UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN

CS411 - Extreme Indexing

illinois.edu

Announcements

- Get MP2 finished
- Enjoy your break!

Extreme Indexing

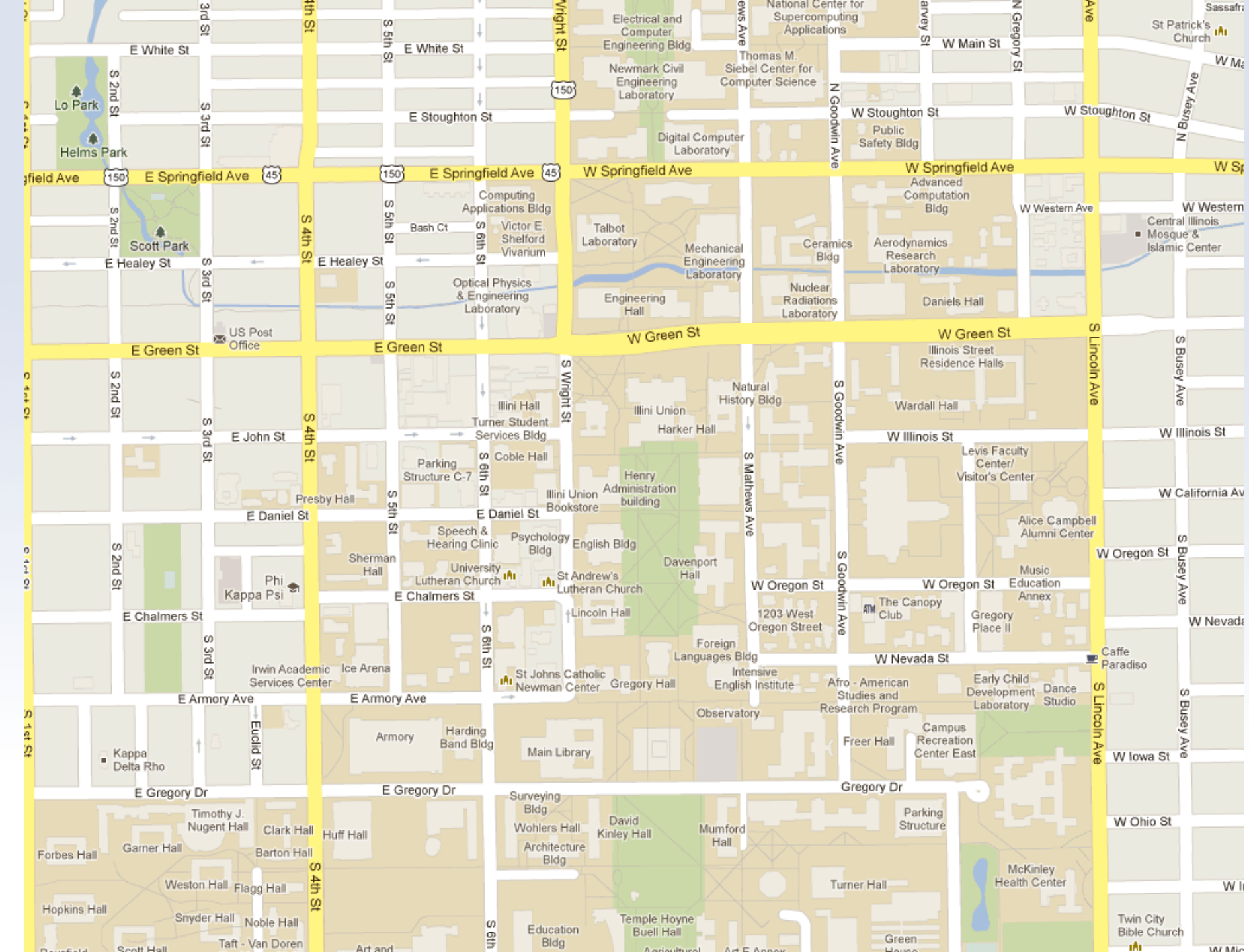
- Indexing tricks we've used so far are great for relational data.
- What if we wanted to query other kinds of data quickly?

Examples

- Where is the nearest grocery store?
- What coffee shop am I in now?
- What web pages contain the words “Belle” and “Sebastian”?
- Where in the human genome (a string of 3 billion characters) does the string “GATTACA” appear?

Multidimensional Data

- We're focus on spatial data
- Queries involve points, lines, and polygons
  - Range queries: find points/shapes in a region
  - Nearest-neighbor: find the nearest point
  - Location queries: find shapes containing a point
- Our examples will be two dimensions: x,y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S Wright St
Lo Park
Helms Park
E Stoughton St
Electrical and Computer Engineering Bldg
National Center for Supercomputing Applications
Thomas M. Siebel Center for Computer Science
Newmark Civil Engineering Laboratory
W Main St
N Gregory St
St Patrick's Church
N Goodwin Ave
W Stoughton St
Public Safety Bldg
Digital Computer Laboratory
E Springfield Ave
W Springfield Ave
Advanced Computation Bldg
W Western Ave
Central Illinois Mosque & Islamic Center
Computing Applications Bldg
Victor E. Shelford Vivarium
Bash Ct
Scott Park
E Healey St
Talbot Laboratory
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Aerodynamics Research Laboratory
Optical Physics & Engineering Laboratory
Engineering Hall
Nuclear Radiations Laboratory
Daniels Hall
US Post Office
E Green St
W Green St
S Lincoln Ave
Illinois Street Residence Halls
Natural History Bldg
Wardall Hall
Illini Hall
Turner Student Services Bldg
Illini Union
Harker Hall
S Busey Ave
E John St
W Illinois St
Parking Structure C-7
Coble Hall
Henry Administration building
Levis Faculty Center/ Visitor's Center
Presby Hall
Illini Union Bookstore
E Daniel St
W California Av
Speech & Hearing Clinic
Psychology Bldg
English Bldg
Alice Campbell Alumni Center
Sherman Hall
University Lutheran Church
St Andrew's Lutheran Church
Davenport Hall
W Oregon St
Music Education Annex
Phi Kappa Psi
E Chalmers St
Lincoln Hall
1203 West Oregon Street
The Canopy Club
Gregory Place II
W Nevada
Foreign Languages Bldg
W Nevada St
Caffe Paradiso
Irwin Academic Services Center
Ice Arena
St Johns Catholic Newman Center
Gregory Hall
Intensive English Institute
Afro - American Studies and Research Program
Early Child Development Laboratory
Dance Studio
E Armory Ave
Observatory
Armory
Harding Band Bldg
Main Library
Freer Hall
Campus Recreation Center East
W Iowa St
Kappa Delta Rho
Euclid St
E Gregory Dr
Gregory Dr
Surveying Bldg
Timothy J. Nugent Hall
Clark Hall
Huff Hall
Wohlers Hall
David Kinley Hall
Mumford Hall
Parking Structure
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McKinley Health Center
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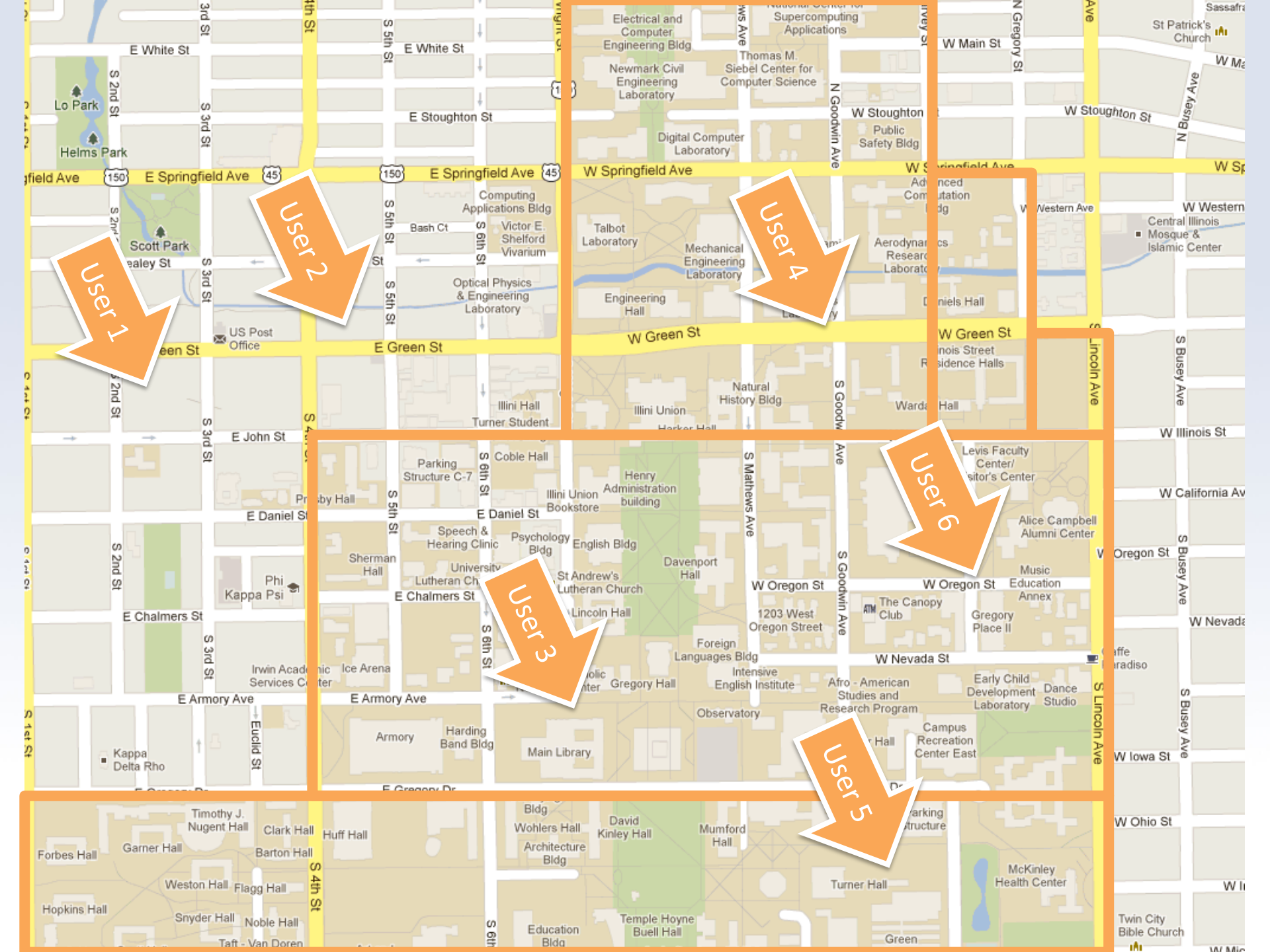
Temple Hoyne Buell Hall
Twin City Bible Church


User 1
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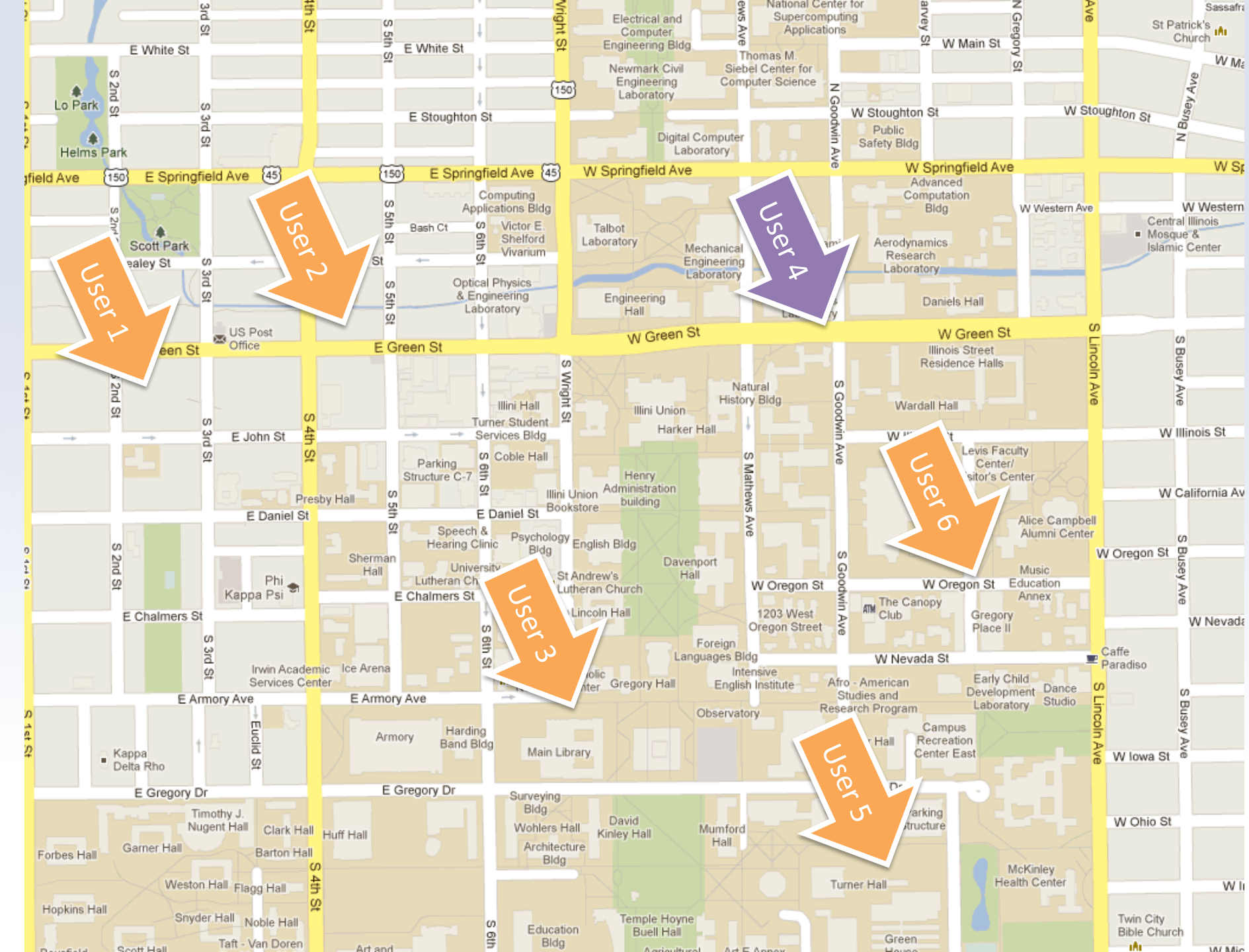
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S Lincoln Ave
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Goodwin Ave
Mathews Ave
Beckman Institute for Advanced Science and Technology
Engineering Parking Garage
Chiller Plant
Electrical and Computer Engineering Bldg
Coordinated Science Laboratory
Oval Allée
Newmark Civil Engineering Laboratory
Thomas M. Siebel Center for Computer Science
Richmond Studio
University High School Gymnasium
Public Safety Bldg
Digital Computer Laboratory
University Place Christian Church
Shlens Edmund N CPA
Niro's Gyros
Mulberry Alley
Dos Reales
Hot Wok Express
Computing Applications Bldg
Grainger Engineering Library Information Center
Oil Chemistry Building
Victor E. Shelford Vivarium
Talbot Laboratory
Mechanical Engineering Laboratory
Transportation Bldg
Aeronautical Laboratory A
Ceramics Kiln House
Ceramics Bldg
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Environmental Health and Safety
Aerodynamics Research Laboratory
Plant Services Bldg North
Engineering Senior Design Studio & Annex Plant Services Storage Bldg
Optical Physics & Engineering Laboratory
Mechanical Engineering Bldg
Nuclear Radiations Laboratory
Everitt Laboratory
Engineering Hall
Materials Science & Engineering
Atmospheric Sciences Annex2
Daniels Hall
Astronomy Bldg
Sushi Ichiban
Follett
User 4

MySQL

```
create table Points (
name VARCHAR(20) PRIMARY KEY,
location Point NOT NULL,
description VARCHAR(200),
SPATIAL INDEX(location)
);
```

MySQL

```
SELECT name, AsText(location)
FROM Points
WHERE Intersects(location, GeomFromText(@bbox) );
+------------+----------------------+
| name       | AsText(location)     |
+------------+----------------------+
| name_243.0 | POINT(6.1908 3.1375) |
| name_429.0 | POINT(6.5194 0.4023) |
| name_533.0 | POINT(7.7479 3.0894) |
| name_808.0 | POINT(4.6818 7.618)  |
+------------+----------------------+
```

Multidimensional Indexing

- The book covers a lot of structures for this:
  - Grid Files
  - Partitioned Hash Tables
  - Multiple-Key Indexes
  - Quad trees
- We'll only cover two

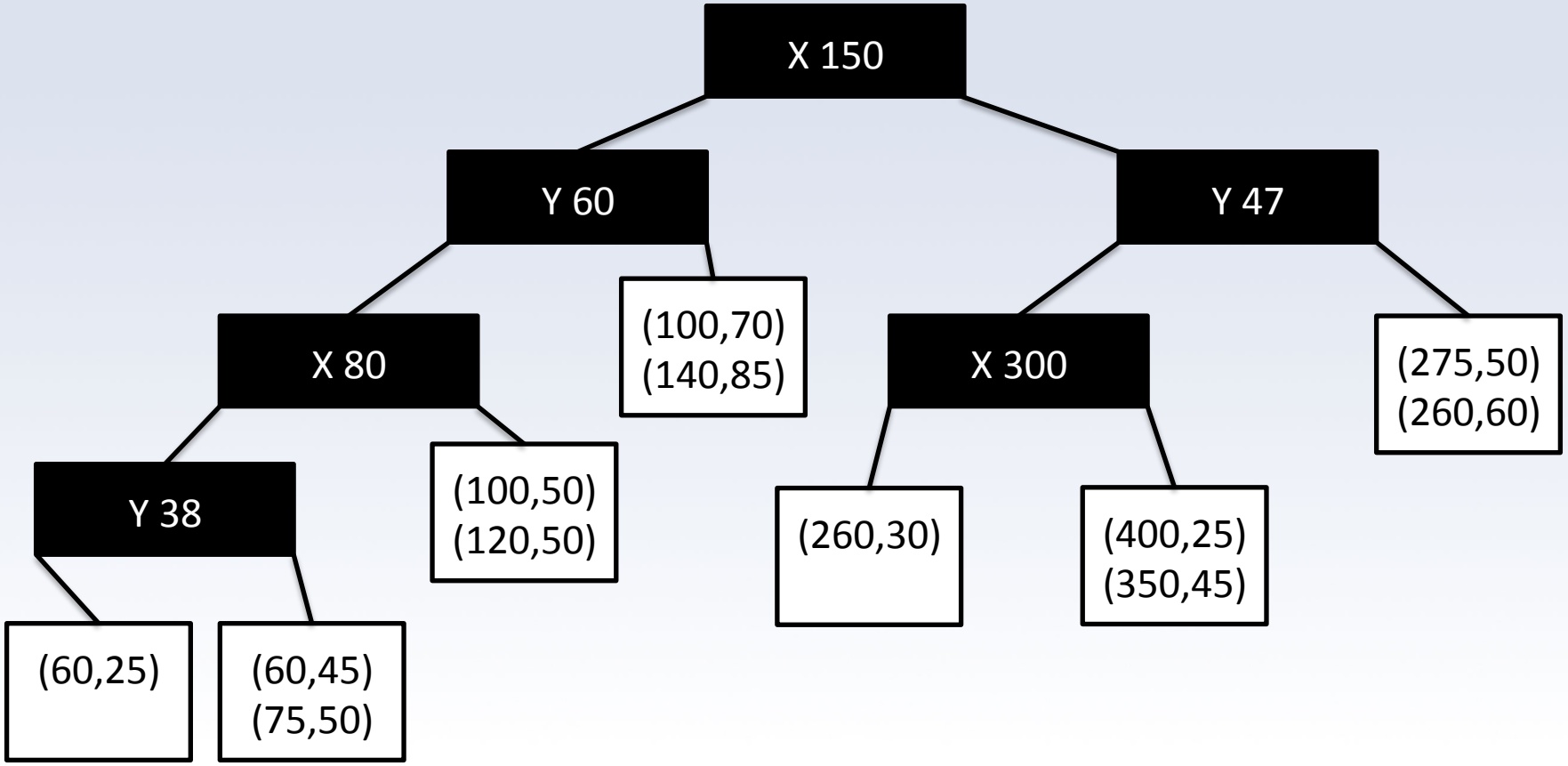
k-d trees

- Useful for range and nearest neighbor queries
- Similar to binary search tree
- Internal nodes alternate dimensions
  - Root splits x
  - Child splits y
  - Grandchild splits x
  - Great grandchild splits y…

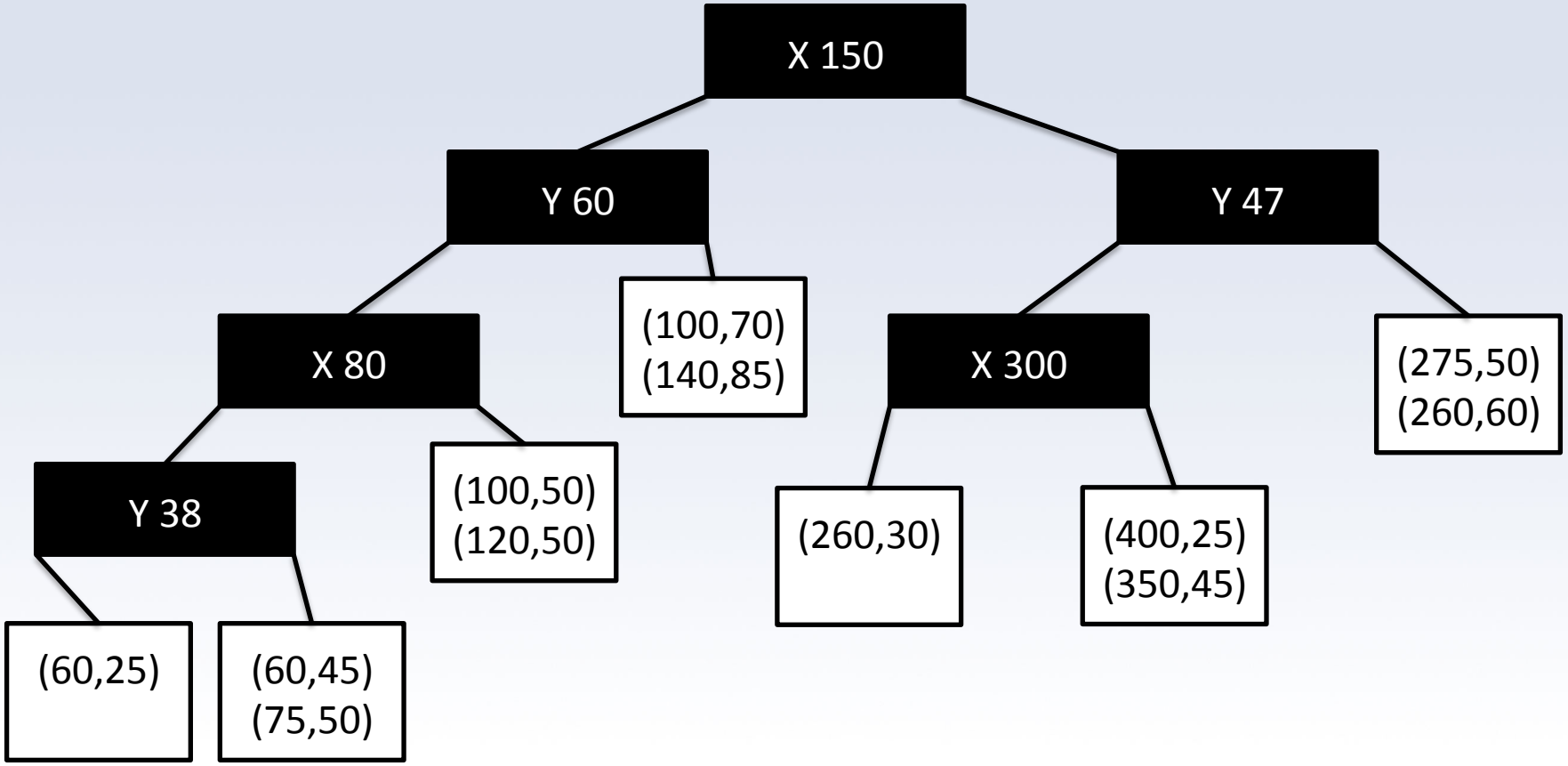
k-d trees

- Each internal node has a dimension D and value V associated with it
  - If R.D < V, point in left subtree
  - If R.D $\geq$ V, point in right subtree
- Leaves contain blocks
  - not typical for memory-based k-d trees

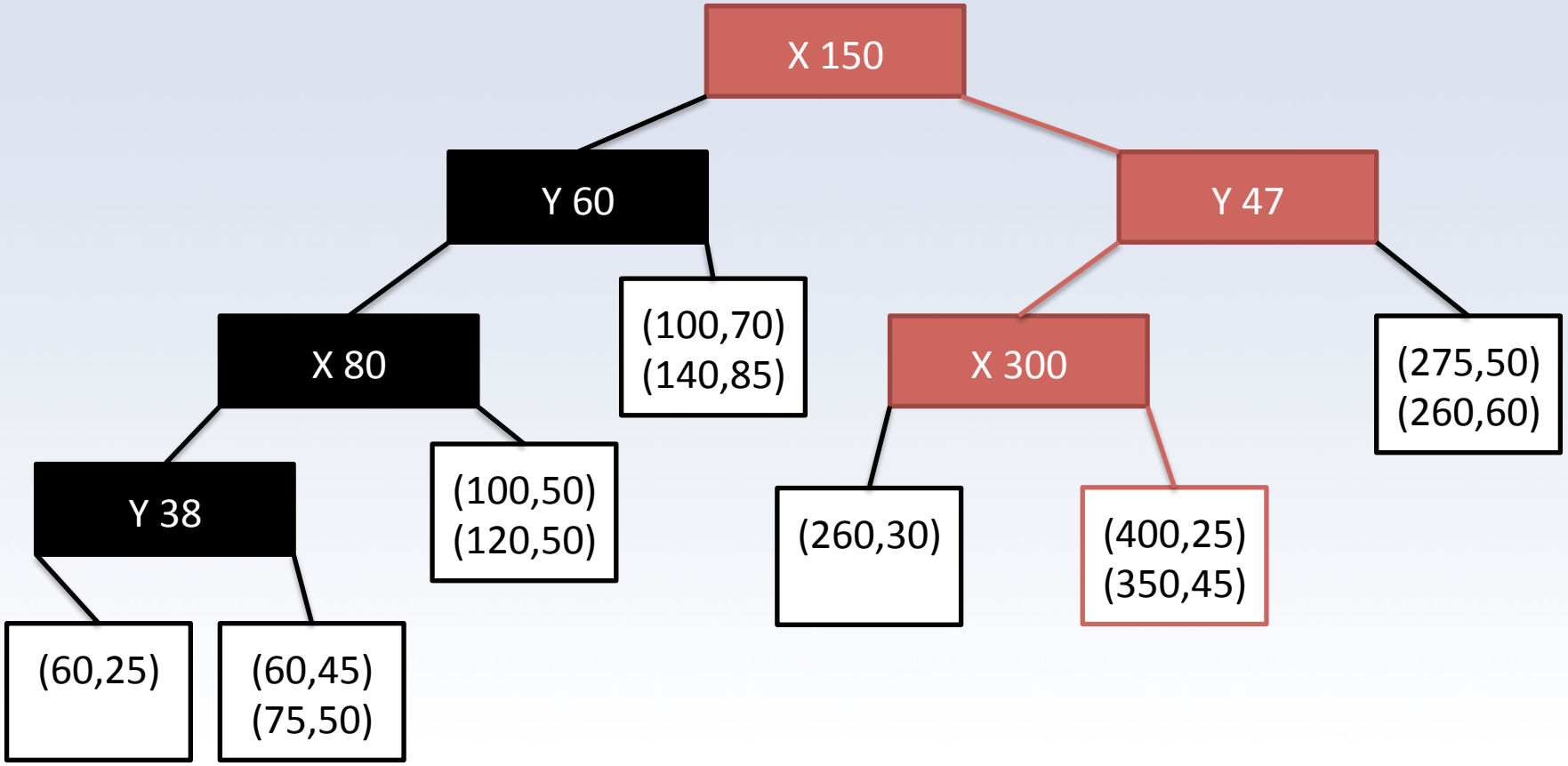
Example


X 150
Y 60
Y 47
X 80
(100,70)
(140,85)
X 300
(275,50)
(260,60)
Y 38
(100,50)
(120,50)
(260,30)
(400,25)
(350,45)
(60,25)
(60,45)
(75,50)

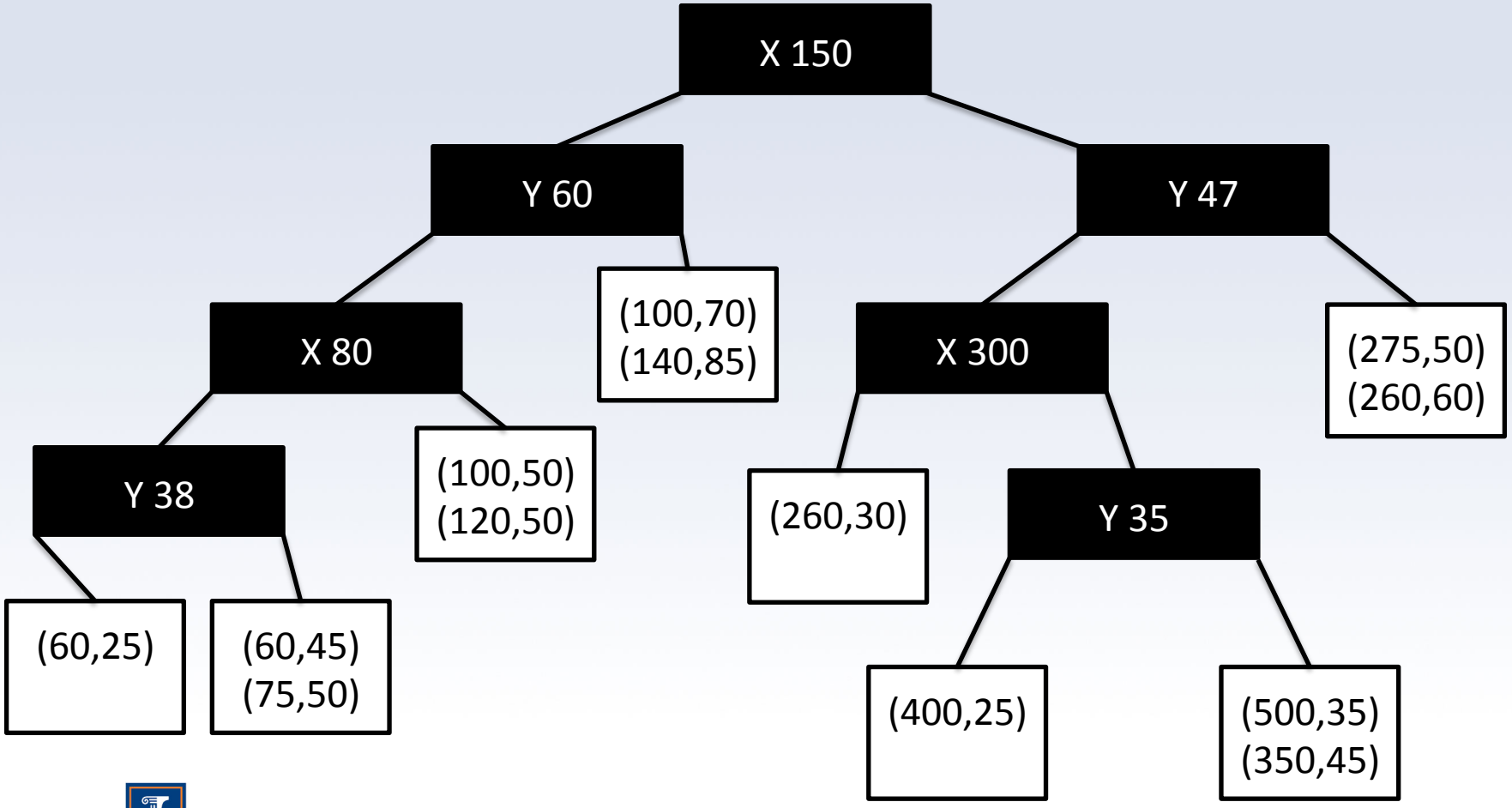
Insert (500,35)



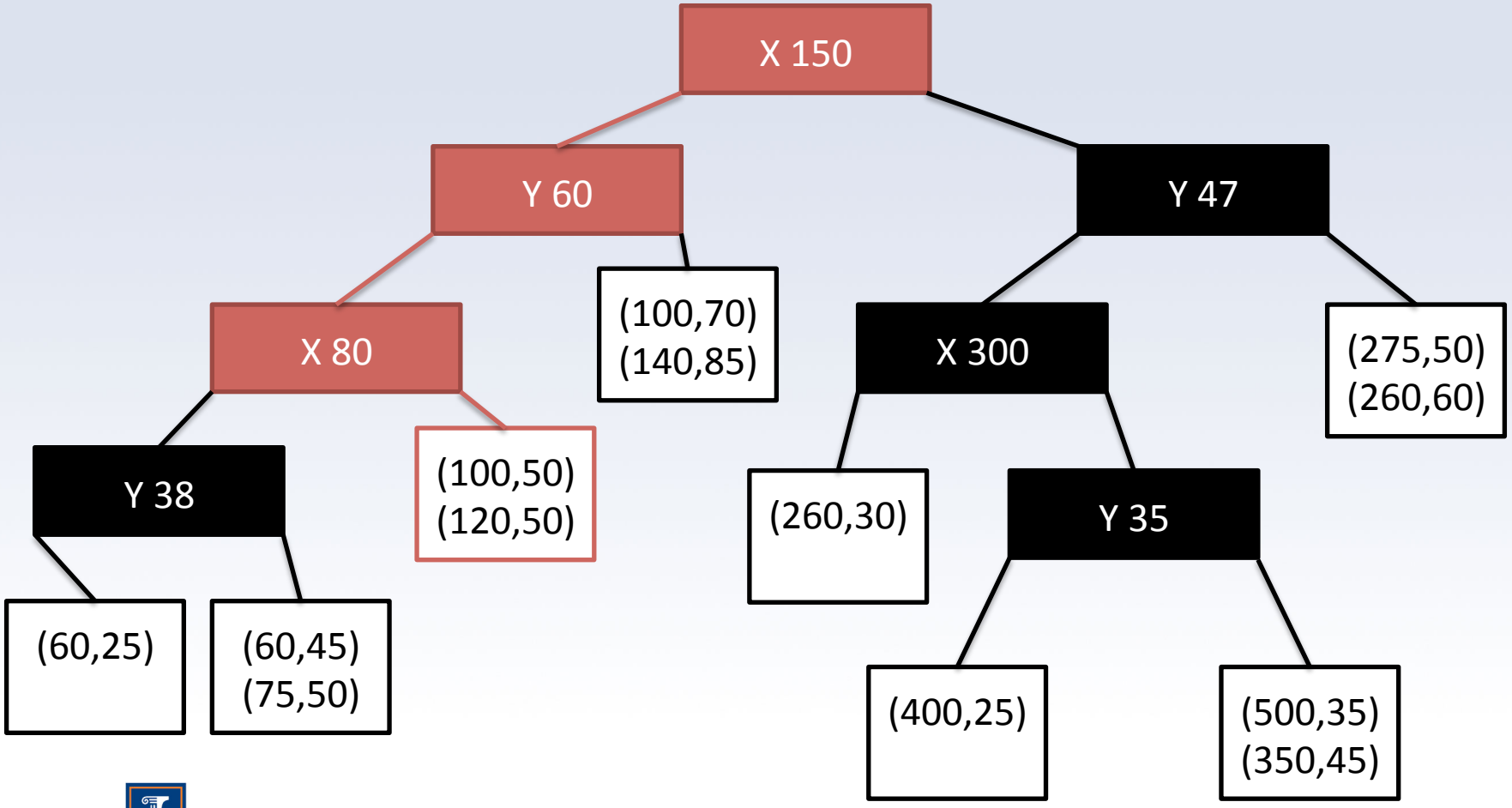
Insert (500,35)



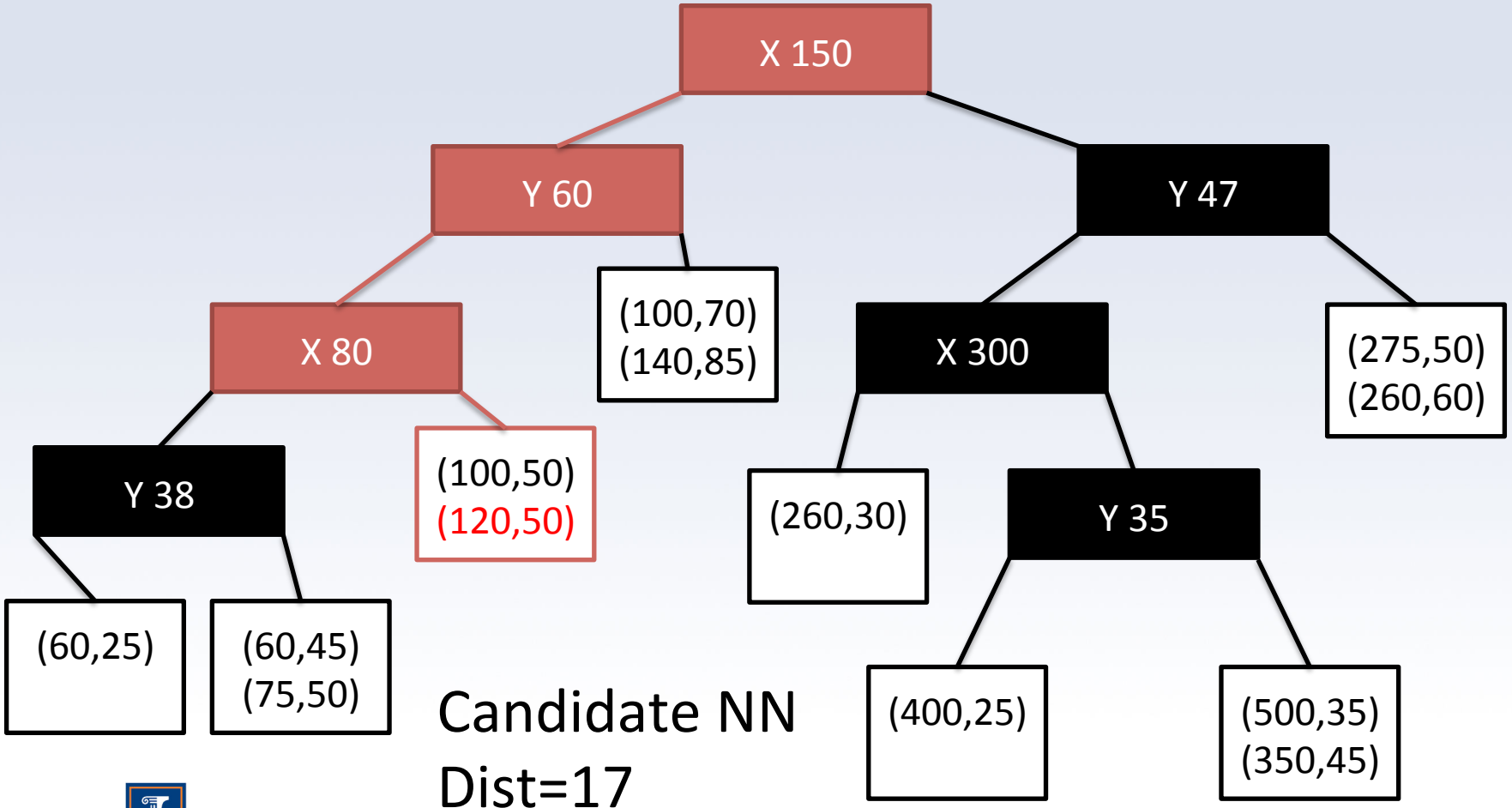
Insert (500,35)



Find NN (120,33)



Find NN (120,33)



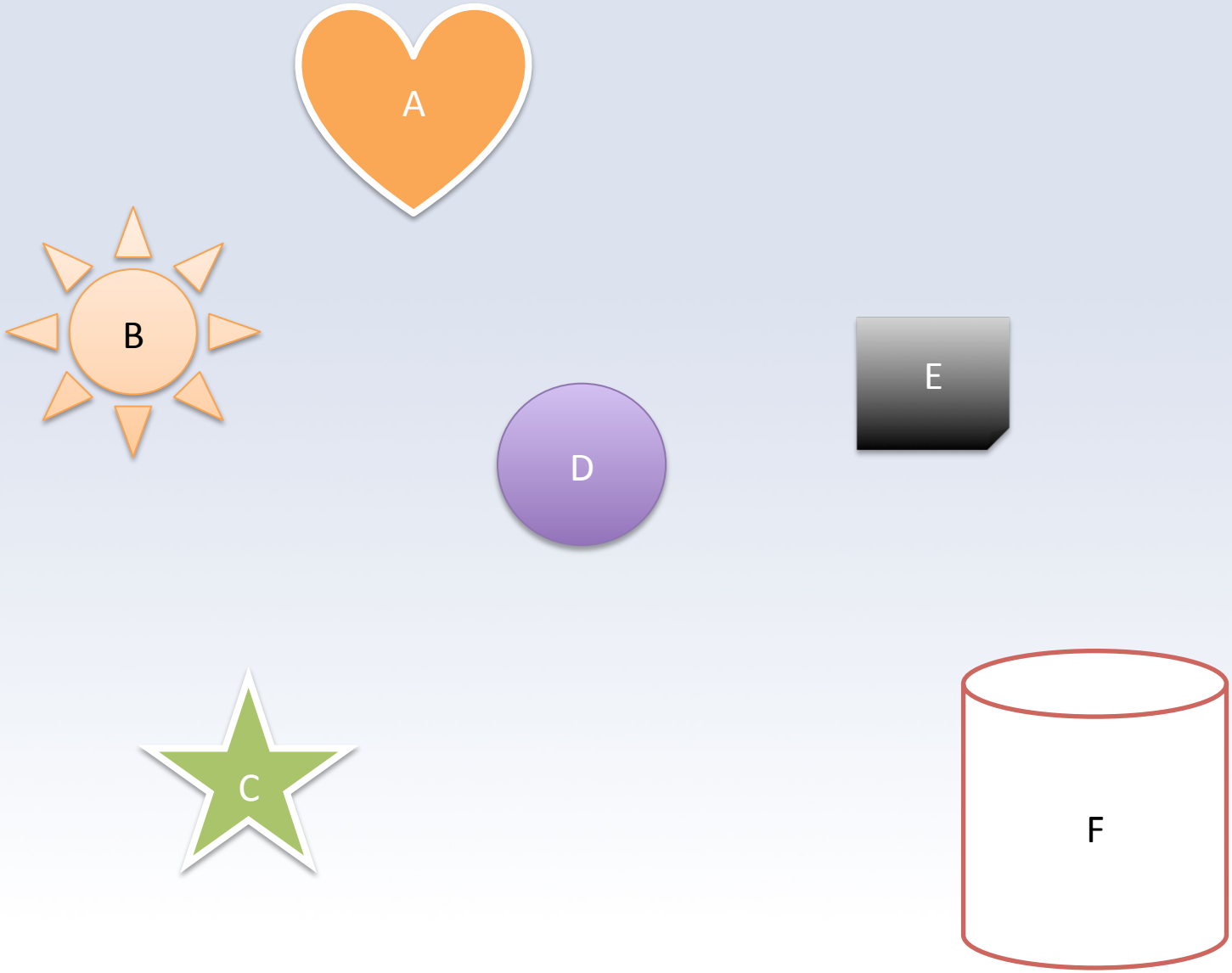
Candidate NN
Dist=17

Find NN (120,33)

- After finding a candidate, we have to "unwind" back up the search tree to check if any points are closer
- Requires more data at internal nodes
- Not going to cover this

R-trees

- Groups nearby objects by *minimum bounding rectangles*
  - Smallest rectangle that contains them
- Groups the groups with more bounding rectangles
- Creates a tree of bounding regions


A
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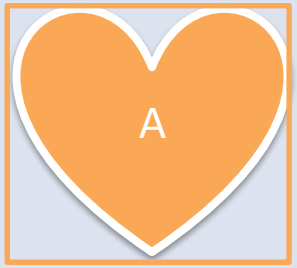
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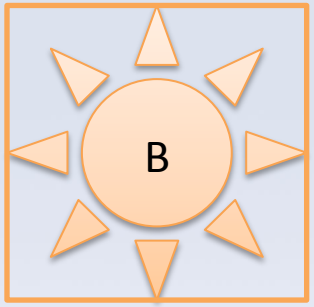
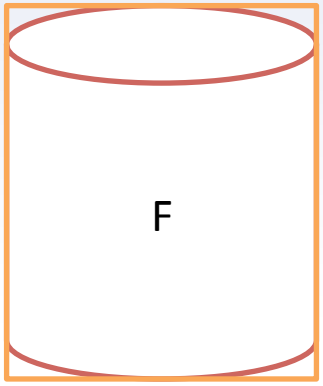

B


D


E


C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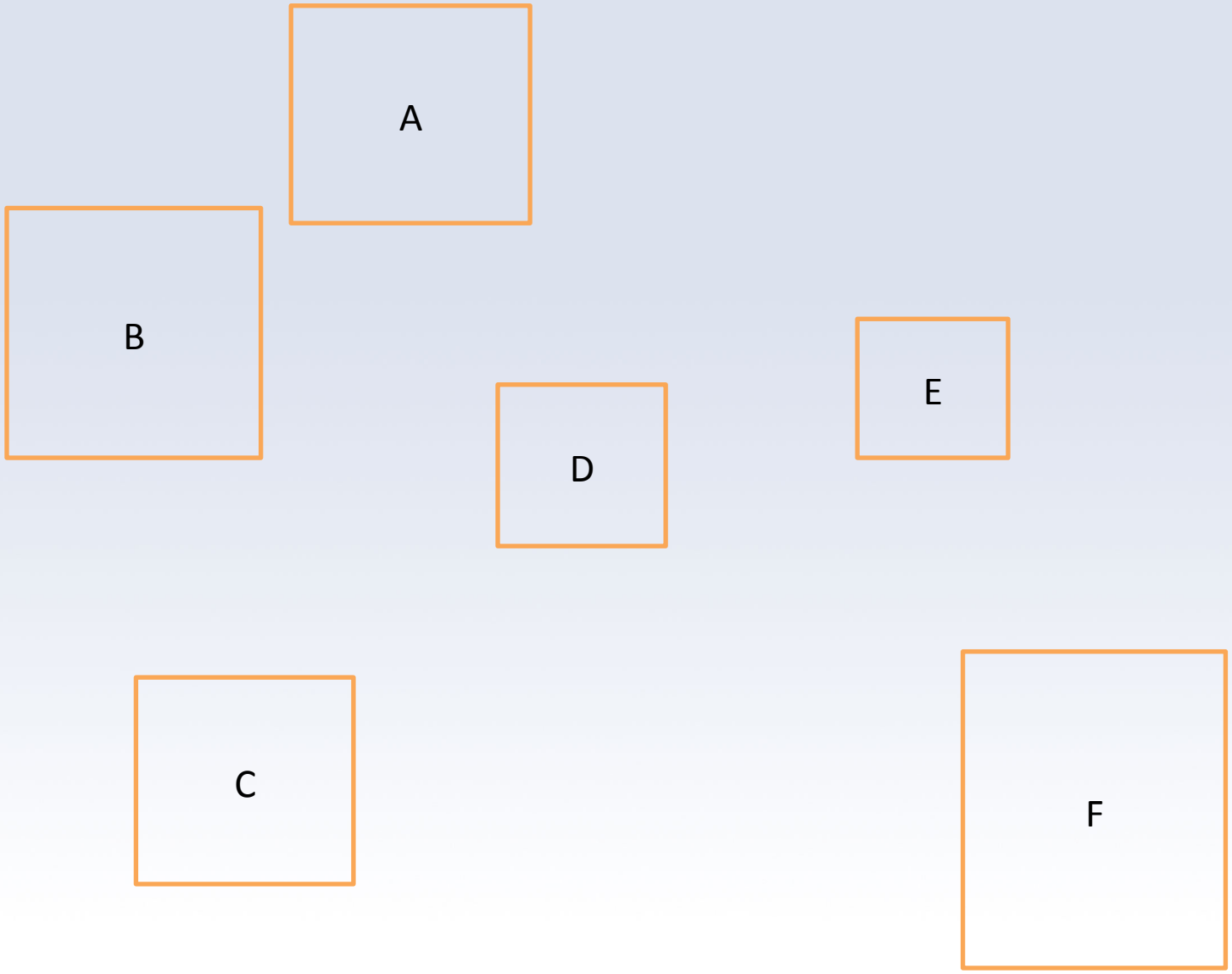
A
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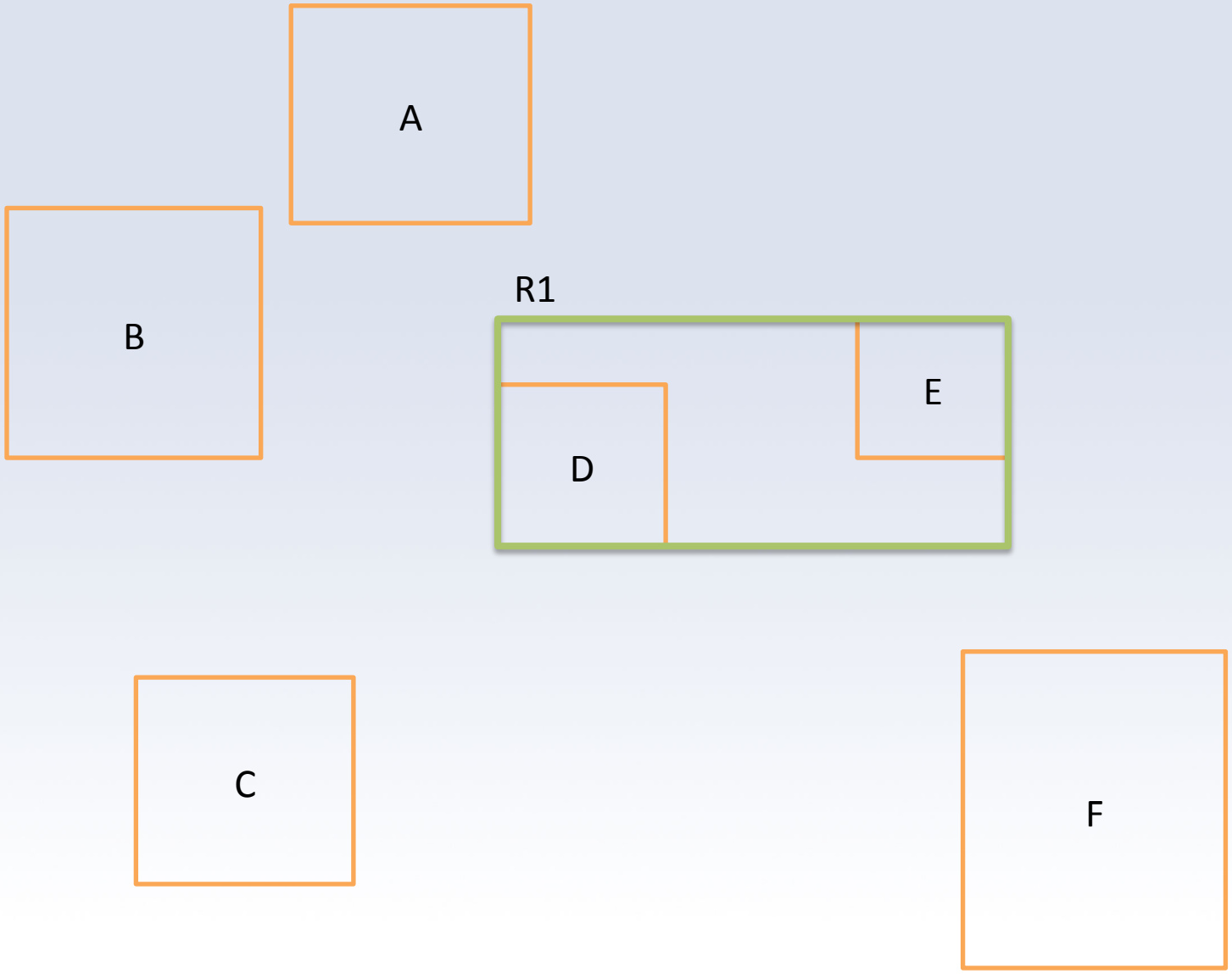
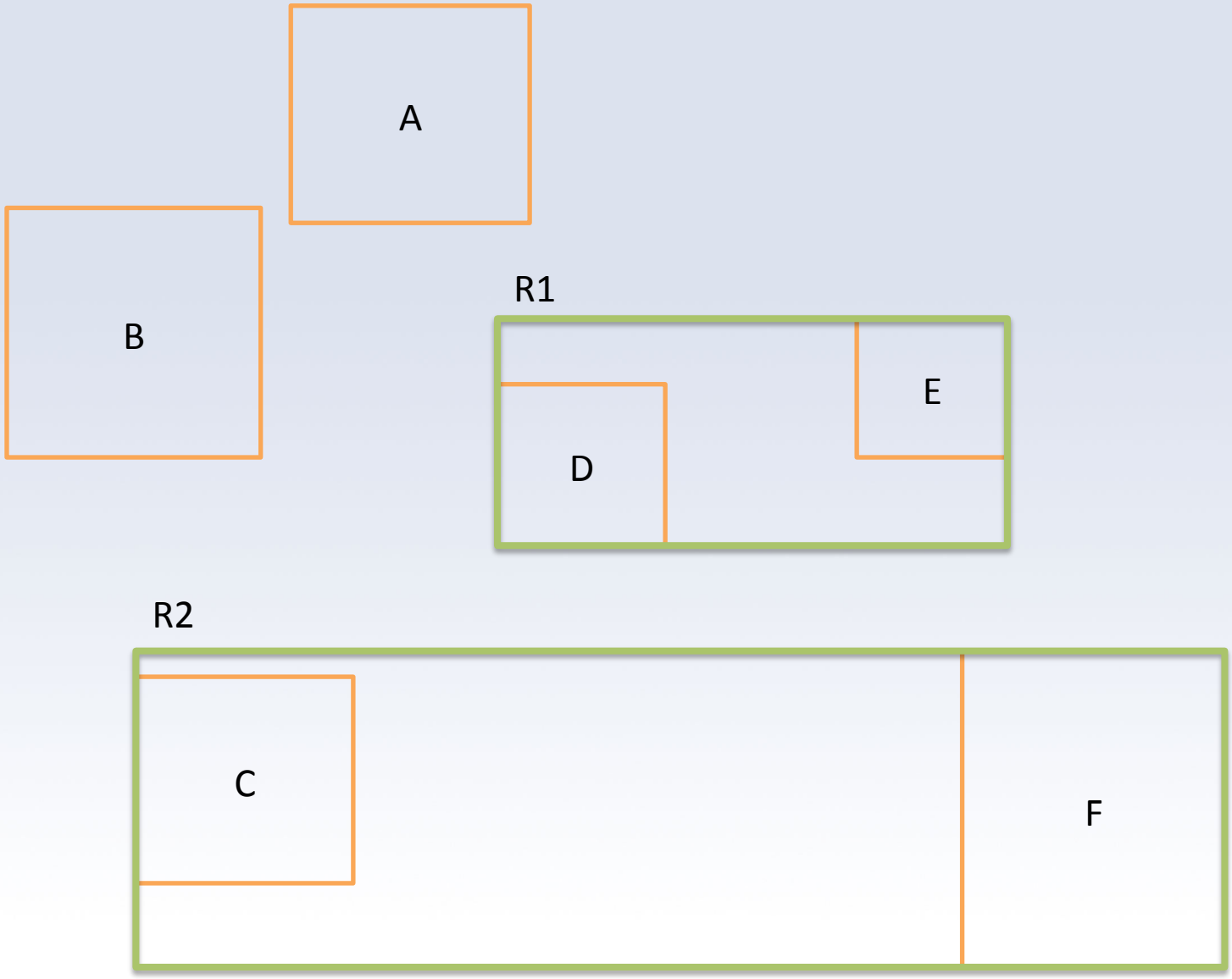
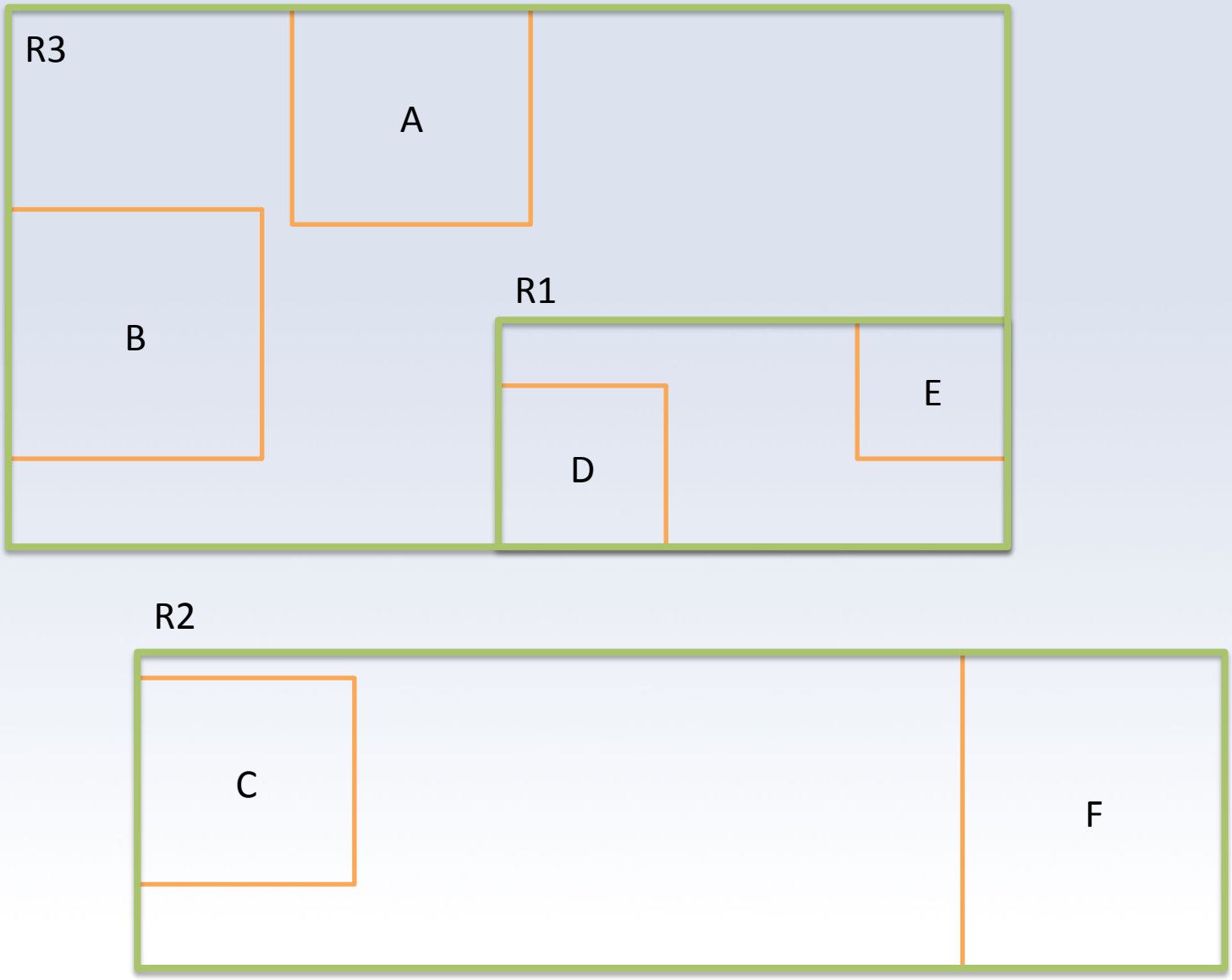
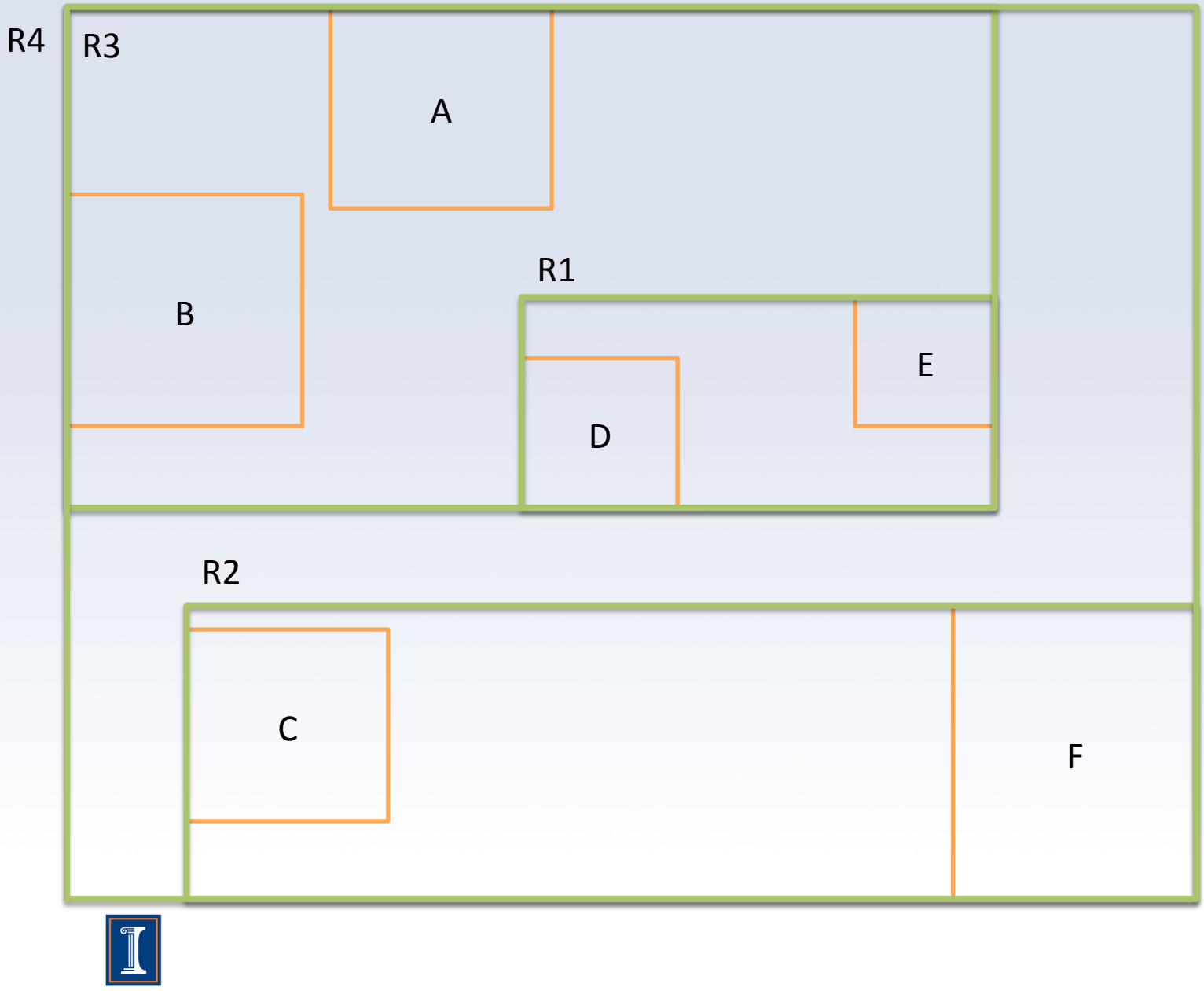

A
B
R1
E
D
C
F


A
B
R1
E
D
R2
C
F


R3
A
B
R1
E
D
R2
C
F


R4
R3
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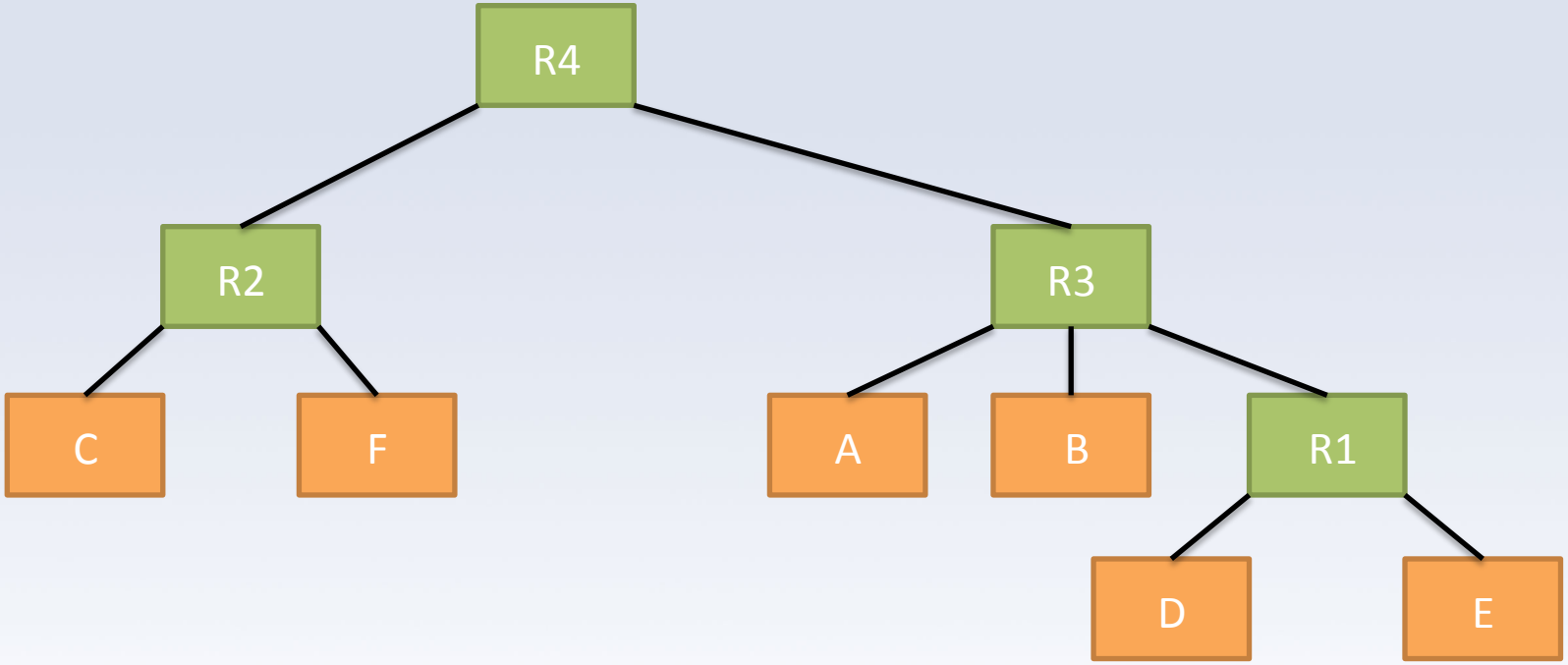
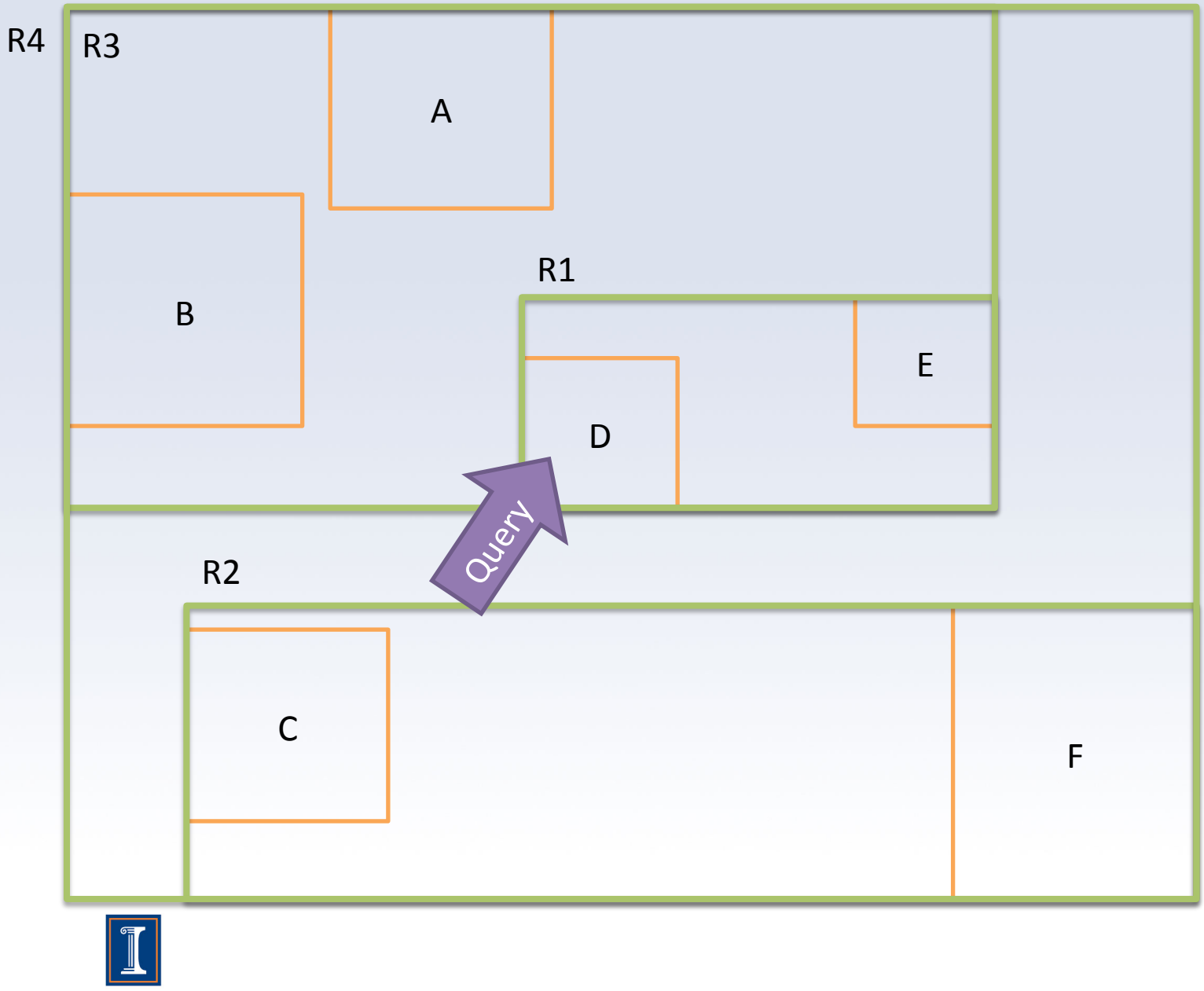
F

R-tree



Location query

- Recursively check if point is in each rectangle


R4
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Query
R2
C
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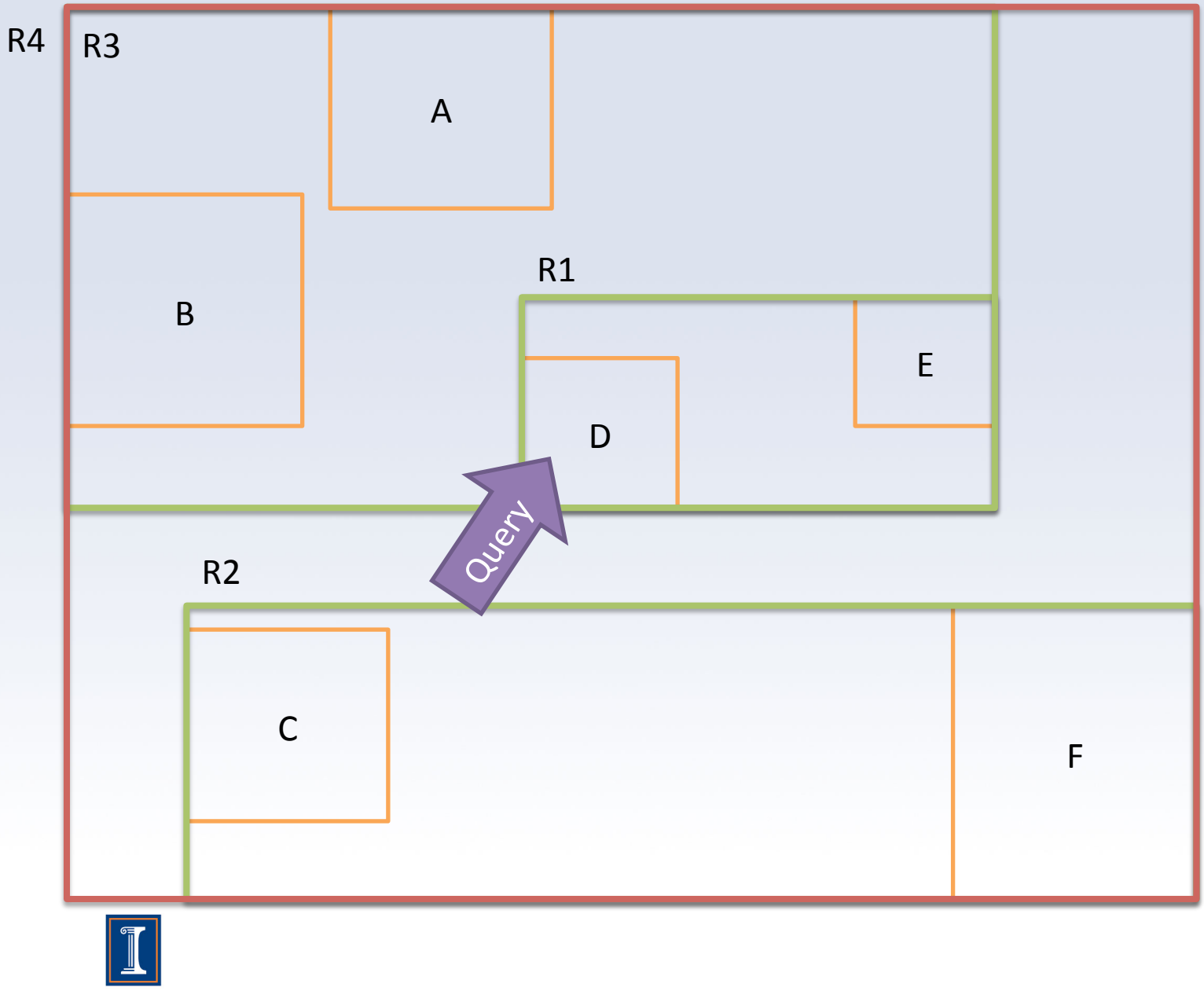
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Query
R2
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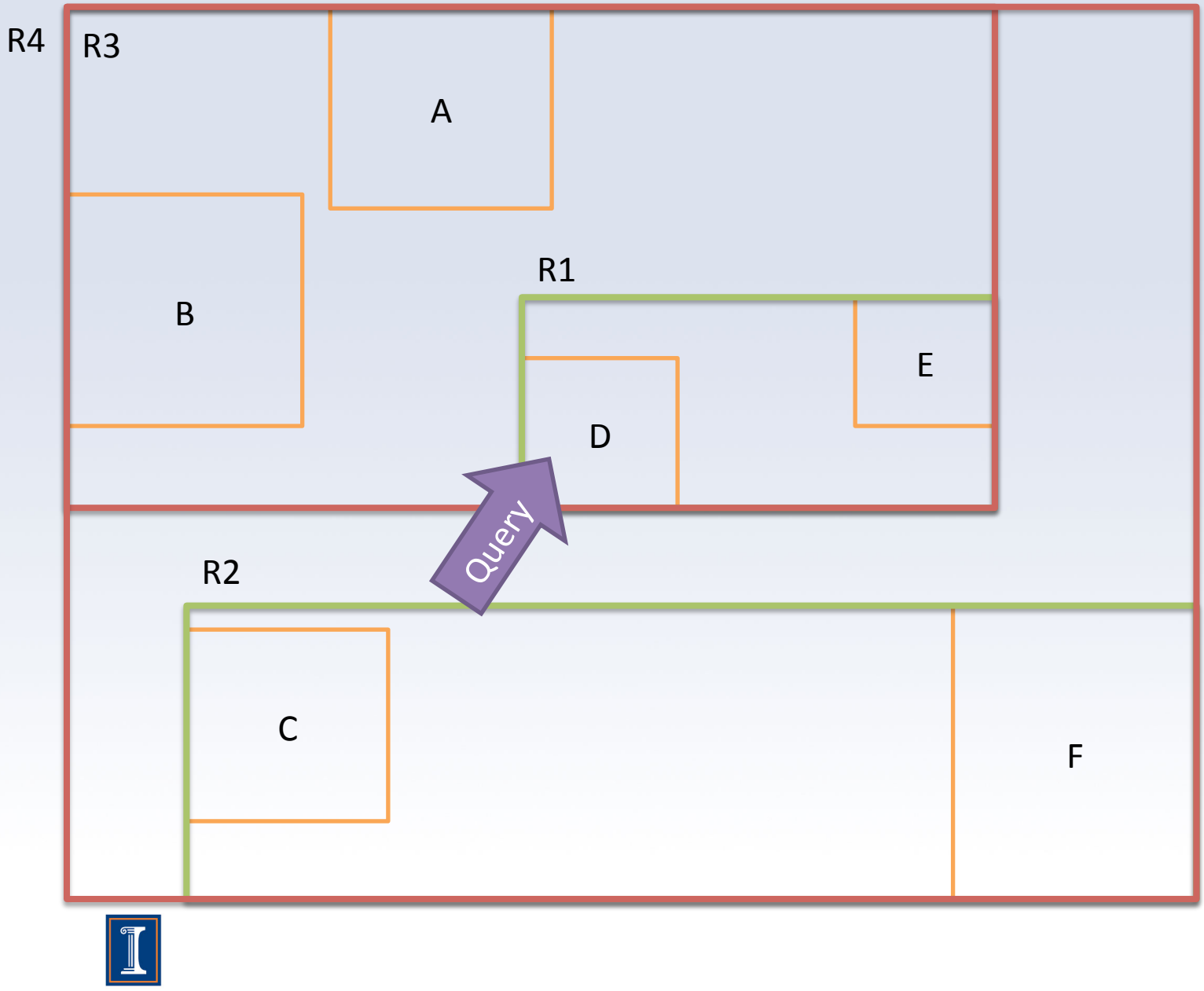
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Query
R2
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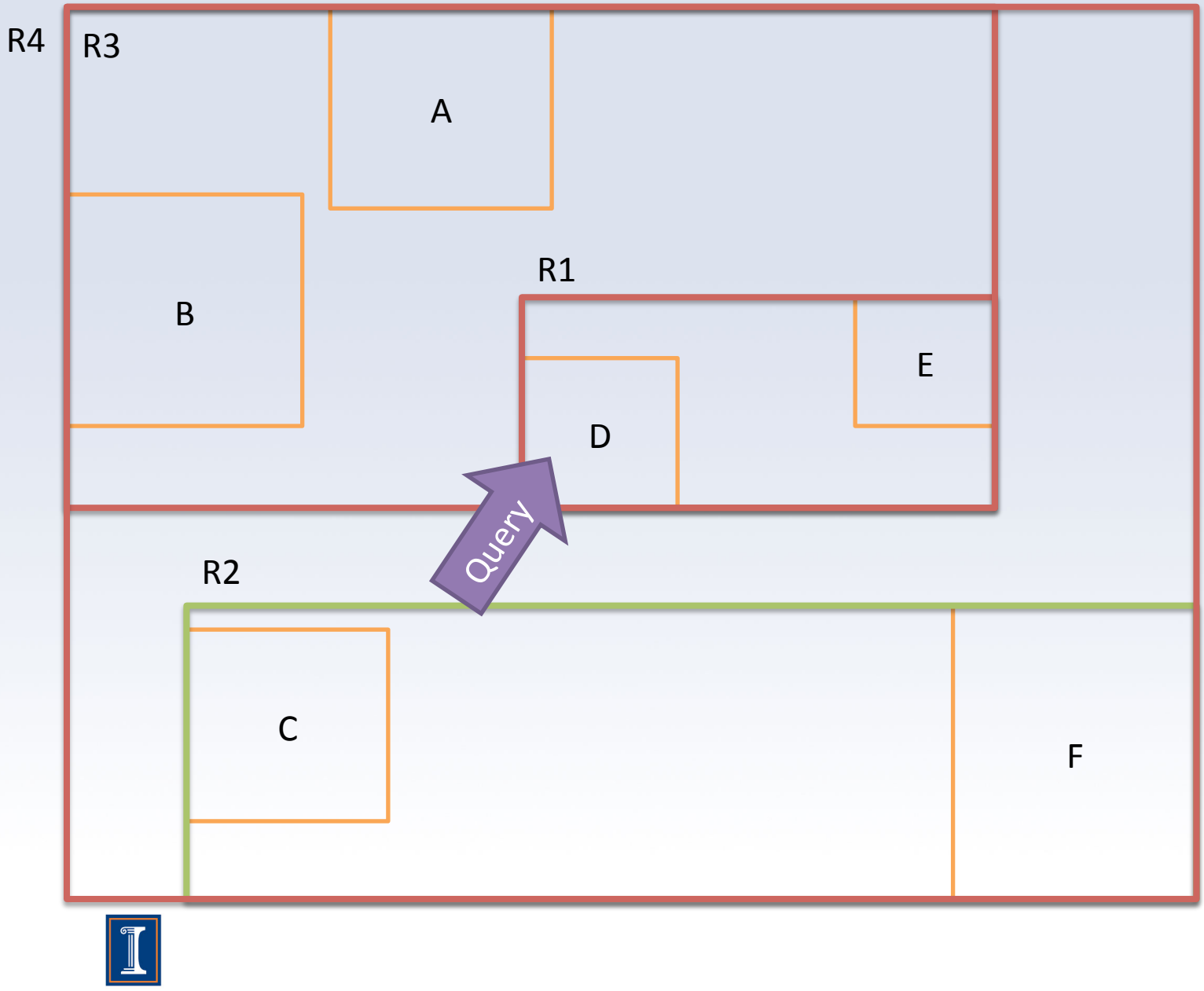
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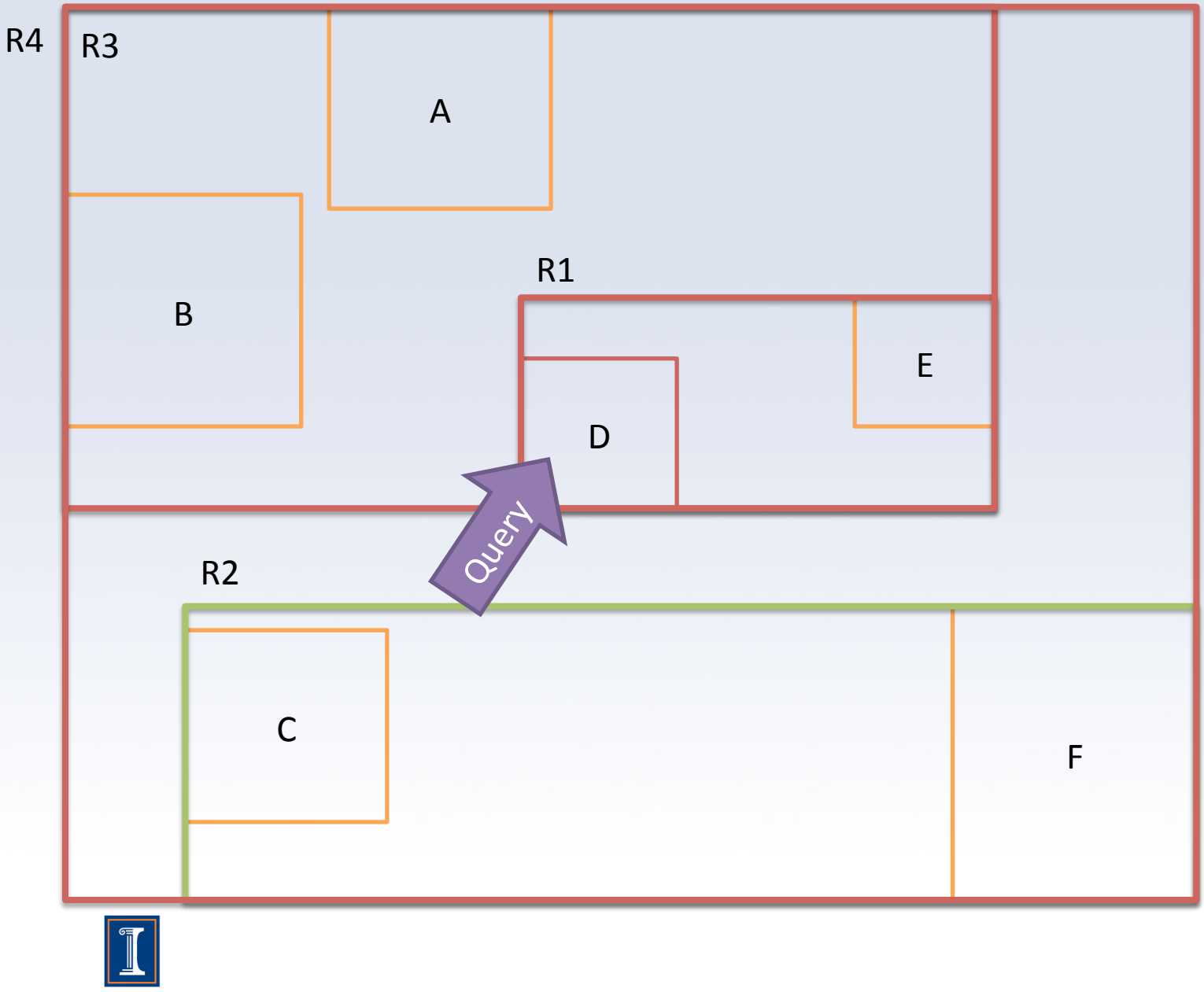
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R2
C
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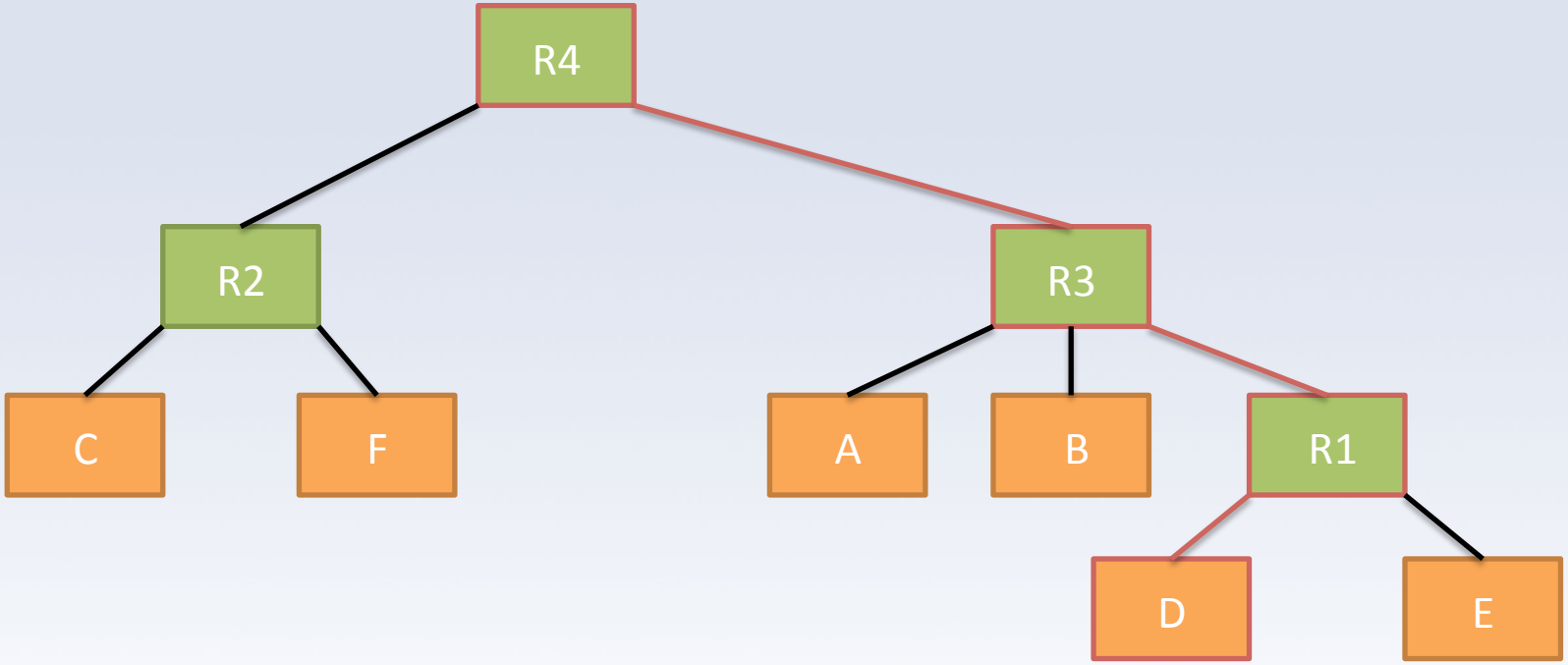
Location Query



R-trees

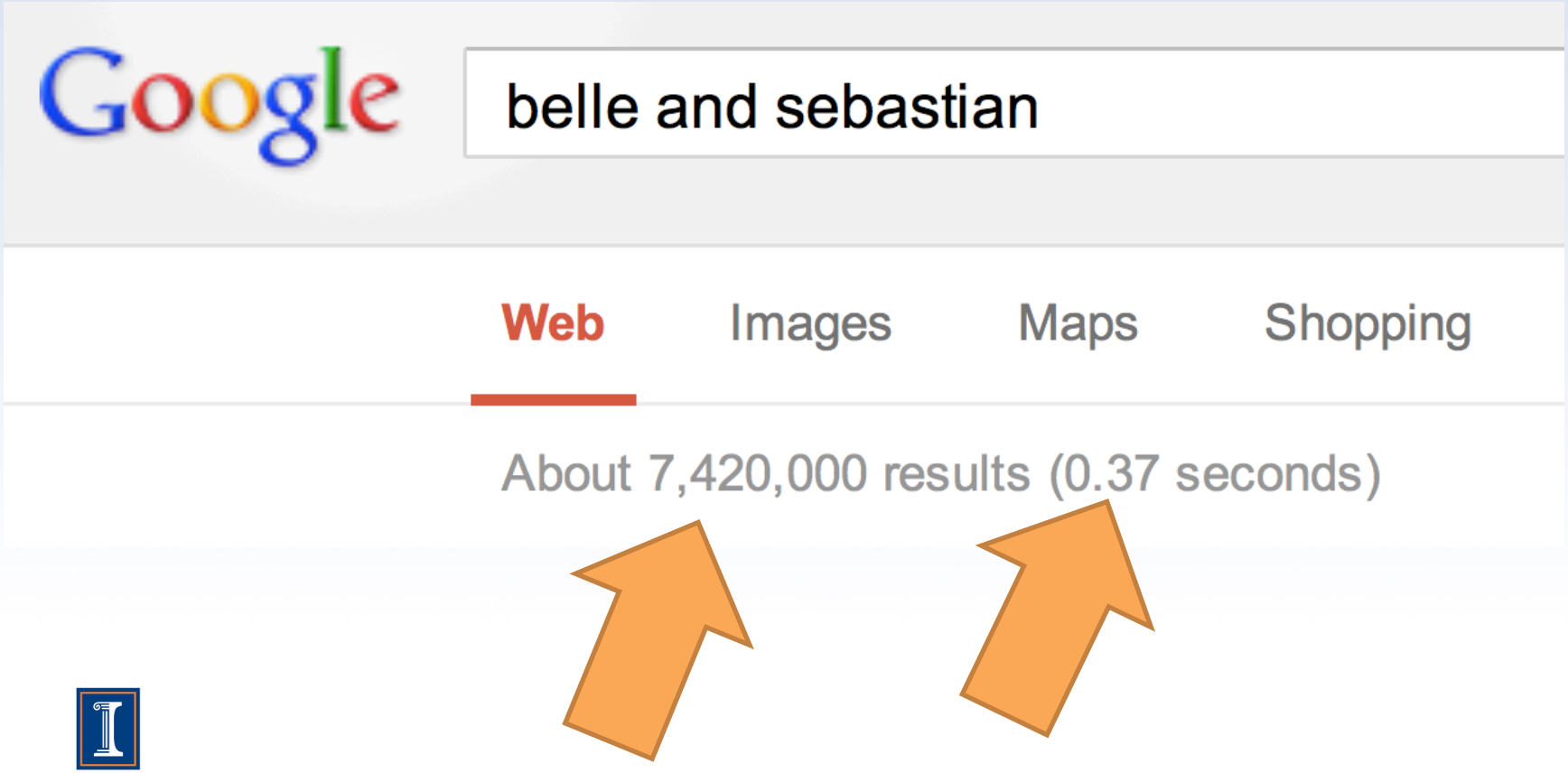
- Actually implemented in MySQL

```
CREATE SPATIAL INDEX spatial_ind
ON Points(location);
```

Document Indexing

- Given a collection of text documents, how can we quickly find all the documents containing a word

Document search



Inverted Index

DOCUMENT 1
Don't you love her madly?

DOCUMENT 2
Say you love me.

DOCUMENT 3
Come as you are.

DOCUMENT 4
All you need is love.

don't	{1}
you	{1,2,3,4}
love	{1,2,4}
her	{1}
madly	{1}
say	{1}

me	{2}
come	{3}
as	{3}
are	{3}
all	{4}
need	{4}

is	{4}

Query: you love

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Don't you love her madly?

DOCUMENT 2
Say you love me.

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Come as you are.

DOCUMENT 4
All you need is love.

don't	{1}
you	{1,2,3,4}
love	{1,2,4}
her	{1}
madly	{1}
say	{1}

me	{2}
come	{3}
as	{3}
are	{3}
all	{4}
need	{4}

is	{4}

Query: you love

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her	{1}
madly	{1}
say	{1}

me	{2}
come	{3}
as	{3}
are	{3}
all	{4}
need	{4}

is	{4}

$$\{1,2,3,4\} \cap \{1,2,4\} = \{1,2,4\}$$

Query: you love

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madly	{1}
say	{1}

me	{2}
come	{3}
as	{3}
are	{3}
all	{4}
need	{4}

is	{4}

$$\{1,2,3,4\} \cap \{1,2,4\} = \{1,2,4\}$$

Inverted Indexing

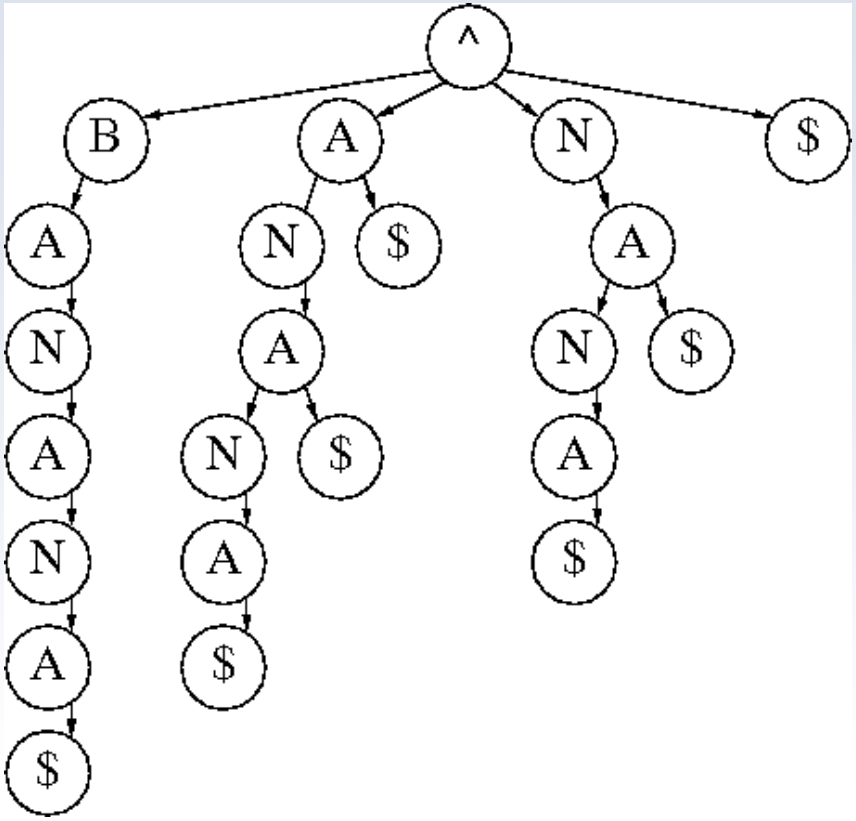
- Real inverted indexes also keep track of the location of the word in each page
- This allows exact phrases to be looked up

String indexing

- How can we index an extremely long string for search?

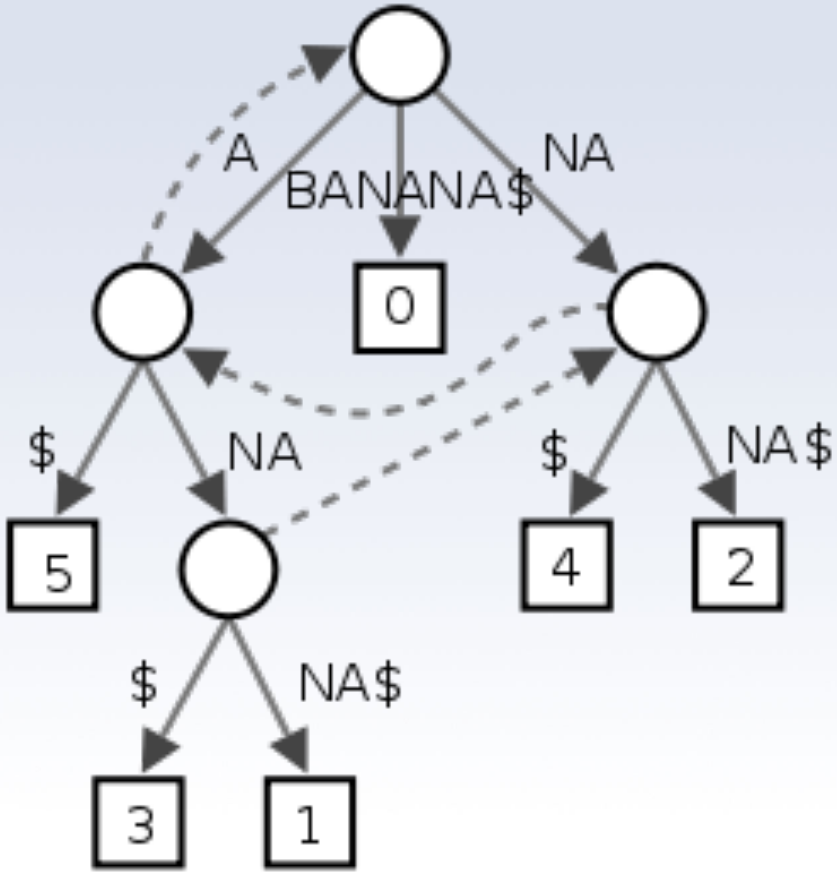
Suffix Trie

- A tree where each path from root to leaf corresponds to a suffix
- O(n) to search
- O(n²) memory



Suffix Tree

- Compressed version of the suffix trie
- Can be searched in O(n)
- Takes O(n) memory
- Can be constructed in O(n) ?!?!?!
- Ukkonen's algorithm



Suffix Array

- SA:
  - Given a text T, add a string terminator character '$'
  - Compute all n rotations (cyclic rotations)
  - Sort the rotations
  - Suffix array is the index of the rotation
- Can be computed in O(n) from a suffix tree

Suffix Array

Suffix	i
banana$	1
anana$	2
nana$	3
ana$	4
na$	5
a$	6
$	7

Suffix Array

Suffix	i
$	7
a$	6
ana$	4
anana$	2
banana$	1
na$	5
nana$	3

Burrows-Wheeler Transform

- BWT:
  - Given a text T, add a string terminator character '$'
  - Compute all n rotations (cyclic rotations)
  - Sort the rotations
  - BWT is last column
- Can be computed in O(n) from a suffix array

A	B	R	A	C	A	D	A	B	R	A

A	B	R	A	C	A	D	A	B	R	A	$

0	A	B	R	A	C	A	D	A	B	R	A	$

0	A	B	R	A	C	A	D	A	B	R	A	$
1	B	R	A	C	A	D	A	B	R	A	$	A

0	A	B	R	A	C	A	D	A	B	R	A	$
1	B	R	A	C	A	D	A	B	R	A	$	A
2	R	A	C	A	D	A	B	R	A	$	A	B

0	A	B	R	A	C	A	D	A	B	R	A	$
1	B	R	A	C	A	D	A	B	R	A	$	A
2	R	A	C	A	D	A	B	R	A	$	A	B
3	A	C	A	D	A	B	R	A	$	A	B	R

0	A	B	R	A	C	A	D	A	B	R	A	$
1	B	R	A	C	A	D	A	B	R	A	$	A
2	R	A	C	A	D	A	B	R	A	$	A	B
3	A	C	A	D	A	B	R	A	$	A	B	R
4	C	A	D	A	B	R	A	$	A	B	R	A

0	A	B	R	A	C	A	D	A	B	R	A	$
1	B	R	A	C	A	D	A	B	R	A	$	A
2	R	A	C	A	D	A	B	R	A	$	A	B
3	A	C	A	D	A	B	R	A	$	A	B	R
4	C	A	D	A	B	R	A	$	A	B	R	A
5	A	D	A	B	R	A	$	A	B	R	A	C
6	D	A	B	R	A	$	A	B	R	A	C	A
7	A	B	R	A	$	A	B	R	A	C	A	D
8	B	R	A	$	A	B	R	A	C	A	D	A
9	R	A	$	A	B	R	A	C	A	D	A	B
10	A	$	A	B	R	A	C	A	D	A	B	R
11	$	A	B	R	A	C	A	D	A	B	R	A

11	$	A	B	R	A	C	A	D	A	B	R	A
10	A	$	A	B	R	A	C	A	D	A	B	R
7	A	B	R	A	$	A	B	R	A	C	A	D
0	A	B	R	A	C	A	D	A	B	R	A	$
3	A	C	A	D	A	B	R	A	$	A	B	R
5	A	D	A	B	R	A	$	A	B	R	A	C
8	B	R	A	$	A	B	R	A	C	A	D	A
1	B	R	A	C	A	D	A	B	R	A	$	A
4	C	A	D	A	B	R	A	$	A	B	R	A
6	D	A	B	R	A	$	A	B	R	A	C	A
9	R	A	$	A	B	R	A	C	A	D	A	B
2	R	A	C	A	D	A	B	R	A	$	A	B

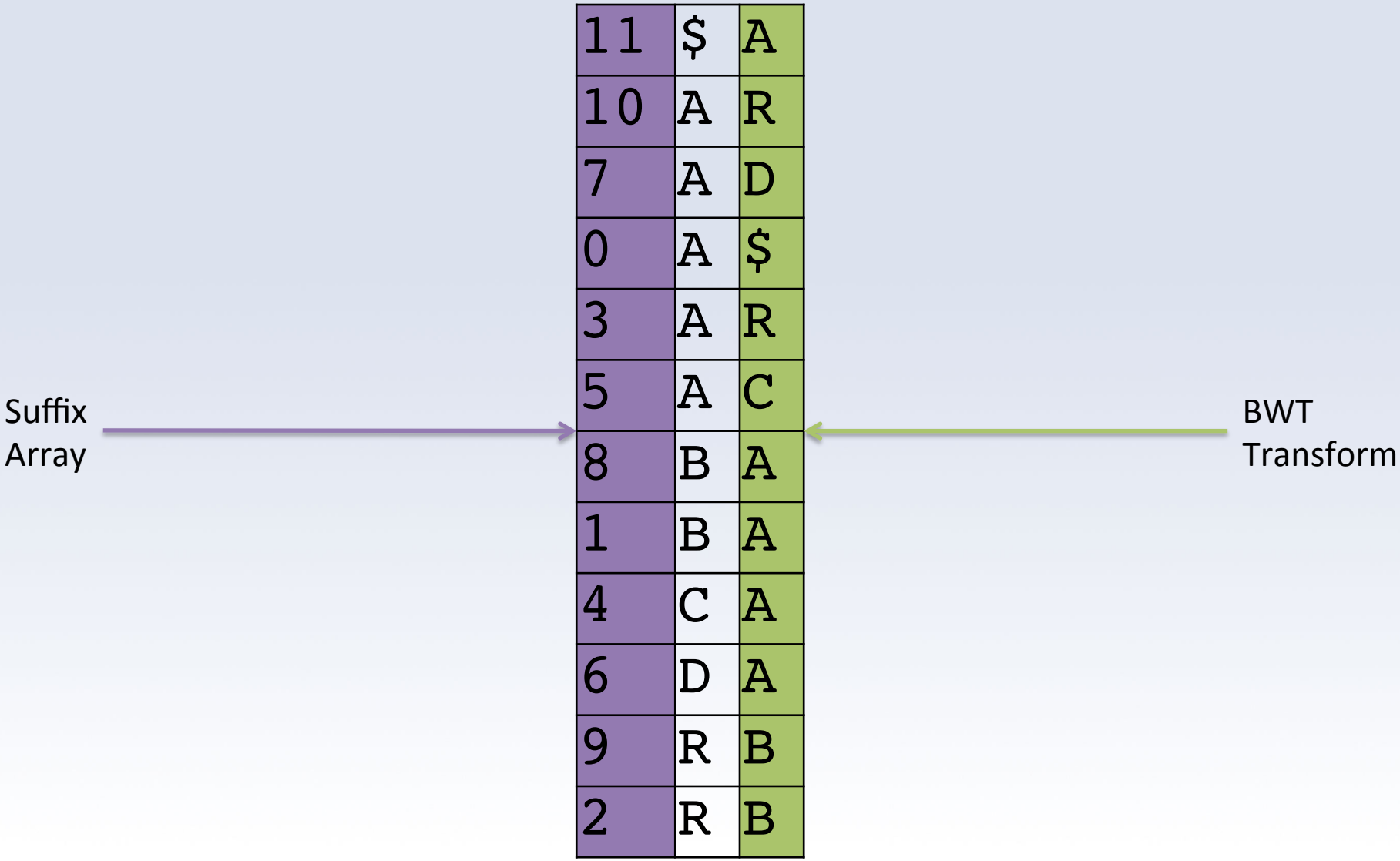
Suffix Array

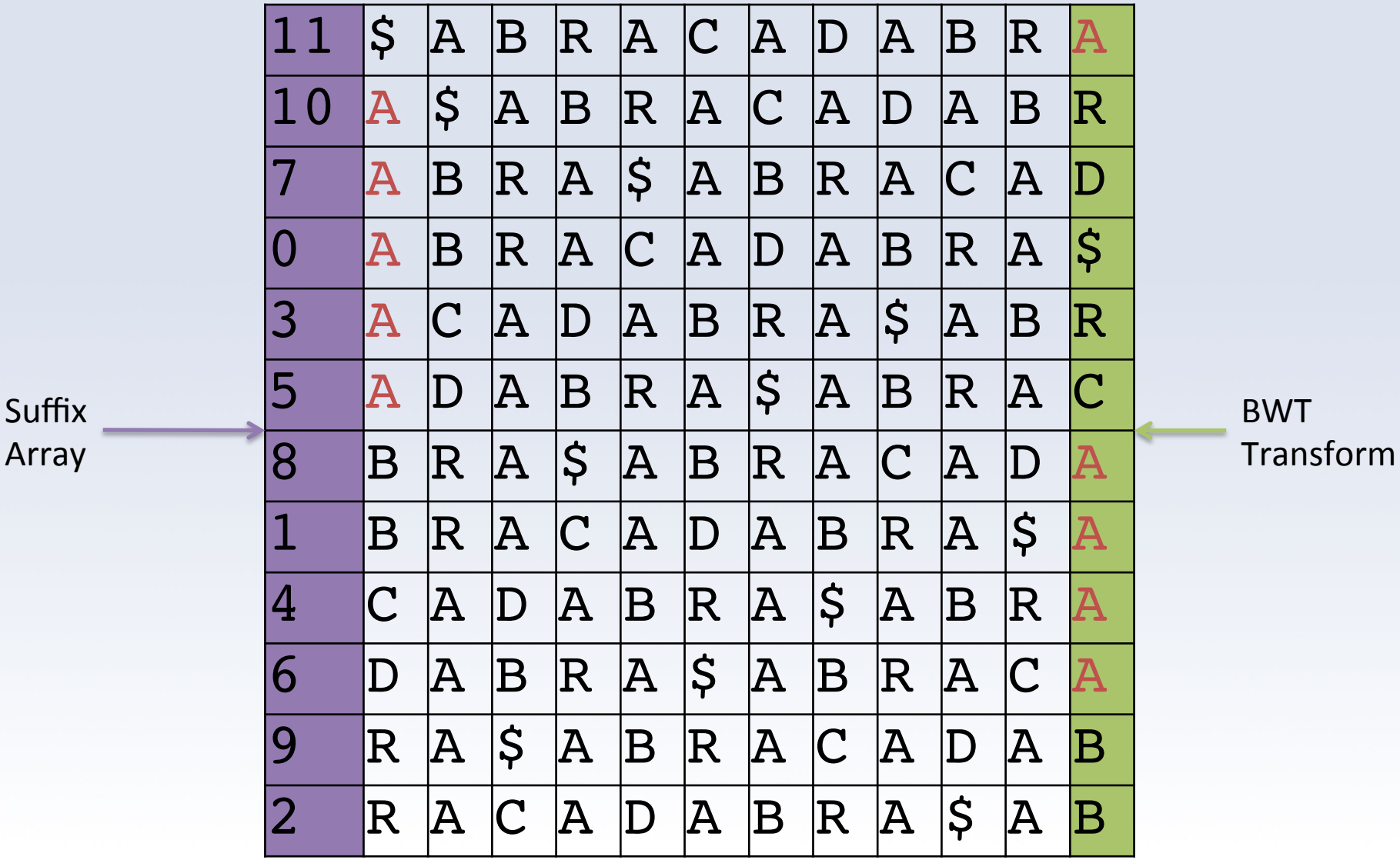
BWT Transform

illinois.edu

ABRA

11	$	A	B	R	A	C	A	D	A	B	R	A
10	A	$	A	B	R	A	C	A	D	A	B	R
7	A	B	R	A	$	A	B	R	A	C	A	D
0	A	B	R	A	C	A	D	A	B	R	A	$
3	A	C	A	D	A	B	R	A	$	A	B	R
5	A	D	A	B	R	A	$	A	B	R	A	C
8	B	R	A	$	A	B	R	A	C	A	D	A
1	B	R	A	C	A	D	A	B	R	A	$	A
4	C	A	D	A	B	R	A	$	A	B	R	A
6	D	A	B	R	A	$	A	B	R	A	C	A
9	R	A	$	A	B	R	A	C	A	D	A	B
2	R	A	C	A	D	A	B	R	A	$	A	B


Suffix
Array
11 $ A
10 A R
7 A D
0 A $
3 A R
5 A C
8 B A
1 B A
4 C A
6 D A
9 R B
2 R B
BWT
Transform


Suffix Array
BWT Transform
11 $ A B R A C A D A B R A
10 A $ A B R A C A D A B R
7 A B R A $ A B R A C A D
0 A B R A C A D A B R A $
3 A C A D A B R A $ A B R
5 A D A B R A $ A B R A C
8 B R A $ A B R A C A D A
1 B R A C A D A B R A $ A
4 C A D A B R A $ A B R A
6 D A B R A $ A B R A C A
9 R A $ A B R A C A D A B
2 R A C A D A B R A $ A B

	$	A	B	R	A	C	A	D	A	B	R	A	1
1	A	$	A	B	R	A	C	A	D	A	B	R	
2	A	B	R	A	$	A	B	R	A	C	A	D	
3	A	B	R	A	C	A	D	A	B	R	A	$	
4	A	C	A	D	A	B	R	A	$	A	B	R	
5	A	D	A	B	R	A	$	A	B	R	A	C	
	B	R	A	$	A	B	R	A	C	A	D	A	2
	B	R	A	C	A	D	A	B	R	A	$	A	3
	C	A	D	A	B	R	A	$	A	B	R	A	4
	D	A	B	R	A	$	A	B	R	A	C	A	5
	R	A	$	A	B	R	A	C	A	D	A	B	
	R	A	C	A	D	A	B	R	A	$	A	B	

Search Interval: (0,11)

Search String: ABRA

11	$	A
10	A	R
7	A	D
0	A	$
3	A	R
5	A	C
8	B	A
1	B	A
4	C	A
6	D	A
9	R	B
2	R	B

Search Interval: (0,11)

Search String: ABRA

11	$	A
10	A	R
7	A	D
0	A	$
3	A	R
5	A	C
8	B	A
1	B	A
4	C	A
6	D	A
9	R	B
2	R	B

Search Interval: (0,11)

Search String: ABRA

11	$	A
10	A	R
7	A	D
0	A	$
3	A	R
5	A	C
8	B	A
1	B	A
4	C	A
6	D	A
9	R	B
2	R	B

Search Interval: (0,11)

Search String: ABRA

11	$	A	
10	A	R	1st R
7	A	D	
0	A	$	
3	A	R	2nd R
5	A	C	
8	B	A	
1	B	A	
4	C	A	
6	D	A	
9	R	B	
2	R	B	

Search Interval: (10,11)

Search String: ABRA

11	$	A	
10	A	R	1st R
7	A	D	
0	A	$	
3	A	R	2nd R
5	A	C	
8	B	A	
1	B	A	
4	C	A	
6	D	A	
9	R	B	
2	R	B	

Search Interval: (10,11)

Search String: ABRA

11	$	A	
10	A	R	
7	A	D	
0	A	$	
3	A	R	
5	A	C	
8	B	A	
1	B	A	
4	C	A	
6	D	A	
9	R	B	1st B
2	R	B	2nd B

Search Interval: (6,7)

Search String: ABRA

11	$	A
10	A	R
7	A	D
0	A	$
3	A	R
5	A	C
8	B	A
1	B	A
4	C	A
6	D	A
9	R	B
2	R	B

Search Interval: (6,7)

Search String: ABRA

11	$	A	
10	A	R	
7	A	D	
0	A	$	
3	A	R	
5	A	C	
8	B	A	2nd A
1	B	A	3rd A
4	C	A	
6	D	A	
9	R	B	
2	R	B	

Search Interval: (2,3)

Search String: ABRA

11	$	A
10	A	R
7	A	D
0	A	$
3	A	R
5	A	C
8	B	A
1	B	A
4	C	A
6	D	A
9	R	B
2	R	B

Two occurrences found
At indexes 0 and 7

01234567
ABRACADABRA

Improvements

- This algorithm can be improved:
  - We can make lookup tables for the “rank” of each symbol in BWT
  - We can make lookup tables for the location of each symbol in the middle array
  - Lookup is O(n) (with a very small constant)
  - Memory footprint is just 3 arrays for each character in the original string!

Improvements

- This algorithm can be expanded
  - Allow for mismatches

HAVE A GOOD BREAK!

- GO HAVE FUN!!!!!!