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COMP6463

Assignment "Automated Reasoning in First-Order Logic"

Due date: November 23, 2011

Marking scheme: Full marks for a formulation that correctly answers the question and clearly shows the steps to obtain the solution.

Solutions to be submitted electronically by Email to `Peter.Baumgartner@nicta.com.au`. Scans of handwritten solutions are of course acceptable.

Question 1 (2 + 2 pts). True or false? Prove or find a counterexample:

1. For all propositional logic formulas F and G and suitable assignments $\mathcal{A}$, if $(F \rightarrow G)$ is valid and $\mathcal{A} \not\models G$ then $\neg F$ is satisfiable.
2. For all propositional logic formulas F and G, if $(F \rightarrow G)$ is satisfiable and F is satisfiable then G is satisfiable.

Question 2 (6 pts). Is the following propositional clause set M satisfiable? Justify your answer by a proof. (*Hint:* inductive proof, compactness.)

$$M = \{A_1 \lor A_2,\ \neg A_2 \lor \neg A_3,\ A_3 \lor A_4,\ \neg A_4 \lor \neg A_5, \ldots\}\ .$$

Question 3 (10 pts). In class, the proof of Theorem 30 (completeness of propositional Resolution) was sketched by means of an example. Carry out the proof in its generality.

Question 4 (4 pts). In a criminal case the following facts have been shown to hold true:

1. At least one of the persons X,Y,Z is guilty.
2. If X is guilty and Y is not guilty, then Z is guilty.
3. If Y is guilty then Z is guilty.

Use propositional resolution to prove that one of X, Y, Z is guilty (who?). What can be said about the others?

Question 5 (3 + 3 pts). Convert these formulas to clause normal form:

1. $(((A \rightarrow B) \lor C) \rightarrow ((A \leftrightarrow B) \land C))$
2. $\forall x\ ((\forall x\ P(x)) \rightarrow \exists y\ Q(x, y))$

Question 6 (3 + 3 pts). Apply the unification algorithm presented in class to these sets of equations and read off the result, i.e., either FAIL or the unifier (a is a constant, x and y are variables):

1. $U = \{x = y,\ f(f(x)) = f(y)\}$

2. $U = \{a = x,\ f(x, z) = y,\ f(z, x) = y\}$

Question 7 (4 pts). Give a resolution refutation of the clause set

$$M = \{P(x) \vee P(y),\ Q(x, f(x)) \vee \neg P(x),\ \neg Q(g(y), z)\}\ .$$

Question 8 (5 + 5 pts). Given the following facts.

(i) Every barber shaves all persons who do not shave themselves.

(ii) No barber shaves a person who shaves himself.

(iii) There is no barber.

1. Formalize (i), (ii) and (iii) in first-order logic. Use $B(x)$ for "x is a barber" and $S(x, y)$ for "x shaves y".

2. Use the automated theorem prover "Otter" to prove that (iii) follows from (i) and (ii).

   Otter is available from `http://www.cs.unm.edu/~mccune/otter/` . Otter is usually very easy to install and comes with a good manual. Looking at the example problem `examples/auto/steam.in` should give you enough clues to get started. Remember that otter reads its input from standard input. This means otter needs to be invoked as "`otter < myproblem.in`", not "`otter myproblem.in`".