Logic

Symbols: $\geq \leq \neq \neg \sim \wedge \vee \oplus \equiv \rightarrow \leftrightarrow$ □ $\exists\ \forall$

Identities:

$\sim(\sim p) \equiv p$		Double Negation
$p \wedge \mathbf{T} \equiv p$	$p \vee \mathbf{F} \equiv p$	Identity
$p \vee \mathbf{T} \equiv \mathbf{T}$	$p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination
$p \wedge p \equiv p$	$p \vee p \equiv p$	Idempotent
$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$	Commutative
$(p \vee q) \vee r \equiv p \vee (q \vee r)$		Associative
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$		“
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$		Distributive
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$		“
$\sim(p \wedge q) \equiv \sim p \vee \sim q$		DeMorgan's
$\sim(p \vee q) \equiv \sim p \wedge \sim q$		“
$p \vee (p \wedge q) \equiv p$		Absorption
$p \wedge (p \vee q) \equiv p$		“
$p \rightarrow q \equiv \sim q \rightarrow \sim p$		Contrapositive
$p \oplus q \equiv q \oplus p$		“
$p \rightarrow q \equiv \sim p \vee q$		Implication
$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$		Biconditional Equivalence
$(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$		Exportation
$(p \rightarrow q) \wedge (p \rightarrow \sim q) \equiv \sim p$		Absurdity
$p \vee q \equiv \sim p \rightarrow q$		Alternate Implication
$p \wedge q \equiv \sim(p \rightarrow \sim q)$		“
$\sim(p \rightarrow q) \equiv p \wedge \sim q$		“
$\sim\forall x\, P(x) \equiv \exists x\, \sim P(x)$		DeMorgan's for Quantifiers
$\sim\exists x\, Q(x) \equiv \forall x\, \sim Q(x)$		“

A conditional statement $p \rightarrow q$ can be also read as -::
--If p then q
-- p implies q
-- If p, q
-- p only if q
-- q if p
-- q unless ~p
-- q when p
-- q whenever p
-- q follows from p
-- p is a sufficient condition for q (p is sufficient for q)
-- q is a necessary condition for p (q is necessary for p)

Proofs

Direct Assume P and prove Q

Contrapositive: Assume Not Q and prove Not P

Contradiction: Assume P and Not Q and prove a contradiction

Induction: Prove base(s), assume P(m), prove P(k+1)

Sets

Symbols: $\in \notin \subseteq \subset \supseteq \supset \emptyset \cup \cap \times$

Common Sets:

$\mathbf{N} = \{0, 1, 2, \ldots\}$ = natural numbers

$\mathbf{Z} = \{\ldots, -1, 0, 1, 2, \ldots\}$ = integers

$\mathbf{Z}^+ = \{1, 2, 3, \ldots\}$ = positive integers

$\mathbf{Q} = \{p/q \mid p \in \mathbf{Z}, q \in \mathbf{Z}, q \neq 0\}$ rational numbers

$\mathbf{U} = \{*\}$ universal

Identities:

$A \cup \emptyset = A$	$A \cap \mathbf{U} = A$	Identity
$A \cup \mathbf{U} = \mathbf{U}$	$A \cap \emptyset = \emptyset$	Domination
$A \cup A = A$	$A \cap A = A$	Idempotent
$A \cup A^C = \mathbf{U}$	$A \cap A^C = \emptyset$	Complement
$A \cup B = B \cup A$	$A \cap B = B \cap A$	Commutative
$(A \cup B) \cup C = A \cup (B \cup C)$		Associative
$(A \cap B) \cap C = A \cap (B \cap C)$		“
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$		Distributive
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$		“
$(A \cup B)^C = A^C \cap B^C$		DeMorgan's
$(A \cap B)^C = A^C \cup B^C$		“
$A \cup (A \cap B) = A$		Absorption
$A \cap (A \cup B) = A$		“

Series & Sums

Symbols: $\sum \cdot$

Sum equations:

Geometric Progression	$a_n = a \cdot r^n$	$\sum_{k=0}^{n} r^k = \frac{(r^{(n+1)} - 1)}{(r-1)}$
Arithmetic Progression	$a_n = a + d \cdot n$	$\sum_{k=1}^{n} k = \frac{1}{2}(n^2 + n)$

Counting

Symbols: λ

Equations:

Sum Rule: for a union of disjoint sets the total number of elements is:

$S(m) = |S_1| + |S_2| + \ldots + |S_m|$

Product Rule: for a sequence of k choices the total number of elements is:

$P(m) = |P_1| \cdot |P_2| \cdot \ldots \cdot |P_m|$

Subset Exclusion: when a set to be counted is the difference of two sets A – B, where A ⊆B

$C(m) = |A| - |B|$

Inclusion/Exclusion: when a set to be counted is the union of non-disjoint sets A and B

$D(m) = |A| + |B| - |A \cap B|$

- r-permutations $P(n,r) = \frac{n!}{(n-r)!}$
- r-permutations with repetition allowed - n^r
- The number of different permutations of n objects, where there are n_1 indistinguishable objects of type 1, n_2 indistinguishable objects of type 2, . . . , and n_k indistinguishable objects of type k, is $\frac{n!}{n_1! n_2! \cdots n_k!}$
- r-combinations $C(n,r) = \frac{n!}{r!(n-r)!}$
- r-combinations with repetition allowed - $C(n+r-1, r) = \frac{(n+r-1)!}{r!(n+r-1-r)!} = \frac{(n+r-1)!}{r!(n-1)!}$