18 successes. In terms of productivity, the average whole seed growth when successful shows the same general pattern as in Table 5, but the spread between the three types of relations is less extreme.

Insert Table 6 here.

The similar pattern of results in the two tables is encouraging, because it tells us that the results are not particularly sensitive to how we define *success*. Both tables confirm that manager-worker relations provide the best balance of stability and productivity.

### **5 Future Work and Limitations**

The natural next step in this line of research is to study what happens when we have three or more parts in a symbiote. Layer 4, the symbiotic layer of Model-S, always joins two seed patterns at a time, which entails that the evolutionary history of all past fusions in the line of descent of a symbiote will form a binary tree. In the current article, we have focused on the top of this binary tree, ignoring the lower levels; that is, we have ignored the smaller parts that may be inside a second-level part. A binary tree tells us the order in

which the parts were fused over the course of their evolutionary history. Therefore it seems that the simplest way to analyze the interactions of three or more parts would be to recursively descend through the binary tree of the history of past fusions.

A binary tree of the evolutionary history of past fusions is useful for identifying the parts in a symbiote, since each part was once functioning as an independent entity, and is therefore a natural candidate for a functional module in the symbiote. However, manager-worker relations are about interactions among parts of a symbiote in the present, which are not necessarily determined by the binary tree of the history of past fusions. We believe that the binary tree of the evolutionary history of past fusions should be used to identify modules (natural parts), but then another method should be used to study the interactions of the modules, in order to identify managers and workers in the symbiote.

Let us assume that we have decomposed a symbiote into a set of modules, based on the binary tree of the evolutionary history of past fusions of the symbiote. If there are n modules, then there are n(n-1)/2 possible pairs of modules. In general, it seems that we must examine all of these pairs to obtain a complete picture of manager-worker relations. Often two members of a pair will be spatially separated by other modules between them, so they might never have an opportunity to interact; therefore we may not need to consider all possible pairs, but it is likely that there will still be many pairs to consider.

Consider the extensions that must be made to the Management Game in order to analyze the interactions among n > 2 modules. Suppose we have n modules in an initial seed pattern. We will then have n colours in the initial pattern, one colour for each module. When the game begins and the modules start to interact with each other, we need n(n-1)/2 colours to mark cases where a module is playing a supporting role (it provides one of the three live cells required for birth) and we need another n(n-1)/2 colours to mark cases where a module is playing the main role (it provides two of the three live cells required for birth). Thus we will need  $n^2 - n$  new colours for each interacting pair of modules.

Furthermore, with only two colours (red and blue), the B3 rule is simple (see Table 2), because ties are not possible; there is always a clear majority. With n > 2, we need to decide how to handle ties. Ties could be resolved randomly, but it would be cleaner to have a deterministic rule for ties. This may require introducing more colours for handling ties.

We believe that it is entirely feasible to extend the Management Game to analyze manager-worker relations for large values of n, but it will take time to work out the best way to approach this problem. There are many details to address.

### **6 Conclusion**

In this article, we have presented a simple computational model of Stewart's (2020) Management Theory. We introduced the Management Game, an extension of the Game of Life (Gardner, 1970) and the Immigration Game (Wainwright, 1971), to support a formal model of manager-worker relations in evolved cellular automata (Section 3.2). Our goal was to demonstrate that Management Theory is not limited to human societies and eusocial organisms, in which it is easy to see a hierarchical structure of managers and workers. In the current article, we have demonstrated that Management Theory is applicable to an elementary cellular automaton (the Management Game) combined with an evolutionary algorithm (Model-S) that supports the formation of symbiotic entities (Turney, 2020).

Stewart (1995; 2020) claimed that Management Theory could explain the major cooperative transitions in evolution. In essence, his argument is that, if evolution were able to create something analogous to a hierarchical system of managers and workers, then cooperation could be self-sustaining, without requiring group selection to prevent the cooperation from falling apart. This is a reasonable hypothesis, but the hypothesis lacks detailed empirical support. We hope that the computational model presented here provides some empirical support for Management Theory and that it suggests ways that Management Theory could be rigorously, empirically tested in biological and cultural evolution.

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### References

Buss, L. W. (1987). The evolution of individuality. Princeton: Princeton University Press.

Douglas, A. (2010). The symbiotic habit. Princeton, New Jersey: Princeton University Press.

Gardner, M. (1970). Mathematical games: The fantastic combinations of John Conway's new solitaire game 'Life'. *Scientific American*, 223(4), 120-123.

Ikegami, T. (1994). From genetic evolution to emergence of game strategies. *Physica D: Nonlinear phenomena*, 75(1-3), 310-327.

Margulis, L. (1970). Origin of eukaryotic cells. New Haven, CT: Yale University Press.

Margulis, L. (1981). Symbiosis in cell evolution. San Francisco, CA: W. H. Freeman.

- Martin, B. D., & Schwab, E. (2013). Current usage of symbiosis and associated terminology. *International Journal of Biology*, *5*(1), 32-45.
- Maynard Smith, J. (1988). Evolutionary progress and levels of selection. In M. H. Nitecki, *Evolutionary progress?* (pp. 219-236). Chicago, IL: University of Chicago Press.
- Maynard Smith, J., & Szathmáry, E. (1995). *The major transitions in evolution*. Oxford, UK: Oxford University Press.
- Michod, R. E. (1999). *Darwinian dynamics: Evolutionary transitions in fitness and individuality*. Princeton, NJ: Princeton University Press.
- Nolan, P., & Lenski, G. E. (2010). *Human societies: An introduction to macrosociology* (11th ed.). Boulder, CO: Paradigm Publishers.
- Stewart, J. E. (1995). Metaevolution. Journal of Social and Evolutionary Systems, 18(2), 113-147.
- Stewart, J. E. (1997). Evolutionary transitions and artificial life. Artificial Life, 3(2), 101-120.
- Stewart, J. E. (2014). The direction of evolution: The rise of cooperative organization. *BioSystems*, 123, 27-36. doi:https://doi.org/10.1016/j.biosystems.2014.05.006
- Stewart, J. E. (2020). Towards a general theory of the major cooperative evolutionary transitions. *BioSystems*, 198, 1-12. doi:https://doi.org/10.1016/j.biosystems.2020.104237
- Szathmáry, E. (2015). Toward major evolutionary transitions theory 2.0. *Proceedings of the National Academy of Sciences*, 112(33), 10104–10111.
- Trevorrow, A., Rokicki, T., Hutton, T., Greene, D., Summers, J., Verver, M., ... Rowett, C. (2021, February 19). *Golly*. Retrieved February 19, 2021, from SourceForge: http://golly.sourceforge.net/
- Turney, P. D. (2018). Conditions for major transitions in biological and cultural evolution. *Proceedings of the Third Workshop on Open-Ended Evolution (OEE3) at the 2018 Conference on Artificial Life*, (pp. 1-9). Tokyo, Japan. Retrieved from https://arxiv.org/abs/1806.07941v1
- Turney, P. D. (2020). Symbiosis promotes fitness improvements in the Game of Life. *Artificial Life*, 26(3), 338-365. doi:https://doi.org/10.1162/artl\_a\_00326
- Turney, P. D. (2021, March 27). *Management theory: Source code*. Retrieved from GitHub: https://github.com/pdturney/management-theory

- Wainwright, R. T. (1971, June). *Lifeline volume* 2. Retrieved February 19, 2021, from LifeWiki: https://www.conwaylife.com/wiki/Lifeline\_Volume\_2
- West, S. A., Fisher, R. M., Gardner, A., & Kiers, E. T. (2015). Major evolutionary transitions in individuality. *Proceedings of the National Academy of Sciences*, 112, 10112-10119.
- Whitley, D. (1989). The GENITOR algorithm and selective pressure: Why rank-based allocation of reproductive trials is best. In D. Schaffer (Ed.), *Third International Conference on Genetic Algorithms* (pp. 116–121). San Francisco: Morgan Kaufmann.
- Wilson, D. S. (2015). *Does altruism exist? Culture, genes, and the welfare of others.* New Haven, CT: Yale University Press.
- Wilson, D. S. (2019). *This view of life: Completing the Darwinian revolution*. New York, NY: Pantheon Books.
- Wilson, D. S., & Wilson, E. O. (2007). Rethinking the theoretical foundation of sociobiology. *Quarterly Review of Biology*, 82, 327-348.

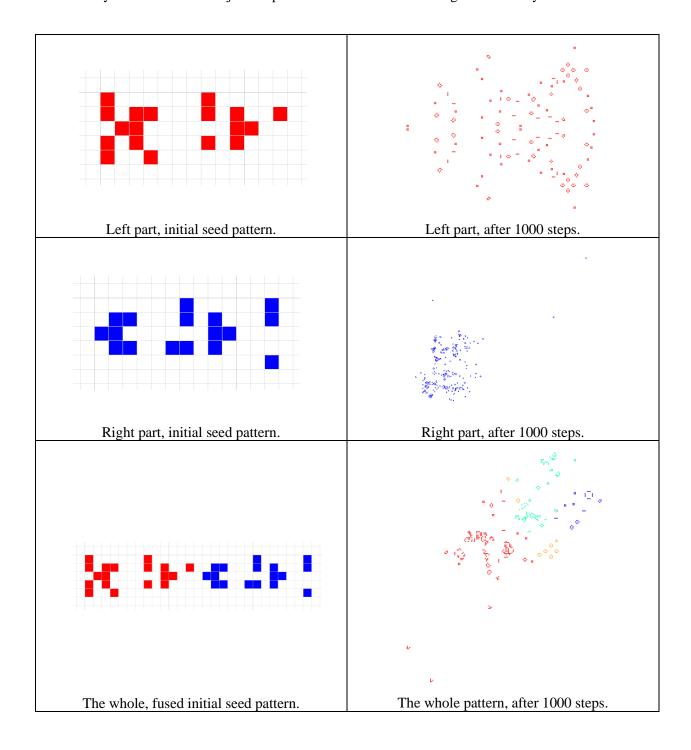


Figure 1. These images show the two parts of a symbiotic fusion, growing separately (in the first two rows) and growing together (in the last row). The first column displays the initial seed patterns and the second column shows the final patterns after 1000 steps of the Management Game. When red and blue make contact (in the last row and last column), they produce green or orange, as determined by the rules of the Management Game.

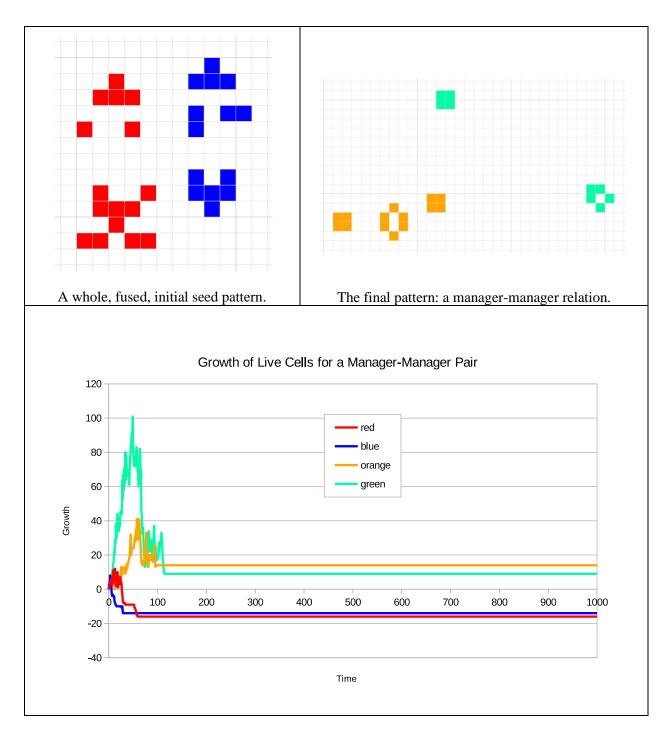


Figure 2. The type of relation (manager-manager, manager-worker, or worker-worker) usually changes over the course of the Management Game, from the initial seed pattern to the final step. This figure shows a typical example of a pattern that settles into a manager-manager relation. The top left shows the initial seed pattern at time t = 0. The top right shows the final seed pattern at time t = 1000. The bottom graph shows the growth of the four colours from t = 0 to t = 1000. The growth of red and blue is negative.

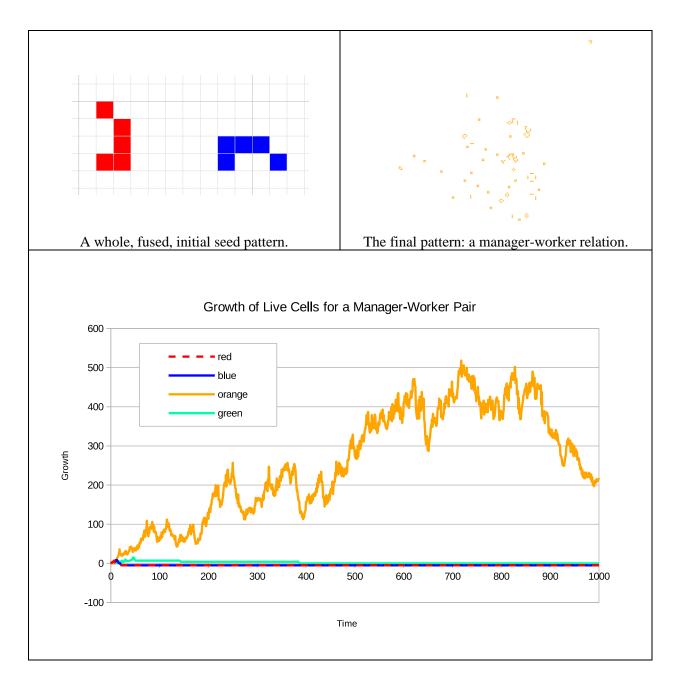


Figure 3. This figure shows a typical example of a pattern that settles into a manager-worker relation. The growth of red and blue is negative. The growth of green is zero. Only orange remains alive at time t = 1000. The total growth, summed over the four colours, is considerably larger here than in the manager-manager relation in Figure 2.

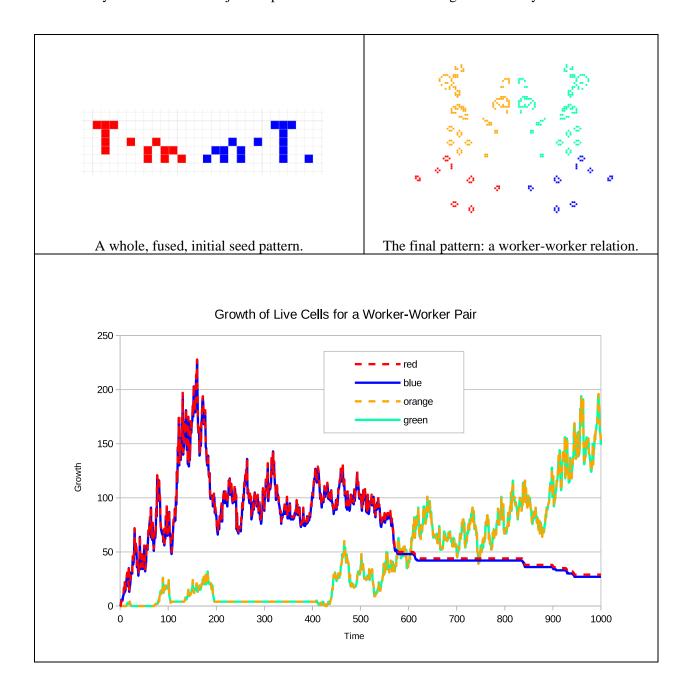


Figure 4. This figure shows a typical example of a pattern that settles into a worker-worker relation. This type of relation tends to decay into another type, unless the two workers are nearly equal in their growth over time.

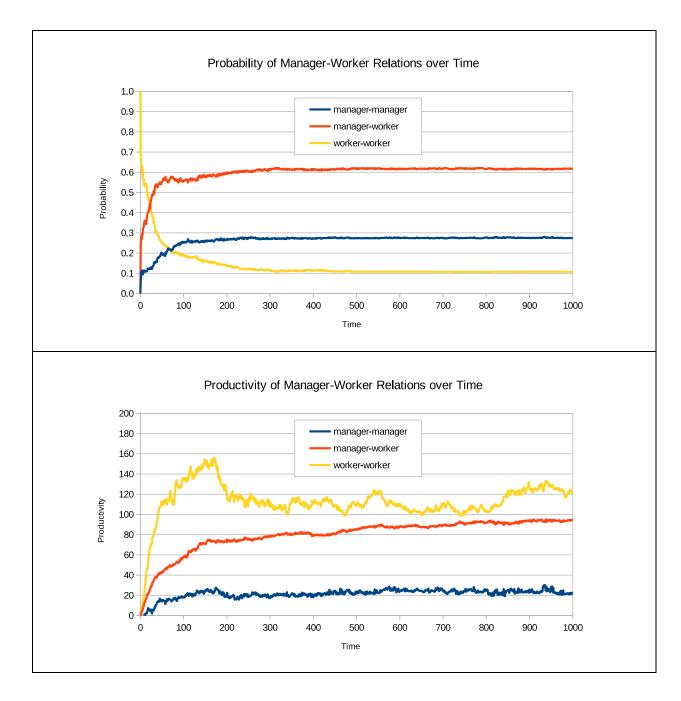


Figure 5. These graphs show the probability and productivity of the three types of relations (manager-manager, manager-worker, or worker-worker) over time. The top graph shows that the most *probable* relation, in the long run, is the manager-worker relation. The bottom graph shows that the most *productive* relation is the worker-worker relation. The productivity of a pattern at time *t* is the growth of all its living cells (cells that are red, blue, orange, or green) at that time. Although the worker-worker relation is the most productive relation (bottom graph), it is also the least probable relation (top graph). Worker-worker relations depend on a balance of growth in the two workers, but the balance is difficult to maintain.

Table 1. This table introduces some terminology that we will use to define the rules of the Management Game. In our experiments, the initial seeds at time t = 0 are always red or blue. Orange and green only appear later at t > 0 as red and blue interact with each other. Red and orange are close relatives and blue and green are close relatives. As a mnemonic device, consider orange as red with some yellow and consider green as blue with some yellow.

States	Colours	Descriptions
0	white	the dead background colour
1	red	initial seed colour or a child born from 3 red parents
2	blue	initial seed colour or a child born from 3 blue parents
3	orange	red origins but with some past non-red contact
4	green	blue origins but with some past non-blue contact
2, 3, 4	non-red	blue, orange, or green
1, 3, 4	non-blue	red, orange, or green
2, 4	blue/green	blue or green
1, 3	red/orange	red or orange

Table 2. This table presents a summary of the rules of the Management Game. We focus here on the rules for the birth of a new cell, since this is the only case where the Management Game differs from the Game of Life and the Immigration Game. At time t, we have an empty (dead, white) cell with eight neighbouring cells around it. For birth to happen in the empty central cell, exactly three of the eight neighbours must be alive. The table specifies the colour the central cell will have at time t + 1, based on the colours of the neighbouring cells at time t. Table 1 explains the terminology used here.

State of 8 neighbouring cells at time <i>t</i>	State of central cell at time <i>t</i>	State of central cell at time $t + 1$
3 red neighbours	white (dead)	red (birth)
3 blue neighbours	white (dead)	blue (birth)
2 red/orange + 1 non-red neighbour	white (dead)	orange (birth)
2 blue/green + 1 non-blue neighbour	white (dead)	green (birth)

Table 3. This table defines the three types of relations. A worker focuses on making more of its own of type. A red/orange worker focuses on making red/orange cells. A blue/green worker focuses on making blue/green cells. On the other hand, a manager focuses on helping its symbiotic partner. A red/orange manager helps its blue/green partner make green cells. (Red/orange cells cannot contribute to blue births.) A blue/green manager helps its red/orange partner make orange cells. (Blue/green cells cannot contribute to red births.)

Term	Definition
red/orange cells are managers	green growth > (red growth + orange growth)
red/orange cells are workers	green growth $\leq$ (red growth + orange growth)
blue/green cells are managers	orange growth > (blue growth + green growth)
blue/green cells are workers	orange growth $\leq$ (blue growth + green growth)
a manager-manager relation exists	red/orange and blue/green are both managers
a manager-worker relation exists	one part is a manager and the other part is a worker
a worker-worker relation exists	red/orange and blue/green are both workers

Table 4. The expected productivity for a group is the sum of the multiplication of the average productivity of each relation type in the group with the corresponding probability of the relation type. For example, if manager-workers in Group 1 produce an average growth of 73.81 live cells and are able to work with a probability of 63.51%, then the expected productivity of the workers is  $46.88 (73.81 \times 0.6351)$ . The four groups in this table were generated by ranking all 844 symbiotes according to their order of birth in their particular run of Model-S and then splitting them into four equal sized groups ( $4 \times 211 = 844$ ). As we can see in the bottom row of this table, the later a group is in the evolutionary order (the more recently it evolved), the greater its expected productivity.

	Relation types	Group 1	Group 2	Group 3	Group 4
Average productivity for	manager-manager	-1.23	5.24	45.70	41.29
each relation type	manager-worker	73.81	85.80	106.85	116.63
each relation type	worker-worker	40.75	83.90	95.16	178.08
Duchahility for anah	manager-manager	0.3270	0.2370	0.2654	0.2654
Probability for each relation type	manager-worker	0.6351	0.6635	0.6161	0.5592
relation type	worker-worker	0.0379	0.0995	0.1185	0.1754
Average productivity	manager-manager	-0.40	1.24	12.13	10.96
multiplied by probability	manager-worker	46.88	56.93	65.83	65.22
for each relation type	worker-worker	1.55	8.35	11.27	31.23
Expected productivity	all relations	48.02	66.52	89.23	107.41

Table 5. Let us say that a seed is *successful* if the growth of the whole seed is greater than the *sum* of the growths of its parts, growing separately. For example, in Figure 1, the two parts (left and right) are shown growing separately in the first two rows and they are shown together, in the third row, growing as a whole. This table examines the degree of success (as defined above) for each of the three relations: managermanager, manager-worker, and worker-worker.

Measure	Manager-	Manager-	Worker-	All three
Wieasure	manager	worker	worker	relations
relation count in final time step	231	522	91	844
relative size of each relation in final step	0.27	0.62	0.11	1.00
average whole seed growth of relation	21.9	94.9	121.5	77.8
average sum of parts growth	268.3	270.6	582.0	303.5
number of successes: whole > sum of parts	7	84	8	99
probability of success for each relation	0.03	0.16	0.09	0.12
average whole seed growth when successful	274.6	285.3	532.9	304.6
average whole seed growth when not successful	14.0	58.4	81.8	47.7

Table 6. Let us say that a seed is *successful* if the growth of the whole seed is greater than the *maximum* of the growths of its parts, growing separately. This criterion for success is less demanding than the criterion of Table 5. The table examines the degree of success for each of the three relations: manager-manager, manager-worker, and worker-worker.

Measure	Manager-	Manager-	Worker-	All three
Weasure	manager	worker	worker	relations
relation count in final time step	231	522	91	844
relative size of each relation in final step	0.27	0.62	0.11	1.00
average whole seed growth of relation	21.9	94.9	121.5	77.8
average maximum of parts growth	152.7	174.1	298.7	181.7
number of successes: whole > maximum of parts	20	129	18	167
probability of success for each relation	0.09	0.25	0.20	0.20
average whole seed growth when successful	226.6	236.6	350.7	247.7
average whole seed growth when not successful	2.4	48.4	65.0	35.9