

# Power Functions

## Basics

### Definition

A power function is a function of the form,

$$f(x) = ax^p,$$

where  $a \neq 0$  is a constant and  $p$  is a real number. Some examples of power functions include:

$$\begin{aligned}f_1(x) &= 3x^2, \\f_2(x) &= x^{\frac{1}{3}}, \\f_3(x) &= -3\sqrt{x}, \\f_4(x) &= \frac{1}{2}x^{-\frac{1}{4}} = \frac{1}{2x^{\frac{1}{4}}}.\end{aligned}$$

Root functions, such as  $f(x) = \sqrt{x}$  and  $g(x) = \sqrt[3]{x}$ , are examples of power functions.

Graphically, power functions can resemble exponential or logarithmic functions for some values of  $x$ . However, as  $x$  gets very large, power functions and exponential or logarithmic functions begin to diverge from one another. An exponentially growing function will overtake a growing power function for large values of  $x$ . On the other hand, growing power functions will overtake logarithmic functions for large values of  $x$ .

### Domain and Range

The [domain](#) of a power function depends on the value of the power  $p$ . We will look at each case separately.

**1.**  $p$  is a non-negative integer

*The domain is all real numbers (i.e.  $(-\infty, \infty)$ ).*

**2.**  $p$  is a negative integer

The domain is all real numbers not including zero (i.e.  $(-\infty, 0) \cup (0, \infty)$  or  $\{x|x \neq 0\}$ ). We will revisit this case when we study rational functions.

**3.**  $p$  is a rational number expressed in lowest terms as  $r/s$  and  $s$  is **even**

**A.**  $p > 0$

*The domain is non-negative real numbers (i.e.  $[0, \infty)$  or  $\{x|x \geq 0\}$ ).*

**B.**  $p < 0$

*The domain is positive real numbers (i.e.  $(0, \infty)$  or  $\{x|x > 0\}$ ).*

**4.**  $p$  is a rational number expressed in lowest terms as  $r/s$  and  $s$  is **odd**

**A.**  $p > 0$

*The domain is all real numbers.*

**B.**  $p < 0$

*The domain is all real numbers not including zero.*

**5.**  $p$  is an irrational number

**A.**  $p > 0$

*The domain is all non-negative real numbers.*

**B.**  $p < 0$

*The domain is all positive real numbers.*

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**In the next section we will study the graphs of power functions.**

[Graphing power functions](#)

[The Biology Project](#) > [Biomath](#) > [Power Functions](#) > **Basics**

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[The Biology Project](#)  
[Department of Biochemistry and Molecular Biophysics](#)  
[The University of Arizona](#)  
April 2006  
[Contact the Development Team](#)

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