American Computer Science League

2019-2020 **ACSL Finals**

Junior Shorts Solutions

1. Boolean Algebra

$\overline{A\,\overline{B} + C}\;A\,B\,\overline{\overline{B} + C}$

$= \overline{A\,\overline{\overline{B}}}\;\overline{C}\,A\,B\,B\,\overline{C}$

$= (\overline{A} + B)\;\overline{C}\,A\,B\,\overline{C}$

$= \overline{A}\,\overline{C}\;A\,B\,\overline{C} + B\,\overline{C}\,A\,B\,\overline{C}$

$= 0 + A\,B\,\overline{C}$

$= A\,B\,\overline{C}$ *which is TRUE if* $A = 1,\ B = 1,\ C = 0$

1. 0 (A)

2. Boolean Algebra

First $A\ \$\ B = \overline{A}\,B + \overline{B}$

A \$ B + ($\overline{A}$ \$ B) ($\overline{A}$ \$ $\overline{B}$)

$= (\overline{A}\,B + \overline{B}) + \left(\overline{\overline{A}}\,B + \overline{B}\right)\left(\overline{\overline{A}}\,\overline{B} + \overline{\overline{B}}\right)$

$= (\overline{A}\,B + \overline{B}) + (A\,B + \overline{B})\,(A\,\overline{B} + B)$

$= (\overline{A}\,B + \overline{B}) + \left(A\,B\,A\,\overline{B} + A\,B\,B + \overline{B}\,A\,\overline{B} + \overline{B}\,B\right)$

$= \overline{A}\,B + \overline{B} + 0 + A\,B + A\,\overline{B} + 0$

$= B\,(\overline{A} + A) + \overline{B}\,(1 + A)$

$= B + \overline{B}$

$= 1$

This is always TRUE.

2. 4 (D)

3. Bit-String Flicking

(RSHIFT-2 (LCIRC-1 (NOT 0111001))) AND
(NOT (RCIRC-2 (LSHIFT-1 1100011)))

= (RSHIFT-2 (LCIRC-1 1000110)) AND
(NOT (RCIRC-2 1000110))

= (RSHIFT-2 0001101) AND (NOT 1010001)

= 0000011 AND 0101110

= 0000010

3. 0000010 (C)

4. Bit-String Flicking Let X = abcde and NOT X = ABCDE LHS = (LCIRC-2 01010) OR (RSHIFT-1 ((LCIRC-2 abcde) AND 01110)) = 01001 OR (RSHIFT-1 (cdeab AND 01110)) = (01001 OR (RSHIFT-1 0dea0) = 01001 OR 00dea = 01de1 LHS = RHS → 01de1 = 01101 → d = 1, e = 0, a = *, b = *, c = * → b = 1, c = 1, e = 1 → a = *, d = * Therefore X = abcde = ***10 8 solutions	4. 8 (C)
5. Recursive Functions $f(17) = 2 \cdot f(17-3) + 4 = 2 \cdot f(14) + 4 = 2 \cdot 188 + 4 = 380$ $f(14) = 2 \cdot f(14-3) + 4 = 2 \cdot f(11) + 4 = 2 \cdot 92 + 4 = 188$ $f(11) = 2 \cdot f(11-3) + 4 = 2 \cdot f(8) + 4 = 2 \cdot 44 + 4 = 92$ $f(8) = 2 \cdot f(8-3) + 4 = 2 \cdot f(5) + 4 = 2 \cdot 20 + 4 = 44$ $f(5) = 2 \cdot f(5-3) + 4 = 2 \cdot f(2) + 4 = 2 \cdot 8 + 4 = 20$ $f(2) = 3 \cdot 2 + 2 = 8$	5. 380 (D)
6. Recursive Functions $f(25) = 2 + f\left(\left[\frac{25}{2}\right]\right) = 2 + f(12) = 2 + 42 = 44$ $f(12) = 2 + f\left(\left[\frac{12}{2}\right]\right) = 2 + f(6) = 2 + 40 = 42$ $f(6) = f(6-1) + f(6-2) = f(5) + f(4) = 25 + 15 = 40$ $f(5) = f(5-1) + f(5-2) = f(4) + f(3) = 15 + 10 = 25$ $f(4) = f(4-1) + f(4-2) = f(3) + f(2) = 10 + 5 = 15$ $f(3) = 3^2 + 1 = 10$ $f(2) = 2^2 + 1 = 5$	6. 44 (C)

7. Digital Electronics

The digital circuit translates to:

$$\overline{\left(A + \overline{\overline{(A + B)}\,\overline{(B\,C)}}\right)}\,C$$
$$= \left(\overline{A}\left(\overline{A + B}\right)\left(\overline{B\,C}\right)\right)C$$
$$= (\overline{A}\,\overline{A}\,\overline{B}\,(\overline{B} + \overline{C}))\,C$$
$$= \overline{A}\,\overline{B}\,C\,(\overline{B} + \overline{C})$$
$$= \overline{A}\,\overline{B}\,C\,\overline{B} + \overline{A}\,\overline{B}\,C\,\overline{C}$$
$$= \overline{A}\,\overline{B}\,C + 0$$
$$= \overline{A}\,\overline{B}\,C \textit{ which is TRUE if } A = 0,\ B = 0 \textit{ and } C = 1$$

7. 001 (D)

8. Digital Electronics

The circuit translates to:

(A)($\square$(A, B, C)) + ((($\square$ (A, B, C)) + C)

Let X = $\square$ (A, B, C).

The expression is now: A X + (X + C)

A	B	C	X	AX	X + C	AX + (X+C)
0	0	0	0	0	0	0
0	0	1	1	0	1	1
0	1	0	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	1	1	1
1	0	1	0	0	1	1
1	1	0	0	0	0	0
1	1	1	0	0	1	1

Therefore there are 6 triples that make the expression TRUE.

8. 6 (D)

9. Prefix-Infix-Postfix

- % - + 2 ^ 3 2 4 * + / 8 4 $ 2 0 / / + 8 2 $ 2 5 % - 3 8

= - % - + 2 (^ 3 2) 4 * + (/ 8 4) ($ 2 0) / / (+ 8 2) ($ 2 5) % (- 3 8)

= - % - (+ 2 9) 4 * (+ 2 0) / (/ 10 2) (% (- 5))

= - % (- 11 4) * 2 (/ 5 5)

= - (% 7) (* 2 1)

= - 7 2 = 5

9. 5 (B)

10. Prefix-Infix-Postfix

If A = 5, B = 3, and C = 2:

A B C + / B C ^ ^ B A + C B ^ / A * +

= 5 3 2 + / 3 2 ^ ^ 3 5 + 2 3 ^ / 5 * +

= 5 (3 2 +) / (3 2 ^) ^ (3 5 +) (2 3 ^) / 5 * +

= (5 5 /) 9 ^ (8 8 /) 5 * +

= (1 9 ^) (1 5 *) +

= 1 5 +

= 6

10. 6 (A)

11. Computer Number Systems

$2020_8 - 202_8 - 20_8 + 2_8 = 1600_8$

Convert each bit to binary: 001 110 000 000

Group 4 at a time: 0011 1000 0000

Convert to hex: 3 8 0

11 380 (C)

12. Computer Number Systems

Change each to its binary representation:

50: 110010	55: 110111	60: 111100
51: 110011	56: 111000	61: 111101
52: 110100	57: 111001	62: 111110
53: 110101	58: 111010	63: 111111
54: 110110	59: 111011	64: 1000000

Therefore there are 60 1's.

12. 60 (B)

13. Data Structures

The stack is constructed using LIFO as follows:

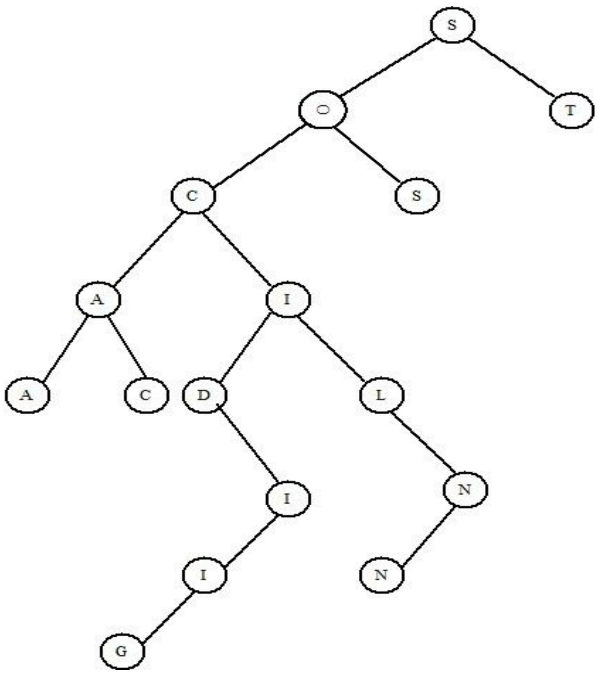
G, GE, GER, GERB, GER~~B~~, GE~~R~~, GEE, GE~~E~~, GER, GERA,
GERAD, GERA~~D~~, GERAA, GERAAI, GERAA~~I~~, GERA~~A~~,
GERAS, GERASY, GERAS~~Y~~, GERA~~S~~, GER~~A~~

The next item popped would be R.

13. R (D)

14. Data Structures

The binary search tree for SOCIALDISTANCING is:

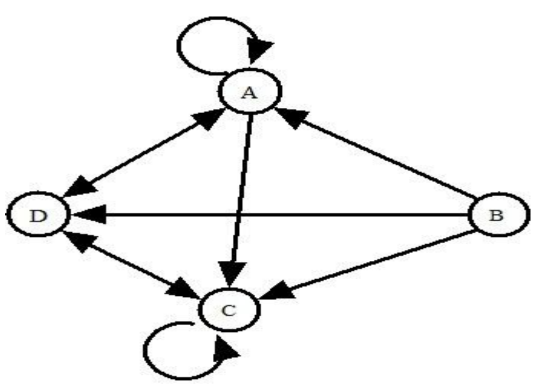


It has a depth of 7 since the root node has depth 0.

14. 7 (D)

15. Graph Theory

The graph that the adjacency matrix represents is:



The cycles are: AA, ACDA, ADA, CC, and CDC.

15. 5 (C)

16. Graph Theoryoio

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 1 & 0 & 2 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

By adding the entries in the squared matrix you get 13 paths of length 2.

16. 13 (D)

17. What Does This Program Do?

This program counts the number of increasing factors of 2020 that sum to less than 2020. They are 1, 2, 4, 5, 10, 20, 101, 202, 404 and 505.

17. 10 (C)

18. What Does This Program Do?

This program matches the letters in each string and sums the product of their locations.

Matching Letter	Position in A	Position in B	Sum
C	0	0	0
O	1	1	1
O	3	1	4
V	6	2	16
I	7	3	37

18. 37 (B)

19. What Does This Program Do?

This program adds all the entries in the array (352), then finds the average (39). It counts the entries in the first half that are less than the average (19, 21) and the entries in the upper half that are greater than the average (33, 11) and the middle entry (28). There are 5.

19. 5 (C)

20. What Does This Program Do?

This program finds the sum of specific input locations from 1 to 15.

N	V	R	C	A(R,C)	S
1	4	0	4	1	1
2	8	1	3	3	4
3	11	2	1	2	6
4	2	0	2	2	8
5	5	1	0	4	12
6	13	2	4	1	13
7	6	1	1	5	18

20. 18 (A)