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Mass Fractionation in Exoplanet Atmospheres

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ABSTRACT

Exoplanets with orbits close to their host star are subject to intense stellar radiation. High energy radiation from x-ray to ultraviolet drives photochemistry and atmospheric escape in the upper atmosphere of planets. If the escape rate of a lighter constituent (i.e. H) is high enough, it can drag heavier species along with it thus fractionating the atmosphere. Here I propose to use hydrodynamic escape in an energy limited regime to explore the effects of drag on carrying away heavier species (Z>1). I compute the energy limited escape rate for GJ 436 b, GJ 1214 b and HD 97658 b and find mass loss rates around $\sim 10^9$g/s for each planet. The escape flux is used to estimate the crossover mass, the mass at which a constituent would have zero escape flux if it interacted with escaping Hydrogen through collisions. Each system has a crossover mass larger than $\sim$10 amu suggesting, after enough time, the planet's atmosphere should be enriched in metals due to the atmospheric escape of lighter species (e.g. H & He).

Keywords: miscellaneous — atmospheric escape

1. INTRODUCTION

Recent exoplanet surveys have discovered an abundance of planets in tightly bound orbits. The atmosphere of a close-in exoplanet is subject to a more extreme environment than witnessed by planets in our own Solar System. X-ray and EUV flux irradiate the planet altering the upper atmospheric structure through photochemistry and atmospheric escape (Lammer et al. 2003; Yelle 2004; Erkaev et al. 2007).

If the escape rate is very high, the escaping flux of a lighter constituent (e.g. H) can be large enough to drag a heavier species along with it (e.g. He, C). The limiting mass for a constituent to be dragged off, the crossover mass, is discussed in past literature for Solar System bodies (Hunten et al. 1987). Here I estimate the crossover mass for some close-in exoplanets subject to energy limited escape rates. Mass-dependent fractionation, depending on the scale, leads to vary

Table 1. Planetary Parameters

Name	Radius	Mass	Semi-major Axis	Equilibrium Temp	Distance	Star Temp.
	$R_\oplus$	$M_\oplus$	AU	K	parsec	K
HD 97658 b	2.29±0.16	7.87±0.73	0.0796±0.0013	732.6	21.11±0.33	5119.0
GJ 436 b	4.13±0.10	23.11±1.01	0.0287±0.0005	648.6	10.14±0.24	3350.0
GJ 1214 b	2.62±0.13	6.47±1.00	0.0143±0.0006	559.4	12.95±0.91	3026.0

Note—The equilibrium temperature is estimated using an albedo of 0.5 and heat redistribution factor of 0.25 (globally averaged output).

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different molecular compositions and thus distinctive spectral features in transmission and emission measurements of exoplanets (Koskinen et al. 2013; Hu et al. 2015). The next generation of space telescopes may be able to detect atmospheric evolution by probing for signatures of fractionated atmospheres (e.g. mass-dependent depletion of gases).

This paper is organized as follows. Our hydrodynamic equation leading to the cross over mass is setup in section 1. Next the energy limited escape rate is derived in section 2. In section 3, I adopt a diffusion coefficient from kinetic theory of non-uniform gases. Lastly we discuss our results and conclude in section 5.

2. HYDRODYNAMIC ESCAPE

The nature of mass fractionation in hydrodynamic escape has been investigated previously (Zahnle & Kasting 1986; Hunten et al. 1987). Here we revisit the problem with energy limited escape rates for close-in exoplanets. We assume the primary escaping constituent to be atomic Hydrogen because photochemistry models show that atomic hydrogen is the domination form of H in the upper atmosphere (Yelle 2004; Moses et al. 2011).

2.1. *Momentum Conservation*

We start with the equation of momentum conservation to examine the behavior of two fluids interacting with one another. The net flow of momentum out of our system will be dominated by the forces within

$$mn\frac{D}{Dt}\mathbf{u} + \bigtriangledown P - mn\mathbf{G} = mn\sum_i \nu_i\phi_i(\mathbf{u}_i - \mathbf{u}) \tag{1}$$

Here m is mass of a particle in the fluid, n is number density, $\frac{D}{Dt}$ is the convective derivative ($\frac{d}{dt} + u\bigtriangledown$) and $\mathbf{G}$ is our gravity vector. The right hand term represents the exchange of momentum for colliding fluids where ν_{1i} is the momentum transfer collision frequency, ϕ_{1i} is the velocity correction factor but for hard spheres this value is 1, and u represents the bulk flow velocity.

Dealing with planetary atmospheres allows us to make some simplifications to the momentum equation. Particularly, we exclude the effects of electricity and magnetism in momentum conservation and exclude ions. Additionally, we assume a steady state atmosphere such that the time derivative ($\partial u/\partial t$) is zero. We assume subsonic flow and steady state so that the convective derivative in our atmosphere is zero. Additionally, we focus on the parallel component of the outward flow and express equation 1 with our assumptions in 1D below

$$\frac{d}{dr}P_2 + m_2 n_2 g = n_2 m_2 \nu_{21}(u_1 - u_2) \tag{2}$$

.

The collision frequency is related to our diffusion coefficient using the relation in equation 3 (Schunk & Nagy 2004). Computing the number density as a function of altitude is as simple as differentiating the ideal gas law and substituting $\frac{d}{dr}P$ back into equation 2. For simplicity we assume the portion of the atmosphere we're dealing with has low densities and will thus remain isothermal as a result of thermal conduction. Removing the temperature derivative and substituting in the diffusion coefficient into equation 2 yields

$$D_{21} = \frac{kT}{m_2\nu_{21}} \tag{3}$$

$$\frac{d}{dr}n_2 = \frac{-n_2}{H_2} + \frac{n_2}{D_{21}}(u_1 - u_2) \tag{4}$$

.

Here H_i is the scale height defined by $\frac{kT}{m_i g}$. Multiplying the diffusion term by n_1/n_1, substituting the flux of particles $F = nu$ and letting $b_1 = 1/n_1 D_{21}$ Will give us equation 4 from Hunten et al. 1987. From here I follow the same derivation as Hunten et al. 1987 for the critical mass. The critical mass can be obtained by taking the derivative of the mole fraction with respect to altitude and setting that equal zero. Setting the derivative equal to zero implies the mole fractions are constant with respect to altitude. When the flux of constituent 1 is large, constituent 2 is carried along. We can then derive the crossover mass, a mass limit such that above this mass constituent 1 is no longer capable of imparting enough momentum to transport constituent 2.

$$m_c = m_1 + \frac{kTF_1}{b_1 g X_1} \tag{5}$$

Here k is the boltzmann constant, T is the temperature, F is the number flux of particles leaving the atmosphere, g is gravity and b_1 is our binary diffusion coefficient between constituent 1 and 2.

3. ENERGY LIMITED ESCAPE

Energy from incident stellar radiation can excite particles in the atmosphere to near escape speeds. Energy limited escape is defined as the balance of incident stellar flux with the gravitational potential of a planet. I adopt stellar flux measurements from the MUSCLES survey[1] where the EUV luminosity of the system is define as

$$L_{EUV} = 4\pi d^2 \int_5^{912} F(\lambda) d\lambda \tag{6}$$

. Here d is the distance to the star from Earth, $F(\lambda)$ are flux measurements from a MUSCLES spectrum between 5 and 912 A (see Figure 1). The total mass loss flux from our planet is define as

$$\dot{M} = \eta \frac{L_{EUV}}{4\pi a^2} \frac{R_{UV}^2}{4R^2} \frac{1}{\kappa} \frac{R}{GM} \tag{7}$$

.

$\dot{M}$ is the mass loss flux from the planet, R$_{UV}$ is the absorption radius (where $\tau \sim 1$) in the UV for HI between 13.6 eV and 20 eV ($\sigma \sim$2-7e-18 cm^2). η is the efficiency of heating and literature values are cited between 0.01-0.3 (Koskinen et al. 2013; Hu et al. 2015), we adopt a constant value of 0.2 for each of our planets. κ is the reduction factor in gravitational potential due to the Roche lobe effect (Erkaev et al. 2007). The total mass loss rate, $4\pi R^2 \dot{M}$, is reported in table 2.

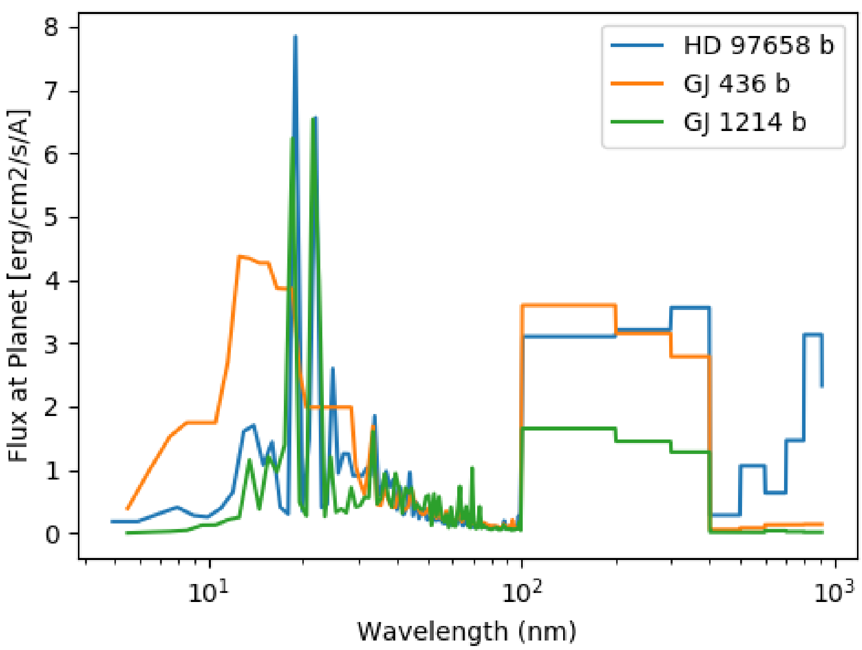

Figure 1. Stellar spectra from the MUSCLES treasury survey in the X-Ray and EUV (5-912A). All flux values have been scaled for the distance of each respective planet (i.e. this is the flux the planet receives).

4. DIFFUSION COEFFICIENT

Diffusion of non-uniform gases depends on the temperature of the mixture, diameters and masses of the constituents. We adopt a formulation for the diffusion coefficient based on kinetic theory for hard sphere collisions in a non-uniform

[1] https://archive.stsci.edu/prepds/muscles/

gas (See equation 9; Chamberlain 1969). The van der Waal radius is used as the kinetic radius in the diffusion coefficient (σ_i is the kinetic diameter in equation 8). Figure 2 has the van der Waal radii of our particles as a function of mass (Batsanov 2001).

$$Q = \frac{\pi}{16}(\sigma_1 + \sigma_2)^2 \tag{8}$$

$$b_1 = \frac{3}{64Q}\left(2\pi kT\frac{(m_1+m_2)}{m_1 m_2}\right)^{1/2} \tag{9}$$

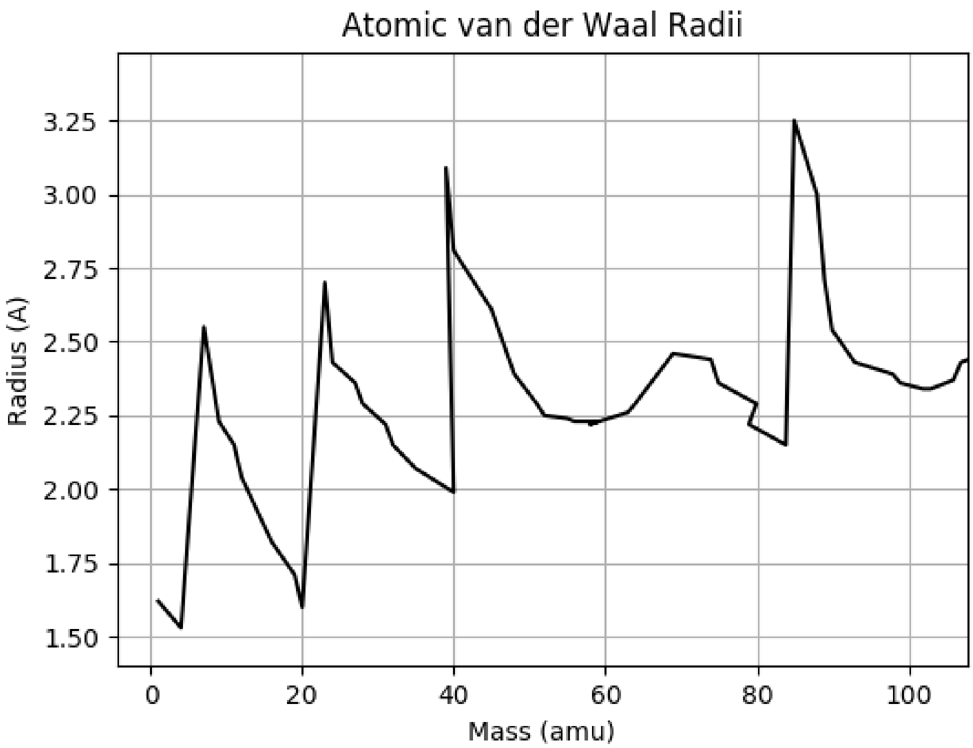

Figure 2. The van der Waal radii of atoms plotted versus their mass. We use the van der Waal radii as the kinetic radii in our diffusion coefficient calculation. The data comes from Batsanov 2001.

5. CROSSOVER MASS

Combining our equations for energy limited escape and diffusion coefficient produces the crossover mass equation below,

$$m_2 = m_1 + \frac{kT\dot{M}}{b_1 g X_1 m_1} \tag{10}$$

. This equation is transcendental because the diffusion coefficient scales with the crossover mass. The equation is solved by finding the root using a numerical algorithm (e.g. Brent 1973). We linearly interpolate between the atomic mass and radius in our diffusion coefficient calculation. Expanding the crossover mass equation in terms of planetary parameters results in the crossover mass having an R^3, M^{-2} and $T^{1/2}$ dependence.

6. RESULTS

The crossover mass is computed for the three transiting exoplanets in the MUSCLES survey using the planetary parameters in Table 1.

Table 2. Atmospheric Escape Parameters

Name	L_{euv}	Mass loss	Crossover Mass	$(R_{UV}/R)^2$	Roche Effect
	erg/s	g/s	amu		κ
GJ 436 b	2.e+27	2.42e+09	12.7	1.34	0.75
GJ 1214 b	2.87e+26	1.46e+09	24.5	1.73	0.66
HD 97658 b	3.10e+28	1.97e+09	30.8	1.66	0.91

GJ 436 b is suspected to have a He dominated atmosphere (Hu et al. 2015). Hu+2014 compute an escape rate of ~7e8 g/s (combining the escape of H and He) for GJ 436 b compared to my value of 2.4e9 g/s. The Hu escape rate yields a cross over mass of 4.5 suggesting there should be some escape of He along with the H in the atmosphere. My calculation for the escape rate is not consistent with the HST observation of the GJ 436 system in the Lyα absorption (Kulow et al. 2014). The escape rate of H derived from the Lyα transit light curve is $\sim 10^7$ g/s, assuming an ionization fraction of 0.1.

Mass fractionation of GJ 436 b's atmosphere would result in transit measurements of heavier constituents because the lighter constituents are escaping. 3-30 μm observations with Spitzer revealed a CH_4 poor atmosphere with a large CO absorption feature (Stevenson et al. 2010; Lanotte et al. 2014). Additionally, 1–2 μm transmission spectrum measurements from HST/WFC3 show no H_2O or CH_4 features (Knutson et al. 2014). Optical HST/STIS measurements from 0.5-1 μm display no scattering slope or molecular features (Lothringer et al. 2018). Models in the (Lothringer et al. 2018) suggest a high metallicity atmosphere is the best fit to their data and all prior transmission measurements. Mass fractionation from atmospheric escape could be a possible explanation for the relatively featureless transit measurements. Atmospheric escape would remove H and He from the atmosphere, decreasing the scale height by increasing the metallicity.

7. CONCLUSION

In conclusion, the energy limited escape rates are calculated for 3 transiting exoplanets with X-ray to EUV spectra from the MUSCLES survey. The mass loss rates for energy limited escape are $\sim 10^9$ g/s using a heating efficiency of 0.2 for each planet. The crossover mass using the mass loss flux for each planet yields at least ~12 amu or greater. A large crossover mass suggests the planet's atmosphere may be fractionated due to atmospheric escape of light constituents (e.g. H, He).

REFERENCES

Batsanov, S. S. 2001, Inorganic Materials, 37, 9

Brent, R. P. 1973, SIAM Journal on Numerical Analysis, 10, 327

Chamberlain, J. W. 1969, ApJ, 155, 711

Erkaev, N. V., Kulikov, Y. N., Lammer, H., et al. 2007, A&A, 472, 329

Hu, R., Seager, S., & Yung, Y. L. 2015, ApJ, 807, 8

Hunten, D. M., Pepin, R. O., & Walker, J. C. G. 1987, Icarus, 69, 532

Knutson, H. A., Benneke, B., Deming, D., & Homeier, D. 2014, Nature, 505, 66

Koskinen, T. T., Harris, M. J., Yelle, R. V., & Lavvas, P. 2013, Icarus, 226, 1678

Kulow, J. R., France, K., Linsky, J., & Loyd, R. O. P. 2014, ApJ, 786, 132

Lammer, H., Selsis, F., Ribas, I., et al. 2003, ApJL, 598, L121

Lanotte, A. A., Gillon, M., Demory, B.-O., et al. 2014, A&A, 572, A73

Lothringer, J. D., Benneke, B., Crossfield, I. J. M., et al. 2018, AJ, 155, 66

Moses, J. I., Visscher, C., Fortney, J. J., et al. 2011, ApJ, 737, 15

Schunk, R. W., & Nagy, A. F. 2004, Ionosphere, 570

Stevenson, K. B., Harrington, J., Nymeyer, S., et al. 2010, Nature, 464, 1161

Yelle, R. V. 2004, Icarus, 170, 167

Zahnle, K. J., & Kasting, J. F. 1986, Icarus, 68, 462