

Regular expression

Strings and languages

- Regular expression are suitable for describing tokens
- As string is a finite sequence of symbols such as “aabb” or “010”
- The length of a string is denoted by $|x|$
- The term language means a set of strings formed from some specific alphabet
- Simple sets
 - ϕ which denote an empty set
 - $\{\epsilon\}$ which denotes containing only the empty string

Strings and languages

- Concatenation
 - If L and M are languages then
 - $LM = \{xy \mid x \text{ is in } L \text{ and } y \text{ is in } M\}$
- union
 - $L \cup M = \{x \mid x \text{ is in } L \text{ or } x \text{ is in } M\}$
 - Let $L = \{0, 01, 110\}$
 - Let $M = \{00, 111, 1101, 10\}$
 - $L \cup M = \{0, 01, 110, 00, 111, 1101, 10\}$
- $\phi \cup L = L \cup \phi = L$
- $\phi L = L \cup \phi = \phi$

Strings and languages

- closure
- $L^* = \bigcup_{i=0}^{\infty} L^i$

Definition of regular expression

- The regular expression over alphabet Σ are exactly those expressions that can be constructed from the following rules
 - Each regular expression denotes a language
 - Any set represented by a regular expression is called a regular set
1. ϵ is a regular expression denoting $\{\epsilon\}$, that is, the language containing the empty string
 2. For each a in Σ , a is a regular expression denoting $\{a\}$
 3. If R and S are regular expression denoting L_R and L_S respectively, then
 - i. $(R) \mid (S)$ is a regular expression denoting $L_R \cup L_S$
 - ii. $(R) \cdot (S)$ is a regular expression denoting $L_R \cdot L_S$
 - iii. $(R)^*$ is a regular expression denoting L_R^*

Definition of regular expression

- Identifier=letter (letter | digit)*
- $(a \mid b)^*$
- $a \mid ba^*$
- Two regular expression are considered equivalent
 - If?
- **Identities**
 - $(PQ)^*R = P(QP)^*$
 - $(P+Q)R = PR+QR$ and $R(Q+P) = RP+RQ$
 - $R^*R^* = R^*$

Class work

- Represent the following sets by regular expression
 1. $\{0,1,2\}$
 2. $\{1^{2n+1} \mid n > 0\}$
 3. $\{a^2, a^5, a^8, \dots\}$
 4. The set of all strings over $\{a,b\}$ beginning with a and ends with b
 5. The set of all strings over $\{0,1\}$ which has at most two zeros

Next classes topics

- Finite automata
- Deterministic automata
- Non deterministic automata
- Conversion a deterministic automata from a non deterministic automata
- Converting non deterministic finite automata from a regular expression
- Minimized the number of states of a DFA

solution

- $0+1+2$
- $111(11)^*$
- $aa(aaa)^*$
- $a(a+b)^*b$
- $1^*+1^*01^*+1^*01^*01^*$