Quasienergy and Shaken Optical lattice

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I. INTRODUCTION

Optical lattice is a clean and highly controllable platform in quantum field. It becomes one of the most popular platform to simulate quantum many-body system.[1–3]

In fact, optical lattice is a kind of artificial periodic structure. In this way, the optical lattice can simulate the crystal structure in condensed matter physics. Moreover, the properties of cold atoms in the optical lattice are very similar to those of electrons in the solid lattice, so the cold atoms in the optical lattice can be used to simulate the complex crystal model, such as Hubbard and spin models, disordered systems, topological order and quantum computation, fraction quantum Hall states and so on.[4–7]

The optical lattice system can be used to study the quantum phase transition as well. In 2002, Bloch and others observed the superfluid Mott phase transition of boson in the Bose Einstein condensate in three-dimensional optical lattice.[8–10]

The cold atoms also has great development potential in time measurement, because the cold atoms in the optical lattice are not easily disturbed by the outside world. [11, 12]

If we modulate the potential and make the potential is not only spatial period but also temporal period, the potential well will move left and right. We call that Shaken Optical Lattice.[13–15]

In shaken optical lattice system, we use the Floquet theroy to study the problem of time period. Floquet dynamics of a quantum system subject to periodic modulations of system parameters provide a powerful tool for engineering new quantum matter with exotic properties.[16–18]

One of the recent theoretical advances is the introduction of Floquet topological insulators. It turns shaken optical lattice into a new platform for simulating topological phases. [19–21]

In this paper, we study the properties and modulation methods of the shaken optical lattice system.

II. SHAKEN OPTICAL LATTICE

The optical lattice is produced by a pair of laser beams as Fig. 1. The potential function of one-dimensional optical lattice is as follows:

$$V = \frac{V_0}{2}\cos(2kx) \tag{1}$$

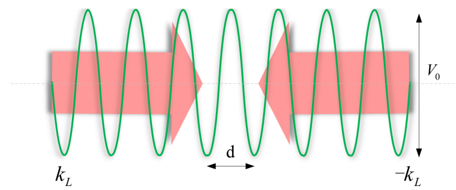

FIG. 1. The optical lattice is produced by a pair of laser beams.The two laser beams propagate in the opposite direction form a standing wave field. This force is periodic. The corresponding periodic optical potential well is obtained.

However, when the two lasers do not have exactly the same parameter(for example, there is a difference between two lasers on frequency or phase), the form of the potential function will change. The standing wave will move(or shaking), so the system needs to be rederived. We call the system shaken optical lattice. Shaken optical lattice can simulate quantum systems, its advantage is that the potential is spatiotemporal period. Many novel physical phenomena can be induced and deduced by spatiotemporal period in the quantum physics. Shaken optical lattice opens up new possibilities for studying in quantum matter.[22–25]

Processing the parameters of two laser beams is also called modulation. The potential function will also change:

$$V = \frac{V_0}{2}\cos\left(2kx + X_0(t)\right) \tag{2}$$

Now time-dependent term in the potential function which could affect the whole optical lattice system. Then the Hamiltonian of the system becomes:

$$H = \frac{p^2}{2M} + V_{lat}(x,t) \tag{3}$$

$V_{lat}(x,t)$ is not only a spatial period, but also temporal period, for the convenience of later calculation. We need to simplify the Eqs. 3. After simplification, we can get[26]:

$$H' = \frac{\left[\widetilde{\boldsymbol{p}} + M\dot{X}_0\right]^2}{2M} + V_{lat}(x) - \frac{1}{2}M\dot{X}_0^2 \tag{4}$$

Time dependent vector potential is induced into the system of shaken optical lattice according to the above equation Eqs. 4.

$$A = -M\dot{X}_0 \tag{5}$$

The vector potential can deduce a force field as the following:

$$\mathbf{F} = \frac{\partial \mathbf{A}}{\partial \mathrm{t}} = -\mathrm{M}\ddot{X}_0 \tag{6}$$

III. THE MODULATION

Different modulation methods will lead to different forms of potential functions[27, 28].

Here are some common modulation methods as follow.

A. Frequency Modulation

First, it is frequency modulation. When the frequency difference between two lasers is $\Delta v = \Delta v_{\max} \sin(\omega t)$. The actual potential field of the shaken optical lattice is

$$V_{\text{lat}} = \frac{V_0}{2} \cos\left(2k\left[x - \frac{\lambda \Delta v_{\max}}{2\omega} \cos(\omega t)\right]\right) \quad (7)$$

B. Phase Modulation

Phase modulation is another modulation method. The system consists of a standing wave formed by the interference of the reflected light of a laser beam reflected by a plane mirror with the initial incident light. When the plane mirror is periodically shifted, it will change the phase of the reflected light, so as to achieve the purpose of periodic movement of the optical lattice.

The displacement equation of plane mirror is set as $X(t) = \Delta x_{\max} \cos(\omega t)$, then the potential field in the optical lattice is

$$V_{lat} = \frac{V_0}{2} \cos\left(2k\left[x - \Delta x_{\max} \cos(\omega t)\right]\right) \quad (8)$$

IV. DYNAMIC LOCALIZATION

In the case of tight binding approximation of single particle model, the corresponding time-dependent Schrodinger equation is given by the so-called Houston State. These states give the time-dependent quasi momentum relations as follows

$$q(t) = -\frac{F_0}{\hbar\omega} \cos(\omega t) \quad (9)$$

According to the relation of the lowest energy level in the tight binding approximation, we can get

$$E^{(0)}(q) = -2J_0 \cos(dq) \quad (10)$$

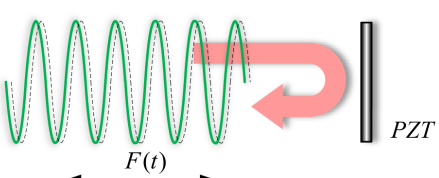

FIG. 2. Frequency modulation in thr shaken optical lattice. The frequency difference between the two laser beams results in the shaken optical lattice.

We can get the equivalent energy band relation under tight binding approximation by taking the above relation into and taking the time average in a period.

$$E_{eff}(q) = \frac{1}{T}\int_0^T dt E^{(0)}(q(t)) = -2J_{eff}\cos(dq) \quad (11)$$

Where J_{eff} is the equivalent transition matrix element, which is obtained by rescaling the normal transition matrix element through the zero order Bessel function in the Fig. 4

$$J_{eff} = J_0 \mathcal{J}_0\left(\frac{dF_0}{\hbar\omega}\right) = J_0 \mathcal{J}_0\left(K_0\right) \quad (12)$$

K_0 is a dimensionless parameter, which describes the intensity of modulation mode.

From the Eqs. 7, we can get the modulation intensity

$$K_0 = \frac{\mathrm{F}_0 d}{\hbar\omega} = \frac{M d^2 \omega \Delta v_{\max}}{\hbar\omega} = \frac{\pi^2}{2} \cdot \frac{\Delta v_{\max}}{\omega_{rec}} \quad (13)$$

where $\omega_{rec} = \frac{E_{rec}}{\hbar}$ is recoil frequency. From the above formula, we can see the relationship between frequency modulation mode and modulation intensity. The modulation intensity K_0 is directly proportional to the amplitude of frequency modulation and inversely proportional to the recoil frequency. It is not related to the frequency of modulation. Of course, the recoil frequency is related to the selected cold atoms. Once selected, it will not change. For example, ^{87}Rb its recoil frequency is $3.24k \times 2\pi Hz$. The relationship between modulation intensity and frequency modulation amplitude can be analyzed. As Fig. 5.

In this modulation mode, the modulation intensity is

$$K_0 = \frac{\mathrm{F}_0 d}{\hbar\omega} = \frac{M\omega^2 \omega \Delta x_{\max} d}{\hbar\omega} = \frac{\pi^2}{2} \frac{\omega}{\omega_{rec}} \frac{\Delta x_{\max}}{d} \quad (14)$$

The modulation intensity is related to modulation frequency, modulation amplitude, recoil frequency and lattice constant. Similarly, the recoil frequency and lattice constant are related to the constructed shaken optical lattice system. In this way, we analyze the relationship between modulation intensity and modulation frequency.

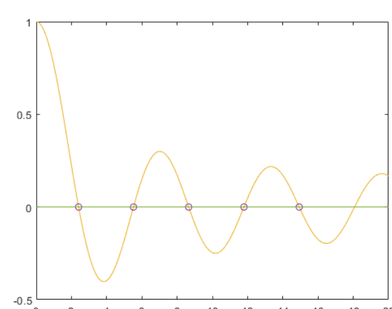

FIG. 3. The phase modulation. The phase difference between laser beam and its reflected wave emerge shaken optical lattice.

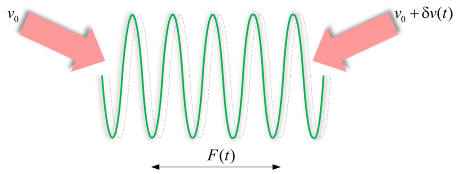

FIG. 4. Zero order Bessel function.The positions of zero points of Zero order Bessel function

[1] D. Jaksch and P. Zoller, The cold atom hubbard toolbox, Annals of Physics (New York) **315**, 52 (2005)
.

[2] I. Bloch, Ultracold quantum gases in optical lattices, Nature physics **1**, 23 (2005)
.

[3] M. Lewenstein, A. Sanpera, V. Ahufinger, B. Damski, A. Sen, and U. Sen, Ultracold atomic gases in optical lattices: mimicking condensed matter physics and beyond, Advances in Physics **56**, 243 (2007)
.

[4] O. Morsch and M. Oberthaler, Dynamics of bose-einstein condensates in optical lattices, Reviews of modern physics **78**, 179 (2006)
.

[5] B. Damski, J. Zakrzewski, L. Santos, P. Zoller, and M. Lewenstein, Atomic bose and anderson glasses in optical lattices, Physical review letters **91**, 080403 (2003)
.

[6] A. Micheli, G. Brennen, and P. Zoller, A toolbox for lattice-spin models with polar molecules, Nature Physics **2**, 341 (2006)
.

[7] N. Wilkin and J. Gunn, Condensation of "composite bosons" in a rotating bec, Physical review letters **84**, 6 (2000)
.

[8] D. Jaksch, C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller, Cold bosonic atoms in optical lattices, Physical Review Letters **81**, 3108 (1998)
.

[9] M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Quantum phase transition from a superfluid to a mott insulator in a gas of ultracold atoms, Nature **415**, 39 (2002)
.

[10] R. Jördens, N. Strohmaier, K. Günter, H. Moritz, and T. Esslinger, A mott insulator of fermionic atoms in an optical lattice, Nature **455**, 204 (2008)
.

[11] M. Takamoto, F.-L. Hong, R. Higashi, and H. Katori, An optical lattice clock, Nature **435**, 321 (2005)
.

[12] B. Bloom, T. Nicholson, J. Williams, S. Campbell, M. Bishof, X. Zhang, W. Zhang, S. Bromley, and J. Ye, An optical lattice clock with accuracy and stability at the 10- 18 level, Nature **506**, 71 (2014)
.

[13] A. Eckardt, C. Weiss, and M. Holthaus, Superfluid-insulator transition in a periodically driven optical lattice, Physical review letters **95**, 260404 (2005)
.

[14] L.-C. Ha, L. W. Clark, C. V. Parker, B. M. Anderson, and C. Chin, Roton-maxon excitation spectrum of bose condensates in a shaken optical lattice, Physical review letters **114**, 055301 (2015)
.

[15] A. Keleş, E. Zhao, and W. V. Liu, Effective theory of interacting fermions in shaken square optical lattices, Physical Review A **95**, 063619 (2017)
.

[16] A. Eckardt, M. Holthaus, H. Lignier, A. Zenesini, D. Ciampini, O. Morsch, and E. Arimondo, Exploring dynamic localization with a bose-einstein condensate, Physical Review A **79**, 013611 (2009)
.

[17] W. Zheng and H. Zhai, Floquet topological states in shaking optical lattices, Physical Review A **89**, 061603 (2014)
.

[18] X.-W. Luo and C. Zhang, Self-adapted floquet dynamics of ultracold bosons in a cavity, Physical review letters **120**, 263202 (2018)
.

[19] F. Mei, J.-B. You, D.-W. Zhang, X. Yang, R. Fazio, S.-L. Zhu, and L. C. Kwek, Topological insulator and particle pumping in a one-dimensional shaken optical lattice, Physical Review A **90**, 063638 (2014)
.

[20] A. G. Grushin, A. Gómez-León, and T. Neupert, Floquet

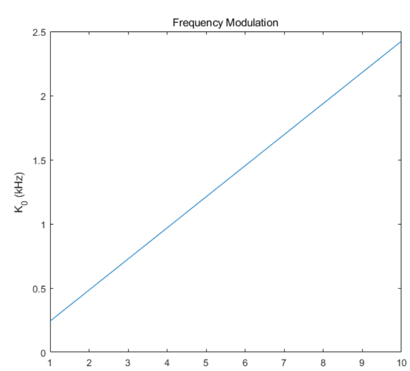

FIG. 5. The relationship between frequency modulation amplitude and modulation intensity

fractional chern insulators, Phys. Rev. Lett. **112**, 156801 (2014)
.

[21] B. Béri and N. R. Cooper, $\mathbb{Z}_2$ topological insulators in ultracold atomic gases, Phys. Rev. Lett. **107**, 145301 (2011)
.

[22] L. W. Clark, B. M. Anderson, L. Feng, A. Gaj, K. Levin, and C. Chin, Observation of density-dependent gauge fields in a bose-einstein condensate based on micromotion control in a shaken two-dimensional lattice, Physical review letters **121**, 030402 (2018)
.

[23] C. Weidner and D. Z. Anderson, Experimental demonstration of shaken-lattice interferometry, Physical review letters **120**, 263201 (2018)
.

[24] S.-L. Zhang and Q. Zhou, Manipulating novel quantum phenomena using synthetic gauge fields, Journal of Physics B: Atomic, Molecular and Optical Physics **50**, 222001 (2017)
.

[25] M. Di Liberto, O. Tieleman, V. Branchina, and C. M. Smith, Finite-momentum bose-einstein condensates in shaken two-dimensional square optical lattices, Physical Review A **84**, 013607 (2011)
.

[26] E. Arimondo, D. Ciampini, A. Eckardt, M. Holthaus, and O. Morsch, Kilohertz-driven bose–einstein condensates in optical lattices, in *Advances in Atomic, Molecular, and Optical Physics*, Vol. 61 (Elsevier, 2012) pp. 515–547
.

[27] C. Sias, H. Lignier, Y. Singh, A. Zenesini, D. Ciampini, O. Morsch, and E. Arimondo, Observation of photon-assisted tunneling in optical lattices, Physical review letters **100**, 040404 (2008)
.

[28] A. Zenesini, H. Lignier, C. Sias, O. Morsch, D. Ciampini, and E. Arimondo, Tunneling control and localization for bose-einstein condensates in a frequency modulated optical lattice, Laser physics **20**, 1182 (2010).