

**2018-2019**

**American Computer Science League**

**Senior Division - Short Round Solutions**

**All-Star Contest**

**1, Boolean Algebra**

$$\begin{aligned}
 & \overline{A(\overline{B+C} + \overline{BC})} \oplus \left( \overline{B(\overline{A+C}(\overline{AC+B}))} \right) \\
 &= \overline{A\overline{B+C} + \overline{BC}} \oplus \left( \overline{B(\overline{A+C} + \overline{AC+B})} \right) \\
 &= \left( \overline{A\overline{B}} \overline{AC} \overline{BC} \right) \oplus \left( \overline{A\overline{B}C} + \overline{A\overline{B}C} \right) \\
 &= \left( (\overline{A+B})(\overline{A+C})(\overline{B+C}) \right) \oplus \left( \overline{A\overline{B}}(\overline{C+C}) \right) \\
 &= (\overline{AB} + \overline{AC} + \overline{BC}) \oplus \overline{AB}
 \end{aligned}$$

To be FALSE, LHS = RHS.

If LHS = RHS = 0 and  $A = 1$ , then

$$\begin{aligned}
 0B + 0\overline{C} + \overline{BC} &= 0 \quad \wedge \quad 1\overline{B} = 0 \\
 B &= 1
 \end{aligned}$$

$$\begin{aligned}
 0*1 + 0 + 1*\overline{C} &= 0 \\
 C &= 1 \quad \Rightarrow (1, 1, 1)
 \end{aligned}$$

If LHS = RHS = 0 and  $A = 0$ , then

$$\begin{aligned}
 1*B + 1*\overline{C} + \overline{BC} &= 0 \quad \wedge \quad 0*\overline{B} = 0 \\
 B=0 \wedge C=1 & \quad B=* \\
 \Rightarrow (0, 0, 1)
 \end{aligned}$$

If LHS = RHS = 1 and  $A = 1$ , then

$$\begin{aligned}
 0*B + 0*\overline{C} + \overline{BC} &= 1 \quad \wedge \quad 1*\overline{B} = 1 \\
 B = 1 \wedge C = 0 & \quad B = 0 \\
 & \text{which is impossible.}
 \end{aligned}$$

IF LHS = RHS = 1 and  $A = 0$ , then

$$\begin{aligned}
 1*B + 1*C + \overline{BC} &= 1 \quad \wedge \quad 0*\overline{B} = 1 \\
 1 &= 1 \quad \quad 0 = 1 \\
 & \text{which is impossible.}
 \end{aligned}$$

C. (0, 0, 1) (1, 1, 1)

## 2. Bit-String Flicking

Let  $x = abcdef$

$x \text{ OR } (\text{LCIRC-4 } x) \text{ AND } (\text{RSHIFT-1 } x)$

$= abcdef \text{ OR } (\text{LCIRC-4 } abcdef)$

$\text{AND } (\text{RSHIFT-1 } abcdef)$

$= abcdef \text{ OR } efabcd \text{ AND } 0abcde$

$= abcdef \text{ OR } e0 \text{ af ab bc cd de}$

This sets up six equations:

$$a + e0 = 0 \rightarrow a = 0$$

$$b + af = 0 \rightarrow b = 0$$

$$c + ab = 1 \rightarrow c = 1$$

$$d + bc = 1 \rightarrow d = 1$$

$$e + cd = 1 \rightarrow e + 1 = 1$$

$$f + de = 0 \rightarrow f + e = 0 \rightarrow e = 0 \text{ and } f = 0$$

Therefore  $x = 001100$

A. 1

## 3. Recursive Functions

$$f(4, -1, 2) = f(4 - 2, -1 + 1, \lceil -\frac{1}{4} \rceil) + 2$$

$$= f(2, 0, -1) + 2 = 1 + 2 = 3$$

$$f(2, 0, -1) = f(2 - 2, 0 + 1, \lceil 0/2 \rceil) + (-1)$$

$$= f(0, 1, 0) - 1 = 2 - 1 = 1$$

$$f(0, 1, 0) = f(3, 1 + 1, 0 - 1) + 1 * 0$$

$$= f(3, 2, -1) + 0 = 2 + 0 = 2$$

$$f(3, 2, -1) = f(3 - 2, 2 + 1, \lceil \frac{2}{3} \rceil) + (-1)$$

$$= f(1, 3, 0) - 1 = 3 - 1 = 2$$

$$f(1, 3, 0) = f(1 + 1, 3 - 2, 3 - 1) - 1 * 3$$

$$= f(2, 1, 2) - 3 = 6 - 3 = 3$$

$$f(2, 1, 2) = f(2 - 2, 1 + 1, \lceil \frac{1}{2} \rceil) + 2$$

$$= f(0, 2, 0) + 2 = 4 + 2 = 6$$

$$f(0, 2, 0) = f(3, 2 + 1, 0 - 2) - 0 * 2$$

$$= f(3, 3, -2) - 0 = 4 - 0 = 4$$

$$f(3, 3, -2) = 3 + 3 + (-2) = 4$$

C. 3

#### 4. Prefix-Infix-Postfix

$$\begin{aligned}
 & \$ / - ^ 5 2 * 2 2 + 1 @ 6 4 / - + ^ 3 2 \\
 & \quad * \& 8 4 4 \$ 1 2 * * @ 3 2 2 \& 6 4 \\
 & = \$ / - (^ 5 2) (* 2 2) + 1 (@ 6 4) / - + (^ 3 2) \\
 & \quad * (\& 8 4) 4 (\$ 1 2) * * (@ 3 2) 2 (\& 6 4) \\
 & = \$ / (- 25 4) (+ 1 6) / - + 9 (* 6 4) 3 * (* 3 2) 5 \\
 & = \$ (/ 21 7) / - (+ 9 24) 3 (* 6 5) \\
 & = \$ 3 / - 33 3 30 \\
 & = \$ 3 / (- 33 3) 30 \\
 & = \$ 3 (/ 30 30) \\
 & = \$ 3 1 \\
 & = -8
 \end{aligned}$$

A. -8

#### 5. Digital Electronics

The digital circuits translates to:

$$\overline{(A((AB) \oplus (B+C)))} \oplus \overline{((B+C)(C+D)) + D}$$

The LHS of  $\oplus$  simplifies to:  $\bar{A} + AB + \bar{B}\bar{C}$

The RHS of  $\oplus$  simplifies to:  $D$

To be TRUE the sides of  $\oplus$  must be opposites.

If  $D = 1$ , then  $\bar{A} + AB + \bar{B}\bar{C} = 0$ .

$$\rightarrow \bar{A} = 0 \wedge AB = 0 \wedge \bar{B}\bar{C} = 0$$

$$\rightarrow A = 1, B = 0, C = 1 \quad (1, 0, 1, 1)$$

If  $D = 0$ , then  $\bar{A} + AB + \bar{B}\bar{C} = 1$ .

If  $A = 0$ , then  $1 + 0B + \bar{B}\bar{C} = 1 \rightarrow B = * \wedge C = *$ .

$$(0, *, *, 0)$$

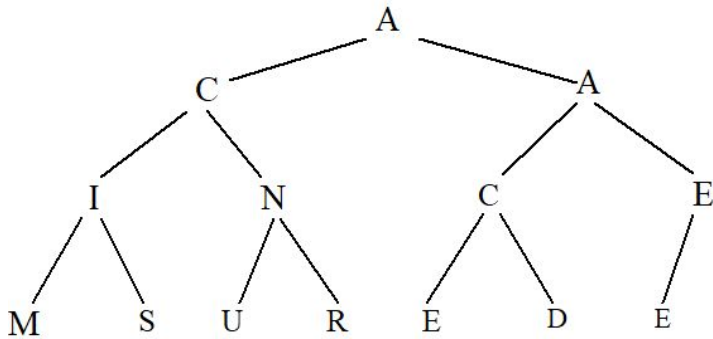
If  $A = 1$ , then  $0 + B + \bar{B}\bar{C} = 1$ .

$$\text{If } B = 1, \text{ then } C = *. \quad (1, 1, *, 0)$$

$$\text{If } B = 0, \text{ then } C = 0. \quad (1, 0, 0, 0)$$

Therefore there are 8 ordered quadruples that make it TRUE

A. 8

<p><b>6. Computer Number Systems</b></p> <p> <math>2000_{10} = 3720_8</math> and <math>2199_{10} = 4227_8</math>  41 appears once from <math>3720_8</math> to <math>3777_8</math> : <math>3741_8</math>  41 appears once from <math>4000_8</math> to <math>4077_8</math> : <math>4041_8</math>  41 appears at the beginning of each from <math>4100_8</math> to <math>4177_8</math> which adds 64 and <math>4141_8</math> adds one more.  The total is 67. </p>	<p>C. 67</p>
<p><b>7. What Does This Program Do?</b></p> <p> This program counts the number of deficient, perfect, and abundant numbers that are inputted. There were 3 deficient numbers (2, 21, 59), 2 perfect numbers (6, 28), and 5 abundant numbers (80, 36, 12, 100, 24) </p>	<p>D. 3 2 5</p>
<p><b>8. Data Structures</b></p> <p>The min-heap for <b>ICECREAMSUNDAE</b> is:</p>  <pre> graph TD     A1[A] --- C1[C]     A1 --- A2[A]     C1 --- I[I]     C1 --- N[N]     I --- M[M]     I --- S[S]     N --- U[U]     N --- R[R]     A2 --- C2[C]     A2 --- E1[E]     C2 --- E2[E]     C2 --- D[D]     E1 --- E3[E] </pre>	<p>C. I, N, C, E</p>
<p><b>9. Graph Theory</b></p> <p> The round trips from A with just ACSL Air are: ADA, ABA, ABCDA, ABDA, ACDA, ACBA, and ACBDA. After merging with CompSci Air 5 new round trips were added: ABGFCDA, ACGFBA, ACFBA, ACFBDA, and ACGFBDA. </p>	<p>B. 5</p>

<p><b>10. LISP</b></p> <pre> (CDADR (REVERSE (CDADDR '(a (b e) (a (b (c d)                         (e f (g h))) f (d a)))))) = (CDADR (REVERSE (CDADR '((b e) (a (b (c d)                         (e f (g h))) f (d a)))))) = (CDADR (REVERSE (CDAR '((a (b (c d)                         (e f (g h)))f (d a)))))) = (CDADR (REVERSE (CDR '(a (b (c d) (e f (g h)))))) = (CDADR (REVERSE '((b (c d) (e f (g h)))))) = (CDADR '((e f (g h)) (b (c d)))) = (CDAR '((b (c d)))) = (CDR '(b (c d))) = ((c d)) </pre>	<p>D. ((c d))</p>
<p><b>11. FSAs and Regular Expressions</b></p> <p><b>[0-9]*[[A-Z][a-z]]*[[&amp;,@,/,].[^d, e, t]]*</b></p> <p>a) 1978RICSLwasestablished Missing special character after the "d"</p> <p>b) AllStar@NJ MATCHES</p> <p>c) 41Consecutive/yr MATCHES</p> <p>d) 12Categories@SR MATCHES</p> <p>e) Programmingisfun&amp;challenging. Fails at the "e" in challenging</p> <p>f) ACSL.org MATCHES</p>	<p>D. b, c, d, f</p>
<p><b>12. Assembly Language</b></p> <p>This program counts the number of even factors of the numbers from 1 to 10, inclusive. There are 10 of them.</p>	<p>A. 10</p>