Introduction to Commutative Algebra

and affine algebraic varieties

Amal M

February 10, 2021

Table of Contents

Table of Contents

Introduction

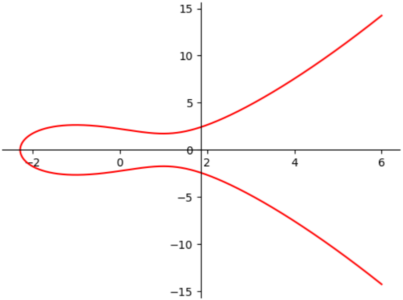
The Plan

- Study undergraduate algebraic geometry
- Read and do the exercies from Atiyah-Macdonald, Introduction to Commutative Algebra
- Read first chapter of Hartshorne's Algebraic Geometry

Table of Contents

Curves

$y^2 = x^3 3x + 5$ is a polynomial in $\mathbb{R}[x, y]$. The set of zeros of this polynomial looks like this



Polynomial Ring

Given a algebraicaly closed field k we can form the polynomial ring in n indeterminants

$$k[x_1, \cdots, x_n]$$

Every polynomial $p(x_1, \cdots, x_n) \in k[x_1, \cdots, x_n]$ can be thought of as a mapping from $k^n \to k$. We call k^n the *affine n-space* and denote it by $\mathbb{A}_k^n$.

Affine Algebraic Varieties

S is a set of polynomial in $k[x_1, \cdots, x_n]$. $V(S)$ is points in $\mathbb{A}_k^n$ at which every polynomial in S *vanishes*. $V(S)$ is called the *affine algebraic variety*.

Table of Contents

The Coordinate Ring

Given a variety V in $\mathbb{A}_k^n$ the *ideal of a variety* is the ideal $I(V)$ which consists of all polynomials in $k[x_1, \cdots, x_n]$ that vanish on V. The Coordinate ring of a variety is the ring

$$P(X) = k[x_1, \cdots, x_n]/I(X)$$

Hilbert's Nullstellensatz

Nullstellensatz means the theorem of zeros.

Algebra	**Geometry**
$k[x_1, \cdots, x_n]$	$\mathbb{A}_k^n \cong k^n$
$I(V)$	$V(I)$
$(x - a_1, \cdots, x - a_n)$	the point $(a_1, \cdots, a_n)$

Algebraic - Geometry

There is a connection between geometric objects such as curves and the algebraical objects like a ring.

Regular mappings

Explain polynomial mapping/regular mapping between varieties

Table of Contents

What sort of Commutative Algebra do we use?

What sort of commutative algebra machinery do we use: (Do not explain any of these. Point out where you use them instead)

1. Modules
2. Tensor products
3. Exact sequnces
4. Direct Limits

Table of Contents

Zariski

Talk about the prime spectrum and the Zariski Topology what sort of machinery would that use?

Constructible Topology

You can have another topology called the Constructible Topology

Table of Contents

Presheaf and Sheaf

Definiton of a Presheaf and Sheaf

Table of Contents

Applications of Algebraic Geometry

Do you really want applications? You could mention in passing string theory, arithmetic geometry, proof of the Fermat's last theorem etc. . .

Table of Contents

Acknowledgement

Hwey Lewis Borat