

Due date: August 29, 2012, 5pm.

Marking scheme: Full marks for a formulation that correctly answers the question and clearly shows the steps to obtain the solution.

Solutions to be submitted electronically by Email to Peter.Baumgartner@nicta.com.au, or on paper to a lecturer of this course. Neatly hand-written solutions are of course acceptable.

Question 1 (Semantics of FOL, 3 pts). Let F and G be first-order logic formulas. Prove that $F \models G$ holds if and only if $F \wedge \neg G$ is unsatisfiable.

Question 2 (Clause Normal Form, 4 pts). Transform the formula $\forall x \exists y (R(x, y) \rightarrow \exists y Q(x, y))$ into clause normal form. All intermediate formulas (Prenex normal form, Skolemized form, CNF) must be given explicitly.

Question 3 (Herbrand Interpretation, 4 pts). Consider the signature $\Sigma = (\{a/0, f/1\}, \{P/1\})$. Find a Herbrand interpretation I such that $I \models \{\neg P(x) \vee P(f(x)), P(x) \vee \neg P(f(x))\}$.

Question 4 (Resolution Model Construction, 4+4 pts). Suppose $A_2 \succ A_1 \succ A_0$. Let N be the clause set consisting of the following clauses:

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|------------------------------|---------------------------------------|
| (1) $\neg A_2 \vee A_1$ | (3) $A_2 \vee \neg A_1 \vee \neg A_1$ |
| (2) $\neg A_2 \vee \neg A_2$ | (4) $A_1 \vee A_0$ |

- (i) List the clauses in N in ascending order (using the clause ordering induced by \succ).
(ii) Compute the candidate interpretation I_N^\succ . Does $I_N^\succ \models N$ hold?

Question 5 (Unification, 3 + 3 pts). Apply the unification algorithm presented in class to these sets of equations and read off the result, i.e., either \perp or the unifier (a is a constant, x and y are variables):

1. $U = \{x = y, g(g(x)) = g(y)\}$
2. $U = \{a = x, f(x, z) = y, f(z, x) = y\}$

Question 6 (5 pts). Find a Resolution refutation of the following clause set. As for the mgus used, it suffices to only state them, you do not need to show the details of the runs of the unification algorithm.

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| (1) $\neg P(x) \vee Q(f(x)) \vee Q(y)$ | (3) $P(a)$ |
| (2) $\neg Q(x) \vee R(x)$ | (4) $\neg R(f(a))$ |