

Introduction to Commutative Algebra

and affine algebraic varieties

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 - 2 min
- 2 Algebraic Varieties
 - 3 min
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 - 5 min
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- 5 Zariski Topology
 - 3 min
- 6 Presheaf and Sheaf
 - 4 min
- 7 Applications
 - 1 min

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Introduction

Talk for two minutes about what you did and introduce yourself

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Curves

Introduce elliptic curves as an example of such a curve. Talk about other simple curves first and what kind of shapes their solutions make.

Polynomial Ring

Introduce the polynomial ring, How a single polynomial makes a curve, how a bunch of polynomials is called an ideal

Affine Algebraic Varieties

Define an affine algebraic variety or set.

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The Coordinate Ring

Define the ideal of a variety and $P(X)$ the coordinate ring

Nullstellensatz

Discuss curves over the complex numbers. If you have such curves then you got the Nullstellensatz which basically gives you a connection between algebra and geometry

Algebraic - Geometry

Thus explain the deep hidden connection between geometry and algebra

Regular mappings

Explain polynomial mapping/regular mapping between varieties

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What sort of Commutative Algebra do we use?

What sort of commutative algebra machinery do we use: (Do not explain any of these. Point out where you use them instead)

- 1 Modules
- 2 Tensor products
- 3 Exact sequences
- 4 Direct Limits

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Zariski

Talk about the prime spectrum and the Zariski Topology what sort of machinery would that use?

Constructible Topology

You can have another topology called the Constructible Topology

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Presheaf and Sheaf

Definiton of a Presheaf and Sheaf

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Applications of Algebraic Geometry

Do you really want applications? You could mention in passing string theory, arithmetic geometry, proof of the Fermat's last theorem etc. . .

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Acknowledgement

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