

- ADTs based on SETS -

NO:

DATE:

* SET

- unique elements
- order DOESN'T MATTER
(unless specified)

* LIST

- can have duplicate elements
- order is IMPORTANT
↳ like in Array list:
shifting of index

* Types

- 1.) ADT VID (Union, Intersection, Difference)
- 2.) Dictionary
- 3.) Priority Queue

* Implementations

- 1.) Linked List
- 2.) Array Implementation
- 3.) Cursor-based
- 4.) Bit-vector Implementation

* Operations

- 1.) Union
- 2.) Intersection
- 3.) Difference
- 4.) MAKENULL

5.) INITIALIZE

A	B
0 1	0 1
1 0	1 0
2 0	2 0
3 1	3 0
4 1	4 1
5 0	5 1
6 0	6 0
7 1	7 0
8 0	8 0
9 1	9 0

$$\text{Set } A = \{3, 7, 0, 4, 9\}$$

$$\text{Set } B = \{0, 4, 5\}$$

$$U = \{0, 1, 2, \dots, 9\}$$

* List range will be the entirety of the universal set, regardless if the elements are present or not.

- BIT-VECTOR IMPLEMENTATION -

NO:

DATE:-

* Bit vector Implementation

2000110100101*

Bit vector = Boolean array implementation (T, F)

- size of set is dependent on Universal Set
- best used:
 - ↳ all given sets are SUBSETS

> when Universal Set U is SHALL

> elements are INTEGERS

(or can be mapped to integers)

Example:

set $A = \{3, 7, 0, 4, 9\}$

$A[x] = 1$ if $x \in A$

$A[x] = 0$ if $x \notin A$

* Place a 1 for every elements that

= to index in set A .

* Assign a UNIQUE integer number

for each non-integer element!

	A
0	1
1	0
2	0
3	1
4	1
5	0
6	0
7	1
8	0
9	1

* Advantages

1.) member(), insert(), delete() = $O(1)$

2.) union(), intersection(), difference() = $O(N)$

3.) If U is sufficiently small &
> can fit in one computer word
= AND can be performed in one logical operation

* Disadvantages

1) Space needed = size of U

(7.7) If Universal Set U is relatively big, then size of set will also be big.

* Insert

$$U = \{0, 1, 2, \dots, 9\}$$

$$A = \{1, 0, 5\}$$

A
1
0
0
0
1
0
1
0
0

• Insert ($A, 7$)

$$A = \{1, 0, 5, 7\}$$

* Union

- combine the sets

$$A = \{1, 0, 5, 7\}$$

$$B = \{0, 4, 5\}$$

: Union (A, B)

$$A \cup B = \{0, 1, 5, 4, 7\}$$

* Set Operations

1.) UNION ()

- belongs to A or B // BOTH / COMBINE
- $A \cup B$

2.) INTERSECTION ()

- common to A and B // both

- $A \cap B$

3.) DIFFERENCE ()

- element in A, but not in B

- $A - B$

→ SET OPERATIONS : UNION -
(BVI)

// Exercise 1 - Union ()

- Definition of datatype SET such that
A is an array of integers of size 10.

```
#define SIZE 10
```

```
typedef int SET [SIZE];
```

SET A; → = A[10]

A	
0	1
1	0
2	0
3	1
4	1
5	0
6	0
7	1
8	0
9	1

A = {0, 3, 4, 7, 9}

- VEN DIAGRAM OPERATIONS - VISUALIZATION

* VD

- Union - combine all elements

$$\text{SET } A = \{7, 0, 4, 2\} \quad A \cup B = \{0, 1, 5, 2, 4, 7\}$$
$$B = \{0, 1, 5\}$$

7 6 5 4 3 2 1 0

OR

1 0 0 1 0 1 0 1 = A
0 0 1 0 0 0 1 1 = B

1 0 1 1 0 1 1 1 = {0, 1, 2, 4, 5, 7}

- Intersection - common elements

$$\text{SET } A = \{7, 0, 4, 1\} \quad A \cap B = \{0, 1\}$$
$$B = \{0, 1, 5\}$$

7 6 5 4 3 2 1 0

AND

& 1 0 0 1 0 0 1 1 = A
0 0 1 0 0 0 1 1 = B

0 0 0 0 0 0 1 1 = {0, 1}

- Difference - element in A, NOT in B

$$\text{SET } A = \{7, 0, 4, 1\}$$

* get complement of B FIRST.

$$B = \{0, 1, 5\}$$

$$0 0 1 0 0 0 1 1 \xrightarrow{\sim} 1 1 0 1 1 1 0 0 \quad A - B = \{7, 4\}$$

COMPLEMENT

* perform AND

7 6 5 4 3 2 1 0

8

1 0 0 1 0 0 1 1 = A
1 1 0 1 1 1 0 0 = B

1 0 0 1 0 0 0 0 = {7, 4}

• Subset - if set's every element is in another set

$$\text{SET } A = \{7, 0, 4, 1\} \quad B \subseteq A$$

$$B = \{0, 1, 4\}$$

7 6 5 4 3 2 1 0
1 0 0 1 0 0 1 1 = A
⑧ 0 0 0 1 0 0 1 1 = B
—————
0 0 0 1 0 0 1 1 != 0 ✓ B is a subset of A.

- SET OPERATIONS -

VISUALIZATION

//member() - returns 1/non-zero if element is member
0 if not member

SET A = {2, 4, 5}

X = 5

*S is a mask value that was shifted to that position from 1.

$$\begin{array}{r} 7654 \quad 3210 \\ 0011 \quad 0100 = A \\ \textcircled{8} \quad 0010 \quad 0000 = 5 \\ \hline 0010 \quad 0000 = 0 \checkmark \end{array}$$

MEMBER

$$\begin{array}{r} 0001 \quad 0100 = A \\ 0010 \quad 0000 = 5 \\ \hline 0000 \quad 0000 == 0 X \\ \text{NOT MEMBER} \end{array}$$

mask value = 0000 0001 << X

//insert() - insert element at appropriate position matching bit position

SET A = {7, 0, 4, 1}

X = 5

OR

$$\begin{array}{r} 7654 \quad 3210 \\ 1001 \quad 0011 = A \\ \textcircled{1} \quad 0010 \quad 0000 = 5 \\ \hline 1011 \quad 0011 = \{7, 0, 4, 1, 5\} \end{array}$$

//delete() - delete element at appropriate position matching bit position
*get complement of X
*perform AND

SET A = {7, 0, 4, 1, 5}

X = 5

$$\begin{array}{r} 7654 \quad 3210 \\ 1011 \quad 0011 = A \\ \textcircled{~} \quad 0010 \quad 0000 = 5 \\ \hline \end{array}$$

$$\begin{array}{r} 1011 \quad 0011 = A \\ \textcircled{8} \quad 1101 \quad 1111 = 5 \\ \hline 1001 \quad 0011 = \{7, 0, 4, 1\} \end{array}$$

- Function Header : Union () - will return new set C w/ elements found in given sets.

~~SET *Union (SET C, SET D);~~

SET * Union (SET C, SET D);

- Function Call (w/ variable declaration & initialization)

// Assumption: U = {0, 1, 2, 3, 4} D = {0, 1, 3}

C = {1, 3, 4} CUD = {0, 1, 3, 4}

* C & D in main()
A & B in Union()

* shortened to SIZE = 5 for convenience

C	0	1	2	3	4
0	0				
1		1			
2	0				
3		1			
4	1				

$$C = \{1, 3, 4\}$$

D	0	1	2	3	4
0	1				
1		1			
2	0				
3		1			
4	0				

$$D = \{0, 1, 3\}$$

- Declaration & Initialization!

SET C = {0, 1, 0, 1, 1}

REMEMBER!

SET D = {1, 1, 0, 1, 0}

keep declaration & initialization

TOGETHER!

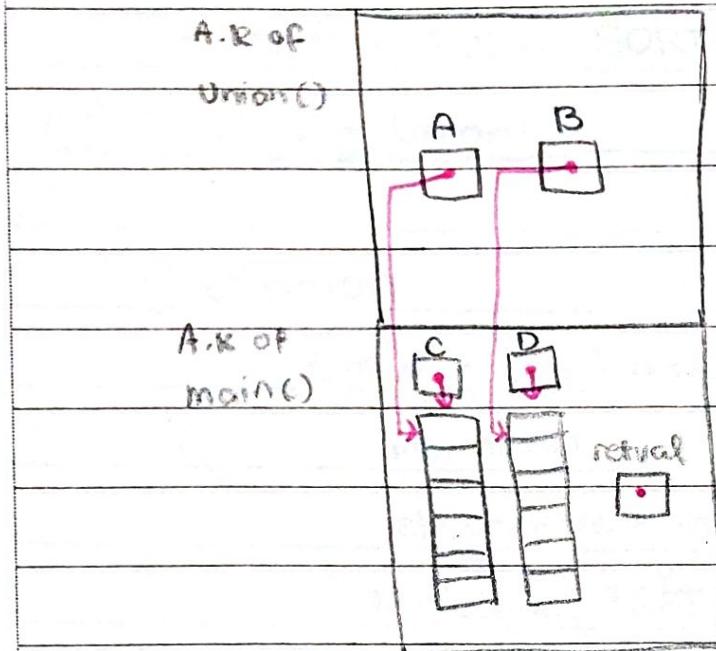
[can be omitted]

SET * retval = Union(C, D);

* Make a catcher variable bcs you're returning a new set.

* its a pointer bcs it will point to the locally created dynamically allocated array in the function.

Execution Stack



* The name of an array is a **POINTER** to first component.

* A & B will get the address of C[0] & D[0]

* Array must be dynamically allocated when returned.

Code

#define SIZE 5

typedef int SET [SIZE];

SET *Union (SET A, SET B) {

SET *C = (SET *) calloc (MAX, sizeof (SET));

int x;

→ initializes array to be all 0 so just change it to 1; eliminates need for else!

if (C != NULL) {

for (x=0; x<SIZE; x++) {

(*C)[x] = (A[x]==1 || B[x]==1)? 1:0;

y

OR:

y (*C)[x] = A[x] || B[x];

y

EXECUTION

STACK

(after)

- SET OPERATIONS: UNION - DATE:

(LINKED LIST)

- uses size of set represented, not size of universal set
- better if list is SORTED

//Exercise 2 - Union()

• Definition

typedef struct node{	SET A; // {1,3,4}
int data;	SET B; // {0,1,3}
struct node *link;	AUB = {0,1,3,4}
}; Nodetype, *SET;	SET C;
	C = union (A,B); // pass by copy

• Code

```
SET Union(SET A, SET B) {
```

```
    SET C, *aptr, *bptr;
```

```
    C = NULL; aptr = &C;
```

```
    aptr = A; bptr = B;
```

```
    while (aptr != NULL && bptr != NULL) {
```

```
        if (aptr->data < bptr->data) { // A < B
```

```
            *aptr = (SET) malloc(sizeof(struct node));
```

```
            (*aptr)->data = aptr->data;
```

```
            aptr = aptr->link;
```

```
        }
```

```
        else if (bptr->data < aptr->data) { // B < A
```

```
            *aptr = (SET) malloc(sizeof(struct node));
```

```
            (*aptr)->data = bptr->data;
```

```
            bptr = bptr->link;
```

```
}
```

else {

// EQUAL

*cptr->elem = aptr->elem;
aptr = aptr->link;
bptr = bptr->link;

3

cptr = &(*cptr)->link)

3

if(aptr == NULL){

aptr = bptr;

3

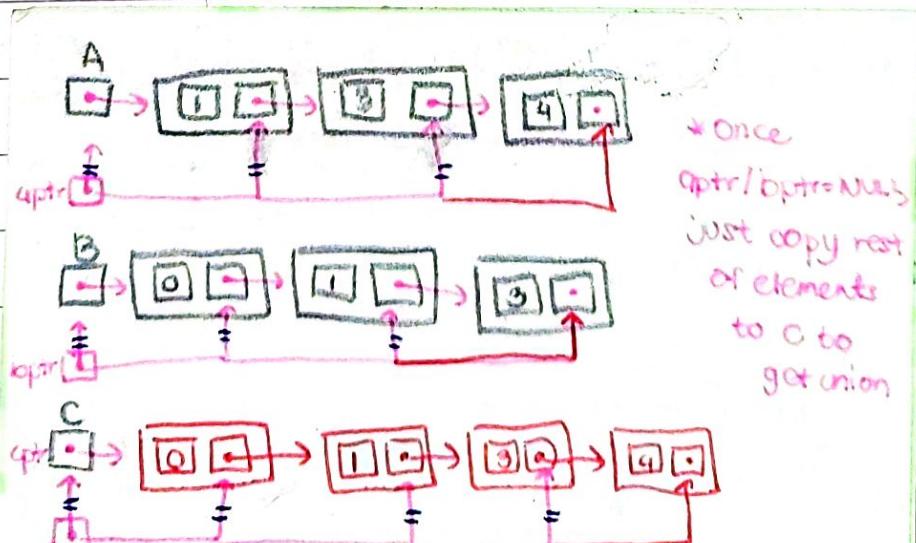
for(; aptr != NULL; aptr = aptr->link, cptr = &(*cptr)->link){
*cptr = malloc(sizeof(struct node));
(*cptr)->elem = aptr->elem;

3

8

*cp = NULL;

return C;



* Once
aptr/bptr=NULL
Just copy rest
of elements
to C to
get union

* aptr, bptr = PN ; *cptr = PPN

* traversal = PN (bcz just searching)

- ① Declare -
- aptr, bptr (P1D) bcs just traversing
- cptr (PPN) bcs inserting
- C (new list/set)
- ② Loop checking process until either ptr becomes
NULL.
- ③ Check if $a < b$ (bcs sorted), if
a is smaller, insert first to C.
- update *cptr & aptr

- COMPUTER WORD IMPLEMENTATION

* Computer Word

* manipulating BITS!

- unit of data of defined bit length
- can be moved & addressed
- can be:

char 8 bits

short int 16 bits

int 32 bits

Example:

* Operations

char ch = 'A'

1.) VOID

char x = -25

2.) member ()

3.) insert()

4.) delete()

ch = - - - - - 8 bits
 128 64 32 16 8 4 2 1

'A' = 65 (ASCII)

$$-\frac{1}{64} \quad \dots \quad \frac{1}{1} = 65$$

$$25 = \frac{0001}{16} \frac{1001}{8} = 25$$

↓ COMPLEMENT (to get -25)

= 0001 1001 → leave first 0 & 1 unchanged
 (none, so skip)

1110 0110 → reverse bits

= 1110 0111 → addl 1

-25 = 1110 0111 (in computer's memory)

//display Bit Pattern

Write the code of the fn() that will display bit pattern of a given integer.

*RECALL

In Bit-vector implementation,

$$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \quad A = \{7, 0, 4, 2\}$$

A
0
1
2
3
4
5
6
7

Facts

- contents are bits (0,1)
- instead of a "bit" datatype, we use char/int
- elements of Universal Set are indices of array
- can FIT INSIDE char since char = 8 bits

* NOT an array, just a representation

* BVI to Computer Word

BVI

7	6	5	4	3	2	1	0
1	0	0	1	0	1	0	1

Computer Word

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	0	1	0	1	0	1

* Elements are INDEX.

* Elements are EXPONENTS.

- If $x \in A$ then bit position w/ place value 2^x is 1.

NO.

DATE:

Code

```
void displayBitPattern (SET A) {
```

```
    int x;
```

```
    int SIZE = sizeof (SET) * 8;
```

bitwise

```
    for (x=1; x < SIZE; x++) {
```

AND

```
        if (A & (1 << x)) {
```

```
            printf ("%d", x);
```

3

2007

// to print what numbers

are IN the set.

3

OR

```
if (A & (1 << x)) {
```

// to print 0000 1111

```
printf ("%d", 1);
```

pattern of the set.

```
3 else
```

```
    printf ("%d", 0);
```

3

① Set $x=1 \rightarrow$ MASK VALUE

fn. will shift from here

② calculate size of datatype.

-remember, must be platform-independent

③ Check bit pattern of A against

$1 << x$. (^{shift 1 place,}
^{2 places})

④ Print.

NO: _____
DATE: _____

- COMPUTER WORD OPERATIONS (VISUALIZATION)

```
typedef unsigned  
char SET;
```

$$2^7 \ 2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$$

----- | A

//member() - VISUALIZATION

- must EXTRACT to retrieve element \rightarrow mask value
- checks if element (exponent) is a MEMBER of set (bits)

Example: $A = \{5, 4, 2\}$

$$= \begin{smallmatrix} 7 & 6 & 4 & 3 & 2 & 1 & 0 \\ 0011 & 0100 \end{smallmatrix}$$

0 1 1 0 1 1₁₀

AND operator

$$\uparrow 0011 \ 0100$$

$$\textcircled{8} \ 0000 \ 0100 \rightarrow \text{mask value}$$

$$0000 \ 0100 != 0$$

MEMBER

$$0011 \ 0000$$

$$\textcircled{8} \ 0000 \ 0100$$

$$0000 \ 0000 == 0$$

NOT MEMBER

mask value: 0000 0001 $\ll 1$

* Move/SHIFT the mask value & use it to check against the elements in set A to check if elements are present.

* Mask value should always start at 1 (0000 0001) & shift LEFT! Function will check against each bit per iteration.

* If resulting bit pattern $\neq 0$ (means it found an element in that position, so 2^x exists)
 $\neq 0$ (means element does not exist in that bit position)

- COMPUTER WORD OPERATIONS -
CODE

NO:

DATE:

* UID

// Union (A, B) - returns AUB

typedef unsigned

char SET;

SET union (SET A, SET B) {

 return (A | B);

}

// Intersection (A, B) - returns A ∩ B

SET intersection (SET A, SET B) {

 return (A & B);

}

// Difference (A, B) - returns A - B

SET difference (SET A, SET B) {

 return (A & ~B);

}

// Subset. (A, B) - returns 1/nonzero if $B \subseteq A$; 0 otherwise

int subset (SET A, SET B) {

 return (A & B) == B ;

}

* Set Functions

//initialize (*A)

```
void initialize(SET *A) {
    *A = 0;
```

3

//member (A, x)

```
int member(SET A, int elem) {
    return (A & 1<<elem) > 0;
```

3

//insert (*A, x)

```
SET insert(SET *A, int x) {
    *A = *A | 1<<x;
```

3

//delete (*A, x)

```
SET delete(SET *A, int x) {
    *A = *A & ~ (1<<x);
```

3