Lecture 2

Unit 3

Closure

- Infer additional functional dependencies from a given set of FDs
- The new FDs are logically implied by F
- The set of all these logically implied FDs including those of F is called closure of F
- Denoted as F^+

Attribute Closure

- **Determine each set of attributes X that appears as a left-hand side of some functional dependency in F and then determine the set of *all attributes* that are dependent on X.**

- Thus, for each such set of attributes X, we determine the set X+ of attributes that are functionally determined by X based on F

- **X+ is called the closure of X under F.**

- Typically, check if a given FD $X \rightarrow Y$ is in F+ (if $X \rightarrow Y$ can be inferred from F).
  - Compute **attribute closure** of X (X+) w.r.t F:
    - X+: Set of all attributes A such that $X \rightarrow A$ can be inferred
    - X+ can be calculated by repeatedly applying IR1, IR2, IR3 using the FDs in F
  - Check if Y is in X+
    - If yes, then $X \rightarrow Y$ is in F+

Algorithm for Computing Attribute Closure

```
Comp_Attr_closure(X)
{
    closure=X;
    repeat until there is no change:
    {
    if there is an FD U→V in F such that U ⊆ closure
    then set closure = closure U V
    }
    return(closure)
}
```

- Example

  F → {SSN → ENAME,
  PNUMBER → {PNAME, PLOCATION},
  {SSN, PNUMBER} → HOURS}

  - Closure sets w.r.t. F are

    {SSN}+ → {SSN, ENAME}
    {PNUMBER}+ → {PNUMBER, PNAME, PLOCATION}
    {SSN, PNUMBER}+ → {SSN, PNUMBER, ENAME, PLOCATION, HOURS}

Minimal Cover – 1/2

- A set of FDs F is **minimal** if it satisfies the following conditions:
  - Every dependency in F has a single attribute for its right-hand side.
  - We cannot remove any attribute from an FD in F and still have a set of FDs that is equivalent to F.
  - We cannot remove any FD in F and still have a set of FDs that is equivalent to F.

Minimal Cover – 2/2

- F is minimal in two respects:
  - Every dependency is as small as possible
  - Every dependency in F is required in order for the closure to be equal to $F+$

- Every set of FDs has an equivalent minimal set

- There can be several equivalent minimal sets

Minimal Cover - Algorithm

1. Rewrite every FD so that every **right side** is a **single attribute** (Atomic)

2. **Remove redundant FDs**

   e.g., check if $X \rightarrow Y$ is redundant

   - compute $X+$ w.r.t $G - (X \rightarrow Y)$
   - if $X+$ contains Y, $X \rightarrow Y$ is redundant and G = $G - (X \rightarrow Y)$

3. **Remove redundant LHS attribute** for every FD

   e.g., $XY \rightarrow A$, check if Y is redundant

   - compute $X+$ w.r.t $\{G - (X \rightarrow Y)\} \cup \{X \rightarrow A\}$
   - if $X+$ contains Y, than Y is redundant and G= G $\cup$ $\{X \rightarrow A\}$

Equivalence of Sets of Functional Dependencies (Go thro)

- Two sets of FDs F and G are **equivalent** if:
  - every FD in F can be inferred from G, *and*
  - every FD in G can be inferred from F
  - Hence, F and G are equivalent if $F+ = G+$

- F **covers** G if every FD in G can be inferred from F (i.e., if $G+ \subseteq F+$)
  - calculate $X+$ w.r.t F for each $X \rightarrow Y$ in G
  - check whether $X+$ includes the attributes in Y
  - if yes for every FD in G, then F covers G

- F and G are equivalent if F covers G and G covers F