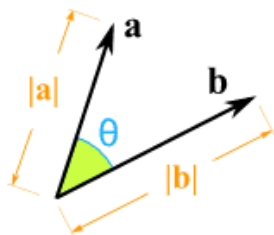# Dot Product

These are [vectors](#):

They can be **multiplied** using the "**Dot Product**" (also see [Cross Product](#)).

Calculating

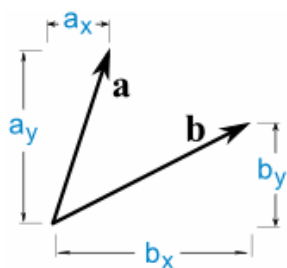You can calculate the Dot Product of two vectors this way:

a · b = |**a**| × |**b**| × cos(θ)

Note: |**a**| means the magnitude (length) of vector **a**

(multiply the length of **a** times the length of **b**, then multiply by the cosine of the angle between **a** and **b**)

OR you can calculate it this way:

a · b = $a_x × b_x + a_y × b_y$

(multiply the x's, multiply the y's, then add)

They both work!

Example: Calculate the dot product of vectors **a** and **b**:
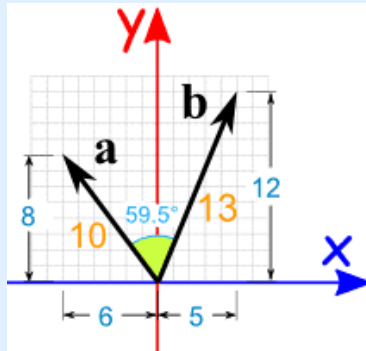
$a \cdot b = |a| \times |b| \times \cos(\theta)$

➤ $a \cdot b = 10 \times 13 \times \cos(59.5°)$

➤ $a \cdot b = 10 \times 13 \times 0.5075...$

➤ $a \cdot b = 65.98... = 66$ (rounded)

$a \cdot b = a_x \times b_x + a_y \times b_y$

➤ $a \cdot b = -6 \times 5 + 8 \times 12$

➤ $a \cdot b = -30 + 96$

➤ $a \cdot b = 66$

Both methods came up with the same result (after rounding)

Also note that we used **minus 6** for a_x (it is heading in the negative x-direction)
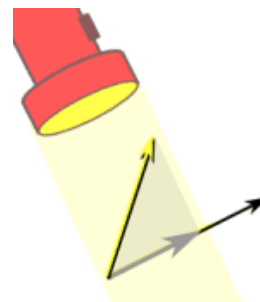
Note: you can use the Vector Calculator to help you.

Why cos(θ) ?

OK, to multiply two vectors it makes sense to multiply their lengths together **but only when they point in the same direction**.

So we make one "point in the same direction" as the other by multiplying by cos(θ):
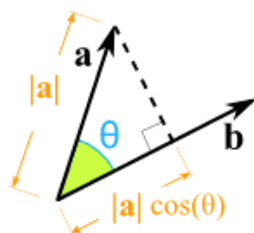
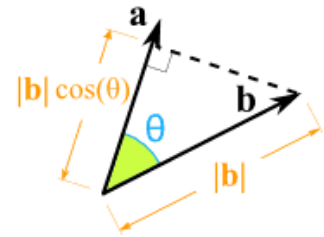We take the component of **a** that lies alongside **b**

Like shining a light to see where the shadow lies

THEN we multiply !

It works exactly the same if we "projected" **b** alongside **a** then multiplied:

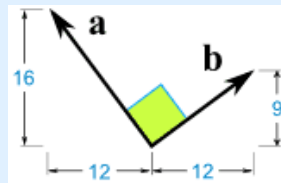Because it doesn't matter which order we do the multiplication:

$$|a| \times |b| \times \cos(\theta) = |a| \times \cos(\theta) \times |b|$$

Right Angles

When two vectors are at right angles to each other the dot product is **zero**.

Example: calculate the Dot Product for:

a · **b** = |a| × |b| × cos(θ)

➜ **a · b** = |a| × |b| × cos(90°)

➜ **a · b** = |a| × |b| × 0

➜ **a · b** = 0

a · b = $a_x \times b_x + a_y \times b_y$

➜ **a · b** = -12 × 12 + 16 × 9
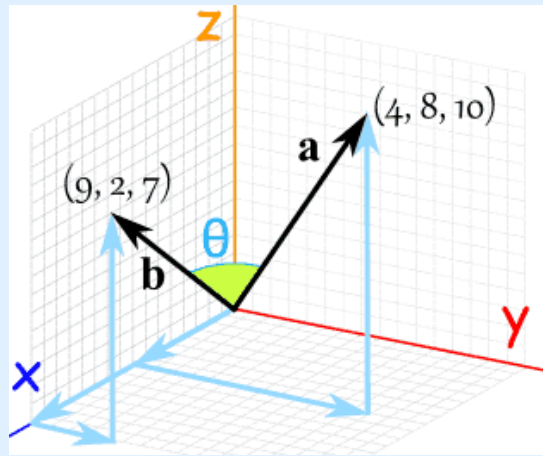
➜ **a · b** = -144 + 144

➜ **a · b** = 0

This can be a handy way to find out if two vectors are at right angles.

Three or More Dimensions

This all works fine in 3 (or more) dimensions, too.

And can actually be very useful!

Example: Sam has measured the end-points of two poles, and wants to know the angle between them:

We have 3 dimensions, so don't forget the z-components:

$\mathbf{a} \cdot \mathbf{b} = a_x \times b_x + a_y \times b_y + a_z \times b_z$

➜ $\mathbf{a} \cdot \mathbf{b} = 9 \times 4 + 2 \times 8 + 7 \times 10$

➜ $\mathbf{a} \cdot \mathbf{b} = 36 + 16 + 70$

➜ $\mathbf{a} \cdot \mathbf{b} = 122$

Now for the other formula:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \times |\mathbf{b}| \times \cos(\theta)$

But what is |**a**| ? It is the magnitude, or length, of the vector **a**. We can use [Pythagoras](#):

- $|\mathbf{a}| = \sqrt{(4^2 + 8^2 + 10^2)} = \sqrt{(16 + 64 + 100)} = \sqrt{180}$

Likewise for |**b**|:

- $|\mathbf{b}| = \sqrt{(9^2 + 2^2 + 7^2)} = \sqrt{(81 + 4 + 49)} = \sqrt{134}$

And we know from the calculation above that **a · b** = 122, so:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \times |\mathbf{b}| \times \cos(\theta)$

➜ $122 = \sqrt{180} \times \sqrt{134} \times \cos(\theta)$

➜ $\cos(\theta) = 122 / (\sqrt{180} \times \sqrt{134})$

➜ $\cos(\theta) = 0.7855...$

➜ $\theta = \cos^{-1}(0.7855...) = 38.2...°$

Done!

I tried a calculation like that once, but worked all in angles and distances ... it was very hard, involved lots of trigonometry, and my brain hurt. The method above is much easier.

Cross Product

The Dot Product gives a **scalar** (ordinary number) answer, and is sometimes called the "scalar product"

But there is also the Cross Product which gives a **vector** as an answer.

Your turn: Question 1 Question 2 Question 3 Question 4 Question 5
Question 6 Question 7 Question 8 Question 9 Question 10

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