Major Cooperative Transitions and Management

Theory in the Game of Life

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Abstract

Biological and cultural evolution show a trend towards increasing hierarchical organization, in which entities at one level combine cooperatively to form a new entity at a higher level of organization. In each case where such a cooperative transition has been studied, we have some understanding of how the transition came about, but it is difficult to formulate a unified theory that covers all of these transitions. John Stewart has proposed a theoretical framework called *Management Theory*, which attempts to explain all of the major cooperative transitions in biological and cultural evolution. The idea is that successful transitions require the integration of managers and workers into a cooperative organization. This theory seems appropriate when we consider the cultural evolution of corporations, where managers and workers are clearly essential, but it seems less plausible when we consider the biological evolution of entities that do not invite anthropomorphic projection. However, in the following article, we define managers and workers in an abstract way that enables us to apply these terms over a broad range of cases, including cultural evolution, biological evolution, and computational simulations of evolution. The core idea is that a manager is an entity that takes a leading role in the production of something and a worker is an entity that plays a supporting role in the production of something. We apply this abstract view of managers and workers to a computational simulation of evolving cooperative transitions in John Conway's Game of Life. The simulation confirms the expectations of Management Theory: (1) A combination of managers and workers performs better than managers alone or workers alone. (2) When there are relatively few workers, one manager is better than two or more managers.

Keywords: Major cooperative transitions, management theory, symbiosis, evolution, cellular automata, hierarchical organization.

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1 Introduction

In past work, we presented *Model-S* (Turney, 2020), a computational simulation of the evolution of symbiosis in John Conway's cellular automaton, the *Game of Life* (Gardner, 1970). Model-S was created to provide some insight into the major cooperative transitions in biological and cultural evolution (Maynard Smith & Szathmáry, 1995; Nolan & Lenski, 2010; Turney, 2018). John Stewart's work on *Management Theory* (Stewart, 2020) was also intended to provide insight into the major cooperative transitions in evolution. The current article attempts to apply Stewart's Management Theory to the symbiotic entities that evolve in Model-S.

Management Theory aspires to account for all of the major cooperative transitions in biological and cultural evolution, yet the examples and arguments for the theory mostly depend on cultural evolution, especially human cultural evolution (Stewart, 1995; 1997; 2014; 2020). On the other hand, Model-S is a highly abstract and simplified mathematical model. As such, it seems most suited to the earliest instances of biological evolution. This suggests that applying Stewart's Management Theory to entities in Model-S is a challenging test for both Management Theory and Model-S.

In this article, we examine entities that are joined to form a new *symbiotic* entity. We consider symbiotic entities with two, three, or four parts. In general, *symbiosis* can mean a relation in which all parts benefit or a relation in which some of the parts do not benefit. Following Douglas (2010), we will use the term *mutualism* when all parts benefit and the term *symbiosis* to cover all cases, with and without benefits.

The first step in applying Management Theory is to define *managers* and *workers* in a way that is precise yet general enough to apply to biology, culture, and cellular automata. Our proposal is that a *worker* is a part of a symbiote that takes the *supporting role* in the production of something and a *manager* is a part that plays a *leading role* in the production something. In general, a part may sometimes act as a worker and other times act as a manager. We will classify parts at a specific time as workers or managers based on whether they mostly act as managers or mostly act as workers at that point in time. As time goes on, a part tends to settle into one role, either manager or worker.

Each part of a symbiotic entity may be either a worker or a manager. Given a symbiotic entity with N parts, there are N+1 possibilities: (1) 0 managers and N workers, (2) 1 manager and N-1 workers, (3) 2 managers and N-2 workers, ..., (N+1) N managers and 0 workers. In our experiments (Section 4), we find that the most *probable* (most likely to occur) and most *productive* (most likely to grow) symbiotes are composed of 1 manager and N-1 workers. Our experiments are limited to $N \in \{2, 3, 4\}$. We expect that there is a value of N greater than 4, such that more than one manager will be needed to optimize growth.

The experiments we present here provide a simple, abstract, mechanistic account of manager-worker relations in symbiotic unions, including the major cooperative transitions in biological and cultural evolution. The results lend support to Stewart's Management Theory. Stewart's arguments are largely based on human societies, human governance mechanisms, and the structure of human corporations, whereas the results presented in this article rely on a mathematical, computational model. It is encouraging that two, quite different approaches to understanding major cooperative transitions agree on the importance of manager-worker relations.

In Section 2, we briefly discuss past work on symbiosis and major cooperative transitions. Section 3 introduces the background that is necessary to understand the experiments: the design of Model-S and how we apply Management Theory to the symbiotic entities that evolve in Model-S. Section 4 presents the experiments, the core of the paper. Section 5 explores future work and limitations of the current work. We conclude in Section 6.

2 Related Work

Douglas (2010) provides an excellent survey of research on symbiosis. Martin and Schwab (2013) give a detailed explanation of the various terms that are used to classify the many different types of symbioses.

The major cooperative transitions in biological and cultural evolution may be seen as cases of particularly successful symbioses. Attempts to explain the processes that drive the major cooperative transitions include the work of Buss (1987), Maynard Smith (1988), Maynard Smith and Szathmáry (1995), Stewart (1995; 2014; 2020), Michod (1999), Wilson and Wilson (2007), Wilson (2015), West et al. (2015), Szathmáry (2015), and Wilson (2019).

Model-S is a computational model of symbiosis with shifting levels of selection (Turney, 2020). In Model-S, when two entities enter a symbiotic relation, they are fused together, hence they live or die as a unit. This kind of fusion, in which selection shifts from the two components to the fused whole, is called *endosymbiosis*. The paradigmatic example of endosymbiosis is the major transition in which two prokaryotic cells merged to form a eukaryote (Margulis, 1970; 1981).

Model-S uses the *Immigration Game* (Wainwright, 1971), created by Don Woods, as a contest for measuring the fitness of evolved entities. The Immigration Game is a two-player variation of John Conway's Game of Life (Gardner, 1970). We chose the Immigration Game for our computational simulation of the evolution of symbiosis because there is a natural way to model symbiosis: two seed patterns can be merged into one symbiote by simply joining them side-by-side and treating them as a unit. We describe Model-S and the Immigration Game in detail in Section 3.

Ikegami (1994) also chose to use game theory in his pioneering computational simulation of the evolution of symbiosis. The fitness of the entities in the population was measured by playing the *Erroneous Iterated Prisoner's Dilemma* game. An entity's game strategy was represented by a tree. Two entities can be merged into one symbiote by fusing one tree onto a branch of the other tree.

3 Background: Individuals and Populations

In this section, we describe the Game of Life, the Immigration Game, the Management Game, and Model-S. All three games involve individuals growing over the course of their lifetimes. Model-S, however, involves populations evolving over the course of many generations. We discuss this distinction between the lifetimes of individuals and evolutionary times of populations in Section 3.1. The three games are presented in Section 3.2. Model-S is described in Section 3.3.

3.1 Lifetimes and Generations

We use three games in this article, the Game of Life, the Immigration Game, and the Management Game. These three games take place in time scales that are different from the time scale of Model-S. The games all run in *lifetimes*, whereas Model-S runs in *evolutionary time*. A living organism grows and develops over the course of its life. The three games involve *individuals* growing over their lifetimes. A *population* in Model-S evolves over a much longer period of time, consisting of many lifetimes.

Model-S has a constant population size of 200. Each newly born entity replaces an existing entity in the population. When 200 new entities have been born, one generation of time has passed. Model-S runs for 100 generations, which results in 20,000 evolved entities (200×100).

The fitness of an entity in Model-S is measured by one-on-one competitions in the Immigration Game, in the Golly cellular automata software, outside of Model-S. Model-S makes a call to Golly when it is time to run an Immigration Game. These games take place in the lifetime of the entities. A lifetime is a specified number of steps in the Immigration Game. In Model-S, the precise number of steps is a function of the size of the entities: larger entities are allowed more time to grow. In the experiments in Section 4, outside of Model-S, we use a fixed number of steps (1000), so that all entities can be compared under the same conditions.

The current article is focused on lifetimes, not evolutionary time. We ran Model-S 20 times, resulting in the birth of 400,000 individuals ($20 \times 200 \times 100$). Of these individuals, 910 were symbiotes. The experiments in this paper examine the lifetime behaviours of these 910 symbiotes.

To increase our understanding of the 910 symbiotes, we analyze them with three different games. In addition to the Immigration Game, which was used by Model-S, we examine the behaviour of the symbiotes

outside of Model-S, using the Game of Life and the Management Game. These games are described in the next section.

3.2 Three Games

John Conway's Game of Life (Gardner, 1970) is played on an infinite, two-dimensional grid of square cells, where each cell is either dead (state 0) or alive (state 1). Often the dead state (the background) is coloured white and the live state (the foreground) is coloured black. The state of a cell changes with time, based on the state of its eight nearest neighbours (the Moore neighbourhood). Time passes in discrete intervals, and the states of the cells at time t uniquely determine the states of the cells at time t + 1. There is only one player in the game and the player's only actions are to choose the initial states of the cells at time t = 0 and the time limit for the game, given by a maximum value for t. The states for t > 0 are calculated by a computer. The initial states form a seed pattern that determines the course of the game. The rules are summarized in Table 1.

Insert Table 1 here.

The rules for updating states in the Game of Life are compactly expressed as B3/S23: A cell is *born* (it switches from state 0 to state 1) if it has exactly three living neighbours (B3). A cell *survives* (it remains in state 1) if it has two or three living neighbours (S23). Otherwise, the cell *dies* (it switches from 1 to 0) or remains *unborn* (it remains in state 0).

Don Woods' Immigration Game (Wainwright, 1971) is almost the same as the Game of Life, except there are two different *live* states (states 1 and 2, usually represented by red and blue colours). The rules for updating remain B3/S23, but there are two new rules for determining colour: (1) Live cells do not change colour unless they die (they become white). (2) When a new cell is born, it takes the colour of the majority of its living neighbours. Since birth requires three live neighbours, there is always a clear majority. The initial states at time t = 0 are chosen by the two players of the game; one player makes a red seed pattern and the other player makes a blue seed pattern. The players agree on a time limit, given by a maximum value for t. The rules are summarized in Table 2.

Insert Table 2 here.

In the Immigration Game, if states 1 and 2 were coloured black, instead of red and blue, the game would appear to be exactly the same as the Game of Life. The purpose of the two colours is to score the two players, to convert the Game of Life from a solitaire game into a two-player competitive game. In Model-S,

we score each player by the *growth* of their initial seed, defined as the final number of cells of their colour minus the initial number of cells of their colour. Growth will be negative if the final number is less than the initial number. The motivation for this method of scoring is to avoid biasing the game in favour of the seed that has the most living cells at the beginning of the game. The winner is the player whose colour has grown the most over the course of the game.

The Management Game (a new game, introduced here) extends the Immigration Game by adding two more *live* states, orange and green. The Management Game also adds one more *dead* state, purple, to serve as a border for separating the parts of a symbiote. This brings the total number of states to six. Table 3 lists the six states and introduces some terminology.

Insert Table 3 here.

Table 4 presents the rules of the Management Game. Like the Game of Life and the Immigration Game, the basic rule is still B3/S23. The only change is how colours are handled. We colour one part of the symbiote red and the other parts blue and then play the Management Game. If red and blue do not interact, then we will only see changing red and blue patterns. If red and blue do interact, then we will start to see orange and green colours appearing.

Insert Table 4 here.

We noted above that, if states 1 and 2 in the Immigration Game were displayed as black, then the Immigration Game would look exactly like the Game of Life. The only difference between the Game of Life and the Immigration Game is colouration. Likewise, in the Management Game, if state 3 (orange) were displayed as red, state 4 (green) were displayed as blue, and state 5 (purple) were displayed as white, then the Management Game would look exactly like the Immigration Game. In the Management Game, orange is used to mark a subset of the cells that would be red in the Immigration Game, green is used to mark a subset of the cells that would be white.

The four rows of Table 4 tell us four things: (1) A newly born red cell is always the work of three other red cells. (2) A newly born blue cell is always the work of three other blue cells. (3) When we see an orange cell, we know that, at some time in the past, a blue or green cell played a *supporting* role (one of three cells for B3) in forming the orange cell (since the initial seed pattern contains no orange). The *leading* role (two of three cells) was played by orange or red. (4) When we see a green cell, we know that, at some time in the past or in the current birth, a red or orange cell played a *supporting* role (one of three cells for B3) in

forming the green cell (since the initial seed contains no green). The *leading* role (two of three cells) was played by green or blue.

Figure 1 gives an example of a symbiote with three parts, separated by purple borders. To classify one of the three parts as a *manager* or a *worker*, we colour the chosen part red and we colour all other parts blue. We then run the Management Game from time t = 0 to time t = 1000. The status of the chosen part (manager or worker) varies as a function of time t. Figure 1 shows the status of the three parts at t = 0 and t = 1000. We can see that the leftmost part and the central part are classified as workers at t = 1000, whereas the rightmost part is classified as a manager.

Insert Figure 1 here.

Table 5 explains how we decide whether the chosen red part is a manager or a worker at time t. If the number of orange cells at time t is greater than the number of green cells at time t, then the red part is acting as a *manager* at time t. If the number of orange cells a time t is less than or equal to the number of green cells at time t, then the red part is acting as a *worker*.

Insert Table 5 here.

3.3 Model-S

This section is a summary of work that was described in detail in our previous article on symbiosis in Model-S with the Immigration Game (Turney, 2020). Model-S is a genetic algorithm for evolving seed patterns that are good at playing the Immigration Game. It is based on a GENITOR-style genetic algorithm (Whitley, 1989), with one-at-a-time reproduction, a constant population size, and rank-based tournament selection.

Model-S uses *Golly* (Trevorrow, et al., 2021), an open source, cross-platform application for exploring cellular automata. Golly can be controlled with Python code. Model-S consists of Python routines that manage a population of entities (seed patterns). A seed pattern is stored in Model-S as a matrix. Whenever it is necessary to measure the relative fitness of two entities by running a contest in the Immigration Game, Model-S passes the seed patterns (with one entity coloured red and the other coloured blue) to Golly and receives a score from Golly when the contest is over (the score is based on the growth of red and blue). In addition to providing the initial seed patterns for time t = 0, Model-S also sets the duration of the contest by giving a maximum value for t. The maximum value increases in proportion to the size of the seed patterns.

Model-S is constructed with four layers, each subsequent layer building on the previous layers, as shown in Figure 2. The purpose of having four layers is to measure each layer's contribution to the fitness of the evolving population, by selectively enabling or disabling layers. In the current article, we use all four layers.

Insert Figure 2 here.

Layer 1 implements a simple form of asexual reproduction, with a fixed genome size (that is, a fixed matrix size). A member of the population is selected for reproduction using tournament selection (Whitley, 1989). The chosen seed pattern is mutated by randomly flipping some of the bits in the seed matrix and it then competes in a series of one-on-one Immigration Games with the other members of the population. Its fitness is the average fraction of games it wins.

Layer 2 implements a slightly more sophisticated form of asexual reproduction. A member of the population is selected for reproduction using tournament selection. Layer 2 allows the seed matrix to grow or shrink by appending or removing a row or column to or from the seed matrix. Layer 2 then passes the seed on to Layer 1 for mutation by flipping bits.

Layer 3 selects two seeds from the population using tournament selection and then combines them with genetic crossover (sexual reproduction). Layer 3 requires the two seeds to be somewhat similar for crossover to proceed. If a suitable match is found, then Layer 3 combines the two seeds with crossover and passes the new seed on to Layer 2 for row or column adjustments. Otherwise, if no suitable match is found, Layer 3 passes only one of the two seeds on to Layer 2, without making any changes to the chosen seed.

Layer 4 adds symbiosis to Model-S. Two seeds are selected from the population and they are fused together, side-by-side, creating a new symbiotic genome. This new fused seed is treated as a whole; that is, selection shifts from the level of the two parts to the level of the whole. The parameters in Model-S are set so that fusion is rare. Most of the time, Layer 4 makes no changes and simply passes control to Layer 3. The main result of our past work (Turney, 2020) is evidence that symbiosis (Layer 4) promotes fitness improvements in the Immigration Game.

Each symbiote created by Layer 4 is composed of parts (seed patterns) that have been fused together to make a new whole (a new, fused seed pattern). In Layer 4, seed patterns are fused two at a time. In the earliest generations of a run of Model-S, when two seeds are selected for fusion, they will not have experienced fusion before; therefore the selected seeds will not be composed of parts. The result of fusion with these seeds will be a new seed with two parts. Over the course of a long run of Model-S, eventually one or both of the two seeds that are selected for fusion will have experienced fusion in the past, so the

new, fused seed will have three or more parts. In our 20 runs of Model-S, there are seeds with two, three, and four parts.

The state of a seed pattern at time t = 0 is analogous to the *genome* of an organism. When a seed pattern enters a game, it develops over time, following the rules of the game, as t increases. The developing pattern is analogous to the developing *phenome* of an organism. The genome is static and the phenome is dynamic. Model-S records the genome of each organism (the pattern at time t = 0), but it keeps no record of the phenome (any patterns at t > 0). All the information that Model-S requires for evolving new genomes is provided by the fitness score of the phenome. The phenome itself is ephemeral.

When Model-S is running, symbiotes and non-symbiotes compete against other symbiotes and non-symbiotes, in order to measure the fitness of the entities in the population. The competitions use the Immigration Game to compare two entities sampled from the population. These competitions determine which entities can reproduce; that is, the competitions determine life and death in the population. All entities, symbiotic or not, are treated the same way. Model-S does not analyze the parts inside a symbiote in any way: a symbiote is treated as a whole.

After Model-S has finished running, we can analyze the stored record of all seed patterns (genomes), across all generations of the run. We inspect the record of seed patterns and extract all symbiotic seed patterns from the record, skipping over non-symbiotes, since our focus in this article is symbiotes. We then analyze the internal parts of these extracted symbiotic seed patterns using the Management Game, as shown in Figure 1. This inspection of the stored symbiotic patterns takes place outside of Model-S, at the end of a run. When we apply the Management Game to the parts of a symbiote, our goal is to understand the interactions among the parts in the symbiote. The goal of this analysis is *not* to calculate the fitness of the parts (unlike the purpose of Model-S); the goal is to gain insight into the internal hierarchical structure of the symbiote.

To test Management Theory, we need a way to analyze the interaction between the parts of the symbiote, in order to label the parts as workers or managers. To classify a given part, we colour the given part red and we colour all other parts blue (see Figure 1). We could run the Immigration Game with these coloured seeds, but the Immigration Game can only tell us how much the parts grow; it cannot distinguish different types of growth. For example, if a new red cell appears in the Immigration Game, we cannot tell from its colour alone whether it was created by three red neighbours (that is, the new red cell is the result of red cells working alone) or by two red neighbours and one blue neighbour (thus the new red cell is the result of red cells and blue cells cooperating). Therefore we created the Management Game.

In Figure 1, the first column shows the symbiote at t = 0, as a genome. The second column shows the symbiote at t = 1000, as a fully developed phenome. The genome is composed of three distinct and non-overlapping parts, separated by purple borders. The borders disappear at t = 1. In the phenome, the parts interact with each other. Although the parts have usually mixed together by t = 1000, we can still detect which part of the genome is responsible for the cells in the phenome at t = 1000, due to their colours in the Management Game. This is the motivation for introducing the new colours, orange and green. The new colours allow us to track the contributions of each of the parts of the symbiote to the final organism.

4 Experiments: Manager-Worker Relations

In this section, we analyse 20 runs of Model-S. Each run evolves a fixed-size population of 200 entities for 100 generations. A generation is defined as 200 births (including new symbiotes as *births*, although they are not *born* in the usual sense), so a full run yields 20,000 births of evolved entities (200×100) . One run takes about two weeks with a typical desktop computer. We executed 20 runs in parallel on three computers. Table 6 gives a summary of the runs.

Insert Table 6 here.

Most of the births in Model-S are asexual (Layer 2) or sexual (Layer 3). Our assumption is that symbioses need a variety of non-symbiotic entities as components for creating fit symbiotic entities. Therefore we chose the parameters in Model-S so that new symbiotic entities (Layer 4) would be relatively rare, just as they are relatively rare in biological evolution.

As we can see from Table 6, the symbiotes may consist of two, three, or four parts. Analysis of the symbiotes depends on the number of parts in each symbiote. Therefore we examine two-part symbiotes in Section 4.1, three-part symbiotes in Section 4.2, and four-part symbiotes in Section 4.3. For symbiotes with N parts, Section 4.4 discusses the value of having 1 manager and N-1 workers. Section 4.5 examines the conflicts and benefits of parts in a symbiote.

The details of Model-S are presented elsewhere (Turney, 2020) and most of the details are not important for understanding the current article. Whenever we talk about running an experiment in the current article, it will be an experiment with running a game in the *lifetime* of a seed pattern, in a cellular automaton (in Golly), not an experiment with evolving a population of seed patterns in Model-S.

Software for reproducing the results presented here is freely available for downloading (Turney, 2021).

4.1 Two-Part Symbiotes

A total of 585 two-part symbiotes were generated in the 20 runs of Model-S (see Table 6). With two-part symbiotes, there are three possible combinations of managers and workers: (A) 2 managers and 0 workers, (B) 1 manager and 1 worker, and (C) 0 managers and 2 workers. The status of a part of a symbiote as a manager or worker varies as the Management Game runs from step t = 0 to step t = 1000, thus the populations of the three possible combinations change as t changes.

Figure 3 shows three different graphs of the three possible combinations of managers and workers. The top graph shows the fraction of the population that belong to each of the three possible combinations of managers and workers at each time t. We may think of the fraction of the population for a given combination (A, B, or C) as the probability that a randomly selected symbiote belongs to the given combination. The three probability curves reach a stable state very quickly, at about t = 40. The top graph shows that the most frequent combination of managers and workers is (B), 1 manager and 1 worker, with a probability of about 0.7. Second is group (C), 0 managers and 2 workers, with a probability of about 0.3. Last is group (A), 2 managers and 0 workers, with a probability of 0.0.

Insert Figure 3 here.

The middle graph in Figure 3 shows the productivity of the three possible combinations of managers and workers. The productivity of a symbiote is growth of the four colours, red, blue, orange, and green. The growth of a colour at time t is the number of living cells of the given colour at time t minus the number of living cells of the given colour at time t minus the number of living cells of the given colour at time t = 0. For each of the three possible combinations of managers and workers (A, B, and C), we calculate the average growth of the symbiotes in the given group. The middle graph in Figure 3 indicates that group (B), 1 manager and 1 worker, has the greatest productivity. The average growth of (B) rises steadily from t = 0 to t = 500 and then hovers around 60 living cells from t = 500 to t = 1000. Second is group (C), 0 managers and 2 workers, with an average growth that is similar to the growth of group (B) up to about t = 140, at which point it collapses and eventually settles at an average growth around 30 living cells. Last is (A), 2 managers and 0 workers, with a growth of zero.

The probabilities in the top graph tell us how likely it is for a given group (A, B, or C) to occur in the population. The productivities in the middle graph tell us how hardy the growth is for a given group (A, B, or C). A robust symbiote should be both likely to occur in the population and capable of strong, hardy growth. The natural way to measure this is to multiply the probability of a group with the average growth of the group. The bottom graph in Figure 3 shows the probability multiplied by productivity for each of the three possible combinations of managers and workers (A, B, and C). The bottom graph also shows the 95%

confidence intervals for the three groups. For t > 20, group (B), 1 manager and 1 worker, is by far the most robust group of the three.

In the middle graph in Figure 3, it is interesting that group (C), 0 managers and 2 workers, is able to match group (B), 1 manager and 1 worker, up to about time t = 140, but then group (C) collapses to a lower level of productivity. It seems that a manager is not essential in the early stages of growth, but a manager becomes essential as time goes on. This is suggestive of some early-stage technology start-ups, which begin with a flat structure but soon find that a more hierarchical structure is required for continued progress.

4.2 Three-Part Symbiotes

In the 20 runs of Model-S, 129 symbiotes had three parts (see Table 6). Given three parts, there are four possible combinations of managers and workers: (A) 3 managers and 0 workers, (B) 2 managers and 1 worker, (C) 1 manager and 2 workers, and (D) 0 managers and 3 workers. The populations of the four possible combinations change as *t* changes.

The top graph in Figure 4 displays the probability of the four possible combinations of managers and workers. As with two-part symbiotes (see Figure 3), 1 manager (group C) is the most probable group, with a probability that stabilizes at around 0.79. Next is 0 managers (group D), with a probability stabilizing at about 0.13. Not far behind is 2 managers (group B) at 0.08. Last is 3 managers, with a probability of zero.

Insert Figure 4 here.

The middle graph in Figure 4 gives the productivity of the four groups. The group with 1 manager (group C) has the most robust productivity, stabilizing at around 80 living cells. The groups with 2 managers and 0 managers (groups B and D) begin with trajectories that match the group with 1 manager (group C), but then they collapse at about t = 540 to much lower levels of productivity.

The bottom graph in Figure 4 shows the probability multiplied by productivity for each of the four possible combinations of managers and workers. As in Figure 3, the group with one manager (group C) is significantly the most robust of the four groups. It has both high probability and high productivity.

Note that the productivity curves for groups B and D are quite noisy (see the middle graph). This is because their sample sizes are small (see the top graph). The curve for the productivity of group C is smoother because its sample size is much larger. When t = 1000, group C has a population of $102 (129 \times 0.79)$, whereas B has a population of $10 (129 \times 0.08)$ and D has a population of $17 (129 \times 0.13)$. Therefore the difference in the noise in these curves is not surprising and it does not have any unusual implication.

4.3 Four-Part Symbiotes

The 20 runs of Model-S yielded 196 four-part symbiotes (see Table 6). With four parts, there are five possible combinations of managers and workers: (A) 4 managers and 0 workers, (B) 3 managers and 1 worker, (C) 2 managers and 2 workers, (D) 1 manager and 3 workers, and (E) 0 managers and 4 workers.

The top graph in Figure 5 presents the probabilities of the five groups. As with two-part symbiotes (Figure 3) and three-part symbiotes (Figure 4), 1 manager (group D) is the most probable group. The probability of 1 manager stabilizes at about 0.74. The probability of 0 managers stabilizes at about 0.16 and the probability of 2 managers stabilizes at about 0.10. The probability of 3 managers is almost zero and the probability of 4 managers is exactly zero.

Insert Figure 5 here.

The middle graph in Figure 5 presents the productivities of the five groups. The most productive is 1 manager (group D), but 0 managers (group E) and 2 managers (group C) are not far behind.

The bottom graph in Figure 5 gives the probability multiplied by productivity for each of the five possible combinations of managers and workers. As with Figure 3 and Figure 4, the group with one manager (group D) has both high probability and high productivity. For t > 20, group (D), 1 manager and 3 workers, is the most robust, as measured by probability times productivity.

4.4 The Value of One Manager

Seed patterns with N parts yield N+1 possible combinations of managers and workers. For example, a seed pattern with three parts yields four types of combinations: (A) 0 managers and 3 workers, (B) 1 manager and 2 workers, (C) 2 managers and 1 worker, and (D) 3 managers and 0 workers. Suppose we have seed patterns with N parts, resulting in N+1 types of seed patterns, s_0 , s_1 , s_2 , ..., s_N , consisting of varying combinations of managers and workers. Let *probability* of type s_i be $p(s_i)$ and let the productivity (*growth* of cells) of type s_i be $g(s_i)$. The statistical expectation for the average productivity of the patterns is $\sum_{i=0}^{N} p(s_i)g(s_i)$, the sum of the probability of each type of pattern multiplied by the productivity of each type of pattern. Let s_1 be the type of seed pattern that has 1 manager and N-1 workers at time t. The productivity of s_1 relative to s_0 , s_1 , s_2 , ..., s_N is $p(s_1)g(s_1)/\sum_{i=0}^{N} p(s_i)g(s_i)$. (We omit time t here for simplicity, but it should be noted that the values of $p(s_i)$ and $g(s_i)$ vary with time.)

Figure 6 gives the productivity contribution of seed patterns of type s_1 (1 manager and N-1 workers) relative to the whole set of types $(s_0, s_1, s_2, ..., s_N)$. There are three curves, for N = 2, 3, and 4. The curves begin to stabilize when t > 300. At this point, the contribution of s_1 relative to the whole is greater than 70%

for all three values of N. It is clear in Figure 6 that the cases in which there is 1 manager and N-1 workers account for most of the productivity of the seed patterns. This result fits with our intuition, at least for relatively small values of N: A small group of people working together typically works best with one manager, rather than zero, two, or more managers.

Insert Figure 6 here.

4.5 Conflicts and Benefits

Douglas (2010, p. 56) wrote, "Conflict is inherent to the reciprocal exchange of benefits that underpin symbiosis." Chapter 3 of her book, *The Symbiotic Habit*, describes the complex balance of conflicts and benefits in symbiosis. Martin and Schwab (2013, p. 32) wrote, "Confusion has afflicted the definition of symbiosis for over 130 years." They propose a set of terms based on whether the two species in a symbiotic relationship experience a beneficial effect (+), a harmful effect (-), or a neutral effect (0). This yields six types of symbiosis: neutralism (0/0), antagonism (-/-), amensalism (0/-), agonism (+/-), commensalism (+/-), and mutualism (+/+) (Martin & Schwab, 2013, p. 41).

Evolution is competitive. When entities combine to become a symbiote, they do not immediately become cooperative. They continue to compete with each other, to some degree, especially when environmental conditions are not optimal (Douglas, 2010). Likewise, when people join together to form a new company, they make an effort to cooperate and work towards a common goal, but there is also inevitable competition among them. We hypothesize that the distinction between managers and workers arises from this interaction between cooperation and competition. Douglas (2010, p. 90) wrote, "... many symbioses are not associations of equals, but involve one organism that can control many of the traits of its partners." Therefore we expect that most symbiotes in our model will have exactly one manager, and the manager will have the power to control the workers.

We see in the probability graphs in Figures 3, 4, and 5 that most symbiotes have settled into a stable state by the time t = 1000, such that there is 1 manager and N - 1 workers. We would like to know whether there is some characteristic that distinguishes the part that becomes the manager from the parts that become workers. Here we test three hypotheses about the characteristics of managers.

Hypothesis 1: We hypothesize that the part that grows the most in isolation from the other parts is likely to be the part that eventually (by time t = 1000) becomes the manager of the symbiote in the Management Game. To test this hypothesis, we remove each part from the symbiote and run each part one-at-a-time in

the Game of Life to time t = 1000. Growth of each part is measured by the final size count minus the initial size count, counting the black (live) cells in the Game of Life.

Hypothesis 2: We hypothesize that the part that wins the most one-on-one competitions in the Immigration Game, with competitions between all possible pairs of parts, is likely to be the part that eventually becomes the manager of the symbiote in the Management Game. For each possible pair of parts, extracted from the symbiote, we hold 500 competitions in the Immigration Game. In the cellular grid, the initial (t = 0) relative position and rotation of the two competing parts varies randomly with each competition. The fitness of a part is the fraction of one-on-one competitions that it wins (at t = 1000). This is the measure of fitness that is used for evolution in Model-S, except that fitness in Model-S is based on competitions among all entities in the population, whereas fitness here is measured by competitions among only the parts of the given symbiote. In each competing pair, one part is coloured red and the other part is coloured blue. The winner of a one-on-one competition is the colour that grows the most.

Hypothesis 3: We hypothesize that the part that wins the most one-on-one competitions in the Management Game, with competitions between all possible pairs of parts, is likely to be the part that eventually becomes the manager of the symbiote in the Management Game. For each possible pair of parts, extracted from the symbiote, we hold 500 competitions in the Management Game. In each competing pair, at t = 0, one part is coloured red and the other part is coloured blue. This is much like Hypothesis 2, except the scoring is different. The winner of a competition is the part that tends to contribute two cells to a birth, rather than one. If a given part is red, it is scored by the number of orange cells it produces at t = 1000. If a given part is blue, it is scored by the number of green cells it produces at t = 1000. If the orange count is greater than the green count, then red wins. If the green count is greater than the orange count, then blue wins. If there is a tie, each part gets half a point.

Table 7 shows the results for the three hypotheses. Rows 3 to 6 are conditional probabilities. Recall that we are restricting our attention to the cases where there is 1 manager and N-1 workers. Row 3 in the table gives the probability that random guessing will successfully identify the manager. This serves as a baseline for comparison with Rows 4 to 6.

Insert Table 7 here.

Hypothesis 1: Row 4 shows that Hypothesis 1 is predicting better than random guessing (Row 3) when there are two or three parts, but Hypothesis 1 is worse than random guessing when the symbiotes have four parts. It seems that growth in isolation from the other parts becomes a less reliable sign of managers as the number of parts increases.

Hypothesis 2: Row 5 shows that Hypothesis 2 is superior to Hypothesis 1 (Row 4) in all cases (2, 3, or 4 parts). This indicates that growth in one-on-one competitions is a better predictor for managers than growth in isolation.

Hypothesis 3: Row 6 shows that Hypothesis 3 is superior to Hypothesis 1 (Row 4) in all cases. It is inferior to Hypothesis 2 (Row 5) in two cases (symbiotes with 2 parts and symbiotes with 4 parts). It is superior to Hypothesis 2 in one case (symbiotes with 3 parts).

Of the three hypotheses, Hypothesis 2 is the most successful. The symbiotes were created by evolution in Model-S, where fitness is measured by one-on-one competitions in the Immigration Game, so it is not surprising that success in the Immigration Game is a good predictor of which part will be a manager. Consider the rules of the Immigration Game in Table 2. If red contributes two cells to the birth of a new cell, then the new cell will be red and red's growth will increase by one cell. If red contributes one cell to the birth of a new cell, then the new cell will be blue and blue's growth will increase by one cell. Therefore evolution in Model-S will favour seed patterns that are more likely to contribute two cells to the birth of a new cell over seed patterns that are more likely to contribute one cell.

5 Future Work and Limitations

Model-S evolves a population of seed patterns using a fitness measure based on competitive growth. Seed patterns compete in the Immigration Game, where the winner is the pattern that grows the most. The fitness of a seed pattern in Model-S is measured by the average number of games that it wins, competing against all of the other seeds in the population. This fitness measure would naturally favour symbiotes with exactly one manager, because additional managers would not support growth as efficiently as one manager, and zero managers would not grow as efficiently as one manager. Thus we believe that the results reported in Section 4 are a consequence of the fitness measure used in Model-S.

In future work, we plan to test our hypothesis that the fitness measure in Model-S (based on the Immigration Game) is responsible for the dominance of symbiotes with exactly one manager. It is not yet clear to us how to test the hypothesis. One possibility would be to screen every newly born symbiote, checking to see how many managers it has. If it has exactly one manager, the symbiote would be eliminated; otherwise, it would enter the population as usual. We expect that this modified version of Model-S would greatly reduce the fitness of the population, when compared with the current version of Model-S. This reduction in fitness would be evidence that exactly one manager is optimal for maximizing fitness, as measured by a series of one-on-one competitions.

In Section 4, for all three types of symbiotes (two-part symbiotes, three-part symbiotes, and four-part symbiotes), we observed that robustness, as measured by probability times productivity, is maximized with exactly one manager. However, we believe that more managers will eventually be necessary as the number of parts increases beyond four parts. Unfortunately, Model-S runs much more slowly as the number of parts increases, so testing this hypothesis will require more time, more computers, or more efficiency. We plan to explore all three of these options.

Model-S is most suited to modeling the earliest stages of life, as opposed to modeling human social organizations. Human managers and workers are clearly much more complex and difficult to model than simple cellular automata. From this perspective, it is somewhat surprising that such a simple model seems to be consistent with what we might expect for human organizations. The Management Game should be used in the spirit of Prisoner's Dilemma game (Axelrod, 1984; Poundstone, 1993; Ikegami, 1994), as a simple model that may teach us something about the real world, but should not be mistaken as an accurate representation of the real world.

6 Conclusion

In this article, we have presented a simple computational model of Stewart's (2020) Management Theory. We introduced the Management Game, an extension of Conway's Game of Life (Gardner, 1970) and Woods' Immigration Game (Wainwright, 1971), to support a model of manager-worker relations in evolved cellular automata (Section 3). Our goal was to demonstrate that Management Theory is not limited to human societies and eusocial organisms, in which it is easy to see a hierarchical structure of managers and workers. In the current article, we have demonstrated that Management Theory is applicable to an elementary cellular automaton (the Management Game) combined with an evolutionary algorithm (Model-S) that supports the formation of symbiotic entities (Section 4).

Stewart (1995; 2020) claimed that Management Theory could explain the major cooperative transitions in evolution. In essence, his argument is that, if evolution were able to create something analogous to a hierarchical system of managers and workers, then cooperation would be self-sustaining. This is a reasonable hypothesis, but the hypothesis lacks detailed empirical support, especially for the earliest major transitions, such as the evolution of eukaryotes by symbiosis of prokaryotes (Margulis, 1970; Margulis, 1981). We hope that the computational model presented here provides some empirical support for Management Theory and that it suggests ways that Management Theory could be rigorously, empirically tested in biological and cultural evolution.

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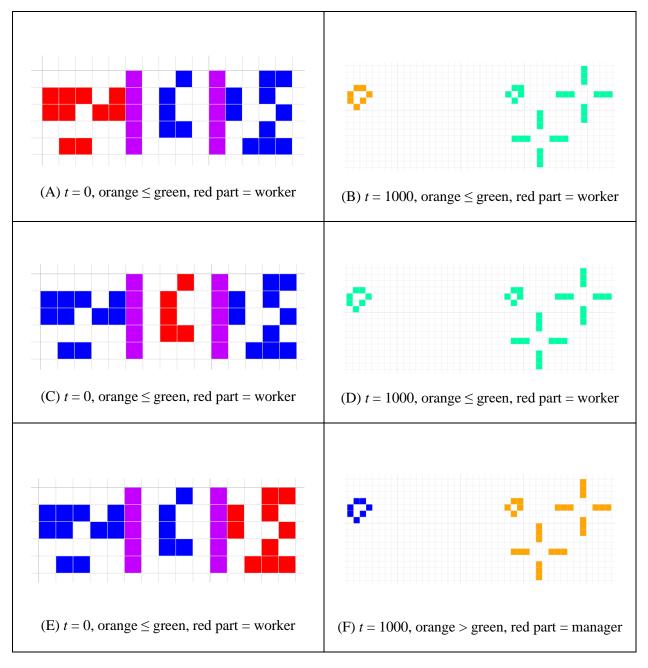


Figure 1. Image A shows an initial seed pattern (t = 0) with three parts, one red part and two blue parts. The three parts are separated by purple borders. We focus on the red part and watch how it interacts with the two blue parts. Image B shows the final pattern after one thousand steps (t = 1000) in the Management Game. Since there are more green cells than orange cells, the focal red part is classified as a worker when t = 1000. Image C shifts the focus to the central part, which is now red. Image D shows that the central part is also classified as a worker in the final pattern (t = 1000). Image E shifts the focus to the rightmost part, which is now red. Image F shows that the rightmost part is classified as a manager in the final pattern.

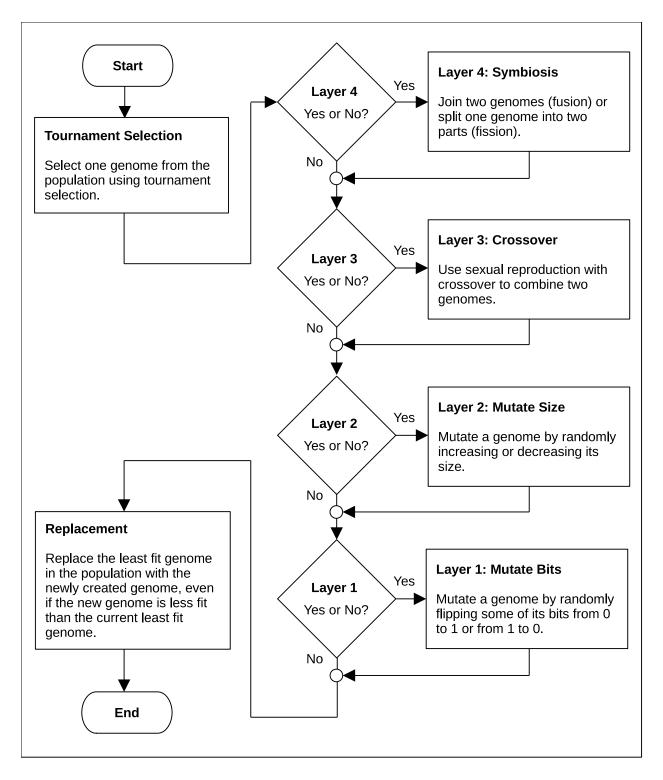


Figure 2. This flowchart outlines the process for selecting an individual's genome from the population and creating a new genome. This process is a subroutine in a loop that produces a series of new individuals. The decision to use a given layer is determined by the parameters of Model-S. This flowchart first appeared in a previous article (Turney, 2021).

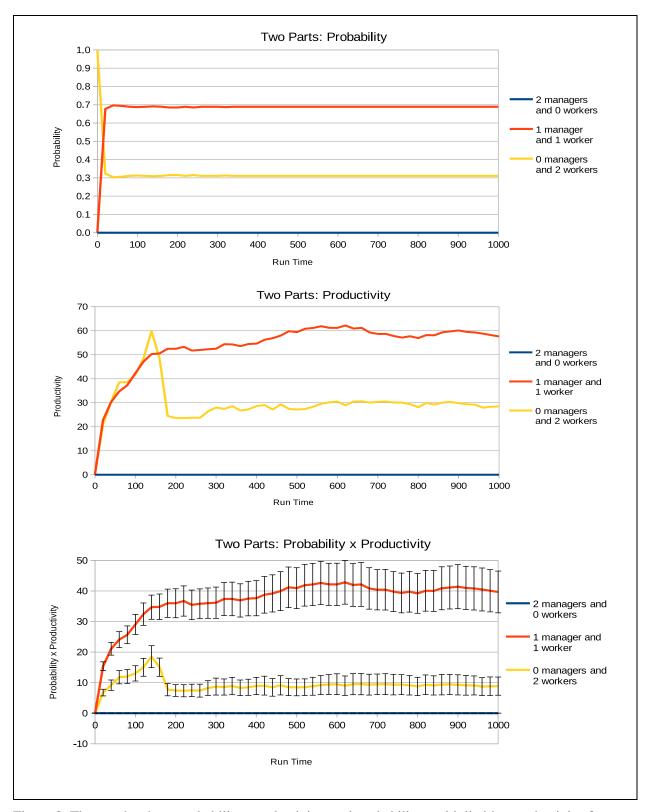


Figure 3. The graphs show probability, productivity, and probability multiplied by productivity for two-part seed patterns in the Management Game. The third graph gives 95% confidence intervals for probability times productivity. The most probable and productive combination is one manager and one worker.

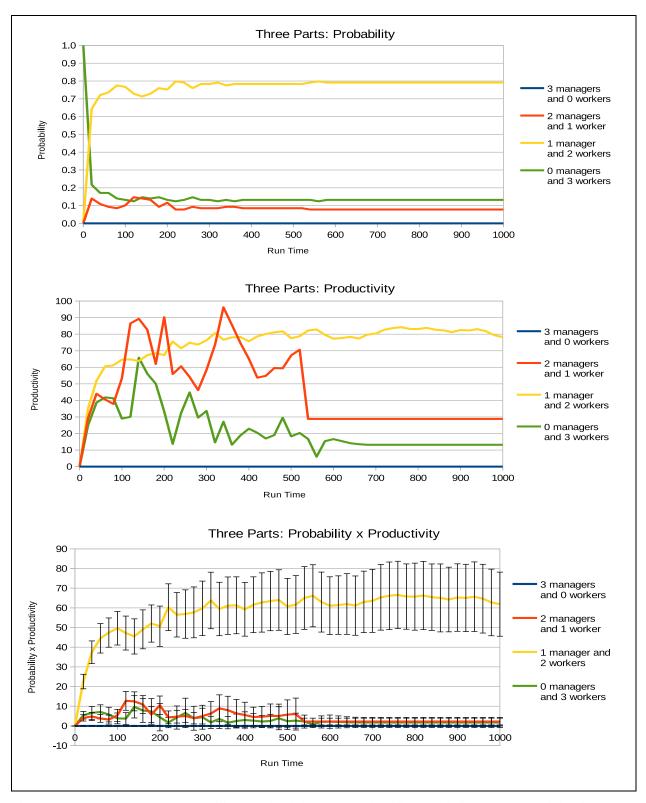


Figure 4. The graphs show probability, productivity, and probability multiplied by productivity for three-part seed patterns in the Management Game. The third graph gives 95% confidence intervals for probability times productivity. The most probable and productive combination is one manager and two workers.

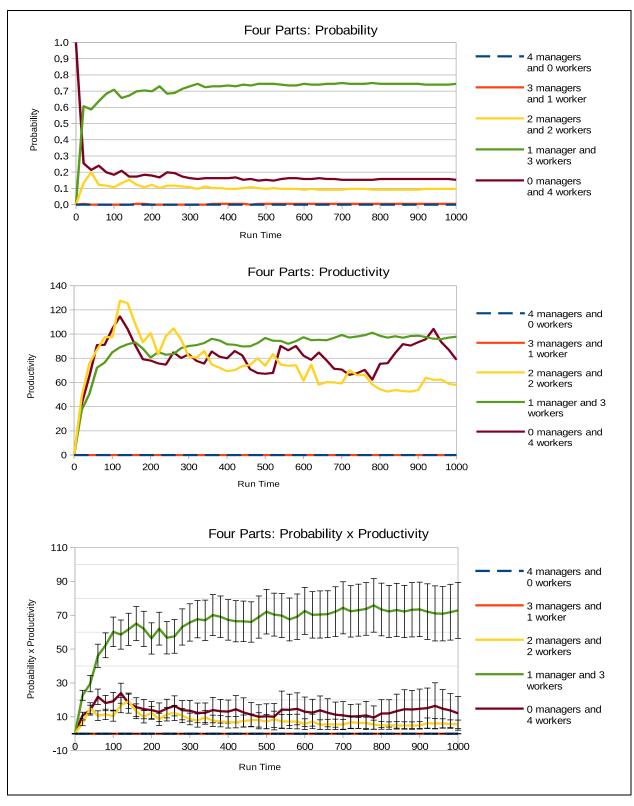


Figure 5. The graphs show probability, productivity, and probability multiplied by productivity for four-part seed patterns in the Management Game. The third graph gives 95% confidence intervals for probability times productivity. The most probable and productive combination is one manager and three workers.

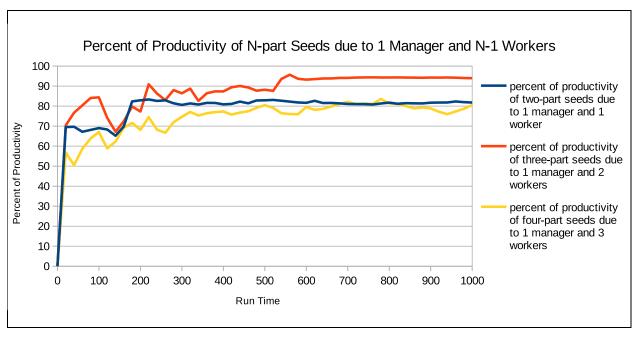


Figure 6. This graph summarizes the main results of the Management Games in Figures 3, 4, and 5. When t > 300, seed patterns with 1 manager and N-1 workers account for more than 70% of the total productivity of the set of all seeds with N parts.

Table 1. This table presents the rules of the Game of Life. Black cells are *alive* (represented as state 1 in the cellular automata grid) and white cells are *dead* (represented as state 0 in the grid). Let B represent *born*, S represent *survive*, D represent *die*, and U represent *unborn*. We use the convention that B3 implies its complement U01245678 and S23 implies its complement D0145678. This convention allows us to summarize the rules of the Game of Life as B3/S23.

Description of	State of 8 neighbouring	State of central	State of central	Abbreviation
change	cells at time <i>t</i>	cell at time t	cell at time $t + 1$	
born	3 black neighbours	white	black	B3
survive	2 or 3 black neighbours	black	black	S23
die from overpopulation	4 or more black neighbours	black	white	D45678
die from underpopulation	0 or 1 black neighbours	black	white	D01
unborn	anything other than 3 black neighbours	white	white	U01245678

Table 2. This table presents a summary of the rules of the Immigration Game. The Immigration Game follows the rules of the Game of Life, except when a new cell is born (B3); therefore the table only shows the rules for birth. White cells are *dead* (state 0) and red and blue cells are *alive* (states 1 and 2). At time t, we have an empty central cell (state 0) with eight neighbouring cells around it. The empty central cell comes alive (it is born) at time t + 1 if exactly three of the neighbouring cells are alive at time t. This much is the same as the Game of Life. The Immigration Game diverges from the Game of Life in the way that the colour of the central cell at time t + 1 is determined.

State of 8 neighbouring cells at time <i>t</i>	State of central cell at	State of central cell at	
	time t (dead)	time $t + 1$ (alive)	
3 red neighbours	white	red	
3 blue neighbours	white	blue	
2 red neighbours + 1 blue neighbour	white	red	
2 blue neighbours + 1 red neighbour	white	blue	

Table 3. This table introduces some terminology that we will use to define the rules of the Management Game. In our experiments, the initial seeds at time t = 0 are always red or blue. Orange and green only appear later at t > 0 as red and blue interact with each other. The purple borders function as if they were dead, like the white background. The borders only appear at time t = 0. They turn white at time t = 1.

States	Colours	Descriptions	
0	white	the background colour	dead
1	red	initial seed colour or a child born from 3 red parents	alive
2	blue	initial seed colour or a child born from 3 blue parents	alive
3	orange	red origins but with some past non-red contact	alive
4	green	blue origins but with some past non-blue contact	alive
5	purple	the colour of the borders that separate parts of a seed	dead
2, 3, 4	non-red	blue, orange, or green	alive
1, 3, 4	non-blue	red, orange, or green	alive
2, 4	blue/green	blue or green	alive
1, 3	red/orange	red or orange	alive

Table 4. This table presents a summary of the rules of the Management Game. We focus here on the rules for the birth of a new cell (B3), since this is the only case where the Management Game differs from the Game of Life and the Immigration Game. At time t, we have an empty (white or purple) cell with eight neighbouring cells around it. For birth to happen in the empty central cell, exactly three of the eight neighbours must be alive. The table specifies the colour the central cell will have at time t+1, based on the colours of the neighbouring cells at time t. Table 3 explains the terminology used here.

State of 8 neighbouring cells at time <i>t</i>	State of central cell at time <i>t</i> (dead)	State of central cell at time $t + 1$ (alive)
3 red neighbours	white or purple	red
3 blue neighbours	white or purple	blue
2 red/orange + 1 non-red neighbour	white or purple	orange
2 blue/green + 1 non-blue neighbour	white or purple	green

Table 5. This table looks at the Management Game from the perspective of the red part of the initial seed pattern. (Due to the symmetry of the rules of the game, we would get the same result if we swapped the roles of red and blue.) We are only interested in instances of cooperation. It is not meaningful to talk of managers and workers when there is no cooperation. Therefore we ignore the count of red and blue cells. We say that red is a *manager* when the count of orange cells is greater than the count of green cells. Otherwise, red is a *worker*. A cell can only be orange or green if there was some cooperation between red and blue at some time in the past.

State of 8 neighbouring cells at time <i>t</i>	State of central cell at time $t + 1$	Interpretation	Cooperation between red and blue
3 red neighbours	red	red made a new red cell alone	no cooperation
3 blue neighbours	blue	blue made a new blue cell alone	no cooperation
2 red/orange + 1 non-red neighbour	orange	red took the <i>leading</i> role to make a new orange cell; red acted as a <i>manager</i>	cooperation
2 blue/green + 1 non-blue neighbour	green	red took the <i>supporting</i> role to make a new green cell; red acted as a <i>worker</i>	cooperation

Table 6. This table summarizes the number of symbiotic seeds that were generated over the 20 runs of Model-S. An average of 45.5 symbiotes were generated per run. With a fixed population size of 200 seeds and evolution proceeding for 100 generations, Model-S generates 20,000 entities. Thus an average run results in 0.002275 new symbiotes per entity (45.5 symbiotes / 20,000 evolved entities).

Number of parts in	Number of symbiotic	Average seeds per	
seed pattern	seeds	run	
2	585	29.3	
3	129	6.5	
4	196	9.8	
Total	910	45.5	

Table 7. This table examines three hypotheses about the characteristics that distinguish a manager from a worker. We focus on the cases where a symbiote has exactly one manager, since these cases form the majority, as we can see in Figure 6. We attempt to predict which one of the N parts in a symbiote will be the manager at time t = 1000, based on the characteristics of each part. By the time t = 1000, the status of the parts of the symbiotes as managers or workers has stabilized.

		Symbiotes	Symbiotes	Symbiotes
		with 2 parts	with 3 parts	with 4 parts
1	Number of symbiotes with N parts ($N = 2, 3, 4$)	585	129	196
2	Number of symbiotes with exactly one manager at time $t = 1000$	403	102	146
3	Probability of randomly guessing the manager correctly (probability = $1/N$), given exactly one manager at time $t = 1000$	0.500	0.333	0.250
4	Probability that the part with the most black growth will be the manager, given exactly one manager at time $t = 1000$ (Game of Life)	0.546	0.441	0.219
5	Probability that the part that wins the most red-versus- blue contests will be the manager, given exactly one manager at time $t = 1000$ (Immigration Game)	0.581	0.451	0.356
6	Probability that the part that wins the most orange-versus-green contests will be the manager, given exactly one manager at time $t = 1000$ (Management Game)	0.576	0.480	0.295