

THE STEERABLE PYRAMID: A FLEXIBLE ARCHITECTURE FOR MULTI-SCALE DERIVATIVE COMPUTATION

Eero P Simoncelli

GRASP Laboratory, Room 335C
3401 Walnut St
Philadelphia, PA 19104-6228

William T Freeman

Mitsubishi Electric Research Laboratories
201 Broadway
Cambridge, MA 02139

ABSTRACT

*We describe an architecture for efficient and accurate linear decomposition of an image into scale and orientation subbands. The basis functions of this decomposition are directional derivative operators of any desired order. We describe the construction and implementation of the transform.*¹

Differential algorithms are used in a wide variety of image processing problems. For example, gradient measurements are used as a first stage of many edge detection, depth-from-stereo, and optical flow algorithms. Higher-order derivatives have also been found useful in these applications. Extraction of these derivative quantities may be viewed as a decomposition of a signal via terms of a local Taylor series expansions [1].

Another widespread tool in signal and image processing is multi-scale decomposition. Apart from the advantages of decomposing signals into information at different scales, the typical recursive form of these algorithms leads to large improvements in computational efficiency.

Many authors have combined multi-scale decompositions with differential measurements (eg., [2, 3]). In these cases, a multi-scale pyramid is constructed, and then differential operators (typically, differences of neighboring pixels) are applied to the subbands of the pyramid. Since both the pyramid decomposition and the derivative operation are linear and shift-invariant, we may combine them into a single operation. The advantages of doing so are that the resulting derivatives may be more accurate (see [4]). In this paper, we propose a simple, efficient decomposition architecture for combining these two operations.

The decomposition is the latest incarnation of

“steerable pyramid”, as developed in [5, 6]. Similar representations have been developed by Perona and collaborators [7]. In this linear decomposition, an image is subdivided into a collection of subbands localized in both scale and orientation. The scale tuning of the filters is constrained by a recursive system diagram (described below). The orientation tuning is constrained by the property of steerability [5]. A set of filters form a steerable basis if (1) they are rotated copies of each other, and (2) a copy of the filter at any orientation may be computed as a *linear combination* of the basis filters. The simplest example of a steerable basis is a set of $N + 1$ N th-order directional derivatives.

In addition to having steerable orientation subbands, the transform we describe is designed to be “self-inverting” (i.e., the matrix corresponding to the inverse transformation is equal to the transpose of the forward transformation matrix)², and is essentially aliasing-free. Most importantly, the pyramid can be designed to produce any number of orientation bands, k . The resulting transform will be overcomplete by a factor of $4k/3$.

A summary of these properties, in comparison with two well-known multi-scale decompositions is given in table 1. Note that the steerable pyramid retains some of the advantages of orthonormal wavelet transforms (eg., basis functions are localized in space and spatial-frequency; the transform is a tight frame), but improves on some of their disadvantages (eg., aliasing is eliminated; steerable orientation decomposition). One obvious disadvantage is in computational efficiency: the steerable pyramid is substantially overcomplete.

We now describe the steerable pyramid in more detail. The decomposition is most easily defined in the Fourier domain, where it is (ideally) polar-

¹Source code and filter kernels for implementation of the steerable pyramid are available via anonymous ftp from `ftp.cis.upenn.edu:pub/eero/steerpyr.tar.Z`

²In the wavelet literature, such a transform is known as a *tight frame* [8]

	Laplacian Pyramid	Dyadic QMF/Wavelet	Steerable Pyramid
self-inverting (tight frame)	no	yes	yes
overcompleteness	4/3	1	4k/3
aliasing in subbands	perhaps	yes	no
rotated orientation bands	no	only on hex lattice [9]	yes

Table 1: Properties of the Steerable Pyramid relative to two other well-known multi-scale representations.

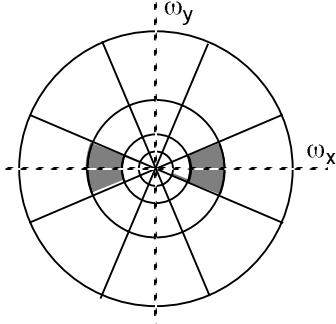


Figure 1. Idealized illustration of the spectral decomposition performed by a steerable pyramid with $k = 4$. Frequency axes range from $-\pi$ to π . The basis functions are related by translations, dilations and *rotations* (except for the initial highpass subband and the final low-pass subband). For example, the shaded region corresponds to the spectral support of a single (vertically-oriented) subband.

separable. Figure 1 contains a diagram of the idealized frequency response of the subbands, for $k = 4$. We write the Fourier magnitude of the i th oriented bandpass filter in polar-separable form:

$$B_i(\vec{\omega}) = A(\theta - \theta_i)B(\omega),$$

where $\theta = \tan^{-1}(\omega_y/\omega_x)$, $\theta_i = \frac{2\pi}{k}$ and $\omega = |\vec{\omega}|$. Below, we describe the constraints on the two components $A(\theta)$ and $B(\omega)$.

1. ANGULAR DECOMPOSITION

The angular portion of the decomposition, $A(\theta)$, is determined by the desired derivative order. A directional derivative operation in the spatial domain corresponds to multiplication by a linear ramp function in the Fourier domain, which we rewrite in polar coordinates as follows:

$$-j\omega_x = -j\omega \cos(\theta)$$

(note that we have described a derivative operator in the x direction). We **ignore the imaginary constant**, and the factor of ω , which is absorbed into the radial portion of the function. The relevant angular portion of the first derivative operator (in the x direction) is thus $\cos(\theta)$.

Higher-order directional derivatives correspond to multiplication in the Fourier domain by the

ramp raised to a power, and thus the angular portion of the filter is $\cos(\theta)^N$ for an N th-order directional derivative. Knutson and Granlund have also developed polar-separable filters with such angular components [10]. The steerability of such functions has been discussed in our previous work [5, 6].

2. RADIAL DECOMPOSITION

The radial function, $B(\omega)$, is constrained by both the desire to build the decomposition recursively (i.e., using a “pyramid” algorithm), and the need to prevent aliasing from occurring during subsampling operations. The recursive system diagram for $B(\omega)$ is given in figure 2³.

The filters $H_0(\omega)$ and $L_0(\omega)$ are necessary for pre-processing the image in preparation for the recursion. The recursive portion of the diagram corresponds to the subsystem contained in the dashed box. This subsystem decomposes a signal into two portions (lowpass and highpass). The low-pass portion is subsampled, and the recursion is performed by repeatedly applying the recursive transformation to the lowpass signal.

The constraints on the filters in the diagram are as follows:

1. Bandlimiting (to prevent aliasing in the subsampling operation):

$$L_1(\omega) = 0 \quad \text{for } |\omega| > \pi/2.$$

2. Flat System Response:

$$|H_0(\omega)|^2 + |L_0(\omega)|^2 [|L_1(\omega)|^2 + |B(\omega)|^2] = 1.$$

3. Recursion:

$$|L_1(\omega/2)|^2 = |L_1(\omega/2)|^2 [|L_1(\omega)|^2 + |B(\omega)|^2].$$

Typically, we choose $L_0(\omega) = L_1(\omega/2)$, so that the initial lowpass shape is the same as that used within the recursion. An idealized illustration of filters that satisfy these constraints is given in figure 3. Note that $L_1(\omega)$ is strictly bandlimited, and $B(\omega)$ is power-complementary.

³This system diagram is modified from that of [6].

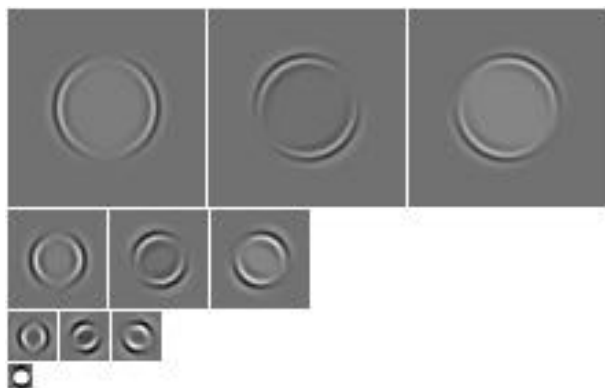


Figure 5. A 3-level $k = 3$ (second derivative) steerable pyramid. Shown are the three band-pass images at each scale and the final lowpass image.

4. REFERENCES

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