Classical Assumptions

EC 320: Introduction to Econometrics

Philip Economides

Winter 2022

Prologue

Housekeeping

- Reminder: **Problem Set 3** due by 11:59pm
- This lecture last one relevant to **Midterm exam**
- Revise material, have questions ready for **review session**
- Anything after this lecture will not come up in midterm exam

Agenda

Last Week

How does OLS estimate a regression line?

- **Minimize RSS**.

What are the direct consequences of minimizing RSS?

- Residuals sum to zero.
- Residuals and the explanatory variable X are uncorrelated.
- Mean values of X and Y are on the fitted regression line.

Whatever do we mean by *goodness of fit*?

- What information does R^2 convey?

Agenda

Today

Under what conditions is OLS *desirable*?

- **Desired properties:** Unbiasedness, efficiency, and ability to conduct hypothesis tests.
- **Cost:** Six **classical assumptions** about the population relationship and the sample.

Returns to Schooling

Policy Question: How much should the state subsidize higher education?

- Could higher education subsidies increase future tax revenue?
- Could targeted subsidies reduce income inequality and racial wealth gaps?
- Are there positive externalities associated with higher education?

Empirical Question: What is the monetary return to an additional year of education?

- Focuses on the private benefits of education. Not the only important question!
- Useful for learning about the econometric assumptions that allow causal interpretation.

Returns to Schooling

Step 1: Write down the population model.

$$\log(\text{Earnings}_i) = \beta_1 + \beta_2 \text{Education}_i + u_i$$

Step 2: Find data.

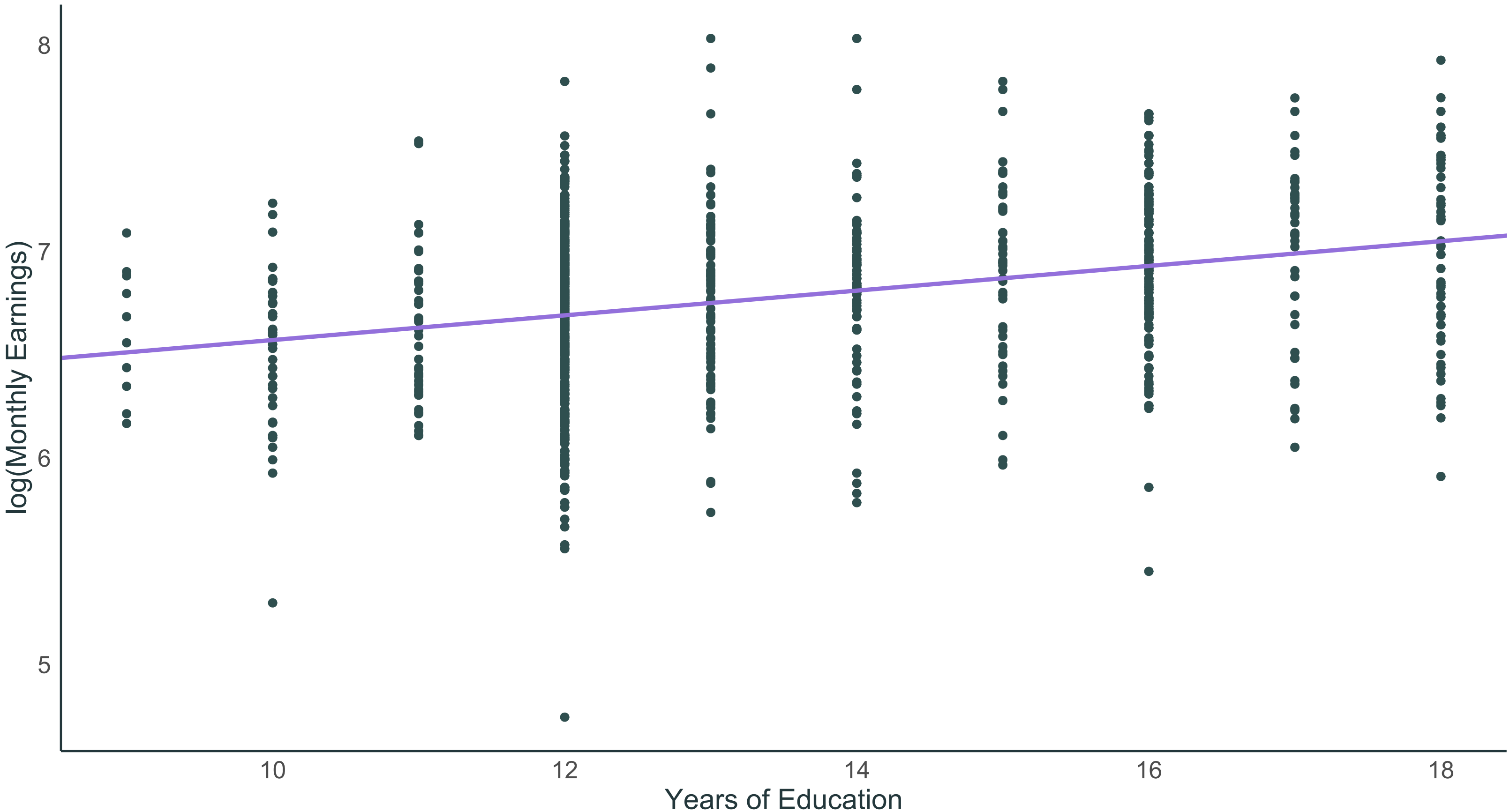
- *Source:* Blackburn and Neumark (1992).

Step 3: Run a regression using OLS.

$$\widehat{\log(\text{Earnings}_i)} = \hat{\beta}_1 + \hat{\beta}_2 \text{Education}_i$$

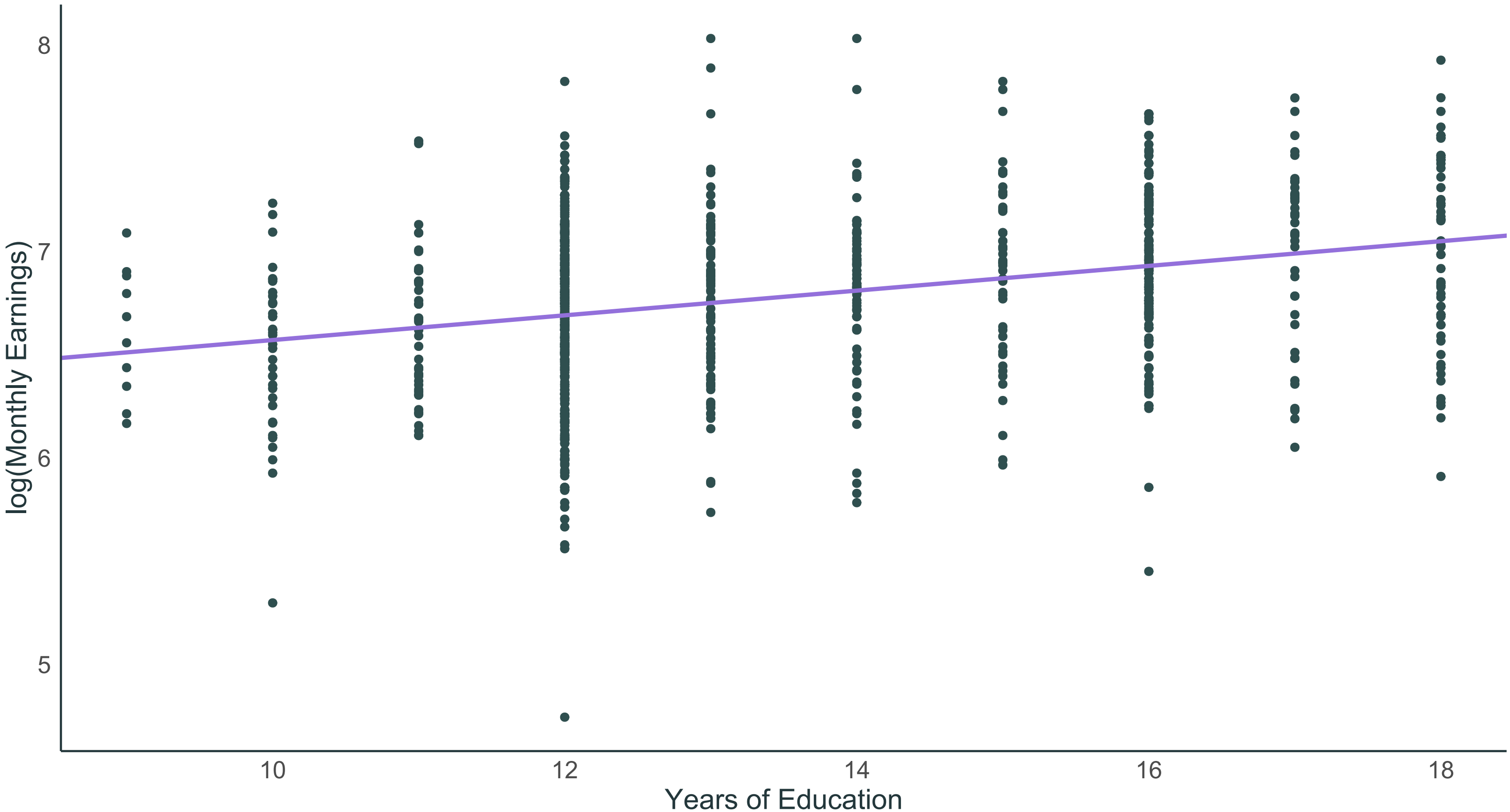
Returns to Schooling

$$\widehat{\log(\text{Earnings}_i)} = \mathbf{5.97} + \mathbf{0.06} \times \text{Education}_i.$$



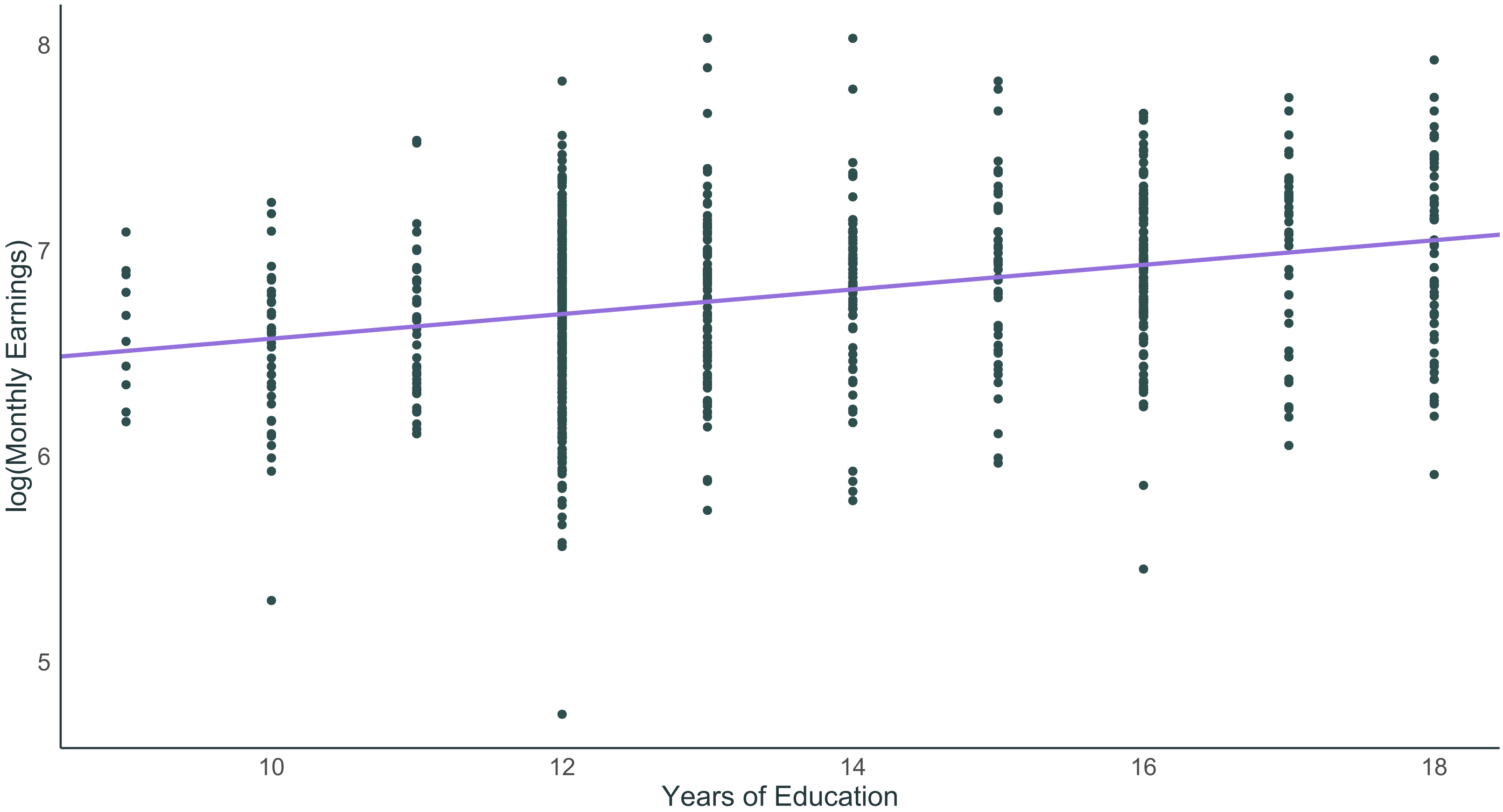
Returns to Schooling

Additional year of school associated with a **6%** increase in earnings.



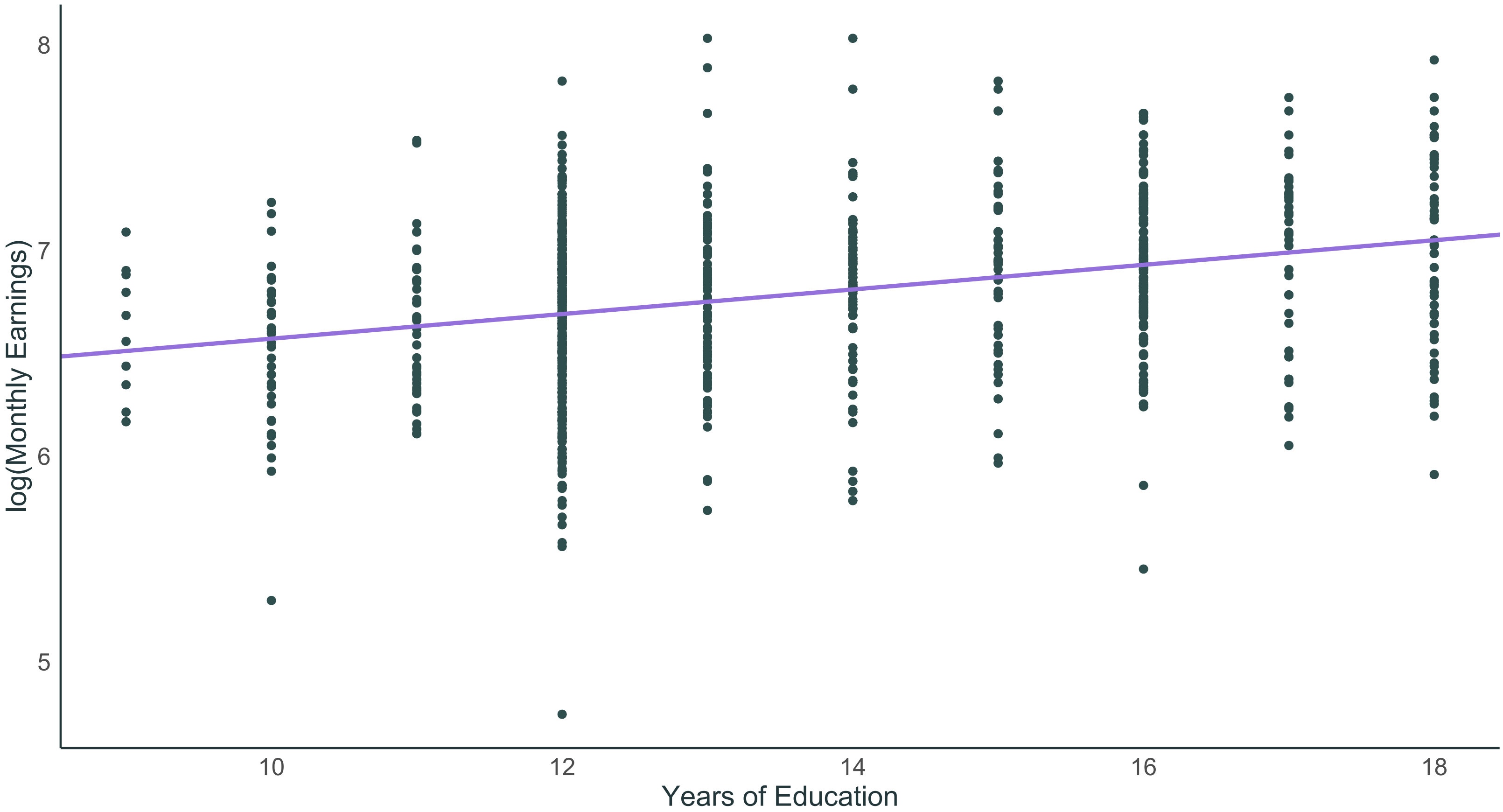
Returns to Schooling

$R^2 = \mathbf{0.097}$.



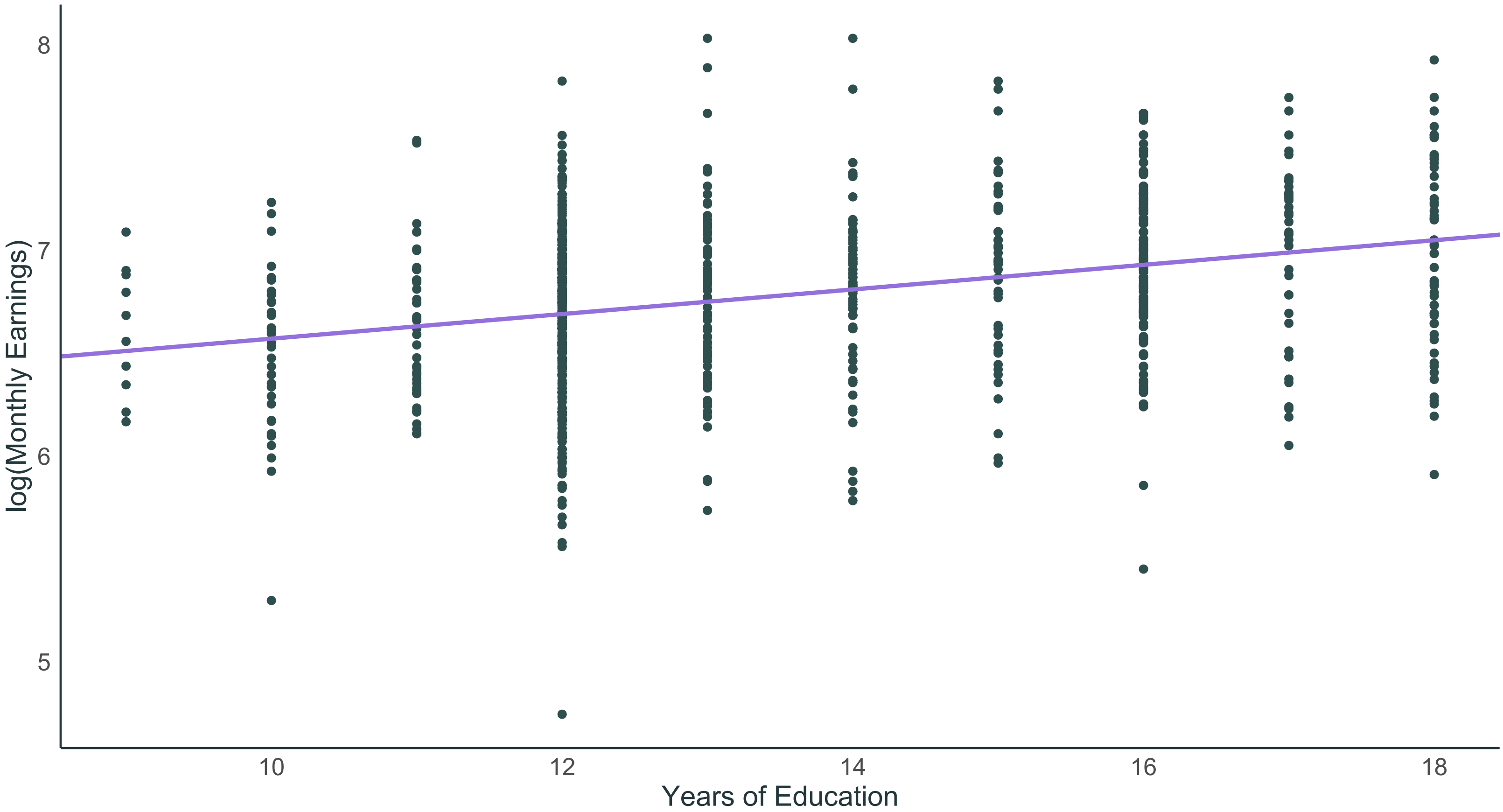
Returns to Schooling

Education explains **9.7%** of the variation in wages.



Returns to Schooling

What must we **assume** to interpret $\hat{\beta}_2 = \mathbf{0.06}$ as the return to schooling?



Residuals *vs.* Errors

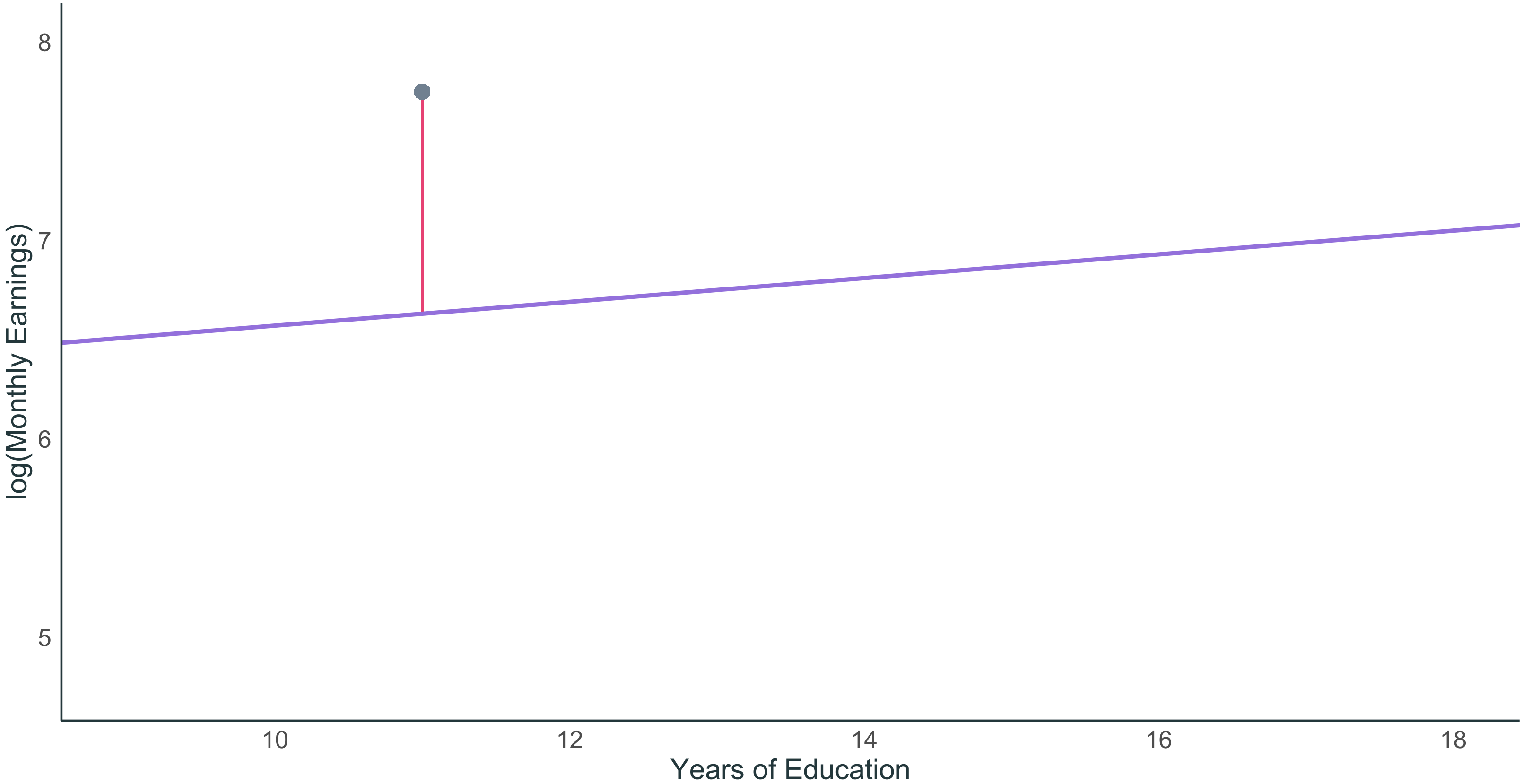
The most important assumptions concern the error term u_i.

Important: An error u_i and a residual $\hat{u}_i$ are related, but different.

- **Error:** Difference between the wage of a worker with 16 years of education and the **expected wage** with 16 years of education.
- **Residual:** Difference between the wage of a worker with 16 years of education and the **average wage** of workers with 16 years of education.
- **Population *vs.* sample.**

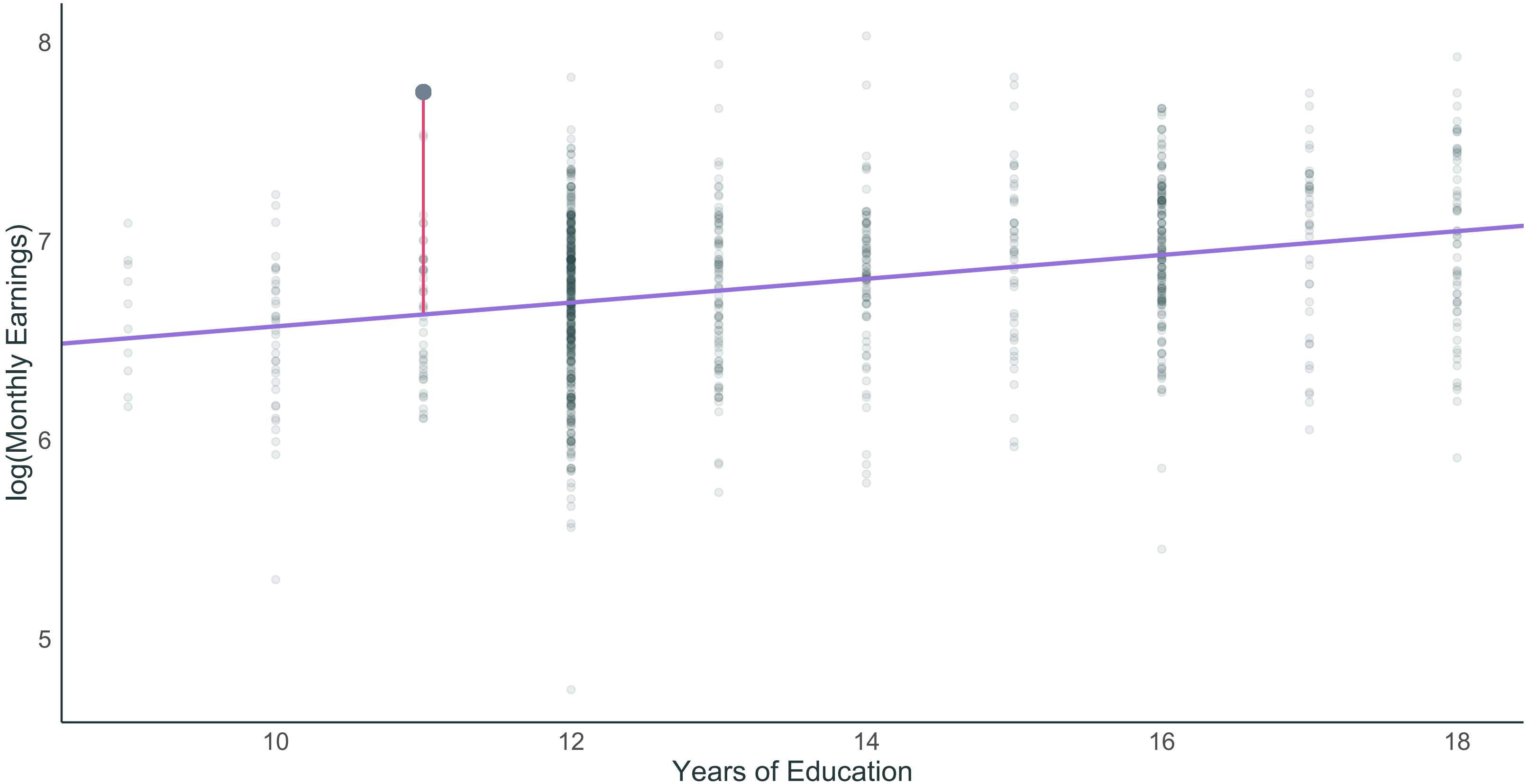
Residuals *vs.* Errors

A **residual** tells us how a **worker**'s wages compare to the average wages of workers in the **sample** with the same level of education.



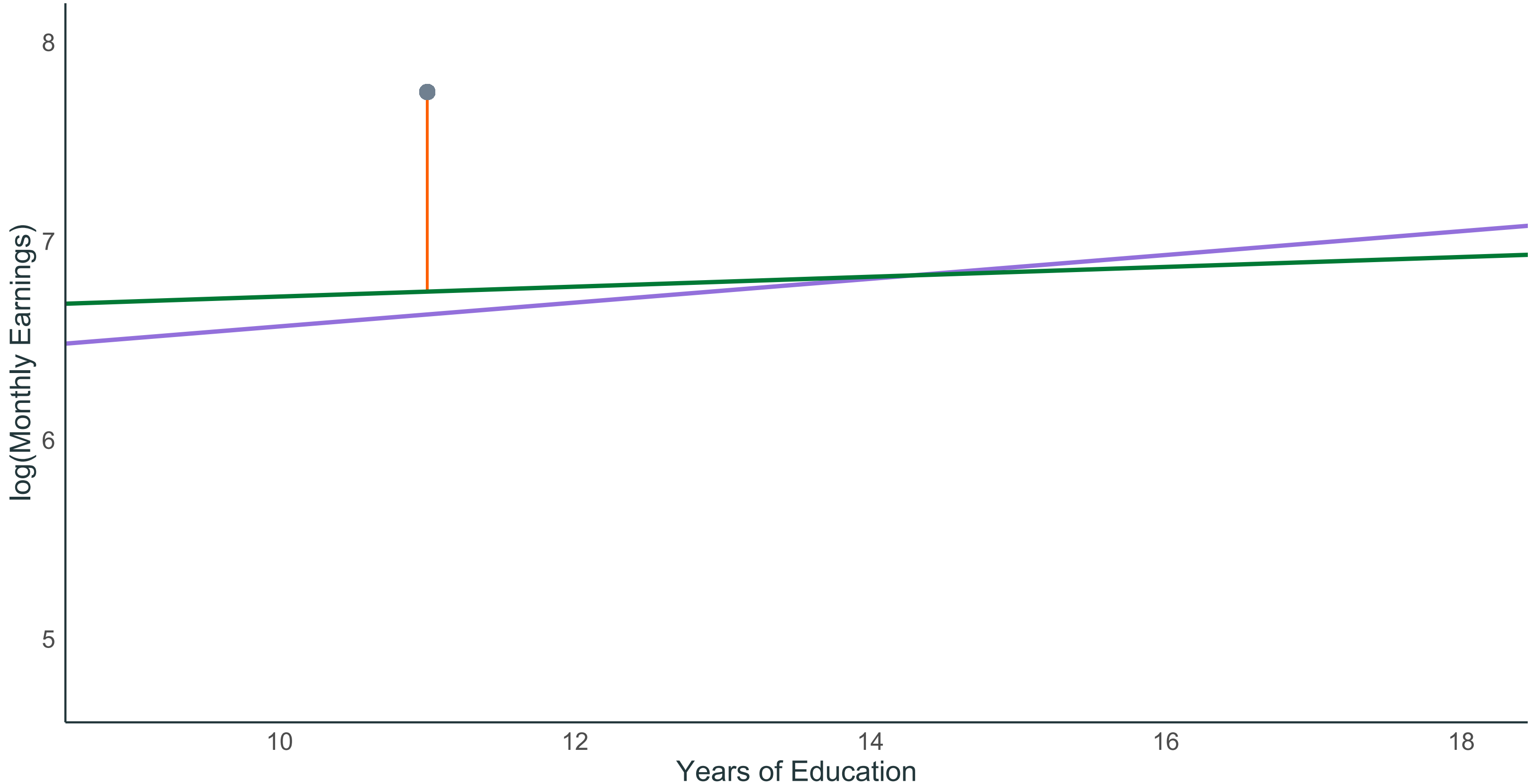
Residuals *vs.* Errors

A **residual** tells us how a **worker**'s wages compare to the average wages of workers in the **sample** with the same level of education.



Residuals *vs.* Errors

An **error** tells us how a **worker**'s wages compare to the expected wages of workers in the **population** with the same level of education.



Classical Assumptions

Classical Assumptions of OLS

1. **Linearity:** The population relationship is **linear in parameters** with an additive error term.

2. **Sample Variation:** There is variation in X.

3. **Exogeneity:** The X variable is **exogenous** (*i.e.*, $\mathbb{E}(u|X) = 0$).†

4. **Homoskedasticity:** The error term has the same variance for each value of the independent variable (*i.e.*, $\text{Var}(u|X) = \sigma^2$).

5. **Non-autocorrelation:** The values of error terms have independent distributions (*i.e.*, $E[u_i u_j] = 0, \forall i \text{ s.t. } i \neq j$)

6. **Normality:** The population error term is normally distributed with mean zero and variance σ^2 (*i.e.*, $u \sim N(0, \sigma^2)$)

† Implies assumption of **Random Sampling:** We have a random sample from the population of interest.

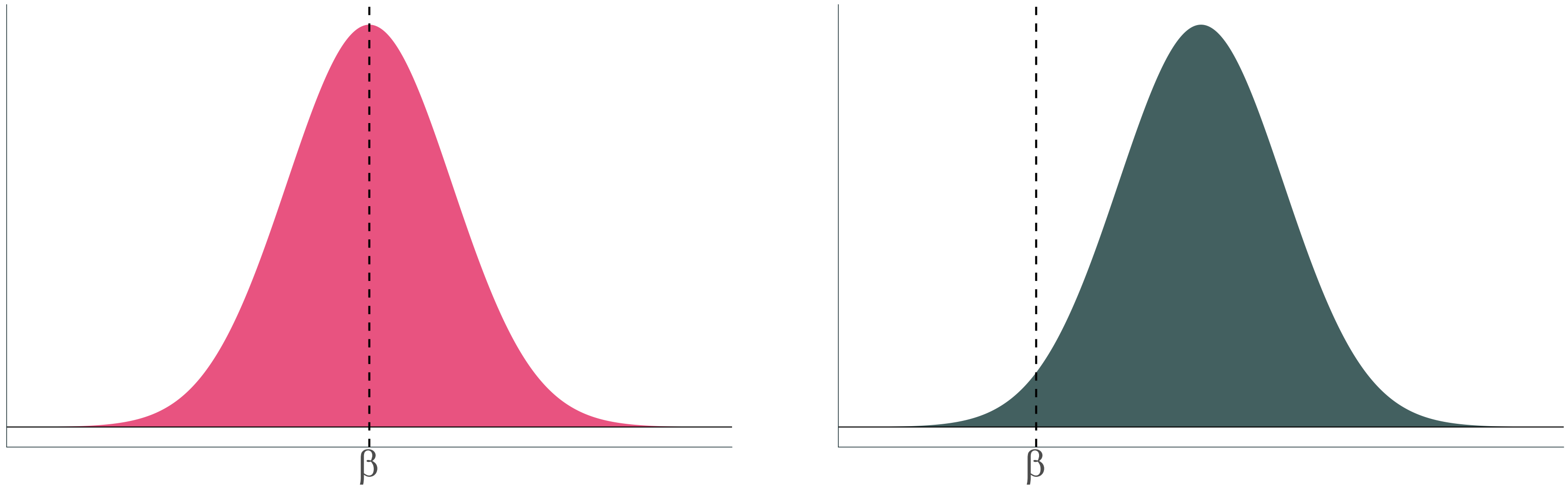
When Can We Trust OLS?

Bias

An estimator is **biased** if its expected value is different from the true population parameter.

Unbiased estimator: $\mathbb{E}\left[\hat{\beta}\right] = \beta$

Biased estimator: $\mathbb{E}\left[\hat{\beta}\right] \neq \beta$



When is OLS Unbiased?

Required Assumptions

1. **Linearity:** The population relationship is **linear in parameters** with an additive error term.

2. **Sample Variation:** There is variation in X.

3. **Exogeneity:** The X variable is **exogenous** (*i.e.*, $\mathbb{E}(u|X) = 0$).

☛ (3) implies **Random Sampling**. Without, the internal validity of OLS uncompromised, but our external validity becomes uncertain.[†]

† **Internal Validity:** relates to how well a study is conducted (does it satisfy OLS assumptions?).
External Validity: relates to how applicable the findings are to the real world.

Result

OLS is unbiased.

Linearity (A1.)

Assumption

The population relationship is **linear in parameters** with an additive error term.

Examples

- $\text{Wage}_i = \beta_1 + \beta_2 \text{Experience}_i + u_i$
- $\log(\text{Happiness}_i) = \beta_1 + \beta_2 \log(\text{Money}_i) + u_i$
- $\sqrt{\text{Convictions}_i} = \beta_1 + \beta_2 (\text{Early Childhood Lead Exposure})_i + u_i$
- $\log(\text{Earnings}_i) = \beta_1 + \beta_2 \text{Education}_i + u_i$

Linearity (A1.)

Assumption

The population relationship is **linear in parameters** with an additive error term.

Violations

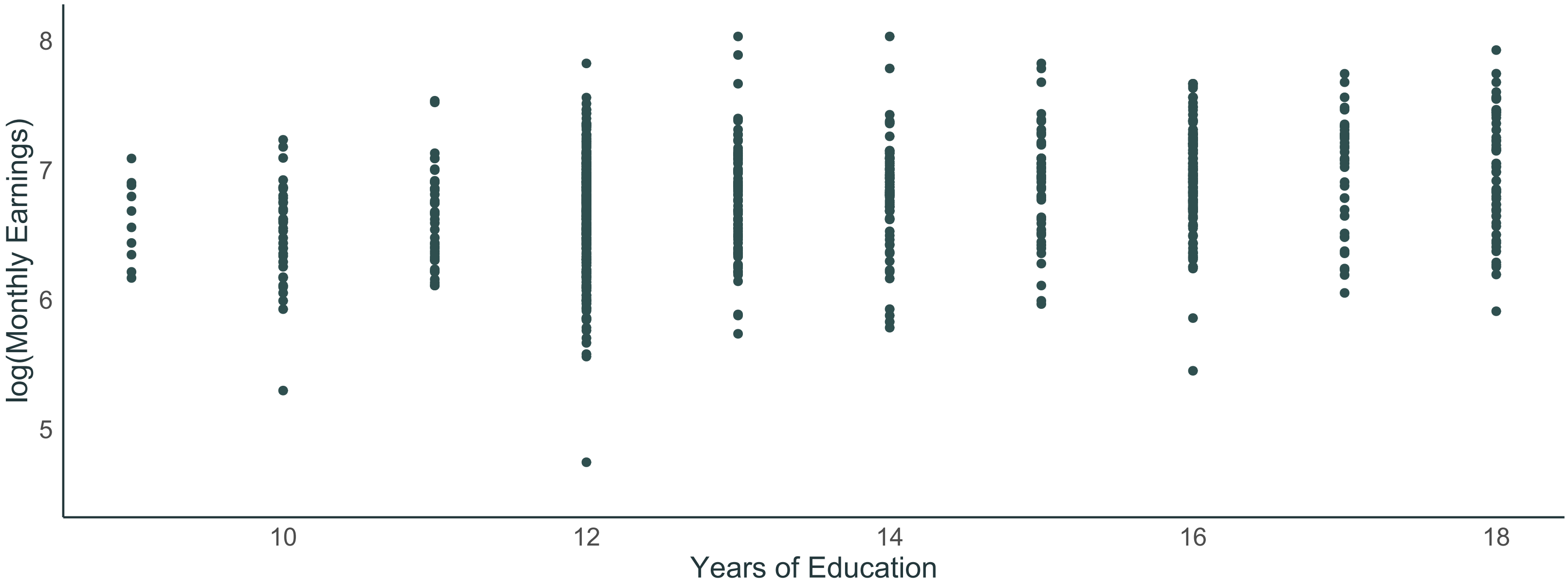
- $\text{Wage}_i = (\beta_1 + \beta_2 \text{Experience}_i) u_i$
- $\text{Consumption}_i = \frac{1}{\beta_1 + \beta_2 \text{Income}_i} + u_i$
- $\text{Population}_i = \frac{\beta_1}{1 + e^{\beta_2 + \beta_3 \text{Food}_i}} + u_i$
- $\text{Batting Average}_i = \beta_1 (\text{Wheaties Consumption})_i^{\beta_2} + u_i$

Sample Variation (A2.)

Assumption

There is variation in X.

Example

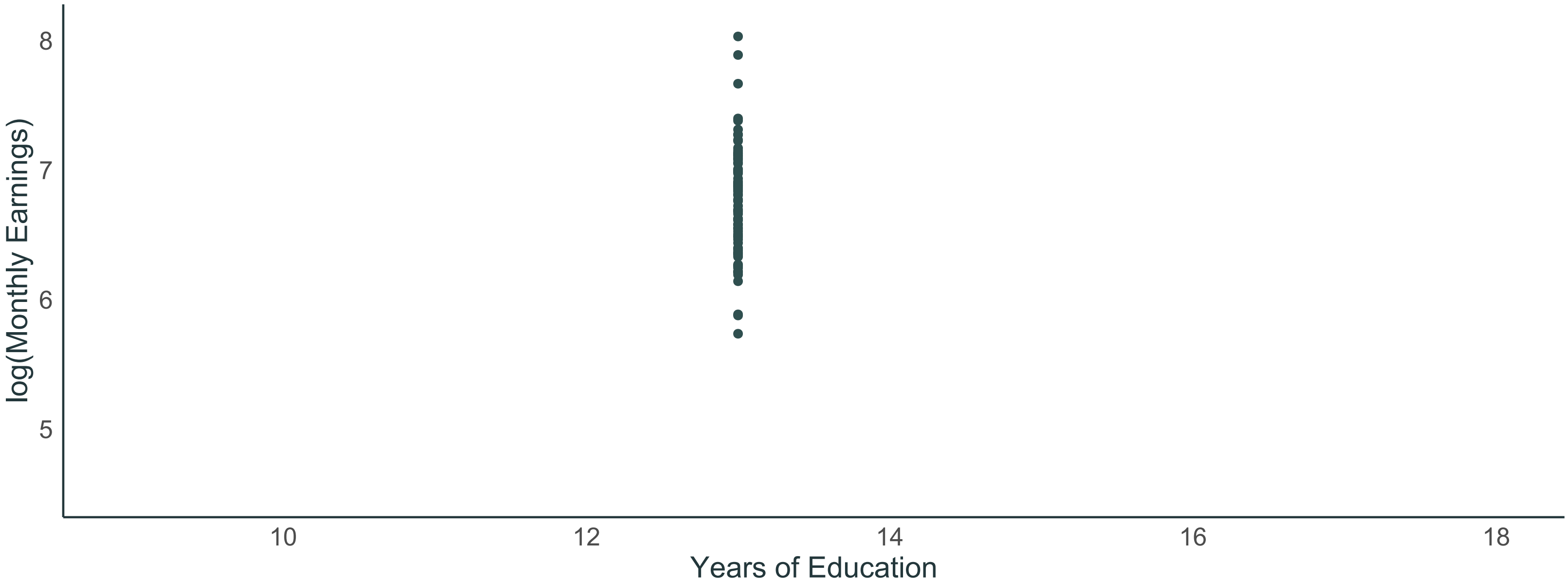


Sample Variation (A2.)

Assumption

There is variation in X.

Violation



Exogeneity (A3.)

Assumption

The X variable is **exogenous:** $\mathbb{E}(u|X) = 0$.

- For *any* value of X, the mean of the error term is zero.

The most important assumption!

Really two assumptions bundled into one:

1. On average, the error term is zero: $\mathbb{E}(u) = 0$.
2. The mean of the error term is the same for each value of X: $\mathbb{E}(u|X) = \mathbb{E}(u)$.

Exogeneity (A3.)

Assumption

The X variable is **exogenous:** $\mathbb{E}(u|X) = 0$.

- The assignment of X is effectively random.
- **Implication: no selection bias** and **no omitted-variable bias**.

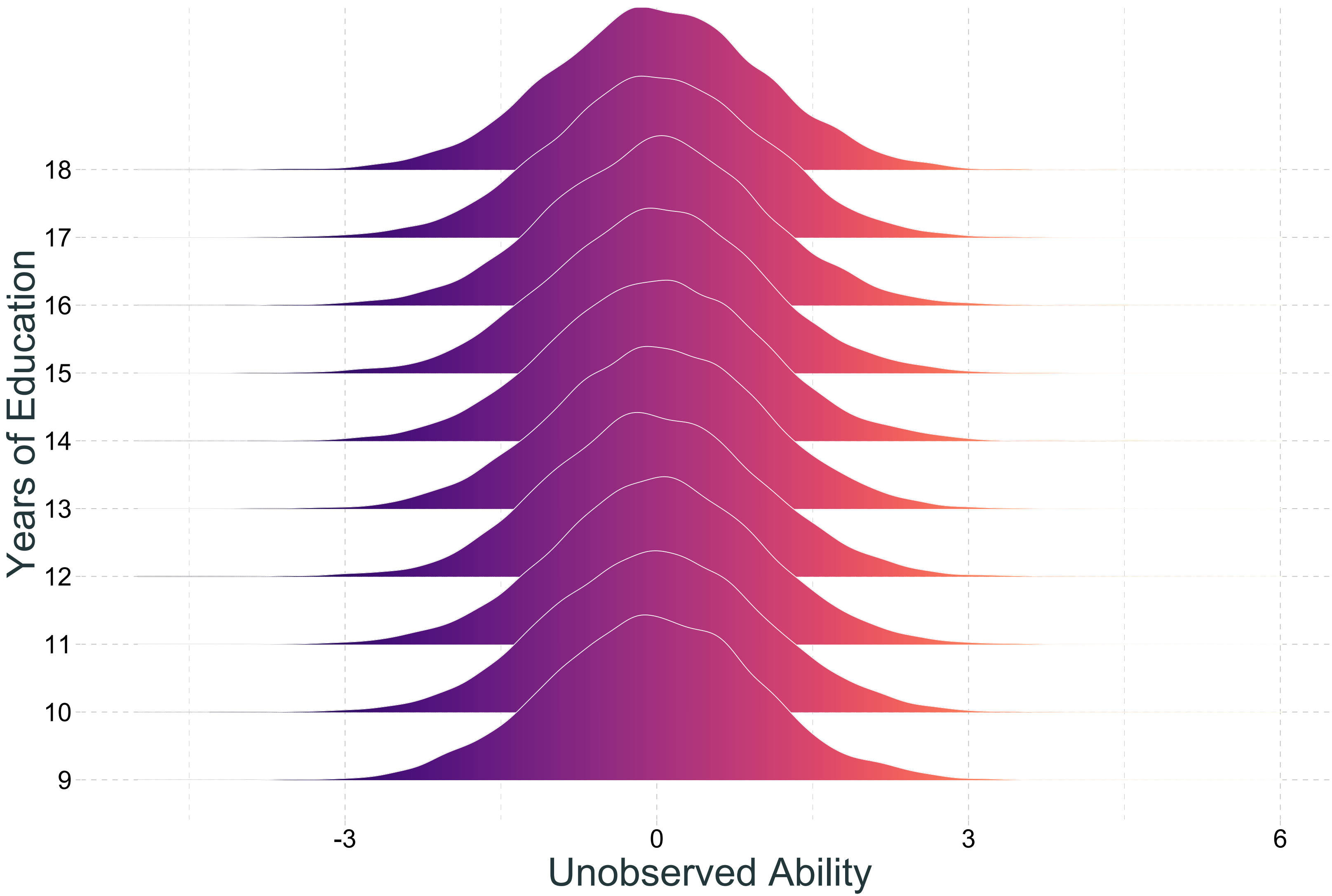
Examples

In the labor market, an important component of u is unobserved ability.

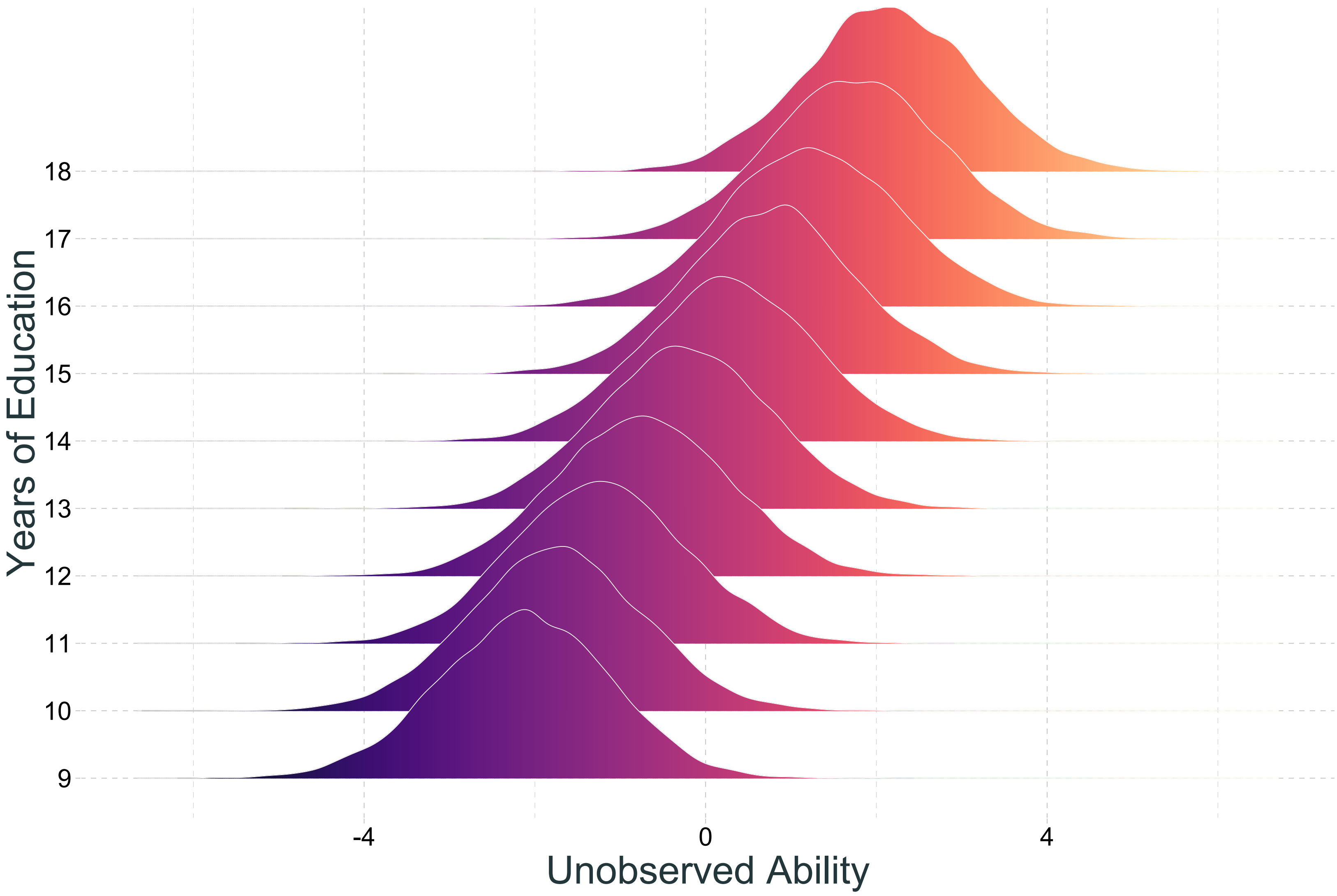
- $\mathbb{E}(u|\text{Education} = 12) = 0$ and $\mathbb{E}(u|\text{Education} = 20) = 0$.
- $\mathbb{E}(u|\text{Experience} = 0) = 0$ and $\mathbb{E}(u|\text{Experience} = 40) = 0$.
- **Do you believe this?**

Graphically...

Valid exogeneity, *i.e.*, $\mathbb{E}(u \mid X) = 0$



Invalid exogeneity, *i.e.*, $\mathbb{E}(u \mid X) \neq 0$



Variance Matters, Too

Why Variance Matters

Unbiasedness tells us that OLS gets it right, *on average*.

- But we can't tell whether our sample is "typical."

Variance tells us how far OLS can deviate from the population mean.

- How tight is OLS centered on its expected value?
- This determines the **efficiency** of our estimator.

The smaller the variance, the closer OLS gets, **on average**, to the true population parameters *on any sample*.

- Given two unbiased estimators, we want the one with smaller variance.
- If (A4.) and (A5.) are satisfied as well, we are using the **most efficient** linear estimator.

OLS Variance

To calculate the variance of OLS, we need:

1. The same four assumptions we made for unbiasedness.
2. **Homoskedasticity.**
3. **Non-autocorrelation**

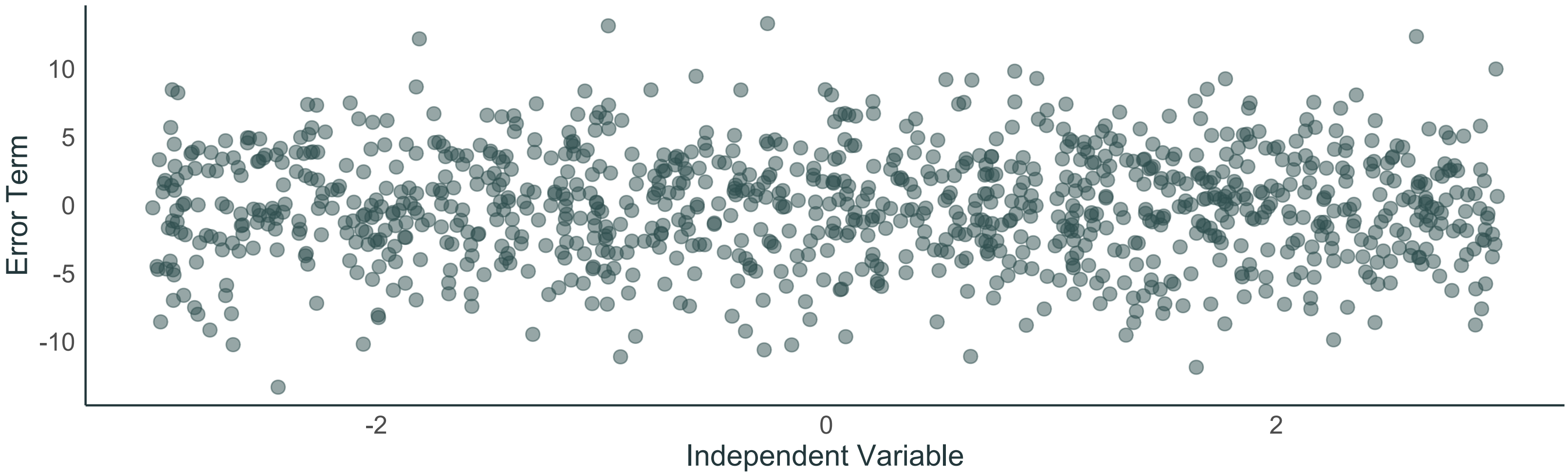
Homoskedasticity (A4.)

Assumption

The error term has the same variance for each value of the independent variable:

$$\mathrm{Var}(u|X) = \sigma^2.$$

Example



Homoskedasticity (A4.)

Assumption

The error term has the same variance for each value of the independent variable:

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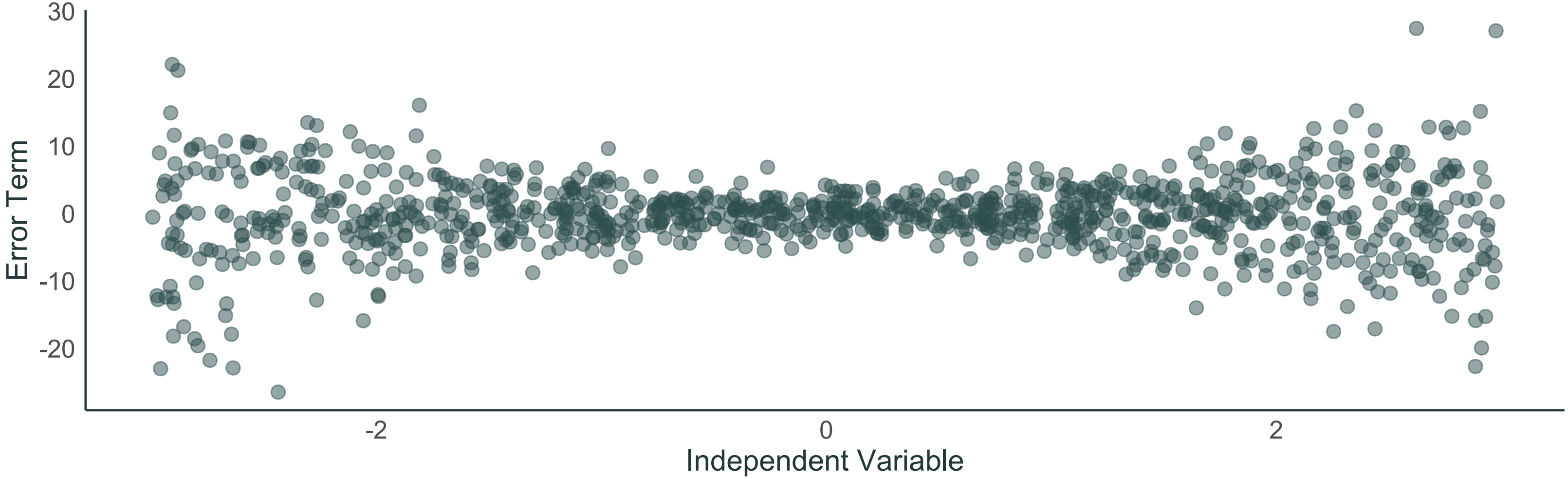
Violation: Heteroskedasticity



Non-Autocorrelation

Assumption

Any individual's error term is drawn independently of other error terms.

$$\begin{aligned}\text{Cov}(u_i, u_j) &= E[(u_i - \mu_u)(u_j - \mu_u)] \\ &= E[u_i u_j] = E[u_i]E[u_j] = 0, \textbf{ where } i \neq j\end{aligned}$$

- This implies no systematic association between error term values for any pair of individuals
- In practice, there is always some correlatio in unobservables across individuals (e.g. common correlation in unobservables among individuals within a given US state)
- Referred to as **clustering** problem. Standard errors can be adjusted to address

OLS Variance

Variance of the slope estimator:

$$\mathbf{Var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}.$$

- As the error variance increases, the variance of the slope estimator increases.
- As the variation in X increases, the variance of the slope estimator decreases.
- Larger sample sizes exhibit more variation in $X \implies \mathbf{Var}(\hat{\beta}_2)$ falls as n rises.

Gauss-Markov

Gauss-Markov Theorem

OLS is the **Best Linear Unbiased Estimator (BLUE)** when:

1. **Linearity:** The population relationship is **linear in parameters** with an additive error term.

2. **Sample Variation:** There is variation in X.

3. **Exogeneity:** The X variable is **exogenous** (*i.e.*, $\mathbb{E}(u|X) = 0$).

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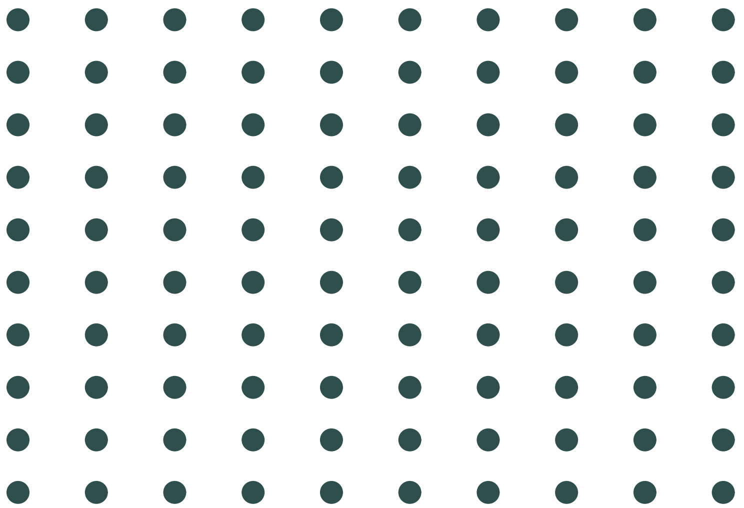
Gauss-Markov Theorem

OLS is the **Best Linear Unbiased Estimator (BLUE)**

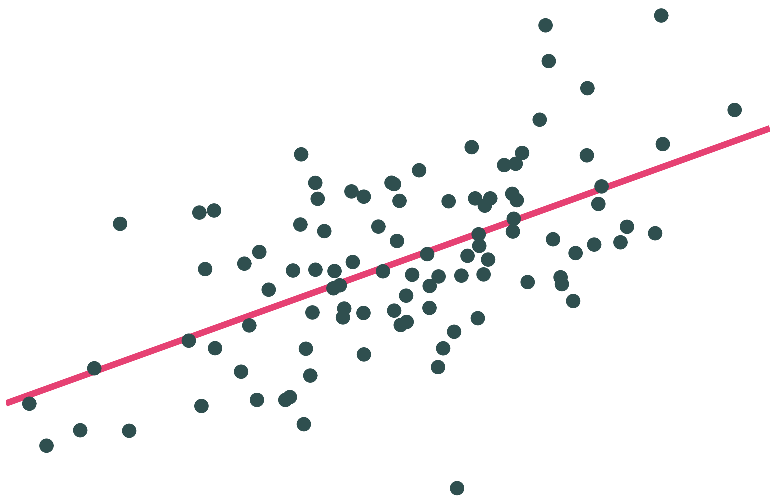
Population *vs.* Sample, Revisited

Population *vs.* Sample

Question: Why do we care about *population vs. sample*?



Population



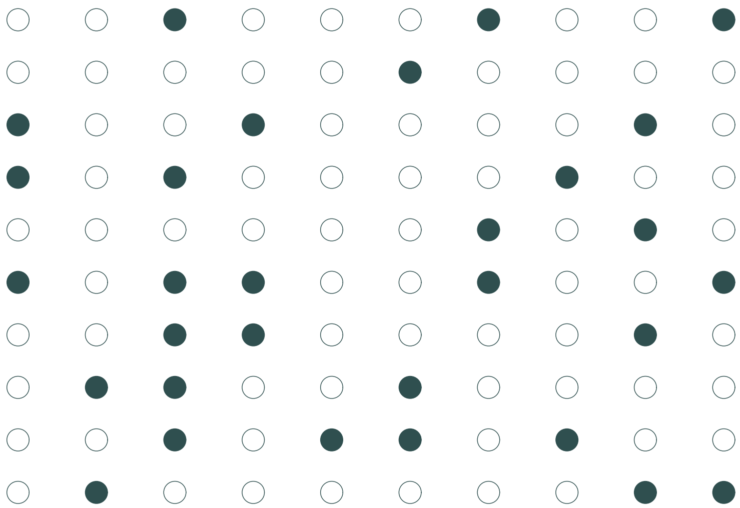
Population relationship

$$y_i = 2.53 + 0.57x_i + u_i$$

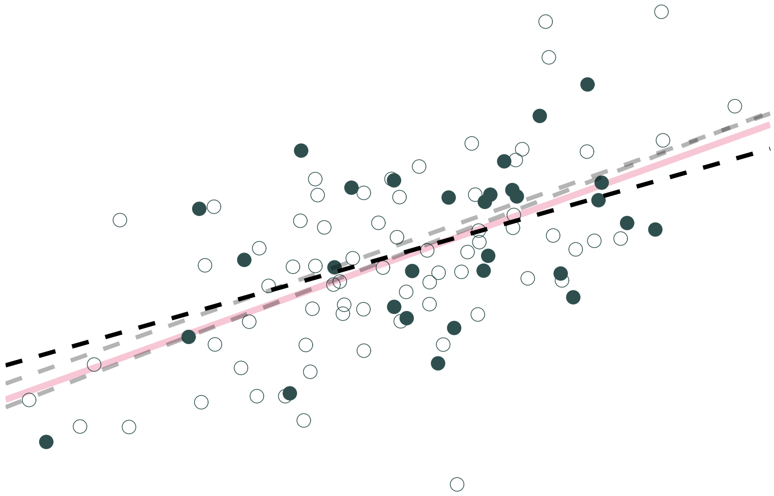
$$y_i = \beta_1 + \beta_2 x_i + u_i$$

Population *vs.* Sample

Question: Why do we care about *population vs. sample*?



Sample 3: 30 random individuals



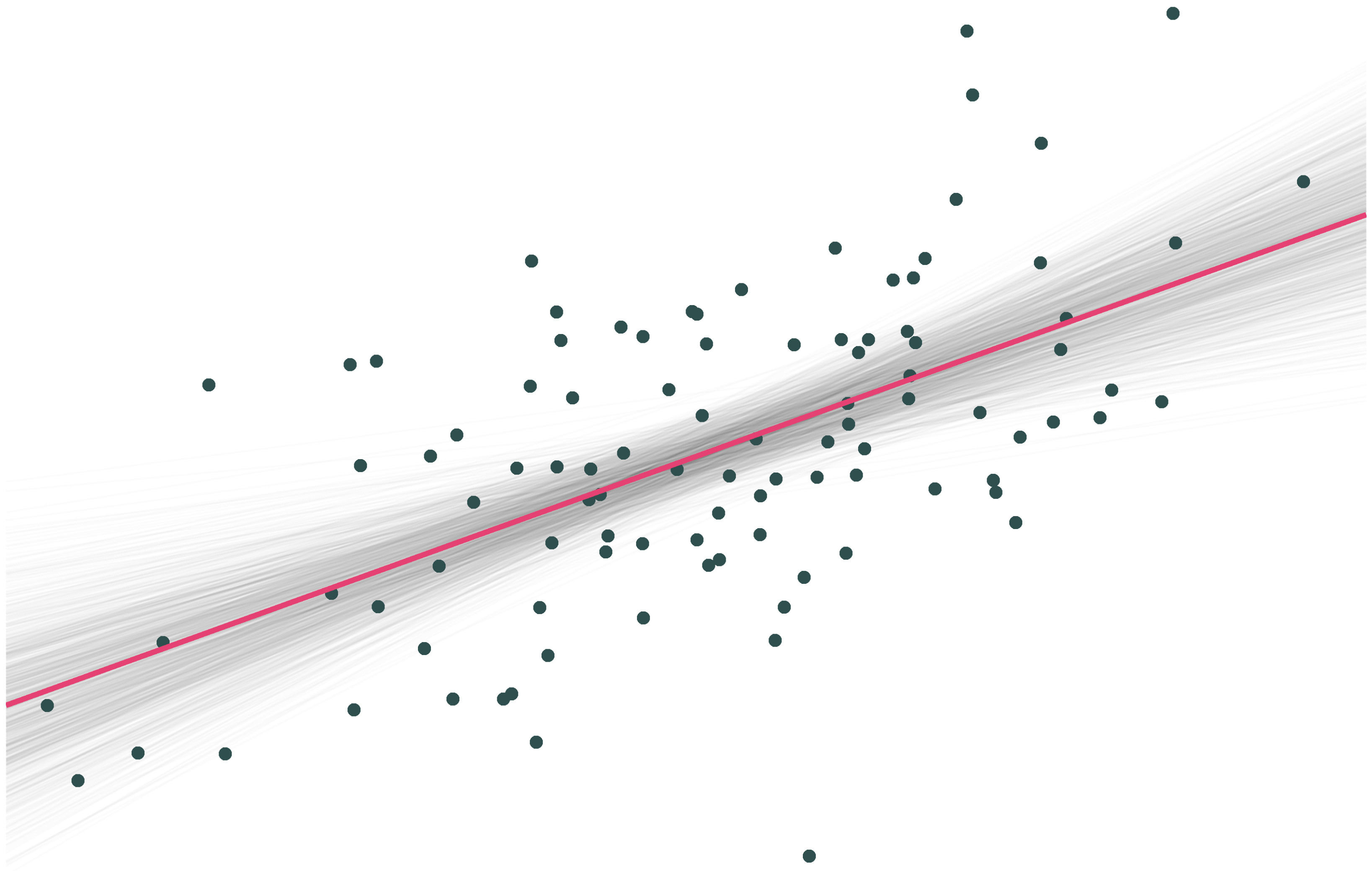
Population relationship

$$y_i = 2.53 + 0.57x_i + u_i$$

Sample relationship

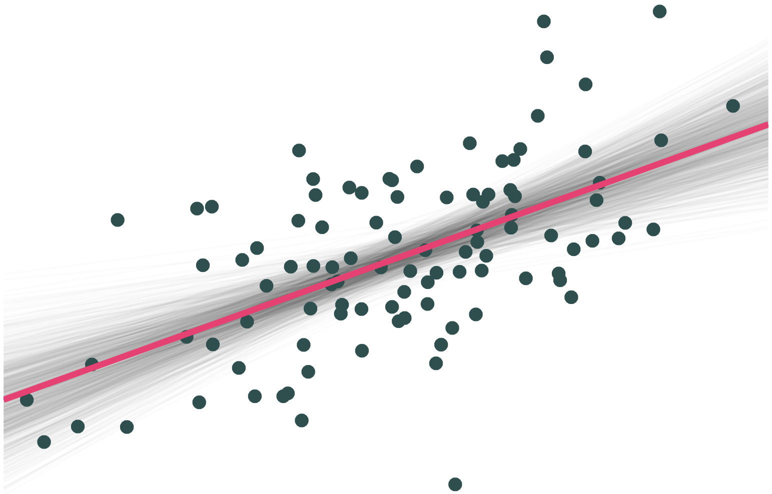
$$\hat{y}_i = 3.21 + 0.45x_i$$

Repeat **10,000 times** (Monte Carlo simulation).



Population *vs.* Sample

Question: Why do we care about *population vs. sample*?



- On **average**, the regression lines match the population line nicely.
- However, **individual lines** (samples) can miss the mark.
- Differences between individual samples and the population create **uncertainty**.

Population *vs.* Sample

Question: Why do we care about *population vs. sample*?

Answer: Uncertainty matters.

$\hat{\beta}_1$ and $\hat{\beta}_2$ are random variables that depend on the random sample.

We can't tell if we have a "good" sample (similar to the population) or a "bad sample" (very different than the population).

Next time, we will leverage all six classical assumptions, including **normality**, to conduct hypothesis tests.