MBA@772

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Week 1 - Time Value of Money

Present & Future Values

Present value of future cash flow at time t

$$PV_0 = \frac{FV_t}{(1+r)^t}$$

Present value of annuity with growth

$$PV_0 = \mathrm{PMT}_1 \times \mathrm{ADF}\,(t, r, g)$$
$$\mathrm{ADF}\,(t, r, g) = \left[\frac{1 - \left(\frac{1+g}{1+r}\right)^t}{r-g}\right]$$

Future value of annuity with growth

$$FV_t = \mathrm{PMT}_1 \times \mathrm{ACF}\,(t, r, g)$$
$$\mathrm{ACF}\,(t, r, g) = \left[\frac{(1+r)^t - (1+g)^t}{r-g}\right]$$

Present value of perpetuity with growth

$$PV_0 = \frac{\mathrm{PMT}_1}{r-g}$$

APR, EAR & Converting Rates

The stated annual rate (also known as the APR) is the rate that is stated. It is a fake rate because it does not include the effects of compounding interest. It must be converted to a true rate. The APR is written as _____% per year compounded _____. You can only divide the APR by the compounding frequency (semi-annual, quarterly, weekly, etc.).

The Effective Annual Yield (EAR) is a true rate and includes the effects of compounding interest. You can NOT divide EAR by 2, 4, 12 to get the semi-annual, quarterly, monthly rate.

If your cash flows are at a different frequency then the compounding frequency, then you need to calculate the EAR and then convert the EAR into the rate that aligns with the frequency that match the cash flows.

Example:

Your bank offers you a loan of $100 at an annual rate of 12%, compounded monthly. How much interest would you pay in one year?

Note that the stated rate is 12%. This is our fake rate, i.e. APR=12%.

Since the compounded frequency is monthly, we must calculate the true monthly rate. Our generic formulas are:

$$1 + APR = 1 + monthly\ rate \times 12$$
$$1 + EAR = (1 + monthly\ rate)^{12}$$

Using the APR formula, we have

$$monthly\ rate = \frac{\text{APR}}{12} = \frac{12\%}{12} = 1\%$$

The monthly rate of 1% is a true rate that incorporates compounding of interest. We could calculate the interest paid over the course of a year (i.e. over the next 12 1-month periods) as

$$\$100 \times (1 + 1\%)^{12} = \$112.68$$

and we see that we would pay \$12.68 in interest over the course of a year. We could also calculate the EAR using the EAR formula

$$EAR = (1 + monthly\ rate)^{12} - 1 = (1 + 1\%)^{12} - 1 = 12.68\%$$

We could also calculate the interest paid over the course of a year as

$$\$100 \times (1 + 12.68\%)^{1} = \$112.68$$

We see that the interest is again \$12.68. We see that the monthly rate and EAR both take into compounding of interest.

Valuation

Price any asset by discounting its expected future cash flows back at an appropriate, risk adjusted discounted rate

$$Price = \sum_{t=0}^{\infty} \frac{CF_t}{(1+r)^t}$$

Excel Formulas

$$\begin{aligned}
PV &= CF_0 + NPV\,(r, CF_t) \\
PV &= PV\,(r, n, PMT, FV) \\
FV &= FV\,(r, n, PMT, PV) \\
PMT &= PMT\,(r, n, PV, FV) \\
r &= IRR\,(\text{CF}) \\
r &= RATE\,(n, PMT, PV)
\end{aligned}$$

Week 2 - Bond Valuation

Cash flows

- Coupons
  - Payments over time
  - Coupon rate determines the coupon payments
  - Coupon = Coupon Rate × Face
- Face (or par) value
  - Single lump sum payment at bond's maturity
- Coupons & face do not change

Discount rate

- Yield to maturity (ytm)
- Market rate and can change over time

Note that both the coupon rate and ytm should be treated as stated rates. So if we have a bond with \$1000 face that makes semi-annual payments with a stated annual coupon rate of 6% and annual yield to maturity of 10% then

- 6-month coupon payment $= \frac{6\%}{2} \times \$1000 = \$30$
- 6-month discount rate for cash flows $= \frac{10\%}{2} = 5\%$

Time to maturity

- How long the bond lives

Price

$$\begin{aligned} Price &= PV\,(\text{Cash Flows}) \\ &= PV\,(\text{Coupons}) + PV\,(\text{Face}) \\ &= \text{Coupon} \times \left[\frac{1 - \frac{1}{(1+r)^n}}{r}\right] + \frac{\text{Face}}{(1+r)^n} \end{aligned}$$

- Note we use annuity formula to price coupons and single cash flow formula to price face
- If coupon rate goes up, price goes up (you like getting more future cash flows)
- If ytm goes up, price goes down (you dislike more risk)

Spot Rates

The n-year spot rate r_n is found from a n-year, zero-coupon bond

$$Price = \frac{\text{Par}}{(1+r_n)^n}$$

Yield Curve

The yield curve plots the spot rates (y-axis) against the time to maturity (x-axis). Shows evolution of spot rates across time.

Week 3 - Stock Valuation

Cash flows

- Dividends
  - Payments over time
  - Can be irregular
- Future stock price if sell stock
  - Known as capital gains

Discount rate

- Cost of equity (sometimes denote k_e)
- Can decompose total return = capital gains + dividend yield
- $r = g + \frac{D_1}{P_0}$

Price

Ex-dividend price is the present value today of future dividends paid at time 1, 2, 3, ...

$$\begin{aligned} Price &= PV\left(\text{Cash Flows}\right) \\ &= PV\left(D_1, D_2, D_3, \ldots\right) \\ &= \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + \cdots \end{aligned}$$

Note that we cannot write down the infinite summation, so we must make assumptions about the dividends to use our perpetuity formulas

Constant Growth Model

- Assume dividends grow at a constant rate of g
- $D_{t+1} = D_t \times (1+g)$
- Note the timing difference in our formula (price today, dividend tomorrow)

$$P_0 = \frac{D_1}{r-g}$$

Differential Growth Model

- Assume short run dividends followed by long run dividends following constant growth
- To value stock, we need to:
  1. Estimate future dividends in the foreseeable future
  2. Estimate the future stock price when the stock becomes a constant growth stock

Compute the total present value of the estimated future dividends and future stock price at the appropriate discount rate

Week 4 - Portfolio Theory

Center

- Mean, average and expected return are synonymous in this section
- Expected Return of an Asset

$$E[R] = \sum_{j=1}^{s} \text{Prob}_j \times R_j$$

- Expected Return Portfolio

$$E[R_P] = \sum_{i=1}^{n} w_i \times E[R_i]$$

Dispersion

- Variance of an Asset

$$\text{Var}(R) = \sigma^2 = \sum_{j=1}^{s} \text{Prob}_j \times (R_j - E[R])^2$$

- Variance of a Portfolio of Two Assets

$$\sigma_P^2 = w_1^2 \cdot \sigma_1^2 + w_2^2 \cdot \sigma_2^2 + 2 \cdot w_1 \cdot w_2 \cdot \sigma_{1,2}$$

- Standard Deviation

$$\text{SD}(R) = \sigma = \sqrt{\text{Var}(R)} = \sqrt{\sigma^2}$$

Comovement

- Covariance

$$\text{Cov}(R_1, R_2) = \sigma_{1,2} = \sum_{j=1}^{s} \text{Prob}_j \times (R_{1,j} - E[R_1]) \times (R_{2,j} - E[R_2])$$

- Correlation
  - Bounded between -1 and 1

$$\text{Corr}(R_1, R_2) = \rho_{1,2} = \frac{\sigma_{1,2}}{\sigma_1 \sigma_2}$$

Week 5 - CAPM

CAPM

$R_i = R_f + \beta_i \times (R_M - R_f)$

- Required return for asset i: R_i
- Risk-free asset: R_f
- Asset i's beta: β_i
- Risk Premium: $R_M - R_f$

Security's β

$$\beta_i = \frac{\mathrm{Cov}\,(R_i, R_M)}{\mathrm{Var}\,(R_M)} = \frac{\sigma_{\mathrm{iM}}}{\sigma_M^2} = \frac{\sigma_i \sigma_M \rho_{\mathrm{iM}}}{\sigma_M^2} = \frac{\sigma_i}{\sigma_M} \rho_{\mathrm{iM}}$$

If efficient portfolio (i.e. on CML), then $\rho_{\mathrm{iM}} = 1$ and hence

$$\beta_i = \frac{\sigma_i}{\sigma_M} \rho_{\mathrm{iM}} = \frac{\sigma_i}{\sigma_M}$$

Portfolio β_P

$$\beta_P = \sum_{i=1}^{n} w_i \times \beta_i$$

Capital Market Line

CML: Plot R vs σ

- Slope: Sharpe $\frac{R_M - R_f}{\sigma_M}$
- An efficient portfolio falls on the CML and hence

$$\begin{aligned} R_i &= R_f + \beta_i \times (R_M - R_f) \\ &= R_f + \frac{\sigma_i}{\sigma_M} \times (R_M - R_f) \\ &= R_f + \sigma_i \times \frac{R_M - R_f}{\sigma_M} \end{aligned}$$

Security Market Line

SML: Plot R vs β

- Slope: $RP_M = R_M - R_f$
- $R_i = R_f + \beta_i \times (R_M - R_f)$