American Computer Science League

2018-2019 **Contest #4**

INTERMEDIATE DIVISION SOLUTIONS

1. Graph Theory

To find the number of paths of length 3, add the entries in the cube of the adjacency matrix. The sum is 41.

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix}^3 = \begin{vmatrix} 7 & 5 & 5 \\ 3 & 2 & 2 \\ 7 & 5 & 5 \end{vmatrix}$$

1. 41

2. Graph Theory

There are 9 cycles from A: ABA, ABDCA, ABEDCA, ACA, ACBA, ACEDBA, AEDBA, AEDCA, AEDCBA

2. 9

3. Digital Electronics

The circuit translates to: $(\overline{A} + \overline{AB + \overline{BC}}) \oplus ((C + D)\overline{D})$

Note: operands may be commuted.

3. As shown

4. Digital Electronics

The Boolean expression represented by the circuit is: $(\overline{A} + \overline{AB + \overline{C}})C$

$$(\overline{A} + \overline{AB + \overline{C}})C = (\overline{A} + \overline{AB + \overline{C}})C = (\overline{A} + \overline{AB}\,\overline{\overline{C}})C$$
$$= (\overline{A} + (\overline{A} + \overline{B})C)C = (\overline{A} + \overline{A}C + \overline{B}C)C$$
$$= \overline{A}C + \overline{A}CC + \overline{B}CC = \overline{A}C + \overline{B}C = C(\overline{A} + \overline{B})$$

To be TRUE: $C(\overline{A} + \overline{B}) = 1 \rightarrow C = 1 \wedge \overline{A} + \overline{B} = 1$

$\overline{A} + \overline{B} = 1$ except when $A = 1 \wedge B = 1$

Therefore 3 ordered triples make the circuit TRUE.

(0, 0, 1), (0, 1, 1) and (1, 0, 1)

4. 3

5. Assembly Language

This program takes a two-digit number, 36, divides it by 10 to separate the digits. B = 3 and D = 6. Then it calculates a new number, 10 * 6 + 3, the original number reversed. After finding the difference, 63 - 36, it divides it by 9 to get 3. Note the difference will always be a multiple of 9.

5. 3