Interim Report

Predicting Algae Bloom spread in the Marlborough Sounds using Bayesian Inference

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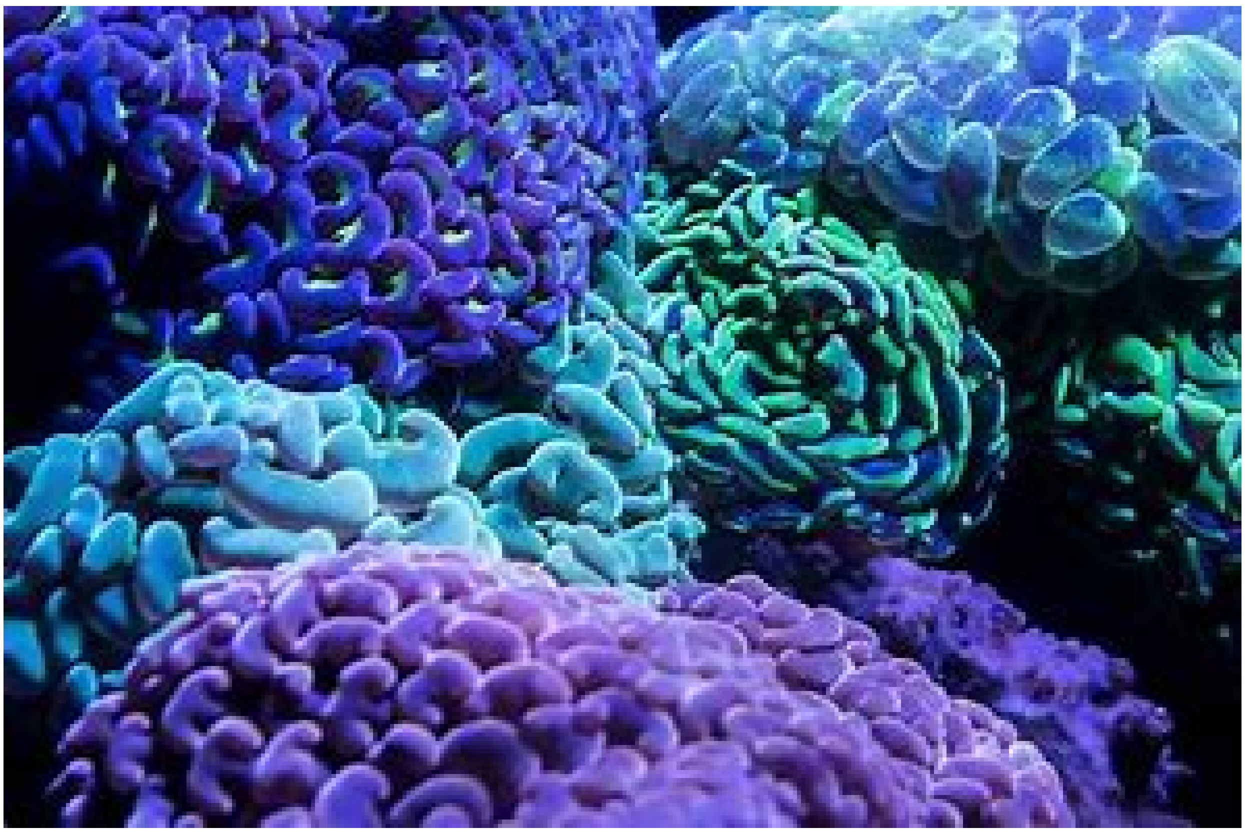


Introduction

This is an interim technical report describing the current state of my Metropolis-Hastings algorithm for Predicting Algae Bloom spread, using a simplified model involving a singular event.

Our Model

Mass

Mass is observed as:

- n Sites and T discrete Times
- Growth parameter α
- Decay parameter β
- Dispersion Matrix P where p_{ij} is the proportion of Mass at site i that travels to site j (Such that $\sum_{j=1}^{n} p_{ij} = 1$ for all $i \in \{1,..., n\}$)
- N events (Simplified to 1)
- Each of $k \in \{1,..., N\}$ events has:
  - Starting site I_k with uniform probability $\frac{1}{n}$
  - Starting time $T_k \in \{1,..., T\}$
  - Length of time W_k
  - Starting size S_k

Where the equation for $M_{i,t}$ is:

$$M_{i,t} = \alpha A_{i,t} M_{i,t-1} + \beta\left(\sum_{j=1}^{n} p_{i,j} M_{j,t-1}\right) + B_{i,t}$$

Where

$$A_{i,t} = I\left(\sum_{k=1}^{n} I\left(I_k = i\right) I\left(T_k \leq t \leq T_k + W_k\right) > 0\right)$$

$$B_{i,t} = \sum_{k=1}^{n} I\left(I_k = i\right) I\left(T_k = t\right) S_k$$

Concentration

Concentration is calculated from Mass where:

- Each site $i \in \{1,..., n\}$ has volume V_i
- δ is a concentration offset of approximately 0^+
- Precision parameter τ

Where the equation for $C_{i,t}$ is

$$log(Y_{i,t}) \sim N\left(log\left(\delta + \frac{M_{i,t}}{V_i}\right), \frac{1}{\tau}^2\right)$$

Parameters

Known Parameters:

- Number of sites, n
- Time period, T
- Dispersion Matrix, P
- Number of events, N
- Volumes, V_i
- Concentration offset, δ

Parameters to be estimated

- Growth Parameter, α
- Decay Parameter, β
- Event site, I_k
- Event Starting time, T_k
- Event time length, W_k
- Event size, S_k
- Precision τ

Priors

$\alpha \sim Gamma\left(a_{\alpha}, b_{\alpha}\right)$

$\beta \sim Beta\left(a_{\beta}, b_{\beta}\right)$

$N = 1$

$I_{k} \sim DiscreteUniform(1, n)$

$T_{k} \sim DiscreteUniform(1, T)$

$W_{k} \sim NegBinomial\left(r_{W}, p_{W}\right)$

$S_{k} \sim Gamma\left(a_{S}, b_{S}\right)$

$\tau \sim Gamma\left(a_{\tau}, b_{\tau}\right)$

Where:

$a, b = 1$

$r_{W} = 6$

$p_{W} = 0.75$

Pseudocode

FindM$_{New}$

Inputs:

- M
- α
- β
- P
- A
- B

$growth = \alpha \times (AM)$

$transport = \beta \times (P \cdot M)$

$M_{New} = B + growth + transport$

Outputs:

- M_{New}

RunSimulation

Inputs:

- n
- T
- P

$\alpha = 2$

$\beta = 0.8$

$N = 1$

$For\ x\ in\ 1{:}\ N{:}$

$$i \sim DiscreteUniform(1,\ n)$$

$$t \sim DiscreteUniform(1,\ T)$$

$$w \sim NegBinom(6,\ 0.75)\ +\ 1$$

$$s \sim Gamma(1,\ 1)$$

$$A_{t:min(t+w,T),\,i} = 1$$

$$B_{t,\,i} = s$$

$$\left[M_{0,\,0}, ...,\ M_{n,\,0}\right] = 0$$

$For\ t\ in\ 1{:}\ T{:}$

$$M_{New} = FindM_{New}\left(\left[M_{0,\,t-1}, ...,\ M_{n,\,t-1}\right],\ \alpha,\ \beta,\ P_{i:n,\,i:n},\ \left[A_{t,\,1}, ..., A_{t\,,n}\right],\ \left[B_{t,\,1}, ..., B_{t,\,n}\right]\right)$$

$$\left[M_{0,\,t}, ...,\ M_{n,\,t}\right] = M_{New}$$

Outputs:

- $\left[\left[M_{0,\,1}, ...,\ M_{n,\,1}\right], ..., \left[M_{0,\,T}, ...,\ M_{n,\,T}\right]\right]$
- α
- β
- N
- $\left[I_{0}, ...,\ I_{N}\right]$
- $\left[T_{0}, ...,\ T_{N}\right]$
- $\left[W_{0}, ...,\ W_{N}\right]$
- $\left[S_{0}, ...,\ S_{N}\right]$

GetSample

Inputs:

- n
- T
- P
- α
- β
- N
- $\left[I_{1}, \ldots, I_{N}\right]$
- $\left[t_{1}, \ldots, t_{N}\right]$
- $\left[W_{1}, \ldots, W_{N}\right]$
- $\left[s_{1}, \ldots, s_{N}\right]$

$For\ x\ in\ 1: N:$

$$A_{t_{x}:min(t_{x}+W_{x}, t), I_{x}} = 1$$

$$B_{t, i} = s_{x}$$

$$\left[M_{0, 0}, \ldots, M_{n, 0}\right] = 0$$

$For\ t\ in\ 1: T:$

$$M_{New} = FindM_{New}\left(\left[M_{0, t-1}, \ldots, M_{n, t-1}\right], \alpha, \beta, P_{i:n, i:n}, \left[A_{t, 1}, \ldots, A_{t, n}\right], \left[B_{t, 1}, \ldots, B_{t, n}\right]\right)$$

$$\left[M_{0, t}, \ldots, M_{n, t}\right] = M_{New}$$

Outputs:

- $\left[\left[M_{0, 1}, \ldots, M_{n, 1}\right], \ldots, \left[M_{0, T}, \ldots, M_{n, T}\right]\right]$

ConvertConcentration

Inputs:

- n
- T
- M
- V
- δ

$For\ i\ in\ 1:n$:

$$C_{i,\,1:T} = \frac{M_{i,\,1:T}}{V_i}$$

$$LogC = log(C + \delta)$$

Outputs:

- $LogC$

ConvertY

Inputs:

- C
- τ

$$LogY \sim N\left(C,\ \frac{1}{\sqrt{\tau}}\right)$$

Outputs:

- $LogY$

Metropolis-Hastings Algorithm

Inputs:

- P

$n = 10$

$T = 100$

$\delta = 1^{-10}$

$\tau_{True} \sim Gamma(1, 1)$

$\left[V_1, ..., V_n\right] \sim NegBinom(1, 1) + 1$

$\left[M_{True}, \alpha_{True}, \beta_{True}, N_{True}, I_{True}, T_{True}, W_{True}, S_{True}\right] = runSimulation(n, T, P)$

$C_{True} = ConvertCalculation\left(n, T, M_{True}, V, \delta\right)$

$Y_{True} = ConvertY\left(C_{True}, \tau_{True}\right)$

$\alpha_1 \sim Gamma(1, 1)$

$\beta_1 \sim Beta(1, 1)$

$N = N_{True}$

$\left[I_{0,1}, ..., I_{0,N}\right] \sim DiscreteUniform(1, n)$

$\left[t_{1,1}, ..., t_{0,N}\right] \sim DiscreteUniform(1, T)$

$\left[w_{0,1}, ..., w_{0,N}\right] \sim NegBinom(6, 0.75)$

$\left[s_{0,1}, ..., s_{0,N}\right] \sim Gamma(1, 1)$

$\tau_0 \sim Gamma(1, 1)$

$\phi = \left[\alpha_0, \beta_0, N, \left[I_{0,1}, ..., I_{0,N}\right], \left[t_{0,1}, ..., t_{0,N}\right], \left[w_{0,1}, ..., w_{0,N}\right], \left[s_{0,1}, ..., s_{0,N}\right], \tau_0\right]$

$For\ each\ parameter_0\ in\ \phi$:

$\quad Array_{parameter} = parameter_0$

$\quad Accept_{parameter} = 0$

$For\ s\ in\ 1{:}\,100000{:}$

Alpha

$$\alpha_{Candidate} \sim N\left(\alpha_{Candidate},\ 1\right)$$

$$M_{Candidate} = GetSample\left(n,\ T,\ \alpha_{Candidate},\ \phi_2,\ \phi_3,\ \phi_4,\ \phi_5,\ \phi_6\right)$$

$$C_{Candidate} = ConvertConcentration\left(n,\ T,\ M_{Candidate},\ V,\ \delta\right)$$

$$M_{Current} = GetSample\left(n,\ T, \phi_1,\ \phi_2,\ \phi_3,\ \phi_4,\ \phi_5,\ \phi_6\right)$$

$$C_{Current} = ConvertConcentration\left(n,\ T,\ M_{Current},\ V,\ \delta\right)$$

$$p_{prior_{Candidate}} = Gamma(1,\ 1).PDF\left(\alpha_{Candidate}\right)$$

$$p_{prior_{Current}} = Gamma(1,\ 1).PDF\left(\phi_1\right)$$

$$p_{Likelihood_{Candidate}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Candidate},\ \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$$p_{Likelihood_{Current}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Current},\ \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$$p_{Q_{Candidate}} = N\left(\phi_1,\ \frac{1}{4}\right).PDF\left(\alpha_{Candidate}\right)$$

$$p_{Q_{Current}} N\left(\alpha_{Candidate},\ \frac{1}{4}\right).PDF\left(\phi_1\right)$$

$$ratio_{\alpha} = min\left(\frac{p_{prior_{Candidate}} \times p_{Likelihood_{Candidate}}}{p_{prior_{Current}} \times p_{Likelihood_{Current}}} \times \frac{p_{Q_{Current}}}{p_{Q_{Candidate}}},\ 1\right)$$

$$u \sim U(0,\ 1)$$

$if\ ratio > u$:

$$\phi_1 = \alpha_{Candidate}$$

$$Accept_\alpha = Accept_\alpha + 1$$

$$Likelihood_{\alpha_s} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Candidate}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$else$:

$$Likelihood_{\alpha_s} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Current}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

Beta

$$\beta_{Candidate} \sim Beta(1, 1)$$

$$M_{Candidate} = GetSample\left(n, T, \phi_1, \beta_{Candidate}, \phi_3, \phi_4, \phi_5, \phi_6\right)$$

$$C_{Candidate} = ConvertConcentration\left(n, T, M_{Candidate}, V, \delta\right)$$

$$M_{Current} = GetSample\left(n, T, \phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6\right)$$

$$C_{Current} = ConvertConcentration\left(n, T, M_{Current}, V, \delta\right)$$

$$p_{prior_{Candidate}} = Beta(1, 1).PDF\left(\beta_{Candidate}\right)$$

$$p_{prior_{Current}} = Beta(1, 1).PDF\left(\phi_2\right)$$

$$p_{Likelihood_{Candidate}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Candidate}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$$p_{Likelihood_{Current}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Current}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$$p_{Q_{Candidate}} = Beta\left(\phi_2 + 1, 1\right).PDF\left(\beta_{Candidate}\right)$$

$$p_{Q_{Current}} \; Beta\left(\beta_{Candidate} + 1, 1\right).PDF\left(\phi_2\right)$$

$$ratio_{\beta} = min\left(\frac{p_{prior_{Candidate}} \times p_{Likelihood_{Candidate}}}{p_{prior_{Current}} \times p_{Likelihood_{Current}}} \times \frac{p_{Q_{Current}}}{p_{Q_{Candidate}}}, 1\right)$$

$$u \sim U(0, 1)$$

$$if\; ratio > u:$$

$$\phi_2 = \beta_{Candidate}$$

$$Accept_{\beta} = Accept_{\beta} + 1$$

$$Likelihood_{\beta_s} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Candidate}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$$else:$$

$$Likelihood_{\beta_s} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Current}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$$for\; k\; in\; 1: n:$$

$$I_{k_{Candidate}} \sim DiscreteUniform(1, n)$$

$$I_{Candidate} = \phi_3$$

$$I_{Candidate_k} = I_{k_{Candidate}}$$

$$M_{Candidate} = GetSample\left(n, T, \phi_1, \phi_2, I_{Candidate}, \phi_4, \phi_5, \phi_6\right)$$

$$C_{Candidate} = ConvertConcentration\left(n, T, M_{Candidate}, V, \delta\right)$$

$$M_{Current} = GetSample\left(n, T, \phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6\right)$$

$$C_{Current} = ConvertConcentration\left(n, T, M_{Current}, V, \delta\right)$$

$$p_{prior_{Candidate}} = DiscreteUniform(1, n).PMF\left(I_{Candidate_k}\right)$$

$$p_{prior_{Current}} = DiscreteUniform(1, n).PMF\left(\phi_{3_k}\right)$$

$$p_{Likelihood_{Candidate}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Candidate}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$$p_{Likelihood_{Current}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Current}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$$p_{Q_{Candidate}} = DiscreteUniform(1, n).PDF\left(I_{Candidate_k}\right)$$

$$p_{Q_{Current}} \; DiscreteUniform(1, n).PDF\left(\phi_{3_k}\right)$$

$$ratio_{I_k} = min\left(\frac{p_{prior_{Candidate}} \times p_{Likelihood_{Candidate}}}{p_{prior_{Current}} \times p_{Likelihood_{Current}}} \times \frac{p_{Q_{Current}}}{p_{Q_{Candidate}}}, 1\right)$$

$$u \sim U(0, 1)$$

$$if\; ratio > u:$$

$$\phi_3 = I_{Candidate}$$

$$Accept_I = Accept_I + \frac{1}{N}$$

$$Likelihood_{I_{s,k}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Candidate}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$else:$

$$Likelihood_{I_{s,k}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Current}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

T

$for\ k\ in\ 1:n:$

$$T_{k_{Candidate}} \sim DiscreteUniform(1,\ T - 1)$$

$$T_{Candidate} = \phi_4$$

$$T_{Candidate_k} = T_{k_{Candidate}}$$

$$M_{Candidate} = GetSample\left(n,\ T,\ \phi_1,\ \phi_2,\ \phi_3,\ T_{Candidate},\ \phi_5,\ \phi_6\right)$$

$$C_{Candidate} = ConvertConcentration\left(n,\ T,\ M_{Candidate},\ V,\ \delta\right)$$

$$M_{Current} = GetSample\left(n,\ T, \phi_1,\ \phi_2,\ \phi_3,\ \phi_4,\ \phi_5,\ \phi_6\right)$$

$$C_{Current} = ConvertConcentration\left(n,\ T,\ M_{Current},\ V,\ \delta\right)$$

$$p_{prior_{Candidate}} = DiscreteUniform(1,\ T).PMF\left(T_{Candidate_k}\right)$$

$$p_{prior_{Current}} = DiscreteUniform(1,\ T).PMF\left(\phi_{4_k}\right)$$

$$p_{Likelihood_{Candidate}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Candidate}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$$p_{Likelihood_{Current}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Current}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$$p_{Q_{Candidate}} = DiscreteUniform(1,\ T - 1).PDF\left(T_{Candidate_k}\right)$$

$$p_{Q_{Current}} = DiscreteUniform(1,\ T - 1).PDF\left(\phi_{4_k}\right)$$

$$ratio_{T_k} = min\left(\frac{p_{prior_{Candidate}} \times p_{Likelihood_{Candidate}}}{p_{prior_{Current}} \times p_{Likelihood_{Current}}} \times \frac{p_{Q_{Current}}}{p_{Q_{Candidate}}},\ 1\right)$$

$$u \sim U(0,\ 1)$$

$$if\ ratio > u:$$

$$\phi_4 = T_{Candidate}$$

$$Accept_T = Accept_T + \frac{1}{N}$$

$$Likelihood_{T_{s,k}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Candidate},\ \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$$else:$$

$$Likelihood_{T_{s,k}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Current},\ \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

W

$$for\ k\ in\ 1:n:$$

$$W_{k_{Candidate}} \sim Binom\left(T - \phi_{4_k},\ \frac{1 + \phi_{5_k}}{2 + T}\right)$$

$$W_{Candidate} = \phi_5$$

$$W_{Candidate_k} = W_{k_{Candidate}}$$

$$M_{Candidate} = GetSample\left(n,\ T,\ \phi_{1},\ \phi_{2},\ \phi_{3},\ \phi_{4},\ W_{Candidate},\ \phi_{6}\right)$$

$$C_{Candidate} = ConvertConcentration\left(n,\ T,\ M_{Candidate},\ V,\ \delta\right)$$

$$M_{Current} = GetSample\left(n,\ T, \phi_{1},\ \phi_{2},\ \phi_{3},\ \phi_{4},\ \phi_{5},\ \phi_{6}\right)$$

$$C_{Current} = ConvertConcentration\left(n,\ T,\ M_{Current},\ V,\ \delta\right)$$

$$p_{prior_{Candidate}} = DiscreteUniform(1,\ T).PMF\left(W_{Candidate_{k}}\right)$$

$$p_{prior_{Current}} = DiscreteUniform(1,\ T).PMF\left(\phi_{5_{k}}\right)$$

$$p_{Likelihood_{Candidate}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Candidate},\ \frac{1}{\sqrt{\phi_{7}}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$$p_{Likelihood_{Current}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Current},\ \frac{1}{\sqrt{\phi_{7}}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$$p_{Q_{Candidate}} = Binom\left(T - 1,\ \frac{1+\phi_{5_{k}}}{2+T}\right).PMF\left(W_{Candidate_{k}}\right)$$

$$p_{Q_{Current}} Binom\left(T - 1,\ \frac{1+W_{Candidate_{k}}}{2+T}\right).PMF\left(\phi_{5_{k}}\right)$$

$$ratio_{W_{k}} = min\left(\frac{p_{prior_{Candidate}} \times p_{Likelihood_{Candidate}}}{p_{prior_{Current}} \times p_{Likelihood_{Current}}} \times \frac{p_{Q_{Current}}}{p_{Q_{Candidate}}},\ 1\right)$$

$$u \sim U(0,\ 1)$$

$if\ ratio > u$:

$$\phi_{5} = W_{Candidate}$$

$$Accept_{W} = Accept_{W} + \frac{1}{N}$$

$$Likelihood_{W_{s,k}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Candidate}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$else$:

$$Likelihood_{W_{s,k}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Current}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

S

$for\ k\ in\ 1{:}\ n$:

$$S_{k_{Candidate}} \sim N\left(\phi_{6_k}, 1\right)$$

$$S_{Candidate} = \phi_6$$

$$S_{Candidate_k} = S_{k_{Candidate}}$$

$$M_{Candidate} = GetSample\left(n, T, \phi_1, \phi_2, \phi_3, \phi_4, \phi_5, S_{Candidate}\right)$$

$$C_{Candidate} = ConvertConcentration\left(n, T, M_{Candidate}, V, \delta\right)$$

$$M_{Current} = GetSample\left(n, T, \phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6\right)$$

$$C_{Current} = ConvertConcentration\left(n, T, M_{Current}, V, \delta\right)$$

$$p_{prior_{Candidate}} = Gamma(1, 1).PDF\left(S_{Candidate_k}\right)$$

$$p_{prior_{Current}} = Gamma(1, n).PDF\left(\phi_{6_k}\right)$$

$$p_{Likelihood_{Candidate}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Candidate}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$$p_{Likelihood_{Current}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Current}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$$p_{Q_{Candidate}} = N\left(\phi_{6_k}, 1\right).PDF\left(S_{Candidate_k}\right)$$

$$p_{Q_{Current}} N\left(S_{Candidate_k}, 1\right).PDF\left(\phi_{6_k}\right)$$

$$ratio_{S_k} = min\left(\frac{p_{prior_{Candidate}} \times p_{Likelihood_{Candidate}}}{p_{prior_{Current}} \times p_{Likelihood_{Current}}} \times \frac{p_{Q_{Current}}}{p_{Q_{Candidate}}}, 1\right)$$

$$u \sim U(0, 1)$$

$if\ ratio > u$:

$$\phi_6 = S_{Candidate}$$

$$Accept_S = Accept_S + \frac{1}{N}$$

$$Likelihood_{S_{s,k}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Candidate}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$else$:

$$Likelihood_{S_{s,k}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Current}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

Tau

$$\tau_{Candidate} \sim N\left(\phi_7, 1\right)$$

$$M_{Candidate} = GetSample\left(n, T, \phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6\right)$$

$$C_{Candidate} = ConvertConcentration\left(n, T, M_{Candidate}, V, \delta\right)$$

$$M_{Current} = GetSample\left(n, T, \phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6\right)$$

$$C_{Current} = ConvertConcentration\left(n, T, M_{Current}, V, \delta\right)$$

$$p_{prior_{Candidate}} = Gamma(1, 1).PDF\left(\tau_{Candidate}\right)$$

$$p_{prior_{Current}} = Gamma(1, 1).PDF\left(\phi_7\right)$$

$$p_{Likelihood_{Candidate}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Candidate}, \frac{1}{\sqrt{\tau_{Candidate}}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$$p_{Likelihood_{Current}} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Current}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$$p_{Q_{Candidate}} = N\left(\phi_7, 1\right).PDF\left(\tau_{Candidate}\right)$$

$$p_{Q_{Current}} \; N\left(\tau_{Candidate}, 1\right).PDF\left(\phi_7\right)$$

$$ratio_{\alpha} = min\left(\frac{p_{prior_{Candidate}} \times p_{Likelihood_{Candidate}}}{p_{prior_{Current}} \times p_{Likelihood_{Current}}} \times \frac{p_{Q_{Current}}}{p_{Q_{Candidate}}}, 1\right)$$

$$u \sim U(0, 1)$$

$if\ ratio > u$:

$$\phi_8 = \tau_{Candidate}$$

$$Accept_\tau = Accept_\tau + 1$$

$$Likelihood_{\tau_s} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Candidate}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

$else$:

$$Likelihood_{\tau_s} = \sum_{y=1}^{T}\left(\sum_{x=1}^{N}\left(Log\left(\delta + N\left(C_{Current}, \frac{1}{\sqrt{\phi_7}}\right).PDF\left(Y_{True}\right)\right)\right)\right)$$

Proposal Distributions

$$\alpha^{*} \sim N(\phi_{1}, 1)$$

$$\beta^{*} \sim Beta(1, 1)$$

$$I_{k}^{*} \sim DiscreteUniform(1, n)$$

$$T_{k}^{*} \sim DiscreteUniform(1, T - 1)$$

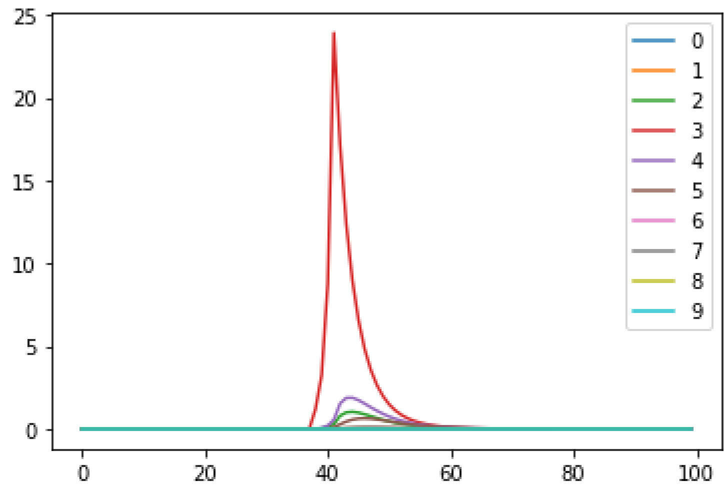
$$W_{k}^{*} \sim Binom\left(T - \phi_{4_{k}}, \frac{1 + \phi_{5_{k}}}{2 + T}\right)$$

$$S_{k}^{*} \sim N\left(\phi_{6_{k}}, 1\right)$$

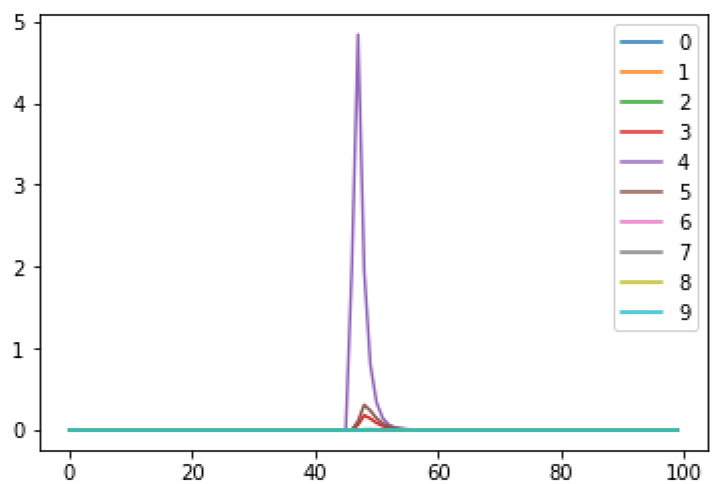
$$\tau^{*} \sim N(\phi_{7}, 1)$$

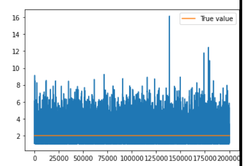
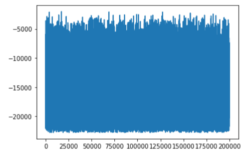
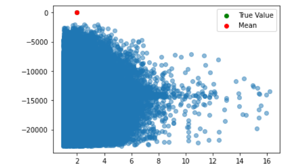
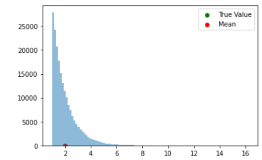
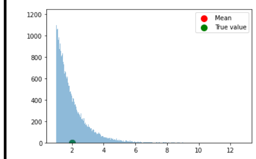
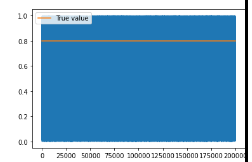
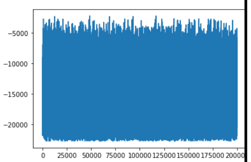
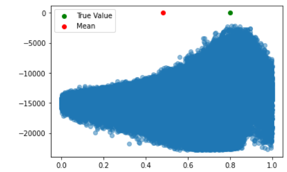
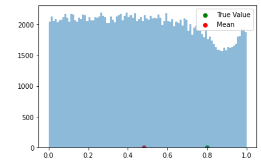
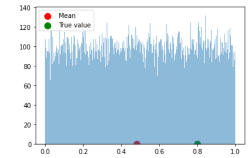
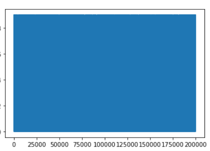
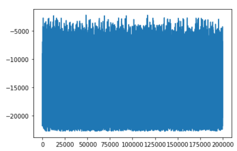
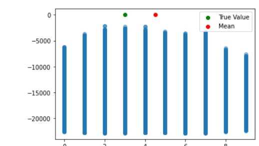
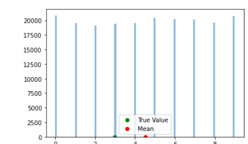
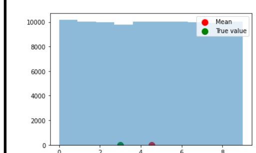
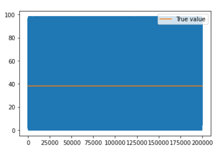
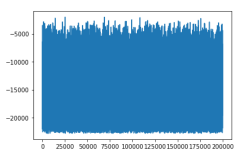
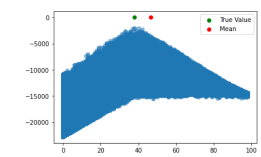
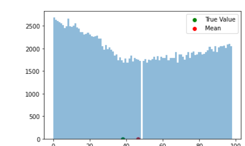
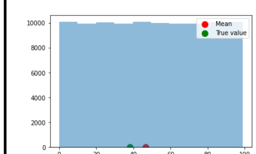
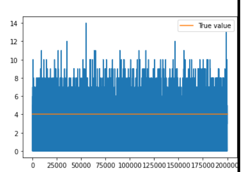
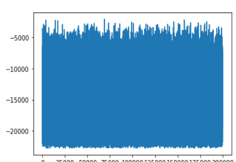
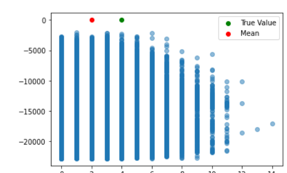
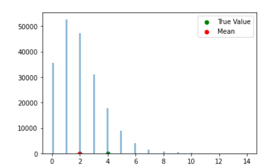
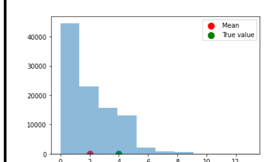
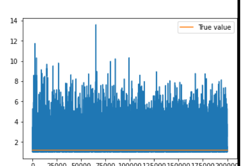
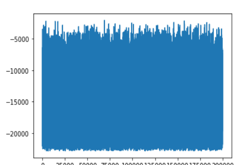
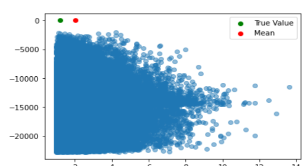
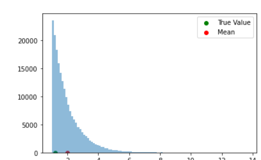
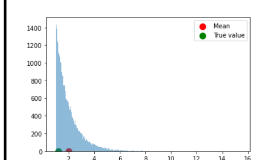
Results

Mass - True values



Mass - Estimated Parameters



Parameter	P(Acceptance)	Walk of value	Walk of Likelihood	Scatter Plot of Likelihood	Histogram	Prior PDF
α	0.5216					
β	0.9296					
I_0	0.1004					
T_0	0.9095					
W_0	0.6281					
S_0	0.5242					
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