CS 225

Symbols and Formulas

Fall 2020

Logic

Symbols: $\geq$ $\leq$ $\neq$ $\neg$ $\sim$ $\wedge$ $\vee$ $\oplus$ $\equiv$ $\rightarrow$ $\leftrightarrow$ $\Box$ $\exists$ $\forall$

Identities:

$\sim(\sim p) \equiv p$	Double Negation
$p \wedge \mathbb{T} \equiv p \quad p \vee \mathbb{F} \equiv p$	Identity
$p \vee \mathbb{T} \equiv \mathbb{T} \quad p \wedge \mathbb{F} \equiv \mathbb{F}$	Domination
$p \wedge p \equiv p \quad p \vee p \equiv p$	Idempotent
$p \vee \sim p \equiv \mathbb{T} \quad p \wedge \sim p \equiv \mathbb{F}$	Negation
$p \vee q \equiv q \vee p \quad p \wedge q \equiv q \wedge p$	Commutative
$(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive
$\sim(p \wedge q) \equiv \sim p \vee \sim q$	DeMorgan's
$\sim(p \vee q) \equiv \sim p \wedge \sim q$	DeMorgan's
$p \vee (p \wedge q) \equiv p$	Absorption
$p \wedge (p \vee q) \equiv p$	Absorption
$p \rightarrow q \equiv \sim q \rightarrow \sim p$	Contrapositive
$p \oplus q \equiv q \oplus p$	Contrapositive
$p \rightarrow q \equiv \sim p \vee q$	Implication
$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	Biconditional Equivalence
$(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$	Exporation
$(p \rightarrow q) \wedge (p \rightarrow \sim q) \equiv \sim p$	Absurdity
$p \vee q \equiv \sim p \rightarrow q$	Alternate Implication
$p \wedge q \equiv \sim(p \rightarrow \sim q)$	Alternate Implication
$\sim(p \rightarrow q) \equiv p \wedge \sim q$	Alternate Implication
$\sim\Big(\forall\ x\ P(x)\Big) \equiv\ \exists\ x\ \sim P(x)$	DeMorgan's for Quantifiers
$\sim\Big(\exists\ x\ Q(x)\Big) \equiv\ \forall\ x\ \sim Q(x)$	DeMorgan's for Quantifiers

A conditional statement $p \rightarrow q$ can also be read as:

- If p then q
- p implies q
- If p, q
- p only if q
- q if p
- q unless $\sim p$
- q when p
- q whenever p
- q follows from p
- p is a sufficient condition for q (p is sufficient for q)
- q is a necessary condition for p (q is necessary for p)

Proofs:

- Direct: Assume P and prove Q.
- Contrapositive: Assume Not Q and prove Not P.
- Contradiction: Assume P and Not Q and prove a contradiction.
- Induction: Prove base(s), assume $P(m)$, prove $P(m+1)$.

Sets

Symbols: $\in \notin \subseteq \subset \supseteq \supset \ \varnothing \ \cup \ \cap \ \times$

Common Sets:

$\mathbb{N} = \{0, 1, 2, 3, \dots\}$	natural numbers
$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$	integers ($\mathbb{Z}$ for German Zahlen, meaning "integers")
$\mathbb{Z}^+ = \{1, 2, 3, \dots\}$	positive integers
$\mathbb{Q} = \left\{ \frac{p}{q} \;\middle|\; p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0 \right\}$	rational numbers
$\mathbb{U} = \{*\}$	universal set

Identities:

$A \cup \varnothing = A \quad A \cap \mathbb{U} = A$	Identity
$A \cup \mathbb{U} = \mathbb{U} \quad A \cap \varnothing = \varnothing$	Domination
$A \cup A = A \quad A \cap A = A$	Idempotent
$A \cup A^c = \mathbb{U} \quad A \cap A^c = \varnothing$	Complement
$A \cup B = B \cup A \quad A \cap B = B \cap A$	Commutative
$(A \cup B) \cup C = A \cup (B \cup C)$	Associative
$(A \cap B) \cap C = A \cap (B \cap C)$	Associative
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive
$(A \cup B)^c = A^c \cap B^c$	DeMorgan's
$(A \cap B)^c = A^c \cup B^c$	Demorgan's
$A \cup (A \cap B) = A$	Absorption
$A \cap (A \cup B) = A$	Absorption

Series and Sums

Symbols: $\sum \cdot$

Geometric Progression	$a_k = a \cdot r^k$
Arithmetic Progression	$a_k = a + d \cdot k$

Sum Formulas:

Formula #	Sum	Closed Form
1	$\sum_{k=0}^{n} ar^k \ (r \neq 0)$	$\frac{ar^{n+1} - a}{r - 1}, \ r \neq 1$
2	$\sum_{k=i}^{n} ar^k \ (r \neq 0)$	$\frac{ar^{n+1} - ar^i}{r - 1}, \ r \neq 1$
3	$\sum_{k=i}^{n} c$	$c \cdot (n - i + 1)$
4	$\sum_{k=1}^{n} k$	$\frac{n(n+1)}{2}$
5	$\sum_{k=1}^{n} k^2$	$\frac{n(n+1)(2n+1)}{6}$
6	$\sum_{k=1}^{n} k^3$	$\frac{n^2(n+1)^2}{4}$

Counting

Symbols: λ

Equations:

- Sum Rule:

  if S is the union of m disjoint sets $S_1, S_2, \ldots, S_m$, then the number of elements in S is:

  $$S = |S_1| + |S_2| + \cdots + |S_m|$$

- Product Rule:

  for a sequence of m choices the total number of elements is:

  $$P(m) = |P_1| \cdot |P_2| \cdot \cdots \cdot |P_m|$$

- Subset Exclusion:

  if $C = A - B$ is the difference of two sets, where $B \subseteq A$, then the number of elements in C is:

  $$|C| = |A| - |B|$$

- Inclusion/Exclusion:

  when a set D to be counted is the union of non-disjoint sets A and B, the number of elements in D is:

  $$|D| = |A| + |B| - |A \cap B|$$

Permutations and Combinations:

- r-permutations: $P(n, r) = \dfrac{n!}{(n-r)!}$
- r-permutations with repetition allowed: n^r
- The number of different permutations of n objects, where there are n_1 indistinguishable objects of type 1, and n_2 indistinguishable objects of type 2, …and n_k indistinguishable objects of type k is $\dfrac{n!}{n_1! n_2! \ldots n_k!}$
- r-combinations $C(n, r) = \dfrac{n!}{r!(n-r)!}$
- r-combinations with repetition allowed:

  $$C(n+r-1, r) = \frac{(n+r-1)!}{r!(n+r-1-r)!} = \frac{(n+r-1)!}{r!(n-1)!}$$