

ADT Based on SETS

SET → collection of elements where each member is distinct, unique. No two elements that are the same are contained within a set.

SET vs. LIST

SET

- unique elements
- order is not significant (unless specified)

vs.

LIST

- elements can be duplicated
- order is important

"ADTs based on SETS" aka no duplicated elements in the ADTs

- ↳ ADT VID
- ↳ Dictionary
- ↳ Priority Queue (save for later)

SET VID

SET ADT with operations

- ① Set Union
- ② Set Intersection
- ③ set Difference

IMPLEMENTATIONS

- Ⓐ Array
- Ⓑ Linked list - sorted or unsorted
- Ⓒ Cursor Based
- Ⓓ Bit-vector implementation

BIT VECTOR IMPLEMENTATION

- the set is represented by a bit vector ('bit' i.e binary digit - the component of a 1-D vector, 'vector' i.e a 1-D array) in which the n^{th} bit is true or '1' if n is part of the set.

Eq. SET A = {3, 7, 0, 4, 9}

A
0
1
2
3
4
5
6
7
8
9

$$A[x] = 1 \text{ if } x \in A$$

$$A[x] = 0 \text{ if } x \notin A$$

When to use Bit-vector implementation?

- when the elements are integers because they represent the indices of the set array
- when the universal set U is small because the size of the set is dependent on the size of the universal set.

BIT VECTOR IMPLEMENTATION PROS VS. CONS.

PROS.

- * `ismember()`, `insert()` & `delete()` can be performed with $O(1)$ by directly addressing the appropriate bit.

CONS.

- * Space needed for the set is proportional to the size of the universal set.
If universal set is large then more space is needed.

- * `Union()`, `intersection()` & `difference()` can be performed in time proportional to the size of the universal set

- * if universal set is small & can fit a computer word, then `AND` operations can be performed with one logical operation

SET Union - set of all elements which belong to A or to B or to both. $A \cup B \rightarrow OR \parallel |$

SET Intersection - set of all elements that are common to A and to B. $A \cap B \rightarrow AND \&$

SET Difference - all elements in A but no in B.
 $A - B \rightarrow NOT ! \sim$

EXERCISE - WRITE THE CODE OF UNION

SET A

0	1
1	0
2	0
3	1
4	1
5	0
6	0
7	1
8	0
9	1

① Write the appropriate definition of datatype SET such that in the declaration SET A, A is an array of integers of size 10.

```
#define MAX 10  
typedef int SET[MAX];
```

② Write the code of Union, given two sets C & D.

$$C = \{1, 3, 4, 7, 9\}$$

$$D = \{0, 2, 6, 8, 9\}$$

Initialize & declare the variables used in the function call & write the function call.

Draw the execution stack of the function call.

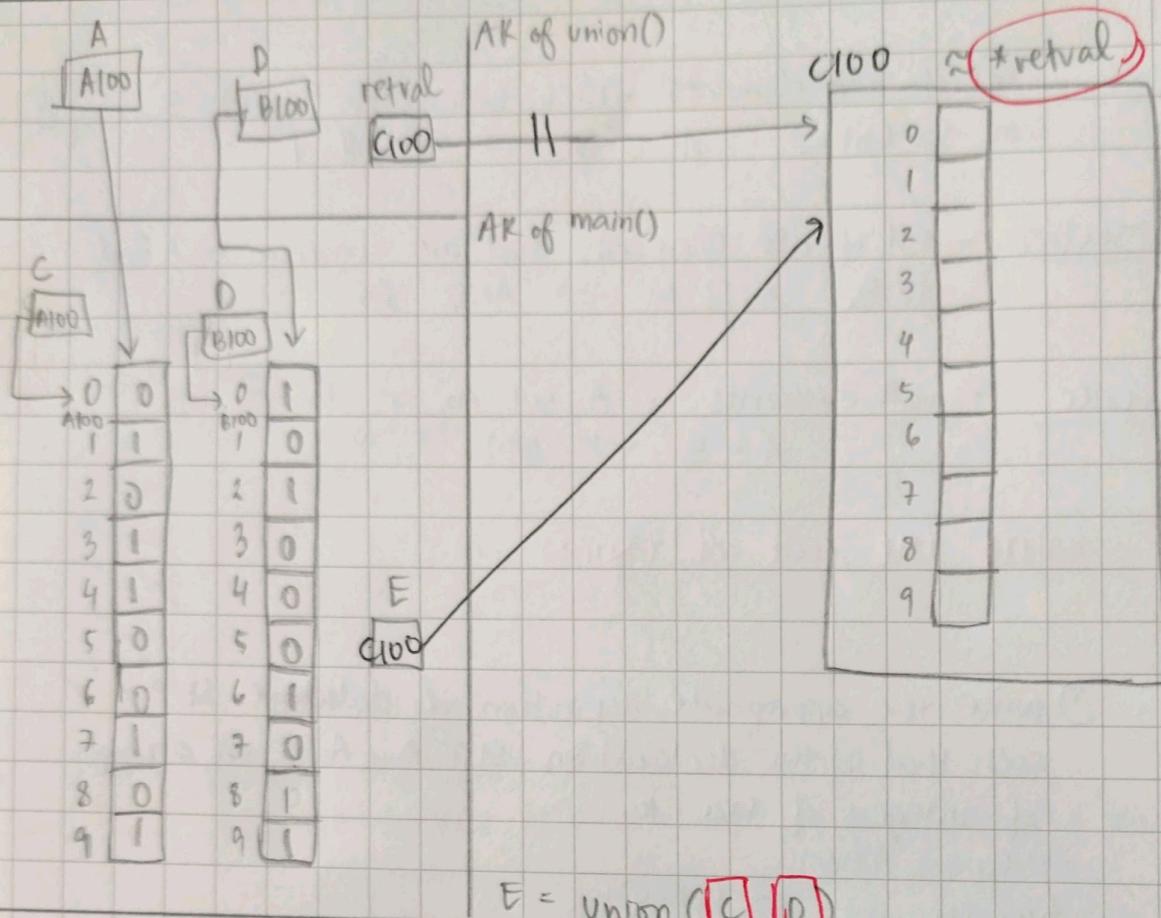
SET C = {0, 1, 0, 1, 1, 0, 0, 1, 0, 1}
SET D = {1, 0, 1, 0, 0, 1, 0, 1, 1}

SET * E;

E = Union(C, D);

→ to initialize an array,
always declare it on the
same line
(or any aggregate
variable)

→ do not initialize a function
call on the same line as
declaration. Do it separately.



$E = \text{Union}(C, D)$

the name of an array is
a pointer to the first component

SET * Union (SET A, SET B) // A and B are pointers to the first component
of their SET

SET * retval = (SET *) malloc (size of (SET));

catcher variable, retval is a pointer to an entire array, not the
first component

```

int ndx;
for (ndx = 0; ndx < MAX; ndx++) {
    (*retval)[ndx] = A[ndx] || B[ndx];
}
return retval;
    
```

X	Y	X		Y
0	0			
0	1			
1	0			
1	1			

EXERCISE - WRITE THE CODE OF BIT VECTOR
IMPLEMENTATION OPERATIONS INTERSECTION
& SET DIFFERENCE

A] INTERSECTION

```
SET * intersect (SET A, SET B) {  
    SET * retval = (SET *) malloc (sizeof (SET));  
    int ndx;  
    for (ndx=0; ndx < MAX; ndx++) {  
        (*retval)[ndx] = A[ndx] && B[ndx];  
    }  
    return retval;  
}
```

B] SET DIFFERENCE

```
SET * Difference (SET A, SET B) {  
    SET * retval = (SET *) malloc (sizeof (SET));  
    int ndx;  
    for (ndx=0; ndx < MAX; ndx++) {  
        (*retval)[ndx] = A[ndx] && !B[ndx];  
    }  
    return retval;  
}
```

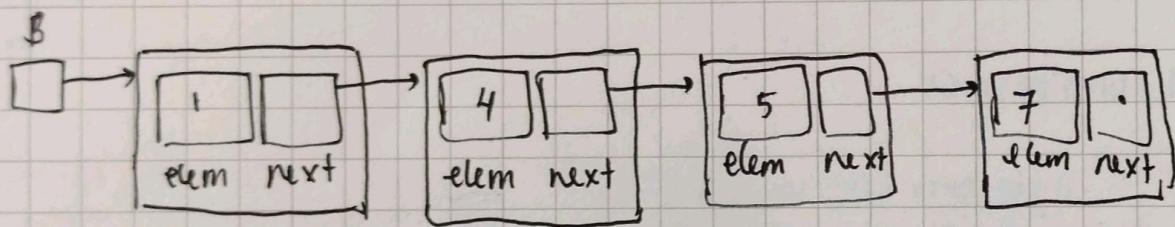
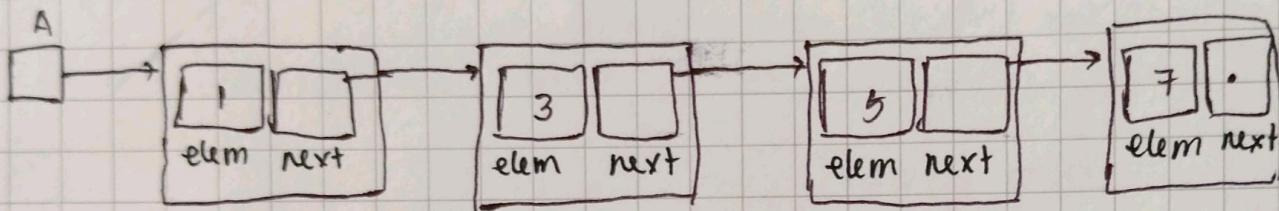
LINKED LIST (POINTER) IMPLEMENTATION)

```
typedef struct cell {  
    int elem;  
    struct cell * next;  
} ctype, * SET;
```

GIVEN:

$$\text{SET A} = \{1, 3, 5, 7\}$$

$$\text{SET B} = \{1, 4, 5, 7\}$$



Write the codes for Union, Intersection, Difference & isSubset.

UNION

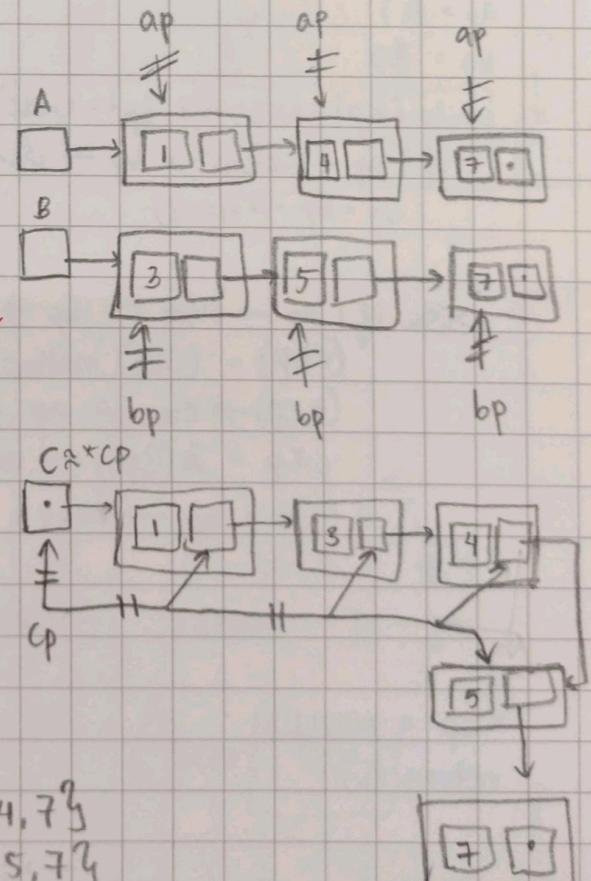
- Subtasks :
- ① Assign pointers to the first node for the 2 given set
 - ② Initialize a 3rd set (the list that contains the union) & assign a pointer to the head.
 - ③ Start traversing the sets & compare the elems. If one elem is lesser than the other, copy the data into the new list.
 - ④ Assign the pointers to point to the next node. For the set to be returned, set up the pointer to point to the next field.
 - ⑤ If one list reaches null, copy the rest of the remaining data into the new linked list.
 - ⑥ Null the "next" field of the new linked list.
 - ⑦ Return to the calling function.

UNION

```

SET Union (SET A, SET B) {
    SET C, *cp, ap, bp;
    C=NULL;
    cp=&C;
    ap=A;
    bp=B;
    while (ap!=NULL && bp!=NULL) {
        (*cp)=(SET) malloc(sizeof(ctype));
        if (ap->elem < bp->elem) {
            (*cp)->elem = ap->elem;
            ap=ap->next;
        } else {
            if (ap->elem == bp->elem) {
                ap=ap->next;
            } (*cp)->elem = bp->elem;
            bp=bp->next;
        }
        cp=&(*cp)->next;
    }
    if (bp!=NULL) // A came to NULL
        ap=bp;
    }
    while (ap!=NULL) {
        (*cp)->elem = ap->elem;
        ap=ap->next;
        cp=&(*cp)->next;
    }
    *cp=NULL;
    return C;
}

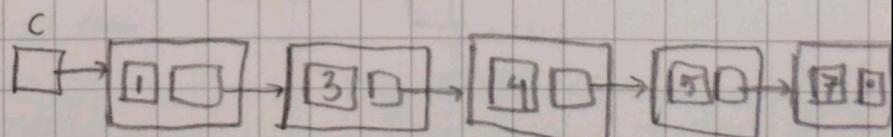
```



$$A = \{1, 4, 7\}$$

$$B = \{3, 5, 7\}$$

RESULT:



INTERSECTION

- subtasks:
- ① Assign pointers to the first node for the 2 given sets
 - ② Initialize a 3rd set (the intersection set i.e set to be returned) & assign pointer to the head.
 - ③ Start comparing elements in both sets. If one element is greater than the other, traverse to the next element.
 - ④ Else if both elements are equal, copy the data into the new list. Traverse both sets.
 - ⑤ When one set reaches NULL, exit loop
 - ⑥ NULL the "next" field of the new linked list
 - ⑦ Return to the calling function.

INTERSECTION

```
SET Intersection ( SET A, SET B ) {
```

```
SET C, * cp, ap, bp;
```

```
C = NULL;
```

```
cp = &C;
```

```
ap = A;
```

```
bp = B;
```

```
while ( ap != NULL && bp != NULL ) {
```

```
    if ( ap -> elem > bp -> elem ) {
```

```
        bp = bp -> next;
```

```
}
```

```
if ( ap -> elem == bp -> elem ) {
```

```
( *cp ) = ( SET ) malloc ( sizeof ( ctypd ) );
```

```
( *cp ) -> elem = ap -> elem;
```

```
cp = &( *cp ) -> next;
```

```
bp = bp -> next;
```

```
}
```

```
} ap = ap -> next;
```

```
* cp = NULL;
```

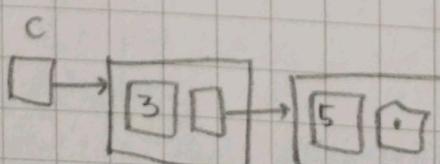
```
return C;
```

```
}
```

$$A = \{ 1, 3, 5 \}$$

$$B = \{ 3, 4, 5 \}$$

RESULT:



DIFFERENCE

- Subtasks :
- ① Assign pointers to the first node for the 2 given sets
 - ② Initialize a 3rd set (the difference set i.e set to be returned) & assign pointer to the head.
 - ③ Start comparing elements in both sets. If both elements are the same, traverse both sets.
 - ④ If one element is lesser than the other, traverse one set. If greater, copy the data into the new list.
 - ⑤ When one set reaches NULL, copy the rest of the remaining data into the new list.
 - ⑥ NULL the "next" field of the new linked list
 - ⑦ Return to the calling function.

DIFFERENCE

A - B

SET Difference (SET A, SET B) {

SET C, *cp, ap, bp;

C = NULL;

cp = &C;

ap = A;

bp = B;

while (ap != NULL) {

 while (bp != NULL) {

 if (ap->elem == bp->elem) {

 ap = ap->next;

 bp = bp->next;

 } else {

 if (ap->elem > bp->elem) {

 bp = bp->next;

 } else {

 (*cp) = (SET) malloc (sizeof (ctype));

 (*cp)->elem = ap->elem;

 ap = ap->next;

 cp = &(*cp)->next;

}

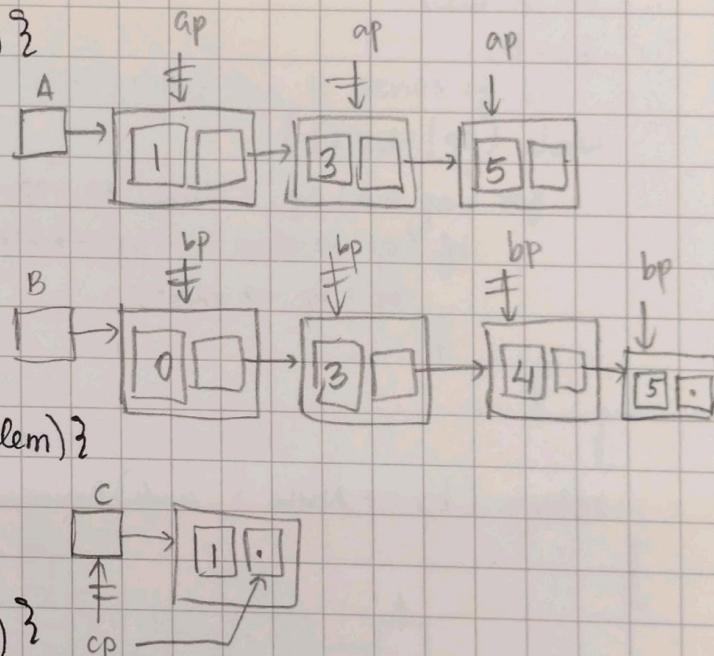
* cp = (SET) malloc (sizeof(ctype));

(*cp)->elem = ap->elem;

ap = ap->next;

cp = &(*cp)->next;

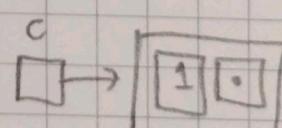
 } continue *



$$A = \{1, 3, 5\}$$

$$B = \{0, 3, 4, 5\}$$

RESULT:



* cp = NULL;
} return C;

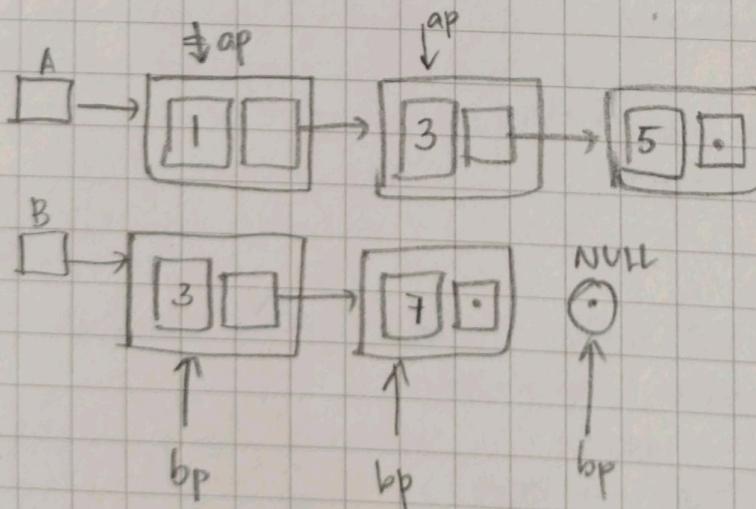
- Subtasks :
- ① Traverse through one set until the end
 - ② If element is found in the second set, move pointer to next node
 - ③ If 2nd pointer reaches NULL, the set is a subset of the larger set.
 - ④ Return 1 or 0

SUBSET is B subset of A?

```
int isSubset (SET A, SET B) {
    SET ap, bp;
    int retval;
    bp = B;

    while (bp != NULL) {
        for (ap = A; ap != NULL; ap = ap->next) {
            if (ap->elem == bp->elem) {
                bp = bp->next;
            }
        }
    }

    return (bp == NULL ? 1 : 0);
}
```



$$\begin{aligned} A &= \{1, 3, 5\} \\ B &= \{3, 5\} \end{aligned}$$

RESULT : 1, B is a subset of A.