# CS 225

Symbols and Formulas

Fall 2020

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# Logic

**Symbols:** $\geq \leq \neq \neg \sim \wedge \vee \oplus \equiv \rightarrow \leftrightarrow \Box \exists \forall$

**Identities:**

| | |
|---|---|
| $\sim(\sim p) \equiv p$ | Double Negation |
| $p \wedge \mathbb{T} \equiv p \quad p \vee \mathbb{F} \equiv p$ | Identity |
| $p \vee \mathbb{T} \equiv \mathbb{T} \quad p \wedge \mathbb{F} \equiv \mathbb{F}$ | Domination |
| $p \wedge p \equiv p \quad p \vee p \equiv p$ | Idempotent |
| $p \vee \sim p \equiv \mathbb{T} \quad p \wedge \sim p \equiv \mathbb{F}$ | Negation |
| $p \vee q \equiv q \vee p \quad p \wedge q \equiv q \wedge p$ | Commutative |
| $(p \vee q) \vee r \equiv p \vee (q \vee r)$ | Associative |
| $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ | Associative |
| $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ | Distributive |
| $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ | Distributive |
| $\sim(p \wedge q) \equiv \sim p \vee \sim q$ | DeMorgan's |
| $\sim(p \vee q) \equiv \sim p \wedge \sim q$ | Demorgan's |
| $p \vee (p \wedge q) \equiv p$ | Absorption |
| $p \wedge (p \vee q) \equiv p$ | Absorption |
| $p \rightarrow q \equiv \sim q \rightarrow \sim p$ | Contrapositive |
| $p \oplus q \equiv q \oplus p$ | Contrapositive |
| $p \rightarrow q \equiv \sim p \vee q$ | Implication |
| $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ | Biconditional Equivalence |
| $(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$ | Exporation |
| $(p \rightarrow q) \wedge (p \rightarrow \sim q) \equiv \sim p$ | Absurdity |
| $p \vee q \equiv \sim p \rightarrow q$ | Alternate Implication |
| $p \wedge q \equiv \sim(p \rightarrow \sim q)$ | Alternate Implication |
| $\sim(p \rightarrow q) \equiv p \wedge \sim q$ | Alternate Implication |
| $\sim \forall x P(x) \equiv \exists x \sim P(x)$ | DeMorgan's for Quantifiers |
| $\sim \exists x Q(x) \equiv \forall x \sim Q(x)$ | DeMorgan's for Quantifiers |

A conditional statement $p \rightarrow q$ can also be read as:

- If $p$ then $q$
- $p$ implies $q$
- If $p$, $q$
- $p$ only if $q$
- $q$ if $p$
- $q$ unless $\sim p$
- $q$ when $p$
- $q$ whenever $p$
- $q$ follows from $p$
- $p$ is a sufficient condition for $q$ ($p$ is sufficient for $q$)
- $q$ is a necessary condition for $p$ ($q$ is necessary for $p$)

**Proofs:**

- Direct: Assume $P$ and prove $Q$.
- Contrapositive: Assume Not $Q$ and prove Not $P$.
- Contradiction: Assume $P$ and Not $Q$ and prove a contradiction.
- Induction: Prove base(s), assume $P(m)$, prove $P(m+1)$.

# Sets

**Symbols:** $\in \notin \subseteq \subset \supseteq \supset \ \varnothing \ \cup \ \cap \times$

**Common Sets:**

| | |
|---|---|
| $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ | natural numbers |
| $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ | integers ($\mathbb{Z}$ for German Zahlen, meaning "integers") |
| $\mathbb{Z}^+ = \{1, 2, 3, \dots\}$ | positive integers |
| $\mathbb{Q} = \left\{ \frac{p}{q} \;\middle|\; p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0 \right\}$ | rational numbers |
| $\mathbb{U} = \{*\}$ | universal set |

**Identities:**

| | |
|---|---|
| $A \cup \varnothing = A \quad A \cap \mathbb{U} = A$ | Identity |
| $A \cup \mathbb{U} = \mathbb{U} \quad A \cap \varnothing = \varnothing$ | Domination |
| $A \cup A = A \quad A \cap A = A$ | Idempotent |
| $A \cup A^c = \mathbb{U} \quad A \cap A^c = \varnothing$ | Complement |
| $A \cup B = B \cup A \quad A \cap B = B \cap A$ | Commutative |
| $(A \cup B) \cup C = A \cup (B \cup C)$ | Associative |
| $(A \cap B) \cap C = A \cap (B \cap C)$ | Associative |
| $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ | Distributive |
| $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ | Distributive |
| $(A \cup B)^c = A^c \cap B^c$ | DeMorgan's |
| $(A \cap B)^c = A^c \cup B^c$ | Demorgan's |
| $A \cup (A \cap B) = A$ | Absorption |
| $A \cap (A \cup B) = A$ | Absorption |

# Series and Sums

**Symbols:** $\sum \ \cdot$

**Sum equations:**

| | | |
|---|---|---|
| Geometric Progression | $a_k = a \cdot r^k$ | $\sum_{k=0}^{n} a_k = a \cdot \sum_{k=0}^{n} r^k = a \cdot \frac{r^{n+1} - 1}{r - 1}$ |
| Arithmetic Progression | $a_k = a + d \cdot k$ | $\sum_{k=1}^{n} k = \frac{1}{2}(n^2 + n)$ |

# Counting

**Symbols:** $\lambda$

**Equations:**

- Sum Rule:

  if $S$ is the union of $m$ disjoint sets $S_1, S_2, \ldots, S_m$, then the number of elements in $S$ is:
  $$S = |S_1| + |S_2| + \cdots + |S_m|$$

- Product Rule:

  for a sequence of $m$ choices the total number of elements is:
  $$P(m) = |P_1| \cdot |P_2| \cdot \cdots \cdot |P_m|$$

- Subset Exclusion:

  if $C = A - B$ is the difference of two sets, where $B \subseteq A$, then the number of elements in $C$ is:
  $$|C| = |A| - |B|$$

- Inclusion/Exclusion:

  when a set $D$ to be counted is the union of non-disjoint sets $A$ and $B$, the number of elements in $D$ is:
  $$|D| = |A| + |B| - |A \cap B|$$

**Permutations and Combinations:**

- $r$-permutations: $P(n,r) = \dfrac{n!}{(n-r)!}$
- $r$-permutations with repetition allowed: $n^r$
- The number of different permutations of $n$ objects, where there are $n_1$ indistinguishable objects of type 1, and $n_2$ indistinguishable objects of type 2, …and $n_k$ indistinguishable objects of type $k$ is $\dfrac{n!}{n_1!n_2!\ldots n_k!}$
- $r$-combinations $C(n,r) = \dfrac{n!}{r!(n-r)!}$
- $r$-combinations with repetition allowed: $C(n+r-1,r) = \dfrac{(n+r-1)!}{r!(n+r-1-r)!} = \dfrac{(n+r-1)!}{r!(n-1)!}$