- if/else
- loops
- functions
- Arrays
- Strings

Prime No. Any positive no that has exactly 2 factors

2, 3, 5, 7, 11, 13, 17...

24 ⇒

$1 \times 24 = 24$
$2 \times 12 = 24$
$3 \times 8 = 24$
$4 \times 6 = 24$
$6 \times 4 = 24$
$8 \times 3 = 24$
$12 \times 2 = 24$
$24 \times 1 = 24$

$1 \times 0 = 0$
$2 \times 0 = 0$
$3 \times 0 = 0$
$4 \times 0 = 0$
⋮
∞

Q Given a no.. Write a fn that returns true if the no is prime.

```
boolean checkPrime (int N) {
    int c = 0;
    for (int i = 1; i <= N; i++) {
        if (N % i == 0) {
            c++;
        }
    }
    if (c == 2)
        ret true;
    else
        ret false;
}
```

N iterations (Worst case)

i : [1, N]

Assumption : 10^8 iterations in 1 sec

$N \approx 10^9$ ⟶ 10^9 iteration

$\Rightarrow \frac{10^9}{10^8} = 10$ sec

$N \approx 10^{18}$ ⟶ 10^{18} iteration

3 Pence
10^8 iterations ——— 10 Rupee
30 P
10^{18} iterations ——— 1 sec

$\frac{10^{18} \times 1 \text{sec}}{10^8} = 10^{10}$ se

≈ 317 years

You ⟶ Kids ⟶ Grand Kids ⟶ 4th Gen ⟶ 5th ⟶ Gen

↓

Shoib Malik retire

If a, b & N are +ve integers

if $a \times b = N$

$\Rightarrow b = N/a$

a is a factor of N

b is a factor of N

{a, b} factors of N

{a, N/a} factors of N

If a is a factor of N

$\Rightarrow$ N/a is also a factor

N = 24

i	N/i
1	24
2	12
3	8
4	6
6	4
8	3
12	2
24	1

C = 0 → +2 → 2 → +2 → 4 → +2 → 6 → +2 → 8

N = 100

i	N/i	C=0
1	100	2
2	50	4
4	25	6
5	20	8
10	10	9 ↓11
20	5	
25	4	
50	2	
100	1	

```
boolean checkPrime(N){
    int c = 0;
    for(int i = 1; i<=√N ; i++){   // i*i <= N
        // if i is a factor
        then N/i is also a factor
        if(N%i == 0){
            if(i == N/i)
                c++;
            else
                c = c+2;
        }
    if(c==2)
        ret true;
    else ret false;
}
```

$i <= \frac{N}{i}$

$\Rightarrow \boxed{i \times i <= N}$

$i^2 <= N$

$i <= \sqrt{N}$

$i : [1, \sqrt{N}]$

$\Rightarrow \sqrt{N}$ iterations

if(c>2) ret false

$N \approx 10^{18}$ ⟶ $\sqrt{10^{18}}$ iterations $\Rightarrow 10^9$ iteration

10^8 iterations ⟶ 1 sec

10^9 iteration ⟶ 10 sec

1000000000 ↖

1000000000

$= 10^{18}$

$\sqrt{16} =$

$\sqrt{100} \Rightarrow \sqrt{10^2} \longrightarrow 10^1$

$\sqrt{10000} \Rightarrow \sqrt{10^4} \longrightarrow 10^2$

$\sqrt{100000000} \Rightarrow \sqrt{10^8} \longrightarrow 10^4$

Carl Friedrich Gauss
(4th Grade)

$1 + 2 + 3 + 4 \text{ ---- } + 100$

$$
\begin{aligned}
S &= 1 + 2 + 3 + 4 \quad \text{- - - - -} \quad + 97 + 98 + 99 + 100 \\
+ \quad S &= 100 + 99 + 98 + 97 \quad \text{- - -} \quad + 4 + 3 + 2 + 1 \\
\hline
2S &= 101 + 101 + 101 + 101 \quad \text{- - -} \quad + 101 + 101 + 10 + 10 \\
2S &= 101 \times 100 \\
S &= \frac{101 \times 100}{2} = 5050
\end{aligned}
$$

$\log_a b$

$\log_2 N$

$N = 7 \xrightarrow{\div 2} 3 \xrightarrow{\div 2} 1$

$\log_2 7 \longrightarrow 2$

$N = 1$

$\log_2 1 \longrightarrow 0$

$N = 2 \xrightarrow{\div 2} 1$

$\log_2 2 \longrightarrow 1$

$N = 4 \xrightarrow{\div 2} 2 \xrightarrow{\div 2} 1$

$\log_2 4 \longrightarrow 2$

$N = 32 \longrightarrow 16 \longrightarrow 8 \longrightarrow 4 \longrightarrow 2 \longrightarrow 1$ $\log_2 32 \longrightarrow 5$

$N = 55 \longrightarrow 27 \longrightarrow 13 \longrightarrow 6 \longrightarrow 3 \longrightarrow 1$ $\log_2 55 \longrightarrow 5$

$N = 100 \longrightarrow 50 \longrightarrow 25 \longrightarrow 12 \longrightarrow 6 \longrightarrow 3 \longrightarrow 1$ $\log_2 100 \longrightarrow 6$

$N = 64 \longrightarrow 32 \longrightarrow 16 \longrightarrow 8 \longrightarrow 4 \longrightarrow 2 \longrightarrow 1$ $\log_2 64 \longrightarrow 6$

$\log_2 32 \longrightarrow 5$

$\log_2 27 \longrightarrow 4.xyzx$ $(2^x = 27)$

Amazon

Q Given a number which is a perfect sq. Write a fn that returns the sq root.

Perfect sq — N is a perfect sq if $\sqrt{N}$ is an integer

i.e. there exists an integer x

$x \times x = N$

$N = 25 \longrightarrow \sqrt{25} \longrightarrow 5$ ($5 \times 5 = 25$)

$N = 100 \longrightarrow \sqrt{100} \longrightarrow 10$ ($10 \times 10 = 100$)

$N = 31$ ✗

$i \times i/2 == 0$

16

$N = 36$

$1 \times 1 \longrightarrow 1$

$2 \times 2 \longrightarrow 4$

$3 \times 3 \longrightarrow 9$

$4 \times 4 \longrightarrow 16$

$5 \times 5 \longrightarrow 25$

$6 \times 6 \longrightarrow 36$

$N = 144$

$1 \times 1 \longrightarrow 1$

$2 \times 2 \longrightarrow 4$

$3 \times 3 \longrightarrow 9$

⋮

$10 \times 10 \longrightarrow 100$

$11 \times 11 \longrightarrow 121$

$12 \times 12 \longrightarrow 144$

$4 \times 4/2 == 0$?

$\Rightarrow 8$ ✗

```
int getSqRoot (int N){
    for(i=1; i<=N; i++){
        if (i*i == N){
            ret i;
        }
    }
}
```

→ $i*i <= N$

$\sqrt{N}$ iterations

$N = 2^{32} \longrightarrow 2^{16}$ iterations

10^8 iterations $\longrightarrow$ 1 sec

2^{16} iterations $\longrightarrow \frac{2^{16}}{10^8}$

$2^{10} = 1024 \approx 10^3$

$2^{10} \longrightarrow 10^3$

$$\frac{2^{10} \times 2^6}{10^8}$$

$$\frac{10^3 \times 2^6}{10^8} = \frac{64}{10^5} \text{ sec}$$

$N = 2^{64} \longrightarrow 2^{32}$ iterations

$$\frac{2^{32}}{10^8} \text{ sec} \Rightarrow \frac{2^{10} \times 2^{10} \times 2^{10} \times 2^2}{10^8} \text{ sec}$$

$$\Rightarrow \frac{10^9 \times 2^2}{10^8} \text{ sec}$$

$\Rightarrow$ 40 sec

N = 36

i	i x i	i x i == N
1	1	✗
2	4	✗
3	9	✗
4	16	✗
5	25	✗
6	36	✓

N = 100

[1, 100]

⟵ 100 ⟶

50 x 50 > 100

1, 2, 3, 4, 5, ---- 49, 50, 51, ---- 100

[1, 49]

⟵ 50 ⟶

25 x 25 > 100

1, 2, 3, 4, 5, ---- 24, 25 --- 49

[1, 24]

⟵ 25 ⟶

12 x 12 > 100

1, 2, 3, ---- 11, 12, 13 --- 24

[1, 11]

⟵ 12 ⟶

6 x 6 < 100

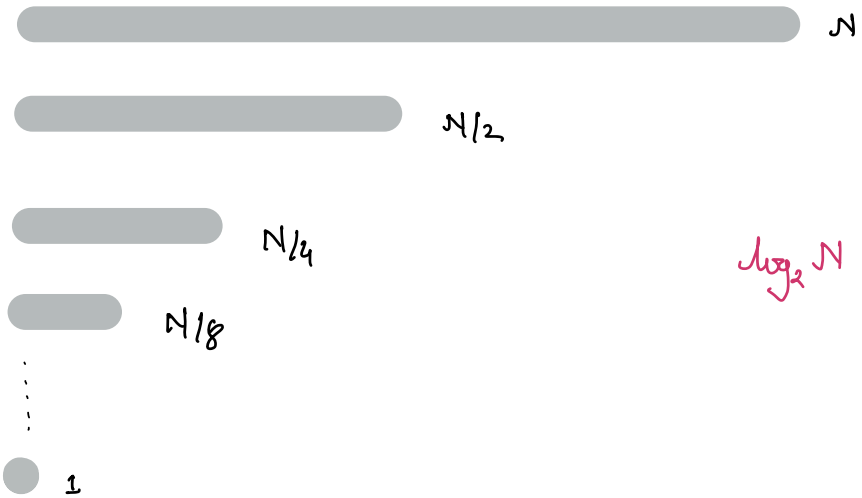
[7, 11]

⟵ ≈ 6 ⟶

1, 2, 3, --- 5, 6, 7 --- 11

9 x 9 < 100

7, 8, 9, 10, 11



$\log_2 N$ iterations

$N = 1024\ (2^{10})$

$N = 2^{32}$

$N = 2^{64}$

	$\sqrt{N}$	$\log_2 N$
$N = 2^{10}$	$\sqrt{2^{10}} \longrightarrow 32$	$\log_2 2^{10} \longrightarrow 10$
$N = 2^{32}$	$\sqrt{2^{32}} \longrightarrow 2^{16}$ (65536)	$\log_2 2^{32} \longrightarrow 32$
$N = 2^{64}$	$\sqrt{2^{64}} \longrightarrow 2^{32}$ (4294,67,296)	$\log_2 2^{64} \longrightarrow 64$