Connected for Better or Worse? The Role of Production Networks in Financial Crises*

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Abstract

Production networks shape the severity of financial crises by influencing how sectoral shocks propagate. We develop a small open economy model with intersectoral linkages and an occasionally binding collateral constraint to study this mechanism. In advanced economies, stronger linkages between tradable and nontradable sectors create input price adjustments that stabilize profits, reducing financial distress. In contrast, weaker linkages in emerging markets amplify profit contractions, increasing crisis severity. Using industry-level input-output data, we show that sectoral connectivity systematically differs between emerging and advanced economies, reinforcing its role in Sudden Stops. Our findings suggest that macroprudential policies should consider production structures alongside financial vulnerabilities, as network connectivity influences financial stability.

Keywords: Production Networks, Financial Crises, Sudden Stops, Macroprudential Policy

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1 Introduction

Financial crises occur more frequently and with greater severity in emerging markets than in advanced economies (e.g., Calvo et al., 2006; Mendoza, 2010; Bianchi and Mendoza, 2020). Episodes of Sudden Stops–sudden reversals in capital inflows–often trigger deep recessions and protracted recoveries. The prevailing view in the literature attributes these disparities primarily to differences in financial development (Garcia-Cicco et al., 2010) and to the greater volatility of external shocks faced by emerging markets (Aguiar and Gopinath, 2007). While these factors are undoubtedly important, they do not fully explain the systematic cross-country variation in crisis severity.

This paper highlights a neglected structural factor in the analysis of financial crises: the role of production networks in amplifying or mitigating economic downturns.[1] We argue that differences in intersectoral linkages–that is, the extent to which firms in tradable and nontradable sectors depend on each other for intermediate inputs–play a critical role in shaping the propagation and amplification of sectoral shocks.

In advanced economies, production networks tend to be more internally connected, with nontradable sectors relying more heavily on both tradable and nontradable inputs. This denser interdependence creates transmission channels through which adverse shocks can be partially absorbed. When a sector contracts, its reduced input demand lowers input prices, easing cost pressures elsewhere in the economy. These price adjustments act as automatic stabilizers, cushioning profit declines and mitigating the tightening of financial constraints. In contrast, many emerging markets feature sparser production linkages, particularly in the nontradable sector, which relies more heavily on commodity inputs. This weaker internal buffering amplifies the propagation of shocks, leading to sharper profit contractions and more severe financial crises.

To formalize this mechanism, we develop an analytical framework that integrates production networks into a small open economy with an occasionally binding collateral constraint. Our model features a two-sector structure, comprising a tradable and a nontradable sector, where each uses the other's output as an intermediate input. This setup allows us to examine how intersectoral connectivity influences the severity of financial crises.

Using a perfect foresight framework similar to that of Mendoza (2005), we show that network structure plays a critical role in shaping the economy's response to sectoral shocks. Specifically, we demonstrate that two economies with identical levels of finan-

[1] Fadinger et al. (2022), McNerney et al. (2022), and Gloria et al. (2024) document how production network heterogeneity accounts for income differences across countries. Miranda-Pinto (2021) and Miranda-Pinto et al. (2023) emphasize how production network structures drive cross-country differences in output volatility and skewness.

cial development but differing only in their production network structures can exhibit markedly different outcomes in response to the same shock. In economies with strong intersectoral linkages–that is, where sectors rely heavily on inputs from other sectors–negative shocks trigger input price adjustments that act as automatic stabilizers. These stabilizers help cushion profit losses and mitigate financial constraints. In contrast, when sectors are more self-contained–as is more common in emerging markets–shocks propagate more directly, resulting in deeper contractions and prolonged financial distress.

To validate this mechanism empirically, we combine industry-level input–output data with macroeconomic outcomes during Sudden Stops. We first document that sectoral connectivity systematically differs between emerging and advanced economies: advanced economies exhibit denser networks, particularly through greater use of nontradable inputs. We then construct a quantitative measure of network complexity–the distance of a country's input–output matrix from a diagonal structure–and show that this metric predicts the severity of Sudden Stops. In panel regressions with country and year fixed effects, we find that countries with more interconnected production structures experience significantly smaller declines in GDP and smaller current account reversals when capital flows suddenly dry up. These findings imply that the structure of production networks–not just financial development or external volatility–helps explain the heterogeneity in crisis outcomes across countries. As a result, macroprudential policy design should consider not only financial sector vulnerabilities but also the real economy's architecture of intersectoral linkages.

To assess the quantitative role of network structure, we develop a multi-sector DSGE model featuring three sectors: a commodity-producing sector, a non-commodity tradable sector, and a nontradable sector. This richer framework allows us to examine how production structure interacts with macroprudential policy in environments that differ systematically in both financial depth and sectoral connectivity. We calibrate the model to match key empirical features of emerging and advanced economies. Crucially, the quantitative model employs CES production functions rather than Cobb-Douglas. This formulation allows the input mix to adjust with relative prices, meaning that policy interventions that shift sectoral prices–such as taxes or external shocks–can endogenously alter the effective production network.

We conduct a counterfactual in which we endow an advanced economy with the network structure of an average emerging market while keeping all other parameters and shock processes unchanged. This experiment reveals that the modified economy experiences a 30% larger GDP decline and a 28% larger current account reversal during a Sudden Stop. It is also nearly 50% more likely to encounter such an episode (1.5% probability versus 1%). These findings highlight the quantitative importance of network structure in shaping both the likelihood and severity of financial crises.

We then turn to a normative analysis and characterize the problem faced by a social planner. Production networks increase the dimensionality of the planner's problem by expanding the set of input choices, which both introduces more distorted margins and creates new policy instruments in the form of sectoral taxes and subsidies. We show that the presence of networks, combined with the occasionally binding borrowing constraint, distorts input allocations in ways not internalized by decentralized agents. As in Bianchi (2011), a pecuniary externality arises because agents fail to account for how their choices affect prices and hence the value of collateral. In our model, this mechanism operates not only at the level of aggregate borrowing but also through sectoral allocations. Moreover, the planner has incentives to manipulate the country's terms of trade by shifting the relative price of the non-commodity tradable good, which affects export revenues and the tightness of the constraint.

We use the quantitative model to evaluate two simple policy instruments: a flat macroprudential debt tax and a flat tax on purchases of final and intermediate goods. In line with the planner's problem, these instruments influence both borrowing behavior and sectoral allocations through their effects on relative prices. We find that debt taxes can enhance welfare and reduce crisis risk in both emerging and advanced economies, but their effectiveness is highly sensitive to the underlying production structure. In financially developed economies that nonetheless exhibit sparse, emerging-market-like networks, even small taxes can generate welfare losses. This underscores that macroprudential policy is not one-size-fits-all: its success depends on how financial frictions interact with the elasticity of input substitution and the configuration of intersectoral linkages.

We then examine sectoral taxes in the context of an emerging economy. We find that taxing commodity inputs is always welfare-reducing, despite modest gains in financial stability, because it compresses margins in sectors already exposed to volatile global demand. By contrast, taxes on non-commodity tradable and nontradable inputs can improve welfare by shifting production toward more stabilizing input mixes, but they come at the cost of slightly higher Sudden Stop risk. These results reflect a broader tradeoff: policies that alter relative prices can reshape the production network in welfare-enhancing ways, but may also change the distribution of sectoral profits and thus the tightness of borrowing constraints.

All in all, our results point to an important and underappreciated role for production network structure in shaping both the probability and severity of Sudden Stops. They also reveal that the effectiveness of financial policy interventions critically depends on the architecture of production–a structural dimension often overlooked in macroprudential policy design.

Related Literature. Our work contributes to three strands of literature.

First, we contribute to the Sudden Stops literature (e.g., Mendoza, 2010; Bianchi, 2011; Bianchi et al., 2016; Benigno et al., 2013; Bianchi and Mendoza, 2020) by incorporating production network structure into macroeconomic models of financial crises. While canonical models focus on financial frictions and external shocks, we show that differences in domestic production architecture–specifically the density and composition of input linkages–can substantially affect the amplification and severity of crises. In this sense, our paper is complementary to Rojas and Saffie (2022), who emphasize how non-homothetic preferences alter the transmission of Sudden Stops. We also relate to work on policy stabilization under financial frictions, including Bianchi and Sosa-Padilla (2024), who show how international reserves mitigate amplification via collateral constraints, and Sosa-Padilla (2018), who studies feedback between sovereign default and banking fragility.

Second, we contribute to the literature on production networks and macroeconomic fluctuations (e.g., Horvath, 1998; Foerster et al., 2011; Acemoglu et al., 2012; Baqaee and Farhi, 2019; Bigio and La'o, 2020; McNerney et al., 2022). Our innovation is to embed these network structures in a dynamic, multi-sector open economy with occasionally binding collateral constraints. Existing models with production networks are typically static or solved using local perturbation methods (Long and Plosser, 1983; Pasten et al., 2020; Vom Lehn and Winberry, 2022), or formulated in continuous time (Afrouzi and Bhattarai, 2023; Liu and Tsyvinski, 2024). In contrast, we use a globally solved model with CES production and occasionally binding constraints, following numerical strategies similar to De Groot et al. (2023), which allows us to explore nonlinear dynamics and policy trade-offs in response to large shocks.[2]

Third, we contribute to the international economics literature by emphasizing a dimension that has received far more attention in international trade than in international finance: the structure of domestic production networks. A large literature in trade–including Antràs et al. (2012), Antrás and Chor (2022), and Yi (2010)–has documented how sourcing patterns, offshoring, and global value chains shape aggregate and distributional outcomes. This work highlights how firm-level and sectoral decisions propagate through domestic and international input–output networks. By contrast, models in

[2]Our comparative statics and general equilibrium logic also builds on recent work using sequence-space Jacobians and determinants in heterogeneous-agent models (Auclert et al., 2021; Wolf, 2023; Auclert et al., 2024).

international finance have largely abstracted from these internal production structures, typically assuming endowment economies or disconnected production structures. Our results show that production structure plays a first-order role in shaping macro-financial fragility and the effectiveness of policy interventions.

Outline. The rest of the paper is structured as follows. Section 2 presents our theoretical model, outlining the key assumptions and mechanisms. Section 3 presents empirical evidence on sectoral connectivity in emerging and advanced economies and its role in the severity of Sudden Stops. Section 4 describes the three-sector quantitative model and provides the quantitative results. Section 5 provides a normative analysis. Finally, Section 6 concludes.

2 Analytical Framework

We consider a canonical small open economy model with two goods: tradable and non-tradables. Time is discrete and the horizon is infinite. The economy features a borrowing constraint on the household's side. We consider a deterministic setup.

2.1 Households

There is a continuum of identical households of measure one. They maximize the present discounted value of utility:

$$\sum_{t=0}^{\infty} \beta^t u(c_t(c_t^N, c_t^T)), \text{ where } \beta \in (0,1). \tag{1}$$

Households consume both tradable and nontradable goods according to the following consumption aggregator:

$$c_t(c_t^N, c_t^T) = \left((1-\omega)(c_t^N)^{-\eta} + \omega(c_t^T)^{-\eta}\right)^{-\frac{1}{\eta}}, \text{ where } \eta \in (0,\infty) \text{ and } \omega \in (0,1). \tag{2}$$

They trade one-period non-state contingent bonds with exogenous price q and receive flow income from firms' profits in the tradable (π_t^N) and nontradable sector (π_t^T). The budget constraint is then given by

$$p_t^N c_t^N + c_t^T + q b_{t+1}^* = \pi_t^N + \pi_t^T + b_t^*, \tag{3}$$

where the left-hand side represents households' consumption of nontradable and tradable goods plus bond purchases. The right-hand side is households' income, which combines profits and bond positions at the beginning of the period. All prices are denominated in units of tradable goods. Therefore, p_t^N is the relative price of nontradable goods.

Households face an occasionally binding borrowing constraint. More specifically, households can borrow up to a fraction κ of their flow income, which is the sum of total sectoral profits

$$qb_{t+1}^* \geq -\kappa(\pi_t^N + \pi_t^T). \tag{4}$$

Optimality conditions. The first-order condition for nontradable and tradable consumption implies:

$$p_t^N = \frac{(1-\omega)}{\omega} \left(\frac{c_t^T}{c_t^N} \right)^{1+\eta}. \tag{5}$$

Let $\mu_t \geq 0$ be the Lagrange multiplier on the borrowing constraint. The first-order condition for bond purchases implies the following Euler equation for tradable goods and complementary slackness condition:

$$u_T(t) = \beta R^* u_T(t+1) + \mu_t, \tag{6}$$

$$0 = \mu_t \left(qb_{t+1}^* + \kappa(\pi_t^N + \pi_t^T) \right), \tag{7}$$

where $u_T(t) = \frac{\partial u(c_t)}{\partial c_t} \frac{\partial c_t}{\partial c_t^T}$ and $R^* = q^{-1}$.

2.2 Firms

Our economy features production with no labor. Instead, sectors can use sectoral outputs as inputs in production. This bloc generates a network structure in production. We now describe each sector in turn.

2.2.1 Nontradable Sector

The nontradable sector operates according to the following technology

$$y_t^N = A^N z_t^N (m_{Nt}^N)^{\alpha_N^N} (m_{Tt}^N)^{\alpha_T^N}, \text{ where } \alpha_N^N + \alpha_T^N \in [0,1), \tag{8}$$

and $A^N = (\alpha_T^N)^{-\alpha_T^N} (\alpha_N^N)^{-\alpha_N^N}$ is a normalizing constant.

Firms' profits are $\pi_t^N = p_t^N y_t^N - p_t^N m_{Nt}^N - m_{Tt}^N$. The firms' problem is to maximize profits subject to (8) taking p_t^N as given. The solution to this problem delivers the nontradable supply schedule:

$$y_t^N(z_t^N, p_t^N) = \left(z_t^N (p_t^N)^{\alpha_T^N}\right)^{\frac{1}{1-\alpha_N^N-\alpha_T^N}} \tag{9}$$

2.2.2 Tradable Sector

There is a continuum of identical firms in the tradable sector that operates according to the following production function

$$y_t^T = A^T z_t^T (m_{Tt}^T)^{\alpha_T^T} (m_{Nt}^T)^{\alpha_N^T}, \text{ where } \alpha_N^T + \alpha_T^T \in [0,1), \tag{10}$$

and $A^T = (\alpha_T^T)^{-\alpha_T^T}(\alpha_N^T)^{-\alpha_N^T}$ is a normalizing constant.

Firms' profits are $\pi_t^T = y_t^T - p_t^N m_{Nt}^T - m_{Tt}^T$. Tradable firms maximize profits subject to (10) taking p_t^N as given. The solution to this problem delivers the tradable supply schedule:

$$y_t^T(z_t^T, p_t^N) = \left(z_t^T (p_t^N)^{-\alpha_N^T}\right)^{\frac{1}{1-\alpha_T^T-\alpha_N^T}} \tag{11}$$

2.3 Competitive Equilibrium

Market clearing for nontradables requires that demand for nontradables equal supply:

$$c_t^N + m_{Nt}^N + m_{Nt}^T = y_t^N. \tag{12}$$

The aggregate resource constraint satisfies

$$c_t^T + m_{Tt}^T + m_{Tt}^N = y_t^T - q b_{t+1}^* + b_t \tag{13}$$

2.4 Perfect Foresight Analysis.

For simplicity we set $\beta R^* = 1$. We study a wealth-neutral shock to tradable productivity (z_t^T), large enough to make the borrowing constraint binding, triggering a Sudden Stop. When the borrowing constraint binds $q b_{t+1}^* = -\kappa(\pi_t^N + \pi_t^T)$.

Under these assumptions, the model can be characterized by the following two curves:

$$p_t^N = \left(\frac{c_t^T}{c_t^N(p_t^N; z_t^N, z_t^T)}\right)^{1+\eta}\left(\frac{1-\omega}{\omega}\right) \qquad \text{(PP Curve)} \tag{14}$$

$$c_t^T = \kappa_N \pi_t^N(p_t^N; z_t^N) + (1+\kappa_T)\pi_t^T(p_t^N; z_t^T) + b_0 \qquad \text{(BB Curve)}, \tag{15}$$

where

$$\kappa_N = \kappa - \frac{\alpha_T^N}{1-\alpha_N^N-\alpha_T^N}; \quad \kappa_T = \kappa + \frac{(1-\alpha_T^T)}{1-\alpha_T^T-\alpha_N^T},$$
$$\pi_t^N = (1-\alpha_N^N-\alpha_T^N)p_t^N y_t^N; \quad \pi_t^T = (1-\alpha_T^T-\alpha_N^T)y_t^T,$$

and output supplies (y_t^N, y_t^T) are given by equations (9) and (11) and nontradable consumption (c_t^N) follows from the market clearing condition (12). For future reference, nontradable consumption can be expressed as a function of output supplies in both sectors and the nontradable price:

$$c_t^N = (1-\alpha_N^N)y_t^N(p_t^N; z_t^N) - \alpha_N^T \frac{y_t^T(p_t^N; z_t^T)}{p_t^N} \tag{16}$$

The following proposition shows that the PP curve always slopes upward, irrespective of the production network's structure.

Proposition 1. *The PP curve is upward-sloping in the space (c_t^T, p_t^N).*

Proof. See Appendix A.1. □

The BB curve slope instead depends on the network structure. The following proposition provides a characterization of this result for the general case:

Proposition 2. *The BB curve slope depends on the network structure. If*

$$\kappa_N \frac{\partial \pi_t^N}{\partial p_t^N} + (1+\kappa_T)\frac{\partial \pi_t^T}{\partial p_t^N} \geq 0,$$

where $\kappa_N = \left(\kappa - \frac{\alpha_T^N}{(1-\alpha_N^N-\alpha_T^N)}\right)$ and $\kappa_T = (\kappa + \frac{\alpha_N^T}{(1-\alpha_T^T-\alpha_N^T)})$, then the BB curve is upward-sloping. Otherwise, it is downward sloping.

Proof. See Appendix A.2. □

At the core of Proposition 2 is how tradable consumption responds to a change in nontradable price. This has two components: how tradable consumption react to (i) changes in nontradable profits $\left(\kappa_N \frac{\partial \pi_t^N}{\partial p_t^N}\right)$ and (ii) to changes in tradable profits $\left((1+\kappa_T)\frac{\partial \pi_t^T}{\partial p_t^N}\right)$. Changes in nontradable profits in response to a decrease in the nontradable price affect tradable consumption in two ways. First, it tightens the borrowing constraint. This effect is captured by κ and it is the classical mechanism in Sudden Stop models. Second, the nontradable sector inputs' demand for tradable resources decreases. Given an amount of tradable resources, this lower input demand increases tradable consumption by $\alpha_T^N/(1-\alpha_N^N-\alpha_T^N)$. Similarly, a lower nontradable price increases profits in the tradable sector. This relaxes the borrowing constraint by κ and increases tradable production by $\alpha_N^T/(1-\alpha_T^T-\alpha_N^T)$. Both mechanisms go in the same direction: they increase tradable consumption.

Although profits affect household income directly, as in standard Sudden Stop models, intersectoral linkages alter how tradable consumption responds to profit changes. A roundabout production structure ($\alpha_T^N = \alpha_N^T = 0$) coincides exactly with an endowment economy where nontradable profit changes affect tradable consumption by $\kappa_N = \kappa$ and tradable profits affect tradable consumption by $\kappa_T = \kappa$. Thus, intermediate inputs without intersectoral linkages do not affect any of the predictions of standard Sudden Stop models or the slope of the BB curve. They only affect how profits react to changes in the nontradable price. In contrast, intersectoral linkages are key for how changes in profits affect tradable consumption. In an extreme case, strong intersectoral connections can potentially change the sign of the BB curve slope: when $\alpha_T^N \geq \kappa(1-\alpha_N^N)/(1+\kappa)$, the BB curve slope is negative.

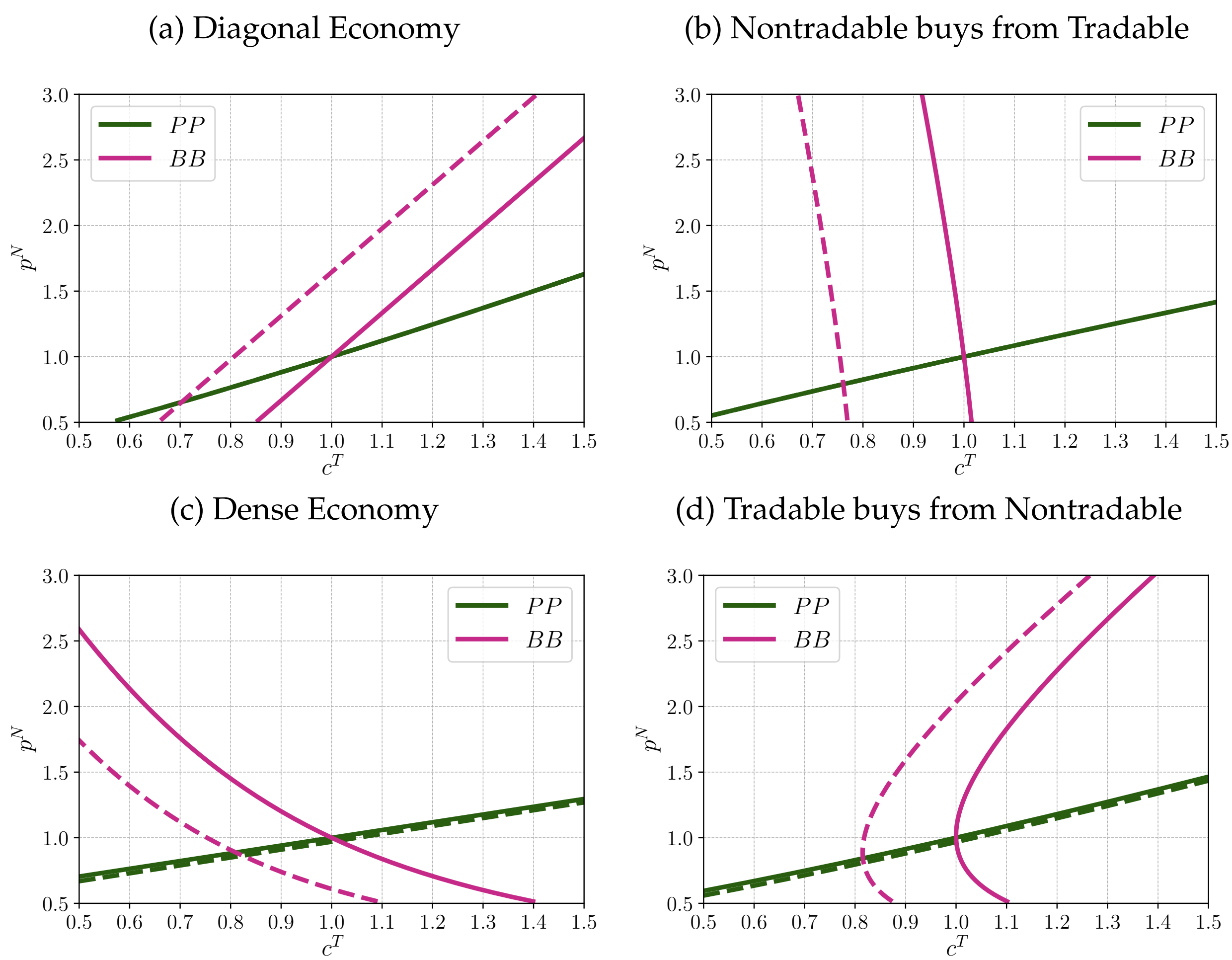
Figure 1 uses a numerical example to illustrate how different network structures can shape the equilibrium relationships in the BB-PP space. We study four different network structures:

$$\Omega^{\text{dense}} = \begin{bmatrix} \frac{\alpha}{2} & \frac{\alpha}{2} \\ \frac{\alpha}{2} & \frac{\alpha}{2} \end{bmatrix}, \quad \Omega^{\text{NT}\leftarrow\text{T}} = \begin{bmatrix} \frac{\alpha}{2} & \frac{\alpha}{2} \\ 0 & \alpha \end{bmatrix},$$
$$\Omega^{\text{diagonal}} = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix}, \quad \Omega^{\text{T}\leftarrow\text{NT}} = \begin{bmatrix} \alpha & 0 \\ \frac{\alpha}{2} & \frac{\alpha}{2} \end{bmatrix},$$

where α is the total intermediate input share that we keep constant across specifications. In the Ω^{dense} economy, the nontradable sector (first row and column) and tradable sector (second row and column) rely on each other in the same proportion. In the Ω^{diagonal} case, each sector only uses intermediates from their same sector. In the $\Omega^{\text{NT}\leftarrow\text{T}}$ economy, the nontradable sector buys from the tradable sector, but the tradable sector does not use nontradable as an input. The $\Omega^{\text{T}\leftarrow\text{NT}}$ economy does the opposite: the tradable sector uses nontradable as an input while the nontradable does not use tradable as an input. We normalize the initial equilibrium to be the same across models $(c^T, p^N) = (1, 1)$.

Figure 1 clearly shows that while the slope and shape of the BB curve (pink line) vary significantly with the network structure, the PP curve (green line) is (almost) linear and upward-sloping across the different network structures.[3]

Figure 1: PP and BB curves under different network structures



Note: This figure depicts the PP and BB curves under different network scenarios. The solid lines represent the initial equilibrium, while the dotted lines represent a 10% decline in the productivity of the tradable sector. To generate these figures we set $\eta = 0.2035$, the intermediate input share $\alpha = 0.4$ (consistent with the average emerging market number), $\kappa = 0.3$ and $b_0 = -0.3$. We assume $\omega = 0.5$ to focus on the role of the production network structure. See Appendix B for more details.

We end this subsection with a general result. We establish that the nontradable price and tradable consumption unambiguously decline in response to a change in tradable

[3] The PP curve is always increasing but can be convex or concave depending on the network structure. In the region near the equilibrium, however, the PP curve is almost linear in the (c^T, p^N) space.

productivity:

Proposition 3. *Consider a wealth-neutral decline in tradable productivity starting from an equilibrium where the borrowing constraint is marginally binding. The response of tradable consumption and nontradable price is*

$$\frac{\mathrm{d}\log c_t^T}{\mathrm{d}\log z_t^T} = \mathcal{K}\left[(\gamma_t^N+\gamma_t^T)(\delta_t^N-1)+\left(\gamma_t^N(\delta_t^N-1)+\delta_t^N\gamma_t^T\right)\varepsilon_{p_t^N}^{y_t^N}+\frac{\gamma_t^T}{1+\eta}\right]\varepsilon_{z_t^T}^{y_t^T}\geq 0$$
$$\frac{\mathrm{d}\log p_t^N}{\mathrm{d}\log z_t^T} = \mathcal{K}\left((\delta_t^N-1)+\gamma_t^T\right)\varepsilon_{z_t^T}^{y_t^T}\geq 0$$

where

$$\gamma_t^N=\frac{\kappa_N\pi_t^N}{c_t^T};\quad \gamma_t^T=\frac{(1+\kappa_T)\pi_t^T}{c_t^T};\quad \delta_t^N=\frac{(1-\alpha_N^N)y_t^N}{c_t^N},$$
$$\varepsilon_{p_t^N}^{y_t^N}=\frac{\alpha_T^N}{1-\alpha_N^N-\alpha_T^N};\quad \varepsilon_{z_t^T}^{y_t^N}=\frac{1}{1-\alpha_N^N-\alpha_T^N};\quad \varepsilon_{p_t^N}^{y_t^T}=\frac{\alpha_N^T}{1-\alpha_T^T-\alpha_N^T},$$
$$\mathcal{K}=\left(\frac{1}{1+\eta}-\gamma_t^N+(\delta_t^N-1)+(\delta_t^N-\gamma_t^N)\varepsilon_{p_t^N}^{y_t^N}+(1-\gamma_t^T-\delta_t^N)\varepsilon_{p_t^N}^{y_t^T}\right)^{-1}.$$

$\mathcal{K}$ and the term in brackets represent general equilibrium effects, and $\varepsilon_{z_t^T}^{y_t^T}$ represents the direct response of tradable output with respect to tradable productivity. $\varepsilon_{p_t^N}^{y_t^N}$ and $\varepsilon_{p_t^N}^{y_t^T}$ represent the elasticity of nontradable and tradable output to a change in the nontradable price, respectively.

Proof. See Appendix A.3. □

Proposition 3 characterizes our model's rich general equilibrium interactions. While the initial tradable productivity impulse consists simply of a change in tradable production, ceteris paribus, this triggers general equilibrium responses of both the production and consumption sides captured by $\mathcal{K}$ and the terms in brackets. Proposition 3 shows that, although rich and, in principle, hard to compute, the effect always goes in the same direction: a decline in tradable productivity reduces tradable consumption and the nontradable price.

Proposition 3 is general enough to sign the direction of tradable consumption and the nontradable price in a model with intersectoral linkages. However, it does not allow us to say anything regarding the importance of different network structures, it just states clearly that networks shape the dynamics of the simple model. We can, however, numerically explore how the same proportional change in tradable productivity across models changes tradable consumption and nontradable price. The dashed lines in Figure 1 show

BB and PP curves in response to a 10 percent decline in tradable productivity. In all four panels, the decline in tradable productivity unambiguously declines tradable consumption and the nontradable price, as Proposition 3 showed and the quantitative impact is different for differnet network structures. However, to make further analytic progress, we explore two particular network structures.

2.4.1 The Role of the Nontradable Sector Linkages

We study a case that allows us to make clear analytical predictions on the importance of production linkages in shaping the dynamics of a Sudden Stop—as described by the response of nontradable price and tradable consumption to a productivity shock in the tradable sector.

We consider the already introduced "nontradable buys from tradable" network structure:

$$\Omega^{\mathrm{NT}\leftarrow\mathrm{T}} = \begin{bmatrix} \alpha_N^N & \alpha_T^N \\ 0 & \alpha_T^T \end{bmatrix}.$$

Under this structure, the tradable sector only uses intermediate inputs from itself. The nontradable sector uses intermediates from itself and from the tradable sector. For comparability, we must maintain the degree of returns to scale constant across economies. Otherwise, the result would be partially driven by heterogeneity in the degree of decreasing returns to scale across sectors. Hence, $\overline{\alpha}^N = \alpha_T^N + \alpha_N^N$ and $\overline{\alpha}^T = \alpha_T^T$ the same for any given economy, where $\bar{\alpha}^i$ is the degree of decreasing returns to scale for sectors $i \in \{N, T\}$. In what follows we call this the N economy.

We analyze how changes in α_T^N and α_N^N alter how the economy responds to shocks to the tradable sector productivity (z^T). We compare two economies. The $\Omega^{\mathrm{diagonal}}$ (D) economy with $\alpha_T^N(D) = 0$, $\alpha_N^N(D) = \overline{\alpha}^N$ and the $\Omega^{\mathrm{NT}\leftarrow\mathrm{T}}$ (N) economy with $\alpha_T^N(N) > 0$, $\alpha_N^N(N) < \alpha_N^N(D)$, and $\overline{\alpha}^N = \alpha_T^N(N) + \alpha_N^N(N)$.

The following proposition characterizes how the production network structure matters for the severity of the Sudden Stop on tradable consumption.

Proposition 4. *Consider a decline in tradable productivity. Let $\varepsilon^{BB}(G)$ and $\varepsilon^{PP}(G)$ be the elasticities of the BB curve and PP curve under a network $G = \{D, N\}$ evaluated at the marginally binding equilibrium. If*

$$\left|\gamma^T(N)\frac{\varepsilon^{BB}(N)}{\varepsilon^{BB}(N) - \varepsilon^{PP}(N)}\right| \leq \left|\gamma^T(D)\frac{\varepsilon^{BB}(D)}{\varepsilon^{BB}(D) - \varepsilon^{PP}(D)}\right|,$$

then

$$\left|\frac{\mathrm{d}\log(c_t^T)^{NT\leftarrow T}}{\mathrm{d}\log z_t^T}\right| \leq \left|\frac{\mathrm{d}\log(c_t^T)^{Diagonal}}{\mathrm{d}\log z_t^T}\right|,$$

and

$$\left|\frac{\mathrm{d}\log(p_t^N)^{NT\leftarrow T}}{\mathrm{d}\log z_t^T}\right| \leq \left|\frac{\mathrm{d}\log(p_t^N)^{Diagonal}}{\mathrm{d}\log z_t^T}\right|.$$

Proof. See Appendix A.4 □

In other words, if the above condition is satisfied, the diagonal economy's tradable consumption and nontradable price decline more than in the intersectoral economy. This result is illustrated in panels (a) and (b) Figure 1, where a 10 percent decline in tradable productivity, tradable consumption declines around six percentage points more in the diagonal economy relative to the N economy. The intuition behind this result is the following. In the diagonal economy case, a decline in tradable productivity lowers tradable profits, which tightens the borrowing constraint (due to a decline in the value of the household's collateral) leading to deleveraging of the household (by lowering tradable consumption). Since, in general, tradable and nontradable goods are complements, this implies that the aggregate demand for nontradable goods will also decline, negatively affecting profits and output of the nontradable sector. This further tightens the borrowing constraint, generating further declines in tradable and nontradable consumption due to the Fisherian deflationary spiral in effect.

What happens in the N economy? We have a similar behavior than in the diagonal economy. However, since tradable goods are used in nontradable production, the decline in tradable production and profits will be milder than in the diagonal case. Hence, the size of the deleveraging that the household needs to implement is small, partially mitigating the drop in the demand for tradable and nontradable goods.

The slope of the PP curve changes with intersectoral linkages because nontradable supply becomes a function of the nontradable price, affecting nontradable consumption in equilibrium, which is part of the PP curve. To see this, the nontradable market clearing conditions in each case are:

$$\begin{aligned} c^N(D) &= (1-\alpha_N^N(D))y^N(z^N(D)), \\ c^N(N) &= (1-\alpha_N^N(N))y^N(z^N(N), p^N). \end{aligned}$$

This mechanism reduces the sensitivity of the nontradable price to changes in tradable consumption when the economy features intersectoral linkages.

We already discussed how intersectoral linkages change the slope of the BB curve in Proposition 2. The benchmark case in the literature considers endowment economies (Bianchi, 2011; Rojas and Saffie, 2022). These economies are a special case of ours. Indeed, without production, our model collapses to $\kappa_N = \kappa_T = \kappa$, $\pi_t^N = p_t^N \bar{y}_t^N$ and $\pi_t^T = \bar{y}_t^T$, where a bar over a variable denotes that it is an endowment. It is not a surprise then that panel (a) of Figure 1 is almost identical to these papers' equilibrium characterization: an increasing and convex PP curve and an increasing and linear BB curve.

Our BB curve highlights sectoral heterogeneity in the effective collateral constraint parameter faced by the household due to production heterogeneity. A connected economy can mitigate the Sudden Stop as it displays a smaller decline in profits, which, through the borrowing constraint, allows households to consume more tradables than in an island economy. This is apparent when we look at panels (c) and (d) of Figure 1, where both interconnected economies exhibit a smaller decline in tradable consumption and nontradable price than the diagonal economy. Moreover, note that whenever the tradable sector buys inputs from the nontradable sector, the PP curve shifts down in response to a tradable productivity decline. A decrease in tradable productivity reduces tradable sector input demand from the nontradable sector, which increases nontradable consumption given nontradable production (a downward shift in the PP curve). The figures make clear that this shift is likely to be small, especially when compared to the BB curve shifts.

An avid reader would argue that intersectoral linkages are unnecessary to generate heterogeneity in κ's across sectors: different decreasing returns to scale across sectors would suffice. This statement is incorrect. Decreasing returns to scale affects how profits respond to changes in nontradable price, $\frac{\partial \pi_t^i}{\partial p_t^N}$. They do not affect how tradable consumption reacts to changes in profits. Under decreasing returns to scale across sectors but no intersectoral linkages, the parameters collapse to $\tilde{\kappa}_N = \kappa$ and $\tilde{\kappa}_T = \kappa$. Exactly as in the endowment economy. With intersectoral linkages these parameters are sector-specific: $\kappa_N = \kappa - \frac{\alpha_T^N}{(1-\alpha_N^N-\alpha_T^N)}$ and $\kappa_T = \kappa + \frac{\alpha_N^T}{(1-\alpha_N^T-\alpha_T^T)}$. As we show in the empirical section, intersectoral connections via intermediate inputs are important across countries: α_T^N and α_N^T are empirically relevant and largely differ across countries.

3 Empirical Analysis

3.1 Stylized Facts: Commodities, Tradables, and nontradables Linkages

In this section, we provide simple descriptive statistics on the network structure of tradable—commodity and non-commodity tradables—and nontradable sectors in advanced and emerging economies. We use the sample of countries in Bianchi and Mendoza (2020) and use OECD input-output data release 2021. Our definition of tradable and nontradable sectors is based on sectoral gross trade intensity. Sectors with a ratio of gross trade to gross output above 20% are classified as tradable sectors. Here, we consider the average ratio across countries so we maintain the same group of sectors for all countries. See Appendix C for details about sectoral classification and country classification groups. All our numbers below are based on year 2018.

Here, we present three facts on sectoral linkages across emerging and advanced economies. For a particular sector i and country k, we measure the input share from sector j as the ratio between the purchases of intermediates $p_j m_{ij}$ over total gross output $p_i y_i$. We consider domestic and imported intermediates in our measure, but similar results hold if we only use domestic intermediates. Table 1 provides the average linkages among sectors for emerging and advanced economies. Three main facts emerge.

Fact 1 In advanced economies, the commodity sector is less connected, as a supplier, to the rest of the economy.

Fact 2 In advanced economies, the tradable sector is less connected, as a supplier, to the nontradable sector.

Fact 3 In advanced economies, the nontradable sector, as a supplier, is more connected to the rest of the sectors of the economy.

To put these numbers into perspective, consider an off-the-shelf production network model (e.g., Acemoglu et al., 2012). In these models, the Leontief inverse elements of a supplier drive the aggregate effects of sectoral shocks. Besides input-output shares, we consider average consumption shares of 4% , 19% , 77% consumption expenditure share in commodities, tradable, and nontradables. We keep them constant across economies to evaluate the differences between production structures. The differences observed in Table 1 imply that a 10% decrease in the TFP of the commodity sector generates a 1.36% decline in advanced economies' real GDP and a 2.25% decline in emerging economies. These differences do not yet consider the main ingredients of our model: the household borrowing constraint and its further interactions with production linkages. These differences do not

Table 1: Average Intermediate Input Shares: Emerging and Advanced (percentage)

	(1)	(2)	(3)
	Emerging	Advanced	% Difference
ω_C^C	26.59	29.48	10.86%
ω_X^C	19.87	22.14	11.43%
ω_N^C	7.53	11.18	48.53%
ω_C^X	11.37	6.66	-41.44%
ω_X^X	34.68	32.60	-5.99%
ω_N^X	11.36	16.22	42.83%
ω_C^N	4.61	1.55	-66.40%
ω_X^N	17.19	13.51	-21.45%
ω_N^N	18.43	24.72	34.13%

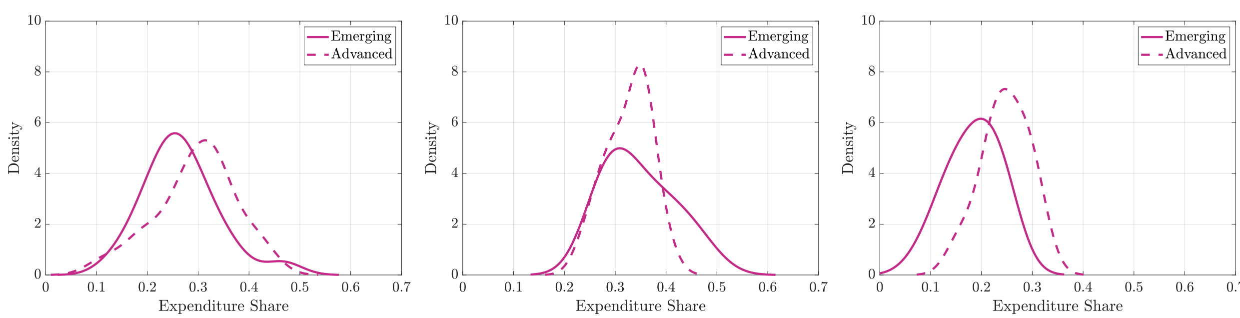
Note: This table reports the average intermediate input share (in percentage points) for emerging economies (column 1) and advanced economies (column 2). Column 3 reports the percent difference between advanced and emerging. ω_X^X, ω_X^N, ω_X^C represent the share of intermediate inputs from sector X in the gross output of sector X, N, and C, respectively.

consider non-linearities generated from non-unitary consumption and production elasticities (Baqaee and Farhi, 2019).

The next figures present the kernel density of intermediate input shares across emerging and advanced economies. Figure 2 plots the own sector intermediate input share. Advanced economies commodity sector and nontradable sector are more connected to itself than emerging economies. The tradable sector is, on average, more connected in emerging countries, but mainly due to the large right tail. The median emerging country and the median advanced country display a similar own tradable sector linkage.

Figure 2: Distribution of intermediate input shares from the same sector

(a) Commodities to Commodities (b) Tradable to Tradable (c) Nontradable to Nontradable



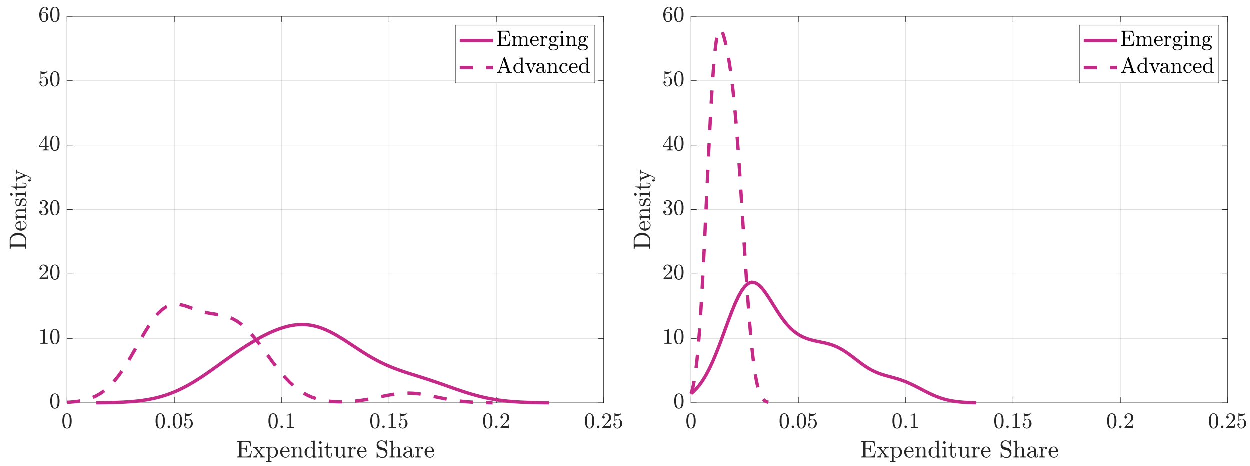
Note: This figure shows the intermediate input usage from the same sector. Solid lines depict emerging markets. Dashed lines depict advanced economies.

Figure 3 shows the importance of commodity inputs in tradable and nontradable sectors' production. There is a clear dominance in emerging economies in how important commodity inputs are in the production of tradables and nontradables.

Figure 3: Use of commodity inputs by tradables and nontradable sectors

(a) Commodities to Tradable

(b) Commodities to Nontradable

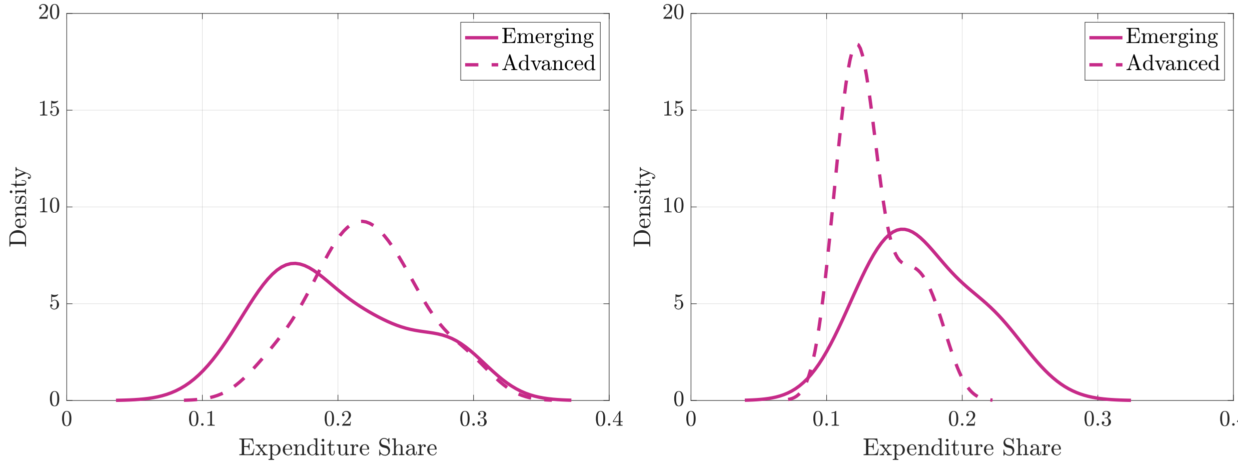


Note: This figure shows the intermediate input usage of commodities by the tradable and nontradable sectors. Solid lines depict emerging markets. Dashed lines depict advanced economies.

The results in Figure 4 highlight a clear heterogeneity in the production structure of commodity sectors and nontradable sectors in emerging vs. advanced economies. While commodity sectors in advanced economies use a larger share of tradable inputs, compared to emerging economies, this is not the case for nontradable sectors. Nontradable sectors in emerging countries use a significantly larger share of intermediates from the tradable sector compared to advanced economies.

Figure 4: Use of tradable inputs by commodity and nontradable sectors

(a) Tradable to Commodities (b) Tradable to Nontradable

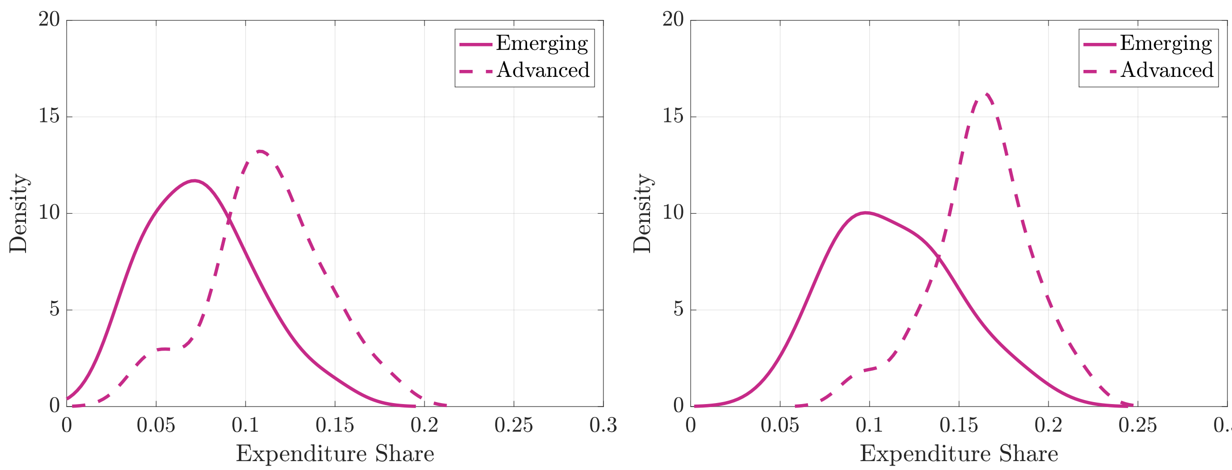


Note: This figure shows the intermediate input usage of tradables by the commodity and nontradable sector. Solid lines depict emerging markets. Dashed lines depict advanced economies.

Finally, Figure 5 shows that the commodity sector and the tradable sector in advanced economies use a significantly larger share of inputs from the nontradable sector, compared to emerging economies.

Figure 5: Use of nontradable inputs by commodities and tradable sectors

(a) Nontradable to Commodities (b) Nontradable to Tradable



Note: This figure shows the intermediate input usage of nontradable by the commodity and tradable sector. Solid lines depict emerging markets. Dashed lines depict advanced economies.

3.2 Regression Analysis

The analytical model in Section 2 highlights that economies closer to a diagonal structure should experience more severe Sudden Stops. Appendix B presents a numerical analysis of the perfect foresight model under several network structures. The key point that emerges from that exploration is that more connected economies can better endure Sudden Stops. In this subsection, we empirically assess this prediction. To that end, we construct a measure of "distance to the diagonal" as follows

$$\text{Distance}_c = \left(\sum_{i,j} (\mathbb{1}_{ij} - \omega_{ijc})^2 \right)^{\frac{1}{2}}, \tag{17}$$

where $\mathbb{1}_{ij}$ is an indicator variable equal to 1 if $i = j$ and zero otherwise, ω_{ijc} represents the expenditure share of sector i on inputs from country j as a fraction of total intermediate input expenditures of sector i, for a given country c. By construction $\sum_j \omega_{ijc} = 1$. Essentially, our measure is an Euclidean distance between the identity matrix and the observed input-output network. Higher values of this metric indicate that countries deviate more from the diagonal network. A related metric is studied in Koren and Tenreyro (2013), which focuses on how the diagonal elements of the input-output network have decreased over time across advanced economies. We complement this view by measuring how, at a given point in time, countries —both advanced and emerging —differ from a diagonal input-output network.

Equipped with this measure, we estimate the following regression specification:

$$\begin{aligned} y_{ct} &= \alpha + \alpha_c + \alpha_t \\ &\quad + \beta_0 \text{SS}_{ct} + \beta_1 \text{SS}_{ct} \times \text{EM}_c + \beta_2 \text{SS}_{ct} \times \text{Distance}_c + \beta_3 \text{SS}_{ct} \times \text{Distance}_c \times \text{EM}_c \\ &\quad + \sum_{j \in \{C,X,N\}} \gamma_j \text{SS}_{ct} \times \text{Size}_{jc} + \varepsilon_{ct}, \end{aligned} \tag{18}$$

where y_{ct} is an outcome variable, SS_{ct} is an indicator variable that equals one if a Sudden Stop occurs in country c at time t. $\text{Size}_{jc} = \frac{\text{Sales}_{jc}}{\text{GDP}_c}$ represents the sales share of sector j in country c GDP. $\text{EM}_c = 1$ if country c is an emerging market. Finally, ε_{ct} is an error term.

We measure both Distance_c and Size_{jc} using information from the Input-Output tables for 1995, the first year for which data are available. We study two outcomes: real GDP and the Current Account to GDP ratio. Both are sourced from the World Development Indicators (WDI) database of the World Bank. We source Sudden Stop episodes from Bianchi and Mendoza (2020). To ease readability of the estimated parameters, we

Table 2: Real GDP and Current Account during Sudden Stops

	GDP Growth			CA/GDP		
	(1)	(2)	(3)	(4)	(5)	(6)
SS_{ct}	-0.0406***	-0.0290***	0.0048	0.0147**	0.0108	-0.0107
	(0.011)	(0.008)	(0.009)	(0.007)	(0.007)	(0.011)
$SS_{ct} \times EM_c$			-0.0648***			0.0397**
			(0.021)			(0.017)
$SS_{ct} \times \text{Distance}_c$	-0.0039	0.0063	0.0155***	-0.0077	-0.0050	-0.0171**
	(0.013)	(0.011)	(0.006)	(0.006)	(0.007)	(0.008)
$SS_{ct} \times \text{Distance}_c \times EM_c$			-0.0149			0.0176*
			(0.016)			(0.010)
$SS_{ct} \times \text{Tradable Size}_c$		0.0841*	0.1116***		-0.0493	-0.0666*
		(0.047)	(0.031)		(0.043)	(0.036)
$SS_{ct} \times \text{Non-Tradable Size}_c$		0.1746***	0.0821		-0.1073*	-0.0514
		(0.053)	(0.065)		(0.064)	(0.060)
$SS_{ct} \times \text{Commodity Size}_c$		-0.0988	0.0078		-0.1537	-0.2139**
		(0.091)	(0.080)		(0.108)	(0.083)
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1517	1517	1517	1434	1434	1434

Note: Time period: 1979 – 2019. Annual frequency. Includes countries that did not experience a Sudden Stop during the full sample. Two-way clustered standard errors at the country-year level. *,** ,*** represents significance at the 10%, 5% and 1% level. The panel is unbalanced, comprising 37 countries: 17 emerging markets and 20 advanced economies for which data are available.

standardized the distance variable to have a unit standard deviation and a mean of zero, while we demeaned the size variable.

Our specification aims to shed light on the following questions: when a Sudden Stop occurs, do countries with a network structure that are less diagonal feature less severe drops in activity and less severe current account reversals? Do these results differ between emerging and advanced economies?

Based on our analytical model, we expect β_2 to be negative for real GDP growth and positive for the current account-to-GDP ratio. We remain agnostic on the sign of β_3 as this triple interaction illustrates the relative effect of distance between advanced and emerging markets.

Table 2 shows the results. Columns (1) to (3) report the results for real GDP growth, while the remaining three columns report the results for the current account to GDP ratio.

We first focus on GDP growth. The table suggests that economies experiencing a Sudden Stop feature an average growth reduction of approximately four percentage points. This effect seems to be driven by emerging markets, as column (3) suggests: relative to advanced economies, GDP growth declines six percentage points. Interestingly, this decline is lower (in absolute value) in economies that exhibit a higher distance from the diagonal, by around 1.5 percentage points for a one-standard-deviation increase in the distance metric. To give a sense of the magnitude, a one standard deviation is akin to replacing the Argentine production network with that of Japan, under our three-sector classification. This effect is present even if we take into account the size of the sectors.

Columns (4) to (6) indicate that current account reversals are more severe in emerging markets, with the current account-to-GDP ratio increasing by four percentage points relative to that of advanced economies. Interestingly, our distance measure mitigates the effect of current account reversals ($\beta_2 < 0$), albeit to a lesser extent for emerging markets, as $\beta_3 > 0$. Similar to the GDP growth case, these effects are present even when we control for measures of sectoral size.

Taking our results together suggests that the production network structure matters for how Sudden Stops translate into aggregate outcomes. It, however, does not address the question of how important it is relative to other mechanisms and features highlighted elsewhere in the literature. In the next section, we enrich the analytical model to address this question.

4 Quantitative Model

This section extends the simple 2-sector model studied in Section 2.4 so that it can capture the differences in the cross-country production structures documented in Section 3. The quantitative model extends the simple framework by allowing for two different tradable goods: a commodity good where the price is fully exogenous, and a differentiated tradable good where the country faces a demand with finite elasticity. The model is also dynamic and features aggregate risk fueled by sector-specific shocks.

4.1 Households

The representative household maximizes the present discounted value of utility:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right], \text{ where } \beta \in (0,1). \tag{19}$$

The household consumes a tradable basket and a nontradable good according to the following consumption aggregator:

$$c_t = \left((1-\omega)(c_t^N)^{-\eta} + \omega(c_t^T)^{-\eta}\right)^{-\frac{1}{\eta}}, \tag{20}$$

where c_t^T is a composite tradable good, which is defined by

$$c_t^T = \left[\omega_T(c_t^C)^{-\eta_T} + (1-\omega_T)(c_t^X)^{-\eta_T}\right]^{-\frac{1}{\eta_T}}, \tag{21}$$

where c_t^C is a commodity tradable good and c_t^X is a non-commodity tradable good.[4] Both tradable goods are produced domestically and can be exported. The household trades one-period non-state contingent bonds with exogenous price q and receives flow income from firms' profits in the tradable (π_t^C and π_t^X) sectors and the nontradable sector (π_t^N). The budget constraint is then given by:

$$p_t^N c_t^N + p_t^C c_t^C + p_t^X c_t^X + q b_{t+1}^* = \pi_t^N + \pi_t^C + \pi_t^X + b_t^*, \tag{22}$$

where the left-hand side represents households' consumption of nontradable and tradable goods plus bond purchases. The right-hand side is the income of the household, which combines profits and bond positions at the beginning of the period. All prices are denominated in units of the commodity good, which is the numeraire. Therefore, p_t^X and p_t^N are the relative price of non-commodity and nontradable goods.

The household faces an occasionally binding borrowing constraint. More specifically, households can borrow a up to a fraction κ of their flow income, which is the sum of total sectoral profits

$$q b_{t+1} \geq -\kappa(\pi_t^N + \pi_t^C + \pi_t^X). \tag{23}$$

Finally, there is a foreign demand for the non-commodity good. Similarly to Benguria et al. (2024) and Saffie et al. (2020), we assume that the foreign demand for the differentiated tradable good has the same price elasticity as the domestic demand. The size of foreign demand is parametrized by the demand shifter Γ. Then, the domestic demand for

[4] To simplify notation, we denote the state contingent consumption plan by c_t instead of $c(s^t)$. The same abuse of notation is applied to other variables.

the non-commodity good is

$$c_t^X = (1-\omega_T)^{\frac{1}{1+\eta_T}} \left(\frac{p_t^X}{p_t^T}\right)^{-\frac{1}{1+\eta_T}} c_t^T, \tag{24}$$

the foreign demand is

$$\hat{c}_t^X = \Gamma \left(\frac{p_t^X}{p_t^T}\right)^{-\frac{1}{1+\eta_T}}, \tag{25}$$

and the market clearing requires:

$$c_t^X + \hat{c}_t^X = y_t^X - \sum_i m_{X,t}^i. \tag{26}$$

4.2 Production Structure

The production of each good $i \in \{C, X, N\}$ uses intermediate goods from every sector. In particular,

$$y_t^i = z_t^i \left[\left(\sum_{j \in \{C,X,N\}} (\omega_j^i)^{\frac{1}{\chi^i}} (m_{jt}^i)^{\frac{\chi^i - 1}{\chi^i}} \right)^{\frac{\chi^i}{\chi^i - 1}} \right]^{\gamma^i}, \tag{27}$$

with $\sum_{j \in \{C,X,N\}} \omega_j^i = 1$ for all i, $\gamma^i \in (0,1)$ and $\chi^i > 0$. The parameters (γ^i, χ^i) are both sector-specific and denote the degree of decreasing returns to scale and elasticity of substitution across inputs, respectively. z_t^i is a sector specific productivity shock that follows an AR(1) with persistence ρ_i and dispersion σ_i.[5]

Given prices $\{p_t^j\}$ and technology, the firms' problem is to choose inputs $\{m_{jt}^i\}$ so as to maximize profits π_t^i subject to the production function in equation (27). Mathematically,

$$\pi_t^i = \max_{\{m_{jt}^i\}_{j \in \{C,X,N\}}} p_t^i y_t^i - \sum_{j \in \{C,X,N\}} p_t^j m_{jt}^i \tag{28}$$

subject to (27)

[5]While our model does not have factors of production, we calibrate the decreasing returns to scale accounting for the effective labor share in the data. From the firm optimality conditions, we now have, in steady-state, that $\frac{p_j m_j^i}{p_i y_i} = \omega_j^i \gamma_i$ and that $\frac{p_j m_j^i}{\sum_k p_k m_k^i} = \omega_j^i$

4.3 Equilibrium Characterization

The equilibrium conditions of the representative households are the following:

$$\frac{p_t^N}{p_t^T} = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{1+\eta} \tag{29}$$

$$p_t^X = \frac{1-\omega_T}{\omega_T}\left(\frac{c_t^C}{c_t^X}\right)^{1+\eta_T} \tag{30}$$

$$u_C(t) = q^{-1}\beta\mathbb{E}_t\left[u_C(t+1)\right] + \mu_t \tag{31}$$

$$c_t^X + \hat{c}_t^X = y_t^X - \sum_i m_{X,t}^i \tag{32}$$

$$c_t^N = y_t^N - \sum_i m_{N,t}^i \tag{33}$$

$$c_t^C + qb_{t+1} = y_t^C - \sum_i m_{C,t}^i + b_t + p_t^X \hat{c}_t^X \tag{34}$$

$$qb_{t+1} \geq -\kappa(\pi_t^N + \pi_t^C + \pi_t^X) \tag{35}$$

$$p_t^T = \left[\omega_T^{\frac{1}{1+\eta_T}} + (1-\omega_T)^{\frac{1}{1+\eta_T}}(p_t^X)^{\frac{\eta_T}{1+\eta_T}}\right]^{\frac{\eta_T+1}{\eta_T}} \tag{36}$$

where $u_C(t)$ denotes the marginal utility of consumption of the commodity good in period t, μ_t represents the Lagrange multiplier of the borrowing constraint, and p_t^T corresponds to the price index of the tradable composite.

4.4 Quantitative Analysis

4.4.1 Calibration

We calibrate the model for an average emerging economy and an average advanced economy. We use the OECD input-output tables to calibrate sectoral production and consumption shares. We use KLEMS sectoral data to measure sectoral gross output's volatility and persistence. We use the percent deviation from the trend using an HP filter for the data and the model. Due to data availability, we calibrate the sectoral output autocorrelation and volatility to match that of Hungary as the average emerging market. For advanced economies, we consider the following list of countries with available data: Austria, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Netherlands, and Portugal.

Table 3 shows our parameter values for the emerging market and advanced economy

Table 3: Parameter values

Parameter	Description	EM	AE
Preference			
β	subjective discount factor	0.9	0.95
σ	curvature utility function	2	–
ω	consumption weight of tradables	0.165	–
ω_T	consumption weight of non-commodity tradables in tradables	0.2	–
κ	collateral constraint parameter	0.32	0.46
Production			
γ	Decreasing returns to scale	0.8	–
ω_C^C	importance of commodity inputs in commodity production	0.5	0.5
ω_X^C	importance of tradable inputs in commodity production	0.37	0.35
ω_N^C	importance of nontradable inputs in commodity production	0.13	0.15
ω_C^X	importance of commodity inputs in tradable production	0.20	0.13
ω_X^X	importance of tradable inputs in tradable production	0.61	0.59
ω_N^X	importance of nontradable inputs in tradable production	0.19	0.28
ω_C^N	importance of commodity inputs in nontradable production	0.18	0.06
ω_X^N	importance of tradable inputs in nontradable production	0.35	0.34
ω_N^N	importance of nontradable inputs in nontradable production	0.47	0.6
χ	elasticity of substitution among varieties	0.6	–
$\frac{1}{1+\eta} = \frac{1}{1+\eta_T}$	trade elasticity of substitution	0.83	–
R^*	steady state of world interest rate	1.04	–
$\{z_L^C, z_H^C\}$	Low-High productivity C	$\{1.9, 2.05\}$	$\{1.735, 1.745\}$
$\{z_L^X, z_H^X\}$	Low-High productivity X	$\{1.95, 2.05\}$	$\{1.71, 1.77\}$
$\{z_L^N, z_H^N\}$	Low-High productivity N	$\{1.98, 2.05\}$	$\{1.71, 1.77\}$
$\mathbb{P}_C$	prob. stay z_L^C	0.75	–
$\mathbb{P}_X$	prob. stay z_L^X	0.9	–
$\mathbb{P}_N$	prob. stay z_L^N	0.88	–

Note: This table shows the parameter values. The calibration is annual. – indicates that values are the same as in the EM calibration.

baseline calibration. Table 4 shows our targeted moments for emerging and advanced economies. In the table, we also show our model's implied moments based on our calibrations.

Table 4: Targeted Moments for Emerging and Advanced Economies

Moment	**Emerging Economies**		**Advanced Economies**	
	Data	**Model**	**Data**	**Model**
Macroeconomic Moments				
Debt to GDP	-0.428	-0.307	-0.609	-0.47
Trade Balance to GDP	-0.0003	0.01	0.007	0.02
σ(CA/GDP)	0.032	0.012	0.031	0.008
σ(GDP)	0.035	0.07	0.020	0.0180
ρ(GDP)	0.617	0.402	0.615	0.54
Sudden Stop Probability	0.03	0.033	0.017	0.0088
Expenditure Shares and Sectoral Moments				
$p_t^C c_t^C / \sum_j p_t^j c_t^j$	0.04	0.05	0.04	0.05
$p_t^X c_t^X / \sum_j p_t^j c_t^j$	0.19	0.17	0.19	0.17
$p_t^N c_t^N / \sum_j p_t^j c_t^j$	0.77	0.78	0.77	0.78
$\sigma(y_t^C)$	0.09	0.06	0.05	0.05
$\sigma(y_t^X)$	0.07	0.06	0.06	0.04
$\sigma(y_t^N)$	0.06	0.06	0.04	0.04
$\rho(y_t^C)$	0.39	0.44	0.23	0.14
$\rho(y_t^X)$	0.57	0.57	0.42	0.62
$\rho(y_t^N)$	0.57	0.54	0.46	0.61
$p^C m_C^C / \sum_k p^k m_k^C$	0.49	0.48	0.47	0.45
$p^X m_X^C / \sum_k p^k m_k^C$	0.37	0.38	0.35	0.38
$p^N m_N^C / \sum_k p^k m_k^C$	0.14	0.14	0.18	0.17
$p^C m_C^X / \sum_k p^k m_k^X$	0.20	0.19	0.12	0.11
$p^X m_X^X / \sum_k p^k m_k^X$	0.60	0.62	0.59	0.60
$p^N m_N^X / \sum_k p^k m_k^X$	0.20	0.19	0.29	0.29
$p^C m_C^N / \sum_k p^k m_k^N$	0.11	0.17	0.04	0.05
$p^X m_X^N / \sum_k p^k m_k^N$	0.43	0.36	0.34	0.34
$p^N m_N^N / \sum_k p^k m_k^N$	0.46	0.48	0.62	0.61

Note: Sectoral moments come from OECD input-output tables and KLEMS. The volatility and persistence of sectoral output correspond to the median in the sample, across sectors and countries. We use annual data for the 41 countries in our network sample for macroeconomic moments from 1970 to 2019. Debt-to-GDP comes from the IMF government gross debt data. Current account to GDP ratio comes from Lane and Milesi-Ferretti (2017) updated 2023 data. The Trade Balance to GDP ratio and GDP data come from the World Bank, World Development Indicators. We use GDP in constant 2015 US dollars and apply an HP filter to the (log) series with $\lambda = 100$. $\sigma(GDP)$ and $\rho(GDP)$ are the standard deviation of the cyclical component and the (median) autoregressive coefficient of the cyclical component across countries in each group. For comparability across exercises, we kept the same consumption shares across economies.

4.4.2 Exercises and Results

Baseline. Our first exercise studies Sudden Stops and the behavior of macro aggregates in our emerging and advanced economy calibration.

The long-run moments results are in Table 5. To create this table, we simulate the model for 100,000 periods, discard the first 1,000, and compute the average over time.

We can see that emerging markets are more volatile, more prone to exhibit a binding collateral constraint, and, importantly, more prone to exhibit Sudden Stops than advanced economies.[6] All of this despite their lower debt-to-GDP ratio. These differences can arise due to many aspects of our quantitative model. In the following subsection, we focus on one key aspect: the production network structure.

Advanced economy with emerging market network structure. We now consider a counterfactual exercise where we change the production structure of the average advanced economy to match that of the average emerging market while keeping all other parameters and processes for the advanced economy with the same value.

Long-run moments. Table 5 shows the long-run moments implied in our simulations.

A striking fact from this table is that while the debt level of the advanced economy with the network of emerging markets relative to the purely advanced economy is roughly similar (-46.23 vs -46.68), their Sudden Stop probability drastically differs. Indeed, in the advanced economy with the emerging market network, the Sudden Stop probability is 1.47 relative to 0.88 in the purely advanced economy. This is also reflected in the likelihood that the borrowing constraint binds in the former case, which is almost twice as large (29.64 vs 15.84). We conclude that Sudden Stops are more likely to happen in AE when they possess a network structure similar to that of emerging markets. In other words, the network structure can affect the probability of a Sudden Stop.

Average Sudden Stop severity. While the above table studies the long-run behavior of the model and highlights how the network structure may affect the probability of a Sudden Stop, we can also use the quantitative model to assess the severity of Sudden Stops. Figure 6 shows the behavior of GDP (in units of commodity) in panel (a) and the current account in panel b during an average Sudden Stop episode. The blue solid line plots their behavior for an average AE. The red dashed line repeats the exercise for an average AE with the EM network. As we can see from panel (a), the AE economy with the EM network exhibits a larger decline in GDP than its AE counterpart. Quantitatively, this decline is around 30% larger in the former case. Panel (b) shows that the current account reversal is also larger in

[6] We define a Sudden Stop as an event where the curret account-to-GDP ratio is two standard deviations above its long-run average and the borrowing constraint is binding.

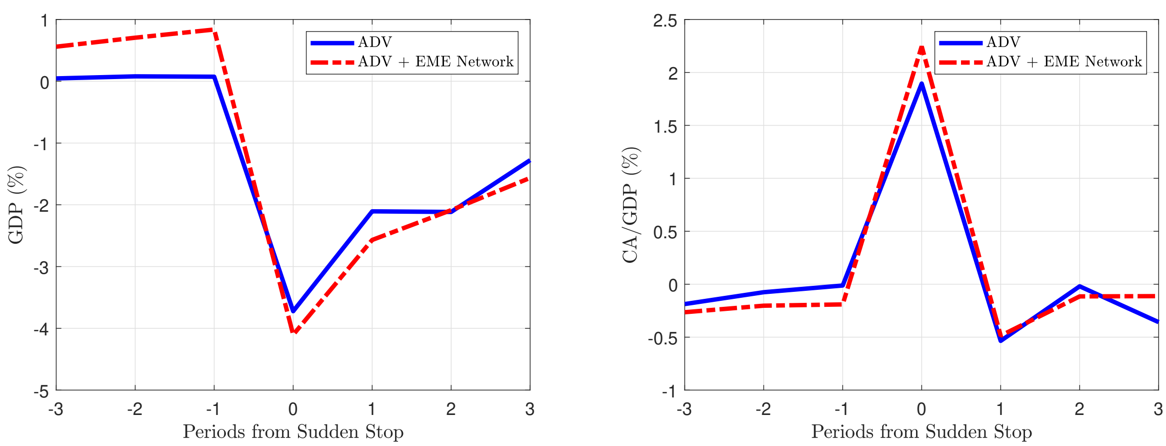
Table 5: Selected long-run moments

Moment	EM	AE with EM Network	AE
$\mathbb{E}(b/Y)$ %	-30.77	-46.23	-46.68
$\sigma(CA/Y)$ %	1.25	0.73	0.84
$(p^C m_C^C / \sum_k p^k m_k^C) \times 100$	48.06	47.81	45.39
$(p^N m_N^C / \sum_k p^k m_k^C) \times 100$	13.91	13.9	16.74
$(p^C m_C^X / \sum_k p^k m_k^X) \times 100$	18.78	18.7	11.1
$(p^N m_N^X / \sum_k p^k m_k^X) \times 100$	19.48	19.48	29.39
$(p^C m_C^N / \sum_k p^k m_k^N) \times 100$	16.59	16.52	5.00
$(p^N m_N^N / \sum_k p^k m_k^N) \times 100$	47.57	47.57	61.5
Sudden Stop Probability %	3.33	1.47	0.88
$\mathbb{P}(\mu_t > 0)$ %	43.68	29.64	15.84
ΔGDP_{SS} %	-12.23	-4.94	-3.8
$\Delta\text{CA/GDP}_{SS}$ %	3.9	2.44	1.91

Note: This table shows long-run moments implied by our model. We simulate the economy for 100000 periods, discard the first 1000, and compute the average over time.

Figure 6: GDP and CA during an average Sudden Stop

(a) GDP (b) CA/GDP



Note: This figure shows the behavior of GDP (panel a) and the current account to GDP ratio (panel b) for an average Sudden Stop episode. The blue line depicts the results for an average AE. The red dashed line depicts the results for an average AE with the same network structure as an average EM.

the AE with EM network case (28% larger). As was the case with the long-run moments, this shows that the network structure is an essential element that can quantitatively affect the probability and severity of Sudden Stops.

5 Normative Analysis

In this section, we characterize the planner's problem using the quantitative framework outlined in the previous section. We consider a planner that can freely choose allocations but faces two implementability constraints, which are the pricing functions for the relative price of nontradable goods and relative price of the non-commoditity good.

5.1 Setup

The planner's problem can be stated in recursive form as follows:

$$V(b, \boldsymbol{e}) = \max_{\boldsymbol{x}} \quad U(c^C, c^X, c^N) + \beta E(V(b', \boldsymbol{e}')),$$

subject to

$$c^C + m_C^C + m_C^X + m_C^N + qb' = y^C + p^X(c^C, c^X)\hat{c}^X(p^X(c^C, c^X)) + b \quad (\lambda_1), \tag{37}$$

$$c^X + \hat{c}^X(p^X(c^C, c^X)) + m_X^C + m_X^X + m_X^N = y^X \qquad (\lambda_2), \tag{38}$$

$$c^N + m_N^C + m_N^X + m_N^N = y^N \qquad (\lambda_3), \tag{39}$$

$$qb' \geq -\kappa(\pi^C + \pi^X + \pi^N) \qquad (\mu), \tag{40}$$

where $\boldsymbol{x} = \{c^C, c^X, c^N, m_N^N, m_X^N, m_C^N, m_N^X, m_X^X, m_C^X, m_N^C, m_X^C, m_C^C, b'\}$ collects all planner's choice variables and $\boldsymbol{e} = \{\{z^i\}_{i \in \{C,X,N\}}\}$ collects the exogenous processes. λ_1 is the Lagrange multiplier on the aggregate resource constraint, λ_2 is the Lagrange multiplier on the market-clearing condition for the tradable non-commodity good, λ_3 is the Lagrange multiplier on the nontradable good market-clearing condition, and μ is the Lagrange multiplier on the borrowing constraint.

Before proceeding to the main equations, we note that we take the dependence of profits on prices and of prices on consumption implicitly. That is

$$\pi^i = \pi^i \left(p^N(c^N, c^C), p^X(c^X, c^C), y^i, \{m_j^i\}_{j \in \{C,X,N\}} \right) \qquad \forall i \in \{C, X, N\}$$

5.2 Key equations

Here we highlight the key differences between the planner's and competitive equilibrium allocations. This short section aims to show how production networks alter the problem relative to previous setups. We discuss some qualitative changes introduced in our setup and relegate the quantitative exercises to section 5.3 and all derivations to Appendix D.

Pecuniary Externality. Our setup features a pecuniary externality as in canonical small open economy models of Sudden Stops with a small twist. In particular, when the planner chooses c^C, its first-order condition satisfies

$$\lambda_1 = U_C + \Theta, \tag{41}$$

where Θ is a wedge defined as

$$\Theta = \underbrace{\frac{\mu\kappa\frac{\partial\pi}{\partial c^C}}{1-\frac{\partial p^X}{\partial c^C}\hat{c}^X\frac{\eta_T}{1+\eta_T}}}_{\text{Pecuniary Externality}} + \underbrace{\frac{U_C\frac{\partial p^X}{\partial c^C}\hat{c}^X\frac{\eta_T}{1+\eta_T}-\lambda_2\frac{\partial\hat{c}^X}{\partial c^C}}{1-\frac{\partial p^X}{\partial c^C}\hat{c}^X\frac{\eta_T}{1+\eta_T}}}_{\text{Terms of Trade Manipulation}}. \tag{42}$$

Hence, the planner internalizes that in choosing commodity consumption, it affects the economy's real income in units of the commodity good ($\pi = \pi^C + \pi^X + \pi^N$) via its effect on the nontradable price p^N and the tradable non-commodity good price p^X. This is reflected in the first term on the right-hand side of equation (42).

The magnitude of the pecuniary externality is also affected by the fact that the planner understands that it can affect p^X, which affects export revenues in this economy $p^X\hat{c}^X$. This is why the denominator in each expression features $\frac{\partial p^X}{\partial c^C}\hat{c}^X\frac{\eta_T}{1+\eta_T}$

How do production networks shape the pecuniary externality component? we can see this in the term $\frac{\partial\pi}{\partial c^C}$, which can be expanded to:

$$\frac{\partial\pi}{\partial c^C} = \frac{\partial p^X}{\partial c^C}\sum_{i\in C,X,T}\frac{\partial\pi^i}{\partial p^X} + \frac{\partial p^N}{\partial c^C}\sum_{i\in C,X,T}\frac{\partial\pi^i}{\partial p^N}. \tag{43}$$

Equation (43) shows that the effect of consumption on the value of the household's collateral is a function of the sensitivity of relative prices to commodity consumption and also of the sensitivity of profits to changes in these prices. Importantly, we see that the shape of the production network has a first-order effect on the magnitude of the response of profits to changes in relative prices. Consider two extreme examples. First, the case of a diagonal economy. Here, we have that profits in the commodity sector are unaffected by changes in relative prices, while profits in the other sectors increase, so the total effect is positive whenever prices increase. In the second case consider a production network where the commodity sector only uses commodities as inputs, the non-commodity tradable sector uses nontradables as inputs, and the nontradable sector uses non-commodity tradables as inputs. In this case, we have that as relative prices increase the profits in the non-commodity tradable and nontradable sectors can decrease depending on what relative price is changing, leading to milder effects on the value of collateral.

Terms of Trade Manipulation. The second term in equation (42) is a pure terms-of-trade manipulation motive absent in the canonical small open economy model. This effect is operative even if the collateral constraint is not binding $\mu = 0$. This considers two effects. The first one is due to a change in the price of p^X, which, given an exported quantity, increases resources available in this economy since $\frac{\partial p^X}{\partial c^C} > 0$. The second effect consider the change in the exported quantity, which reacts negatively since $\frac{\partial \hat{c}^X}{\partial c^C} = \frac{\partial \hat{c}^X}{\partial p^X}\frac{\partial p^X}{\partial c^C} < 0$ as a higher p^X implies a reduction in $\hat{c}^X$. From a resource feasibility perspective, the $\hat{c}^X$ reduction implies more X resources available to be consumed domestically.

Production side. While the above results only used the consumption side of the economy, we close this short characterization by showing how production networks may affect production decisions.

Consider the first-order condition for m_C^N under competitive equilibrium and the planner's problem. The fact that N uses C as an intermediate input to produce encodes the production network idea. An increase in the number of inputs expands the set of decisions the planner must address. A larger set of choices potentially introduces additional distorted margins. These distortions, in turn, may offer the planner more instruments to exploit. We now show that input decision margins are distorted in our setup.

Under the competitive equilibrium, we have

$$p^N \frac{\partial y^N}{\partial m_C^N} = 1,$$

This compares the value of the marginal nontradable unit produced using m_C^N $\left(p^N \frac{\partial y^N}{\partial m_C^N}\right)$ to its marginal cost, which equals one since $p^C = 1$.

Under the planner's solution, we can write the first-order condition of m_C^N as

$$p^N \frac{\partial y^N}{\partial m_C^N} = \frac{\lambda_1 + \mu\kappa}{U_C + \mu\kappa\left(1 + \frac{\partial \pi}{\partial c^N}\frac{1}{p^N}\right)}. \tag{44}$$

When the collateral constraint is not binding, $\mu = 0$, and there are no terms of trade manipulation motives, both private and social relative valuations coincide as $U_C = \lambda_1$. When the constraint is binding $\mu > 0$, the right-hand side can differ from one. This implies that the planner and the competitive equilibrium allocation are different. Hence, there is a distortion.

The reason is similar to the earlier discussion from the consumer's perspective: a pecuniary externality. Relative to the competitive equilibrium, the planner internalizes that its *input* decisions impact aggregate profits, which affects the borrowing capacity of the economy. By internalizing this, the planner can improve the competitive equilibrium allocation. Whether this can be done using an ex-post subsidy or a tax requires more structure.

Lastly, we see in Equation (44) that, as in the consumption side, the magnitude of the reallocation incentives that the planner faces will depend on the shape of the production structure of the economy since the term $\frac{\partial \pi}{\partial c^N}$ is present in the first order condition of the input.

We finally note that there is no room for an ex-ante instrument to affect the production margins, since they are only distorted when $\mu > 0$ (absent terms-of-trade manipulation motives). This point is highlighted elsewhere in the literature for production economies with *imported intermediate inputs* without terms-of-trade manipulation and production networks (e.g. Hernandez and Mendoza, 2017). We show here that this statement generally applies to economies with domestic linkages as well.

5.3 Quantitative Analysis

We now study the optimal policy problem using the fully quantitative model. We start by focusing on a macroprudential debt tax, and then focus on sectoral taxes.

5.3.1 Macroprudential Debt Tax

Our first quantitative experiment is to study how a macroprudential debt tax affects the welfare gains and the probability of Sudden Stops in our setup. To do so, we introduce a non-state contingent tax on household borrowing. Revenues raised via this instrument are later rebated back to households in the form of lump-sum transfers.[7] We explore different values for the tax rate.

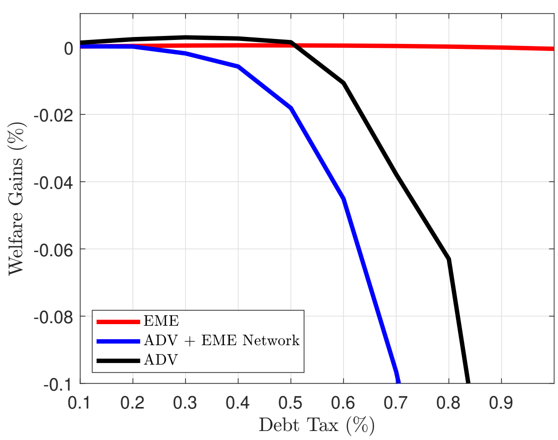
Figure 7 shows the results when introducing a debt tax under different model calibrations. Panel (a) shows the welfare gains, while panel (b) shows the probability of Sudden Stops. In each plot, we have three different models. An average emerging market (red line), an average advanced economy (black line), and an average advanced economy with the network structure of emerging markets (blue line). Panels (a) and (b) show that the

[7]The debt tax introduces a wedge in the bonds Euler equation of households by raising the financial cost of borrowing R to $(1 + \tau^b)R$, where τ^b is the tax rate.

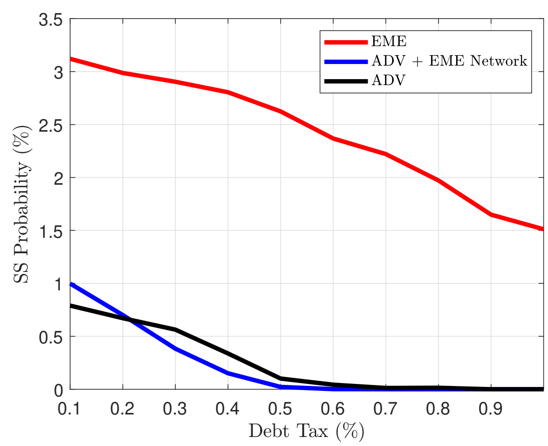
debt tax can generate welfare gains in the emerging and advanced economy cases by reducing the likelihood of Sudden Stops (due to less borrowing as it becomes more expensive), but that we have to be careful with setting taxes that are relatively large (i.e. above 0.5 %). We see that in the advanced economy calibration, welfare losses can be substantial if the tax is set to be relatively large. Interestingly, panel (a) also shows that advanced economies with that exhibit production networks similar to those of emerging markets are hurt more by the debt tax. In particular, we see that even for low levels of the debt tax we have welfare losses. These results show that policymakers should be careful at the moment of designing macroprudential tools to curb borrowing because it is not only important to distinguish across levels of financial development but also types of production structures. A "one-size-fits-all" approach can have quite negative effects for certain types of economies.

Figure 7: Welfare gains and Sudden Stop probability due to a debt tax

(a) Welfare Gains



(b) Sudden Stop Probability



Note: This figure shows the welfare gains (panel a) and the Sudden Stop probability (panel b) for different debt taxes (x-axis). We consider three calibrations: an average emerging market (red line), an average advanced economy (black line) and an average advanced economy with the network structure of emerging markets (blue line).

5.3.2 Sectoral Taxes

We now shift our attention towards sectoral taxes/subsidies.[8] Figure 8 shows the results when introducing sectoral taxes for our baseline emerging market calibration. The black

[8] We assume that the government imposes taxes on the purchases of final goods and inputs. This is, households and firms face these taxes when purchasing goods/inputs. All revenues are rebated back to households in the form of lump-sum transfers.

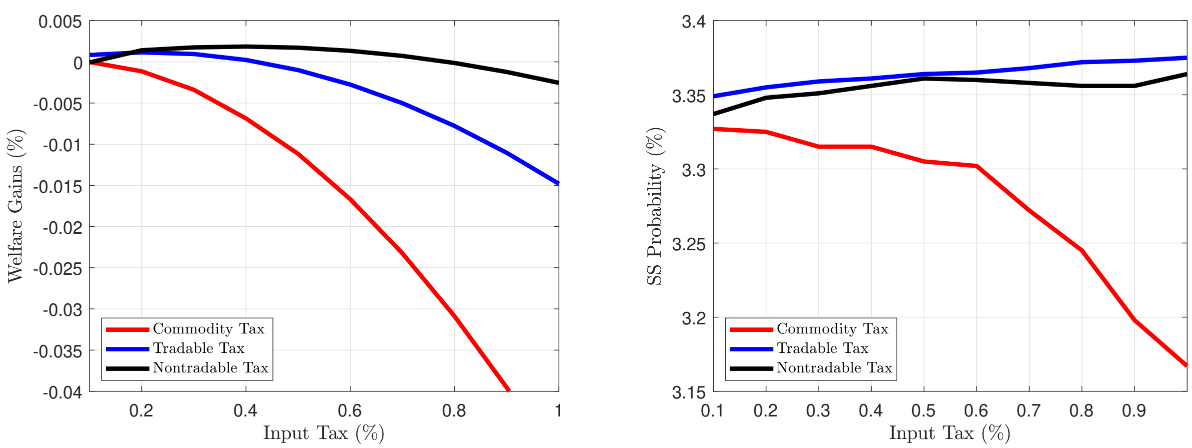
line corresponds to a tax on the nontradable input, the red line is a tax on the commodity input, and the blue line is a tax on the tradable (non-commodity) input.

Panel (a) shows the welfare gains, while panel (b) shows the probability of Sudden Stops. From panel (a) we see that that a tax on the commodity input is welfare-decreasing, suggesting that it is never optimal to tax commodities using a constant tax. On the other hand, we see that taxes on nontradable and tradable inputs can be welfare-increasing, but the effects on welfare are slightly different as taxes on nontradable inputs are less likely to generate welfare losses.

Panel (b) shows that the Sudden Stop probability increases with taxes on the tradable and nontradable inputs, and decreases with a tax on the commodity input. That the tax on the commodity input decreases the Sudden Stop probability is intuitive. Since debt is denominated in units of the commodity good, taxing the commodity input is similar to a direct tax on debt. Taxes on the tradable and nontradable inputs slightly increase the likelihood of experiencing Sudden Stops, mainly because they tilt the use of producton inputs towards the commodity. Since the price of the commodity good the numeraire, we have that the economy becomes more fragile because firms are using less of the inputs whose price can adjust during downturns.

Figure 8: Welfare gains and Sudden Stop probability due to sectoral taxes: baseline EM

(a) Welfare Gains (b) Sudden Stop Probability



Note: This figure shows the welfare gains (panel a) and the Sudden Stop probability (panel b) for different input taxes. The calibration corresponds to our baseline emerging market. The black line corresponds to a tax on the nontradable input, the red line is a tax on the commodity input, and the blue line is a tax on the tradable (non-commodity) input.

6 Conclusion

This study highlights the crucial role of production network structures in shaping the severity and propagation of financial crises. We show how intersectoral linkages influence collateral constraints and macroeconomic stability, offering a novel perspective on the disparities between emerging and advanced economies. Our empirical findings and quantitative model reinforce the idea that network structures, beyond traditional financial indicators, can significantly impact the effectiveness of macroprudential policies and sectoral input taxes. Thus, policymakers should consider production network structures as a key factor in financial stability frameworks.

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A Proofs

A.1 Proof of Proposition 1

Totally differentiating the PP curve equation (14)

$$\mathrm{d}\log p_t^N = (1+\eta)\mathrm{d}\log c_t^T - (1+\eta)\mathrm{d}\log c_t^N.$$

This delivers the solution

$$\frac{\mathrm{d}\log p_t^N}{\mathrm{d}\log c_t^T} = \frac{(1+\eta)}{1+(1+\eta)\frac{\mathrm{d}\log c_t^N}{\mathrm{d}\log p_t^N}}.$$

Since $\eta > -1$ it suffices to show that $\frac{\mathrm{d}\log c_t^N}{\mathrm{d}\log p_t^N} \geq 0$. Differentiating the nontradable market clearing condition

$$\begin{aligned}
\mathrm{d}\log c_t^N &= \frac{(1-\alpha_N^N)y_t^N}{c_t^N}\mathrm{d}\log y_t^N - \alpha_N^T \frac{y_t^T}{p_t^N c_t^N}\left(\mathrm{d}\log y_t^T - \mathrm{d}\log p_t^N\right), \\
\frac{\mathrm{d}\log c_t^N}{\mathrm{d}\log p_t^N} &= \frac{(1-\alpha_N^N)y_t^N}{c_t^N}\frac{\mathrm{d}\log y_t^N}{\mathrm{d}\log p_t^N} - \alpha_N^T \frac{y_t^T}{p_t^N c_t^N}\left(\frac{\mathrm{d}\log y_t^T}{\mathrm{d}\log p_t^N} - 1\right) \geq 0,
\end{aligned}$$

which is positive since $\frac{\mathrm{d}\log y_t^N}{\mathrm{d}\log p_t^N} = \alpha_T^N/(1-\alpha_N^N-\alpha_T^N) \geq 0$ and $\frac{\mathrm{d}\log y_t^T}{\mathrm{d}\log p_t^N} = -\alpha_N^T/(1-\alpha_T^T-\alpha_N^T) \leq 0$. This concludes the proof.

A.2 Proof of Proposition 2

Totally differentiating equation (15)

$$\mathrm{d}c^T = \kappa_N \mathrm{d}\pi_t^N(p_t^N; z_t^N) + (1+\kappa_T)\mathrm{d}\pi_t^T(p_t^N; z_t^T). \tag{45}$$

Since we are only allowing the nontradable price and tradable consumption to change, we have that

$$\mathrm{d}\pi_t^N = \frac{\partial \pi_t^N}{\partial p_t^N}\mathrm{d}p_t^N, \tag{46}$$

$$\mathrm{d}\pi_t^T = \frac{\partial \pi_t^T}{\partial p_t^N}\mathrm{d}p_t^N. \tag{47}$$

Replacing equation (46) and (47) in equation (45) and solving out for $\mathrm{d}p_t^N/\mathrm{d}c_t^T$, we have

$$\frac{\mathrm{d}p_t^N}{\mathrm{d}c_t^T} = \frac{1}{\kappa_N \frac{\partial \pi_t^N}{\partial p_t^N} + (1+\kappa_T)\frac{\partial \pi_t^T}{\partial p_t^N}}. \tag{48}$$

Hence, the slope of the BB curve inherits the sign of $\kappa_N \frac{\partial \pi_t^N}{\partial p_t^N} + (1+\kappa_T)\frac{\partial \pi_t^T}{\partial p_t^N}$, as stated in Proposition 2. This concludes the proof.

A.3 Proof of Proposition 3

The proof involves several steps. For convenience, we rewrite the equilibrium system in terms of outputs and prices below:

$$p_t^N = \left(\frac{c_t^T}{c_t^N(p_t^N; z_t^N, z_t^T)}\right)^{1+\eta}\left(\frac{1-\omega}{\omega}\right) \qquad \text{(PP Curve)} \tag{49}$$

$$\begin{aligned} c_t^T = &\left(\kappa(1-\alpha_N^N-\alpha_T^N)-\alpha_T^N\right) p_t^N y_t^N(p_t^N; z_t^N) \\ &+ (\kappa(1-\alpha_N^T-\alpha_T^T)+(1-\alpha_T^T))y_t^T(p_t^N; z_t^T) + b_0 \qquad \text{(BB Curve)}, \end{aligned} \tag{50}$$

Define elasticities as $\varepsilon_x^y = \frac{\partial \log y}{\partial \log x}$. Totally differentiating both curves, I get

$$\mathrm{d}\log p_t^N = (1+\eta)(\mathrm{d}\log c_t^T - \mathrm{d}\log c_t^N) \tag{51}$$

$$\mathrm{d}\log c_t^T = \gamma_t^N(\mathrm{d}\log p_t^N + \mathrm{d}\log y_t^N) + \gamma_t^T \mathrm{d}\log y_t^T \tag{52}$$

We need to solve out $\mathrm{d}\log y_t^N, \mathrm{d}\log y_t^T, \mathrm{d}\log c_t^N$. Since these are function of p_t^N and (z_t^N, z_t^T), we have

$$\mathrm{d}\log y_t^N = \varepsilon_{p_t^N}^{y_t^N}\mathrm{d}\log p_t^N + \varepsilon_{z_t^N}^{y_t^N}\mathrm{d}\log z_t^N \tag{53}$$

$$\mathrm{d}\log y_t^T = \varepsilon_{p_t^N}^{y_t^T}\mathrm{d}\log p_t^N + \varepsilon_{z_t^T}^{y_t^T}\mathrm{d}\log z_t^T \tag{54}$$

$$\mathrm{d}\log c_t^N = \varepsilon_{p_t^N}^{c_t^N}\mathrm{d}\log p_t^N + \varepsilon_{z_t^N}^{c_t^N}\mathrm{d}\log z_t^N + +\varepsilon_{z_t^T}^{c_t^N}\mathrm{d}\log z_t^T \tag{55}$$

Replacing into equations (51) and (52), we get

$$\mathrm{d}\log c_t^T = \left(\frac{1}{1+\eta} + \varepsilon^{c_t^N}_{p_t^N}\right)\mathrm{d}\log p_t^N + \varepsilon^{c_t^N}_{z_t^N}\mathrm{d}\log z_t^N + \varepsilon^{c_t^N}_{z_t^T}\mathrm{d}\log z_t^T \tag{56}$$

$$\mathrm{d}\log c_t^T = \left(\gamma_t^N(1+\varepsilon^{y_t^N}_{p_t^N}) + \gamma_t^T\varepsilon^{y_t^T}_{p_t^N}\right)\mathrm{d}\log p_t^N + \gamma_t^N\varepsilon^{y_t^N}_{z_t^N}\mathrm{d}\log z_t^N + \gamma_t^T\varepsilon^{y_t^T}_{z_t^T}\mathrm{d}\log z_t^T. \tag{57}$$

For future reference, let

$$\varepsilon^{PP} = \left(\frac{1}{1+\eta} + \varepsilon^{c_t^N}_{p_t^N}\right)^{-1}$$

be the slope of the PP curve and

$$\varepsilon^{BB} = \left(\gamma_t^N(1+\varepsilon^{y_t^N}_{p_t^N}) + \gamma_t^T\varepsilon^{y_t^T}_{p_t^N}\right)^{-1}$$

be the slope of the BB curve.

This system can be expressed as

$$\begin{bmatrix} 1 & -\left(\frac{1}{1+\eta} + \varepsilon^{c_t^N}_{p_t^N}\right) \\ 1 & -\left(\gamma_t^N(1+\varepsilon^{y_t^N}_{p_t^N}) + \gamma_t^T\varepsilon^{y_t^T}_{p_t^N}\right)\end{bmatrix}\begin{bmatrix}\mathrm{d}\log c_t^T \\ \mathrm{d}\log p_t^N\end{bmatrix} = \begin{bmatrix}\varepsilon^{c_t^N}_{z_t^N} & \varepsilon^{c_t^N}_{z_t^T} \\ \gamma_t^N\varepsilon^{y_t^N}_{z_t^N} & \gamma_t^T\varepsilon^{y_t^T}_{z_t^T}\end{bmatrix}\begin{bmatrix}\mathrm{d}\log z_t^N \\ \mathrm{d}\log z_t^T\end{bmatrix} \tag{58}$$

If $\frac{1}{1+\eta} + \varepsilon^{c_t^N}_{p_t^N} \neq \left(\gamma_t^N(1+\varepsilon^{y_t^N}_{p_t^N}) + \gamma_t^T\varepsilon^{y_t^T}_{p_t^N}\right)$, we can invert the matrix in the left-hand side and write it as

$$\begin{bmatrix}\mathrm{d}\log c_t^T \\ \mathrm{d}\log p_t^N\end{bmatrix} = \mathcal{H}\mathcal{P}\begin{bmatrix}\mathrm{d}\log z_t^N \\ \mathrm{d}\log z_t^T\end{bmatrix}, \tag{59}$$

where

$$\mathcal{H} = \begin{bmatrix} 1 & -\left(\frac{1}{1+\eta} + \varepsilon^{c_t^N}_{p_t^N}\right) \\ 1 & -\left(\gamma_t^N(1+\varepsilon^{y_t^N}_{p_t^N}) + \gamma_t^T\varepsilon^{y_t^T}_{p_t^N}\right)\end{bmatrix}^{-1}$$

$$\mathcal{H} = \frac{1}{\left[\frac{1}{1+\eta} + \varepsilon^{c_t^N}_{p_t^N}\right] - \left(\gamma_t^N(1+\varepsilon^{y_t^N}_{p_t^N}) + \gamma_t^T\varepsilon^{y_t^T}_{p_t^N}\right)}\begin{bmatrix} -\left(\gamma_t^N(1+\varepsilon^{y_t^N}_{p_t^N}) + \gamma_t^T\varepsilon^{y_t^T}_{p_t^N}\right) & \frac{1}{1+\eta} + \varepsilon^{c_t^N}_{p_t^N} \\ -1 & 1\end{bmatrix},$$

captures the general equilibrium effects. The second matrix,

$$\mathcal{P} = \begin{bmatrix} \varepsilon^{c^N_t}_{z^N_t} & \varepsilon^{c^N_t}_{z^T_t} \\ \gamma^N_t \varepsilon^{y^N_t}_{z^N_t} & \gamma^T_t \varepsilon^{y^T_t}_{z^T_t} \end{bmatrix}$$

captures partial equilibrium effects. It considers the direct effects of tradable and non-tradable productivity on the shifters of each curve. This represents shifts of both the PP and BB curves. The general equilibrium effects can be thought of as movements along each curve. This way of formulating the problem follows recent work in the heterogeneous agents literature; see, for example, Auclert et al. (2021), Wolf (2023), and Auclert et al. (2024). We apply the same logic to a multi-sector context.

We can find an expression for $\varepsilon^{c^N_t}_{p^N_t}$ as a function of supply elasticities using the non-tradable market clearing condition

$$\varepsilon^{c^N_t}_{p^N_t} = \delta^N_t \varepsilon^{y^N_t}_{p^N_t} + (1 - \delta^N_t)(\varepsilon^{y^T_t}_{p^N_t} - 1),$$

where $\delta^N_t = (1 - \alpha^N_N) y^N_t / c^N_t$ and $1 - \delta^N_t = -(\alpha^T_N y^T_t / p^N_t) / c^N_t$. Note that by construction $\delta^N_t \geq 1$. This is important to sign the determinant. Using this result, the determinant can be written as

$$\begin{aligned} \left[\frac{1}{1+\eta} + \varepsilon^{c^N_t}_{p^N_t}\right] - \left(\gamma^N_t (1 + \varepsilon^{y^N_t}_{p^N_t}) + \gamma^T_t \varepsilon^{y^T_t}_{p^N_t}\right) &= \left[\frac{1}{1+\eta} + \delta^N_t \varepsilon^{y^N_t}_{p^N_t} + (1 - \delta^N_t)(\varepsilon^{y^T_t}_{p^N_t} - 1)\right] \\ &\quad - \left(\gamma^N_t (1 + \varepsilon^{y^N_t}_{p^N_t}) + \gamma^T_t \varepsilon^{y^T_t}_{p^N_t}\right) \\ &= \frac{1}{1+\eta} - \gamma^N_t + (\delta^N_t - 1) + (\delta^N_t - \gamma^N_t)\varepsilon^{y^N_t}_{p^N_t} \\ &\quad + (1 - \gamma^T_t - \delta^N_t)\varepsilon^{y^T_t}_{p^N_t} \end{aligned}$$

Now, notice that we know the following $\eta \in (-1, \infty)$, $\gamma^N_t \in [0, 1]$, $\gamma^T_t \geq 0$, $\delta^N_t \geq 1$, $\varepsilon^{y^N_t}_{p^N_t} \geq 0$ and $\varepsilon^{y^T_t}_{p^N_t} \leq 0$. For γ^N_t we need a restriction that we discuss below. Therefore, the determinant is **positive** since $(1+\eta)^{-1} - \gamma^N_t + (\delta^N_t - 1) > 0$, $(\delta^N_t - \gamma^N_t)\varepsilon^{y^N_t}_{p^N_t} > 0$ and

$(1-\gamma_t^T-\delta_t^N)\varepsilon_{p_t^N}^{y_t^T} > 0$. In what follows, let:

$$\mathcal{K} = \frac{1}{\left[\frac{1}{1+\eta} + \varepsilon_{p_t^N}^{c_t^N}\right] - \left(\gamma_t^N(1+\varepsilon_{p_t^N}^{y_t^N}) + \gamma_t^T\varepsilon_{p_t^N}^{y_t^T}\right)} \tag{60}$$

We can now assess the effects of tradable productivity on tradable consumption and the nontradable price. The effect of tradable productivity on tradable consumption is

$$\begin{aligned}
\frac{\mathrm{d}\log c_t^T}{\mathrm{d}\log z_t^T} &= \mathcal{K}\left(-\left(\gamma_t^N(1+\varepsilon_{p_t^N}^{y_t^N}) + \gamma_t^T\varepsilon_{p_t^N}^{y_t^T}\right)\varepsilon_{z_t^T}^{c_t^N} + \left(\frac{1}{1+\eta} + \varepsilon_{p_t^N}^{c_t^N}\right)\gamma_t^T\varepsilon_{z_t^T}^{y_t^T}\right) \\
\frac{\mathrm{d}\log c_t^T}{\mathrm{d}\log z_t^T} &= \mathcal{K}\left(-\left(\gamma_t^N(1+\varepsilon_{p_t^N}^{y_t^N}) + \gamma_t^T\varepsilon_{p_t^N}^{y_t^T}\right)(1-\delta_t^N)\varepsilon_{z_t^T}^{y_t^T} + \left(\frac{1}{1+\eta} + \delta_t^N\varepsilon_{p_t^N}^{y_t^N} + (1-\delta_t^N)(\varepsilon_{p_t^N}^{y_t^T}-1)\right)\gamma_t^T\varepsilon_{z_t^T}^{y_t^T}\right) \\
&= \mathcal{K}\left(\left[-\left(\gamma_t^N(1+\varepsilon_{p_t^N}^{y_t^N}) + \gamma_t^T\varepsilon_{p_t^N}^{y_t^T}\right)(1-\delta_t^N) + \left(\frac{1}{1+\eta} + \delta_t^N\varepsilon_{p_t^N}^{y_t^N} + (1-\delta_t^N)(\varepsilon_{p_t^N}^{y_t^T}-1)\right)\gamma_t^T\right]\varepsilon_{z_t^T}^{y_t^T}\right) \\
&= \mathcal{K}\left(\left[-\gamma_t^N(1+\varepsilon_{p_t^N}^{y_t^N})(1-\delta_t^N) - \gamma_t^T(1-\delta_t^N)\varepsilon_{p_t^N}^{y_t^T} + \delta_t^N\gamma_t^T\varepsilon_{p_t^N}^{y_t^N} + (1-\delta_t^N)\gamma_t^T(\varepsilon_{p_t^N}^{y_t^T}-1)\right]\varepsilon_{z_t^T}^{y_t^T}\right) \\
&\quad + \mathcal{K}\left(\frac{\gamma_t^T}{1+\eta}\varepsilon_{z_t^T}^{y_t^T}\right) \\
&= \mathcal{K}\left[-\gamma_t^N(1+\varepsilon_{p_t^N}^{y_t^N})(1-\delta_t^N) + \delta_t^N\gamma_t^T\varepsilon_{p_t^N}^{y_t^N} - (1-\delta_t^N)\gamma_t^T + \frac{\gamma_t^T}{1+\eta}\right]\varepsilon_{z_t^T}^{y_t^T} \\
&= \mathcal{K}\left[-(\gamma_t^N+\gamma_t^T)(1-\delta_t^N) - \left(\gamma_t^N(1-\delta_t^N) - \delta_t^N\gamma_t^T\right)\varepsilon_{p_t^N}^{y_t^N} + \frac{\gamma_t^T}{1+\eta}\right]\varepsilon_{z_t^T}^{y_t^T} \\
\frac{\mathrm{d}\log c_t^T}{\mathrm{d}\log z_t^T} &= \mathcal{K}\left[(\gamma_t^N+\gamma_t^T)(\delta_t^N-1) + \left(\gamma_t^N(\delta_t^N-1) + \delta_t^N\gamma_t^T\right)\varepsilon_{p_t^N}^{y_t^N} + \frac{\gamma_t^T}{1+\eta}\right]\varepsilon_{z_t^T}^{y_t^T} > 0
\end{aligned}$$

Hence, tradable consumption moves in the same direction as tradable productivity. For the nontradable price, we have:

$$\begin{aligned}
\mathrm{d}\log p_t^N &= \mathcal{K}\left(-\varepsilon_{z_t^T}^{c_t^N} + \gamma_t^T\varepsilon_{z_t^T}^{y_t^T}\right)\mathrm{d}\log z_t^T \\
\mathrm{d}\log p_t^N &= \mathcal{K}\left(-(1-\delta_t^N)\varepsilon_{z_t^T}^{y_t^T} + \gamma_t^T\varepsilon_{z_t^T}^{y_t^T}\right)\mathrm{d}\log z_t^T \\
\mathrm{d}\log p_t^N &= \mathcal{K}\left(-(1-\delta_t^N) + \gamma_t^T\right)\varepsilon_{z_t^T}^{y_t^T}\mathrm{d}\log z_t^T \\
\frac{\mathrm{d}\log p_t^N}{\mathrm{d}\log z_t^T} &= \mathcal{K}\left((\delta_t^N-1) + \gamma_t^T\right)\varepsilon_{z_t^T}^{y_t^T} > 0
\end{aligned}$$

Hence, in response to a negative tradable productivity shock, nontradable prices and tradable consumption declines in this economy. This concludes the proof.

Justification for $\gamma_t^N \in [0,1]$. Note that the key objects in all the above expressions are

$$
\begin{aligned}
\delta_t^N &= \frac{(1-\alpha_N^N)y_t^N}{c_t^N}; \qquad (1-\delta_t^N) = -\alpha_N^T \frac{y_t^T/p_t^N}{c_t^N} \\
\gamma_t^N &= \frac{\xi^N p_t^N y_t^N}{c_t^T} = \frac{((\kappa(1-\alpha_N^N-\alpha_T^N)-\alpha_T^N))p_t^N y_t^N}{c_t^T} \\
\gamma_t^T &= \frac{\xi^T y_t^T}{c_t^T} = (\kappa(1-\alpha_N^T-\alpha_T^T)+(1-\alpha_T^T))\frac{y_t^T}{c_t^T} \\
\varepsilon_{z_t^T}^{y_t^T} &= \frac{1}{1-\alpha_T^T-\alpha_N^T}
\end{aligned}
$$

We now justify why $\gamma_t^N \in [0,1]$. For this to hold, it requires $\kappa(1-\alpha_N^N-\alpha_T^N)-\alpha_T^N \geq 0$. What is this restriction? It turns out that this has a nice expression

$$
\begin{aligned}
\kappa(1-\alpha_N^N-\alpha_T^N)-\alpha_T^N \geq 0 & \\
\kappa &\geq \frac{\alpha_T^N}{(1-\alpha_N^N-\alpha_T^N)} \\
\kappa &\geq \frac{m_{Tt}^N}{p_t^N y_t^N(1-\alpha_N^N-\alpha_T^N)} \\
\kappa &\geq \frac{m_T^N}{\pi_t^N} = \tilde{\alpha}_T^N
\end{aligned}
$$

This restriction says that provided that κ is larger than the expenditure of nontradable on tradables as a share of profits, then $\gamma_t^N \geq 0$. Otherwise, it can be negative. This happens for large values of α_T^N. In other words, for $\gamma_t^N \geq 0$, we require the response of tradable consumption to increase with changes in nontradable profits. In an endowment economy or an economy where the tradable sector does not buy from the nontradable sector, this is equivalent to assume that the BB curve is upward sloping, as we discussed in Proposition 2.

Can $\gamma_t^N \geq 1$? It cannot. Suppose it is

$$
\begin{aligned}
\gamma_t^N &\geq 1 \\
\frac{(\kappa(1-\alpha_N^N-\alpha_T^N)-\alpha_T^N)p_t^N y_t^N}{c_t^T} &\geq 1 \\
(\kappa(1-\alpha_N^N-\alpha_T^N)-\alpha_T^N)p_t^N y_t^N &\geq c_t^T \\
(\kappa(1-\alpha_N^N-\alpha_T^N)-\alpha_T^N)p_t^N y_t^N &\geq (\kappa(1-\alpha_N^N-\alpha_T^N)-\alpha_T^N)p_t^N y_t^N + ((\kappa(1-\alpha_N^T-\alpha_T^T)+(1-\alpha_T^T)))y_t^T + b_0 \\
0 &\geq (\kappa(1-\alpha_N^T-\alpha_T^T)+(1-\alpha_T^T))y_t^T + b_0,
\end{aligned}
$$

which is a contradiction since $(\kappa(1-\alpha_N^T-\alpha_T^T)+(1-\alpha_T^T))y_t^T + b_0 \geq 0$.

Hence, $\gamma_t^N \in [0,1]$ iff $\kappa \geq \tilde{\alpha}_T^N$.

A.4 Proof of Proposition 4

To derive this expression, let us start again from the PP and BB curve general definitions:

$$
p_t^N = \left(\frac{c_t^T}{c_t^N(p_t^N; z_t^N, z_t^T)}\right)^{1+\eta}\left(\frac{1-\omega}{\omega}\right) \tag{61}
$$

$$
c_t^T = \kappa_N \pi_t^N(p_t^N; z_t^N) + (1+\kappa_T)\pi_t^T(p_t^N; z_t^T) + b_0 \tag{62}
$$

Taking total differential with respect to both p_t^N and c_t^T, we get for the PP curve:

$$
\mathrm{d}\log p_t^N = (1+\eta)\mathrm{d}\log c_t^T - (1+\eta)\frac{\partial \log c_t^N}{\partial \log p_t^N}\mathrm{d}\log p_t^N
$$

Define $\varepsilon_t^{PP} = \frac{\mathrm{d}\log p_t^N}{\mathrm{d}\log c_t^T}$ as the elasticity of the PP curve, and so

$$
\varepsilon_t^{PP} = \frac{(1+\eta)}{1+(1+\eta)\frac{\partial \log c_t^N}{\partial \log p_t^N}}
$$

Similarly, differentiate the BB curve with respect to c_t^T and p_t^N to get

$$
\mathrm{d}\log c_t^T = \frac{\kappa_N \pi_t^N}{c_t^T}\frac{\partial \pi_t^N}{\partial p_t^N}\frac{p_t^N}{\pi_t^N}\mathrm{d}\log p_t^N + (1+\kappa_T)\frac{\pi_t^T}{c_t^T}\frac{\partial \pi_t^T}{\partial p_t^N}\frac{p_t^N}{\pi_t^T}\mathrm{d}\log p_t^N
$$

$$
\begin{aligned}
\mathrm{d}\log c_t^T &= \frac{\kappa_N \pi_t^N}{c_t^T} \varepsilon_{p_t^N}^{\pi_t^N} \mathrm{d}\log p_t^N + (1+\kappa_T) \frac{\pi_t^T}{c_t^T} \varepsilon_{p_t^N}^{\pi_t^T} \mathrm{d}\log p_t^N \\
\mathrm{d}\log c_t^T &= \gamma_t^N \varepsilon_{p_t^N}^{\pi_t^N} \mathrm{d}\log p_t^N + \gamma_t^T \varepsilon_{p_t^N}^{\pi_t^T} \mathrm{d}\log p_t^N \\
\frac{\mathrm{d}\log p_t^N}{\mathrm{d}\log c_t^T} &= \frac{1}{\gamma_t^N \varepsilon_{p_t^N}^{\pi_t^N} + \gamma_t^T \varepsilon_{p_t^N}^{\pi_t^T}} \\
\varepsilon_t^{BB} &= \frac{1}{\gamma_t^N \varepsilon_{p_t^N}^{\pi_t^N} + \gamma_t^T \varepsilon_{p_t^N}^{\pi_t^T}}.
\end{aligned}
$$

Using these results we can rewrite the general expression in equation (59) as

$$
\begin{bmatrix} \mathrm{d}\log c_t^T \\ \mathrm{d}\log p_t^N \end{bmatrix} = \frac{1}{(\varepsilon_t^{PP})^{-1} - (\varepsilon_t^{BB})^{-1}} \begin{bmatrix} -(\varepsilon_t^{BB})^{-1} & (\varepsilon_t^{PP})^{-1} \\ -1 & -1 \end{bmatrix} \begin{bmatrix} \varepsilon_{z_t^N}^{c_t^N} & \varepsilon_{z_t^T}^{c_t^N} \\ \gamma_t^N \varepsilon_{z_t^N}^{y_t^N} & \gamma_t^T \varepsilon_{z_t^T}^{y_t^T} \end{bmatrix} \begin{bmatrix} \mathrm{d}\log z_t^N \\ \mathrm{d}\log z_t^T \end{bmatrix}
$$

Now, consider the case where $\mathrm{d}\log z_t^T \neq 0$ and $\mathrm{d}\log z_t^N = 0$. Then,

$$
\begin{aligned}
\begin{bmatrix} \mathrm{d}\log c_t^T \\ \mathrm{d}\log p_t^N \end{bmatrix} &= \frac{1}{(\varepsilon_t^{PP})^{-1} - (\varepsilon_t^{BB})^{-1}} \begin{bmatrix} -(\varepsilon_t^{BB})^{-1} & (\varepsilon_t^{PP})^{-1} \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{z_t^T}^{c_t^N} \\ \gamma_t^T \varepsilon_{z_t^T}^{y_t^T} \end{bmatrix} \mathrm{d}\log z_t^T \\
\begin{bmatrix} \mathrm{d}\log c_t^T \\ \mathrm{d}\log p_t^N \end{bmatrix} &= \frac{1}{(\varepsilon_t^{PP})^{-1} - (\varepsilon_t^{BB})^{-1}} \begin{bmatrix} -\varepsilon_{z_t^T}^{c_t^N} (\varepsilon_t^{BB})^{-1} + \gamma_t^T \varepsilon_{z_t^T}^{y_t^T} (\varepsilon_t^{PP})^{-1} \\ -\varepsilon_{z_t^T}^{c_t^N} + \gamma_t^T \varepsilon_{z_t^T}^{y_t^T} \end{bmatrix} \mathrm{d}\log z_t^T
\end{aligned}
$$

Therefore, the total effect on tradable consumption after a change in tradable productivity is

$$
\frac{\mathrm{d}\log c_t^T}{\mathrm{d}\log z_t^T} = \frac{1}{(\varepsilon_t^{PP})^{-1} - (\varepsilon_t^{BB})^{-1}} \left(-\varepsilon_{z_t^T}^{c_t^N} (\varepsilon_t^{BB})^{-1} + \gamma_t^T \varepsilon_{z_t^T}^{y_t^T} (\varepsilon_t^{PP})^{-1} \right).
$$

Under the diagonal and the diversified example, we have that $\varepsilon_{z_t^T}^{c_t^N} = 0$ since the tradable sector does not directly demand from the nontradable sector. For the same reason, we have that across the two economies: $\varepsilon_{z_t^T}^{y_t^T}(I) = \varepsilon_{z_t^T}^{y_t^T}(D) = \frac{1}{1-\alpha_T^T(I)} = \frac{1}{1-\alpha_T^T(D)}$. Solving out,

$$
\frac{\mathrm{d}\log c_t^T}{\mathrm{d}\log z_t^T} = \frac{1}{(\varepsilon_t^{PP})^{-1} - (\varepsilon_t^{BB})^{-1}} \left(\gamma_t^T \varepsilon_{z_t^T}^{y_t^T} (\varepsilon_t^{PP})^{-1} \right),
$$

$$\frac{\mathrm{d}\log c_t^T}{\mathrm{d}\log z_t^T} = \frac{\varepsilon_t^{BB}\varepsilon_t^{PP}}{\varepsilon_t^{BB} - (\varepsilon_t^{PP})}\left(\gamma_t^T \varepsilon_{z_t^T}^{y_t^T}(\varepsilon_t^{PP})^{-1}\right),$$
$$\frac{\mathrm{d}\log c_t^T}{\mathrm{d}\log z_t^T} = \frac{\varepsilon_t^{BB}}{\varepsilon_t^{BB} - \varepsilon_t^{PP}}\gamma_t^T \varepsilon_{z_t^T}^{y_t^T}.$$

Now, we wish to compare the two economies. For tradable consumption in the diversified economy to react less than the island economy, we require

$$\left|\frac{\mathrm{d}\log(c_t^T)^{\mathrm{NT}\leftarrow\mathrm{T}}}{\mathrm{d}\log z_t^T}\right| \leq \left|\frac{\mathrm{d}\log(c_t^T)^{\mathrm{Diagonal}}}{\mathrm{d}\log z_t^T}\right|.$$
$$\left|\frac{\varepsilon_t^{BB}(N)}{\varepsilon_t^{BB}(N) - \varepsilon_t^{PP}(N)}\gamma_t^T(N)\varepsilon_{z_t^T}^{y_t^T}(N)\right| \leq \left|\frac{\varepsilon_t^{BB}(D)}{\varepsilon_t^{BB}(D) - \varepsilon_t^{PP}(D)}\gamma_t^T(D)\varepsilon_{z_t^T}^{y_t^T}(D)\right|,$$
$$\left|\frac{\varepsilon_t^{BB}(N)}{\varepsilon_t^{BB}(N) - \varepsilon_t^{PP}(N)}\gamma_t^T(N)\right| \leq \left|\frac{\varepsilon_t^{BB}(D)}{\varepsilon_t^{BB}(D) - \varepsilon_t^{PP}(D)}\gamma_t^T(D)\right|,$$

evaluating at the initial marginal binding equilibrium delivers the result for tradable consumption.

For nontradable prices, we have

$$\frac{\mathrm{d}\log p_t^N}{\mathrm{d}\log z_t^T} = \frac{1}{(\varepsilon_t^{PP})^{-1} - (\varepsilon_t^{BB})^{-1}}\gamma_t^T \varepsilon_{z_t^T}^{y_t^T}$$
$$\frac{\mathrm{d}\log p_t^N}{\mathrm{d}\log z_t^T} = \frac{\varepsilon_t^{BB}\varepsilon_t^{PP}}{\varepsilon_t^{BB} - \varepsilon_t^{PP}}\gamma_t^T \varepsilon_{z_t^T}^{y_t^T}.$$

Evaluating both sides in both networks' structure

$$\left|\frac{\mathrm{d}\log(p_t^N)^{\mathrm{NT}\leftarrow\mathrm{T}}}{\mathrm{d}\log z_t^T}\right| \leq \left|\frac{\mathrm{d}\log(p_t^N)^{\mathrm{Diagonal}}}{\mathrm{d}\log z_t^T}\right|$$
$$\left|\frac{\varepsilon_t^{BB}(N)\varepsilon_t^{PP}(N)}{\varepsilon_t^{BB}(N) - \varepsilon_t^{PP}(N)}\gamma_t^T(N)\right| \leq \left|\frac{\varepsilon_t^{BB}(D)\varepsilon_t^{PP}(D)}{\varepsilon_t^{BB}(D) - \varepsilon_t^{PP}(D)}\gamma_t^T(D)\right|,$$

which is a less stringent condition than that of tradable consumption since $\varepsilon_t^{PP}(N) \leq \varepsilon_t^{PP}(D)$. This concludes the proof.

B Numerical Analysis of Simple Model

In this section, we present a numerical analysis of our perfect foresight model. We calibrate the model's network structure to reflect key characteristics of emerging economies. Following the approach in Section 2.4, we begin by solving for the equilibrium of different economies that face a marginally binding borrowing constraint. We examine four cases. The first is our baseline economy, where the two productive sectors are interlinked. In the second case, we consider an economy in which the tradable and nontradable sectors operate independently, meaning they do not use each other's inputs in production. The third and fourth cases are combinations of the first two, where we consider an isolated tradable sector (only uses tradable input to produce), and another where the isolated sector is the nontradable one.

Our goal is to use the simple model to showcase the role that different productive structures play in the severity of Sudden Stops, which are events in which tradable productivity collapses.[9] For our baseline case, we set $\eta = 1.2035$, $\beta = R = 1/1.04$, $\omega = 0.3$ and $\kappa = 0.3$, following Bianchi (2011). Next, we set the share of tradable and nontradables inputs of the tradables to be $\alpha_T^T = 0.46$ and $\alpha_N^T = 0.1$, respectively. For the nontradable sector, we set $\alpha_T^N = 0.21$ and $\alpha_N^N = 0.18$. Lastly, we set $z^T = z^N = 1$ for simplicity.

The parameters for the counterfactual economies are chosen as follows. In the case where the two sectors operate independently, we set $\alpha_T^T = 0.56$ and $\alpha_N^N = 0.39$. These values ensure that the returns to scale in the corresponding production functions match those of our benchmark economy. However, this parameter choice does not necessarily guarantee that the equilibrium allocation of tradable consumption and the relative price of nontradables remain the same across models. To address this, we calibrate tradable and nontradable productivities to ensure that the initial consumption allocations and the relative price of nontradables are consistent across economies.[10] The rest of the parameters

[9] More specifically, a Sudden Stop is an event where a wealth-neutral time-0 tradable productivity shock occurs. Since the constraint is marginally binding before the shock is materialized, a decline in productivity will lead to a binding borrowing constraint, causing the well-known Fisherian deflationary mechanism characteristic of this family of models.

[10] By imposing the same tradable consumption and relative price of nontradables, we are left with a system of two equations (the BB and PP curves) and two unknowns (tradable and nontradable productivities).

are identical to those of the benchmark economy.

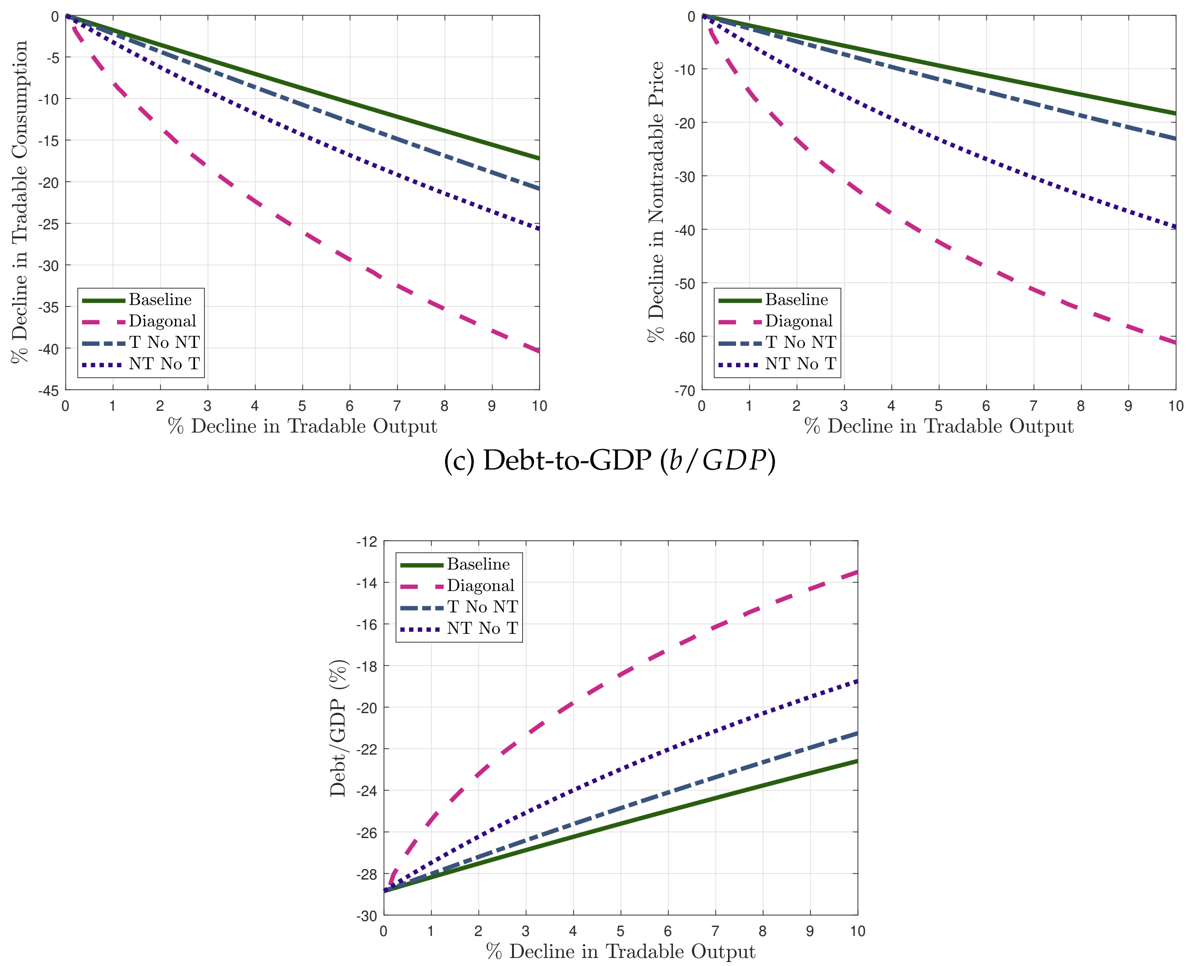
The parametrization of the remaining two cases follows a similar process. For the economy where the tradable sector does not use nontradable inputs in its production function ("T No NT"), we set $\alpha_T^T = 0.56$, $\alpha_N^T = 0$, $\alpha_T^N = 0.21$ and $\alpha_N^N = 0.18$. Lastly, we set $\alpha_T^T = 0.46$, $\alpha_N^T = 0.10$, $\alpha_T^N = 0$ and $\alpha_N^N = 0.39$ for the economy where the nontradable sector does not use tradable inputs in production ("NT No T"). In both cases, we adjust tradable and nontradable productivities to ensure that the initial consumption allocations and the relative price of nontradables remain consistent with those of the baseline economy.

Next, we simulate Sudden Stops by introducing shocks to tradable productivity of varying severity. Figure 9 presents our key results. Panel (a) shows the response of tradable consumption, while Panel (b) displays the relative price of nontradables—both measured relative to their initial steady-state allocations—as tradable productivity changes across the four economies described above. The final panel presents the debt-to-GDP ratio for each economy.

Figure 9: Sudden Stops Across Different Production Structures

(a) Tradable Consumption (c^T) (b) Nontradable Price (p^N)

(c) Debt-to-GDP (b/GDP)



The green solid line illustrates how the benchmark economy responds. The dashed pink line corresponds to the case where the two sectors are independent of each other ("Diagonal"). The dark blue dash-dotted line depicts the economy in which only the nontradable sector uses both tradable and nontradable inputs in production ("T No NT"), while the dotted purple line represents the case where only the tradable sector incorporates both inputs ("NT No T").

Consistent with the model's theoretical predictions, sectoral linkages serve as a hedging mechanism against adverse productivity shocks. Specifically, when tradable productivity declines by 10%, tradable consumption in the baseline economy falls by approximately 17%. In contrast, in the Diagonal economy—where sectors operate independently—the same productivity decline leads to a collapse in tradable consumption of nearly 40%, more than double the drop observed in the baseline case. Similar patterns emerge for the relative price of nontradables and the debt-to-GDP ratio, where the economy without sectoral linkages undergoes a much more severe deflationary spiral and deleveraging process compared to the benchmark case.

What happens when we consider the hybrid cases? We see that when one of the two sectors is not fully linked to the other (i.e. only uses inputs from one sector) we have amplification relative to the baseline economy (T-No-NT and NT-No-T cases) in the responses to tradable productivity declines, but these are substantially milder than the case with no linkages. It is worth noting that for this particular parametrization, the economy where the nontradable sector does not use the tradable good as input is the one that tends to exhibit worse hedging properties (still substantially better than the fully disconnected case).

These results show that the shape of the production structure of the economy can provide substantial attenuation (or amplification) during Sudden Stops. Additionally, these amplification or attenuation effects are nonlinear in the size of shocks: for mild shocks the gaps in responses tend to be relatively smaller than when shocks are large.

C Sector Definitions and Country Classifications

Table 6: Sector Names and Definitions

Sector Name	Definition
Agriculture, hunting, forestry	Commodity
Fishing and aquaculture	Commodity
Mining and quarrying, energy producing products	Commodity
Mining and quarrying, non-energy producing products	Commodity
Mining support service activities	Tradable
Food products, beverages and tobacco	Tradable
Textiles, textile products, leather and footwear	Tradable
Wood and products of wood and cork	Tradable
Paper products and printing	Tradable
Coke and refined petroleum products	Tradable
Chemical and chemical products	Tradable
Pharmaceuticals, medicinal chemical and botanical products	Tradable
Rubber and plastics products	Tradable
Other non-metallic mineral products	Tradable
Basic metals	Commodity
Fabricated metal products	Tradable
Computer, electronic and optical equipment	Tradable
Electrical equipment	Tradable
Machinery and equipment, nec	Tradable
Motor vehicles, trailers and semi-trailers	Tradable
Other transport equipment	Tradable
Manufacturing nec; repair and installation of machinery	Tradable
Electricity, gas, steam and air conditioning supply	Nontradable
Water supply; sewerage, waste management and remediation act.	Nontradable
Construction	Nontradable
Wholesale and retail trade; repair of motor vehicles	Tradable
Land transport and transport via pipelines	Tradable
Water transport	Tradable
Air transport	Tradable
Warehousing and support activities for transportation	Tradable
Postal and courier activities	Nontradable
Accommodation and food service activities	Nontradable
Publishing, audiovisual and broadcasting activities	Nontradable
Telecommunications	Nontradable
IT and other information services	Tradable
Financial and insurance activities	Nontradable
Real estate activities	Nontradable
Professional, scientific and technical activities	Nontradable
Administrative and support services	Nontradable
Public administration and defence; compulsory social security	Nontradable
Education	Nontradable
Human health and social work activities	Nontradable
Arts, entertainment and recreation	Nontradable
Other service activities	Nontradable

Table 7: Country Classification by Development Group

Emerging	**Advanced**
Argentina	Australia
Bulgaria	Austria
Brazil	Canada
Chile	Switzerland
China	Germany
Colombia	Denmark
Croatia	Spain
Hungary	Finland
Indonesia	France
Korea	United Kingdom
Morocco	Greece
Mexico	Iceland
Malaysia	Italy
Peru	Japan
Philippines	Netherlands
Poland	Norway
Russian Federation	New Zealand
Thailand	Portugal
Tunisia	Sweden
Türkiye	United States
South Africa	

D Planner's derivation

The planner's problem is as follows:

$$V(b,e) = \max_{\boldsymbol{x}} \quad U(c^C, c^X, c^N) + \beta E(V(b', e')),$$

subject to

$$c^C + m_C^C + m_C^X + m_C^N + qb' = y^C + p^X(c^C, c^X)\hat{c}^X(p^X(c^C, c^X)) + b, \quad (\lambda_1) \tag{63}$$

$$c^X + \hat{c}^X(p^X(c^C, c^X)) + m_X^C + m_X^X + m_X^N = y^X(m_N^X, m_X^X, m_C^X), \qquad (\lambda_2) \tag{64}$$

$$c^N + m_N^C + m_N^X + m_N^N = y^N(m_N^N, m_X^N, m_C^N), \qquad (\lambda_3) \tag{65}$$

$$qb' \geq -\kappa(\pi^C + \pi^X + \pi^N) = -\kappa\pi, \qquad (\mu) \tag{66}$$

where $\boldsymbol{x} = \{c^C, c^X, c^N, m_N^N, m_X^N, m_C^N, m_N^X, m_X^X, m_C^X, m_N^C, m_X^C, m_C^C, b'\}$ collects all planner's choice variables. We also implicitly let prices of nontradables (p^N) and tradable non-commodity (p^X) to be functions of consumptions relative to commodity consumption:

$$p^X = p^X(c^X, c^C),$$
$$p^N = p^N(c^N, c^C),$$

where their dependence comes from the relative demand equations in the competitive equilibrium problem.

First-order conditions for consumption

$$c^C : \qquad U_C - \lambda_1\left(1 - \frac{\partial p^X}{\partial c^C}\hat{c}^X - p^X\frac{\partial \hat{c}^X}{\partial p^X}\frac{\partial p^X}{\partial c^C}\right) - \lambda_2\frac{\partial \hat{c}^X}{\partial p^X}\frac{\partial p^X}{\partial c^C} + \kappa\mu\frac{\partial \pi}{\partial c^C} = 0 \tag{67}$$

$$c^X : \quad U_X + \lambda_1\left(\frac{\partial p^X}{\partial c^X}\hat{c}^X + p^X\frac{\partial \hat{c}^X}{\partial p^X}\frac{\partial p^X}{\partial c^X}\right) - \lambda_2\left(1 + \frac{\partial \hat{c}^X}{\partial p^X}\frac{\partial p^X}{\partial c^X}\right) + \kappa\mu\frac{\partial \pi}{\partial c^X} = 0 \tag{68}$$

$$c^N : \qquad U_N - \lambda_3 + \kappa\mu\frac{\partial \pi}{\partial c^N} = 0 \tag{69}$$

First-order conditions for inputs of nontradable

$$m_N^N: \quad -\lambda_3 + \lambda_3 \frac{\partial y^N}{\partial m_N^N} + \mu\kappa \frac{\partial \pi^N}{\partial m_N^N} = 0 \tag{70}$$

$$m_X^N: \quad -\lambda_2 + \lambda_3 \frac{\partial y^N}{\partial m_X^N} + \mu\kappa \frac{\partial \pi^N}{\partial m_X^N} = 0 \tag{71}$$

$$m_C^N: \quad -\lambda_1 + \lambda_3 \frac{\partial y^N}{\partial m_C^N} + \mu\kappa \frac{\partial \pi^N}{\partial m_C^N} = 0 \tag{72}$$

First-order conditions for inputs of tradable non-commodity

$$m_N^X: \quad -\lambda_3 + \lambda_2 \frac{\partial y^X}{\partial m_N^X} + \mu\kappa \frac{\partial \pi^X}{\partial m_N^X} = 0 \tag{73}$$

$$m_X^X: \quad -\lambda_2 + \lambda_2 \frac{\partial y^X}{\partial m_X^X} + \mu\kappa \frac{\partial \pi^X}{\partial m_X^X} = 0 \tag{74}$$

$$m_C^X: \quad -\lambda_1 + \lambda_2 \frac{\partial y^X}{\partial m_C^X} + \mu\kappa \frac{\partial \pi^X}{\partial m_C^X} = 0 \tag{75}$$

First-order conditions for inputs of tradable commodity

$$m_N^C: \quad -\lambda_3 + \lambda_1 \frac{\partial y^C}{\partial m_N^C} + \mu\kappa \frac{\partial \pi^C}{\partial m_N^C} = 0 \tag{76}$$

$$m_X^C: \quad -\lambda_2 + \lambda_1 \frac{\partial y^C}{\partial m_X^C} + \mu\kappa \frac{\partial \pi^C}{\partial m_X^C} = 0 \tag{77}$$

$$m_C^C: \quad -\lambda_1 + \lambda_1 \frac{\partial y^C}{\partial m_C^C} + \mu\kappa \frac{\partial \pi^C}{\partial m_C^C} = 0 \tag{78}$$

First order condition for bond b'

$$b': \quad \beta E \frac{\partial V'}{\partial b'} - \lambda_1 q + q\mu = 0 \tag{79}$$

Envelope

$$b: \quad \frac{\partial V}{\partial b} = \lambda_1$$

Combining the last two equations yields the Euler equation

$$\lambda_1 = \frac{\beta}{q} E\lambda_1' + \mu \tag{80}$$

Slackness condition

$$(qb' + \kappa\pi)\mu = 0$$

D.1 Solution for $(\lambda_1, \lambda_2, \lambda_3)$ using consumption conditions.

Define $\chi = \frac{\partial \hat{c}^X}{\partial p^X} \frac{p^X}{\hat{c}^X}$ as the export demand elasticity. Also recall $\pi = \pi^C + \pi^N + \pi^X$.

Rewrite equation (69) as

$$U_N = \lambda_3 - \kappa\mu \frac{\partial \pi}{\partial c^N} \implies \lambda_3 = U_N + \kappa\mu \frac{\partial \pi}{\partial c^N},$$

this gives us a solution for λ_3 as a function of μ and other endogenous variables.

Rewrite equation (67) as

$$\lambda_2 = \frac{U_C - \lambda_1 \left(1 - \frac{\partial p^X}{\partial c^C} \hat{c}^X (1+\chi)\right) + \kappa\mu \frac{\partial \pi}{\partial c^C}}{\frac{\partial \hat{c}^X}{\partial c^C}}$$

Rewrite equation (68) as

$$\lambda_2 = \frac{U_X + \lambda_1 \frac{\partial p^X}{\partial c^X} \hat{c}^X (1+\chi) + \kappa\mu \frac{\partial \pi}{\partial c^X}}{\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right)}$$

Combine the two expressions,

$$\frac{U_C - \lambda_1 \left(1 - \frac{\partial p^X}{\partial c^C} \hat{c}^X (1+\chi)\right) + \kappa\mu \frac{\partial \pi}{\partial c^C}}{\frac{\partial \hat{c}^X}{\partial c^C}} = \frac{U_X + \lambda_1 \frac{\partial p^X}{\partial c^X} \hat{c}^X (1+\chi) + \kappa\mu \frac{\partial \pi}{\partial c^X}}{\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right)}$$

$$\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) \left(U_C - \lambda_1 \left(1 - \frac{\partial p^X}{\partial c^C} \hat{c}^X (1+\chi)\right) + \kappa\mu \frac{\partial \pi}{\partial c^C}\right) = \frac{\partial \hat{c}^X}{\partial c^C} \left(U_X + \lambda_1 \frac{\partial p^X}{\partial c^X} \hat{c}^X (1+\chi) + \kappa\mu \frac{\partial \pi}{\partial c^X}\right)$$

$$\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) U_C - \frac{\partial \hat{c}^X}{\partial c^C} U_X + \kappa\mu \left(\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) \frac{\partial \pi}{\partial c^C} - \frac{\partial \hat{c}^X}{\partial c^C} \frac{\partial \pi}{\partial c^X}\right) = \lambda_1 \left(\frac{\partial \hat{c}^X}{\partial c^C} \frac{\partial p^X}{\partial c^X} \hat{c}^X (1+\chi) + \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) \left(1 - \frac{\partial p^X}{\partial c^C} \hat{c}^X (1+\chi)\right)\right),$$

which solving for λ_1 yields

$$\lambda_1 = \frac{\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) U_C - \frac{\partial \hat{c}^X}{\partial c^C} U_X + \kappa\mu \left(\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) \frac{\partial \pi}{\partial c^C} - \frac{\partial \hat{c}^X}{\partial c^C} \frac{\partial \pi}{\partial c^X} \right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C} \frac{\partial p^X}{\partial c^X} \hat{c}^X (1+\chi) + \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) \left(1 - \frac{\partial p^X}{\partial c^C} \hat{c}^X (1+\chi)\right)\right)} \tag{81}$$

To simplify the expressions let $b = 1 - \frac{\partial p^X}{\partial c^C} \hat{c}^X (1+\chi)$ and $a = \frac{\partial p^X}{\partial c^X} \hat{c}^X (1+\chi)$. With this, the above expression is

$$\lambda_1 = \frac{\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) U_C - \frac{\partial \hat{c}^X}{\partial c^C} U_X + \kappa\mu \left(\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) \frac{\partial \pi}{\partial c^C} - \frac{\partial \hat{c}^X}{\partial c^C} \frac{\partial \pi}{\partial c^X} \right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C} a + \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) b\right)} \tag{82}$$

Replacing into any of the other expression of λ_2, we get

$$\lambda_2 \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) = U_X + \lambda_1 a + \kappa\mu \frac{\partial \pi}{\partial c^X}$$

$$\lambda_2 \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) = U_X + \left[\frac{\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) U_C - \frac{\partial \hat{c}^X}{\partial c^C} U_X + \kappa\mu \left(\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) \frac{\partial \pi}{\partial c^C} - \frac{\partial \hat{c}^X}{\partial c^C} \frac{\partial \pi}{\partial c^X} \right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C} a + \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) b\right)} \right] a + \kappa\mu \frac{\partial \pi}{\partial c^X}$$

$$\lambda_2 \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) = U_X \left(1 - \frac{a \frac{\partial \hat{c}^X}{\partial c^C}}{\left(\frac{\partial \hat{c}^X}{\partial c^C} a + \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) b\right)}\right) + U_C \frac{\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) a}{\left(\frac{\partial \hat{c}^X}{\partial c^C} a + \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) b\right)} + \kappa\mu \left(\left[\frac{\left(\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) \frac{\partial \pi}{\partial c^C} - \frac{\partial \hat{c}^X}{\partial c^C} \frac{\partial \pi}{\partial c^X} \right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C} a + \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) b\right)} \right] a + \frac{\partial \pi}{\partial c^X} \right)$$

First term on the right-hand side

$$\left(1 - \frac{a \frac{\partial \hat{c}^X}{\partial c^C}}{\left(\frac{\partial \hat{c}^X}{\partial c^C} a + \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) b\right)}\right) = \frac{\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) b}{\left(\frac{\partial \hat{c}^X}{\partial c^C} a + \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) b\right)}$$

Third term

$$\begin{aligned}
\left[\frac{\left(\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) \frac{\partial \pi}{\partial c^C} - \frac{\partial \hat{c}^X}{\partial c^C} \frac{\partial \pi}{\partial c^X} \right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C} a + \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) b\right)} \right] a + \frac{\partial \pi}{\partial c^X} &= \frac{\left(\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) \frac{\partial \pi}{\partial c^C} - \frac{\partial \hat{c}^X}{\partial c^C} \frac{\partial \pi}{\partial c^X} \right) a + \left(\frac{\partial \hat{c}^X}{\partial c^C} a + \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) b\right) \frac{\partial \pi}{\partial c^X}}{\left(\frac{\partial \hat{c}^X}{\partial c^C} a + \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) b\right)} \\
&= \frac{\left(\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) \frac{\partial \pi}{\partial c^C} \right) a + \left(\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) b\right) \frac{\partial \pi}{\partial c^X}}{\left(\frac{\partial \hat{c}^X}{\partial c^C} a + \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) b\right)} \\
&= \frac{\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C} a + \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) b\right)} \left(\frac{\partial \pi}{\partial c^C} a + \frac{\partial \pi}{\partial c^X} b \right)
\end{aligned}$$

Using these expressions we get

$$\lambda_2\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)=U_X\frac{\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}+U_C\frac{\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)a}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}+\kappa\mu\frac{\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}\left(\frac{\partial \pi}{\partial c^C}a+\frac{\partial \pi}{\partial c^X}b\right)$$

$$\lambda_2=U_X\frac{b}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}+U_C\frac{a}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}+\kappa\mu\frac{\left(\frac{\partial \pi}{\partial c^C}a+\frac{\partial \pi}{\partial c^X}b\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}$$

$$\lambda_2=\frac{bU_X+aU_C+\kappa\mu\left(\frac{\partial \pi}{\partial c^C}a+\frac{\partial \pi}{\partial c^X}b\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}$$

Hence, the lagrange multipliers, as functions of μ, from the demand side are

$$\lambda_1^d=\frac{\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)U_C-\frac{\partial \hat{c}^X}{\partial c^C}U_X+\kappa\mu\left(\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)\frac{\partial \pi}{\partial c^C}-\frac{\partial \hat{c}^X}{\partial c^C}\frac{\partial \pi}{\partial c^X}\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}, \tag{83}$$

$$\lambda_2^d=\frac{bU_X+aU_C+\kappa\mu\left(\frac{\partial \pi}{\partial c^C}a+\frac{\partial \pi}{\partial c^X}b\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}, \tag{84}$$

$$\lambda_3^d=U_N+\kappa\mu\frac{\partial \pi}{\partial c^N}, \tag{85}$$

with

$$a=\frac{\partial p^X}{\partial c^X}\hat{c}^X\left(1+\chi\right),$$
$$b=1-\frac{\partial p^X}{\partial c^C}\hat{c}^X\left(1+\chi\right),$$
$$\chi=\frac{\partial \log \hat{c}^X}{\partial \log p^X}.$$

D.2 Supply side conditions solutions for $(\lambda_1, \lambda_2, \lambda_3)$

We can use the supply-side conditions to solve for the same multipliers. We first note that the own demand first-order conditions (m_i^i) given by equations (70), (74), and (78) require

$$\frac{\partial y^N}{\partial m_N^N}-1=0,$$
$$\frac{\partial y^X}{\partial m_X^X}-1=0,$$

$$\frac{\partial y^C}{\partial m_C^C} - 1 = 0.$$

Otherwise, multipliers would be ill-defined.

Now take equations (71) and (73) and rewrite them as

$$\lambda_2 = \lambda_3 \frac{\partial y^N}{\partial m_X^N} + \mu\kappa \frac{\partial \pi^N}{\partial m_X^N},$$
$$\lambda_2 = \frac{\lambda_3 - \mu\kappa \frac{\partial \pi^X}{\partial m_N^X}}{\frac{\partial y^X}{\partial m_N^X}}.$$

Combining these two equations

$$\lambda_3 \frac{\partial y^N}{\partial m_X^N} + \mu\kappa \frac{\partial \pi^N}{\partial m_X^N} = \frac{\lambda_3 - \mu\kappa \frac{\partial \pi^X}{\partial m_N^X}}{\frac{\partial y^X}{\partial m_N^X}},$$
$$\lambda_3 \frac{\partial y^N}{\partial m_X^N} \frac{\partial y^X}{\partial m_N^X} + \mu\kappa \frac{\partial \pi^N}{\partial m_X^N} \frac{\partial y^X}{\partial m_N^X} = \lambda_3 - \mu\kappa \frac{\partial \pi^X}{\partial m_N^X},$$
$$\lambda_3 = \mu\kappa \frac{\left(\frac{\partial \pi^N}{\partial m_X^N} \frac{\partial y^X}{\partial m_N^X} + \frac{\partial \pi^X}{\partial m_N^X} \right)}{\left(1 - \frac{\partial y^N}{\partial m_X^N} \frac{\partial y^X}{\partial m_N^X} \right)}.$$

Replacing this into the expression for λ_2,

$$\lambda_2 = \lambda_3 \frac{\partial y^N}{\partial m_X^N} + \mu\kappa \frac{\partial \pi^N}{\partial m_X^N},$$
$$\lambda_2 = \mu\kappa \left[\frac{\left(\frac{\partial \pi^N}{\partial m_X^N} \frac{\partial y^X}{\partial m_N^X} + \frac{\partial \pi^X}{\partial m_N^X} \right)}{\left(1 - \frac{\partial y^N}{\partial m_X^N} \frac{\partial y^X}{\partial m_N^X} \right)} \frac{\partial y^N}{\partial m_X^N} + \frac{\partial \pi^N}{\partial m_X^N} \right],$$
$$\lambda_2 = \mu\kappa \left[\frac{\left(\frac{\partial \pi^N}{\partial m_X^N} \frac{\partial y^X}{\partial m_N^X} + \frac{\partial \pi^X}{\partial m_N^X} \right) \frac{\partial y^N}{\partial m_X^N} + \frac{\partial \pi^N}{\partial m_X^N} \left(1 - \frac{\partial y^N}{\partial m_X^N} \frac{\partial y^X}{\partial m_N^X} \right)}{\left(1 - \frac{\partial y^N}{\partial m_X^N} \frac{\partial y^X}{\partial m_N^X} \right)} \right],$$
$$\lambda_2 = \mu\kappa \left[\frac{\frac{\partial \pi^X}{\partial m_N^X} \frac{\partial y^N}{\partial m_X^N} + \frac{\partial \pi^N}{\partial m_X^N}}{\left(1 - \frac{\partial y^N}{\partial m_X^N} \frac{\partial y^X}{\partial m_N^X} \right)} \right].$$

Now rewrite equations (72) and (76)

$$\lambda_1 = \lambda_3 \frac{\partial y^N}{\partial m_C^N} + \mu\kappa \frac{\partial \pi^N}{\partial m_C^N},$$
$$\lambda_1 = \frac{\lambda_3 - \mu\kappa \frac{\partial \pi^C}{\partial m_N^C}}{\frac{\partial y^C}{\partial m_N^C}}.$$

Equating both sides

$$\lambda_3 \frac{\partial y^N}{\partial m_C^N} + \mu\kappa \frac{\partial \pi^N}{\partial m_C^N} = \frac{\lambda_3 - \mu\kappa \frac{\partial \pi^C}{\partial m_N^C}}{\frac{\partial y^C}{\partial m_N^C}}$$
$$\lambda_3 \frac{\partial y^N}{\partial m_C^N} \frac{\partial y^C}{\partial m_N^C} + \mu\kappa \frac{\partial \pi^N}{\partial m_C^N} \frac{\partial y^C}{\partial m_N^C} = \lambda_3 - \mu\kappa \frac{\partial \pi^C}{\partial m_N^C}$$
$$\lambda_3 = \mu\kappa \frac{\left(\frac{\partial \pi^N}{\partial m_C^N} \frac{\partial y^C}{\partial m_N^C} + \frac{\partial \pi^C}{\partial m_N^C} \right)}{\left(1 - \frac{\partial y^N}{\partial m_C^N} \frac{\partial y^C}{\partial m_N^C} \right)},$$

Using this to solve for λ_1,

$$\lambda_1 = \lambda_3 \frac{\partial y^N}{\partial m_C^N} + \mu\kappa \frac{\partial \pi^N}{\partial m_C^N}$$
$$\lambda_1 = \mu\kappa \frac{\left(\frac{\partial \pi^N}{\partial m_C^N} \frac{\partial y^C}{\partial m_N^C} + \frac{\partial \pi^C}{\partial m_N^C} \right)}{\left(1 - \frac{\partial y^N}{\partial m_C^N} \frac{\partial y^C}{\partial m_N^C} \right)} \frac{\partial y^N}{\partial m_C^N} + \mu\kappa \frac{\partial \pi^N}{\partial m_C^N}$$
$$\lambda_1 = \mu\kappa \left[\frac{\left(\frac{\partial \pi^N}{\partial m_C^N} \frac{\partial y^C}{\partial m_N^C} + \frac{\partial \pi^C}{\partial m_N^C} \right) \frac{\partial y^N}{\partial m_C^N}}{\left(1 - \frac{\partial y^N}{\partial m_C^N} \frac{\partial y^C}{\partial m_N^C} \right)} + \frac{\partial \pi^N}{\partial m_C^N} \right]$$
$$\lambda_1 = \mu\kappa \left[\frac{\left(\frac{\partial \pi^N}{\partial m_C^N} \frac{\partial y^C}{\partial m_N^C} + \frac{\partial \pi^C}{\partial m_N^C} \right) \frac{\partial y^N}{\partial m_C^N} + \left(1 - \frac{\partial y^N}{\partial m_C^N} \frac{\partial y^C}{\partial m_N^C} \right) \frac{\partial \pi^N}{\partial m_C^N}}{\left(1 - \frac{\partial y^N}{\partial m_C^N} \frac{\partial y^C}{\partial m_N^C} \right)} \right]$$

$$\lambda_1 = \mu\kappa \left[\frac{\frac{\partial \pi^C}{\partial m_N^C} \frac{\partial y^N}{\partial m_C^N} + \frac{\partial \pi^N}{\partial m_C^N}}{\left(1 - \frac{\partial y^N}{\partial m_C^N} \frac{\partial y^C}{\partial m_N^C}\right)} \right]$$

Finally, rewrite equations (75) and (77),

$$\lambda_1 = \lambda_2 \frac{\partial y^X}{\partial m_C^X} + \mu\kappa \frac{\partial \pi^X}{\partial m_C^X},$$
$$\lambda_1 = \frac{\lambda_2 - \mu\kappa \frac{\partial \pi^C}{\partial m_X^C}}{\frac{\partial y^C}{\partial m_X^C}}.$$

Solving for λ_2

$$\lambda_2 \frac{\partial y^X}{\partial m_C^X} + \mu\kappa \frac{\partial \pi^X}{\partial m_C^X} = \frac{\lambda_2 - \mu\kappa \frac{\partial \pi^C}{\partial m_X^C}}{\frac{\partial y^C}{\partial m_X^C}},$$
$$\lambda_2 \frac{\partial y^X}{\partial m_C^X} \frac{\partial y^C}{\partial m_X^C} + \mu\kappa \frac{\partial \pi^X}{\partial m_C^X} \frac{\partial y^C}{\partial m_X^C} = \lambda_2 - \mu\kappa \frac{\partial \pi^C}{\partial m_X^C},$$
$$\lambda_2 = \mu\kappa \frac{\left(\frac{\partial \pi^X}{\partial m_C^X} \frac{\partial y^C}{\partial m_X^C} + \frac{\partial \pi^C}{\partial m_X^C} \right)}{\left(1 - \frac{\partial y^X}{\partial m_C^X} \frac{\partial y^C}{\partial m_X^C}\right)}.$$

Replacing into λ_1,

$$\lambda_1 = \lambda_2 \frac{\partial y^X}{\partial m_C^X} + \mu\kappa \frac{\partial \pi^X}{\partial m_C^X},$$
$$\lambda_1 = \mu\kappa \frac{\left(\frac{\partial \pi^X}{\partial m_C^X} \frac{\partial y^C}{\partial m_X^C} + \frac{\partial \pi^C}{\partial m_X^C} \right)}{\left(1 - \frac{\partial y^X}{\partial m_C^X} \frac{\partial y^C}{\partial m_X^C}\right)} \frac{\partial y^X}{\partial m_C^X} + \mu\kappa \frac{\partial \pi^X}{\partial m_C^X},$$
$$\lambda_1 = \mu\kappa \left[\frac{\left(\frac{\partial \pi^X}{\partial m_C^X} \frac{\partial y^C}{\partial m_X^C} + \frac{\partial \pi^C}{\partial m_X^C} \right)}{\left(1 - \frac{\partial y^X}{\partial m_C^X} \frac{\partial y^C}{\partial m_X^C}\right)} \frac{\partial y^X}{\partial m_C^X} + \frac{\partial \pi^X}{\partial m_C^X} \right],$$

$$\lambda_1 = \mu\kappa \left[\frac{\left(\frac{\partial \pi^X}{\partial m_C^X} \frac{\partial y^C}{\partial m_X^C} + \frac{\partial \pi^C}{\partial m_X^C} \right) \frac{\partial y^X}{\partial m_C^X} + \left(1 - \frac{\partial y^X}{\partial m_C^X} \frac{\partial y^C}{\partial m_X^C} \right) \frac{\partial \pi^X}{\partial m_C^X}}{\left(1 - \frac{\partial y^X}{\partial m_C^X} \frac{\partial y^C}{\partial m_X^C} \right)} \right],$$

$$\lambda_1 = \mu\kappa \left[\frac{\frac{\partial \pi^C}{\partial m_X^C} \frac{\partial y^X}{\partial m_C^X} + \frac{\partial \pi^X}{\partial m_C^X}}{\left(1 - \frac{\partial y^X}{\partial m_C^X} \frac{\partial y^C}{\partial m_X^C} \right)} \right].$$

Collecting all solutions from the supply side

$$\lambda_3 = \mu\kappa \frac{\left(\frac{\partial \pi^N}{\partial m_X^N} \frac{\partial y^X}{\partial m_N^X} + \frac{\partial \pi^X}{\partial m_N^X} \right)}{\left(1 - \frac{\partial y^N}{\partial m_X^N} \frac{\partial y^X}{\partial m_N^X} \right)},$$

$$\lambda_2 = \mu\kappa \left[\frac{\frac{\partial \pi^X}{\partial m_N^X} \frac{\partial y^N}{\partial m_X^N} + \frac{\partial \pi^N}{\partial m_X^N}}{\left(1 - \frac{\partial y^N}{\partial m_X^N} \frac{\partial y^X}{\partial m_N^X} \right)} \right],$$

$$\lambda_3 = \mu\kappa \frac{\left(\frac{\partial \pi^N}{\partial m_C^N} \frac{\partial y^C}{\partial m_N^C} + \frac{\partial \pi^C}{\partial m_N^C} \right)}{\left(1 - \frac{\partial y^N}{\partial m_C^N} \frac{\partial y^C}{\partial m_N^C} \right)},$$

$$\lambda_1 = \mu\kappa \left[\frac{\frac{\partial \pi^C}{\partial m_N^C} \frac{\partial y^N}{\partial m_C^N} + \frac{\partial \pi^N}{\partial m_C^N}}{\left(1 - \frac{\partial y^N}{\partial m_C^N} \frac{\partial y^C}{\partial m_N^C} \right)} \right],$$

$$\lambda_2 = \mu\kappa \frac{\left(\frac{\partial \pi^X}{\partial m_C^X} \frac{\partial y^C}{\partial m_X^C} + \frac{\partial \pi^C}{\partial m_X^C} \right)}{\left(1 - \frac{\partial y^X}{\partial m_C^X} \frac{\partial y^C}{\partial m_X^C} \right)},$$

$$\lambda_1 = \mu\kappa \left[\frac{\frac{\partial \pi^C}{\partial m_X^C} \frac{\partial y^X}{\partial m_C^X} + \frac{\partial \pi^X}{\partial m_C^X}}{\left(1 - \frac{\partial y^X}{\partial m_C^X} \frac{\partial y^C}{\partial m_X^C} \right)} \right].$$

These equations imply the following equalities

$$\lambda_1 = \lambda_1$$

$$\mu\kappa\left[\frac{\frac{\partial\pi^C}{\partial m_N^C}\frac{\partial y^N}{\partial m_C^N}+\frac{\partial\pi^N}{\partial m_C^N}}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}\right]=\mu\kappa\left[\frac{\frac{\partial\pi^C}{\partial m_X^C}\frac{\partial y^X}{\partial m_C^X}+\frac{\partial\pi^X}{\partial m_C^X}}{\left(1-\frac{\partial y^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}\right)}\right]$$
$$\left[\frac{\frac{\partial\pi^C}{\partial m_N^C}\frac{\partial y^N}{\partial m_C^N}+\frac{\partial\pi^N}{\partial m_C^N}}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}\right]=\left[\frac{\frac{\partial\pi^C}{\partial m_X^C}\frac{\partial y^X}{\partial m_C^X}+\frac{\partial\pi^X}{\partial m_C^X}}{\left(1-\frac{\partial y^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}\right)}\right]$$

$$\lambda_2=\lambda_2$$
$$\mu\kappa\left[\frac{\frac{\partial\pi^X}{\partial m_N^X}\frac{\partial y^N}{\partial m_X^N}+\frac{\partial\pi^N}{\partial m_X^N}}{\left(1-\frac{\partial y^N}{\partial m_X^N}\frac{\partial y^X}{\partial m_N^X}\right)}\right]=\mu\kappa\frac{\left(\frac{\partial\pi^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}+\frac{\partial\pi^C}{\partial m_X^C}\right)}{\left(1-\frac{\partial y^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}\right)}$$
$$\left[\frac{\frac{\partial\pi^X}{\partial m_N^X}\frac{\partial y^N}{\partial m_X^N}+\frac{\partial\pi^N}{\partial m_X^N}}{\left(1-\frac{\partial y^N}{\partial m_X^N}\frac{\partial y^X}{\partial m_N^X}\right)}\right]=\frac{\left(\frac{\partial\pi^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}+\frac{\partial\pi^C}{\partial m_X^C}\right)}{\left(1-\frac{\partial y^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}\right)}$$

$$\lambda_3=\lambda_3$$
$$\mu\kappa\frac{\left(\frac{\partial\pi^N}{\partial m_X^N}\frac{\partial y^X}{\partial m_N^X}+\frac{\partial\pi^X}{\partial m_N^X}\right)}{\left(1-\frac{\partial y^N}{\partial m_X^N}\frac{\partial y^X}{\partial m_N^X}\right)}=\mu\kappa\frac{\left(\frac{\partial\pi^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}+\frac{\partial\pi^C}{\partial m_N^C}\right)}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}$$
$$\frac{\left(\frac{\partial\pi^N}{\partial m_X^N}\frac{\partial y^X}{\partial m_N^X}+\frac{\partial\pi^X}{\partial m_N^X}\right)}{\left(1-\frac{\partial y^N}{\partial m_X^N}\frac{\partial y^X}{\partial m_N^X}\right)}=\frac{\left(\frac{\partial\pi^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}+\frac{\partial\pi^C}{\partial m_N^C}\right)}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}$$

Hence, the supply side conditions imply

$$\lambda_1^s=\mu\kappa\left[\frac{\frac{\partial\pi^C}{\partial m_N^C}\frac{\partial y^N}{\partial m_C^N}+\frac{\partial\pi^N}{\partial m_C^N}}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}\right], \tag{86}$$

$$\lambda_2^s = \mu\kappa \frac{\left(\frac{\partial \pi^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C} + \frac{\partial \pi^C}{\partial m_X^C}\right)}{\left(1 - \frac{\partial y^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}\right)}, \tag{87}$$

$$\lambda_3^s = \mu\kappa \frac{\left(\frac{\partial \pi^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C} + \frac{\partial \pi^C}{\partial m_N^C}\right)}{\left(1 - \frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}, \tag{88}$$

$$\frac{\left(\frac{\partial \pi^C}{\partial m_N^C}\frac{\partial y^N}{\partial m_C^N} + \frac{\partial \pi^N}{\partial m_C^N}\right)}{\left(1 - \frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)} = \frac{\left(\frac{\partial \pi^C}{\partial m_X^C}\frac{\partial y^X}{\partial m_C^X} + \frac{\partial \pi^X}{\partial m_C^X}\right)}{\left(1 - \frac{\partial y^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}\right)}, \tag{89}$$

$$\frac{\left(\frac{\partial \pi^X}{\partial m_N^X}\frac{\partial y^N}{\partial m_X^N} + \frac{\partial \pi^N}{\partial m_X^N}\right)}{\left(1 - \frac{\partial y^N}{\partial m_X^N}\frac{\partial y^X}{\partial m_N^X}\right)} = \frac{\left(\frac{\partial \pi^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C} + \frac{\partial \pi^C}{\partial m_X^C}\right)}{\left(1 - \frac{\partial y^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}\right)}, \tag{90}$$

$$\frac{\left(\frac{\partial \pi^N}{\partial m_X^N}\frac{\partial y^X}{\partial m_N^X} + \frac{\partial \pi^X}{\partial m_N^X}\right)}{\left(1 - \frac{\partial y^N}{\partial m_X^N}\frac{\partial y^X}{\partial m_N^X}\right)} = \frac{\left(\frac{\partial \pi^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C} + \frac{\partial \pi^C}{\partial m_N^C}\right)}{\left(1 - \frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}, \tag{91}$$

$$\frac{\partial y^N}{\partial m_N^N} - 1 = 0, \tag{92}$$

$$\frac{\partial y^X}{\partial m_X^X} - 1 = 0, \tag{93}$$

$$\frac{\partial y^C}{\partial m_C^C} - 1 = 0. \tag{94}$$

D.3 Combining demand and supply side conditions

Take the solutions for $(\lambda_1, \lambda_2, \lambda_3)$ under the demand and supply side and solve for μ.

Start with λ_1:

$$\lambda_1^d = \lambda_1^s$$

$$\frac{\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) U_C - \frac{\partial \hat{c}^X}{\partial c^C} U_X + \kappa\mu \left(\left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right)\frac{\partial \pi}{\partial c^C} - \frac{\partial \hat{c}^X}{\partial c^C}\frac{\partial \pi}{\partial c^X}\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C} a + \left(1 + \frac{\partial \hat{c}^X}{\partial c^X}\right) b\right)} = \mu\kappa \left[\frac{\frac{\partial \pi^C}{\partial m_N^C}\frac{\partial y^N}{\partial m_C^N} + \frac{\partial \pi^N}{\partial m_C^N}}{\left(1 - \frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}\right],$$

$$\frac{\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)U_C-\frac{\partial \hat{c}^X}{\partial c^C}U_X}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}=\mu\kappa\left[\frac{\frac{\partial \pi^C}{\partial m^C_N}\frac{\partial y^N}{\partial m^N_C}+\frac{\partial \pi^N}{\partial m^N_C}}{\left(1-\frac{\partial y^N}{\partial m^N_C}\frac{\partial y^C}{\partial m^C_N}\right)}-\frac{\left(\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)\frac{\partial \pi}{\partial c^C}-\frac{\partial \hat{c}^X}{\partial c^C}\frac{\partial}{\partial}\right.}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right.}\right.$$

which implies

$$\mu=\frac{\frac{\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)U_C-\frac{\partial \hat{c}^X}{\partial c^C}U_X}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}}{\kappa\left[\frac{\frac{\partial \pi^C}{\partial m^C_N}\frac{\partial y^N}{\partial m^N_C}+\frac{\partial \pi^N}{\partial m^N_C}}{\left(1-\frac{\partial y^N}{\partial m^N_C}\frac{\partial y^C}{\partial m^C_N}\right)}-\frac{\left(\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)\frac{\partial \pi}{\partial c^C}-\frac{\partial \hat{c}^X}{\partial c^C}\frac{\partial \pi}{\partial c^X}\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}\right]}$$

Now, consider λ_2

$$\lambda_2^d=\lambda_2^s$$

$$\frac{bU_X+aU_C+\kappa\mu\left(\frac{\partial \pi}{\partial c^C}a+\frac{\partial \pi}{\partial c^X}b\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}=\mu\kappa\frac{\left(\frac{\partial \pi^X}{\partial m^X_C}\frac{\partial y^C}{\partial m^C_X}+\frac{\partial \pi^C}{\partial m^C_X}\right)}{\left(1-\frac{\partial y^X}{\partial m^X_C}\frac{\partial y^C}{\partial m^C_X}\right)}$$

$$\frac{bU_X+aU_C}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}=\mu\kappa\left[\frac{\left(\frac{\partial \pi^X}{\partial m^X_C}\frac{\partial y^C}{\partial m^C_X}+\frac{\partial \pi^C}{\partial m^C_X}\right)}{\left(1-\frac{\partial y^X}{\partial m^X_C}\frac{\partial y^C}{\partial m^C_X}\right)}-\frac{\left(\frac{\partial \pi}{\partial c^C}a+\frac{\partial \pi}{\partial c^X}b\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}\right],$$

which solving for μ yields

$$\mu=\frac{\frac{bU_X+aU_C}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}}{\kappa\left[\frac{\left(\frac{\partial \pi^X}{\partial m^X_C}\frac{\partial y^C}{\partial m^C_X}+\frac{\partial \pi^C}{\partial m^C_X}\right)}{\left(1-\frac{\partial y^X}{\partial m^X_C}\frac{\partial y^C}{\partial m^C_X}\right)}-\frac{\left(\frac{\partial \pi}{\partial c^C}a+\frac{\partial \pi}{\partial c^X}b\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}\right]}.$$

Finally, consider λ_3

$$\lambda_3^d=\lambda_3^s$$

$$U_N + \kappa\mu\frac{\partial\pi}{\partial c^N} = \mu\kappa\frac{\left(\frac{\partial\pi^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C} + \frac{\partial\pi^C}{\partial m_N^C}\right)}{\left(1 - \frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}$$

$$U_N = \mu\kappa\left(\frac{\left(\frac{\partial\pi^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C} + \frac{\partial\pi^C}{\partial m_N^C}\right)}{\left(1 - \frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)} - \frac{\partial\pi}{\partial c^N}\right),$$

which yields

$$\mu = \frac{U_N}{\kappa\left(\frac{\left(\frac{\partial\pi^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C} + \frac{\partial\pi^C}{\partial m_N^C}\right)}{\left(1 - \frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)} - \frac{\partial\pi}{\partial c^N}\right)}.$$

Taking stock, the three solutions for μ are

$$\mu = \frac{\frac{\left(1+\frac{\partial\hat{c}^X}{\partial c^X}\right)U_C - \frac{\partial\hat{c}^X}{\partial c^C}U_X}{\left(\frac{\partial\hat{c}^X}{\partial c^C}a + \left(1+\frac{\partial\hat{c}^X}{\partial c^X}\right)b\right)}}{\kappa\left[\frac{\frac{\partial\pi^C}{\partial m_N^C}\frac{\partial y^N}{\partial m_C^N} + \frac{\partial\pi^N}{\partial m_C^N}}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)} - \frac{\left(\left(1+\frac{\partial\hat{c}^X}{\partial c^X}\right)\frac{\partial\pi}{\partial c^C} - \frac{\partial\hat{c}^X}{\partial c^C}\frac{\partial\pi}{\partial c^X}\right)}{\left(\frac{\partial\hat{c}^X}{\partial c^C}a + \left(1+\frac{\partial\hat{c}^X}{\partial c^X}\right)b\right)}\right]}$$

$$\mu = \frac{\frac{bU_X + aU_C}{\left(\frac{\partial\hat{c}^X}{\partial c^C}a + \left(1+\frac{\partial\hat{c}^X}{\partial c^X}\right)b\right)}}{\kappa\left[\frac{\left(\frac{\partial\pi^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C} + \frac{\partial\pi^C}{\partial m_X^C}\right)}{\left(1-\frac{\partial y^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}\right)} - \frac{\left(\frac{\partial\pi}{\partial c^C}a + \frac{\partial\pi}{\partial c^X}b\right)}{\left(\frac{\partial\hat{c}^X}{\partial c^C}a + \left(1+\frac{\partial\hat{c}^X}{\partial c^X}\right)b\right)}\right]}$$

$$\mu = \frac{U_N}{\kappa\left(\frac{\left(\frac{\partial\pi^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C} + \frac{\partial\pi^C}{\partial m_N^C}\right)}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)} - \frac{\partial\pi}{\partial c^N}\right)}$$

As before, we can keep one of these and consider equalities, this requires

$$\frac{U_N}{\left(\frac{\left(\frac{\partial\pi^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C} + \frac{\partial\pi^C}{\partial m_N^C}\right)}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)} - \frac{\partial\pi}{\partial c^N}\right)} = \frac{\frac{bU_X + aU_C}{\left(\frac{\partial\hat{c}^X}{\partial c^C}a + \left(1+\frac{\partial\hat{c}^X}{\partial c^X}\right)b\right)}}{\left[\frac{\left(\frac{\partial\pi^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C} + \frac{\partial\pi^C}{\partial m_X^C}\right)}{\left(1-\frac{\partial y^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}\right)} - \frac{\left(\frac{\partial\pi}{\partial c^C}a + \frac{\partial\pi}{\partial c^X}b\right)}{\left(\frac{\partial\hat{c}^X}{\partial c^C}a + \left(1+\frac{\partial\hat{c}^X}{\partial c^X}\right)b\right)}\right]}$$

$$\frac{U_N}{\left(\frac{\left(\frac{\partial \pi^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}+\frac{\partial \pi^C}{\partial m_N^C}\right)}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}-\frac{\partial \pi}{\partial c^N}\right)}=\frac{\frac{\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)U_C-\frac{\partial \hat{c}^X}{\partial c^C}U_X}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}}{\left[\frac{\frac{\partial \pi^C}{\partial m_N^C}\frac{\partial y^N}{\partial m_C^N}+\frac{\partial \pi^N}{\partial m_C^N}}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}-\frac{\left(\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)\frac{\partial \pi}{\partial c^C}-\frac{\partial \hat{c}^X}{\partial c^C}\frac{\partial \pi}{\partial c^X}\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}\right]}.$$

Hence,

$$\mu=\frac{U_N}{\kappa\left(\frac{\left(\frac{\partial \pi^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}+\frac{\partial \pi^C}{\partial m_N^C}\right)}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}-\frac{\partial \pi}{\partial c^N}\right)} \tag{95}$$

$$\frac{U_N}{\left(\frac{\left(\frac{\partial \pi^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}+\frac{\partial \pi^C}{\partial m_N^C}\right)}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}-\frac{\partial \pi}{\partial c^N}\right)}=\frac{\frac{bU_X+aU_C}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}}{\left[\frac{\left(\frac{\partial \pi^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}+\frac{\partial \pi^C}{\partial m_X^C}\right)}{\left(1-\frac{\partial y^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}\right)}-\frac{\left(\frac{\partial \pi}{\partial c^C}a+\frac{\partial \pi}{\partial c^X}b\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}\right]} \tag{96}$$

$$\frac{U_N}{\left(\frac{\left(\frac{\partial \pi^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}+\frac{\partial \pi^C}{\partial m_N^C}\right)}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}-\frac{\partial \pi}{\partial c^N}\right)}=\frac{\frac{\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)U_C-\frac{\partial \hat{c}^X}{\partial c^C}U_X}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}}{\left[\frac{\frac{\partial \pi^C}{\partial m_N^C}\frac{\partial y^N}{\partial m_C^N}+\frac{\partial \pi^N}{\partial m_C^N}}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}-\frac{\left(\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)\frac{\partial \pi}{\partial c^C}-\frac{\partial \hat{c}^X}{\partial c^C}\frac{\partial \pi}{\partial c^X}\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}\right]}. \tag{97}$$

D.4 Taking Stock

The system is then

$$\lambda_1=\frac{\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)U_C-\frac{\partial \hat{c}^X}{\partial c^C}U_X+\kappa\mu\left(\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)\frac{\partial \pi}{\partial c^C}-\frac{\partial \hat{c}^X}{\partial c^C}\frac{\partial \pi}{\partial c^X}\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}, \tag{98}$$

$$\lambda_2=\frac{bU_X+aU_C+\kappa\mu\left(\frac{\partial \pi}{\partial c^C}a+\frac{\partial \pi}{\partial c^X}b\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}, \tag{99}$$

$$\lambda_3=U_N+\kappa\mu\frac{\partial \pi}{\partial c^N}, \tag{100}$$

$$\frac{\left(\frac{\partial \pi^C}{\partial m_N^C}\frac{\partial y^N}{\partial m_C^N}+\frac{\partial \pi^N}{\partial m_C^N}\right)}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}=\frac{\left(\frac{\partial \pi^C}{\partial m_X^C}\frac{\partial y^X}{\partial m_C^X}+\frac{\partial \pi^X}{\partial m_C^X}\right)}{\left(1-\frac{\partial y^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}\right)}, \tag{101}$$

$$\frac{\left(\frac{\partial \pi^X}{\partial m_N^X}\frac{\partial y^N}{\partial m_X^N}+\frac{\partial \pi^N}{\partial m_X^N}\right)}{\left(1-\frac{\partial y^N}{\partial m_X^N}\frac{\partial y^X}{\partial m_N^X}\right)}=\frac{\left(\frac{\partial \pi^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}+\frac{\partial \pi^C}{\partial m_X^C}\right)}{\left(1-\frac{\partial y^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}\right)}, \tag{102}$$

$$\frac{\left(\frac{\partial \pi^N}{\partial m_X^N}\frac{\partial y^X}{\partial m_N^X}+\frac{\partial \pi^X}{\partial m_N^X}\right)}{\left(1-\frac{\partial y^N}{\partial m_X^N}\frac{\partial y^X}{\partial m_N^X}\right)}=\frac{\left(\frac{\partial \pi^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}+\frac{\partial \pi^C}{\partial m_N^C}\right)}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}, \tag{103}$$

$$\frac{\partial y^N}{\partial m_N^N}-1=0, \tag{104}$$

$$\frac{\partial y^X}{\partial m_X^X}-1=0, \tag{105}$$

$$\frac{\partial y^C}{\partial m_C^C}-1=0, \tag{106}$$

$$\mu=\frac{U_N}{\kappa\left(\frac{\left(\frac{\partial \pi^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}+\frac{\partial \pi^C}{\partial m_N^C}\right)}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}-\frac{\partial \pi}{\partial c^N}\right)} \tag{107}$$

$$\frac{U_N}{\left(\frac{\left(\frac{\partial \pi^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}+\frac{\partial \pi^C}{\partial m_N^C}\right)}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}-\frac{\partial \pi}{\partial c^N}\right)}=\frac{\frac{bU_X+aU_C}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}}{\left[\frac{\left(\frac{\partial \pi^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}+\frac{\partial \pi^C}{\partial m_X^C}\right)}{\left(1-\frac{\partial y^X}{\partial m_C^X}\frac{\partial y^C}{\partial m_X^C}\right)}-\frac{\left(\frac{\partial \pi}{\partial c^C}a+\frac{\partial \pi}{\partial c^X}b\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}\right]} \tag{108}$$

$$\frac{U_N}{\left(\frac{\left(\frac{\partial \pi^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}+\frac{\partial \pi^C}{\partial m_N^C}\right)}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}-\frac{\partial \pi}{\partial c^N}\right)}=\frac{\frac{\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)U_C-\frac{\partial \hat{c}^X}{\partial c^C}U_X}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}}{\left[\frac{\frac{\partial \pi^C}{\partial m_N^C}\frac{\partial y^N}{\partial m_C^N}+\frac{\partial \pi^N}{\partial m_C^N}}{\left(1-\frac{\partial y^N}{\partial m_C^N}\frac{\partial y^C}{\partial m_N^C}\right)}-\frac{\left(\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)\frac{\partial \pi}{\partial c^C}-\frac{\partial \hat{c}^X}{\partial c^C}\frac{\partial \pi}{\partial c^X}\right)}{\left(\frac{\partial \hat{c}^X}{\partial c^C}a+\left(1+\frac{\partial \hat{c}^X}{\partial c^X}\right)b\right)}\right]}, \tag{109}$$

$$\lambda_1=\frac{\beta}{q}E\lambda_1'+\mu \tag{110}$$

$$c^C+m_C^C+m_C^X+m_C^N+qb'=y^C+p^X(c^C,c^X)\hat{c}^X(p^X(c^C,c^X))+b \tag{111}$$

$$c^X+\hat{c}^X(p^X(c^C,c^X))+m_X^C+m_X^X+m_X^N=y^X \tag{112}$$

$$c^N+m_N^C+m_N^X+m_N^N=y^N \tag{113}$$

$$(qb' + \kappa(\pi^C + \pi^X + \pi^N))\mu = 0 \tag{114}$$

The expression for the profit derivatives satisfy

$$\frac{\partial \pi_t^i}{\partial m_{jt}^i} = p_t^i \frac{\partial y_t^i}{\partial m_{jt}^i} - p_t^j \qquad \forall i, j \in \{C, X, N\}$$

Using the pricing schedules, we also have

$$p_t^i = \frac{U_i(t)}{U_C(t)}.$$

Hence, profit derivatives with respect to intermediate inputs are

$$\frac{\partial \pi_t^i}{\partial m_{jt}^i} = \frac{U_i(t)}{U_C(t)} \frac{\partial y_t^i}{\partial m_{jt}^i} - \frac{U_j(t)}{U_C(t)} \qquad \forall i, j \in \{C, X, N\}.$$

Finally, we seek expressions for how aggregate profits respond to changes in consumption. These are

$$\frac{\partial \pi}{\partial c^N} = \frac{\partial p^N}{\partial c^N} \left(y^N - \sum_{j \in \{C,X,N\}} m_N^j \right) = \frac{\partial p^N}{\partial c^N} c^N,$$

$$\frac{\partial \pi}{\partial c^X} = \frac{\partial p^X}{\partial c^X} \left(y^X - \sum_{j \in \{C,X,N\}} m_X^j \right) = \frac{\partial p^X}{\partial c^X} (c^X + \hat{c}^X),$$

$$\frac{\partial \pi}{\partial c^C} = \frac{\partial p^N}{\partial c^C} \left(y^N - \sum_{j \in \{C,X,N\}} m_N^j \right) + \frac{\partial p^X}{\partial c^C} \left(y^X - \sum_{j \in \{C,X,N\}} m_X^j \right) = \frac{\partial p^N}{\partial c^C} c^N + \frac{\partial p^X}{\partial c^C} \left(c^X + \hat{c}^X \right).$$