

# **Day 1**

# **Discrete maths:**

# **The first chapter**

Lecturer: Msc. Le Minh Tan

# Introduction

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# So what's the plan?

- Progress score: Weekly homework + 2 tests (each x2)
- Final score: Exam
- Preparation: Laptop (**VS C++, Python ≥ 3.10**), e. socket
- Please check out the **subject curriculum on UTE**.

# Textbooks

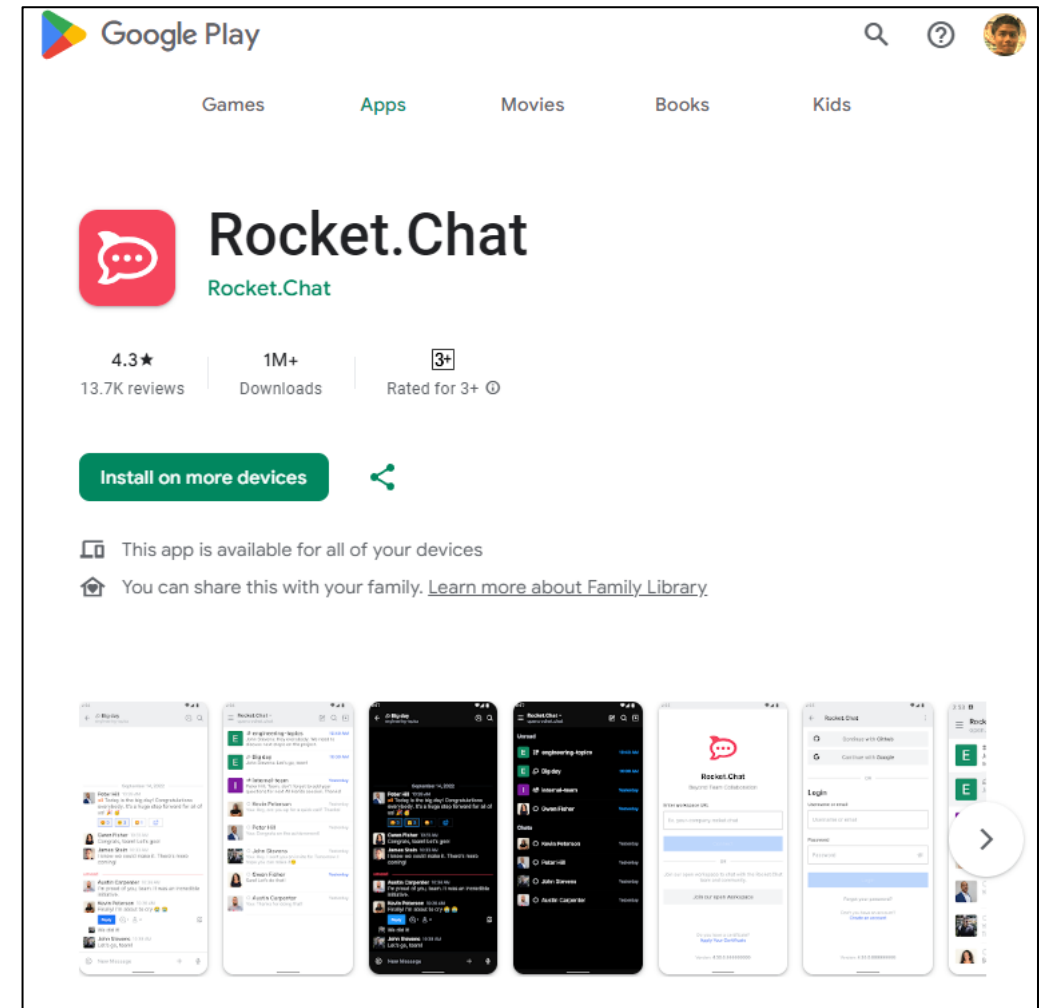
- Nguyễn Hữu Anh "Toán rời rạc"
- Đặng Trường Sơn & Lê Văn Vinh et al. "Giáo trình lý thuyết đồ thị"
- Nguyễn Thành Sơn & Đặng Trường Sơn et al. "Giáo trình toán rời rạc và lý thuyết đồ thị"
- Kenneth H. Rosen & Kamala Krithivasan "Discrete Mathematics and Its Applications"
- Adrian Bondy & U.S.R. Murty "Graph theory"



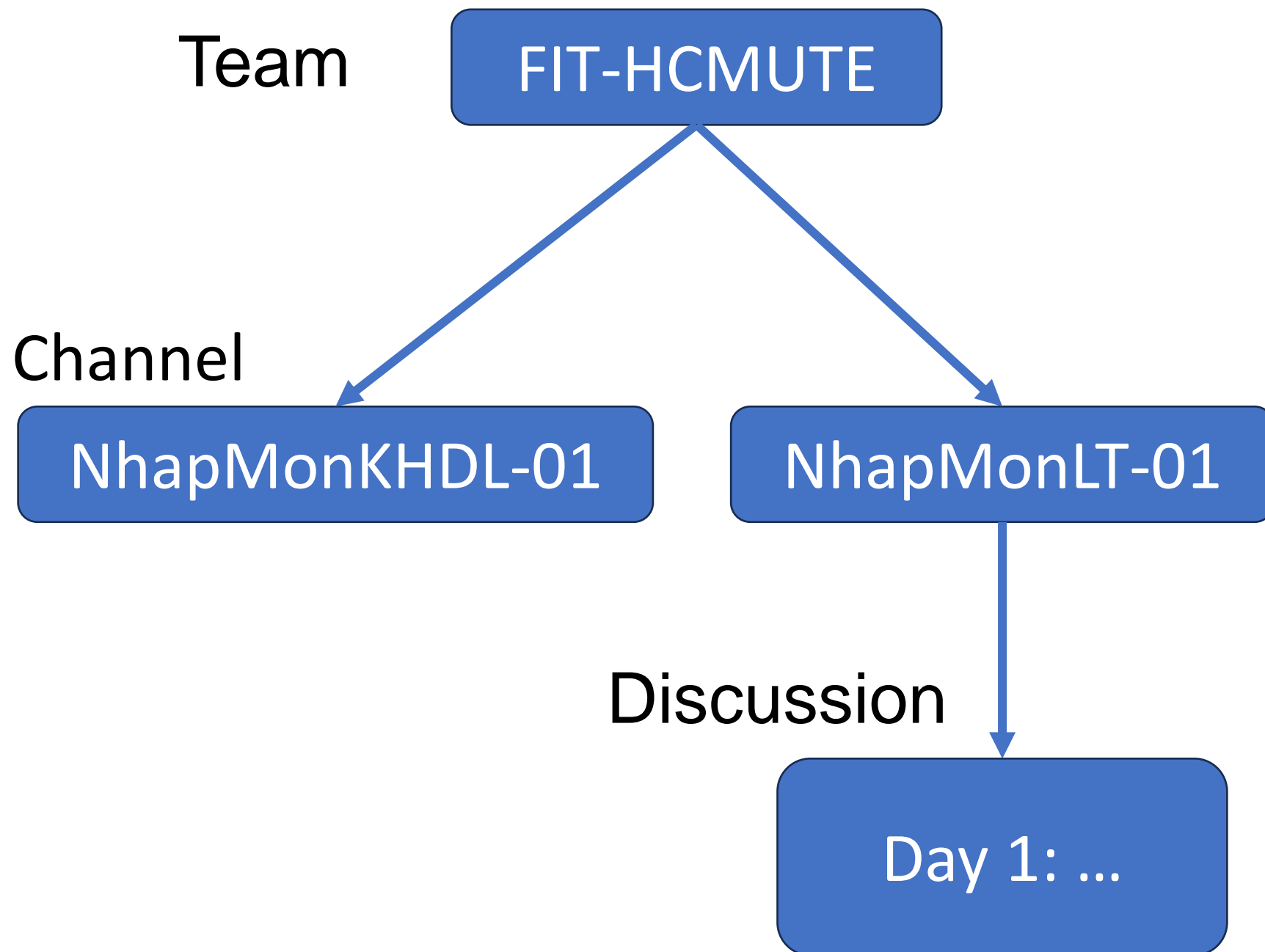
# How we work

- By chatting platform:
  - **Website:** <https://chat.cntt.io>
  - **Android/iOS:** Rocket.Chat
  - **Desktop:**

<https://github.com/RocketChat/Rocket.Chat.Electron/releases>




10



# You all need a group!

- Yes, each needs to be in a group for some activities.
- Steps:
  1. Gather 2-3 members.
  2. Vote for a leader.
  3. Post the list of members (ids + full names), leader id to FB group.
- Rules:
  1. No multiple groups per student.
  2. The smaller size, the better.
  3. Deadline: Before class ends.
  4. Those with no group need to contact me in one week.



A meme featuring two Spider-Man characters. The character on the left is in a red and blue suit, looking serious with his hand to his chin. The character on the right is in a red and blue suit, looking surprised with wide eyes and his hand to his chin. The background is a blurred outdoor scene with trees and a building.

**GUY WHO DOES  
MOST OF THE  
WORK IN THE  
GROUP PROJECT**

**ME TRYING  
TO  
LOOK  
USEFUL**

# Outline

- I. Discrete mathematics
- II. Propositional logic: Concepts
- III. Logical equivalences and rules of inference

# I. Discrete mathematics

|   |   |   |   |
|---|---|---|---|
| A | B | C | D |
|---|---|---|---|

|   |   |
|---|---|
| A | B |
| C | D |

A

C

B

D

Discrete objects

# Discrete maths problems

1. How many ways are there to choose a valid password on a computer system?
2. What is the probability of winning a lottery?
3. Is there a link between two computers in a network?
4. How can I identify spam e-mail messages?

*Page xviii*

*Kenneth H. Rosen & Kamala Krithivasan "Discrete Mathematics and Its Applications"*



# Discrete maths problems

1. How many ways are there to choose a valid password on a computer system? **Quantity**
2. What is the probability of winning a lottery? **Events**
3. Is there a link between two computers in a network? **Yes/No**
4. How can I identify spam e-mail messages? **Spam/Not spam**

*Discrete mathematics is used when ever objects are counted,  
when relationships between finite (or countable) sets are studied,  
and when processes involving a finite number of steps are  
analyzed.*

## II. Propositional logic: Concepts

- A **proposition** is a declarative sentence (that is, a sentence that declares a fact) that is either **true or false, but not both**.
- Some easy examples:
  - Joe Biden is the president of the United States. ✓
  - $1 + 1 = 2$  ✓
  - What time is it? ✗
  - Read this carefully. ✗
  - $x + 1 = 2$  ✗

- *“In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the two other sides”*
- *“If  $a > c$  then  $a + b > c$ ”*
- *“Every even integer greater than 2 can be written as the sum of two primes”*

1. What are the above statements about?
2. Can you prove them?
3. Is conjecture a proposition?

In discrete maths, we don't work with sentences, but variables and symbols:

| Order | Symbol                     | Feature                                | Expression            |
|-------|----------------------------|--|-----------------------|
| 1     | $\neg, -$                  | NOT (Negation)                         | $\neg p, \bar{p}$     |
| 2     | $\wedge$                   | AND/but                                | $p \wedge q$          |
| 3     | $\vee$                     | OR (inclusive or)                      | $p \vee q$            |
| 4     | $\oplus, \underline{\vee}$ | XOR (exclusive or)                     | $p \oplus q$          |
| 5     | $\rightarrow$              | Conditional statement/implication      | $p \rightarrow q$     |
| 6     | $\leftrightarrow$          | Biconditional statement/bi-implication | $p \leftrightarrow q$ |
|       | $\equiv, \Leftrightarrow$  | Equivalence                            | $p \equiv q$          |

Compound propositions

- Let  $p, q, r, s$  are the following propositions:

- *I'm sick.*
- *I finished my homework.*
- *I am going to stay home.*
- *I will attend today's class.*

1. How can you express these in sentence?

- $\neg p$
- $p \wedge q$
- $q \rightarrow r$

2. Is there anything illogical?



- A proposition always has a true value: True or false.
- Truth table helps determine the truth of compound propositions.

| $p$ | $q$ | $\neg q$ | $p \wedge q$ |
|-----|-----|----------|--------------|
| T   | T   | F        | T            |
| T   | F   | T        | F            |
| F   | T   | F        | F            |
| F   | F   | T        | F            |

- 1/0 can also be interpreted as T/F.
- Truth table can be useful as a proof.

*Exercise: Proving the below equivalence is always true*

$$p \oplus q \equiv (p \vee q) \wedge \neg(p \wedge q)$$

*Tips: A truth table could help.*

*Exercise: Proving the below **equivalence** is always true*

$$p \oplus q \equiv (p \vee q) \wedge \neg(p \wedge q)$$

*Tips: A truth table could help.*

| $p$ | $q$ | $p \oplus q$ | $r := p \vee q$ | $s := p \wedge q$ | $r \wedge \neg s$ |
|-----|-----|--------------|-----------------|-------------------|-------------------|
| 1   | 0   | 1            | 1               | 0                 | 1                 |
| 0   | 1   | 1            | 1               | 0                 | 1                 |
| 0   | 0   | 0            | 0               | 0                 | 0                 |
| 1   | 1   | 0            | 1               | 1                 | 0                 |

# III. Logical equivalences and rules of inference

| No. | Law             | Expression   |
|-----|-----------------|--|
| 1   | Double negation | $\neg\neg p \equiv p$  |
| 2   | De Morgan       | $\neg(p \wedge q) \equiv \neg p \vee \neg q$<br>$\neg(p \vee q) \equiv \neg p \wedge \neg q$                             |
| 3   | Commutative     | $p \wedge q \equiv q \wedge p$<br>$p \vee q \equiv q \vee p$   |
| 4   | Associative     | $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$<br>$p \vee (q \vee r) \equiv (p \vee q) \vee r$                     |
| 5   | Distributive    | $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$<br>$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ |

| No. | Law        | Expression   |
|-----|------------|--|
| 6   | Idempotent | $\neg\neg p \equiv p$  |
| 7   | Identity   | $p \wedge 1 \equiv p$<br>$p \vee 0 \equiv p$                     |
| 8   | Negation   | $p \wedge \neg p \equiv 0$<br>$p \vee \neg p \equiv 1$           |
| 9   | Domination | $p \wedge 0 \equiv 0$<br>$p \vee 1 \equiv 1$                     |
| 10  | Absorption | $p \wedge (p \vee q) \equiv p$<br>$p \vee (p \wedge q) \equiv p$ |

***- End of logical equivalences -***



| No. | Law                    | Expression   |
|-----|------------------------|--|
| 1   | Substitution           | $p \rightarrow q \equiv \neg p \vee q$   |
| 2   | Modus Ponens           | $(p \wedge (p \rightarrow q)) \rightarrow q$   |
| 3   | Modus Tollens          | $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$   |
| 4   | Hypothetical syllogism | $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$   |
| 5   | Disjunctive syllogism  | $((p \vee q) \wedge \neg p) \rightarrow q$   |
| 6   | Addition               | $p \rightarrow (p \vee q)$   |
| 7   | Simplification         | $(p \wedge q) \rightarrow p$   |
| 8   | Resolution             | $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$   |
| 9   | Contradictory          | $[(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \rightarrow q] \equiv [(p_1 \wedge p_2 \wedge \cdots \wedge p_n \wedge \neg q) \rightarrow 0]$ |
| 10  | Conditionally proving  | $((p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow ((p \vee q) \rightarrow r)$  |

***- rules of inference -***

# Bài tập tại lớp

Xác định suy diễn sau là đúng hay sai:

- Nếu Toàn tới lớp kịp lúc, bạn ấy sẽ không bị đánh văng.
- Nếu Toàn tới lớp kịp lúc, bạn ấy sẽ hiểu bài hơn.
- Vậy, nếu Toàn tới lớp kịp lúc, bạn ấy sẽ không bị đánh văng và sẽ hiểu bài hơn.

| $P$ | $Q$ | $P \rightarrow Q$ |
|-----|-----|-------------------|
| 0   | 0   | 1                 |
| 0   | 1   | 1                 |
| 1   | 0   | 0                 |
| 1   | 1   | 1                 |

| $P$ | $Q$ | $P \leftrightarrow Q$ |
|-----|-----|-----------------------|
| 0   | 0   | 1                     |
| 0   | 1   | 0                     |
| 1   | 0   | 0                     |
| 1   | 1   | 1                     |

# Homework

- Homework: 27, 28, 29, 30, 31/80 (2pts each)
- Work in group
- Deadline: 1 week
- At least one student per group must submit a docx file containing the answers. Copying from other group is strictly prohibited.