Time Value of Money

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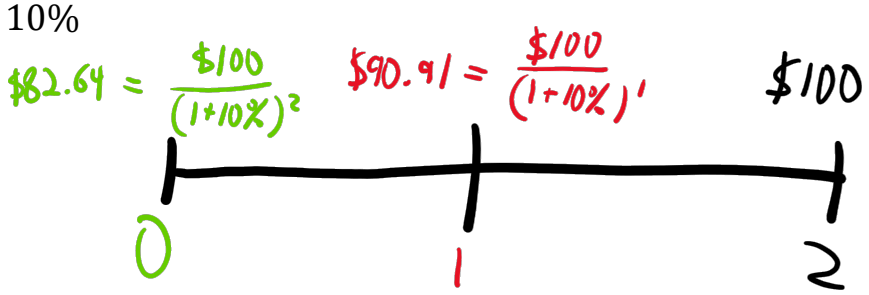
Agenda

- Discounting cash flows
  - Single
  - Annuity
  - Perpetuity
- Excel TVM functions
- Interest rates
  - EAR & IRR
- Challenge problems
- Current event discussion

Present & Future Value Terminology

- Present Value: value of cash flows today
- Future Value: value of cash flows in the future
- Discount rate: relates the value of money across time periods and risky states

- $r = 10\%$



Tools

We will learn how to solve these problems:

- Analytically with formulas: I do not want you to view Excel as a black box
- Excel: I want you to be efficient

Tips

- Do not round intermediate calculations
- Draw timelines
  - Help you see where cash flows happen
  - Give you a sense of what info you have, and what you will be solving for
- Interest rates and periods need to be in sync
  - If you're talking about monthly cash flows, then interest rate and periods needs to be in monthly terms

TVM Formulas

- Present value of future cash flow

$$PV_0 = \frac{FV_t}{(1+r)^t}$$

- Present value of annuity

$$PV_0 = \sum_{t=1}^{n} \frac{PMT}{(1+r)^t}$$

- Present value of perpetuity

$$PV_0 = \frac{PMT_1}{r-g}$$

Excel TVM Functions

Regular cash flows

- $PV = PV(r, n, PMT, FV)$
- $FV = FV(r, n, PMT, PV)$
- $PMT = PMT(r, n, PV, FV)$
- $r = RATE(n, PMT, PV, FV)$

Irregular cash flows

- $PV = CF_0 + NPV(r, CF_t)$
- $r = IRR(CF)$

Interest Rate Formula

- EAR: Effective Annual Rate
  - True cost of borrowing on an annualized frequency
- IRR: Internal rate of return
  - r effective period rate

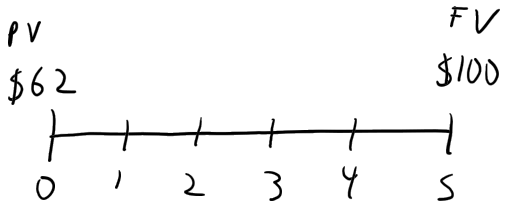
$$1 + EAR = (1 + r)^n$$

Present Value of Single Cash Flow

$$PV_0 = \frac{FV_t}{(1+r)^t}$$

Example:

- Receive $100 in 5 years
- Discount rate 10%
- Present value today?



$$PV_0 = \frac{\$100}{(1+10\%)^5} = \$62.09$$

Excel

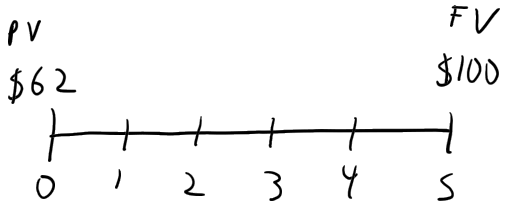
- $PV(r = 10\%, n = 5, , FV = 100) = 62.09$

Future Value of a Single Cash Flow

$$PV_0 = \frac{FV_t}{(1+r)^t} \Leftrightarrow FV_t = PV_0 \times (1+r)^t$$

Example:

- Invest $62.09 today
- Discount rate 10%
- Future value in year 5?



$$FV_5 = \$62.09 \times (1+10\%)^5 = \$100$$

Excel

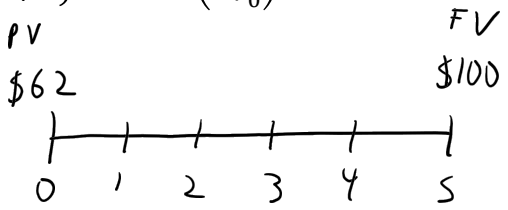
- $FV(r = 10\%, n = 5, , PV = 62.09) = 100$

Internal Rate of Return (IRR) of a Single Cash Flow

$$PV_0 = \frac{FV_t}{(1+r)^t} \Leftrightarrow r = \left(\frac{FV_t}{PV_0}\right)^{1/t} - 1$$

Example:

- Pay $62.09 today
- Receive $100 in 5 years
- What is the return?



$$r = \left(\frac{\$100}{\$62.09}\right)^{1/5} - 1 = 10\%$$

Excel

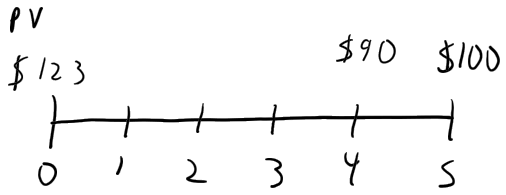
- $RATE(n = 5,, PV = -62.09, FV = 100) = 10\%$

Challenge Problem

An asset will pay $300 in year 7. What is the value of asset in year 4 if the annual discount rate is 8%?

Present Value of Multiple Cash Flows

$$PV_0 = \sum_t \frac{FV_t}{(1+r)^t}$$



Example:

- Receive $90 and $100 in 4 and 5 years
- Discount rate 10%
- Present value today?

$$PV_0 = \frac{\$90}{(1+10\%)^4} + \frac{\$100}{(1+10\%)^5} = \$123.56$$

Annuity

- An annuity is an asset that pays a fixed cash flow for a fixed number of time periods
  - The price is the present value of the payments at time 0
  - The default timing assumption is that the first payment occurs at time 1

- For example: You will receive $100 per year for 3 years

Annuity – Present Value

$$PV_0 = \sum_{t=1}^{n} \frac{PMT}{(1+r)^t}$$
$$= PV(r, n, PMT)$$

Annuity Example

- Receive \$100 per year for three years
- Discount rate 10%

- Present value of annuity

$$PV_0 = \frac{\$100}{(1+10\%)^1} + \frac{\$100}{(1+10\%)^2} + \frac{\$100}{(1+10\%)^3}$$
$$= PV(r = 10\%, n = 3, PMT = \$100)$$
$$= \$248.69$$

- Future value of annuity

$$FV_3 = \$100 \times (1+10\%)^2 + \$100 \times (1+10\%)^1 + \$100$$
$$= FV(r = 10\%, n = 3, PMT = \$100)$$
$$= \$331$$

Annuity – Payments

- Pay $248.69 for a 3-year annuity
- Discount rate 10%
- What are the payments?

$$\$248.69 = \frac{PMT}{(1+10\%)^1} + \frac{PMT}{(1+10\%)^2} + \frac{PMT}{(1+10\%)^3}$$
$$\Rightarrow PMT = \$100$$

Excel

- $PMT(r = 10\%, n = 3, PV = -248.69) = 100$

Annuity – IRR

- Pay $248.69 for a 3-year annuity
- Receive $100 per year for three years
- What is the return?

$$\$248.69 = \frac{\$100}{(1+r)^1} + \frac{\$100}{(1+r)^2} + \frac{\$100}{(1+r)^3}$$
$$\Rightarrow r = 10\%$$

Excel

- $RATE(n = 3, PMT = 100, PV = -248.69) = 10\%$

Annuity Challenge Problem

You are 25 years old. Starting next year, you want to start making equal annual contributions for your retirement. You plan to retire at age 60 (and your last annual contribution will be when you turn 60). You want to have $1,000,000 saved when you retire at 60. With an annual discount rate of 6%, how much must you contribute each year to meet your retirement goal?

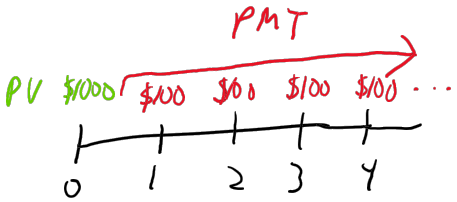
Perpetuity

- A perpetuity is an asset that pays a fixed cash flow for an infinite number of time periods
  - The price is the present value of the payments at time 0
  - The default timing assumption is that the first payment occurs at time 1
  - Payments may grow at fixed growth rate g

$$PV_0 = \frac{PMT_1}{r - g}$$

Example:

- Receive $100 per year, forever
- Discount rate 10%
- Present value today?



$$PV_0 = \frac{\$100}{10\%} = \$1000$$

Perpetuity Challenge Problem

You are 30 years old. You buy an asset that will begin paying your family annual payments in perpetuity when you turn 61. The first payment is $12,000 and will be made at age 61, and the payments will grow 2% per year. If the annual discount rate is 6%, what is the price of the asset today?

Excel TVM Functions

Regular cash flows

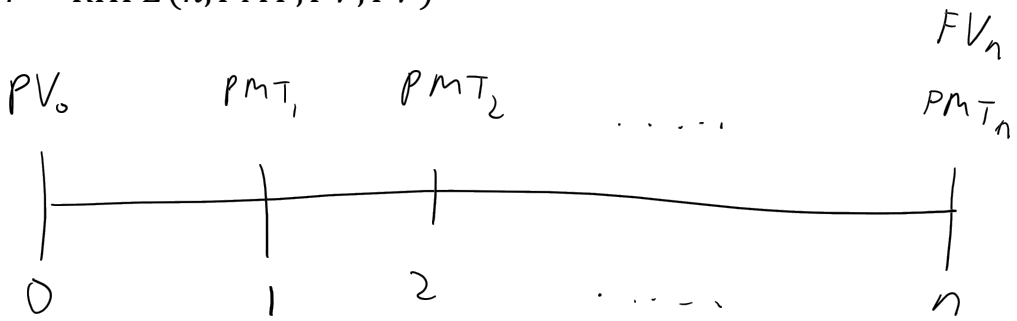
- $PV = PV(r, n, PMT, FV)$
- $FV = FV(r, n, PMT, PV)$
- $PMT = PMT(r, n, PV, FV)$
- $r = RATE(n, PMT, PV, FV)$

Irregular cash flows

- $PV = CF_0 + NPV(r, CF_t)$
- $r = IRR(CF)$

Standard TVM Functions

- $PV = PV(r, n, PMT, FV)$
- $FV = FV(r, n, PMT, PV)$
- $PMT = PMT(r, n, PV, FV)$
- $r = RATE(n, PMT, PV, FV)$



Timing type

Optional input *type* that controls timing of first payment, e.g.

- $PV = PV(r, n, PMT, FV, type)$
- Default: *type*=0 ⟹ First payment at time 1
- Override: *type*=1 ⟹ First payment at time 0
- Note PV and FV do NOT move

Generic TMV Functions

If we have an irregular stream of cash flows, we can use:

- $PV = CF_0 + NPV(r, CF_t)$
- $r = IRR(CF)$

EAR & IRR

- EAR: Effective Annual Rate
  - True cost of borrowing on an annualized frequency
- IRR: Internal rate of return
  - r effective period rate

$$1 + EAR = (1 + r)^n$$

Example 1

Loan with 1% monthly rate

- Effective Annual Rate: $EAR = 12.7\%$

$$1 + EAR = (1 + 1\%)^{12}$$
$$EAR = 12.7\%$$

Example 2

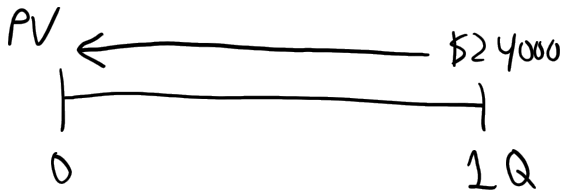
Your opportunity cost of capital is 1.5% quarterly. Calculate the following monthly and annual rates:

$$\text{monthly rate} = (1 + 1.5\%)^{\frac{1}{3}} - 1 = 0.4975\%$$
$$\text{annual rate} = (1 + 1.5\%)^{4} - 1 = 6.1364\%$$

Example 3

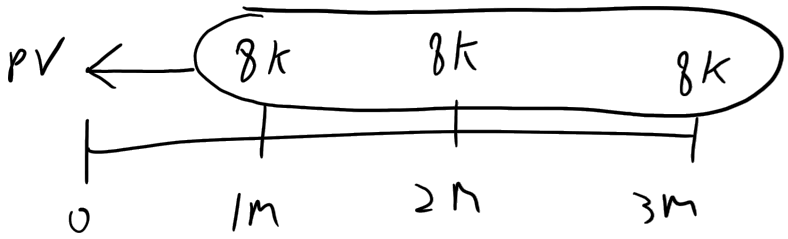
Your opportunity cost of funds is1.5% quarterly. What is the value today of an internship that pays $24,000 in three months?

$$PV = \frac{\$24{,}000}{(1 + 1.5\%)^1} = \$23{,}645$$



Example 4

Your opportunity cost of funds is 1.5% quarterly. What is the value today of an internship that pays $8,000 per month for three months?



Example 4

Your opportunity cost of funds is 1.5% quarterly. What is the value today of an internship that pays $8,000 per month for three months?

$$\begin{aligned}\text{monthly rate} &= 0.4975\% \\ \text{PV} &= PV(r = 0.4975\%, n = 3, PMT = \$8000) \\ &= \$23{,}763\end{aligned}$$

IRR

The IRR can be found with the Excel functions

- $r = IRR(CF)$
- $r = RATE(n, PMT, PV, FV)$

IRR Example

You have been offered the following opportunity: Receive a tax credit of $10,000 today, and in return, you must make higher tax payments of $1,000 per year for 15 years. The first of the annual payments will come in one year's time. What is the internal rate of return on this investment?

$$PV = \sum_{t=1}^{n} \frac{PMT}{(1+r)^t}$$

$$\$10{,}000 = \sum_{t=1}^{15} \frac{\$1{,}000}{(1+r)^t}$$

$$\Rightarrow r = 5.56\%$$

Excel

- $IRR(-10000{,}1000, \cdots, 1000\ ) = 5.56\%$
- $RATE(n = 15, PMT = 1000, PV = -10{,}000) = 5.56\%$

Challenge Problem 1

Starting today, you are going to save $100 per year. Including the $100 payment today, you will make a total of 10 payments. At an interest rate of 5%, what is the present value of the 10 payments?

Challenge Problem 2

You buy an asset that pays $100 payments in years 3, 4 and 5. In year 5, the asset also pays an additional $1,000. The discount rate is 5%. What is the price of the asset today?

Challenge Problem 3

You are 21 years old. Starting next year, you want to start making equal annual contributions for a survivor's benefit. You plan to retire at age 62 (and your last annual contribution will be when you turn 62). You estimate you will pass at age 85. When you pass, you want to leave your spouse a survivor's benefit that will pay them $12,000 per year in perpetuity. The first payment made to your spouse will be at age 86. With an annual discount rate of 6%, how much must you contribute each year to fund the survivor's benefit?

Challenge Problem 4

You buy an asset that will pay you $100 per month for the next 24 months. The asset's rate of return is 6% semiannually. What is the price of the asset?

Challenge Problem 5

You pay $100 for an asset today. It will pay you $300, $400 and $500 in years 3, 4 and 5. What is the asset's IRR?

Challenge Problem 6

You are buying a new car. The dealership offers you two loan options. Loan 1 has a 0.03% daily rate. Loan 2 has a 6% semi-annual rate. Which loan is cheaper?