THE VELOCITY OF LIGHT IN THE MAGNETIC FIELD.

By Henry T. Eddy, Edward W. Morley, and Dayton C. Miller.

Part I.

Upon the Electromagnetic Theory of the Rotation of the Plane of Polarization in a Magnetic Field.

By Henry T. Eddy.

IN a note upon this subject read before Section B at the Toronto meeting of the A. A. A. S., August, 1889,[1] the present writer developed the essential points in the present paper with the exception of the numerical estimate of the amount of the suspected effect.

At the same meeting[2] Professor Morley pointed out a method (See Part II) by which it might be possible to demonstrate experimentally the existence of the suspected effect, which consisted in a decrease in the velocity of the light passing through any magnetic field in a medium causing rotation of the plane of polarization.

An appropriation was generously granted from the funds of the A. A. A. S. to construct apparatus in accordance with the proposal of Professor Morley for the purpose of trying to discover, if possible, whether such an effect as theory indicates was, in fact, observable.

The report finally presented to the A. A. A. S. at their Boston meeting, August, 1898, shows that no such effect was observable with the apparatus employed. The numerical estimate at the end of this paper makes it evident that the effect to be looked for was only two one-hundred-millionths of a wave-length, while the smallest effect capable of observation was five one-hundredths of a wave-length. Hence the effect to be looked for was only four one-millionths of the smallest effect capable of observation with the apparatus. This sufficiently accounts for the failure to observe any

[1] Proc. A. A. A. S., 1889, Vol. 38, p. 129. [2] Ibid., p. 130.

such effect. But it was not known at the time of observation what might be the possible magnitude of the effect.

The effect is, therefore, one which would, in the field employed in this apparatus, amount to about one one-thousandth of an Ångström unit with sodium light.

It is the hope of the writer that the effect in question may not prove to be finally beyond the reach of experimental demonstration by spectroscopic methods, which, as stated in the note above referred to, seem, perhaps, the most promising for rendering the effect observable.

The differential equations which were proposed by Professor Rowland[1] to express the propagation of plane polarized light as an electro-magnetic disturbance in a magnetic non-conducting field in the direction of the lines of force taken as parallel to the axis of z, are :

$$\left.\begin{aligned} K\mu\left(\frac{d^2F}{dt^2} - \frac{c\gamma}{4\pi\mu}\cdot\frac{d^3G}{dtdz^2}\right) &= \frac{d^2F}{dz^2} \\ K\mu\left(\frac{d^2G}{dt^2} + \frac{c\gamma}{4\pi\mu}\cdot\frac{d^3F}{dtdz^2}\right) &= \frac{d^2G}{dz^2} \end{aligned}\right\} \qquad (1)$$

in which F and G express the electro-magnetic momenta in the directions of x and y respectively, and they are so named because they are defined, in case $c = 0$, with respect to the actual electromotive forces P and Q per unit of length in the directions of x and y by the equations :

$$dF / dt = -P \quad \text{and} \quad dG / dt = -Q,$$

and these equations are analogous to the mechanical equations for forces and momenta.

K is the specific inductive capacity of the medium, μ is its magnetic permeability, $\mu\gamma$ is the magnetic induction parallel to the axis of z, and c is the coefficient of proportionality for the Hall effect. There is no electromotive force along z, and there is no magnetic induction except along z.

In case $c = 0$, eqs. (1) reduce to those originally proposed[2] by Maxwell.

[1] Am. Jour. Math., 1880, vol. 3, p. 109.

[2] Maxwell's Elect. and Mag., Vol. 2, 3d ed., p. 439.

Rowland says that we may well suppose one solution of eqs. (1) to be:

$$\left.\begin{aligned} F &= r\cos(nt - qz)\cos mt \\ G &= r\cos(nt - qz)\sin mt \end{aligned}\right\} \tag{2}$$

in which $\cos(nt - qz)$ is the factor on which depends the transverse vibration of the electromagnetic disturbance which is propagated with a velocity v, a wave-length λ, and a period of oscillation T, such that

$$v = n/q, \quad \lambda = 2\pi/q, \quad T = 2\pi/n. \tag{3}$$

The periodic factors $\cos mt$, and $\sin mt$ are those which would express a continuously progressing rotation of the plane of polarization directly proportional to the time but independent of space. So that as time passes the rotation continues without cessation though space be arbitrarily assumed to be constant.

While it might be possible to arrange an apparatus for experiment in which such a continuously progressive rotation of the plane of polarization could be imparted mechanically to that plane at the place where the light enters the medium, nevertheless it is evident that that is not what is intended to be treated by eqs. (2).

The writer would therefore propose instead, the following solutions as applicable to the case in hand, viz., the case of a stationary plane of polarization which is twisted in its passage through the medium in the magnetic field. It is this stationary twist which is described by the word "rotation" used in this paper. The eqs. proposed are:

$$\left.\begin{aligned} F &= r\cos(nt - qz)\cos pz \\ G &= r\cos(nt - qz)\sin pz \end{aligned}\right\} \tag{4}$$

in which the factors $\cos pz$ and $\sin pz$ express a continuously progressive twisting of the plane of the polarization increasing uniformly with the space traversed by the ray. If the angle of twist be θ, and the distance in which the plane makes one complete turn be σ, then

$$\theta = pz, \quad \sigma = 2\pi/p = 2\pi z/\theta, \quad \text{and } \sigma/\lambda = q/p. \tag{5}$$

In order to obtain the equations of condition which must exist between the constants n, q and p in order that eqs. (4) may be solutions of eqs. (1), let the values of F and G be substituted in eqs. (1). The result may then be written in the form:

$$\left. \begin{array}{l} A \cos(nt - qz) \cos pz + B \sin(nt - qz) \sin pz = 0 \\ A \cos(nt - qz) \sin pz - B \sin(nt - qz) \cos pz = 0 \end{array} \right\} \qquad (6)$$

in which the values of A and B are those given in eqs. (7).

Eqs. (6) are identically equal to zero for all values of t and z, and eqs. (4) are consequently solutions of eqs. (1), when and only when $A = 0$ and $B = 0$. Hence, writing the values of A and B in full and equating these values to zero we have:

$$\left. \begin{array}{l} q^2 + p^2 - K\mu n^2 - Knqp \dfrac{c\gamma}{2\pi} = 0 \\ 2qp - Kn(q^2 + p^2) \dfrac{c\gamma}{4\pi} = 0 \end{array} \right\} . \qquad (7)$$

By taking the sum and difference of eqs. (7):

$$\left. \begin{array}{l} K\mu n^2 = (q - p)^2 (1 + ne) \\ K\mu n^2 = (q + p)^2 (1 - ne) \end{array} \right\} \qquad (8)$$

in which eqs. for brevity,

$$e = K \frac{c\gamma}{4\pi} . \qquad (9)$$

In eq. (9), since c is small, e is small also.

Now, in eq. (5) σ is very large, compared with λ, hence q is very large, compared with p. The positive sign must, therefore, be taken in extracting the square roots of eqs. (8); hence by the binomial theorem

$$\left. \begin{array}{l} n\sqrt{K\mu} = (q - p)(1 + ne - \tfrac{1}{2} n^2 e^2 + \text{etc.}) \\ n\sqrt{K\mu} = (q + p)(1 - ne - \tfrac{1}{2} n^2 e^2 - \text{etc.}) \end{array} \right\} . \qquad (10)$$

By taking the sum and difference of eqs. (10) we have approximately

$$\left. \begin{array}{r} n\sqrt{K\mu} = q(1 - \tfrac{1}{2} n^2 e^2) - pne \\ 0 = qne - p(1 - \tfrac{1}{2} n^2 e^2) \end{array} \right\} . \qquad (11)$$

From the last of eqs. (11) $p = qne$ nearly, and this substituted in the first of eqs. (11) gives:

$$n\sqrt{K\mu} = q\left(1 - \frac{3}{2} \frac{p^2}{q^2}\right) . \qquad (12)$$

If $v_0 = 1 / \sqrt{K\mu}$ be the velocity of light in the medium when the magnetic field vanishes, we have by eqs. (3) and (5):

$$v = v_0 \left(1 - \tfrac{3}{2} \frac{\lambda^2}{\sigma^2} \right) \tag{13}$$

$$\frac{\delta v_0}{v_0} = \frac{v_0 - v}{v_0} = \tfrac{3}{2} \frac{\lambda^2}{\sigma^2}. \tag{14}$$

If z_0 be the length of path in the medium occupied by a certain number of wave-lengths when the field vanishes, and $z_0 - \delta z_0$ be the length of path occupied by the same number of wave-lengths when the field has diminished the velocity,

then,
$$\frac{\delta v_0}{v_0} = \frac{\delta z_0}{z_0} = \tfrac{3}{2} \frac{\lambda^2}{\sigma^2} \tag{15}$$

and
$$\frac{\delta z_0}{\lambda} = \tfrac{3}{2} \frac{\lambda z_0}{\sigma^2}. \tag{16}$$

Eq. (16) expresses the fraction of a wave-length by which light is retarded in traversing a length of path z_0 through the field in the medium.

Eqs. (13) to (16) state that the assumption of eq. (4) as the solution of eqs. (1) leads to the result that the velocity of light is less while the plane of polarization is being twisted in a medium by a magnetic field than when the magnetic field vanishes and the plane undergoes no twisting.

In case we take the wave-length of sodium light as approximately $\lambda = 6 \times 10^{-5}$ cm., and let $z_0 = 120$ cm., and $\sigma = 240$ cm., as they were in the apparatus employed, we find from eq. (15).

$$\delta z_0 = 11 \times 10^{-12} \text{ cm.}$$

or approximately one one-thousandth of an Ångström unit, which last is 10^{-8} cm.

$$\therefore \quad \delta z_0 / \lambda = 2 \times 10^{-7} \text{ nearly,}$$

or two one-hundred-millionths of a wave-length of sodium light is the total amount of the retardation of sodium light during its passage through the apparatus with which the experiment was made:

and
$$\therefore \quad \delta v_0 / v_0 = \delta z_0 / z_0 = 10^{-13} \quad \text{nearly,}$$

or the retardation to be looked for in the field employed is one part in one hundred thousand times one hundred million.

Professor Rowland has shown in his discussion of eqs. (2) that they involve an increase in the velocity of light instead of a decrease such as follows from eqs. (4); and in the further discussion of the matter he has assumed that he is at liberty to write the equation $\theta = mz/v$, whereas in his equations $\theta = mt$ is the only value of θ which is admissible, since z and t are independent. It would seem, therefore, that so much of his discussion as depends upon this assumed value of θ may not be valid.

MINNEAPOLIS, OCTOBER, 1898.

PART II.

EXPERIMENTS ON THE VELOCITY OF LIGHT IN A MAGNETIC FIELD.[1]

BY EDWARD W. MORLEY AND DAYTON C. MILLER.

At the Toronto meeting of the American Association for the Advancement of Science in 1889, Professor Henry T. Eddy (see Part I), discussed the partial differential equations of the motion of plane polarized light in a magnetic field. These equations contain terms "expressing the transverse electromotive force due to the Hall effect. The particular solution of these equations which Rowland proposes contains a periodic factor dependent upon the time." In Professor Eddy's paper, "a different particular solution, containing a periodic factor dependent upon the space which the ray traverses in the field, is discussed at length and compared with the solution proposed by Rowland." It is shown that the velocity of the ray would be increased if there is a periodic factor dependent upon the time, but would be decreased by a factor dependent upon the space.

In the discussion which followed the reading of Professor Eddy's paper, one of the present writers suggested a form of apparatus which would detect the suspected change of velocity, provided it amounted to one part in fifty millions. The whole matter was of

[1] Report of an experiment made with the aid of a grant from the research fund of the American Association for the Advancement of Science.

such interest that the section of the Association before which the paper was read, obtained, from the research funds of the Association, a grant of money with which to construct the apparatus and make the experiment. The apparatus was finished in the summer of 1890, and Professor Eddy came to Cleveland to take part in the experiment.

The optical part of the apparatus consists of the interferential refractometer used by Michelson in his experiments to determine

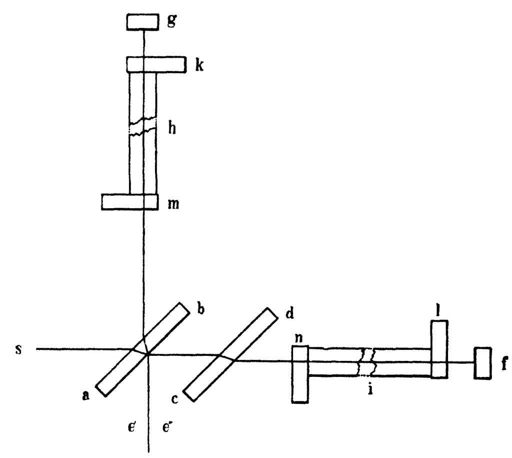


Fig. 1. Diagram, showing arrangement of optical parts.

whether light moves with the same velocity in different directions in the solar system. A train of waves of light from the source s, (Fig. 1) is divided at the second surface of ab, which is coated with silver of such thickness that the division is nearly equal. If the mirrors, f and g, are properly adjusted, the two parts reunite at ab, and interference bands can be seen by the eye at e, if the distances of f and g from ab are properly related. If, now, any force acts intermittently to change the velocity of one part of the ray while the paths are separated it may be detected by a motion of the interference bands, provided the change is not too small.

If we put two tubes, h and i, in the paths of the divided ray, make these of very nearly the same length, close them with plane parallel glasses of equal thickness, and fill the tubes with carbon bisulphide, we shall still obtain interference bands. If we surround each tube with a coil of copper wire, use polarized light, and close the circuit through one coil, there will be a rotation of the plane of polarization of the ray which passes through the magnetic field. If the rotation be not so much as to prevent, or too much affect, the interference of the parts of the divided train of waves, a change of velocity of light while in the magnetic field will produce a displacement of the interference bands. If we complete the circuit in the two coils alternately, we double the displacement which is sought to detect.

If any displacement is observed, it will be necessary to determine whether it may not be due to strain or displacement of parts of the apparatus. In the case of the plates *ab*, *cd*, *f* and *g*, this may be accomplished by watching the effect of closing alternately the circuits through the two coils while tubes *i* and *h* contain only air. But a disturbance which should affect the position or length of one of these tubes needs special means for its detection. The plates which close the ends of these tubes are accordingly placed as shown at *k*, *l*, *m* and *n*, and are silvered except where they cover the tubes. When proper adjustments are made, the observer can see interference bands through the tube by placing the eye at *e*, interference bands at the covers of the nearer end of the tubes by placing the eye at e', and interferences at the covers of the farther ends of the tubes by placing the eye at e''. Since the interferences produced in these three cases are due to rays which have passed through only glass and air, they can be so adjusted as to detect with ease and certainty a displacement which would be entirely insensible in the irregular and most unstable interference bands produced after a ray of light has passed through a liquid so optically unstable as carbon bisulphide. Our means of detecting the effect of mechanical disturbances were therefore greatly superior to those of detecting the change of velocity in question, and the statement that no displacement of interference bands was caused by mechanical disturbance due to the opening and closing of the current through the coils is open to no doubt.

The plates *ab* and *cd* were 9.3 by 2.0 cm.; *f* and *g* were 2.0 cm. by 2.0 cm.; *k*, *l*, *m* and *n* were 4.3 cm. by 2.0 cm. The tubes *h* and *i* were 1.7 cm. in inside diameter, and were 38.1 cm. long. They lay in adjustable supports, and were not in contact with the coils which surrounded them. The core around which the wire of the coils was wound was rectangular in section. Around a free space of 4.2 cm. by 2.2 cm. was first a nonconducting layer of 0.6 cm. in thickness, then a space 0.6 cm. thick for the circulation of water; and, lastly, a coil of wire. Each coil was made of two wires laid side by side, so that when the same current was sent through the two wires in opposite directions, its effect was null. A commutator was so arranged as to send a constant current through one wire of each coil, while the same current was so sent through the other wires of the two coils as to neutralize the effect of the current in the first wire of one coil, and to double it in the other coil. The heating due to the passage of the current was therefore identical in the two coils. The wire of each coil weighed about 82 kilogrammes, and the length covered by the wire was 30 cm.

The flame of a Bunsen lamp was colored by the introduction of some sodium compound as the usual source of light, because the interference bands can, in homogeneous light, more easily be given forms which permit delicate discrimination; a luminous gas flame and the electric arc were also employed. A Nicol prism was sometimes interposed, and the plane of polarization was placed in various positions. The optical parts of the apparatus were so related that the visible interference bands coincided with the surface of the more distant mirrors; their displacement can then be detected by referring their position to lines ruled on the mirrors.

With this apparatus, one observer adjusted the interference bands in width and position; when these were made suitable, the second observer used the commutator to produce a magnetic field, first in one coil and then in the other. He announced the change from one to the other in such a way that the first observer did not know which coil was acting, so that the effect of any possible bias might be excluded.

With this apparatus, both observers suspected a slight displacement of the interference bands, and both satisfied themselves that this displacement was in the direction which indicated acceleration in the light passing through the magnetic field. The amount of the acceleration thus suspected was not more than one part in one hundred millions. But the displacement was so slight that they could not be confident of its existence.

In the examination of the effect of a magnetic field on a ray passing through it, the appearance on making and breaking the circuit suggested the possibility of a change in the density of carbon bisulphide while in the field. There was produced a momentary displacement of the interference bands, which was unmistakable, being many times as great as the displacement which it was suspected continued as long as the current passed. It was easy to imagine that the force which quickly brought the molecules of the liquid into the new positions in which they produce rotation of the plane of polarization, might carry them beyond the position of equilibrium, to which they would quickly return. To examine this phenomenon, a thick brass box filled with carbon bisulphide was placedin each coil, and the two boxes were connected with each other and with a glass tube of small diameter. After all parts of the apparatus had come to a uniform temperature, the level of the liquid was brought to a convenient part of the glass tube, and a current of twenty-seven ampères was sent through the two coils, in such direction as to produce a magnetic field in both coils. Each time the circuit was completed, there was a momentary rise of the liquid by 0.06 mm., and an instantaneous fall to 0.03 mm. On breaking the circuit, there was a momentary fall to −0.03 mm., and an instantaneous rise to 0.00 mm. This displacement could be well measured three or four times by means of a reading micrometer with an amplification of about sixty diameters, the probable error of a reading being less than 0.005 mm. After three or four measurements, the heat developed in the coils made it necessary to wait many hours for equilibrium of temperature to be established again. There were 380 cubic centimeters of carbon bisulphide in the magnetic field; the area of the capillary tube was 0.55 square millimeters; the lasting change of density or the change in the volume of the containing envelope was therefore one part in twenty-three million. The intensity of the field was 1,650 centimeter-gramme-second units.

This matter was afterward investigated with another brass box made to fit loosely between the poles of a dynamo-electric machine, whose coils had been so connected that the box was placed between a north and south magnetic pole. The material of this box was free from magnetic admixture, and was supported so as not to be in contact with any part of the dynamo. In this envelope, we have not yet been able to detect any such change of density or volume as was suspected before. The cause of the momentary displacement of the interference bands which was so obvious will therefore need further examination.

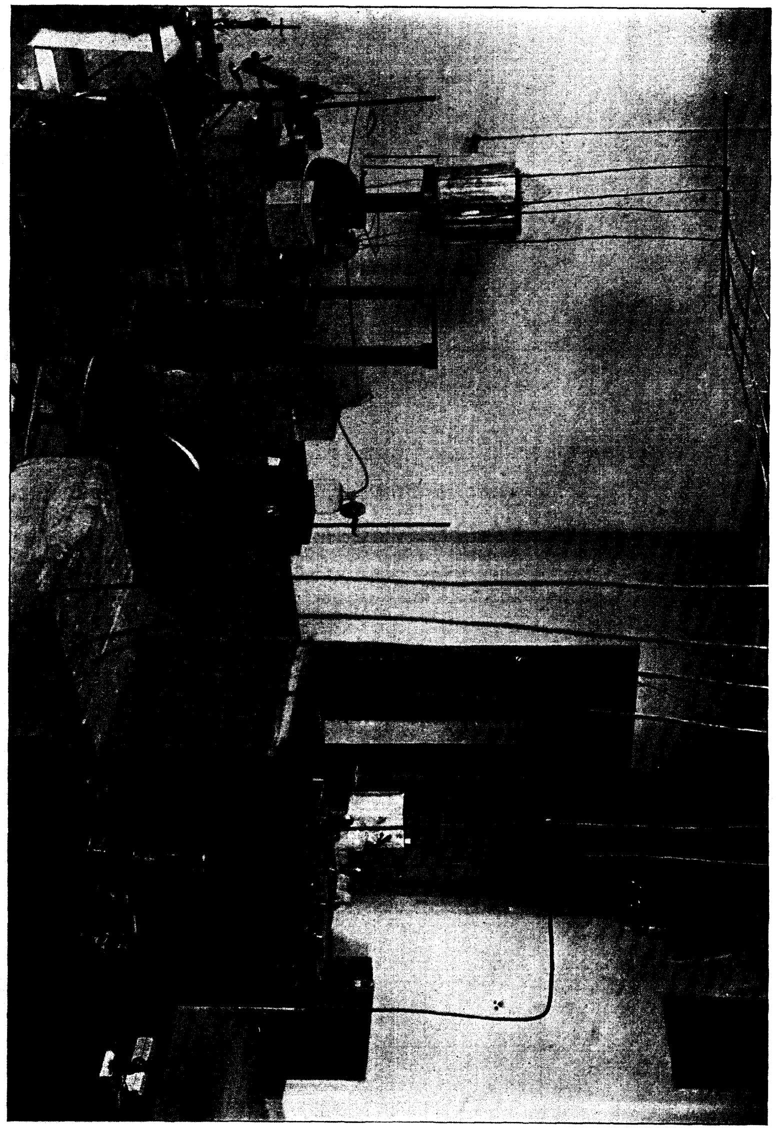
At the meeting of the American Association for the Advancement of Science in Indianapolis, in 1890, Professors Morley and Eddy made a report of the result of their experiment. The matter seemed of such interest to the section that it obtained a further grant with which to continue the experiment with more powerful apparatus. Owing to other occupation, the construction of this apparatus has been delayed till the present year. With the assistance of Professor Dayton C. Miller, now associated with Professors Eddy and Morley, a new apparatus has been set up with which the acceleration or retardation produced would be three times as great as with the former apparatus. Further provision has been made for securing much greater thermal and optical stability of the carbon bisulphide used in the apparatus. As a result of these two modifications, our power of detecting small changes of velocity of light in a magnetic field is probably five times as great as in the case of the former apparatus.

The second apparatus differs from the former in two respects. The optical parts remain as before; the only difference in their use depends on the fact that the coils are now twice as long as before, so that the mirrors f and g are farther from the mirror ab. The tubes i and h are therefore 30 cm. longer and the column of carbon bisulphide in the magnetic field is twice as long as at first. Further, these tubes are so connected that a current of bisulphide can be passed through them at pleasure; and around them are concentric tubes through which passes a current of water, intended to prevent the heating of the bisulphide by the passage of the electric current through the coils. A tank, placed in the room with the apparatus, supplies water at constant temperature, for this purpose; in this tank is immersed another tank, which supplies a current of bisulphide having the same temperature as the water. This arrangement for preventing changes of temperature in the column of carbon bisulphide in the tubes of the apparatus greatly increases the steadiness of the interference bands to be scrutinized, and so increases the delicacy of the observations.

The illustration taken from a photograph shows the apparatus ready for use. At the extreme right are seen the commutator, amperemeter, and resistance coils, used in managing the electric current. The wooden stand at the extreme left carries the source of light and the condensing lens. The adjacent stone pier carries the coils; between them is seen the cubical block of stone which supports the diagonally placed mirrors *ab* and *cd* of Fig. 1. Apparently just above this block, but really some yards to the rear, is a double tank supplying water and carbon bisulphide to the apparatus. The reading telescope with which the observations were made is marked by the hanging cloth. On the left edge of the pier is seen an iron stand carrying a Nicol's prism for polarizing the ray of light sent through the apparatus; on the wooden stand to the right of the pier is seen the analyzer, by means of which the rotation produced was measured while the current was adjusted so as to secure the rotation desired.

When a current of twenty-seven ampères is sent through the coils of this apparatus, there is produced a rotation of the plane of polarization of light by one-half a circumference; in the previous apparatus, we could utilize a rotation no larger than about fifty degrees of arc. The rotation utilized now is therefore three and one-half times as great as that in the former case, involving a corresponding increase in the change of velocity which it is hoped to detect.

With this apparatus, a current was sent through the coils at the call of the observer at the telescope. The current had been adjusted so that it should produce a rotation by half a circumference of the plane of polarization of the light employed at the time. This rotation caused the interference bands, which had been seen before the passage of the current, to shift their position by half a wave-length; this was due to the reversal of phase of the ray which had



suffered rotation. A micrometer wire was set on a maximum or a minimum, the current was reversed, and the coincidence of the wire with the maximum or the minimum was scrutinized. As a result of such scrutiny, often repeated, both observers are confident that there was no displacement amounting to one-twentieth of a wave-length. They also are of opinion that there is no advantage to be expected from an increase in the size of the apparatus, and a repetition of the experiment with carbon bisulphide, unless it shall be kept thermally and optically homogeneous by rapid motion. If dense glass, sufficiently transparent in a long cylinder, could be used in place of the bisulphide, it would probably be possible to detect a quantity which, if a liquid is employed, could not be detected except at an expense which would be prohibitory.

The result reported at the Boston meeting of the American Association is, that we are confident that when light corresponding to the solar D line is passed through one hundred and twenty centimeters of carbon bisulphide in a magnetic field which produces rotation by half a circumference in the plane of its polarization, there is no such change of velocity as one part in sixty million, and probably no such change as one part in a hundred million.