Math 415 - Lecture 38

Applications of SVD

Monday December 7th 2015

Textbook reading: Chapter 6.3

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Strang lecture: Lecture 29: Singular Value Decomposition

- Final exam is on Thursday 12/17/2015 8am - 11am. We will announce the room assignment this week.

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- Please check your scores online. If incorrect, contact TA.

Review

Singular Value Decomposition:

$$A = \underbrace{\begin{bmatrix} | & & | \\ \mathbf{u}_1 & \dots & \mathbf{u}_m \\ | & & | \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} \sigma_1 & 0 & \\ 0 & \sigma_2 & \\ & & \ddots \end{bmatrix}}_{\Sigma} \underbrace{\begin{bmatrix} - & \mathbf{v}_1^T & - \\ & \vdots & \\ - & \mathbf{v}_n^T & - \end{bmatrix}}_{V^T}$$

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Image Compression

Idea

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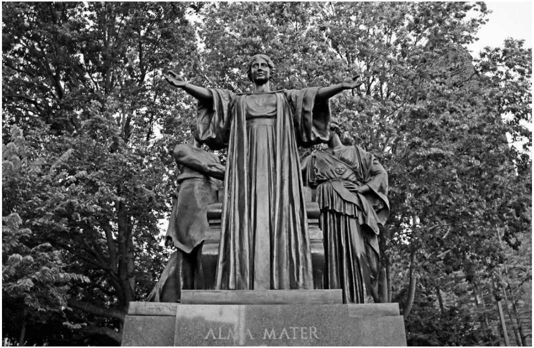

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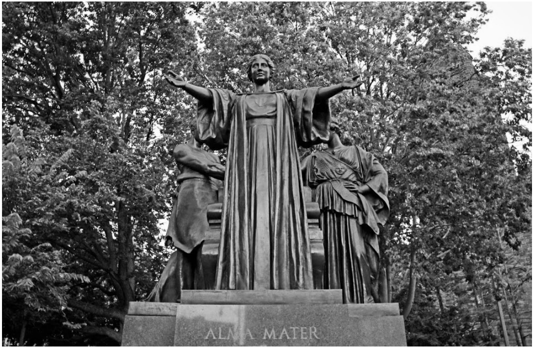

Each pixel is a shade of gray from 0 (black) to 255 (white). This gives an $m \times n$ matrix A. Each entry of A is one pixel of the image; that entry is some integer from 0 to 255, giving the brightness of that pixel.

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Question

Can we do better?

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Recall we can rewrite this as

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Throw away the term $\mathbf{u}_i\sigma_i\mathbf{v}_i^T$ when σ_i is small. If $k \leq r$, define

$$A_k = \mathbf{u}_1\sigma_1\mathbf{v}_1^T + \mathbf{u}_2\sigma_2\mathbf{v}_2^T + \ldots + \mathbf{u}_k\sigma_k\mathbf{v}_k^T$$

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The matrix A_k is very close to the matrix A, if $\sigma_{k+1}, \dots, \sigma_r$ are small.

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- If $k = 100$, then to store the matrix A_{100} we need the numbers $\sigma_1, \ldots, \sigma_{100}$, the vectors $\mathbf{u}_1, \ldots, \mathbf{u}_{100}$ and $\mathbf{v}_1, \ldots, \mathbf{v}_{100}$.

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We reduced the file size by a factor of four!

Examples

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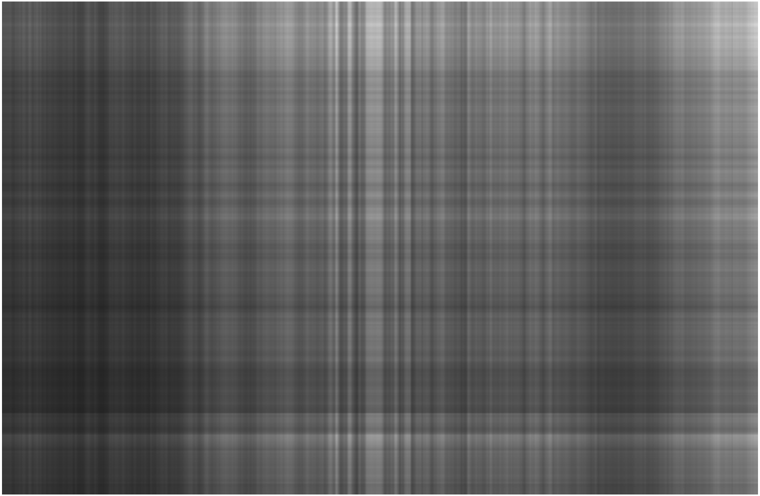
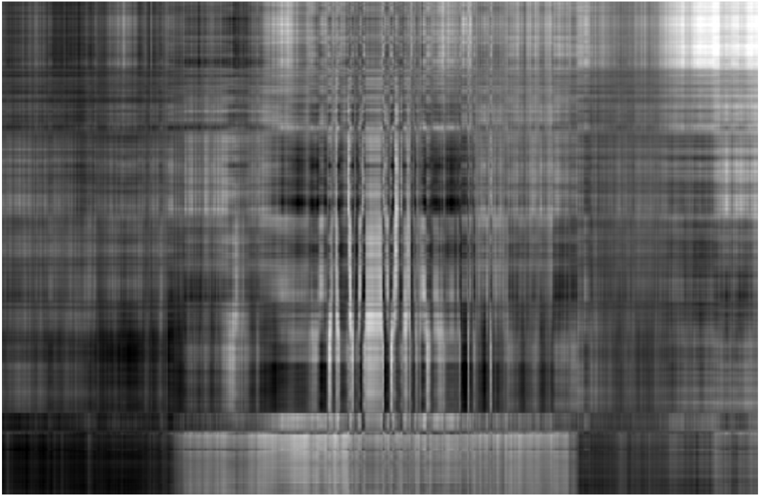


A_1

Examples



A_5

Examples

A_{25}

Examples

A_{50}

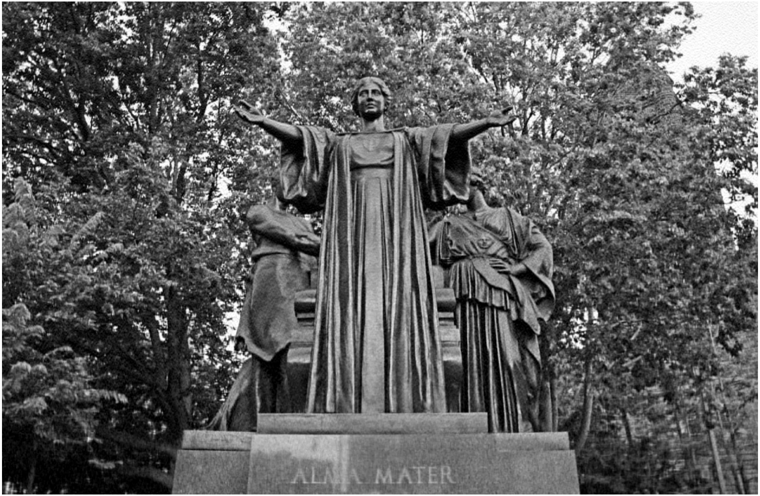
Examples

A_{100}

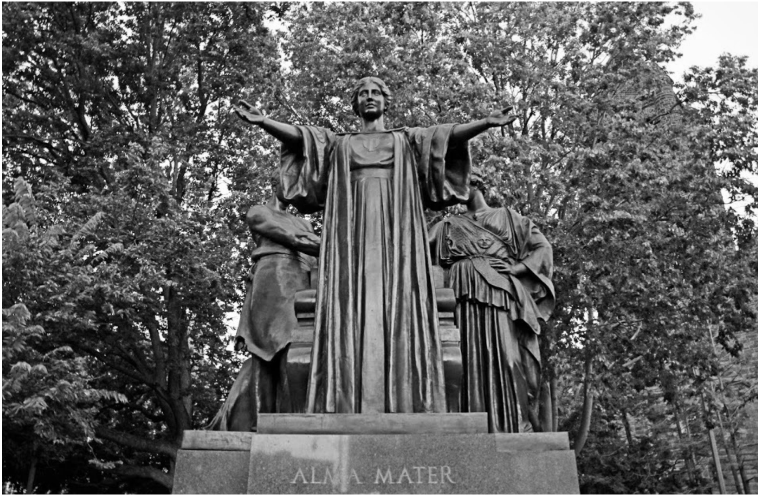
Examples

A_{150}

Examples



A_{200}



A. Probably rank A is 625, so $A = A_{625}$.

Face recognition

Idea

A set of eigenfaces can be generated by performing a mathematical process called principal component analysis (PCA) on a large set of images depicting different human faces. Informally, eigenfaces can be considered a set of "standardized face ingredients", derived from statistical analysis of many pictures of faces. Any human face can be considered to be a combination of these standard faces. For example, one's face might be composed of the average face plus 10% from eigenface 1, 55% from eigenface 2, and even -3% from eigenface 3.

Remarkably, it does not take many eigenfaces combined together to achieve a fair approximation of most faces. Also, because a person's face is not recorded by a digital photograph, but instead as just a list of values (one value for each eigenface in the database used), much less space is taken for each person's face.

Algorithm

Part I - Set up

Step 1. Start with a set of d face images - same resolution, say $r \times c$. Each image is treated as one vector by concatenating the columns of pixels in the original image, resulting in vector in $\mathbb{R}^{rc}$. Set $n = rc$. This gives us d vectors $\mathbf{x}_1, \ldots, \mathbf{x}_d$ in $\mathbb{R}^n$.

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Step 3. Calculate the singular value decomposition of X. Say $X = U\Sigma V^T$. Note that $U = \begin{bmatrix} \mathbf{u}_1 & \ldots & \mathbf{u}_n \end{bmatrix}$ is a $n \times n$-matrix, V is $d \times d$-matrix.

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Step 4. The vectors $\mathbf{u}_1, \ldots, \mathbf{u_n}$ are called eigenfaces. Forget all eigenfaces, where the corresponding singular value is small. Let's say, we keep $\mathbf{u}_1, \ldots \mathbf{u}_s$ for some $s < n$.

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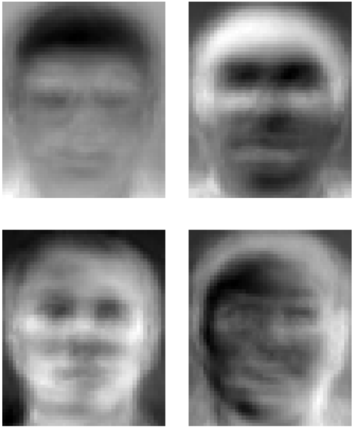
Note that $\mathbf{u}_1, \ldots, \mathbf{u}_\mathbf{n}$ are the eigenvalues (principal components/axes) of XX^T. We can think of XX^T as the matrix of all possible combination of the faces. Therefore $\mathbf{u}_1, \ldots, \mathbf{u}_\mathbf{n}$ are the principal components of all the possible faces.



Some eigenfaces from AT&T Laboratories Cambridge

Part II - Learning new faces

Suppose we want to add a face $\mathbf{f} \in \mathbb{R}^n$ to the database. Then instead of saving f, we save the following vector in $\mathbb{R}^s$

$$\begin{bmatrix} (\mathbf{f}-\mathbf{a}) \cdot \mathbf{u}_1 \\ \vdots \\ (\mathbf{f}-\mathbf{a}) \cdot \mathbf{u}_s \end{bmatrix}.$$

We call this vector $\mathbf{w}_f$.

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Do you know what the entries represent? The weights used in the projection of $\mathbf{f} - \mathbf{a}$ onto $\text{span}(\mathbf{u}_1, \ldots, \mathbf{u}_s)$! So $\mathbf{f}$ is composed of the average face $\mathbf{a}$ plus scalar $(\mathbf{f} - \mathbf{a}) \cdot \mathbf{u}_1$ times eigenface 1 plus $(\mathbf{f} - \mathbf{a}) \cdot \mathbf{u}_2$ times eigenface 2, etc.

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Suppose we are now given a photo of a person $\mathbf{p} \in \mathbb{R}^n$. First calculate $w_{\mathbf{p}}$. How do we know which person in the database is most likely this person? Easy, simply take the face f in the database such that $\|w_p - w_f\|$ is minimal!