

# Quiz 2 → Question 6

~~Back~~

Forward

Back

①

$$L = -y \ln(a) - (1-y) \ln(1-a)$$

$$\frac{dL}{da} = \frac{dL}{d\sigma(z^n)} = \frac{d}{d\sigma(z^n)} (L) = -\frac{y}{\sigma(z^n)} + \frac{1-y}{1-\sigma(z^n)}$$

$$\frac{dL}{d\sigma(z^n)} = -\frac{1}{\sigma(z^n)} + \frac{1-y}{1-\sigma(z^n)}$$

$$y=1$$

$$\sigma(z^n) = \cancel{\frac{1}{2}} \frac{1}{2}$$

$$\frac{dL}{d\sigma(z^n)} = -\frac{1}{1/2} = -2$$

②

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\sigma(z^n) = \frac{1}{2}$$

$$\frac{d\sigma}{dz} = \sigma(1-\sigma)$$

$$\frac{dL}{dz^n} = \frac{dL}{d\sigma(z^n)} \frac{d\sigma(z^n)}{dz^n} = (-2) \left( \frac{1}{2} \right) \left( 1 - \frac{1}{2} \right) = -0.5$$

③

$$z^n = \cancel{w_1 w_2 w_3} \langle w_3^n, h \rangle$$

$$= w_1 h_1 + w_2 h_2 + w_3 h_3$$

$$= w_1 \sigma(z_1) + w_2 \sigma(z_2) + w_3 \sigma(z_3)$$

$$\frac{dz^n}{d\sigma(z_1)} = \frac{d}{d\sigma(z_1)} (w_1 \sigma(z_1) + \dots + w_3 \sigma(z_3))$$

$$= w_1$$

$$\frac{dL}{d\sigma(z_1)} = \frac{dL}{dz^n} \frac{dz^n}{d\sigma(z_1)} = (-0.5)(w_1) = (-0.5)(-1)$$

$$\frac{dL}{d\sigma(z_1)} = 0.5$$

④

$$\sigma(z_1) = \frac{1}{1+e^{-z_1}}$$

$$z_1 = x_1 w_{11} + x_2 w_{21} = 0$$

$$\frac{dL}{dz_1} = \frac{dL}{d\sigma(z_1)} \frac{d\sigma(z_1)}{dz_1}$$

$$= (0.5) (\sigma(z_1) * (1 - \sigma(z_1))) = (0.5)(0.5)(0.5)$$

$$\frac{dL}{dz_1} = 0.125$$

FORWARD

(5)

$$z_1 = x_1 w_{11} + x_2 w_{21} = 0$$

Back

$$\frac{dL}{dw_{11}} = \left( \frac{dL}{dz_1} \right) \left( \frac{dz_1}{dw_{11}} \right) = (0.125) \left( \frac{x_1}{1} \right)$$

$$\frac{d}{dw_{11}} (z_1) = x_1$$

$$\frac{dL}{dw_{11}} = 0.125$$

Question 7

$$w_{21}^{t+1} \leftarrow w_{21}^t - \eta \frac{dL}{dw_{21}} = \frac{1}{2} - \left( \frac{1}{2} \right) (-0.125) = \frac{1}{2} + \frac{1}{16} = \frac{9}{16} \approx 0.563$$

$$\frac{dL}{dw_{21}} = \frac{dL}{dz_1} \frac{dz_1}{dw_{21}} \xrightarrow{\text{at } z_1 = 0} \frac{dL}{dz_1} x_2 = (0.125)(-1) = -0.125$$