CS 225

Symbols and Formulas

Fall 2020

Logic

Symbols: $\geq \leq \neq \neg \sim \wedge \vee \oplus \equiv \rightarrow \leftrightarrow \square \exists \forall$

Identities:

$\sim(\sim p) \equiv p$	Double Negation
$p \wedge \mathbb{T} \equiv p \quad p \vee \mathbb{F} \equiv p$	Identity
$p \vee \mathbb{T} \equiv \mathbb{T} \quad p \wedge \mathbb{F} \equiv \mathbb{F}$	Domination
$p \wedge p \equiv p \quad p \vee p \equiv p$	Idempotent
$p \vee q \equiv q \vee p \quad p \wedge q \equiv q \wedge p$	Commutative
$(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive
$\sim(p \wedge q) \equiv \sim p \vee \sim q$	DeMorgan's
$\sim(p \vee q) \equiv \sim p \wedge \sim q$	Demorgan's
$p \vee (p \wedge q) \equiv p$	Absorption
$p \wedge (p \vee q) \equiv p$	Absorption
$p \rightarrow q \equiv \sim q \rightarrow \sim p$	Contrapositive
$p \oplus q \equiv q \oplus p$	Contrapositive
$p \rightarrow q \equiv \sim p \vee q$	Implication
$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	Biconditional Equivalence
$(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$	Exporation
$(p \rightarrow q) \wedge (p \rightarrow \sim q) \equiv \sim p$	Absurdity
$p \vee q \equiv \sim p \rightarrow q$	Alternate Implication
$p \wedge q \equiv \sim(p \rightarrow \sim q)$	Alternate Implication
$\sim(p \rightarrow q) \equiv p \wedge \sim q$	Alternate Implication
$\sim \forall x P(x) \equiv \exists x \sim P(x)$	DeMorgan's for Quantifiers
$\sim \exists x Q(x) \equiv \forall x \sim Q(x)$	DeMorgan's for Quantifiers

A conditional statement $p \rightarrow q$ can also be read as:

- If p then q
- p implies q
- If p, q
- p only if q
- q if p
- q unless $\sim p$
- q when p
- q whenever p
- q follows from p
- p is a sufficient condition for q (p is sufficient for q)
- q is a necessary condition for p (q is necessary for p)

Proofs:

- Direct: Assume P and prove Q.
- Contrapositive: Assume Not Q and prove Not P.
- Contradiction: Assume P and Not Q and prove a contradiction.
- Induction: Prove base(s), assume $P(m)$, prove $P(m+1)$.

Sets

Symbols: $\in \notin \subseteq \subset \supseteq \supset \ \varnothing \ \cup \ \cap \ \times$

Common Sets:

$\mathbb{N} = \{0, 1, 2, 3, \dots\}$	natural numbers
$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$	integers ($\mathbb{Z}$ for German Zahlen, meaning "integers")
$\mathbb{Z}^+ = \{1, 2, 3, \dots\}$	positive integers
$\mathbb{Q} = \left\{ \frac{p}{q} \,\middle\vert\, p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0 \right\}$	rational numbers
$\mathbb{U} = \{*\}$	universal set

Identities:

$A \cup \varnothing = A \quad A \cap \mathbb{U} = A$	Identity
$A \cup \mathbb{U} = \mathbb{U} \quad A \cap \varnothing = \varnothing$	Domination
$A \cup A = A \quad A \cap A = A$	Idempotent
$A \cup A^c = \mathbb{U} \quad A \cap A^c = \varnothing$	Complement
$A \cup B = B \cup A \quad A \cap B = B \cap A$	Commutative
$(A \cup B) \cup C = A \cup (B \cup C)$	Associative
$(A \cap B) \cap C = A \cap (B \cap C)$	Associative
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive
$(A \cup B)^c = A^c \cap B^c$	DeMorgan's
$(A \cap B)^c = A^c \cup B^c$	Demorgan's
$A \cup (A \cap B) = A$	Absorption
$A \cap (A \cup B) = A$	Absorption

Sets

Symbols: $\sum \ \cdot$

Sum equations: