1 idk

1.1 Notation

Let's first consider the general case of a mapping of the form $f_c : z \mapsto f(z) + c$ for a function $f : \mathbb{C} \to \mathbb{C}$ partially differentiable almost everywhere, and for which 0 is a zero of order $d \geq 2$. This would for example be $f : z \mapsto z^n$ for the n-th-power mandelbrot set, $f : z \mapsto (z^n)^*$ for the n-th-power mandelbar set, $f : |\Re(z^2)| + i\,|\Im(z^2)|$ for the burning ship fractal and so on. For any starting point z_0, let's also define z_n as $f^n_{z_0}(z_0)$; the fractal set in question is then the set of values $z_0 \in \mathbb{C}$ for which (z_n) does not tend towards infinity. Also, from here on we'll use $\frac{\partial z'}{\partial z}$ to denote the Jacobian matrix

$$\frac{\partial z'}{\partial z} := \begin{pmatrix} \frac{\partial \Re(z')}{\partial \Re(z)} & \frac{\partial \Re(z')}{\partial \Im(z)} \\ \frac{\partial \Im(z')}{\partial \Re(z)} & \frac{\partial \Im(z')}{\partial \Im(z)} \end{pmatrix}$$

of $(\Re(z'), \Im(z')) \in \mathbb{R}^2$ with respect to $(\Re(z), \Im(z))$, and $M(z)$ to denote the matrix

$$M(z) := \begin{pmatrix} \Re(z) & -\Im(z) \\ \Im(z) & \Re(z) \end{pmatrix}.$$

Now, let z_0 be a point with $z_{n-1} = 0$ for an $n \in \mathbb{N}$ and $z_k \neq 0$ for all $k \in \{1, ..., n-2\}$. Then for all $x_0 \simeq z_0$, the orbit of x_0 evolves as follows:

$$\begin{aligned} x_{n-1} &= z_{n-1} + \frac{\partial f^{n-1}_{z_0}(z_0)}{\partial z_0} \cdot (x_0 - z_0) + o((x_0 - z_0)^2) \\ &= 0 + \frac{\partial f^{n-1}_{z_0}(z_0)}{\partial z_0} \cdot (x_0 - z_0) + o((x_0 - z_0)^2) \\ &=: d_0 + o((x_0 - z_0)^2), \\ x_n &= x^2_{n-1} + x_0 \end{aligned}$$