STOR 435 Midterm 2

Hannah Le Cao

TOTAL POINTS

10.75 / 12

QUESTION 1

1 Problem 1 **1 / 1**

✓ **- 0 pts** Correct

- **- 0.25 pts** Wrong counting
- **- 1 pts** Wrong setup
- **- 0.25 pts** Wrong number of balls
- **- 0.75 pts** Wrong after finding event
- **- 0.25 pts** Calculation mistake
- **- 0.5 pts** Missing reverse order
- **- 0.5 pts** Balls are chosen without replacement
- **- 0.5 pts** Wrong event

QUESTION 2

2 Problem 2 **1 / 1**

✓ **- 0 pts** Correct

- **- 0.5 pts** X is not a binomial with average probability
- **- 0.25 pts** Missing binomial coefficients
- **- 0.25 pts** Calculation mistake
- **- 1 pts** Different distribution
- **- 0.5 pts** Missing 20 students
- **- 0.25 pts** Should have used binomial distribution
- **- 0.5 pts** Wrong conditioning

QUESTION 3

3 Problem 3 **0.5 / 1**

- **- 0 pts** Correct
- **- 1 pts** Wrong answer and no work shown
- **- 0.25 pts** Calculation mistake
- **- 0.5 pts** Wrong after finding z

✓ **- 0.5 pts Wrong value of z**

- **- 0.75 pts** Used Phi instead of z
- **- 0.5 pts** Wrong std deviation

QUESTION 4

4 Problem 4 **1 / 1**

✓ **- 0 pts** Correct

- **- 1 pts** E(g(X)) is not g(E(X))
- **- 0.5 pts** Missing density
- **- 1 pts** Wrong answer and no work shown
- **- 0.25 pts** Incomplete integration
- **- 0.25 pts** Integration mistake
- **- 1 pts** No integral was setup
- **- 0.5 pts** Wrong integral
- **- 1 pts** X is a not a discrete RV
- **- 0.5 pts** Wrong range

QUESTION 5

5 Problem 5 **1 / 1**

✓ **- 0 pts** Correct

- **- 0 pts** Correct (with no continuity correction)
- **- 1 pts** Wrong answer and no work shown
- **- 0.25 pts** Wrong continuity correction
- **- 1 pts** No solution
- **- 0.5 pts** Wrong standard deviation
- **- 1 pts** No solution
- **- 0.25 pts** Forgot complement
- **- 0.25 pts** Calculation mistake
- **- 0.75 pts** Wrong after getting normal parameters

QUESTION 6

6 Problem 6 **1 / 1**

✓ **- 0 pts** Correct

- **- 1 pts** Wrong solution and no work shown
- **- 0.5 pts** Wrong parameters
- **- 1 pts** No solution
- **- 1 pts** Wrong distribution
- **- 0.25 pts** Should include 4 hurricanes
- **- 0.25 pts** Wrong number of years
- **- 0.5 pts** Wrong event
- **- 0.25 pts** Calculation mistake

QUESTION 7

7 Problem 7 **0.25 / 1**

- **- 0 pts** Correct
- **- 1 pts** No solution
- **- 1 pts** X is not Binomial
- **- 0.5 pts** Either team can win
- **- 1 pts** X is not Geometric
- **- 0.25 pts** Calculation mistake
- ✓ **- 0.75 pts Incorrect pmf**
- **- 0.75 pts** Must show your work
- **- 1 pts** EX cannot be greater than 5
- **- 1 pts** EX cannot be smaller than 3
- **- 1 pts** X is not a continuous RV
- **- 0.5 pts** To play a 4th or 5th game no team can win the first 3 games
- **- 0.5 pts** Incomplete solution

QUESTION 8

8 Problem 8 **1 / 1**

- ✓ **- 0 pts Correct**
- **- 1 pts** Wrong

QUESTION 9

9 Problem 9 **1 / 1**

- ✓ **- 0 pts Correct**
- **- 1 pts** Wrong

QUESTION 10

10 Problem 10 **1 / 1**

- ✓ **- 0 pts Correct**
- **- 1 pts** Wrong

QUESTION 11

11 Problem 11 **1 / 1**

- ✓ **- 0 pts Correct**
- **- 1 pts** Wrong

QUESTION 12

12 Problem 12 **1 / 1**

- ✓ **- 0 pts Correct**
- **- 1 pts** Wrong

STOR 435: Midterm #2

Instructor: Nicolas Fraiman

Date: 26 March 2019

Exam instructions:

- Sign the pledge
- Write your name and PID
- You have 75 minutes
- No books or notes are allowed
- You may use a calculator

Pledge: *I have neither given nor received any unauthorized aid during this exam.*

Signature:

Name: Hannah Cao

PID: 730156589

Problem 1

Two balls are chosen randomly from an urn containing 6 white, 3 black, and 2 orange balls. Suppose that we win \$2 for each black ball selected and we lose \$1 for each white ball selected. Let X denote our winnings. What is the probability $\mathbf{P}(X = 1)$?

Solution

\$1 winning = 1 black + 1 white

$$P(X=1) = \frac{\binom{3}{1}\binom{6}{1}}{\binom{11}{2}} = \frac{18}{55} = 0.3\overline{27}$$

Problem 2

If it is raining, students come to class independently with probability 0.8. If it is not raining this probability increases to 0.9. The chance of rain tomorrow is 0.3. Find the chance that at least 19 students will show up tomorrow in a class of 20 students.

Solution

$$P(X \geq 19) = P(X \geq 19 \mid R)\,P(R) + P(X \geq 19 \mid R')\,P(R')$$

$P(R) = 0.3$, $P(R') = 0.7$

$X \sim B(20, 0.8)$

$$P(X \geq 19) = P(X=19) + P(X=20) = \binom{20}{19}0.8^{19}\,0.2 + \binom{20}{20}0.8^{20} = 0.069175$$

$X \sim B(20, 0.9)$

$$= \binom{20}{19}0.9^{19}(0.1) + \binom{20}{20}0.9^{20} = 0.3917$$

$$P(X \geq 19) = 0.069175 \times 0.3 + 0.3917 \times 0.7 = 0.295$$

Problem 3

The length L of a certain type of chain is Normally distributed with mean 3.2 cm and standard deviation 0.2 cm. Find the value of ℓ such that $P(L > \ell) = 0.01$.

Solution

$$L \sim N(3.2, 0.2^2)$$

$$P(L > \ell) = 0.1 \Rightarrow P\left(Z > \frac{\ell - 3.2}{0.2}\right) = 0.1$$

$$P\left(Z < \frac{\ell - 3.2}{0.2}\right) = 0.9$$

$$\frac{\ell - 3.2}{0.2} = 1.28 \qquad \ell = 0.2(1.28) + 3.2$$

$$= 3.456$$

Problem 4

A student sleeps a random number of hours X that is uniform between 6 and 8. The chance they fall asleep in an early morning class depends on the amount of sleep they got and is given by $(8 - X)^3/8$. What is the expected chance that they fall asleep on the class?

Solution

$$X \sim U(6, 8) \quad f(x) = \frac{1}{2}$$

$$E\left(\frac{(8-X)^3}{8}\right) = \int_6^8 \frac{(8-X)^3}{8} \times \frac{1}{2}\, dx = \frac{1}{16} \int_6^8 (8-X)^3\, dX$$

$$= \frac{1}{16}\left(-\frac{(8-X)^4}{4}\Big|_6^8\right) = \frac{1}{16}\left(\frac{-(8-8)^4}{4} - \frac{-(8-6)^4}{4}\right) = \frac{1}{16}(4) = \frac{1}{4}$$

Problem 5

A basketball player makes 80% of his free throw attempts on average. Use the normal approximation to find out the probability that in 100 attempts he will be successful at least 90 times.

Solution

$$X \sim B(100, 0.8) \approx Y \sim N(80, 16)$$

$$P(X \geq 90) \approx P(Y \geq 89.5) = P\left(Z \geq \frac{89.5 - 80}{4}\right) = P(Z \geq 2.375)$$

$$= 1 - P(Z \leq 2.375) = 1 - \Phi(2.38) = 1 - 0.9913 = 0.0087$$

Problem 6

From 1851 to 2018, exactly 140 hurricanes have hit Florida (including both years 1851 and 2018). In 2005, Florida was hit by four hurricanes: Cindy, Dennis, Katrina, and Wilma. If the probability of hurricane strikes has remained the same since 1851, what is the probability of Florida being struck by four or more hurricanes in the same year?

Solution

140 hurricanes in 168 years.

$0.8\bar{3}$ hurricanes per year

$X \sim \text{Pois}(0.8\bar{3})$: # hurricanes in a year

$$P(X \geq 4) = 1 - P(X \leq 3) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3)$$

$$= 1. - 0.9896$$

$$= 0.01042$$

Problem 7

Two teams play a series of games. The series is finished as soon as one of the teams wins 3 games. Suppose that both teams have probability 1/2 of winning each game. Let X be the number of games played. Find the mass function of X and its mean $\mathbf{E}X$.

Solution

$3 \leq X \leq 5 \ni$ 3-0, 3-1, 3-2

_ _ _ $\underline{A}$

total # of outcomes

_ _ _ _ $\underline{A}$

$$2\left(1 + \binom{3}{1} + \binom{4}{2}\right) = 2(1+3+6) = 20$$

PMF = $P(X=3) = \frac{2}{20}$ $P(X=4) = \frac{6}{20}$ $P(X=5) = \frac{12}{20}$

$$E(X) = 3\left(\frac{2}{20}\right) + 4\left(\frac{6}{20}\right) + 5\left(\frac{12}{20}\right) = 4.5$$

For the following multiple choice problems fill the correct circle.

Problem 8

Let X be uniform on the interval $(0, 1)$ and set $Y = X^2$. For $0 < y < 1$, the cumulative distribution function $F_Y(y)$ of Y is:

Solution

- ○ **A.** y^2
- ● **B.** $\sqrt{y}$
- ○ **C.** 1
- ○ **D.** $\frac{1}{2\sqrt{y}}$
- ○ **E.** $2y$

Problem 9

Let X be a uniform random variable on the interval $(1, 6)$. Which interval contains the standard deviation of X?

Solution

- ○ **A.** $[0, 1)$
- ● **B.** $[1, 2)$
- ○ **C.** $[2, 3)$
- ○ **D.** $[3, 4)$
- ○ **E.** $[4, \infty)$

$\frac{(6-1)^2}{12} = \frac{25}{12}$

Problem 10

People enter a gambling casino according to a Poisson process of rate 1 every 2 minutes. What is the probability that no one enters between 10:00 pm and 10:10 pm? $\lambda = 5$ $P(X=0)$

Solution

- ○ **A.** e^{-20}
- ● **B.** e^{-5}
- ○ **C.** $e^{-1/5}$
- ○ **D.** $1 - e^{-5}$
- ○ **E.** $1 - e^{-20}$

Problem 11

Let X be a random variable with probability density function $f(x) = 6(x - x^2)$ for $0 < x < 1$.
In which range does the probability $P(2/3 < X < 5/4)$ fall?

Solution

- ○ **A.** $(0, 0.1]$
- ○ **B.** $(0.1, 0.2]$
- ● **C.** $(0.2, 0.3]$
- ○ **D.** $(0.3, 0.4]$
- ○ **E.** $(0.4, 1]$

Problem 12

Approximately 100,000 marriages took place in the state of New York last year. One can use the Poisson approximation to binomial to estimate the probability that for exactly two of these couples, both partners were born on March 26. Assuming there are 365 days in a year, this (approximate) probability is $e^{-\lambda}\lambda^2/2!$. What is the value of λ?

Solution

- ○ **A.** $\frac{1}{365}$
- ○ **B.** $\frac{1}{(365)^2}$
- ○ **C.** $\frac{2}{(365)^2}$
- ○ **D.** $\frac{100,000}{365}$
- ● **E.** $\frac{100,000}{(365)^2}$