Contract Enforcement and Young Firm Capital Structure: A Global Perspective*

Gonzalo Basante †

University of New Hampshire

Ina Simonovska ‡

University of California Davis, NBER, CEPR

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Abstract

We study young firms' short- and long-term leverage dynamics across developed and developing countries to infer the severity of firm-level financial constraints and their implications for firm growth. Using balance-sheet data for private firms, we document that stronger contract enforcement raises long-term leverage for young firms more so than for mature ones. We build a model where heterogeneous firms borrow funds in a limited contract-enforcement environment, and show that age is a more robust signal of constraint status than size. We confirm two testable predictions in the data: (i) short-term leverage increases while long-term leverage decreases over the early life cycle of firms; and (ii) the rise in short-term leverage persists longer in less developed economies. Guided by the model, we quantify firm-level financial constraints across countries of different levels of development using short- and long-leverage data.

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†Email: gonzalo.basante@unh.edu. Address: 10 Garrison Avenue Durham, NH, 03824, USA.

‡Email: inasimonovska@ucdavis.edu. Address: One Shields Avenue, Davis, CA, 95616, USA.

1 Introduction

Young firms drive economic growth and dynamism, accounting for a disproportionate share of job creation across economies (Haltiwanger et al., 2013; Ayyagari et al., 2014). These firms predominantly finance their growth through debt (Didier and Cusolito, 2024), making it essential to understand the dynamics of young firms' leverage and the constraints they face in accessing credit. Moreover, the quantitative role that firm financing plays in driving firm growth, relative to other persistent factors, is critical in the optimal design of credit assistance programs worldwide. In this paper, we develop a novel theoretical and empirical framework to quantify financial constraints for young firms by examining firm capital structure evolution during early life stages, and to evaluate the role these constraints play in driving firm growth.

Using firm-level balance sheet data from ORBIS and cross-country measures of contract enforcement provided by the World Bank's Cost of Doing Business Database, we document that long-term leverage increases in the country's loan recovery rate more sharply for young firms than for mature ones. This motivates a theory of firm growth in the presence of financial constraints in a limited enforcement environment. In our model, firms choose their blueprint scale at inception and adjust labor each period while facing exogenous exit risk. Firms must raise debt to finance operations, but can strategically default on their obligations, making enforcement quality crucial for their financial decisions.

Under two key assumptions—that firms face small fixed costs for each financial contract and that first-period cash flows are insufficient to cover both the productive upfront and the per-period variable input costs fully—we derive a unique optimal financial structure featuring long-term debt for initial scale and sequential short-term loans for working capital needs. This uniqueness result has critical empirical implications: by proving that firms optimally use distinct debt instruments with different maturities, we show why examining only total leverage (as in models where capital is financed by rolling over short-term debt) obscures the dynamics needed to measure financial frictions. Our finding that short- and long-term leverage follow opposing trajectories during a firm's early stage of life provides a novel empirical strategy to identify the duration of constrained financing and to quantify the severity of financial frictions. Theoretically, our result contributes to the dynamic contract literature (Albuquerque and Hopenhayn (2004); Clementi and Hopenhayn (2006); DeMarzo and Fishman (2007); Verani (2018)) by eliminating the typical multiplicity of implementations. The optimal contract exhibits front-loaded long-term debt payments and increasing short-term debt until constraints relax, reflecting a fundamental trade-off: higher long-term debt enables larger scale but delays the onset of dividend payments.

We derive sharp theoretical predictions from the model regarding leverage dynamics across countries at different levels of development, where the latter correlate strongly with contract enforcement levels in the data. The model predicts that a typical firm in a less developed economy has lower initial scale, lower short- and long-term leverage, and lower debt maturity. In turn, short-term leverage is increasing over the life cycle of the firm, and this increase persists for longer in less developed economies. We verify that these patterns are present in firm-level data from 25 countries using the Orbis database.

We use the model to quantify the role of cross-country institutional differences in accounting for young firm growth rates around the globe. Specifically, cross-country differences in young firm performance could reflect either institutional constraints that distort optimal choices or fundamental differences in entrepreneurial productivity. This distinction is crucial for policy: if profit gaps stem from financial frictions that prevent firms from choosing efficient scales, institutional reforms could unlock substantial growth. If they reflect immutable differences in entrepreneurial talent, such reforms would have limited impact.

Building on our theoretical framework, we develop a novel decomposition that separates these channels using a key insight: initial leverage ratios depend on enforcement quality but are invariant to entrepreneurial talent. This exclusion restriction allows us to identify institutional parameters and decompose observed performance variation into correctable institutional distortions versus fundamental productivity differences.

Our approach reveals that what empirical studies (ex. Sterk et al. (2021)) commonly observe as ex-ante heterogeneity in firm outcomes actually combines two distinct sources: truly exogenous entrepreneurial talent and endogenously chosen initial scale that depends on the institutional environment. Only the latter component is amenable to policy intervention through institutional reform. Our calibration exercise suggests that the gap in exogenous entrepreneurial talent between least and most developed economies is 40%, whereas middle income economies fall behind the most developed ones by only 7%. Moreover, scale distortions across most and least developed countries amount to 10%.

A key prediction of our model is that long-term leverage is higher in more financially developed countries, as documented in the data. This is in contrast to the standard model of endogenous financial constraints, which assumes that all firms must pay the same unproductive set-up cost and predicts that firms will repay the same face value of long-term debt in less developed (lower enforcement) countries but over a longer period of time. Furthermore, the value of the firm and the total surplus of the contract between the firm and financiers (the value of assets) is increasing in enforcement. A fixed face value of long-term debt for firms across development levels and the fact that asset values are increasing in enforcement imply that in less developed countries, leverage will be higher due to a lower value of assets of the firm. To fix this counterfactual prediction, we endogenize the unproductive set-up cost by making it a productive choice. In our preferred interpretation, young firms buy upfront the blueprint scale they want to operate during the early life cycle. This implies that firms in less developed economies will face a higher cost of capital due to the endogenous financial frictions and choose a smaller scale. Thus, we resolve the counterfactual prediction of the traditional model and introduce a new dimension to the measurement of financial frictions.

Our methodological approach requires that financial constraints emerge endogenously from the environment, rather than being imposed as exogenous assumptions. To this end, we build on the dynamic contracting literature, particularly Albuquerque and Hopenhayn (2004) and Cooley et al. (2004), and develop a model in which financing frictions arise from limited contract enforcement. This framework delivers a well-defined, model-consistent measure of constraint severity, which we exploit by comparing firm outcomes across different enforcement environments. While alternative microfoundations for borrowing constraints exist—such as agency frictions (Clementi and Hopenhayn, 2006; DeMarzo and Fishman, 2007; Verani, 2018)—we focus on enforcement, motivated by extensive evidence that institutional quality and legal enforcement play a central role in shaping access to credit.

Seminal work by La Porta et al. (1997); Porta et al. (1998) and Levine (2005) documents how legal origins and the strength of enforcement institutions explain cross-country variation in financial development and, in turn, economic growth. Building on these insights, quantitative models such as Amaral and Quintin (2010), Buera et al. (2011), and Buera and Shin (2013) show that limited enforcement can lead to persistent capital misallocation and help explain large cross-country income differences. More recent contributions by Kaboski and Townsend (2012) and Buera and Moll (2015) highlight the role of access to external finance in driving firm dynamics and aggregate productivity, with enforcement emerging as a key constraint.

We complement this literature by developing a tractable framework that directly connects contract enforcement to young firms' capital structure choices. Rather than focusing on aggregate misallocation or firm entry, we isolate the early life cycle of firms—where financial frictions are particularly salient (Hadlock and Pierce, 2010)—and identify leverage dynamics as a clean empirical object for quantifying constraint severity. Our model highlights persistent intensive-margin effects through scale distortions, in contrast to the extensive-margin emphasis of Midrigan and Xu (2014), who find that firms in Korea gradually grow out of financial constraints by accumulating internal funds. By endogenizing a productive initial scale choice rather than assuming an unproductive fixed cost, we align our model with evidence on the lasting effects of entry conditions (Moreira, 2019) and ex-ante heterogeneity (Sterk et al., 2021). These choices allow us to treat firm age—not size—as the more robust indicator of constraint status. While we abstract from firm entry decisions, the scale distortion at inception generates persistent effects over the 5–7 year horizon that we study.

Empirically, we relate to a growing literature that uses ORBIS to examine firm dynamics and financial frictions across countries (Kalemli-Özcan et al., 2024; Gopinath et al., 2017; Arellano et al., 2012). A more recent strand of work focuses on the evolution of corporate financing over the life cycle. This literature finds that total leverage—defined as the sum of short- and long-term debt—tends to decline with firm age. For instance, Dinlersoz et al. (2018) use a quadratic specification for U.S. private firms and show that short-term leverage declines monotonically with age. Derrien et al. (2021) document a similar downward pattern in total leverage for French firms during their first ten years, while Kochen (2022) shows that total leverage declines with age across several European countries. In contrast to this literature, we separately examine short- and long-term leverage dynamics in the early life cycle and document a novel pattern: short-term leverage rises while long-term leverage falls during a firm's initial years. This compositional shift is masked in aggregate leverage measures.

In our model, both capital structure and the severity of financial constraints are endogenous outcomes that vary systematically with enforcement quality. Lower enforcement leads to tighter borrowing constraints, which in turn distort firms' capital structure—specifically, resulting in lower long-term leverage and shorter long-term debt maturity. Thus, cross-country differences in capital structure reflect underlying differences in constraint severity. This insight links directly to the international corporate finance literature, which establishes two robust facts. First, traditional capital structure determinants developed using U.S. data apply broadly across countries (Rajan and Zingales, 1995; Booth et al., 2001). Second, and more importantly for our purposes, institutional quality—particularly contract enforcement—plays a central role in shaping firm financing decisions, often outweighing industry-level factors (Demirgüç-Kunt and Maksimovic, 1999; Fan et al., 2012). Weaker enforcement is consistently associated with lower use of long-term debt, especially for small firms (Beck et al., 2008; Demirgüç-Kunt and Maksimovic, 1999, 1998), and with shorter debt maturity structures more generally (Qian and Strahan, 2007). Our model is designed to capture and interpret these patterns through the lens of endogenous borrowing constraints.

The remainder of the paper is organized as follows: Section 2 presents motivational evidence, Section 3 presents the baseline theory, Section 4 extends the theory to a stochastic setting, Section 5 tests the model's predictions using firm-level data, Section 6 quantifies the role of financial constraints in firm performance, and Section 7 concludes. All proofs, data description and alternative empirical specifications are presented in the Appendix.

2 Leverage and Development across Countries

We begin by documenting a stylized fact that motivates our theoretical framework: young firms' leverage is particularly sensitive to a country's level of contract enforcement.

2.1 Data and Variable Construction

Our analysis uses firm-level data from the ORBIS database compiled by Bureau van Dijk, following the data cleaning methodology of Kalemli-Özcan et al. (2024). ORBIS provides harmonized balance sheet information for private firms across countries, which we supplement with institutional measures from the World Bank's Doing Business indicators.[1]

[1] We defer detailed discussion of sample construction and firm selection criteria to Section 5 following our theoretical framework. The interested reader can find comprehensive details on data coverage and sample restrictions in Appendix A.

We define leverage ratios using a consistent denominator across all measures:

$$\text{Short-term Leverage} = \frac{\text{Short-term Debt}}{\text{Short-term Debt} + \text{Long-term Debt} + \text{Equity}}$$

$$\text{Long-term Leverage} = \frac{\text{Long-term Debt}}{\text{Short-term Debt} + \text{Long-term Debt} + \text{Equity}}$$

This formulation follows Rajan and Zingales (1995) and Welch (2011) in using total financing sources as the denominator, ensuring that leverage ratios remain invariant to changes in non-debt financing sources.[2] Our theoretical framework in Section 3 provides additional justification for this approach: when firms optimally choose distinct debt instruments with different maturities, examining the composition of debt financing becomes essential for identifying financial constraints.

2.2 Leverage Patterns and Institutional Quality

While the international corporate finance literature has established that institutional quality shapes capital structure choices across countries, these findings typically focus on aggregate firm populations or mature firms. We extend this analysis specifically to young firms—the primary object of our study—and document that enforcement quality effects are particularly pronounced during firms' early years.

The established literature shows that firms in more financially developed economies exhibit higher long-term leverage and lower short-term leverage (Demirgüç-Kunt and Maksimovic, 1999; Fan et al., 2012). The economic intuition is that weak enforcement environments favor short-term debt, which provides creditors greater control, while long-term debt contracts rely more heavily on legal institutions for enforcement of covenants and collateral arrangements.

[2] Equity is constructed as total assets minus current liabilities minus non-current liabilities, following Kochen (2022). Our qualitative results are unchanged when using shareholders' capital to measure equity.

Our contribution is to establish that these patterns hold and are amplified for young firms specifically. Since we use capital structure dynamics to measure financial constraints, demonstrating that young firms exhibit systematic cross-country differences in leverage composition is essential for our identification strategy. We formalize this relationship by estimating:

$$\text{Leverage}_{ict} = \alpha + \beta_1 \text{Recovery Rate}_c + \beta_2 \text{Recovery Rate}_c \times \text{Young}_{it} + \mathbf{X}_{it}' \gamma + \varepsilon_{ict} \qquad (1)$$

where Recovery Rate_c measures enforcement quality, Young_{it} indicates firms aged 1–11 years, and $\mathbf{X}_{it}$ includes firm-level controls and fixed effects.

Table 1: Enforcement Quality and Leverage: Young vs. Mature Firms

	Leverage Ratios		
	Long-Term (1)	Short-Term (2)	Total (3)
Recovery Rate	0.00338***	0.000212	0.00359***
(Mature Firms)	(0.000206)	(0.000154)	(0.000196)
Recovery Rate × Young	0.000550***	0.000214***	0.000764***
	(0.0000736)	(0.0000358)	(0.0000765)
Total Effect (Young)	0.00393***	0.000426***	0.00435***
($\beta_1 + \beta_2$)	(0.000246)	(0.000156)	(0.000221)
Observations	36,630,806	36,630,806	36,630,806
R-squared	0.145	0.033	0.123
Countries	46	46	46

Notes: Sample restricted to countries with 4,000 eligible firms (observed $\geq$4 times in ages 0-5). Recovery Rate measured as percentage recovery in insolvency proceedings (World Bank Doing Business). Young firms are those aged 11 years. All regressions include firm controls (log employment, ROA, tangible assets ratio) and industry and year fixed effects. Standard errors (in parentheses) clustered at industry×country level. *** p<0.01, ** p<0.05, * p<0.10.

Table 1 confirms that known cross-country patterns for long-term and total leverage extend to young firms with amplified effects. Long-term leverage increases significantly with enforcement quality for all firms ($\beta_1 = 0.00338$), but the effect is substantially stronger for young firms ($\beta_1 + \beta_2 = 0.00393$). The same is true for total leverage. Short term leverage, on the other hand, is effectively unresponsive to the degree of institutional quality.

These results establish that young firms, precisely those most likely to face financial constraints, exhibit the strongest sensitivity to institutional quality in their capital structure choices. This systematic variation provides the foundation for using leverage dynamics to identify and quantify financial constraints during firms' early life cycles.

3 Model

Motivated by the fact that contract enforcement differences affect young firms' leverage decisions, we develop a dynamic model of firm financing under limited enforcement. Firms borrow externally to finance production and face borrowing constraints that endogenously relax over time as they accumulate repayment histories. We derive theoretical predictions from the model regarding leverage dynamics and we test these predictions using Orbis data in subsequent sections.

3.1 Environment and Optimal Financial Structure

Firms operate a decreasing returns to scale technology with a quasi-fixed factor that captures physical capital or blueprint capacity driven by managerial talent (K) and a flexible factor such as labor (n). The firm can only choose K at inception, after which this factor remains fixed. Each period a is divided into two subperiods, morning and night. During the morning, firms commence production, pay for inputs, and make savings decisions. During the night, production and revenues are realized, and firms decide whether to continue operations or strategically default. Conditional on continuation, firms make dividend, savings, and debt repayment decisions. At the end of each period, firms face an exogenous exit shock with probability $1 - \rho$.

A firm's productivity at time t is given by z_{it}, which we initially treat as constant over time ($z_{it} = z$). In Section 4, we extend the framework to incorporate firm-level heterogeneity where $z_{it} = \chi_i \tilde{z}_{it}$, with χ_i representing a permanent firm-specific component and $\tilde{z}_{it}$ following a persistent stochastic process.

The financial environment is characterized by three key assumptions:

Assumption 1 (Property Rights) *There exists an enforcement technology ξ in the economy that can be applied to promises codified in contracts. Registering a transaction in a contract requires paying a fixed cost $\kappa_C > 0$. The technology ξ imposes a penalty $(1-\xi)$ when the borrower engages in strategic default, but not to default induced by exit shocks.*

Assumption 2 (Transaction Costs) *The contracting costs $\kappa_C > 0$ are not too high:*

$$\kappa_C < S_0(K(\xi_L), z_L)\frac{1-\rho}{\rho}$$

where $S_0(K(\xi_L), z_L)$ denotes the short-term debt that the least productive firm (z_L) would request in the first year of operations in the country with worst enforcement (ξ_L).

Assumption 3 (Minimum Scale) *Parameters are such that:*

$$\frac{\beta\rho(1+r_f)+\beta(1-\rho)}{(1-\beta\rho)(1+r_f)} > \frac{1-\eta}{\eta\alpha}$$

Throughout the main text, we assume that capital has no collateral value ($\phi = 0$) to focus on the pure effects of enforcement frictions. The general model with collateral ($\phi > 0$) is presented in the Online Appendix, where we show that the key mechanisms and empirical predictions remain robust.

Proposition 1 (Financial Structure) *Under Assumptions 1-3, the optimal financial structure consists of:*

1. *Long-term debt to fund the initial capital acquisition*

2. A sequence of short-term loans to fund working capital needs

This result establishes the optimality of different debt instruments for different purposes, corresponding to the corporate finance principle of maturity matching. The model provides a disciplined approach to leverage measurement, with clear empirical counterparts for both types of debt.

For notational simplicity, we denote long-term debt as $L = \{L_0; \{q_a\}_{a \geq 0}\}$ and short-term loans as $S = \{\{S_a; (1 + r_f)S_a\}\}_{a=0}^{\infty}$, where q_a represents the coupon payment at age a for long-term debt, and S_a represents short-term borrowing at age a.

3.2 Firm's Problem Under Limited Enforcement

When enforcement is imperfect ($\xi < 1$), firms can engage in strategic default, keeping $(1-\xi)$ of realized revenues. Financiers only sign contracts that are enforceable, meaning they don't incentivize the firm to strategically default.

Let V_a denote the value of the firm at age a, defined as the expected present value of all future profits:

$$V_a = \sum_{j=0}^{\infty} (\beta\rho)^j \left(z \left[K^\alpha \left(\frac{S_{a+j}}{w} \right)^{1-\alpha} \right]^\eta - (1 + r_f)S_{a+j} - q_{a+j} \right)$$

The firm's problem under limited enforcement is to maximize the expected present value of dividends:

$$V_0 = \max_{\{K, \{q_a, S_a, n_a\}_{a=0}^{\infty}\}} \sum_{a=0}^{\infty} (\beta\rho)^a \left(z \left[K^\alpha \left(\frac{n_a}{w} \right)^{1-\alpha} \right]^\eta - (1 + r_f)S_a - q_a \right)$$

subject to:

$$w_a n_a \leq S_a \tag{WCC}$$

$$p_k K(1+r_f) \leq \sum_{a=0}^{\infty} (\beta\rho)^a q_a \tag{PCF}$$

$$0 \leq q_a \leq z\left[K^{\alpha}\left(\frac{R_a}{w}\right)^{1-\alpha}\right]^{\eta} - (1+r_f)S_a \tag{NNC}$$

$$(1-\xi)z\left[K^{\alpha}\left(\frac{S_a}{w}\right)^{1-\alpha}\right]^{\eta} \leq V_a \quad \forall a \tag{LE}$$

where (WCC) is the working capital constraint, (PCF) is the participation constraint of financiers, (NNC) is the non-negativity constraint on coupons, and (LE) is the limited enforcement constraint.

Proposition 2 (Optimal Financial Contract) *For any scale K, a solution to the problem features:*

1. *Long-term debt payments structure:*

$$q_a = \begin{cases} \pi_a(K) & \textit{if } a < T(K) \\ \gamma\pi_a(K) & \textit{if } a = T(K) \\ 0 & \textit{if } a > T(K) \end{cases}$$

where $\pi_a(K) = z\left[K^{\alpha}\left(\frac{S_a(K)}{w}\right)^{1-\alpha}\right]^{\eta} - (1+r_f)S_a(K)$ and $\gamma \in (0,1]$ is the fraction of profits paid as debt service in the last period of debt repayment. We will refer to $T(K)$ as the maturity of long-term debt (i.e., the last period where $q_a > 0$).

2. *Increasing short-term debt for $a \in [0, \hat{a}]$ where $\hat{a} \leq T(K)$:*

$$S_a(K) = \begin{cases} \Theta_a(\gamma) \cdot S_u(K, z) & \text{if } a < \hat{a} \\ S_u(K, z) & \text{if } a \geq \hat{a} \end{cases}$$

where the constraint multiplier is

$$\Theta_a(\gamma) \equiv \left[\frac{(\beta\rho)^{T-a}[\beta\rho + (1-\beta\rho)(1-\gamma)]}{(1-\beta\rho)(1-\xi)} (1 - \eta(1-\alpha)) \right]^{\frac{1}{\eta(1-\alpha)}}$$

and $S_u(K) = w \left[\frac{z\eta(1-\alpha)}{w(1+r_f)} \right]^{\frac{1}{1-\eta(1-\alpha)}} K^{\frac{\eta\alpha}{1-\eta(1-\alpha)}}$ represents the unconstrained level of short-term debt conditional on K.

3. *Dividends at each age are determined by $d_a = \pi_a - q_a$ and $n_a = S_a/w$.*

The optimal contract has several key features that help explain our empirical findings on leverage dynamics. First, long-term debt payments are front-loaded, with the firm paying all profits toward debt until maturity. Second, dividends are back-loaded, with no dividends paid until debt is fully repaid. Third, and most importantly for our analysis, short-term debt increases until reaching the unconstrained level at age $\hat{a}$, while long-term debt decreases as the firm pays it down. This creates a pattern of increasing short-term leverage and decreasing long-term leverage during what we define as the Early Life Cycle (ELC).

It is worth noting that while the solution is unique up to age $\hat{a}$, multiple implementations are possible afterward. We focus on the implementation that prioritizes early repayment of long-term debt, which maximizes balance sheet capacity. This approach would be optimal if firms faced even a small probability of new investment opportunities in the future. The characterization of the ELC duration ($\hat{a}$) is particularly valuable as it allows us to empirically identify when firms become unconstrained in their short-term borrowing, which we will later use to measure funding gaps across countries with different enforcement levels.

3.3 Early Life Cycle Duration

Proposition 2 implies that firms are financially constrained at age a (i.e., $S_a < S_u$) if:

$$\Theta_a(\gamma) < 1 \Leftrightarrow \beta\rho^{T-a} \cdot [\beta\rho + (1-\beta\rho)(1-\gamma)](1-\eta(1-\alpha)) < (1-\beta\rho)(1-\xi)$$

A key insight of our model is the characterization of the Early Life Cycle (ELC) duration, denoted $\hat{a}$. This represents the age at which firms are no longer financially constrained in their short-term borrowing:

$$\hat{a} = \left\lceil T - \frac{\ln\left(\frac{(1-\beta\rho)(1-\xi)}{(1-\eta(1-\alpha))[\beta\rho+(1-\beta\rho)(1-\gamma)]}\right)}{\ln(\beta\rho)} \right\rceil$$

where $\lceil \cdot \rceil$ denotes the ceiling function.

The derivative of ELC duration with respect to enforcement is:

$$\frac{\partial \hat{a}}{\partial \xi} = \frac{\partial T}{\partial \xi} + \frac{1}{(1-\xi)\ln(\beta\rho)}$$

Since $\ln(\beta\rho) < 0$, the direct effect $\frac{1}{(1-\xi)\ln(\beta\rho)}$ is negative, while the indirect effect through maturity $\frac{\partial T}{\partial \xi}$ is positive.

Proposition 3 (Discrete Changes in Optimal Maturity) *The optimal maturity $T(\xi)$ is a non-decreasing function of enforcement quality ξ. For any continuous change in ξ, the resulting change in optimal maturity ΔT can only take values in $\{0, 1\}$:*

$$\frac{\partial T(\xi)}{\partial \xi} \in \{0, 1\}$$

Proposition 3 has important implications for the relationship between $\hat{a}$ and ξ. Since

$\frac{\partial T(\xi)}{\partial \xi} \in \{0, 1\}$, we can bound the effect of enforcement on ELC duration:

$$\frac{\partial \hat{a}}{\partial \xi} \leq 1 + \frac{1}{(1-\xi)\ln(\beta\rho)}$$

For typical parameter values (e.g., $\beta\rho \approx 0.5$ and $\xi \geq 0.1$), this bound implies that $\frac{\partial \hat{a}}{\partial \xi} \leq 0$, indicating that stronger enforcement reduces ELC duration. Firms in countries with better enforcement institutions exit their early life cycle faster, becoming financially unconstrained earlier.

3.4 Optimal Scale Under Limited Enforcement

Having characterized the ELC duration, we now address the optimal scale determination. Due to the non-differentiable nature of the problem, we first solve an auxiliary problem where we optimize over discrete pairs of scale K and maturity T, assuming full debt repayment at maturity ($\gamma = 1$).

Definition 1 (Feasible Discrete Scale and Maturity) *Let F be the set of pairs of T and maximum feasible scale K_T attainable with long-term debt of maturity T. And define the value of the firm with capital K_T and long-term debt maturity T by $V(T)$.*

Proposition 4 (Optimal Scale and Maturity with Limited Enforcement) *Let $\tilde{T}$ be the discrete maturity solving:*

$$\tilde{T} = \min\left\{T \in \mathbb{N} \mid \frac{V(T+1)}{V(T)} < 1\right\}$$

Then the optimal scale K^* solves:

$$K^* = \arg\max_{K \in [K_{\tilde{T}}, K_{\tilde{T}+1}]} V(\tilde{T}+1, K)$$

where $V(T, K)$ denotes the firm's value function at maturity T and scale K.

Given K^*, the optimal maturity is:

$$T^* = \tilde{T} + \mathbf{1}_{\gamma(K^*)>0}$$

where the repayment share γ^* and enforcement duration $\hat{a}^*$ are determined endogenously from (T^*, K^*) and satisfy the participation and enforcement constraints.

The fundamental trade-off in our setting involves balancing higher scale against earlier dividend payments. By choosing a scale K below the maximum feasible scale $K_{\tilde{T}+1}$ for maturity $\tilde{T}+1$, the firm can receive partial dividends at time $\tilde{T}+1$ (when $\gamma(K) < 1$) rather than waiting until $\tilde{T}+2$ for the first dividend payment.

Proposition 5 (Enforcement, Scale, and Maturity) *The maximum feasible scale K_T for any given maturity T is strictly increasing in the enforcement parameter ξ. Additionally, the optimal maturity $T^*(\xi)$ is weakly increasing in ξ. Consequently, economies with lower enforcement exhibit both lower optimal firm scale K^* and shorter debt maturities.*

Interestingly, the relationship between scale and ELC duration operates primarily through the debt maturity channel. This happens because the direct effects of scale on enforcement constraints largely offset each other. Specifically, both the scale effect (increased short-term borrowing needs) and the value effect (enhanced continuation value) are proportional to $K^{\frac{\eta\alpha}{1-\eta(1-\alpha)}}$, causing them to cancel out in our framework.

This creates a clean theoretical prediction: the effect of scale on ELC duration works primarily through its impact on the optimal debt maturity structure. Consequently, countries with stronger enforcement institutions will have firms with larger scale and longer debt maturities, but shorter ELC durations. This reflects how enforcement quality affects both the firm's initial investment decision and its subsequent financial constraints.

3.5 Capital Structure and Leverage Dynamics

To analyze how capital structure evolves, we define leverage ratios to match our empirical measures. Let ℓ_a^S and ℓ_a^L denote short-term and long-term leverage at age a respectively:

$$\ell_a^S = \frac{S_a}{A_a} \quad \text{and} \quad \ell_a^L = \frac{L_a}{A_a}$$

where S_a represents short-term debt, L_a represents long-term debt, V_a net worth, and $A_a = V_a + S_a + L_a$ represents total assets. Based on our characterization of the optimal financial contract, we derive the following leverage dynamics:

Proposition 6 (Leverage Dynamics) *In an environment with enforcement quality ξ, the leverage ratios of a firm evolve during the early life cycle (ages $a \leq \hat{a}(\xi)$) as follows:*

$$\frac{\partial \ell_a^L}{\partial a} < 0 \quad \textit{and} \quad \frac{\partial \ell_a^S}{\partial a} > 0$$

The intuition behind these dynamics is that young firms start with high long-term leverage to fund their initial capital investment. As they age, they gradually pay down this long-term debt while simultaneously relaxing their short-term borrowing constraints, which increases short-term leverage. These opposing movements in leverage ratios continue until age $\hat{a}(\xi)$, when firms become unconstrained in their short-term borrowing.

3.6 Capital Structure and Development

Our model generates predictions about how enforcement quality influences leverage ratios across countries. These predictions are captured in the following proposition:

Proposition 7 (Enforcement and Leverage) *For enforcement quality $\xi < \xi^*$, where ξ^* is a threshold above which financial constraints cease to bind significantly, leverage ratios exhibit the following patterns:*

1. *For firms beyond age a^* (where $a^* \approx 1 - 2$ for typical parameter values), long-term leverage is monotonically increasing in enforcement quality:* $\frac{\partial \ell_a^L}{\partial \xi} > 0$ *for all* $a \geq a^*$.

2. *Short-term leverage is monotonically increasing in enforcement quality across all ages:* $\frac{\partial \ell_a^S}{\partial \xi} > 0$ *for all* $a \geq 0$.

The relationship between enforcement quality and leverage operates through a competition of elasticities among different balance sheet components. For very young firms (ages 0 to $a^* - 1$), enforcement quality has a disproportionate effect on short-term debt through the constraint multiplier Θ_a. This may create non-monotonic patterns in long-term leverage for these youngest firms. Beyond age a^*, the elasticity of long-term debt dominates, creating a monotonically positive relationship with enforcement quality.

The cross-country implications of our model offer clear predictions: firms in economies with stronger enforcement institutions exhibit higher long-term leverage compared to their counterparts in weaker enforcement environments. Our model reveals that the optimal scale-maturity choice varies systematically with enforcement quality. As enforcement improves, firms can support both larger initial scale and longer debt maturities simultaneously. This occurs because better enforcement allows firms to credibly commit to repaying larger debt amounts over extended periods, increasing their debt capacity at inception and enabling greater upfront investment.

This scale-maturity relationship contrasts sharply with predictions from traditional models with exogenous setup costs. In those frameworks, firms in less developed economies would paradoxically require longer debt maturities to finance identical fixed costs, producing counterfactual predictions about debt maturity across development levels. Our model with endogenous scale choice correctly predicts the empirical pattern that firms in more developed economies utilize both higher initial scale and longer debt maturities.

Enforcement quality also shapes the dynamic pattern of leverage over firms' life cycles.

In economies with higher enforcement quality, firms exit their early life cycle faster (smaller $\hat{a}(\xi)$), meaning the period characterized by increasing short-term leverage and decreasing long-term leverage is shorter. This explains our empirical finding that the distinctive leverage pattern of young firms persists longer in less developed economies.

These capital structure patterns provide a novel window into the severity of financial constraints across countries. The joint dynamics of short-term and long-term leverage—particularly the duration of their opposite movements—reveal valuable information about firms' financial constraints that cannot be captured by examining total leverage alone.

4 Firm Heterogeneity, Age, and Size: Stochastic Productivity Extension

The baseline model generates clear predictions about leverage dynamics across countries but does not distinguish between age and size effects, as these dimensions are perfectly correlated in a deterministic framework Cooley and Quadrini (2001). To address this limitation and draw more nuanced empirical implications, we now extend our analysis to incorporate firm-level heterogeneity and stochastic productivity.

4.1 Environment with Firm Heterogeneity

A firm is indexed by its initial type $\chi_i \in \mathbb{R}_+$, drawn at entry from a distribution $F(\chi)$, and faces a time-varying idiosyncratic productivity component $\tilde{z}_{it}$. The firm's total productivity at time t is:

$$z_{it} = \chi_i \tilde{z}_{it}$$

We assume that the idiosyncratic component $\{\tilde{z}_{it}\}$ follows a persistent stochastic process that is identical across firms. While each firm experiences its own realization of shocks, the persistence in both the firm-specific component χ_i and the productivity process generates heterogeneity in observed outcomes even among firms of the same age.

The firm operates a production technology:

$$y_{it} = z_{it}\left[K^{\alpha}\left(\frac{n_{it}}{w}\right)^{1-\alpha}\right]^{\eta} = \chi_i \tilde{z}_{it}\left[K^{\alpha}\left(\frac{n_{it}}{w}\right)^{1-\alpha}\right]^{\eta}$$

As in the baseline model, firms maximize expected lifetime value by choosing capital stock, maturity structure, and state-contingent financing.

4.2 Age as a Sufficient Statistic for Constrained Exit

A key insight from our framework is formalized in the following proposition:

Proposition 8 (Age as a Sufficient Statistic for Constrained Exit) *Under the stochastic productivity structure with permanent component χ_i and transitory component $\tilde{z}_{it}$, the early life cycle duration $\hat{a}$ - defined as the age at which a firm exits the constrained regime ($\Theta_a = 1$) - is independent of the firm's permanent productivity component χ_i.*

The enforcement constraint can be expressed as a borrowing limit $S_a \leq \Theta_a \cdot S_u(K, \chi_i, z_a)$, where Θ_a is a constraint multiplier capturing the severity of financial frictions. While a higher χ_i increases a firm's unconstrained borrowing needs S_u, it also proportionally increases the firm's repayment capacity through higher future profits. These two forces exactly offset each other in the determination of Θ_a. Specifically, both S_a and $S_u(K, \chi_i, z_a)$ scale with χ_i^{θ}, causing the permanent productivity term to cancel out in the ratio $\frac{S_a}{S_u(K,\chi_i,z_a)} = \Theta_a$. Thus, the age $\hat{a}$ at which Θ_a reaches 1 depends only on the productivity path $\tilde{z}_a$, contract structure (K, T), and enforcement parameter ξ - not on χ_i.

This result highlights the fundamental identification advantage of using age rather than size to measure financial constraints. Constraint tightness evolves predictably with age as firms accumulate repayment histories, while size conflates constraint status with inherent productivity differences that are unobservable to the econometrician.

4.3 Expected ELC Duration and Empirical Identification

For empirical identification, we consider the expected ELC duration at the cohort or industry level. The expected age at which firms exit their constrained period can be expressed as:

$$E[\hat{a}] = \hat{a}^{det}(\xi, \gamma) - \Omega(z)$$

where $\hat{a}^{det}(\xi, \gamma)$ is the deterministic ELC duration from our baseline model:

$$\hat{a}^{det}(\xi, \gamma) = T - \frac{\ln\left(\frac{(1-\beta\rho)(1-\xi)}{(1-\eta(1-\alpha))[\beta\rho+(1-\beta\rho)(1-\gamma)]}\right)}{\ln(\beta\rho)}$$

and $\Omega(z)$ is an adjustment term that depends on the properties of the productivity process:

$$\Omega(z) = \frac{\ln\left(\frac{E[z_a^\theta]}{M(z_a,T_{min})} \cdot \frac{[\beta\rho+(1-\beta\rho)(1-\gamma)]}{(1-\eta(1-\alpha))}\right)}{\ln(\beta\rho)}$$

where $M(z_a, T_{min})$ captures the weighted expectation of future productivity at maturity:

$$M(z_a, T_{min}) = \sum_{j=0}^{\Delta T} P(T_{min} + j|z_a)(\beta\rho)^j \left((1-\gamma)(1-\beta\rho) \cdot E[z^\theta_{T_{min}+j}|z_a] + \beta\rho \cdot E[z^\theta_t|z_a]\right)$$

When productivity follows an i.i.d. process, the ratio $\frac{E[z_a^\theta]}{M(z_a,T_{min})}$ equals one, yielding $\Omega(z) = 0$, and the expected ELC duration equals the deterministic duration. With an AR(1) process, the adjustment term depends on persistence, volatility, and the gap between constraint exit and maturity. However, the comparative statics with respect to enforcement quality ξ remain robust even with persistent productivity shocks.

This result provides theoretical justification for our empirical approach. When estimating ELC duration at the industry level, we're effectively averaging over many productivity realizations, which approximates the expected duration under the stationary distribution of the productivity process. Cross-country differences in this duration primarily reflect differences in enforcement quality rather than productivity dynamics.

4.4 Size-Based vs. Age-Based Identification

Our stochastic model demonstrates why size-based measures of financial constraints are confounded by unobserved heterogeneity. Two firms with identical constraint status (same Θ_a) may exhibit vastly different sizes due solely to differences in permanent productivity χ_i.

Observed firm size—whether measured by capital K, assets, or revenue—is directly affected by the unobserved heterogeneity term χ_i. Since K scales with $\chi_i^{\theta_K}$ (where $\theta_K = \frac{\theta}{1-\eta\alpha\theta}$), a firm with high χ_i may appear large even while facing binding borrowing constraints. Conversely, a firm with low χ_i may appear small despite having relaxed its enforcement constraints.

This separation between size and constraint status highlights why age offers a more reliable indicator of borrowing constraint tightness in our framework. As we showed in Proposition 8, the age at which firms exit the constrained regime ($\hat{a}$) is independent of firm-level productivity χ_i, making it a robust metric for comparing constraint severity across firms and countries.

While our model uses the unconstrained short-term debt level S_u as a benchmark, empirical studies often employ earnings-based metrics such as EBITDA multiples. As we show in Appendix C, our constraint can be reformulated as $S_a \leq \lambda^a_{EBITDA} \cdot EBITDA_a$, where the multiplier λ^a_{EBITDA} varies with age and enforcement quality. This formulation connects to empirical leverage rules while preserving the insight that constraint tightness evolves dynamically over the firm's life cycle, consistent with the endogenous borrowing constraint literature.

5 Life-Cycle Leverage Dynamics

Our theoretical framework generates sharp predictions about how young firms' capital structure evolves as enforcement constraints gradually relax. This section tests these predictions using firm-level data, focusing on leverage dynamics during the early life cycle.

5.1 Empirical Strategy

We estimate leverage trajectories over the firm life cycle using two complementary specifications that isolate age effects from confounding factors. Our primary approach employs an age-period-cohort (APC) framework that separates life-cycle effects from time-varying economic conditions and cohort-specific characteristics. This methodology has proven particularly valuable in firm dynamics research, as demonstrated by Argente et al. (2024) in their analysis of the life cycle of products and Kochen (2022) in studying financing dynamics over the firm life cycle.

5.1.1 Age-Period-Cohort Specification

Following Argente et al. (2024), the baseline specification takes the form:

$$\text{Leverage}_{ict} = \sum_{a=1}^{A} \beta_a \cdot \mathbf{1}[\text{Age}_{it} = a] + \sum_{c} \delta_c \cdot \text{Cohort}_c^N + \alpha_{jst} + \varepsilon_{ict}$$

where Leverage_{ict} denotes the leverage ratio of firm i in country c at time t. The coefficients β_a capture age effects relative to the reference category (age 0), while Cohort_c^N represents Deaton-normalized cohort effects that constrain two adjacent cohort coefficients to lie on a linear trend (Deaton, 1997). The fixed effects α_{jst} absorb all time-varying shocks at the country-sector-year level.

5.1.2 Within-Firm Specification

To control for unobserved firm heterogeneity, we complement the APC approach with a within-firm specification:

$$\text{Leverage}_{ict} = \sum_{a=1}^{A} \beta_a \cdot \mathbf{1}[\text{Age}_{it} = a] + \mu_i + \alpha_{jst} + \varepsilon_{ict}$$

where μ_i denotes firm fixed effects. This specification identifies age effects exclusively from within-firm variation, eliminating concerns about selection or unobserved heterogeneity driving the results. We present the results from this specification in Appendix A.9.

5.1.3 Leverage Measures

We employ the leverage ratio definitions established in Section 2.1, which follow Rajan and Zingales (1995) and Welch (2011) in using total financing sources as the denominator. This formulation ensures our measures remain invariant to changes in non-debt financing sources and facilitates comparison with the theoretical predictions derived in Section 3.

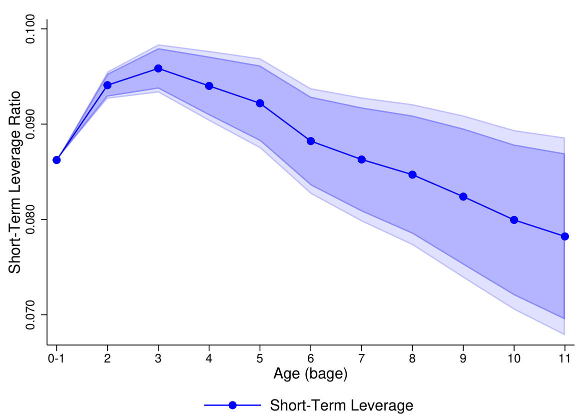
5.2 Identification and Sample Construction

The APC identification strategy addresses the standard collinearity problem through Deaton normalization of cohort effects combined with high-dimensional fixed effects. As shown by Kochen (2022), this approach effectively separates age effects from period and cohort influences in firm financing decisions. Country-sector-year fixed effects absorb period-specific shocks while allowing age effects to be identified through within-cohort variation across calendar years. Standard errors are clustered at the country-sector level to account for potential correlation in financing patterns within industries and countries.

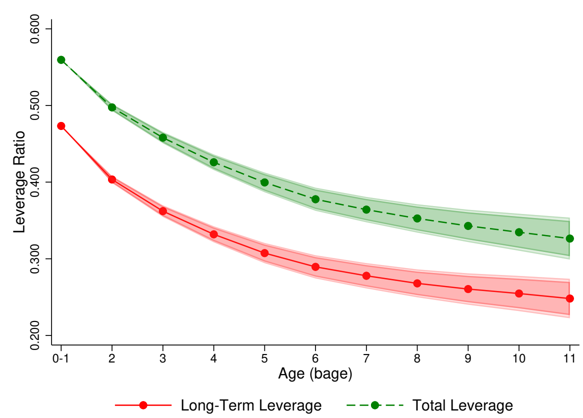
Our sample focuses on firms with demonstrable access to long-term debt markets, defined as reporting positive long-term debt in either of their first two years of operation. This restriction ensures that observed financing patterns reflect responses to institutional constraints rather than heterogeneous entrepreneurial preferences. As documented in Appendix A, firms without early long-term debt access exhibit systematically different growth patterns consistent with lifestyle entrepreneurship rather than binding financial constraints.

5.3 Life-Cycle Patterns in Capital Structure

Figure 1 presents our main results from the APC specification. The patterns provide strong support for our theoretical framework's key predictions.



(a) Short-Term Leverage



(b) Long-Term and Total Leverage

Figure 1: Capital Structure Evolution: Age-Period-Cohort Estimates

Notes: Coefficients from age-period-cohort regressions with country-sector-year fixed effects and Deaton normalization. Age 0 serves as the reference category. Shaded regions represent 90% and 95% confidence intervals. Standard errors clustered at the country-sector level. Sample includes firms with positive long-term debt in their first two years, observed for at least three years.

Panel 1a reveals the distinctive hump-shaped pattern in short-term leverage predicted by our model. Young firms initially exhibit low short-term leverage ratios, reflecting binding enforcement constraints that limit their access to working capital financing. As firms age and accumulate repayment histories, short-term leverage increases, peaking around ages 4–6. This pattern precisely matches our theoretical prediction that the constraint multiplier Θ_a increases during the early life cycle as firms pay down long-term debt.

Panel 1b shows that long-term leverage follows the opposite trajectory, declining monotonically with age. This reflects the front-loaded repayment structure that emerges optimally under enforcement constraints: firms use long-term debt to finance their initial capital investment and gradually pay it down over their early life cycle. Total leverage also declines with age but more gradually, as the rise in short-term leverage partially offsets the decline in long-term leverage during early years.

These opposing dynamics—increasing short-term leverage and decreasing long-term leverage—would be invisible in analyses that focus solely on total leverage. The decomposition reveals the central insight of our framework: enforcement constraints create a systematic rebalancing of capital structure during the early life cycle, specifically, firms decrease long-term debt while increasing short-term financing as constraints relax.

5.4 Cross-Country Evidence: Early Life Cycle Duration and Enforcement Quality

Our theoretical framework predicts that weaker contract enforcement prolongs financial constraints, yielding a longer early life cycle duration ($\partial \hat{a}/\partial \xi < 0$). We test this prediction using grouped Deaton-style age-period-cohort (APC) analyses across countries classified by recovery rates from the World Bank's Doing Business indicators (World Bank, 2020), which measure the percentage of debt recovered by secured creditors in insolvency.

Countries are grouped into low enforcement (recovery rates below 40%), medium enforcement (40–75%), and high enforcement (above 75%). Figure 2 shows the evolution of short-term leverage by firm age for these groups. In low enforcement countries, leverage

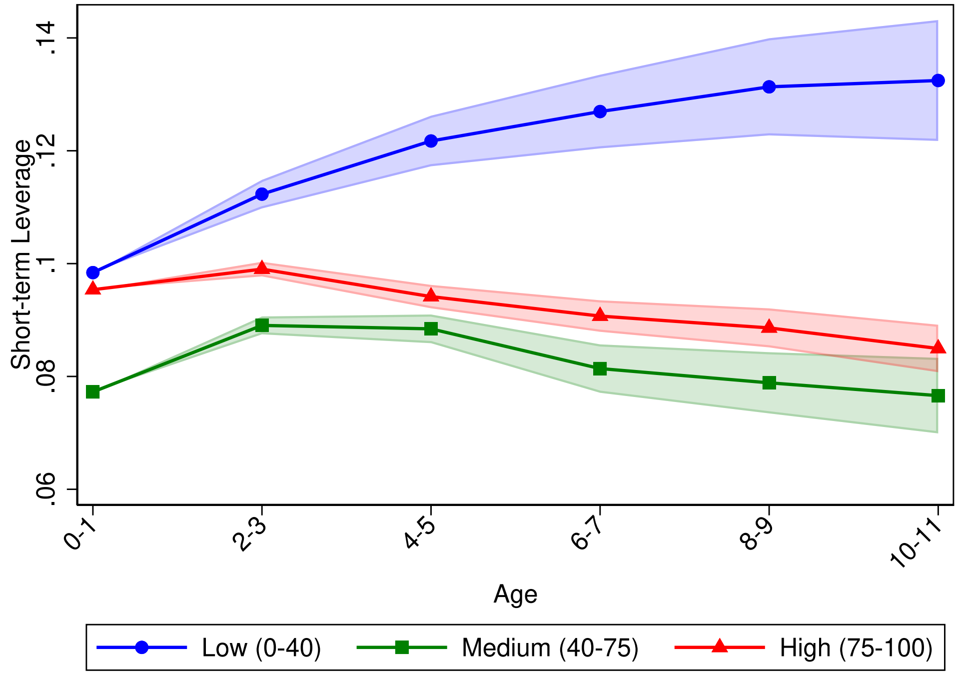


FIGURE 2: SHORT-TERM LEVERAGE LIFE CYCLE BY RECOVERY RATE GROUP

NOTES: Predicted short-term leverage profiles by firm age for country groups classified by recovery rates: low (below 45%), medium (45–75%), and high (above 75%). Age bins are two-year intervals with 90% confidence bands. In low enforcement countries, leverage continues to rise significantly up to ages 4–5. In medium and high enforcement groups, the last significant increase occurs already by ages 2–3, with flatter slopes indicating quicker convergence to target leverage ratios.

continues to rise significantly until agebin 4–5, indicating a prolonged period of elevated financial constraints. By contrast, in both medium and high enforcement groups, the last significantly positive increase in leverage occurs already at agebin 2–3.

Moreover, consistent with our theory that stronger enforcement relaxes borrowing constraints from the outset, we observe a notably flatter life cycle in high enforcement countries: firms start closer to their target leverage and require smaller adjustments over time. This contrasts with the steeper profiles in low enforcement environments, where firms take longer to build up leverage as constraints gradually relax.

These findings provide robust empirical support for the model's prediction that weak enforcement extends the early life cycle duration and shapes the entire leverage trajectory. The systematic differences across institutional environments confirm that contract enforcement plays a central role in determining both the speed and extent of firms' financial convergence.

6 Decomposing Cross-Country Performance Gaps: Financial Frictions versus Entrepreneurial Talent

Cross-country differences in young firm performance could reflect either institutional constraints that distort optimal choices or fundamental differences in entrepreneurial productivity. This distinction is crucial for policy: if profit gaps stem from financial frictions that prevent firms from choosing efficient scales, institutional reforms could unlock substantial growth. If they reflect immutable differences in entrepreneurial talent, such reforms would have limited impact.

Building on our theoretical framework, we develop a novel decomposition that separates these channels using a key insight: initial leverage ratios depend on enforcement quality but are invariant to entrepreneurial talent. This exclusion restriction allows us to identify institutional parameters and decompose observed performance variation into correctable institutional distortions versus fundamental productivity differences.

Our approach reveals that what empirical studies commonly observe as ex-ante heterogeneity in firm outcomes actually combines two distinct sources: truly exogenous entrepreneurial talent and endogenously chosen initial scale that depends on the institutional environment. Only the latter component is amenable to policy intervention through institutional reform.

6.1 Theoretical Foundation

Our focus on long-term leverage as the key identifying moment is grounded in our model's prediction of optimal maturity matching, which aligns with the well-documented practice in corporate finance (Graham and Harvey, 2001). As shown in Proposition 1, our model endogenously generates a financial structure where long-term debt funds initial capital acquisition while short-term loans finance working capital needs. The firm's scale choice K_i represents its blueprint capacity—a long-term, quasi-fixed investment that determines the firm's productive potential throughout its early life cycle. Our theoretical framework demonstrates that such long-term assets should optimally be financed through long-term debt rather than short-term borrowing.

This theoretical connection provides crucial intuition for our identification strategy. If enforcement frictions distort the firm's optimal scale choice, preventing it from choosing the efficient level K^u, this distortion should manifest most clearly in the long-term financing decisions that fund this scale. Moreover, because both the scale choice K_i and its financing through long-term debt L_0 are made simultaneously at entry, the long-term leverage ratio $\ell_0^L = L_0/(L_0 + V_0 + S_0)$ provides a direct window into how enforcement constraints shape this fundamental investment decision. This makes long-term leverage the natural place to look for enforcement-driven distortions in firm scale, consistent with both our theoretical framework and empirical regularities in corporate finance.

Proposition 9 (Leverage Invariance) *Initial long-term leverage ℓ_0^L depends on enforcement ξ_c and initial shock z_{i0} but is invariant to entrepreneurial talent χ_i.*

This invariance property provides our key identifying moment. Both debt L_0 and firm value V_0 scale proportionally with χ_i^θ, as does initial short-term borrowing S_0. The talent term therefore cancels in the leverage ratio, leaving only enforcement and shock effects. This result is central to our identification strategy and distinguishes our approach from existing methods.

6.2 Asset-Normalized Short Term Debt and the Leverage Connection

Consider the unconstrained level of short-term debt:

$$S_{i,u}(K, \chi_i, z) = \frac{w^{1-\theta}}{[(1+r_f)p_k]^{\eta\alpha\theta}} \left(\frac{\eta(1-\alpha)}{(1+r_f)}\right)^{\theta} \cdot (\chi_i)^{\theta} \cdot (z_i)^{\theta} (\ell_{i,0}^{L})^{\eta\alpha\theta} A_{i,0}^{\eta\alpha\theta}$$

Rearranging and taking expectations over z, χ we get:

$$\frac{\mathbb{E}[\frac{S^{LE}}{A_{0,LE}^{\eta\alpha\theta}}]}{\mathbb{E}[\frac{S^{HE}}{A_{0,HE}^{\eta\alpha\theta}}]} = \left(\frac{w^{HE}}{w^{LE}}\right)^{\eta(1-\alpha)\theta} \frac{\mathbb{E}[\chi_{i,LE}^{\theta}]}{\mathbb{E}[\chi_{i,HE}^{\theta}]} \frac{\mathbb{E}[(\ell_{0,LE}^{L})^{\eta\alpha\theta}]}{\mathbb{E}[(\ell_{0,HE}^{L})^{\eta\alpha\theta}]}$$

This expression directly links asset-normalized short-term debt to initial leverage, with a known elasticity $\eta\alpha\theta$ determined by the production technology. This decomposition uses the participation constraint $p_k(1+r_f)K = L_0$, which gives us $K/A_0 = L_0/[p_k(1+r_f)A_0] = \ell_0^L/[p_k(1+r_f)]$. Substituting this relationship and using the fact that leverage is invariant to talent, we can obtain a moment to compute differences in expected ex-ante heterogeneity χ across countries.

6.3 Empirical Implementation: Manufacturing

To quantify the role of entrepreneurial talent, we focus on the manufacturing sector. Table 2 reports the period-0 mean cross-firm leverage L_0 from the Orbis database, the mean wage rare W from the International Labor Organization, the mean scaled leverage moment $\tilde{\ell}_{SA} \equiv (\ell_0^L)^{\eta\alpha\theta}$, and the implied residual χ normalized so that $\chi_{\text{HE}} = 1$.

The residual χ is obtained by inverting

$$\frac{\mathbb{E}[S_{\hat{a}}/A_0^{\eta\alpha\theta} \mid g]}{W^{\,\eta(1-\alpha)\theta}\,\mathbb{E}[(\ell_0^L)^{\eta\alpha\theta} \mid g]} \;=\; \chi\,,$$

Table 2: Manufacturing: Period-0 moments and residual χ

Group	L_0	W	$\tilde{\ell}_{SA}$	χ
Low (0–45)	0.4194	2.9336	1.7302	0.5721
Medium (45–75)	0.4453	3.4368	2.0924	0.9332
High (75–100)	0.4768	1.4469	1.2026	1.0000

with calibration $\eta = 0.95$, $\alpha = 0.40$, $\theta = (1 - \eta(1-\alpha))^{-1} \approx 2.326$ so that $\delta = \eta\alpha\theta \approx 0.86$. Focusing on manufacturing thus isolates enforcement versus talent effects without confounding from sector-mix differences.

7 Measuring Financial Constraints: A Decomposition Approach

Our model provides a direct way to measure constraint severity using observable age profiles. The key insight is that at age $\hat{a}$, when enforcement constraints no longer bind, observed short-term debt reveals the unconstrained borrowing level: $S_{\hat{a},i} = S_u(K_i, \chi_i, z_{i\hat{a}})$.

Using this benchmark, we can measure within-firm constraint tightness as:

$$\frac{S_{a,i}}{S_{\hat{a},i}} = \Theta_a(\xi_c, z_i^a)$$

Taking expectations within a country yields an empirical moment that can be estimated from age profiles:

$$\mathbb{E}\left[\Theta_a(\xi_c, z_i^a)\right] = \mathbb{E}\left[\frac{S_{a,i}}{S_{\hat{a},i}}\right]$$

From Proposition 8, this measure is invariant to permanent productivity differences χ_i, making it a clean indicator of how enforcement constraints evolve with firm age.

However, this within-firm measure does not capture the *full effect* of financial constraints. Weak enforcement also distorts firms' initial scale choices, creating a persistent gap between actual and efficient borrowing capacity that within-firm variation cannot capture.

To capture the total effect, we decompose the borrowing distortion relative to the first-best:

$$\frac{S_{a,i}(K_i, \chi_i, z_{ia})}{S_u(K_{u,i}, \chi_i, z_{ia})} = \underbrace{\frac{S_{a,i}(K_i, \chi_i, z_{ia})}{S_{\hat{a},i}(K_i, \chi_i, z_{ia})}}_{\text{Within-firm constraint}} \cdot \underbrace{\frac{S_{\hat{a},i}(K_i, \chi_i, z_{ia})}{S_u(K_{u,i}, \chi_i, z_{ia})}}_{\text{Scale distortion}}$$

The first term captures within-firm constraint tightness $\Theta_a(\xi_c, z_i^a)$ that evolves as firms age. The second term reflects scale distortion from suboptimal initial capital choice.

Both components are invariant to permanent productivity χ_i. For scale distortion, with the normalization $z_{i0} = 1$, optimal capital choices are:

$$K_i = \kappa(\xi_c) \cdot \chi_i^{\kappa_\chi} \quad \text{and} \quad K_{u,i} = \kappa(1) \cdot \chi_i^{\kappa_\chi}$$

Therefore: $\frac{K_i}{K_{u,i}} = \frac{\kappa(\xi_c)}{\kappa(1)}$, which is constant within country, and gives us:

$$\frac{S_{\hat{a},i}}{S_u(K_{u,i}, \chi_i, z_{ia})} = \left(\frac{\kappa(\xi_c)}{\kappa(1)}\right)^{\eta\alpha\theta}$$

Taking expectations within a country yields our key empirical moment:

$$\mathbb{E}\left[\frac{S_{a,i}}{S_u(K_{u,i}, \chi_i, z_{ia})}\right] = \mathbb{E}\left[\Theta_a(\xi_c, z_i^a)\right] \cdot \left(\frac{\kappa(\xi_c)}{\kappa(1)}\right)^{\eta\alpha\theta}$$

The covariance between these components is zero because constraint tightness varies across firms due to productivity histories, while the scale distortion is constant within country. This multiplicative decomposition allows separate identification: the within-firm component can be estimated from age profiles within countries, while the scale distortion can be identified from cross-country comparisons of enforcement quality.

In practice, we measure these moments by taking averages across firms of the same age within each country, then comparing these averages across countries with different enforcement institutions.

7.1 Results

Table 3 reports constraint tightness patterns across enforcement regimes during firms' early life cycles. We interpret $\Theta_{agebin} > 1$ as indicating unconstrained firms. Constraint severity decreases with enforcement quality. In age bin 0–1, low enforcement economies exhibit the tightest constraints ($\mathbb{E}[\Theta_{agebin}] = 0.410$), with firms accessing only 41% of their unconstrained borrowing capacity. Medium and high enforcement economies show substantially looser initial constraints, with firms accessing 63% and 64% of capacity respectively ($\mathbb{E}[\Theta_{agebin}] = 0.633$ and 0.640).

Constraint relaxation dynamics differ markedly across regimes. High and medium enforcement economies reach unconstrained status ($\mathbb{E}[\Theta_{agebin}] = 1.000$) by age bin 2–3 with zero dropout rates. Low enforcement economies experience gradual relaxation, rising from 41% to 56% of capacity in age bin 2–3 (48.3% dropout rate) before reaching full capacity in age bin 4–5. This extended constraint period confirms that weak institutions prolong financial constraints.

Dropout patterns reveal heterogeneity in financial access. Initial dropout rates are substantial (37–42%) across all regimes, with low enforcement showing the highest rate (42.0%) despite having the most severe average constraints among remaining firms. This suggests greater heterogeneity in weak enforcement environments or data quality.

Panel B shows initial leverage ratios relative to high enforcement economies. Medium (0.907) and low enforcement (0.903) economies begin with nearly identical, slightly lower leverage ratios. This similarity in starting conditions makes the divergent constraint evolution particularly notable—institutional differences manifest through differential access to short-term debt rather than initial leverage positions.

These results demonstrate that enforcement quality determines both constraint severity and relaxation speed, fundamentally altering firm financial development trajectories during the critical early life cycle.

Table 3: Within-Firm Constraint Tightness by Age Bin During Early Life Cycle

Age Bin	High Enforcement (Recovery Rate > 75%)	Medium Enforcement (45% < RR ≤ 75%)	Low Enforcement (RR ≤ 45%)
Panel A: Expected Constraint Tightness $\mathbb{E}[\Theta_{agebin}]$			
0–1	0.640	0.633	0.410
	(133,497)	(131,397)	(16,676)
	[37.4%]	[36.9%]	[42.0%]
2–3	1.000	1.000	0.560
	(264,059)	(277,757)	(16,963)
	[0.0%]	[0.0%]	[48.3%]
4–5	—	—	1.000
			(43,980)
			[0.0%]
Panel B: Relative Scale Distortion Proxy (Long Leverage at age 0-1)			
	1	0.907	0.903

Notes: This table reports within-firm constraint tightness $\mathbb{E}[\Theta_{agebin}]$ for the constrained sample, defined as observations with $\Theta_{agebin} \leq 1$. Θ_{agebin} is the ratio of average short-term debt within age bin to maximum short-term debt during the unconstrained period (age bin 3 for low enforcement, age bin 2 for medium and high enforcement). Age bin 1 covers ages 0–1, age bin 2 covers ages 2–3, and age bin 3 covers ages 4–5. Numbers in parentheses show the number of unique firms in the constrained sample. Numbers in brackets show the percentage of firm-age bin observations excluded due to $\Theta_{agebin} > 1$. Enforcement groups based on World Bank recovery rates in insolvency proceedings. Initial leverage calculated as average long-term leverage ratio during age bin 1 (ages 0–1). Leverage ratios compare initial leverage across enforcement regimes.

8 Conclusion

How important are financial constraints for young firms? This paper proposes a methodology to estimate the full effect of financial constraints emerging from enforcement problems. We have established three key results. First, studying long and short leverage, instead of total leverage, is important to quantify the severity of financial constraints over the early life cycle of firms. The duration of the negative co-movement of short and long leverage defines the duration of the early life cycle. Second, a model of endogenous borrowing constraints that aims to match the stylized facts of international corporate finance requires firms to make a productive initial investment choice. This contrasts with standard models that assume unproductive fixed costs and generates more realistic predictions about debt maturity across countries. Third, the scale distortion can be summarized by long leverage at birth.

Our work has some limitations that future research could address. The model considers a single investment opportunity at firm creation, but future work could examine settings where new projects arrive as firms mature. Additionally, we focus on debt financing rather than equity, which may be an important margin in some contexts. Despite these limitations, our framework provides a novel and empirically implementable approach to measuring financial constraints that accounts for both observable and unobservable components of these frictions.

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A Data Coverage, Sample Selection, and Initial Conditions

This appendix details the construction of our firm-level panel and justifies key sample selection decisions. We characterize corporate capital structure and leverage dynamics over the life cycle of young firms across countries with varying levels of financial development, requiring careful attention to data coverage and firm heterogeneity.

A.1 Country Coverage and Panel Structure

We begin with the cleaned version of ORBIS constructed following Kalemli-Özcan et al. (2024), applying standard protocols to eliminate duplicate records, harmonize firm identifiers, and exclude observations with implausible financial data. These procedures are now standard in the firm dynamics literature using ORBIS (Gopinath et al., 2017).

Tables 4 and 5 report coverage metrics across countries: the number of firms, average years observed per firm, and the share of firms with at least 3, 6, and 9 years of data. These metrics provide a screening mechanism for determining whether a country has sufficient longitudinal depth to support life-cycle analysis.

The substantial cross-country variation in both firm counts and panel quality creates a natural tension in sample construction. Countries with the largest firm populations (Russia, France, United Kingdom) do not necessarily provide the deepest panel coverage, while smaller economies often exhibit superior data quality. This variation reflects differences in reporting requirements, coverage policies across Bureau van Dijk's data collection procedures, and the underlying business environment.

A.2 Sample Restriction: Firms with Early Long-Term Debt Access

Our theoretical framework centers on how enforcement constraints shape young firms' capital structure dynamics. To provide a clean test of these predictions, we focus on firms where financing choices reflect economic fundamentals rather than heterogeneous entrepreneurial preferences. This motivates our key sample restriction.

We define an indicator variable $\mathrm{LTD}_i = 1$ if firm i reports positive long-term debt in either of its first two years in the database. This restriction serves two purposes: it ensures that financing choices are observable at entry, and it excludes firms that may follow fundamentally different growth paths due to either binding constraints or deliberate strategic choices.

Table 6 reveals substantial differences between these groups. Firms with early long-term debt enter significantly larger and grow more over time, though they exhibit lower

Table 4: Country Coverage in NBER Historical ORBIS Data (Part 1)

Country	Firms (000s)	Avg Years per Firm	Panel Coverage (%) 3 years	6 years	9 years	Included in Sample
Russia	3211	5.0	65	36	18	Yes
France	1741	6.4	68	42	28	Yes
United Kingdom	1695	4.8	54	28	17	Yes
India	1606	1.3	4	3	1	Yes
Italy	1470	7.6	75	51	36	Yes
China	1437	2.5	36	7	1	Yes
Spain	910	7.4	72	48	33	Yes
Germany	682	5.0	63	35	19	Yes
Ukraine	608	6.1	67	41	27	Yes
Belgium	539	9.5	82	63	49	Yes
Sweden	523	7.7	81	55	34	Yes
Bulgaria	484	5.4	67	39	24	Yes
South Korea	469	4.7	62	29	15	Yes
Portugal	387	5.3	61	36	22	Yes
Norway	371	6.0	70	44	29	Yes
Japan	315	6.4	73	40	27	Yes
Romania	247	4.3	63	33	12	Yes
Serbia	247	5.5	69	33	22	Yes
Hungary	211	4.4	56	28	14	Yes
Finland	210	5.7	69	42	23	Yes
Czech Republic	179	5.0	61	32	18	Yes
Slovakia	172	5.5	67	39	23	Yes
Denmark	161	4.2	60	31	15	Yes
Croatia	161	7.6	76	53	36	Yes
Colombia	158	3.4	47	20	8	Yes

Notes: This table presents country coverage statistics from the NBER Historical ORBIS database following the data cleaning methodology of Kalemli-Özcan et al. (2024). Countries are ranked by number of unique firms. Firms (000s) shows thousands of unique firms per country. Avg Years per Firm shows mean years of panel data per firm. Panel Coverage columns show the percentage of firms with at least 3, 6, and 9 years of consecutive data, respectively. Our final sample includes countries with at least 10,000 unique firms, providing sufficient statistical power for analyzing young firm dynamics over their early life cycle.

Table 5: Country Coverage in NBER Historical ORBIS Data (Part 2)

Country	Firms (000s)	Avg Years per Firm	Panel Coverage (%) 3 years	6 years	9 years	Included in Sample
Morocco	151	3.7	53	26	7	Yes
Poland	120	5.0	58	32	20	Yes
Latvia	110	5.2	65	38	22	Yes
Austria	102	4.3	63	26	11	Yes
Slovenia	95	3.5	36	19	10	Yes
Netherlands	81	2.4	43	2	1	Yes
Estonia	66	7.2	75	51	34	Yes
Malaysia	64	4.0	60	23	10	Yes
Luxembourg	63	3.7	56	22	5	Yes
Ireland	62	6.1	71	40	24	Yes
Turkey	56	4.4	68	26	12	Yes
Singapore	53	3.1	49	12	4	Yes
Greece	53	8.3	75	53	39	Yes
Philippines	34	5.9	72	52	30	Yes
Iceland	31	5.6	68	39	22	Yes
Bosnia and Herzegovina	30	8.9	88	69	51	Yes
Brazil	22	3.3	46	17	4	Yes
Algeria	19	2.5	39	10	0	Yes
Australia	19	4.9	67	38	13	Yes
Mexico	13	2.1	24	5	1	Yes
Montenegro	10	4.4	62	38	7	Yes
Taiwan	8	4.8	56	28	15	No
Lithuania	8	5.0	57	29	19	No
Malta	7	7.5	78	55	39	No
Belarus	4	2.3	21	6	0	No
Kazakhstan	2	6.1	82	52	27	No
Included (46 countries)	19449		61	34	19	
Excluded (5 countries)	30					

Notes: This table presents country coverage statistics from the NBER Historical ORBIS database following the data cleaning methodology of Kalemli-Özcan et al. (2024). Countries are ranked by number of unique firms. Firms (000s) shows thousands of unique firms per country. Avg Years per Firm shows mean years of panel data per firm. Panel Coverage columns show the percentage of firms with at least 3, 6, and 9 years of consecutive data, respectively. Our final sample includes countries with at least 10,000 unique firms, providing sufficient statistical power for analyzing young firm dynamics over their early life cycle.

return on assets. Firms without early long-term debt remain persistently smaller but earn systematically higher ROA throughout the life cycle.

Table 6: Median Firm Characteristics by Age Bin and Initial Long-Term Debt Status

Age Bin	Total Assets		ROA		Log Compensation	
	LTD = 1	LTD = 0	LTD = 1	LTD = 0	LTD = 1	LTD = 0
0–1	287,168	63,417	1.29	1.57	11.14	10.31
2–3	419,000	123,392	1.23	1.39	11.48	10.69
4–5	512,814	178,918	1.14	1.30	11.66	10.95
6–7	588,399	227,000	1.07	1.24	11.78	11.14
8–9	659,156	283,323	1.01	1.19	11.87	11.33
10–11	723,777	346,934	0.95	1.16	11.92	11.52

Notes: ROA is defined as revenue over assets. All variables are medians by age bin and LTD status.

These patterns suggest two competing interpretations. Firms without early long-term debt may either face binding credit constraints that prevent debt market access (Petersen and Rajan, 1994; Berger and Udell, 1998), or represent lifestyle entrepreneurs who deliberately choose small scale and prioritize autonomy over growth (Pugsley and Hurst, 2011).

A.3 Testing the Lifestyle Firm Hypothesis

To distinguish between financial constraints and lifestyle entrepreneurship, we examine growth patterns by initial size quintile within each group. Under the constraint hypothesis, larger LTD = 0 firms should exhibit higher growth due to greater creditworthiness. Under the lifestyle hypothesis, growth should remain modest across all size groups, as entrepreneurs optimize for different objectives than pure scale maximization.

Table 7 presents asset growth multiples by age and entry-size quintile. The evidence strongly supports the lifestyle hypothesis for LTD = 0 firms. Among firms without early long-term debt, those in the smallest initial quintile grow by 3.93× over the life cycle, while firms in the largest quintile grow only 1.46×. This inverse relationship between entry size and growth directly contradicts predictions from financial constraint models, where larger, more creditworthy firms should grow faster when constraints bind.

Table 7: Asset Growth Multiples by Initial Size Quintile and Long-Term Debt Status

	Without Early LTD (LTD = 0)					With Early LTD (LTD = 1)				
Age Bin	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
0–1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2–3	1.15	1.25	1.22	1.19	1.12	1.22	1.16	1.14	1.12	1.08
4–5	1.74	1.60	1.46	1.38	1.24	1.52	1.34	1.27	1.21	1.13
6–7	2.46	1.88	1.66	1.53	1.32	1.81	1.52	1.38	1.27	1.14
8–9	3.04	2.09	1.81	1.63	1.37	2.09	1.71	1.50	1.32	1.14
10–11	3.93	2.43	2.02	1.79	1.46	2.41	1.91	1.62	1.39	1.17

Notes: This table reports median asset growth multiples (current assets / initial assets) by age bin and initial size quintile. Q1 represents the smallest firms at entry; Q5 the largest.

To formalize this analysis, we estimate regressions where firm outcomes are interacted with age bins and initial asset quintiles, computed separately within each LTD status group. The results, presented in Table 8, confirm large and persistent differences in firm outcomes by early access to long-term debt.

Table 8: Regression Coefficients by Age Bin and Early Long-Term Debt Status

2*Age Bin	Log(Assets)		Log(Compensation)		Log(Sales)		ROA (%)	
	LTD = 0	LTD = 1	LTD = 0	LTD = 1	LTD = 0	LTD = 1	LTD = 0	LTD = 1
0–1	0	0.980***	0	0.273***	0	0.357***	0	−33.20**
2–3	0.477***	1.325***	0.312***	0.396***	0.234***	0.370***	−26.79*	−31.51*
4–5	0.739***	1.496***	0.522***	0.604***	0.384***	0.549***	−28.89*	−25.34**
6–7	0.927***	1.624***	0.690***	0.743***	0.507***	0.668***	−23.47	−29.12
8–9	1.086***	1.732***	0.830***	0.850**	0.617***	0.764***	−31.47*	−28.20**
10–11	1.220***	1.828***	0.953***	0.936	0.719***	0.844***	−32.58**	−27.07**

Notes: Coefficients from regressions with industry-country-year fixed effects. Reported coefficients for LTD = 0 correspond to age bin dummies; coefficients for LTD = 1 are computed as the sum of main effect, age bin, and interaction terms. Standard errors clustered at industry-country-year level. ***$p < 0.01$, **$p < 0.05$, *$p < 0.10$.

A.4 Age-Quintile Analysis of Firm Performance

We further examine these patterns by estimating:

$$y_{ict} = \sum_{a \in \text{AgeBins}} \sum_{q \in \text{Quintiles}} \beta_{a,q} \cdot \mathbf{1}_{\{\text{agebin}_i = a \text{ and quintile}_i = q\}} + \alpha_{csy} + \varepsilon_{ict} \tag{2}$$

where firms are grouped into six age bins and assigned to quintiles based on average total assets in their first two years, computed separately within each LTD status group.

Tables 9 and 10 present the results for ROA and log compensation, respectively. Among LTD $= 0$ firms, the gap between large and small firms remains persistent across the life cycle, with no evidence of catch-up. This static pattern contrasts sharply with the more dynamic convergence observed among LTD $= 1$ firms.

TABLE 9: AGE AND INITIAL FINANCIAL POSITION EFFECTS ON RETURN ON ASSETS

	No Long-term Debt (LTD = 0)					**Early Long-term Debt (LTD = 1)**				
Age Bin	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
0–1	*Reference Category*									
2–3	-109.30^{**}	106.71^{**}	110.40^{**}	108.66^{**}	119.73^{**}	109.93^{**}	109.45^{**}	111.66^{**}	110.10^{**}	149.28^{*}
	(49.63)	(49.57)	(49.61)	(49.46)	(56.42)	(51.20)	(49.58)	(51.75)	(49.73)	(87.67)
4–5	-123.11^{**}	124.43^{**}	121.28^{**}	127.73^{**}	136.03^{**}	123.77^{**}	123.76^{**}	126.13^{**}	124.21^{**}	196.55^{*}
	(50.82)	(50.94)	(50.54)	(51.04)	(62.02)	(52.04)	(50.81)	(53.06)	(51.24)	(102.24)
6–7	-121.96^{**}	117.69^{**}	120.80^{**}	123.74^{**}	133.87^{**}	122.88^{**}	122.60^{**}	125.16^{**}	123.67^{**}	166.92^{*}
	(51.50)	(50.76)	(51.21)	(51.26)	(62.25)	(52.20)	(51.34)	(53.73)	(52.10)	(96.00)
8–9	-119.78^{**}	116.64^{**}	116.78^{**}	116.76^{**}	134.57^{**}	121.31^{**}	120.87^{**}	123.67^{**}	121.56^{**}	174.57^{*}
	(51.92)	(51.00)	(51.27)	(51.35)	(65.38)	(52.18)	(51.72)	(54.15)	(52.67)	(103.96)
10–11	-128.90^{**}	123.09^{**}	127.41^{**}	126.41^{**}	149.85^{**}	130.57^{**}	130.28^{**}	132.68^{**}	131.03^{**}	197.88^{*}
	(52.20)	(50.78)	(51.31)	(51.42)	(65.91)	(52.32)	(51.96)	(54.43)	(52.99)	(117.55)

Notes: Interaction coefficients from ROA regression with industry×year×country fixed effects. Reference category: firms aged 0–1 years with no long-term debt in quintile 1. Standard errors clustered at industry×year×country level. $N = 32,179,346$. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

Table 10: Age and Initial Financial Position Effects on Log Compensation

	No Long-term Debt (LTD = 0)					Early Long-term Debt (LTD = 1)				
Age Bin	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
0–1	*Reference Category*									
2–3	0.458^{***}	-0.072^{***}	-0.130^{***}	-0.145^{***}	-0.154^{***}	-0.035^{**}	-0.117^{***}	-0.159^{***}	-0.167^{***}	-0.142^{***}
	(0.014)	(0.012)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.017)
4–5	0.794^{***}	-0.131^{***}	-0.259^{***}	-0.297^{***}	-0.314^{***}	-0.118^{***}	-0.262^{***}	-0.335^{***}	-0.352^{***}	-0.318^{***}
	(0.021)	(0.017)	(0.021)	(0.022)	(0.022)	(0.021)	(0.022)	(0.022)	(0.022)	(0.024)
6–7	1.015^{***}	-0.162^{***}	-0.325^{***}	-0.380^{***}	-0.405^{***}	-0.156^{***}	-0.346^{***}	-0.445^{***}	-0.472^{***}	-0.423^{***}
	(0.025)	(0.020)	(0.025)	(0.027)	(0.027)	(0.025)	(0.026)	(0.026)	(0.026)	(0.029)
8–9	1.183^{***}	-0.177^{***}	-0.374^{***}	-0.446^{***}	-0.469^{***}	-0.185^{***}	-0.406^{***}	-0.532^{***}	-0.573^{***}	-0.519^{***}
	(0.027)	(0.022)	(0.028)	(0.030)	(0.031)	(0.027)	(0.029)	(0.029)	(0.029)	(0.033)
10–11	1.405^{***}	-0.259^{***}	-0.490^{***}	-0.570^{***}	-0.593^{***}	-0.299^{***}	-0.541^{***}	-0.685^{***}	-0.741^{***}	-0.679^{***}
	(0.032)	(0.028)	(0.034)	(0.035)	(0.036)	(0.033)	(0.034)	(0.034)	(0.035)	(0.040)

Notes: Interaction coefficients from log compensation regression with industry×year×country fixed effects. Reference category: firms aged 0–1 years with no long-term debt in quintile 1. Standard errors clustered at industry×year×country level. $N = 17,175,296$. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

A.5 Compensation Growth Dynamics

Finally, we examine year-on-year compensation growth by estimating:

$$\Delta \log(\text{Compensation}_{it}) = \sum_a \beta_a \cdot \text{AgeBin}_{it}^a + \sum_a \delta_a \cdot (\text{AgeBin}_{it}^a \times \text{LTD}_i) + \mu_i + \lambda_{jct} + \varepsilon_{it} \tag{3}$$

with firm fixed effects μ_i and country-sector-year fixed effects λ_{jct}.

Table 11 shows that firms with early LTD access exhibit systematically faster compensation growth. The gap widens from 2.5 percentage points at age 2–3 to 8.1 percentage points at age 10–11, consistent with cumulative advantages from reduced financing frictions.

Firms with long-term debt enter significantly larger in assets, compensation, and sales. While LTD = 0 firms grow faster—especially in assets—they do not fully converge in scale. Most notably, LTD = 0 firms begin with significantly higher ROA but exhibit declining profitability with age, while LTD = 1 firms show improving ROA over time. This pattern is inconsistent with financial constraints gradually relaxing as small firms accumulate internal

Table 11: Growth in Labor Compensation by Age and Initial LTD Status

Variable	**Coefficient**	**Std. Error**
Age 2–3 (No LTD)	-0.192^{***}	(0.0014)
Age 4–5 (No LTD)	-0.236^{***}	(0.0018)
Age 6–7 (No LTD)	-0.219^{***}	(0.0023)
Age 8–9 (No LTD)	-0.186^{***}	(0.0029)
Age 10–11 (No LTD)	-0.150^{***}	(0.0035)
Age 2–3 × LTD=1	0.025^{***}	(0.0018)
Age 4–5 × LTD=1	0.041^{***}	(0.0019)
Age 6–7 × LTD=1	0.056^{***}	(0.0020)
Age 8–9 × LTD=1	0.067^{***}	(0.0021)
Age 10–11 × LTD=1	0.081^{***}	(0.0022)
Constant	0.255^{***}	(0.0018)
Firm Fixed Effects	Yes	
Sector-Year-Country Fixed Effects	Yes	
Clusters (Firm ID)	3,644,832	
Observations	17,256,731	

Notes: Standard errors clustered at firm level. $^{***}p < 0.01$. Omitted category: Age 0–1 and LTD = 0.

funds, but consistent with lifestyle entrepreneurship optimizing for different objectives.

A.6 Cross-Country Variation in Institutional Quality

The cross-country variation in firm financing patterns provides crucial identification for our enforcement-based theory. Tables 12 and 13 show that the percentage of firms with initial long-term debt ranges from 0.4% in India to 34.5% in Belgium, providing substantial variation for identifying the effects of institutional quality on financing patterns.

Table 12: Cross-Country Sample Coverage: Countries with Highest LTD=1 Firm Counts

Country	Total Firms (000s)	LTD=1 Firms (000s)	% LTD=1	% LTD=1 w/ 3+ yrs	% LTD=1 w/ 6+ yrs	% LTD=1 w/ 9+ yrs	Recovery Rate	GDP p.c. (000s USD)
France	1741.3	585.9	33.6	65.8	37.3	20.4	55.5	35.7
Spain	910.3	310.7	34.1	71.5	44.1	28.6	73.6	25.9
Russia	3210.6	257.8	8.0	62.7	26.3	10.9	41.6	9.3
Belgium	538.7	186.0	34.5	81.3	60.5	45.2	87.6	39.8
Korea	468.9	155.6	33.2	59.4	27.5	13.3	82.9	27.2
Italy	1470.4	155.1	10.5	79.9	52.8	33.7	61.7	31.9
United Kingdom	1695.3	149.8	8.8	56.0	30.7	19.8	86.6	43.8
Sweden	522.7	99.9	19.1	85.3	63.6	43.8	76.6	49.8
Germany	682.0	94.1	13.8	61.0	29.5	13.3	81.6	40.4
Portugal	387.3	93.7	24.2	63.5	33.4	17.4	69.1	19.7
Norway	370.6	43.3	11.7	72.2	44.1	26.5	91.1	74.8
Finland	210.4	34.7	16.5	71.2	43.4	25.3	89.0	43.4
China	1436.7	29.6	2.1	54.7	10.8	2.2	36.6	6.8
Croatia	160.8	25.9	16.1	75.0	43.2	25.1	30.9	12.5
Japan	314.7	24.6	7.8	63.0	31.1	17.9	92.4	34.2
Poland	119.7	18.7	15.6	56.4	26.0	13.5	48.9	12.4
Czech Republic	179.4	18.7	10.4	63.3	33.3	19.2	42.7	16.9
Estonia	66.4	18.6	28.1	81.4	55.8	36.5	38.7	16.5
Denmark	161.4	18.3	11.3	66.2	28.8	8.5	87.8	55.2
Bulgaria	484.1	17.2	3.6	82.1	54.0	34.0	34.4	7.2
Serbia	246.6	15.8	6.4	76.3	45.4	29.8	28.1	5.9
Latvia	110.4	14.1	12.7	67.9	38.4	19.6	43.6	13.2
Ireland	61.7	10.7	17.3	70.8	41.0	21.6	87.0	61.6
Ukraine	607.7	10.4	1.7	65.5	24.3	10.8	9.4	2.2
Slovenia	94.7	8.6	9.0	65.2	30.8	13.2	67.4	22.4

Notes: This table shows the countries with the highest number of firms with initial long-term debt (LTD=1). Countries are ordered by number of LTD=1 firms. Total Firms shows the number of firms (in thousands) in our sample. LTD=1 Firms shows firms with positive long-term debt at ages 0-1. % LTD=1 is the percentage of all firms that are LTD=1. The next three columns show the percentage of LTD=1 firms observed for 3+, 6+, and 9+ years respectively. Recovery Rate is the percentage recovery in insolvency proceedings (World Bank Doing Business). GDP p.c. is GDP per capita in thousands of 2015 USD.

This variation is systematically related to institutional development. Table 14 confirms

Table 13: Cross-Country Sample Coverage: Countries with Lower LTD=1 Firm Counts

Country	Total Firms (000s)	LTD=1 Firms (000s)	% LTD=1	% LTD=1 w/ 3+ yrs	% LTD=1 w/ 6+ yrs	% LTD=1 w/ 9+ yrs	Recovery Rate	GDP p.c. (000s USD)
Austria	102.0	8.4	8.2	45.2	17.2	6.6	75.2	43.4
Morocco	151.2	8.3	5.5	47.7	18.7	3.2	27.8	3.1
Slovakia	171.6	6.8	3.9	74.9	43.5	26.7	50.9	16.1
Iceland	30.5	6.7	22.1	72.7	45.5	28.0	82.2	51.4
India	1606.3	6.5	0.4	84.2	47.5	4.3	32.9	1.9
Greece	52.5	4.3	8.3	74.6	54.5	41.0	40.1	19.9
Turkey	56.0	4.0	7.1	63.7	21.1	7.3	19.4	10.5
Algeria	19.4	3.8	19.4	41.4	6.6	0.0	50.8	4.6
Hungary	211.4	3.5	1.6	68.3	40.5	22.8	39.9	11.8
Bosnia & Herzegovina	30.4	2.8	9.3	94.2	76.0	54.3	35.9	4.3
Singapore	53.0	2.6	4.9	49.8	13.5	2.2	88.9	56.7
Netherlands	81.0	2.6	3.2	24.0	4.8	0.9	86.9	40.5
Luxembourg	63.5	2.0	3.1	64.3	27.2	10.2	43.6	106.7
Australia	18.5	1.9	10.4	58.8	21.0	5.6	82.0	57.2
Romania	247.0	1.7	0.7	86.2	59.2	20.7	15.5	5.8
Colombia	158.4	1.7	1.1	60.5	25.4	11.3	59.9	5.4
Malaysia	64.2	1.3	2.0	71.1	24.3	6.2	75.3	10.1
Lithuania	7.7	1.2	15.7	55.9	31.7	20.9	44.5	13.0
Malta	6.9	1.1	16.5	74.0	48.6	28.7	39.4	23.6
Brazil	21.5	0.9	4.0	51.2	15.3	2.6	17.4	8.8
Montenegro	10.4	0.6	5.3	71.7	38.1	4.3	49.0	6.8
Philippines	34.1	0.4	1.0	65.2	45.0	26.3	14.3	2.8
Belarus	4.4	0.3	6.7	3.7	0.0	0.0	38.4	6.3
Mexico	13.5	0.2	1.5	33.2	7.0	3.0	67.3	9.9

Notes: This table shows the countries with lower numbers of firms with initial long-term debt (LTD=1). Countries are ordered by number of LTD=1 firms. Total Firms shows the number of firms (in thousands) in our sample. LTD=1 Firms shows firms with positive long-term debt at ages 0-1. % LTD=1 is the percentage of all firms that are LTD=1. The next three columns show the percentage of LTD=1 firms observed for 3+, 6+, and 9+ years respectively. Recovery Rate is the percentage recovery in insolvency proceedings (World Bank Doing Business). GDP p.c. is GDP per capita in thousands of 2015 USD.

that countries with higher recovery rates and GDP per capita exhibit significantly higher prevalence of LTD = 1 firms, consistent with our theoretical prediction that stronger enforcement institutions facilitate long-term debt financing.

Table 14: LTD=1 Firm Prevalence and Institutional Quality

	(1) Recovery Rate	(2) ln GDP p.c.	(3) Both
Recovery Rate	0.145**		0.052
	(0.052)		(0.081)
ln GDP per capita		3.935***	2.954
		(1.250)	(1.986)
R-squared	0.142	0.174	0.181
Observations	45	45	45

Notes: Dependent variable is the percentage of firms with initial long-term debt (% LTD=1). Recovery Rate is from World Bank Doing Business indicators. Standard errors in parentheses. Sample includes countries with at least 100 LTD=1 firms.

The positive coefficients on both recovery rates (0.145, significant at 5%) and log GDP per capita (3.935, significant at 1%) suggest that a one-standard-deviation improvement in institutional quality is associated with approximately 5-8 percentage points higher LTD = 1 prevalence.

A.7 Sample Construction for Main Analysis

Based on this analysis, our main empirical work focuses on the LTD = 1 population, where financial constraints are more likely to govern investment and growth decisions rather than heterogeneous entrepreneurial preferences. This sample restriction ensures that observed capital structure dynamics reflect financing considerations, thereby providing a cleaner test of our theoretical predictions about enforcement constraints and firm financial behavior.

For country inclusion, we require two criteria: (1) at least 100 LTD = 1 firms to ensure statistical power, and (2) at least 30% of LTD = 1 firms observed for 6+ years to ensure adequate panel quality for studying financing dynamics. This yields a sample spanning the full range of institutional quality while maintaining sufficient data depth to trace firm capital structure evolution over the early life cycle.

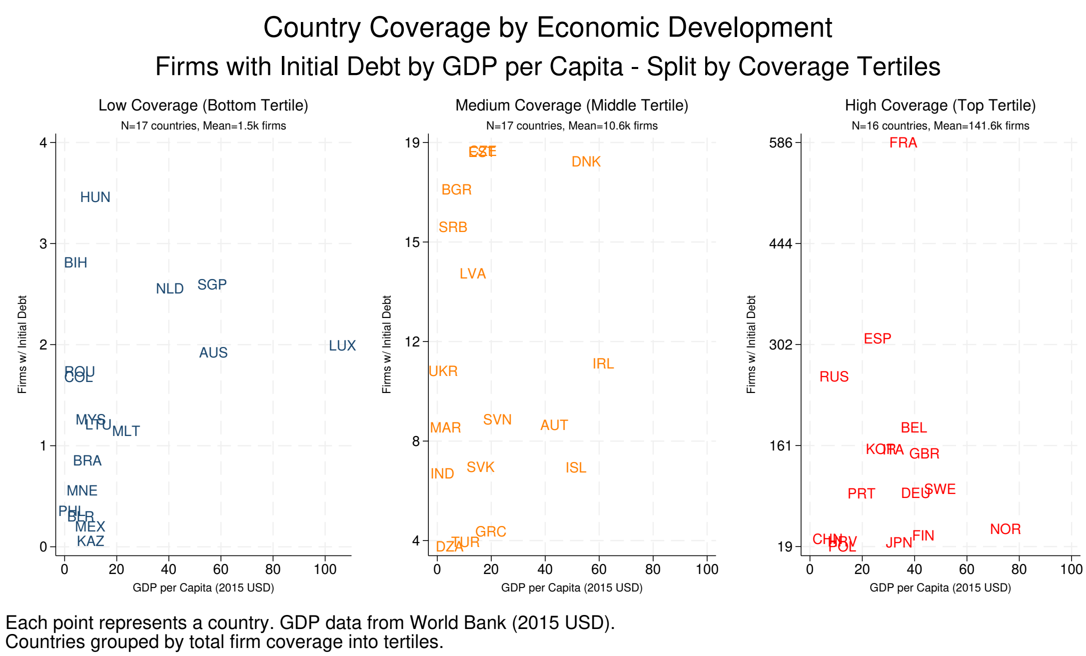


Figure 3: Country Coverage by Economic Development: Total LTD=1 Firms by Coverage Tertiles

Notes: Countries are grouped into tertiles based on total number of firms with initial long-term debt. Each subplot uses a continuous Y-axis scale where the middle tertile begins at the maximum of the bottom tertile, and the top tertile begins at the maximum of the middle tertile. This scaling facilitates identification of natural breakpoints for sample construction. GDP per capita data from World Bank (2015 USD).

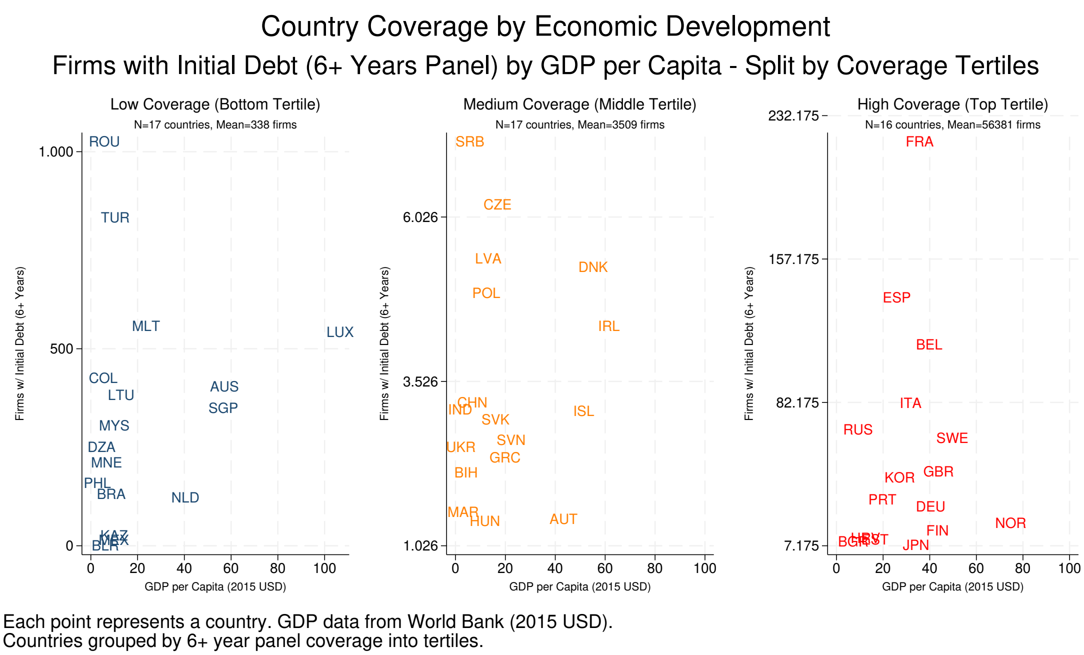


Figure 4: Country Coverage by Economic Development: LTD=1 Firms with 6+ Years Panel Data by Coverage Tertiles

Notes: Countries are grouped into tertiles based on number of LTD=1 firms observed for six or more years, demonstrating the trade-off between sample size and panel quality. Continuous Y-axis scaling across tertiles shows the substantial reduction in coverage when requiring adequate panel depth for life-cycle analysis. GDP per capita data from World Bank (2015 USD).

Figures 3 and 4 illustrate this sample construction strategy. The tertile approach reveals natural breakpoints in the data while maintaining balanced representation across development levels. Countries with fewer than 1,000 LTD = 1 firms provide limited statistical power, while those below 100 firms with adequate panel depth offer insufficient variation for meaningful analysis. The continuous Y-axis scaling across tertiles facilitates identification of these natural cutoff points.

A.8 Implications for Sample Construction

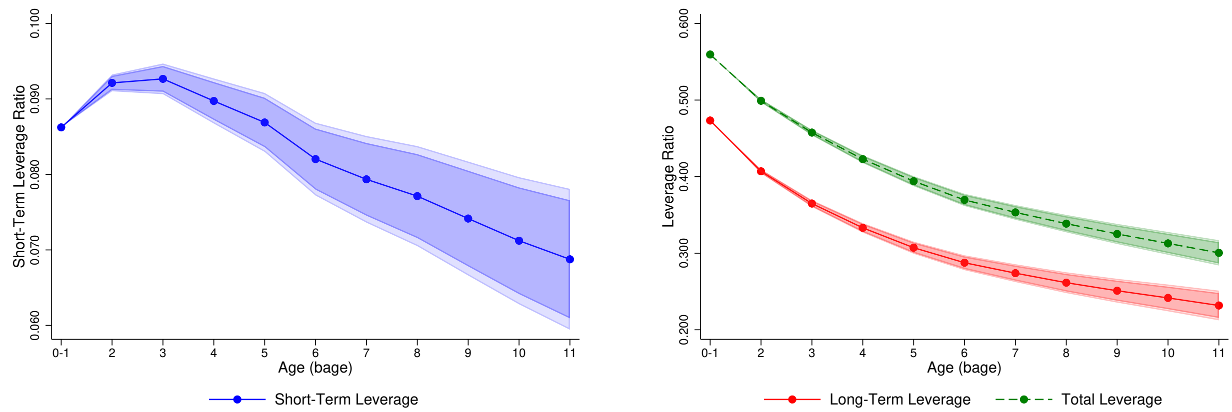
These findings provide strong evidence that LTD = 0 firms constitute a distinct entrepreneurial segment characterized by lifestyle motivations rather than binding financial constraints. The inverse relationship between initial size and growth, persistent performance gaps, and declining profitability trajectories all support this interpretation.

This methodological approach addresses a fundamental challenge in international corporate finance: distinguishing between financing patterns that reflect institutional constraints versus those driven by heterogeneous preferences. By focusing on firms that demonstrably access long-term debt markets, we ensure that our leverage dynamics reflect responses to enforcement quality rather than systematic differences in entrepreneurial motivations across countries.

Our final sample comprises firms from countries meeting both coverage and quality thresholds, spanning the full range of institutional development while maintaining adequate statistical power to identify the causal effects of enforcement quality on young firm capital structure dynamics. This careful sample construction underpins the credibility of our empirical findings and strengthens the external validity of our theoretical framework.

A.9 Within-Firm Evidence

Figure 5 presents results from our within-firm specification, which exploits only variation within firms over time to identify age effects.



(A) Short-Term Leverage (B) Long-Term and Total Leverage

Figure 5: Capital Structure Evolution: Within-Firm Estimates

Notes: Coefficients from regressions with firm and country-sector-year fixed effects. Age 0 serves as the reference category. Shaded regions represent 90% and 95% confidence intervals. Standard errors clustered at the country-sector level. Sample includes firms with positive long-term debt in their first two years, observed for at least three years.

The within-firm results confirm the robustness of our findings. Despite controlling for all time-invariant firm characteristics, we observe the same qualitative patterns: short-term leverage exhibits a hump-shaped profile during early life, while long-term leverage declines monotonically. The magnitudes are somewhat attenuated relative to the APC specification, consistent with the notion that firm fixed effects absorb some of the cross-sectional variation that contributes to age effects in the pooled specification.

Importantly, the persistence of these patterns within firms rules out explanations based on unobserved heterogeneity or selection. The results demonstrate that individual firms systematically rebalance their capital structure as they age, precisely as predicted by our enforcement-constraint model.

B By Country APC Analysis

B.1 Methodology

We implement a Deaton-style Age-Period-Cohort (APC) framework to estimate the leverage lifecycle patterns of young firms across countries, exploiting variation by firm age (grouped into two-year bins), period, and cohort of incorporation.

Specifically, for each country we regress short-term leverage on agebin dummies and normalized cohort effects, absorbing sector-year fixed effects and clustering standard errors at the sector-year level. This follows the robust procedure outlined in Deaton (1997), adapted to our context with explicit normalization using two "anchor cohorts" (typically the third and fourth earliest cohorts in the data). This ensures comparability of leverage profiles over time even in the presence of differing entry patterns across countries.

Formally, the regression we estimate for each country is:

$$\text{Short-Term Leverage}_{i,t} = \sum_{a=2}^{6} \beta_a \mathbf{1}\{\text{agebin}_i = a\} + \sum_{c=5}^{C} \gamma_c \text{cohort}_c + \alpha_{sy} + \varepsilon_{i,t}$$

where α_{sy} are sector-year fixed effects, and the cohort dummies are normalized relative to the means of the two anchor cohorts. Predictions are then generated starting from the average leverage at agebin 1 (0-1 years), incremented by the estimated age effects.

B.2 Cohort coverage and anchor selection

Table 15 reports the number of firms by country and cohort for 1996–2000. We require each country to have at least 200 firms in both 1998 and 1999 cohorts to ensure stable normalization around our anchor cohorts. Countries failing this are excluded from the baseline APC analysis.

Table 15: Firms by Country and Cohort (1996–2000)

Country	1996	1997	1998	1999	2000	Total	Anchor OK
Austria	12	6	11	7	88	124	No
Belgium	31,929	46,803	55,839	61,538	72,499	268,608	Yes
Bulgaria	68	101	116	938	1,385	2,608	No
Bosnia & Herz.		195	479	1,220	1,983	3,877	Yes
China			14	12	10	36	No
Czech Rep.	710	941	966	1,155	2,589	6,361	Yes
Germany	102	322	535	1,212	1,999	4,170	Yes
Denmark			16	28	34	78	No
Spain	57,784	82,156	90,488	100,104	105,710	436,242	Yes
Estonia	1,358	3,227	3,503	3,961	5,603	17,652	Yes
Finland	5,426	7,766	6,577	6,230	7,941	33,940	Yes
France	32,785	56,892	61,629	66,779	70,175	288,260	Yes
UK	14,573	27,956	33,555	36,025	34,061	146,170	Yes
Greece	1,175	1,775	1,371	1,892	2,177	8,390	Yes
Croatia	254	653	644	864	3,575	5,990	Yes
Hungary	293	797	713	514	288	2,605	Yes
India					1	1	No
Ireland	1	88	870	2,552	2,797	6,308	Yes
Iceland	1,235	1,951	1,805	1,908	1,904	8,803	Yes
Italy	772	1,243	1,694	2,735	7,720	14,164	Yes
Japan	661	889	1,220	1,584	2,994	7,348	Yes
S. Korea	186	393	1,811	21,098	34,641	58,129	No
Latvia	613	979	859	742	1,081	4,274	Yes
Poland	490	828	1,178	1,642	1,983	6,121	Yes
Portugal	1,327	3,197	3,650	4,001	5,437	17,612	Yes
Romania	1,116	1,555	1,499	1,829	1,968	7,967	Yes
Russia	436	4,088	6,084	7,623	10,260	28,491	Yes
Serbia	61	1,660	3,231	2,919	4,943	12,814	Yes
Slovakia	133	121	94	195	238	781	No
Sweden	8,648	17,060	17,551	22,653	24,835	90,747	Yes
Turkey	1		1	1	3	6	No
Ukraine	276	340	1,570	2,648	1,415	6,249	Yes
Total	162,425	263,982	299,573	356,609	412,337	1,494,926	

Notes: Number of firms by cohort (incorporation year, 1996–2000) in our final sample of firms with initial long-term debt (ages 0 or 1). We require each country to have at least 1,000 firms observed four times between ages 0–5, and at least 200 firms in both anchor cohorts (1998 and 1999) to ensure reliable cohort normalization.

Online Appendix

C Appendix: Theory

C.1 Proofs of Propositions

C.1.1 Proof of Proposition 1

The optimality of this financial structure follows from comparing the present value of costs across different financing arrangements.

For intra-period funding needs, a single-period loan costs $\kappa_C + (1 + r_f)S_a$ each period. Discounting these recurring costs yields a present value of:

$$\sum_{t=0}^{\infty}(\beta\rho)^t[\kappa_C + (1 + r_f)S_a] = \frac{\kappa_C}{1 - \beta\rho} + \frac{(1 + r_f)S_a}{1 - \beta\rho}$$

In contrast, a multi-period loan covering the same recurring needs costs κ_C once, but charges a premium for firm exit risk:

$$\kappa_C + (1 + r_f)\sum_{t=0}^{\infty}(\beta\rho)^t\frac{S_a}{\rho} = \kappa_C + \frac{(1 + r_f)S_a}{\rho(1 - \beta\rho)}$$

The difference between these costs is:

$$\frac{\kappa_C}{1 - \beta\rho} - \kappa_C + \frac{(1 + r_f)S_a}{1 - \beta\rho} - \frac{(1 + r_f)S_a}{\rho(1 - \beta\rho)} = \frac{\beta\rho\kappa_C}{1 - \beta\rho} - \frac{(1 - \rho)(1 + r_f)S_a}{\rho(1 - \beta\rho)}$$

By Assumption 2, $\kappa_C < S_0(K(\xi_L), z_L)\frac{1-\rho}{\rho}$, which ensures that for typical working capital needs, the transaction cost savings don't outweigh the exit risk premium, making single-period loans optimal.

For capital acquisition, which requires multiple periods to repay (by Assumption 3), a multi-period loan incurs a single transaction cost κ_C, while a sequence of T single-period loans would incur $\sum_{t=0}^{T-1}(\beta\rho)^t\kappa_C$ in transaction costs. Since $T > 1$, the multi-period loan dominates.

Proof of Proposition 2

The firm's problem is to choose $\{q_a, S_a, K\}_{a=0}^{\infty}$ to maximize:

$$\max_{\{q_a,S_a,K\}_{a=0}^{\infty}} \sum_{a=0}^{\infty}(\beta\rho)^a \left(z\left[K^\alpha\left(\frac{S_a}{w}\right)^{1-\alpha}\right]^\eta - (1+r_f)S_a - q_a \right)$$

subject to:

$$(1+r_f)p_k K \leq \sum_{a=0}^{\infty}(\beta\rho)^a q_a \tag{PCF}$$

$$0 \leq q_a \leq z\left[K^\alpha\left(\frac{S_a}{w}\right)^{1-\alpha}\right]^\eta - (1+r_f)S_a \tag{NNC}$$

$$(1-\xi)z\left[K^\alpha\left(\frac{S_a}{w}\right)^{1-\alpha}\right]^\eta \leq V_a \tag{LE}$$

where $V_a = \sum_{j=0}^{\infty}(\beta\rho)^j\left(z\left[K^\alpha\left(\frac{S_{a+j}}{w}\right)^{1-\alpha}\right]^\eta - (1+r_f)S_{a+j} - q_{a+j}\right)$ is the continuation value of the firm at age a.

Let λ denote the Lagrange multiplier for the participation constraint of financiers (PCF), μ_a^H and μ_a^L for the upper and lower bounds of the non-negativity constraint (NNC), and ψ_a for the limited enforcement constraint (LE).

The first-order conditions with respect to q_a and S_a are:

$$\partial q_a \ : \ -(\beta\rho)^a + \lambda(\beta\rho)^a + \mu_a^L - \mu_a^H - \sum_{j=0}^{a}(\beta\rho)^j \psi_{a-j} = 0 \tag{FOC-q}$$

$$\begin{aligned}\partial S_a \ : \ & \left[(\beta\rho)^a + \mu_a^H + \sum_{j=0}^{a}(\beta\rho)^j \psi_{a-j}\right]\left[\frac{\eta(1-\alpha)}{S_a} z \left[K^\alpha \left(\frac{S_a}{w}\right)^{1-\alpha}\right]^\eta - (1+r_f)\right] \\ & = \psi_a(1-\xi)\frac{\eta(1-\alpha)}{S_a} z \left[K^\alpha \left(\frac{S_a}{w}\right)^{1-\alpha}\right]^\eta \end{aligned} \tag{FOC-R}$$

From (FOC-R), we define the unconstrained level of short-term debt:

$$S_u(K,z) \equiv \left[\frac{\eta(1-\alpha)}{(1+r_f)} z \left[K^\alpha \left(\frac{1}{w}\right)^{1-\alpha}\right]^\eta\right]^{\frac{1}{1-\eta(1-\alpha)}}$$

Note that $\psi_a = 0 \iff S_a = S_u(K,z)$, which follows from (FOC-R).

From the first-order condition for q_a (FOC-q), we can isolate λ:

$$\lambda = 1 + \frac{\mu_a^H - \mu_a^L}{(\beta\rho)^a} + \sum_{j=0}^{a} \psi_{a-j}\left(\frac{1}{\beta\rho}\right)^{a-j} \quad \forall a$$

Since λ is a constant multiplier, comparing its expression across periods a and $a+1$ yields:

$$\beta\rho(\mu_a^H - \mu_a^L) - \psi_{a+1} = \mu_{a+1}^H - \mu_{a+1}^L$$

Applying this recursively, we can establish:

$$(\beta\rho)^a \mu_0^H - \sum_{j=1}^{a}(\beta\rho)^{a-j}\psi_j = (\mu_a^H - \mu_a^L)$$

This expression characterizes the dynamics of the contract, we characterize it in following claims:

Claim 1 *In a solution with $K > 0$, we must have $\mu_0^H > 0$.*

Proof. We proceed by contradiction. Suppose that $\mu_0^H = 0$. From the first-order condition with respect to q_0:

$$-1 + \lambda + \mu_0^L - \mu_0^H - \psi_0 = 0$$

Substituting $\mu_0^H = 0$:

$$\lambda = 1 - \mu_0^L + \psi_0$$

Since $\mu_0^L \geq 0$ and $\psi_0 \geq 0$ (as Lagrange multipliers), we have two cases to consider:

Case 1: If $\mu_0^L > 0$, then $q_0 = 0$ by complementary slackness. The participation constraint of financiers requires:

$$(1 + r_f)p_k K \leq \sum_{a=0}^{\infty} (\beta\rho)^a q_a = \sum_{a=1}^{\infty} (\beta\rho)^a q_a$$

From our recursive relationship for the multipliers:

$$(\beta\rho)^a \mu_0^H - \sum_{j=1}^{a} (\beta\rho)^{a-j} \psi_j = (\mu_a^H - \mu_a^L)$$

With $\mu_0^H = 0$, this implies $(\mu_a^H - \mu_a^L) \leq 0$ for all $a \geq 1$ (since all $\psi_j \geq 0$). This means either $q_a = 0$ for all $a \geq 1$ (if $\mu_a^L > 0$), or q_a are below their upper bounds. Either way, the participation constraint cannot be satisfied for any $K > 0$.

Case 2: If $\mu_0^L = 0$, then from our recursive relationship with $\mu_0^H = 0$:

$$-\sum_{j=1}^{a}(\beta\rho)^{a-j}\psi_j = (\mu_a^H - \mu_a^L)$$

Since $\psi_j \geq 0$ for all j, we have $(\mu_a^H - \mu_a^L) \leq 0$ for all $a \geq 1$. For any $K > 0$, the firm must generate positive coupon payments to satisfy the participation constraint:

$$(1 + r_f)p_k K \leq \sum_{a=0}^{\infty}(\beta\rho)^a q_a$$

But with $(\mu_a^H - \mu_a^L) \leq 0$ for all a, the upper bound on coupons never binds, and implies that $q_a = 0$ $\forall a$ is a feasible solution, but this would contradict the PCF for any $K > 0$. Therefore, we must have $\mu_0^H > 0$. ■

Claim 2 *In a solution with $K > 0$, there exists a finite age $\hat{a}$ such that:*

$$(\beta\rho)^{\hat{a}}\mu_0^H \leq \sum_{j=1}^{\hat{a}}(\beta\rho)^{\hat{a}-j}\psi_j$$

Proof. We proceed by contradiction. Suppose that for all ages a:

$$(\beta\rho)^a\mu_0^H > \sum_{j=1}^{a}(\beta\rho)^{a-j}\psi_j \qquad \forall a$$

From our derived recursive relationship, this implies $\mu_a^H - \mu_a^L > 0$ for all a. Since $\mu_a^L \geq 0$, we have $\mu_a^H > 0$ for all a, meaning the upper bound on coupon payments always binds. Consequently, $q_a = \pi_a$ for all a and $V_a = \beta\rho V_{a+1}$ for all a. This implies that the firm pays zero dividends, hence $V_a = 0$. The enforcement constraint then implies $S_a = 0$ $\forall a$ and consequently $\pi_a = q_a = 0$, which contradicts the participation constraint of financiers for any $K > 0$. ■

Claim 2 implies that we can have two cases at $a = \hat{a}$: $\mu^H_{\hat{a}} = 0$ and $\mu^L_{\hat{a}} = 0$, or, $\mu^H_{\hat{a}} = 0$ and $\mu^L_{\hat{a}} > 0$. And it also implies $\mu^H_a - \mu^L_a \leq 0$ for all $a \geq \hat{a}$

Claim 3 *Claims 1 and 2 imply $\mu^H_a - \mu^L_a \leq 0$ for all $a \geq \hat{a}$.*

Proof. Recall the recursive relationship for multipliers derived from the first-order conditions:

$$(\beta\rho)^a \mu^H_0 - \sum_{j=1}^{a} (\beta\rho)^{a-j} \psi_j = (\mu^H_a - \mu^L_a)$$

From Claim 2, we know there exists a finite age $\hat{a}$ such that:

$$(\beta\rho)^{\hat{a}} \mu^H_0 \leq \sum_{j=1}^{\hat{a}} (\beta\rho)^{\hat{a}-j} \psi_j$$

Substituting this inequality into the recursive relationship at age $\hat{a}$:

$$(\beta\rho)^{\hat{a}} \mu^H_0 - \sum_{j=1}^{\hat{a}} (\beta\rho)^{\hat{a}-j} \psi_j = (\mu^H_{\hat{a}} - \mu^L_{\hat{a}})$$

Given the constraint in Claim 2, we have:

$$(\mu^H_{\hat{a}} - \mu^L_{\hat{a}}) \leq 0$$

This establishes the result for $a = \hat{a}$.

To show the inequality holds for all $a > \hat{a}$, we use induction and the recursive relationship between consecutive periods:

$$\beta\rho(\mu^H_a - \mu^L_a) - \psi_{a+1} = \mu^H_{a+1} - \mu^L_{a+1}$$

For the base case $a = \hat{a}$, we have:

$$\beta\rho(\mu_{\hat{a}}^H - \mu_{\hat{a}}^L) - \psi_{\hat{a}+1} = \mu_{\hat{a}+1}^H - \mu_{\hat{a}+1}^L$$

Since $\mu_{\hat{a}}^H - \mu_{\hat{a}}^L \leq 0$ (as established above) and $\psi_{\hat{a}+1} \geq 0$ (being a Lagrange multiplier), we have:

$$\beta\rho(\mu_{\hat{a}}^H - \mu_{\hat{a}}^L) - \psi_{\hat{a}+1} \leq 0 - \psi_{\hat{a}+1} \leq 0$$

Therefore:

$$\mu_{\hat{a}+1}^H - \mu_{\hat{a}+1}^L \leq 0$$

Assuming the inequality holds for some $a \geq \hat{a}$, i.e., $\mu_a^H - \mu_a^L \leq 0$, we can apply the same logic to show it holds for $a + 1$:

$$\beta\rho(\mu_a^H - \mu_a^L) - \psi_{a+1} = \mu_{a+1}^H - \mu_{a+1}^L$$

Since $\mu_a^H - \mu_a^L \leq 0$ by assumption and $\psi_{a+1} \geq 0$, we have:

$$\mu_{a+1}^H - \mu_{a+1}^L \leq 0$$

By induction, we have shown that $\mu_a^H - \mu_a^L \leq 0$ for all $a \geq \hat{a}$. ■

Claim 4 *For $a \geq \hat{a}$, we have $\psi_a = 0$.*

Proof. From Claim 3, we know that $\mu_a^H - \mu_a^L \leq 0$ for all $a \geq \hat{a}$. Since $\mu_a^L \geq 0$ (as a Lagrange multiplier), this implies $\mu_a^H = 0$ for all $a \geq \hat{a}$.

For contradiction, suppose $\psi_a > 0$ for some $a \geq \hat{a}$. Then:

1. From the first-order condition for S_a, when $\psi_a > 0$, we have $S_a < S_u(K, z)$.

2. Since $\mu_a^H = 0$ for $a \geq \hat{a}$, the firm is no longer constrained to pay all profits as coupons, meaning $q_a \leq \pi_a$ and the firm could pay positive dividends.

3. Consider the firm's profit function $\pi(S) = z\left[K^\alpha \left(\frac{S}{w}\right)^{1-\alpha}\right]^\eta - (1+r_f)S$, with derivative:

$$\frac{d\pi}{dS} = \eta(1-\alpha)z\left[K^\alpha \left(\frac{S}{w}\right)^{1-\alpha}\right]^\eta \frac{1}{S} - (1+r_f)$$

   For $S_a < S_u(K, z)$, this derivative is positive, so the firm would strictly benefit from redirecting some dividend funds to increase S_a toward $S_u(K, z)$

4. This reallocation would strictly increase profits, maintaining the same total outflow, contradicting the optimality of $q_a \leq \pi_a$ and $\mu_a^H = 0$ for $a \geq \hat{a}$

Therefore, we must have $\psi_a = 0$ for all $a \geq \hat{a}$. ■

Claim 5 *For any $K > 0$, a coupon structure that repays debt as soon as possible features:*

$$q_a = \begin{cases} \pi_a(K) & \text{if } a < T(K) \\ \in [0, \pi_a(K)] & \text{if } a = T(K) \\ 0 & \text{if } a > T(K) \end{cases}$$

is a solution to the optimal contracting problem, where $T(K) \geq \hat{a}$ and $\pi_a(K) = z\left[K^\alpha \left(\frac{S_a(K)}{w}\right)^{1-\alpha}\right]^\eta - (1+r_f)S_a(K)$. However, any other structure of q_a after $\hat{a}$ that satisfies the participation constraint of financiers (PCF) would also be optimal.

Proof. From Claims 1, 3, and 4, we can characterize the Lagrange multipliers as follows:

$$\mu_a^H > 0 \text{ for } a < \hat{a}$$

$$\mu_a^H = 0 \text{ for } a \geq \hat{a}$$

$$\psi_a \geq 0 \text{ for } a < \hat{a}$$

$$\psi_a = 0 \text{ for } a \geq \hat{a}$$

For $a < \hat{a}$, since $\mu_a^H > 0$, the upper bound on coupon payments binds, meaning $q_a = \pi_a(K)$. For $a \geq \hat{a}$, the pattern of q_a is not uniquely pinned down by the first-order conditions.

From the first-order condition with respect to q_a for $a \geq \hat{a}$:

$$-(\beta\rho)^a + \lambda(\beta\rho)^a + \mu_a^L - \mu_a^H - \sum_{j=0}^{a} (\beta\rho)^j \psi_{a-j} = 0$$

Substituting $\mu_a^H = 0$ and noting that $\psi_j = 0$ for $j \geq \hat{a}$:

$$-(\beta\rho)^a + \lambda(\beta\rho)^a + \mu_a^L - \sum_{j=0}^{\min(a,\hat{a}-1)} (\beta\rho)^j \psi_{a-j} = 0$$

Since λ is a constant multiplier, there are multiple possible coupon structures that could satisfy these conditions. However, we can show that a debt structure where the firm repays its debt as soon as possible is consistent with equilibrium.

Specifically, assume the firm pays the maximum possible coupon $q_a = \pi_a(K)$ for all $a < T(K)$, where $T(K)$ is the earliest period where the participation constraint of financiers is satisfied:

$$
\begin{aligned}
(1+r_f)p_k K &= \sum_{a=0}^{T(K)} (\beta\rho)^a q_a \\
&= \sum_{a=0}^{T(K)-1} (\beta\rho)^a \pi_a(K) + (\beta\rho)^{T(K)} q_{T(K)}
\end{aligned}
$$

Under this debt structure, $q_a = 0$ for all $a > T(K)$ and $q_{T(K)} \in [0, \pi_{T(K)}(K)]$ is determined by the remaining amount needed to satisfy the participation constraint. This corresponds to the firm paying off its debt as quickly as possible.

For $a \geq \hat{a}$, the firm is indifferent between different coupon payment schedules as long as the present value of total payments remains the same, because:

1. For $a < \hat{a}$, we already established that $\mu_a^H > 0$ implies $q_a = \pi_a(K)$. 2. For $a \geq \hat{a}$, the firm is indifferent between paying coupons and dividends when $\lambda = 1$. 3. Even if $\lambda \neq 1$, the firm can adjust the timing of coupon payments to satisfy the PCF without affecting optimality, as long as the present value of the payments equals $(1 + r_f)p_k K$.

The front-loaded structure (repaying debt as soon as possible) is a natural and simple solution, but the firm would be equally satisfied with any other coupon structure that satisfies the PCF after $\hat{a}$. This is because once the enforcement constraint no longer binds (after $\hat{a}$), the time pattern of payments between dividends and coupons becomes irrelevant to firm value maximization, as long as the total present value of payments to financiers satisfies the PCF.

Therefore, while we focus on the debt repayment structure that pays off debt as soon as possible:

$$q_a = \begin{cases} \pi_a(K) & \text{if } a < T(K) \\ \in [0, \pi_a(K)] & \text{if } a = T(K) \\ 0 & \text{if } a > T(K) \end{cases}$$

where $T(K) \geq \hat{a}$. ■

Claim 6 *The optimal dividend structure features:*

$$d_a = \begin{cases} 0 & \textit{if } a < T(K) \\ \in [0, \pi_u(K)] & \textit{if } a = T(K) \\ \pi_u(K) & \textit{if } a > T(K) \end{cases}$$

Proof. The dividend at age a is given by:

$$d_a = \pi_a(K) - q_a$$

From Claim 5, we have:

$$q_a = \begin{cases} \pi_a(K) & \text{if } a < T(K) \\ \in [0, \pi_a(K)] & \text{if } a = T(K) \\ 0 & \text{if } a > T(K) \end{cases}$$

For $a < T(K)$, $q_a = \pi_a(K)$, so $d_a = 0$. For $a = T(K)$, $q_a \in [0, \pi_a(K)]$, so $d_a \in [0, \pi_a(K)]$.

For $a > T(K)$, $q_a = 0$, so $d_a = \pi_a(K)$. Additionally, from Claim 4, we know that $\psi_a = 0$ for $a \geq \hat{a}$, which implies $S_a = S_u(K, z)$ for $a \geq \hat{a}$. Since $T(K) \geq \hat{a}$, we have $S_a = S_u(K, z)$ for $a > T(K)$, and thus $\pi_a(K) = \pi_u(K)$ for $a > T(K)$.

Therefore, the optimal dividend structure is:

$$d_a = \begin{cases} 0 & \text{if } a < T(K) \\ \in [0, \pi_u(K)] & \text{if } a = T(K) \\ \pi_u(K) & \text{if } a > T(K) \end{cases}$$

■

Claim 7 *The optimal short-term debt structure features:*

$$S_a(K) = \begin{cases} \Theta_a(\gamma) \cdot S_u(K,z) & \textit{if } a < \hat{a} \\ S_u(K,z) & \textit{if } a \geq \hat{a} \end{cases}$$

where the constraint multiplier is

$$\Theta_a(\gamma) \equiv \left[\beta\rho^{T(K)-a} \cdot \frac{\beta\rho + (1-\beta\rho)(1-\gamma)}{(1-\beta\rho)(1-\xi)} \cdot (1-\eta(1-\alpha)) \right]^{\frac{1}{\eta(1-\alpha)}}$$

and $S_u(K,z)$ is the unconstrained level of short-term debt:

$$S_u(K,z) = w \left[\frac{z\eta(1-\alpha)}{w(1+r_f)} \right]^{\frac{1}{1-\eta(1-\alpha)}} K^{\frac{\eta\alpha}{1-\eta(1-\alpha)}}$$

Proof. From Claim 4, we know that $\psi_a = 0$ for $a \geq \hat{a}$, which implies $S_a = S_u(K,z)$ for $a \geq \hat{a}$. This establishes the second case in our expression.

For $a < \hat{a}$, the enforcement constraint binds ($\psi_a > 0$), and we have:

$$(1-\xi)z \left[K^\alpha \left(\frac{S_a}{w} \right)^{1-\alpha} \right]^\eta = V_a$$

From Claim 5, we know that $q_a = \pi_a(K)$ for $a < T(K)$, which implies $d_a = 0$ for

$a < T(K)$. Therefore, for $a < \min(\hat{a}, T(K))$:

$$
\begin{aligned}
V_a &= \sum_{j=0}^{\infty} (\beta\rho)^j d_{a+j} = \sum_{j=T(K)-a}^{\infty} (\beta\rho)^j \pi_u(K) \\
&= \frac{(\beta\rho)^{T(K)-a}}{1-\beta\rho} \cdot \pi_u(K)
\end{aligned}
$$

We now substitute the expression for $\pi_u(K)$:

$$
\pi_u(K) = [1 - \eta(1-\alpha)] \cdot \frac{1+r_f}{\eta(1-\alpha)} \cdot S_u(K, z)
$$

Substituting into the enforcement constraint:

$$
(1-\xi)z \left[K^{\alpha} \left(\frac{S_a}{w} \right)^{1-\alpha} \right]^{\eta} = \frac{(\beta\rho)^{T(K)-a}}{1-\beta\rho} \cdot [1 - \eta(1-\alpha)] \cdot \frac{1+r_f}{\eta(1-\alpha)} \cdot S_u(K, z)
$$

Now using the definition of $S_u(K, z)$:

$$
S_u(K, z) = w \left[\frac{z\eta(1-\alpha)}{w(1+r_f)} \right]^{\frac{1}{1-\eta(1-\alpha)}} K^{\frac{\eta\alpha}{1-\eta(1-\alpha)}}
$$

we can simplify both sides to isolate S_a:

$$
\left(\frac{S_a}{S_u(K, z)} \right)^{\eta(1-\alpha)} = \beta\rho^{T(K)-a} \cdot \frac{\beta\rho + (1-\beta\rho)(1-\gamma)}{(1-\beta\rho)(1-\xi)} \cdot (1 - \eta(1-\alpha))
$$

Taking powers:

$$
\frac{S_a}{S_u(K, z)} = \left[\beta\rho^{T(K)-a} \cdot \frac{\beta\rho + (1-\beta\rho)(1-\gamma)}{(1-\beta\rho)(1-\xi)} \cdot (1 - \eta(1-\alpha)) \right]^{\frac{1}{\eta(1-\alpha)}}
$$

Thus:

$$
S_a = \Theta_a(\gamma) \cdot S_u(K, z)
$$

where:

$$\Theta_a(\gamma) \equiv \left[\beta\rho^{T(K)-a} \cdot \frac{\beta\rho + (1-\beta\rho)(1-\gamma)}{(1-\beta\rho)(1-\xi)} \cdot (1-\eta(1-\alpha))\right]^{\frac{1}{\eta(1-\alpha)}}$$

This completes the derivation of the early-life debt policy under the enforcement constraint. ■

Claim 8 *The optimal employment structure follows the dynamics of short-term debt:*

$$n_a = \begin{cases} \frac{S_a(K)}{w} & \textit{if } a \leq \hat{a} \\ \frac{S_u(K)}{w} & \textit{if } a > \hat{a} \end{cases}$$

Proof. This follows directly from the working capital constraint (WCC): $w_a n_a \leq S_a$. Since the firm wants to maximize output, it will use all available short-term financing for labor, implying $w_a n_a = S_a$, or equivalently, $n_a = \frac{S_a}{w}$.

Therefore, the employment pattern directly mirrors the dynamics of short-term debt:

$$n_a = \begin{cases} \frac{S_a(K)}{w} & \text{if } a \leq \hat{a} \\ \frac{S_u(K)}{w} & \text{if } a > \hat{a} \end{cases}$$

■

In summary, Claims 5, 6, 7, and 8 establish the four key features of the optimal financial contract stated in Proposition 2:

1. Front-loading of long-term debt payments (Claim 5) 2. Back-loading of dividends (Claim 6) 3. Increasing short-term debt during the early life cycle (Claim 7) 4. Employment that follows the dynamics of short-term debt (Claim 8)

This completes the proof of Proposition 2.

C.1.2 Proof of Proposition 3

Let $V_0(K, T, \xi)$ denote the firm's value at age 0 as a function of scale K, maturity T, and enforcement quality ξ. For each maturity T, define $K_T(\xi)$ as the maximum feasible scale under enforcement quality ξ.

The optimal maturity $T^*(\xi)$ satisfies:

$$\frac{V_0(K_{T^*+1}(\xi), T^*+1, \xi)}{V_0(K_{T^*}(\xi), T^*, \xi)} < 1 \text{ and } \frac{V_0(K_{T^*}(\xi), T^*, \xi)}{V_0(K_{T^*-1}(\xi), T^*-1, \xi)} \geq 1 \tag{4}$$

First, we show that $T^*(\xi)$ is non-decreasing in ξ. Since higher enforcement quality relaxes the financing constraints, we have $K_T(\xi') \geq K_T(\xi)$ for any $\xi' > \xi$ and any maturity T. Additionally, V_0 is increasing in both K and ξ.

Suppose, by contradiction, that for $\xi' > \xi$, we have $T^*(\xi') < T^*(\xi)$. This implies:

$$\frac{V_0(K_{T^*(\xi')+1}(\xi'), T^*(\xi')+1, \xi')}{V_0(K_{T^*(\xi')}(\xi'), T^*(\xi'), \xi')} < 1 \tag{5}$$

However, since $T^*(\xi') < T^*(\xi)$, we have $T^*(\xi') + 1 \leq T^*(\xi)$. The optimality condition for $T^*(\xi)$ implies:

$$\frac{V_0(K_{T^*(\xi')+1}(\xi), T^*(\xi')+1, \xi)}{V_0(K_{T^*(\xi')}(\xi), T^*(\xi'), \xi)} \geq 1 \tag{6}$$

Given that $K_T(\xi') \geq K_T(\xi)$ and V_0 is increasing in ξ, we have:

$$\frac{V_0(K_{T^*(\xi')+1}(\xi'), T^*(\xi')+1, \xi')}{V_0(K_{T^*(\xi')}(\xi'), T^*(\xi'), \xi')} \geq \frac{V_0(K_{T^*(\xi')+1}(\xi), T^*(\xi')+1, \xi)}{V_0(K_{T^*(\xi')}(\xi), T^*(\xi'), \xi)} \geq 1 \tag{7}$$

This contradicts our initial assumption. Therefore, $T^*(\xi)$ is non-decreasing in ξ.

Next, we show that $\frac{\partial T^*(\xi)}{\partial \xi} \in \{0, 1\}$. Consider an infinitesimal increase from ξ to $\xi + d\xi$. Since T^* is non-decreasing and takes integer values, the change ΔT must be a non-negative integer. We need to show that $\Delta T \leq 1$.

Suppose, by contradiction, that $\Delta T \geq 2$, so $T^*(\xi + d\xi) \geq T^*(\xi) + 2$. This implies:

$$\frac{V_0(K_{T^*(\xi)+1}(\xi + d\xi), T^*(\xi) + 1, \xi + d\xi)}{V_0(K_{T^*(\xi)}(\xi + d\xi), T^*(\xi), \xi + d\xi)} \geq 1 \tag{8}$$

since otherwise $T^*(\xi) + 1$ would be optimal under $\xi + d\xi$.

The change in enforcement from ξ to $\xi + d\xi$ affects the feasible scale K_T through relaxed constraints. However, for an infinitesimal change $d\xi$, the effect on feasible scale is also infinitesimal. Given the smooth nature of the value function V_0 with respect to K and ξ, and the optimality condition at ξ:

$$\frac{V_0(K_{T^*(\xi)+1}(\xi), T^*(\xi) + 1, \xi)}{V_0(K_{T^*(\xi)}(\xi), T^*(\xi), \xi)} < 1 \tag{9}$$

the ratio cannot jump from strictly less than 1 to greater than or equal to 1 for an infinitesimal change in ξ unless $T^*(\xi)$ was exactly at an indifference point.

Therefore, $\Delta T \leq 1$ for any infinitesimal change, which implies $\frac{\partial T^*(\xi)}{\partial \xi} \in \{0, 1\}$.

C.1.3 Proof of Proposition 4

Proof. We first define the set of feasible discrete scales and maturities:

$$F = \{(T, K_T) \mid K_T \text{ is the maximum scale feasible under full repayment } (\gamma = 1) \text{ at maturity } T\} \tag{10}$$

For each $(T, K_T) \in F$, we compute firm value assuming full repayment ($\gamma = 1$):

$$V(T) = \frac{(\beta\rho)^{T+1}}{1 - \beta\rho} \cdot [1 - \eta(1 - \alpha)] \cdot K_T^{\frac{\eta\alpha}{1-\eta(1-\alpha)}} z^{\frac{1}{1-\eta(1-\alpha)}} \left(\frac{\eta(1 - \alpha)}{(1 + r_f)w}\right)^{\frac{\eta(1-\alpha)}{1-\eta(1-\alpha)}} \tag{11}$$

For small T, $V(T)$ is increasing due to rising feasible scale K_T. As T grows, discounting dominates and $V(T)$ eventually declines. Thus, there exists a unique $\tilde{T}$ such that:

$$\tilde{T} = \min \left\{ T \in \mathbb{N} \mid \frac{V(T+1)}{V(T)} < 1 \right\} \tag{12}$$

Let $\hat{a}_0$ denote the early life cycle duration under full repayment at maturity $\tilde{T}$:

$$\hat{a}_0 = \left\lceil \tilde{T} + 1 - \frac{\log\left(\frac{(1-\beta\rho)(1-\xi)}{(1-\eta(1-\alpha))\beta\rho}\right)}{\log(\beta\rho)} \right\rceil \tag{13}$$

For maturity $\tilde{T}+1$, the ELC duration with full repayment ($\gamma = 1$) would be $\hat{a}_0 + 1$.

Now consider scales $K \in [K_{\tilde{T}}, K_{\tilde{T}+1}]$. For any such K, there exists a unique $\gamma(K) \in [0, 1]$ that satisfies the participation constraint of financiers (PCF):

$$(1 + r_f)p_k K = \sum_{a=0}^{\tilde{T}} (\beta\rho)^a \pi_a(K) + (\beta\rho)^{\tilde{T}+1}\gamma(K)\pi_u(K) \tag{14}$$

The function $\gamma(K)$ is continuous and strictly increasing, with $\gamma(K_{\tilde{T}}) = 0$ and $\gamma(K_{\tilde{T}+1}) = 1$.

For maturity $\tilde{T}+1$ with partial repayment factor γ, the ELC duration is:

$$\hat{a}(\gamma) = \left\lceil \tilde{T} + 1 - \frac{\log\left(\frac{(1-\beta\rho)(1-\xi)}{(1-\eta(1-\alpha))[\beta\rho+(1-\beta\rho)(1-\gamma)]}\right)}{\log(\beta\rho)} \right\rceil \tag{15}$$

Crucially, as γ decreases from 1 toward 0, the ELC duration $\hat{a}(\gamma)$ can only take one of two values: either $\hat{a}_0 + 1$ or $\hat{a}_0$. This is because the term $(1-\gamma)(1-\beta\rho) + \beta\rho$ increases from $\beta\rho$ toward 1 as γ decreases, which can decrease the value inside the ceiling function by at most 1.

Let $\bar{\gamma}$ be the threshold value of γ where $\hat{a}(\gamma)$ transitions from $\hat{a}_0 + 1$ to $\hat{a}_0$:

$$\bar{\gamma} = 1 - \frac{(\beta\rho)^{\tilde{T}+1-\hat{a}_0}(1-\xi) - \beta\rho}{(1-\beta\rho)} \tag{16}$$

Given the monotonicity of $\gamma(K)$, there exists a unique threshold scale $\bar{K} \in [K_{\tilde{T}}, K_{\tilde{T}+1}]$ such that $\gamma(\bar{K}) = \bar{\gamma}$, where the ELC duration changes.

The firm value function is:

$$V(K,\gamma) = \frac{(\beta\rho)^{\tilde{T}+1}\pi_u(K)}{1-\beta\rho}[1-(1-\beta\rho)\gamma] \tag{17}$$

The optimal solution can now be found by comparing:

1. The maximum value in the interval $[K_{\tilde{T}}, \bar{K}]$ where $\hat{a}(\gamma) = \hat{a}_0$

2. The maximum value in the interval $[\bar{K}, K_{\tilde{T}+1}]$ where $\hat{a}(\gamma) = \hat{a}_0 + 1$

$$\frac{\partial V(K,\gamma)}{\partial K} = \frac{(\beta\rho)^{\tilde{T}+1}}{1-\beta\rho}[1-(1-\beta\rho)\gamma(K)]\frac{\partial \pi_u(K)}{\partial K} - \frac{(\beta\rho)^{\tilde{T}+1}\pi_u(K)}{1-\beta\rho}(1-\beta\rho)\gamma'(K) \tag{18}$$

$$= \frac{(\beta\rho)^{\tilde{T}+1}}{1-\beta\rho}[1-(1-\beta\rho)\gamma(K)]\frac{\eta\alpha}{1-\eta(1-\alpha)}K^{-1}\pi_u(K) - \frac{(\beta\rho)^{\tilde{T}+1}\pi_u(K)}{1-\beta\rho}(1-\beta\rho)\gamma'(K) \tag{19}$$

Assuming an interior solution, the firm's optimal scale decision balances the marginal benefits of increased scale against the marginal costs of increased debt service. Starting with the first-order condition:

$$\frac{\eta\alpha}{1-\eta(1-\alpha)}\frac{1}{K}[1-(1-\beta\rho)\gamma(K)] = (1-\beta\rho)\gamma'(K) \tag{20}$$

We can rearrange and expand this to better understand the trade-offs:

$$\frac{\eta\alpha}{1-\eta(1-\alpha)}\frac{1}{K} = (1-\beta\rho)\gamma'(K) + (1-\beta\rho)\gamma(K)\frac{\eta\alpha}{1-\eta(1-\alpha)}\frac{1}{K} \tag{21}$$

Multiplying both sides by $\pi_u(K)$ and using the fact that $\frac{\partial \pi_u(K)}{\partial K} = \frac{\eta\alpha}{1-\eta(1-\alpha)}\frac{\pi_u(K)}{K}$:

$$\frac{\partial \pi_u(K)}{\partial K} = (1-\beta\rho)\gamma'(K)\pi_u(K) + (1-\beta\rho)\gamma(K)\frac{\partial \pi_u(K)}{\partial K} \tag{22}$$

This equation has a clear economic interpretation:

1. **Left side:** $\frac{\partial \pi_u(K)}{\partial K}$ represents the full marginal benefit of increased scale - the additional profits generated by a marginal increase in capital.

2. **Right side:** The total marginal cost consists of two components:

   (a) **Increased repayment fraction cost:** $(1-\beta\rho)\gamma'(K)\pi_u(K)$ represents the additional debt service required due to the higher repayment fraction needed to finance the larger scale. As the firm increases K, it needs to borrow more, which increases $\gamma(K)$, delaying dividend payments to shareholders.

   (b) **Existing debt service on new profits:** $(1-\beta\rho)\gamma(K)\frac{\partial \pi_u(K)}{\partial K}$ represents the portion of the additional profits from the scale increase that must be used for debt service rather than distributed as dividends. Since the firm already commits a fraction $\gamma(K)$ of profits at maturity to debt service, any increase in profits also increases the absolute amount going to debt service.

Rearranging to isolate the net benefit:

$$(1-(1-\beta\rho)\gamma(K))\frac{\partial \pi_u(K)}{\partial K} = (1-\beta\rho)\gamma'(K)\pi_u(K) \tag{23}$$

The left side now represents the net marginal benefit of a scale increase - the gross benefit reduced by the portion that goes to increased debt service on existing obligations. At the optimum, this net benefit exactly equals the marginal cost from the increased repayment fraction.

This optimality condition illustrates the fundamental trade-off facing financially constrained young firms: increasing scale requires more debt, which delays dividend payments, but generates higher profits. The optimal scale balances these competing forces.

Given that $\gamma'(K) > 0$, this equation balances the marginal benefit of increased scale against the marginal cost of delayed dividends. The existence of a solution is guaranteed by the continuity of $V(K, \gamma(K))$ over the compact interval $[K_{\tilde{T}}, K_{\tilde{T}+1}]$.

Let K^* be the optimal scale that maximizes $V(K, \gamma(K))$. The optimal maturity T^* is then determined by:

$$T^* = \tilde{T} + \mathbf{1}_{\gamma(K^*)>0} \tag{24}$$

This ensures that if $\gamma(K^*) = 0$ (no repayment at $\tilde{T}+1$), the firm chooses maturity $T^* = \tilde{T}$, while if $\gamma(K^*) > 0$, the firm chooses maturity $T^* = \tilde{T}+1$ with partial repayment at time $\tilde{T}+1$.

The optimal repayment share is $\gamma^* = \gamma(K^*)$, and the resulting ELC duration is:

$$\hat{a}^* = \hat{a}(\gamma^*) = \hat{a}_0 + \mathbf{1}_{\gamma^*>\bar{\gamma}} \tag{25}$$

This formulation captures the fundamental trade-off: choosing a larger scale requires more debt and potentially longer ELC duration, while a smaller scale allows for earlier dividend payments. ■

C.1.4 Proof of Proposition 5

Part 1: Maximum feasible scale is increasing in enforcement

For any fixed maturity T, the maximum feasible scale K_T is determined by the binding enforcement constraint at age $a = 0$:

$$(1-\xi)z\left[K_T^{\alpha}\left(\frac{S_0}{w}\right)^{1-\alpha}\right]^{\eta} = V_0(K_T, T) \tag{26}$$

Given our characterization of short-term debt:

$$S_0(K_T) = \left[\frac{(\beta\rho)^{T+1}}{1-\beta\rho}\frac{1}{(1-\xi)(1-\beta\rho)}\right]^{\frac{1}{\eta(1-\alpha)}} S_u(K_T, z) \tag{27}$$

Taking the total derivative of the enforcement constraint with respect to ξ:

$$-z\left[K_T^{\alpha}\left(\frac{S_0}{w}\right)^{1-\alpha}\right]^{\eta} + (1-\xi)z\eta\left[K_T^{\alpha}\left(\frac{S_0}{w}\right)^{1-\alpha}\right]^{\eta-1} K_T^{\alpha}\left(\frac{1}{w}\right)^{1-\alpha}(1-\alpha)S_0^{-\alpha}\frac{\partial S_0}{\partial \xi} + \tag{28}$$

$$(1-\xi)z\eta\left[K_T^{\alpha}\left(\frac{S_0}{w}\right)^{1-\alpha}\right]^{\eta-1}\left(\frac{S_0}{w}\right)^{1-\alpha}\alpha K_T^{\alpha-1}\frac{\partial K_T}{\partial \xi} = \frac{\partial V_0(K_T,T)}{\partial K_T}\frac{\partial K_T}{\partial \xi} \tag{29}$$

Solving for $\frac{\partial K_T}{\partial \xi}$:

$$\frac{\partial K_T}{\partial \xi} = \frac{z\left[K_T^{\alpha}\left(\frac{S_0}{w}\right)^{1-\alpha}\right]^{\eta} - (1-\xi)z\eta\left[K_T^{\alpha}\left(\frac{S_0}{w}\right)^{1-\alpha}\right]^{\eta-1} K_T^{\alpha}\left(\frac{1}{w}\right)^{1-\alpha}(1-\alpha)S_0^{-\alpha}\frac{\partial S_0}{\partial \xi}}{\frac{\partial V_0(K_T,T)}{\partial K_T} - (1-\xi)z\eta\left[K_T^{\alpha}\left(\frac{S_0}{w}\right)^{1-\alpha}\right]^{\eta-1}\left(\frac{S_0}{w}\right)^{1-\alpha}\alpha K_T^{\alpha-1}} \tag{30}$$

The numerator is positive because: 1. The first term is positive: $z\left[K_T^\alpha\left(\frac{S_0}{w}\right)^{1-\alpha}\right]^\eta > 0$ 2. The second term is negative: $\frac{\partial S_0}{\partial \xi} > 0$ because improved enforcement relaxes the borrowing constraint

The denominator is positive because: 1. $\frac{\partial V_0(K_T,T)}{\partial K_T} > 0$ (firm value increases with scale) 2. The second term is positive

Therefore, $\frac{\partial K_T}{\partial \xi} > 0$, which proves that the maximum feasible scale K_T is strictly increasing in the enforcement parameter ξ.

Part 2: Optimal maturity is weakly increasing in enforcement

Let $\tilde{T}(\xi) = \arg\max_T V(T,\xi)$ denote the discrete optimal maturity as a function of enforcement. We've already proven in our earlier proposition that this is non-decreasing in ξ.

Under our revised framework, the true optimal maturity is $T^*(\xi) = \tilde{T}(\xi) + 1$ with partial repayment fraction $\gamma(K^*(\xi)) < 1$. Since $\tilde{T}(\xi)$ is non-decreasing in ξ, it follows that $T^*(\xi)$ is also non-decreasing in ξ.

Specifically:

$$\frac{\partial T^*(\xi)}{\partial \xi} = \frac{\partial \tilde{T}(\xi)}{\partial \xi} \in \{0, 1\} \tag{31}$$

This follows from the fact that for each maturity T, the maximum feasible scale $K_T(\xi)$ is increasing in ξ, making higher maturities relatively more attractive as enforcement improves.

Part 3: Scale effect and value effect offsetting

To see why the scale effect and value effect largely offset each other, we examine the ELC duration formula:

$$\hat{a} = \left\lceil T^* + 1 - \frac{\ln\left(\frac{(1-\beta\rho)}{(1-\xi)(1-\beta\rho)}\right)}{\ln(\beta\rho)} \right\rceil \tag{32}$$

Note that K does not appear explicitly in this formula. The scale effect increases short-term borrowing needs proportionally to $K^{\frac{\eta\alpha}{1-\eta(1-\alpha)}}$, which tightens the enforcement constraint. However, the value effect increases the continuation value of the firm by the same factor, relaxing the enforcement constraint.

This proportionality can be seen by examining the enforcement constraint:

$$(1-\xi)z\left[K^{\alpha}\left(\frac{S_a}{w}\right)^{1-\alpha}\right]^{\eta} \leq V_a \tag{33}$$

Both the left side (output) and right side (continuation value) scale with $K^{\frac{\eta\alpha}{1-\eta(1-\alpha)}}$, causing the direct effects of scale on the ELC duration to approximately cancel out.

Therefore, the primary effect of scale on ELC duration works through the maturity channel T^*, creating the clean relationship that countries with stronger enforcement have firms with larger scale and longer debt maturities, but shorter ELC durations.

C.1.5 Proof of Proposition 6

We will prove by contradiction that short-term leverage increases and long-term leverage decreases during the early life cycle.

First, recall the definitions of short-term and long-term leverage:

$$\ell_a^S = \frac{S_a}{A_a} = \frac{S_a}{V_a + S_a + L_a}$$
$$\ell_a^L = \frac{L_a}{A_a} = \frac{L_a}{V_a + S_a + L_a}$$

Contradiction Argument for Short-Term Leverage

Suppose, contrary to our claim, that $\ell^S_{a+1} - \ell^S_a \leq 0$ for some $a < \hat{a}$ during the early life cycle. This would imply:

$$\frac{S_{a+1}}{V_{a+1} + S_{a+1} + L_{a+1}} \leq \frac{S_a}{V_a + S_a + L_a}$$

Cross-multiplying, we get:

$$S_{a+1}(V_a + S_a + L_a) \leq S_a(V_{a+1} + S_{a+1} + L_{a+1})$$

Expanding and rearranging:

$$S_{a+1}V_a + S_{a+1}S_a + S_{a+1}L_a \leq S_aV_{a+1} + S_aS_{a+1} + S_aL_{a+1}$$

$$S_{a+1}V_a + S_{a+1}L_a \leq S_aV_{a+1} + S_aL_{a+1}$$

$$S_{a+1}V_a - S_aV_{a+1} \leq S_aL_{a+1} - S_{a+1}L_a$$

$$S_{a+1}V_a - S_aV_{a+1} \leq S_aL_{a+1} - S_{a+1}L_a$$

From Proposition 2, we know that during the ELC:

$$S_{a+1} > S_a \quad \text{(short-term debt increases)}$$

$$V_{a+1} > V_a \quad \text{(firm value increases)}$$

$$L_{a+1} < L_a \quad \text{(long-term debt decreases)}$$

These relationships imply:

$$S_{a+1}V_a - S_aV_{a+1} = S_aV_a\left(\frac{S_{a+1}}{S_a} - \frac{V_{a+1}}{V_a}\right)$$

Since $S_{a+1} > S_a$ and $V_{a+1} > V_a$, the sign of $(S_{a+1}V_a - S_aV_{a+1})$ depends on the relative growth rates of S and V.

For the right-hand side:

$$S_aL_{a+1} - S_{a+1}L_a = S_aL_a\left(\frac{L_{a+1}}{L_a} - \frac{S_{a+1}}{S_a}\right)$$

Since $L_{a+1} < L_a$ and $S_{a+1} > S_a$, we have $\frac{L_{a+1}}{L_a} < 1 < \frac{S_{a+1}}{S_a}$, making $(S_aL_{a+1} - S_{a+1}L_a) < 0$.

Now, from our original assumption $\ell^S_{a+1} - \ell^S_a \leq 0$, we derived:

$$S_{a+1}V_a - S_aV_{a+1} \leq S_aL_{a+1} - S_{a+1}L_a$$

But we've shown that the right-hand side is negative, which means the left-hand side must also be negative:

$$\begin{aligned} S_{a+1}V_a - S_aV_{a+1} &< 0 \\ \frac{S_{a+1}}{S_a} &< \frac{V_{a+1}}{V_a} \end{aligned}$$

This implies that the growth rate of firm value exceeds the growth rate of short-term debt during the ELC.

From Proposition 2, we have:

$$\begin{aligned} S_a &= \left[\frac{(\beta\rho)^{T(K)+1-a}}{1-\beta\rho}\frac{1}{1-\xi}\right]^{\frac{1}{\eta(1-\alpha)}} S_u(K,z) \\ V_a &= (\beta\rho)^{T+1-a}\frac{\pi_u(K,z)}{1-\beta\rho} \end{aligned}$$

Computing the growth rates:

$$\frac{S_{a+1}}{S_a} = \left[\frac{(\beta\rho)^{T(K)-a}}{(\beta\rho)^{T(K)+1-a}}\right]^{\frac{1}{\eta(1-\alpha)}} = \left[\frac{1}{\beta\rho}\right]^{\frac{1}{\eta(1-\alpha)}}$$
$$\frac{V_{a+1}}{V_a} = \frac{(\beta\rho)^{T-a}}{(\beta\rho)^{T+1-a}} = \frac{1}{\beta\rho}$$

Since $\eta(1-\alpha) < 1$ and $\beta\rho < 1$, we have:

$$\left[\frac{1}{\beta\rho}\right]^{\frac{1}{\eta(1-\alpha)}} > \frac{1}{\beta\rho}$$

This means $\frac{S_{a+1}}{S_a} > \frac{V_{a+1}}{V_a}$, which contradicts our derived inequality $\frac{S_{a+1}}{S_a} < \frac{V_{a+1}}{V_a}$.

Therefore, our initial assumption that $\ell^S_{a+1} - \ell^S_a \leq 0$ must be false, and we conclude that $\ell^S_{a+1} - \ell^S_a > 0$ during the early life cycle.

Contradiction Argument for Long-Term Leverage

Similarly, we can prove by contradiction that long-term leverage decreases during the ELC.

Suppose, contrary to our claim, that $\ell^L_{a+1} - \ell^L_a \geq 0$ for some $a < \hat{a}$ during the early life cycle. This would imply:

$$\frac{L_{a+1}}{V_{a+1} + S_{a+1} + L_{a+1}} \geq \frac{L_a}{V_a + S_a + L_a}$$

Following similar steps as above, we derive:

$$L_{a+1}V_a - L_aV_{a+1} \geq L_aS_{a+1} - L_{a+1}S_a$$

Since $L_{a+1} < L_a$, $V_{a+1} > V_a$, and $S_{a+1} > S_a$, we have:

$$L_{a+1}V_a - L_aV_{a+1} = L_aV_a\left(\frac{L_{a+1}}{L_a} - \frac{V_{a+1}}{V_a}\right) < 0$$
$$L_aS_{a+1} - L_{a+1}S_a = L_aS_a\left(\frac{S_{a+1}}{S_a} - \frac{L_{a+1}}{L_a}\right) > 0$$

This leads to the inequality:

$$\underbrace{L_{a+1}V_a - L_aV_{a+1}}_{\text{(negative)}} \geq \underbrace{L_aS_{a+1} - L_{a+1}S_a}_{\text{(positive)}}$$

This is a contradiction. Therefore, our initial assumption that $\ell^L_{a+1} - \ell^L_a \geq 0$ must be false, and we conclude that $\ell^L_{a+1} - \ell^L_a < 0$ during the early life cycle.

This completes our proof by contradiction that during the early life cycle, short-term leverage increases and long-term leverage decreases with firm age.

C.1.6 Proof of Proposition 7

Preliminary version (To Be Updated Soon). What follows is the sketch of the proof

Proof. We derive explicit formulas for leverage ratios from the firm's balance sheet structure. For long-term leverage, we have:

$$\ell^L_a = \frac{L_a}{V_a + (1 + r_f)S_a + L_a} \tag{34}$$

For $a = 0$, we can explicitly characterize this as:

$$\ell^L_0 = \frac{N(\xi)}{D(\xi)} \tag{35}$$

where

$$N(\xi) = \sum_{a=0}^{\hat{a}-1} (\beta\rho)^a \left[\frac{[\Theta_a(\gamma)^{\eta(1-\alpha)} - \eta(1-\alpha) \cdot \Theta_a(\gamma)]}{1-\eta(1-\alpha)} \right] + \sum_{a=\hat{a}}^{T(K)-1} (\beta\rho)^a + (\beta\rho)^T \gamma \tag{36}$$

$$D(\xi) = \sum_{a=0}^{\hat{a}-1} (\beta\rho)^a \left[\frac{[\Theta_a(\gamma)^{\eta(1-\alpha)} - \eta(1-\alpha) \cdot \Theta_a(\gamma)]}{1-\eta(1-\alpha)} \right] + \sum_{a=\hat{a}}^{\infty} (\beta\rho)^a + \frac{\eta(1-\alpha)}{1-\eta(1-\alpha)} \tag{37}$$

For the long-term debt and equity components, we have the elegant formulation:

$$L_0 + V_0 = \frac{\pi_u(K)}{1-\beta\rho} - \pi_u(K) \left[\sum_{a=0}^{\hat{a}-1} (\beta\rho)^a \left[1 - \frac{[\Theta_a(\gamma)^{\eta(1-\alpha)} - \eta(1-\alpha) \cdot \Theta_a(\gamma)]}{1-\eta(1-\alpha)} \right] \right] \tag{38}$$

This formulation shows that the equity and debt components are reduced by a financial constraint penalty term that depends on enforcement quality.

For short-term leverage, we have:

$$\ell_a^S = \frac{(1+r_f)S_a}{V_a + (1+r_f)S_a + L_a} \tag{39}$$

At age 0, this can be expressed as:

$$\ell_0^S = \frac{\frac{\eta(1-\alpha)}{1-\eta(1-\alpha)} \Theta_0(\gamma)}{\frac{\eta(1-\alpha)}{1-\eta(1-\alpha)} \Theta_0(\gamma) + D(\xi)} \tag{40}$$

The non-monotonicity effect at early ages arises from the differential response of components to changes in enforcement quality. For age 0, we analyze how the elasticities of L_0, S_0, and total assets A_0 respond to enforcement changes.

For short-term debt at age 0, the elasticity with respect to ξ has two components:

$$\frac{\partial \ln S_0}{\partial \ln \xi} = \frac{\partial \ln \Theta_0(\gamma)}{\partial \ln \xi} + \frac{\partial \ln S_u(K)}{\partial \ln K} \frac{\partial \ln K}{\partial \ln \xi} \tag{41}$$

The direct effect on the constraint multiplier is:

$$\frac{\partial \ln \Theta_0(\gamma)}{\partial \ln \xi} = \frac{1}{\eta(1-\alpha)} \frac{\xi}{1-\xi} \tag{42}$$

This term grows increasingly sensitive at higher levels of enforcement, causing S_0 to increase disproportionately as ξ approaches 1.

For long-term debt, the elasticity is primarily determined by the scale effect:

$$\frac{\partial \ln L_0}{\partial \ln \xi} \approx \frac{\partial \ln \pi_u(K)}{\partial \ln K} \frac{\partial \ln K}{\partial \ln \xi} \tag{43}$$

The key insight into non-monotonicity comes from comparing these elasticities. At very low levels of enforcement (small ξ), we have:

$$\frac{\partial \ln S_0}{\partial \ln \xi} > \frac{\partial \ln L_0}{\partial \ln \xi} \tag{44}$$

In this regime, the short-term debt elasticity dominates, causing the total assets elasticity to exceed that of long-term debt, potentially creating a negative relationship between long-term leverage and enforcement quality at young ages.

As enforcement quality increases (larger ξ) or firms age (increased a), the elasticities rebalance:

$$\frac{\partial \ln L_a}{\partial \ln \xi} > \frac{\partial \ln S_a}{\partial \ln \xi} \text{ for } a \geq a^* \tag{45}$$

This establishes the monotonic positive relationship between leverage and enforcement quality for firms beyond age a^*.

For short-term leverage, numerical simulations with typical parameter values ($\eta = 0.9$, $\alpha = 0.33$, $\beta\rho \approx 0.81$) indicate the elasticity of S_a dominates that of total assets consistently across all ages, yielding $\frac{\partial \ell_a^S}{\partial \xi} > 0$.

These elasticity considerations provide the formal analytical basis for the relationships stated in the proposition. ■

C.1.7 Proof of Proposition 9

We prove that initial leverage ℓ_0^L is invariant to entrepreneurial talent χ_i.

From the firm's optimal contract, we have:

$$L_0 = \chi_i^\theta \cdot \pi(K,1,1) \cdot [\mathcal{U} \cdot (1 - (\beta\rho)^{T+1}) - \mathcal{D}] \tag{46}$$

$$L_0 + V_0 = \chi_i^\theta \cdot \pi(K,1,1) \cdot [\mathcal{U} - \mathcal{D}] \tag{47}$$

$$(1 + r_f)S_0 = \chi_i^\theta \cdot \pi(K,1,1) \cdot \Theta_0 \cdot z_0^\theta \cdot (\theta - 1) \tag{48}$$

where $\mathcal{U} = \sum_{a=0}^{\infty} (\beta\rho)^a \pi_a^u$ is unconstrained value, $\mathcal{D}$ captures constraint distortions, and all expressions scale with χ_i^θ.

The initial leverage ratio is:

$$\ell_0^L = \frac{L_0}{L_0 + V_0 + (1 + r_f)S_0} = \frac{\chi_i^\theta \cdot [\mathcal{U}(z_0) \cdot (1 - (\beta\rho)^{T(\xi_c)+1}) - \mathcal{D}]}{\chi_i^\theta \cdot [\mathcal{U}(z_0) - \mathcal{D}(\xi_c, z_0)] + \Theta_0 \cdot \chi_i^\theta \cdot z_0^\theta \cdot (\theta - 1)} \tag{49}$$

Factoring out χ_i^θ:

$$\ell_0^L = \frac{\mathcal{U}(z_0) \cdot (1 - (\beta\rho)^{T(\xi_c)+1}) - \mathcal{D}(\xi, z_0)}{\mathcal{U}(z_0) - \mathcal{D}(\xi_c, z_0) + \Theta_0 z_0^\theta (\theta - 1)} \tag{50}$$

where:

$$\mathcal{U}(z_0) = z_0^\theta + \sum_{a=1}^{\infty} (\beta\rho)^a \mathbb{E}[z_a^\theta] \tag{51}$$

$$\mathcal{D}(\xi, z_0) = z_0^\theta [1 - \Theta_0^{\eta(1-\alpha)}] + \sum_{a=1}^{\hat{a}-1} (\beta\rho)^a \mathbb{E}[z_a^\theta][1 - \Theta_a^{\eta(1-\alpha)}] \tag{52}$$

The talent term χ_i^θ cancels completely, proving invariance. The ratio depends only on enforcement ξ_c, contract structure (T, γ), and initial shock z_0.

C.1.8 Explicit Expression for Enforcement Identification

The function $\Psi(\xi)$ that maps enforcement to expected initial leverage can be expressed as:

$$\Psi(\xi) = \mathbb{E}_z[\ell_0^L] = \mathbb{E}_z \left[\frac{\mathcal{U}(z_0) \cdot (1 - (\beta\rho)^{T(\xi_c)+1}) - \mathcal{D}(\xi, z_0)}{\mathcal{U}(z_0) - \mathcal{D}(\xi_c, z_0) + \Theta_0 z_0^\theta(\theta - 1)} \right]$$

$$\mathbb{E}_z \left[\frac{[z_0^\theta + \sum_{a=1}^{\infty} (\beta\rho)^a \mathbb{E}[z_a^\theta]] \cdot (1 - (\beta\rho)^{T(\xi_c)+1}) - [z_0^\theta[1 - \Theta_0^{\eta(1-\alpha)}] + \sum_{a=1}^{\hat{a}-1} (\beta\rho)^a \mathbb{E}[z_a^\theta][1 - \Theta_a^{\eta(1-\alpha)}]]}{[z_0^\theta + \sum_{a=1}^{\infty} (\beta\rho)^a \mathbb{E}[z_a^\theta]] - [z_0^\theta[1 - \Theta_0^{\eta(1-\alpha)}] + \sum_{a=1}^{\hat{a}-1} (\beta\rho)^a \mathbb{E}[z_a^\theta][1 - \Theta_a^{\eta(1-\alpha)}]] + \Theta_0 z_0^\theta(\theta - 1)} \right]$$

The monotonicity of $\Psi(\xi)$ follows from the fact that better enforcement (higher ξ) increases Θ_a for all a, raising both the numerator $F(\xi, z_0)$ and denominator, but the numerator effect dominates, yielding $\Psi'(\xi) > 0$.

This explicit characterization shows that the leverage ratio $\mathbb{E}[\ell_0^L|c]/\mathbb{E}[\ell_0^L|c']$ identifies the relative enforcement quality $\Psi(\xi_c)/\Psi(\xi_{c'})$, which can be inverted to recover $\xi_c/\xi_{c'}$ given the monotonicity of Ψ.

C.2 Comparison to Standard Borrowing Constraint Specifications

To better understand the structure and implications of our dynamic enforcement-based constraint, we now compare it to more commonly used borrowing constraint formulations in the literature.

(a) EBITDA-based Constraint A widely used reduced-form specification is a fixed multiple of earnings before interest, taxes, depreciation, and amortization (EBITDA):

$$S_a \leq \lambda \cdot \text{EBITDA}_a, \quad \text{with } \lambda \text{ constant.}$$

In our framework, this is equivalent to imposing:

$$S_a \leq \lambda \cdot \left(\chi_i z_a K^{\eta\alpha} \left(\frac{S_a}{w} \right)^{\eta(1-\alpha)} \right)$$

$$S_a \leq \left[\lambda \cdot \left(\chi_i z_a K^{\eta\alpha} \left(\frac{1}{w} \right)^{\eta(1-\alpha)} \right) \right]^{\frac{1}{1-\eta(1-\alpha)}}$$

Using $S_u(K, \chi_i, z) = w \left(\frac{\chi_i z \cdot \eta(1-\alpha)}{w(1+r_f)} \right)^{\frac{1}{1-\eta(1-\alpha)}} K^{\frac{\eta\alpha}{1-\eta(1-\alpha)}}$

$$S_a \leq \left[\lambda \frac{1+r_f}{\eta(1-\alpha)} \cdot \right]^{\frac{1}{1-\eta(1-\alpha)}} S_u(K, \chi_i, z)$$

Then an EBITDA based borrowing constraint needs to deliver a $\lambda < \bar{\lambda}$ in order for the ad-hoc borrowing constraint to bind.

$$\bar{\lambda} = \frac{\eta(1-\alpha)}{1+r_f}$$

solved implicitly. Unlike our model, such a constraint does not evolve with repayment history and maturity structure.

(b) Kiyotaki–Moore-Type Collateral Constraints Suppose that defaulting firms can re-enter but must repurchase capital at full cost, and that they draw productivity $z_0 = \chi_i$

at entry. Then default value is:

$$(1-\xi) \cdot \chi_i K^{\eta\alpha} \left(\frac{S_0}{w}\right)^{\eta(1-\alpha)} - p_k(1+r_f)K,$$

and enforceability requires that the continuation value exceed this. This is akin to the net resale value formulation in Kiyotaki and Moore (1997), and mirrors the insight in Rampini and Viswanathan (2010) and Rampini (2013) that lack of commitment can generate collateral constraints even when capital is not physical collateral.

(c) Size-Dependent Constraint Gopinath et al. (2017) propose a reduced-form constraint where borrowing capacity scales with firm size:

$$S_a \leq \phi(S_a) \cdot K, \quad \text{with } \phi'(\cdot) > 0, \text{ and concave.}$$

This captures decreasing marginal access to finance, but abstracts from dynamics of repayment and firm-level enforcement slack.

(d) Comparative Insight Specifications (a) and (b) impose static wedges: λ or ξ are fixed and determine a constant share of earnings or capital. In contrast, our formulation captures the *dynamic* nature of constraint relaxation, as emphasized by Albuquerque and Hopenhayn (2004). The maturity structure T, repayment history z^a, and contract enforcement ξ jointly determine a state-contingent constraint Θ_a.

While the Gopinath et al. (2017) size-dependent constraint partially captures heterogeneity in tightness, it conflates age and size. Our model shows that *age* is a more informative dimension of constraint tightness, since repayment progress accumulates deterministically over time, smoothing idiosyncratic shocks. This insight motivates the use of age as a classification tool when identifying constrained firms.

C.3 EBITDA-Based Interpretation of the Borrowing Constraint

We now show how our enforcement-based borrowing constraint can be rewritten in a form that resembles standard empirical formulations based on EBITDA, but without assuming that the firm is unconstrained.

Start with the general form of the borrowing constraint:

$$S_a \leq \Theta_a(z^a, \gamma) \cdot S_u(K, \chi_i, z_a)$$

Now multiply and divide by EBITDA:

$$S_a = \left(\frac{\Theta_a(z^a, \gamma) \cdot S_u(K, \chi_i, z_a)}{\text{EBITDA}_a} \right) \cdot \text{EBITDA}_a$$

So we define an *endogenous multiplier*:

$$\lambda_a^{dyn} = \frac{\Theta_a(z^a, \gamma) \cdot S_u(K, \chi_i, z_a)}{\text{EBITDA}_a}$$

To express this in terms of parameters, recall:

$$S_u(K, \chi_i, z_a) = w \left(\frac{\chi_i z_a \cdot \eta(1-\alpha)}{w(1+r_f)} \right)^{\theta} K^{\eta\alpha\theta}, \quad \theta = \frac{1}{1-\eta(1-\alpha)}$$

and output at optimal scale is:

$$\text{EBITDA}_a = y_a = \chi_i z_a \left[K^{\alpha} \left(\frac{S_a}{w} \right)^{1-\alpha} \right]^{\eta} = (\chi_i z_a)^{\theta} K^{\eta\alpha\theta} \cdot A_y$$

The constant A_y collects all parameters in the expression for unconstrained output:

$$A_y = (\eta(1-\alpha))^{\theta(1-\alpha)\eta} \cdot w^{-\eta(1-\alpha)} \cdot (1+r_f)^{-\theta(1-\alpha)\eta}$$

So:

$$\lambda_a^{dyn} = \frac{\Theta_a(z^a,\gamma)\cdot w\left(\frac{\chi_i z_a\cdot\eta(1-\alpha)}{w(1+r_f)}\right)^{\theta} K^{\eta\alpha\theta}}{(\chi_i z_a)^{\theta} K^{\eta\alpha\theta}\cdot A_y} \tag{53}$$

$$= \Theta_a(z^a,\gamma)\cdot\frac{w}{A_y}\cdot\left(\frac{\eta(1-\alpha)}{w(1+r_f)}\right)^{\theta} \tag{54}$$

This yields:

$$S_a = \lambda_a^{dyn}\cdot \text{EBITDA}_a, \quad \text{with } \lambda_a^{dyn} = \Theta_a(z^a,\gamma)\cdot\left(\frac{\eta(1-\alpha)}{1+r_f}\right)^{\theta}\cdot\frac{w^{1-\theta}}{A_y}$$

Interpretation. Unlike traditional EBITDA multiples used in practice, our model shows that λ_a^{dyn} is not constant. It evolves endogenously based on:

- **Age** a, through $\Theta_a(z^a,\gamma)$, which grows with repayment progress,
- **Productivity** z_a, affecting both scale and tightness,
- **Enforcement quality** ξ, and interest rate r_f.

This reformulation shows how our model nests and generalizes empirical leverage rules, while grounding them in micro-founded contract dynamics.

Remark: Fixed-λ Constraints Emerge in the Absence of Frictions. In the unconstrained benchmark, the firm's optimal borrowing level satisfies:

$$S_u(K,\chi_i,z_a) = \frac{\eta(1-\alpha)}{1+r_f}\cdot \text{EBITDA}_a$$

Hence, reduced-form constraints of the form $S_a \leq \lambda\cdot\text{EBITDA}_a$ naturally arise even without financial frictions. In practice, λ is often estimated from cross-sectional data without conditioning on age or constraint status, effectively recovering the unconstrained multiplier:

$$\hat{\lambda}^{\text{reduced-form}} \approx \frac{\eta(1-\alpha)}{1+r_f}$$

However, our model shows that the true borrowing constraint is:

$$S_a = \lambda_a^{\text{dyn}} \cdot \text{EBITDA}_a = \Theta_a(z^a, \gamma) \cdot \frac{\eta(1-\alpha)}{1+r_f} \cdot \frac{1}{z_a} \cdot \text{EBITDA}_a$$

The ratio between the estimated and actual constraint slack is:

$$\frac{\hat{\lambda}}{\lambda_a^{\text{dyn}}} = \frac{1}{\Theta_a(z^a, \gamma)} \cdot z_a$$

This expression illustrates how assuming a constant λ leads to systematic bias: When firms are young (low Θ_a) and productive (high z_a), the true constraint is *tighter* than implied by $\hat{\lambda}$, leading to **underestimation of constraint severity**.Thus, failing to account for the dynamics in λ_a masks the link between age, enforcement, and financial slack, and may lead to misclassification of constraint severity in empirical applications.