Interim Report

Predicting Algae Bloom spread in the Marlborough Sounds using Bayesian Inference

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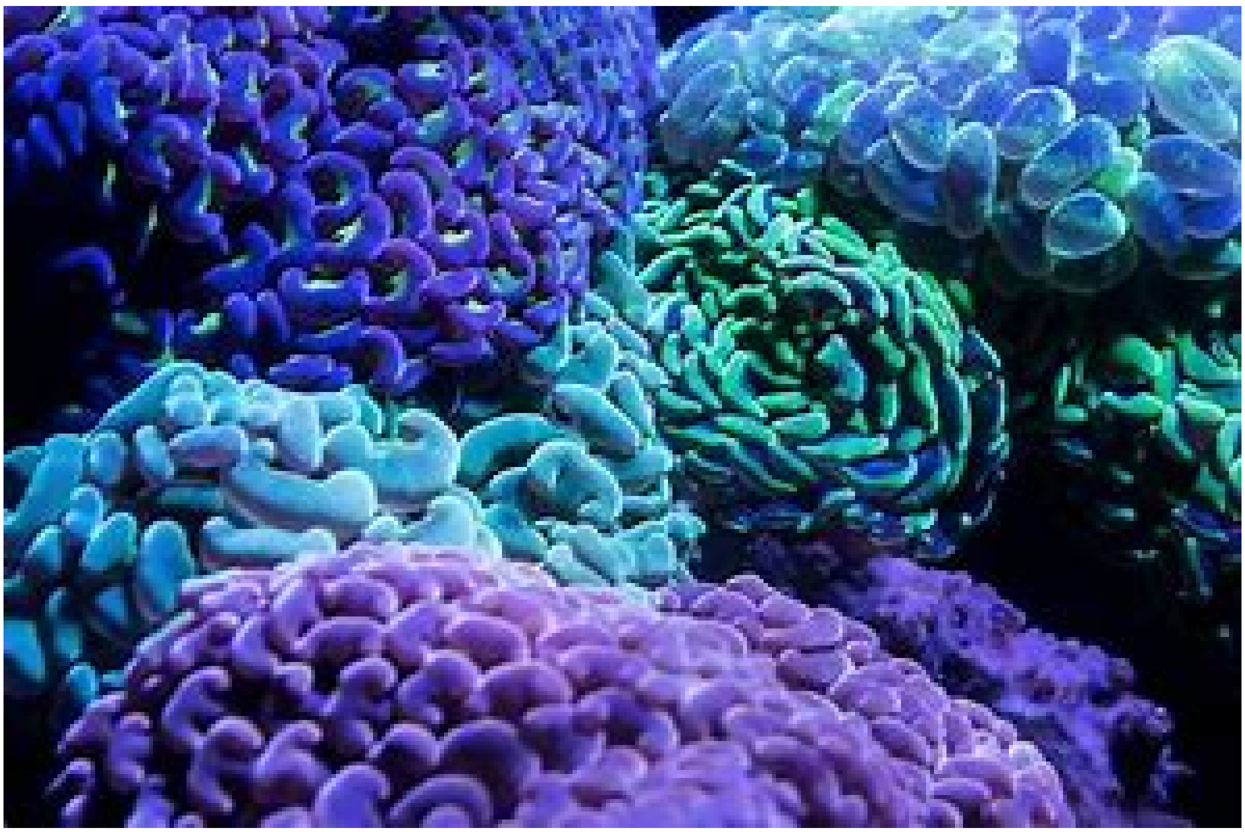


Introduction

This is an interim technical report describing the current state of my Metropolis-Hastings algorithm for Predicting Algae Bloom spread, using a simplified model involving a singular event.

Our Model

Mass

Mass is observed as:

- n Sites and T discrete Times
- Growth parameter α
- Decay parameter β
- Dispersion Matrix P where p_{ij} is the proportion of Mass at site i that travels to site j (Such that $\sum_{j=1}^{n} p_{ij} = 1$ for all $i \in \{1,..., n\}$)
- N events (Simplified to 1)
- Each of $k \in \{1,..., N\}$ events has:
  - Starting site I_k with uniform probability $\frac{1}{n}$
  - Starting time $T_k \in \{1,..., T\}$
  - Length of time W_k
  - Starting size S_k

Where the equation for $M_{i,t}$ is:

$$M_{i,t} = \left(\alpha A_{i,t} + \beta \sum_{j=1}^{n} p_{i,j}\right) M_{i,t-1} + B_{i,t}$$

Where

$$A_{i,t} = I\left(\sum_{k=1}^{n} I\left(I_k = i\right) I\left(T_k \leq t \leq T_k + W_k\right) > 0\right)$$

$$B_{i,t} = \sum_{k=1}^{n} I\left(I_k = i\right) I\left(T_k = t\right) S_k$$

Concentration

Concentration is calculated from Mass where:

- Each site $i \in \{1,..., n\}$ has volume V_i
- δ is a concentration offset of approximately 0^+
- Precision parameter τ

Where the equation for $C_{i,t}$ is

$$log\left(Y_{i,t}\right) \sim N\left(log\left(\delta + \frac{M_{i,t}}{V_i}\right), \frac{1}{\tau}^2\right)$$

Parameters

Known Parameters:

- Number of sites, n
- Time period, T
- Dispersion Matrix, P
- Number of events, N
- Volumes, V_i
- Concentration offset, δ

Parameters to be estimated

- Growth Parameter, α
- Decay Parameter, β
- Event site, I_k
- Event Starting time, T_k
- Event time length, W_k
- Event size, S_k
- Precision τ

Priors

$\alpha \sim Gamma\left(a_{\alpha}, b_{\alpha}\right)$

$\beta \sim Beta\left(a_{\beta}, b_{\beta}\right)$

$N = 1$

$I_{k} \sim DiscreteUniform(1, n)$

$T_{k} \sim DiscreteUniform(1, T)$

$W_{k} \sim NegBinomial\left(r_{W}, p_{W}\right)$

$S_{k} \sim Gamma\left(a_{S}, b_{S}\right)$

$\tau \sim Gamma\left(a_{\tau}, b_{\tau}\right)$

Where:

$a, b = 1$

$r_{W} = 6$

$p_{W} = 0.75$

Metropolis-Hastings Algorithm

$Generate\ Sample\ Y\ \ with\ unknown\ parameters$

$Generate\ initial\ set\ of\ parameters\ \theta\ from\ priors$

$For\ s\ in\ 1{:}\ 10000{:}$

$For\ each\ parameter\ in\ \{\alpha,\ \beta,\ I,\ T,\ W,\ S,\ \tau\}{:}$

$Generate\ candidate\ value\ from\ proposal\ distributions$

$For\ p\ in\ \left\{p\ *,\ p\ _{current}\right\}{:}$

$Generate\ sample\ M\ ^{*}$

$Calculate\ P(p\ )\ from\ prior\ distribution$

$Calculate\ P(Y\mid p,\ \ \theta/p)\ from\ Likelihood\ distribution\ using\ M\ ^{*}$

$Calculate\ probability\ \ =\ P(p)\ \times\ P(Y\mid p,\ \ \theta/p)$

$$Calculate\ ratio\ \ =\ min\left(\frac{probability\ _{p\ ^{*}}}{probability\ _{p\ _{current}}},\ 1\right)$$

$Randomly\ generate\ u \sim U(0,\ 1)$

$If\ ratio\ \ \succ\ u{:}$

$Set\ p\ _{current}\ \ =\ p\ *$

Proposal Distributions

$$\alpha^{*} \sim N\left(\alpha_{current}, 1\right)$$

$$\beta^{*} \sim N\left(\beta_{current}, 0.5\right)$$

$$I_{k}^{*} \sim DiscreteUniform(1, n)$$

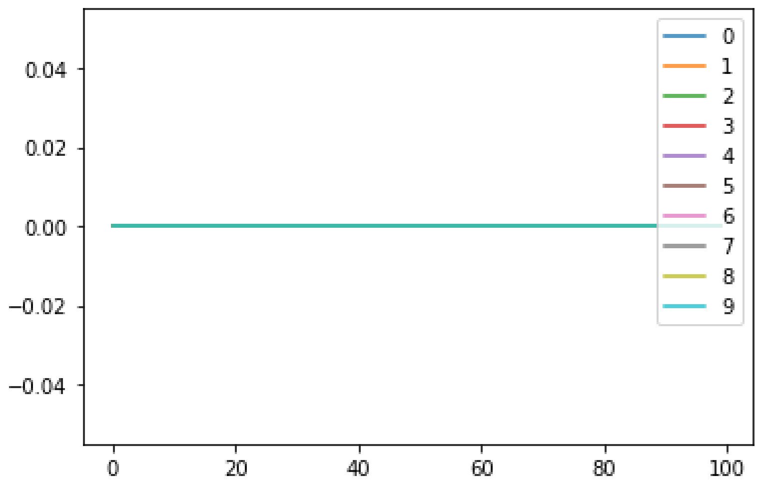
$$T_{k}^{*} \sim DiscreteUniform(1, T)$$

$$W_{k}^{*} \sim DiscreteGamma(1, 1)$$

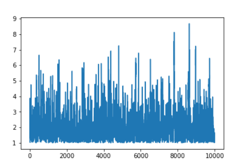
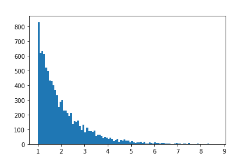
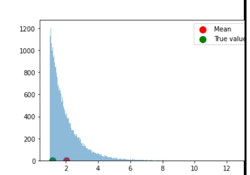
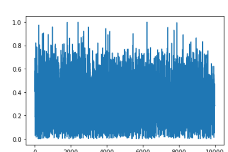
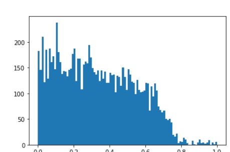
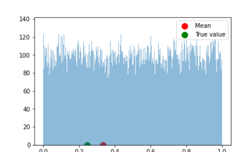
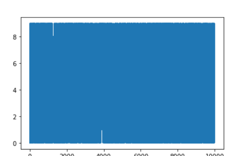
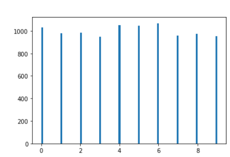
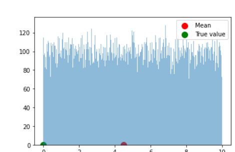
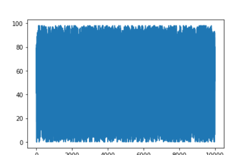
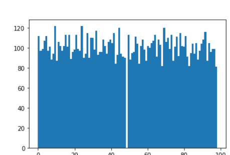
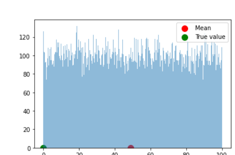
$$S_{k}^{*} \sim Gamma\left(S_{current}, 1\right)$$

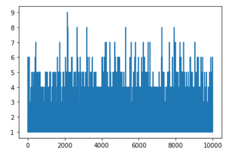
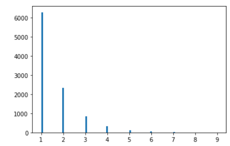
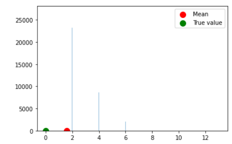
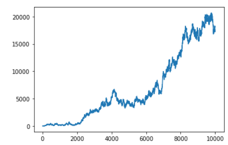
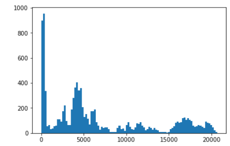
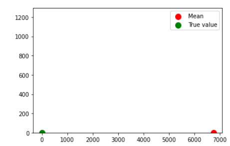
$$\tau^{*} \sim N\left(\tau_{current}, 1\right)$$

Mass - True values



Results

Parameter	Acceptance Probability	Walk	Histogram	Prior PDF
α	0.5237			
β	0.4506			
I_0	0.9999			
T_0	0.9999			

W_0	0.9999			
S_0	0.2234			
τ	0.5767