NEW MEXICO
Public Education Department

New Mexico Instructional Scope
Algebra 2 See Structure in Expressions Guide

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Interpret the structure of expressions.
  - HSA.SSE.A.1
  - HSA.SSE.A.2
- Write expressions in equivalent forms to solve problems
  - HSA.SSE.B.4

Grade	CCSS Domain	CCSS Cluster
A2	See Structure in Expressions	Interpret the structure of expressions

Cluster Standard: HSA.SSE.A.1

Standard	Standards for Mathematical Practice
Interpret expressions that represent a quantity in terms of its context. • HSA.SSE.A.1.A: Interpret parts of an expression, such as terms, factors, and coefficients. • HSA.SSE.A.1.B: Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	• **SMP 4:** Model with mathematics. • **SMP 7:** Look for and make use of structure.
Clarification Statement	Students Who Demonstrate Understanding Can...
• Algebra 1 emphasized linear, exponential and quadratic expressions. The work of Algebra 2 is to generalize that work to polynomial and rational expressions by examining real-world situations that can be modeled by algebraic expressions and explaining how parts of the expression describe different aspects of the situation.	• Identify how parts of an expression relate to a real-world situation. • Interpret how parts of an expression relate to a real-world situation. • Interpret algebraic expressions that describe real-world scenarios, including parts within an expression and use grouping strategies to interpret expressions.
DOK	Blooms
1-2	Remember, Understand, Analyze

Grade	CCSS Domain	CCSS Cluster
A2	See Structure in Expressions	Interpret the structure of expressions

Cluster Standard: HSA.SSE.A.2

Standard	Standards for Mathematical Practice
Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Seeing structure in expressions entails a dynamic view of an algebraic expression, in which potential rearrangements and manipulations are ever present. An important skill for college readiness is the ability to try possible manipulations mentally without having to carry them out, and to see which ones might be fruitful and which not. Emphasize that there are many algebraic properties that can be used to write equivalent forms of an expression. Complex, linear and non-linear equations need to be addressed.	• Identify patterns of factoring. • Factor a polynomial or rational expression. • Classify expressions by method of factoring. • Apply different algebraic properties to an expression to produce an equivalent form.
DOK	**Blooms**
1-2	Remember, Understand, Apply

Common Misconceptions

- Students may confuse equations with expressions. The focus in this cluster is on analyzing expressions.
- Students may confuse the order of operations when they simplify an expression.
- Students may not have a conceptual basis for patterns and therefore struggle to recognize and apply them to new situations.

Grade	CCSS Domain	CCSS Cluster
A2	See Structure in Expressions	Write expressions in equivalent forms to solve problems

Cluster Standard: HSA.SSE.B.4

Standard	Standards for Mathematical Practice
Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*	• **SMP 4:** Model with mathematics.
Clarification Statement	Students Who Demonstrate Understanding Can...
• Introduce geometric sequences. Students need to identify the common ratio, nth term, and previous term. Students calculate the nth term substituting the common ratio and the first term. Students apply the formula for the sum of the finite geometric series by solving for an isolated variable or for a coefficient. Students model real-world applications and should explain in contextual situations.	• Find the sums of finite geometric series; find the common ratio. • Use an infinite series as a model; apply a given formula for the sum of a finite geometric series by solving for the isolated variable. • Apply a given formula for the sum of a finite geometric series to solve for a coefficient. • Apply a given formula for the sum of a finite geometric series to justify real world scenarios.
DOK	Blooms
1-2	Apply

Common Misconceptions

- Geometric series are obtained through a series of additions or subtraction. When applying the sum formula for geometric series, students may subtract the values in the numerator before applying the exponent. Remind them that exponents are evaluated before addition and subtraction in the order of operation.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Seeing Structure in Expressions**	Strand: **Interpret the structure of expressions**

Suggested Student Discourse Questions

- Using a polynomial equation identify the terms, factors and coefficients.
- What strategy would you use to interpret an expression?
- How could you rewrite an expression?
- Explain a different way to factor an equation and share how your strategy was different.
- Find multiple ways to represent the expression.
- When grocery shopping, how could you find the price of 3 items by using a polynomial equation according to the quantity you want to buy?
- Identify at least three careers that their job would require them to make calculations using a polynomial expression. Explain.

Domain: **Seeing Structure in Expressions**	Strand: **Write expressions in equivalent forms to solve problems**

Suggested Student Discourse Questions

- Using a polynomial equation identify the terms, factors and coefficients.
- What strategy would you use to interpret an expression?
- How could you rewrite an expression?
- Explain a different way to factor an equation and share how your strategy was different.
- Find multiple ways to represent the expression.
- Can you find some other careers that would use polynomial expression?

ASSESSMENT GUIDE

- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.

Grade	*CCSS Domain*	*CCSS Strand*
A2	See Structure in Expressions	Interpret the structure of expressions

Sample Task #1 (Constructed Response)

Source: SAT

If $\frac{\sqrt{x^5}}{\sqrt[3]{x^4}} = x^{\frac{a}{b}}$ for all positive values of x, what is the value of $\frac{a}{b}$?

Grade	CCSS Domain	CCSS Strand
A2	See Structure in Expressions	Write expressions in equivalent forms to solve problems

Sample Task #1 (Multiple Choice)

Source: https://satsuitequestionbank.collegeboard.org/

Question ID 19944

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■■	Passport to Advanced Mathematics	Nonlinear functions	2. For a quadratic or exponential function, e. make connections between tabular, algebraic, and graphical representations of the function, by ii. identifying features of one representation given another representation, including maximum and minimum values of the function;	No Calculator

In the quadratic equation above, a is a nonzero constant. The graph of the equation in the xy-plane is a parabola with vertex (c, d). Which of the following is equal to d ?

A. $-9a$

B. $-8a$

C. $-5a$

D. $-2a$

MLSS AND CLR GUIDE

- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.

CCSS Domain	*CCSS Cluster*
See Structure in Expressions	Interpret the structure of expressions

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on interpreting the structure of expressions, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning about the different structures for the number names across the languages in your classroom can lead to a more robust understanding of number for all students by making connections to the different structures of number-names in other languages.	
Cross-Curricular Connections	Science: Earth's Place in the Universe Use mathematical or computational representations to predict the motion of orbiting objects in the solar system. This a connection because each of the terms in the equations model an aspect in the motion of orbiting objects. Let's go to Mars! This activity is designed for students familiar with advanced algebra concepts. In this lesson, students will: • Use algebraic computations to determine the relative positions of Earth and Mars during which an optimal (low energy) transfer of a spacecraft can occur. • Combine this information with planetary-position data to determine the next launch opportunity to Mars.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the*	• Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion, we position them as knowers and doers of mathematics. Using equitable talk moves students to think about the way students talk about who is and isn't capable of mathematics. As a result, we can disrupt the negative images and

	mathematical abilities of students of marginalized cultures and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying, interpreting the structure of expressions facilitating meaningful mathematical discourse is critical because teachers need to lead to some sort of discourse that will ensure that all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration of their own cultures and languages. Interpretation can be critical because students may vary depending on how they learned the previous concepts. Mathematics discourse must lead to digging deeper understanding on why, how and what without deviating from their cultures and beliefs. Answers from the questions may depend on how they perceived and interpreted base from the given activities.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to the work with linear, quadratic, and exponential expressions in Algebra 1 **(HAS.SSE.A)** • Connect to rewriting quadratic functions to find specific key features. **(HSA.SSE.B.3)**	• Connect to rewriting formulas to highlight quantities of interest. **(HSA.CED.4)**	• Connect to work with expressions of all function types.

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that interprets the structure of expressions because with the new vocabulary terms, it can get confusing. Students need to differentiate between an expression and an equation.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	• 5.OA.A.2: This standard provides a foundation for work with interpreting the structure of expressions because students write out the numerical expression without the calculation. Students become comfortable with using the vocabulary words: difference, greater than, multiple, etc. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. • 6.EE.A.4: This standard provides a foundation for work with interpreting the structure of expressions because being able to tell if two expressions are equivalent is the building blocks for being able to construct and deconstruct expressions to use their structure. Being able to tell if what you have done to an expression essentially changes it or not leads to the understanding of how to use these changes to manipulate the expressions and equations to better understand their structure. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• Different forms of an expression can be equivalent and are useful in different contexts.
• The addition, subtraction, multiplication, or division of rational expressions results in another rational expression.
• When a situation and its potential constraints will be represented by all available types of equations/inequalities, including simple root function, or a system of those equations/inequalities.
• When solving graphically/with a table is more efficient than solving algebraically.
	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form.
	• Perform the operations of addition, subtraction, multiplication, and division with rational expressions.
	• Determine reasonable solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities.
	• Solve systems using a graph and a table as well as rewrite an equation as two functions (and vice versa).
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Solving systems of equations / inequalities (8.EE.C.8)
		  ○ Adding / subtracting / multiplying / dividing and simplify fractions
		  ○ Writing and solving one-step and two-step equations (HSA.REI.B.3, HSA.REI.B.4)
		  ○ Modeling linear, exponential, quadratic and absolute value functions (HSF.LE.A, HSF.LE.B)
		  ○ Different forms of linear (linear standard form, point-slope form, slope intercept form) and quadratic equations (quadratic standard form and vertex form) (HSF.LE.A)
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		• Encourage students to use alternative tools to better access the grade level content. Examples include:
		  ○ Desmos.com
		  ○ Graphing calculator
		  ○ Sketch a graph
		  ○ Create a table of values
		  ○ Algebra tiles
		  ○ Graphic organizers

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on interpreting the structure of expressions by examining tasks from a different perspective through a short mini-lesson because students learn differently. Auditory learners may need an explanation and some one-on-one explanations. Students may also learn from one another. Videos and group collaborations are also a great way to help students understand a lesson from a different perspective.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit interprets the structure of expressions by confronting student misconceptions because one-on-one explanations of mistakes made will help the students make the connections to their mistakes. Students may also have their ah-ha moment by recognizing their own mistakes.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension that explores links between various topics because structuring of an expression is the foundation for creating expressions and equations in word problems. Students need to analyze the word problems and pick out the important phrases and create an expression/equation.

CCSS Domain	*CCSS Cluster*
See Structure in Expressions	Write expressions in equivalent forms to solve problems

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on writing expressions in equivalent forms to solve problems, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning about communities having different tax rates and how they are used in the home and community can be a great way to connect schools' tasks with home tasks.	
Cross-Curricular Connections	Art: In the realm of digital art, so many wonderful and playful genres exist that stimulate the imagination, but so few do it with the intricate style of fractal art. Fractal art is achieved through the mathematical calculations of fractal objects being visually displayed, with the use of self-similar transforms that are generated and manipulated with different assigned geometric properties to produce multiple variations of the shape in continually reducing patterns. Sounds extremely technical and not that artistic, true, but these equations create some of the most mesmerizing and inspiring artwork to emerge from the digital art arena. https://fractalfoundation.org/resources/what-are-fractals/ 35 Phenomenal Fractal Art Pictures Engineering: The Invention of Fractal Antennas Dr. Cohen built the first bona fide fractal element antenna in 1988. He is now one of the world's most innovative antenna designers, with 26 years of professional experience, and 53 years of practical experience, stemming from his 'ham' antenna work over many years. Fractal Antennas website: Invention	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of*	• Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it hinders students with strong prior familiarity with school mathematics procedures for solving problems from more methods for solving tasks that occur outside of school mathematics. For example, when studying to write expressions in equivalent forms to solve problems the types of mathematical tasks are critical because

	marginalized cultures and languages? ● *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	teachers need to be aware of the real-world problems that need to be used as tasks for students to solve problems. If teachers use tax and sales real - world solving problems different communities apply tax rates differently. Students' solutions would vary. Instruction should be culturally and linguistically appropriate and relevant to allow students engagement and gain their interest.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
● In Algebra I students have studied exponential growth and decay, so they can identify first terms and common ratios. Students have written arithmetic and geometric sequences both recursively and explicitly. Students have also used arithmetic and geometric sequences to model situations.	● Students will transfer previous learning to geometric series.	● This is an important concept for Calculus when learning about Riemann sums, series, and sequences.

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching about writing expressions in equivalent forms to solve problems because students will see how the prior knowledge and the new lesson will be connected. The formula for the finite geometric series will make more sense when the connection is made by reviewing the binomial expansion or multiplying two binomials.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	7.EE.A.1: This standard provides a foundation for work with writing expressions in equivalent forms because performing operations on binomials is critical. Students need to learn how to add, multiply, and subtract to derive the formula for finite geometric series. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
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• The addition, subtraction, multiplication, or division of rational expressions results in another rational expression.
• When a situation and its potential constraints will be represented by all
	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form.
	• Perform the operations of addition, subtraction, multiplication, and division with rational expressions.
	• Determine
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
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available types of equations/inequalities, including simple root function, or a system of those equations/inequalities. ● When solving graphically/with a table is more efficient than solving algebraically.	reasonable solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities. ● Solve systems using a graph and a table as well as rewrite an equation as two functions (and vice versa).	standard form and vertex form) (HSF.LE.A) ● Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on writing expressions in equivalent forms by critiquing student approaches/solutions to make connections through a short mini-lesson because making connections using different strategies, students are able to communicate using mathematical terms and the more practice, they'll use the terms easily.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit writing expressions in equivalent forms by helping students move from specific answers to generalizations for certain types of problems because seeing the bigger picture to a detailed problem will address the conceptual understanding and students can analyze the formula to the context of the word problems.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics that give opportunities for collaboration and thinking outside the box to make connections. Exploration allows the students to interact and learn from each other.

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Standards Breakdown

- Perform arithmetic operations on polynomials.
  - HSA.APR.A.1
- Understand the relationship between zeros and factors of polynomials.
  - HSA.APR.B.2
  - HSA.APR.B.3
- Use polynomial identities to solve problems.
  - HSA.APR.C.4
- Rewrite rational expressions.
  - HSA.APR.D.6

Grade	CCSS Domain	CCSS Cluster
A2	Arithmetic with Polynomials & Rational Expressions	Perform arithmetic operations on polynomials

Cluster Standard: HSA.APR.A.1

Standard	Standards for Mathematical Practice
Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	• **SMP2:** Reason abstractly and quantitatively.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• The development of **polynomials** and **rational expressions** in high school parallels the development of numbers in elementary and middle grades. In elementary school, students might initially see expressions for the same numbers 8 + 3 and 11, or 3/4 and 0.75, as referring to different entities: 8 + 3 might be seen as describing a calculation and 11 is its answer; 3/4 is a fraction and 0.75 is a decimal. They come to understand that these different expressions are different names for the same numbers, that properties of operations allow numbers to be written in different but **equivalent forms**, and that all these numbers can be represented as points on the number line. In middle grades, they come to see numbers as forming a unified system, the number system, still represented by points on the number line. The whole numbers expand to the integers—with extensions of addition, subtraction, multiplication, and division, and their properties. Fractions expand to the rational numbers—and the four operations and their properties are extended. A similar evolution takes place in algebra. At first, algebraic expressions are simply numbers in which one or	• Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication • Describe the similarities between the set of integers and the system of polynomials • Add, subtract, and multiply polynomials • Determine whether a set or system is closed under a given operation

more letters are used to stand for a number which is either unspecified or unknown. Students learn to use the properties of operations to write expressions in different but equivalent forms. At some point they see equivalent expressions, particularly polynomial and rational expressions, as naming some underlying thing. As they see polynomial expressions as quantities rather than operations to be performed, they can perform operations such as adding, subtracting and multiplying two polynomials and identify that these operations will yield another polynomial, thus making the system of polynomials closed.	
DOK	**Blooms**
1	Remember, Understand

Common Misconceptions

- Students might think polynomials are only monomial, binomial, or trinomial.
- Students may not confuse the impact of adding and subtracting polynomials on the degree of the variable.
- Students may not fully distribute the multiplication of polynomials and only multiply like terms.
- When adding and multiplying like terms students may initially confuse $x + x$ as x^2 instead of $2x$.
- Students may not think $x^2 \bullet x = x^3$ is not an example of closure for polynomial multiplication since the result has a different exponent than the factors.

Grade	CCSS Domain	CCSS Cluster
A2	Arithmetic with Polynomials & Rational Expressions	Understand the relationship between zeros and factors of polynomials

Cluster Standard: HSA.APR.B.2

Standard	Standards for Mathematical Practice
Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	• **SMP3:** Construct viable arguments and critique the reasoning of others.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• The zeros of a polynomial are turned into linear factors and can be used to factor polynomials of any power. The degree of a polynomial will indicate the maximum number of zeros of the polynomial.	• Define the Remainder Theorem. • Use the Remainder Theorem to show the relationship between a factor and a zero.
DOK	**Blooms**
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
A2	Arithmetic with Polynomials & Rational Expressions	Understand the relationship between zeros and factors of polynomials

Cluster Standard: HSA.APR.B.3

Standard	Standards for Mathematical Practice
Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	• **SMP3:** Construct viable arguments and critique the reasoning of others.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• The zeros of a polynomial are turned into linear factors and can be used to factor polynomials of any power. The degree of a polynomial will indicate the maximum number of zeros of the polynomial.	• Determine the degree of a polynomial and the number of possible zeros of that polynomial. • Simplify polynomials into factored forms. • Identify the zeros of the polynomial using the factors. • Plot the zeros of the polynomial on a graph.
DOK	**Blooms**
1-2	Apply, Analyze

Common Misconceptions

- Division problems never have a remainder; it is okay to write R-value.
- Students often forget to distribute the –1, which is equivalent to subtraction, to terms inside the parenthesis.
- Students might make errors in signs when doing synthetic division and synthetic substitution because values are added rather than subtracted as in long division. Remind them that terms are always added for synthetic substitution and synthetic division. When listing the coefficients, there may be missing degrees and students will forget to write a zero.

Grade	CCSS Domain	CCSS Cluster
A2	Arithmetic with Polynomials & Rational Expressions	Use polynomial identities to solve problems

Cluster Standard: HSA.APR.C.4

Standard	Standards for Mathematical Practice
Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	• **SMP7:** Look for and make use of structure. • **SMP8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students make systematic lists of all arrangements and count the number of unique subgroups. Students use prior knowledge of counting techniques to calculate the number of combinations.	• Understand that polynomial identities include but are not limited to the product of the sum and difference of two terms, the difference of two squares, the sum and difference of two cubes, the square of a binomial, etc. • Prove polynomial identities by showing steps and providing reasons. • Illustrate how polynomial identities are used to determine numerical relationships
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- There are no *y*-axis zeros.
- Easily get lost with the different coefficients and degrees.
- Multiplying the degrees.
- Students might incorrectly expand binomial expressions by choosing the wrong row of coefficients in Pascal's triangle. Remind students that for an exponent of *n*, choose row *n* of Pascal's triangle. Row *n* will be the row with the value *n* as the second entry.

Grade	CCSS Domain	CCSS Cluster
A2	Arithmetic with Polynomials & Rational Expressions	Rewrite rational expressions

Cluster Standard: HSA.APR.D.6

Standard	Standards for Mathematical Practice
Rewrite simple rational expressions in different forms; write $^{a(x)}/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP5:** Use appropriate tools strategically. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Rational expressions can be rewritten using properties of fractions and elementary numerical algorithms.	• Divide polynomials using long division. • Divide polynomials using synthetic division. • Relate the algorithm of dividing multi-digit integers with polynomial long division. • Perform partial fraction decomposition. • Determine the quotient and remainder of rational expressions using inspection, long division, and/or a computer algebra system
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Students may forget to write the polynomial in descending order.
- Students may not recognize a missing term in the divisor or dividend and forget to insert a zero for the missing term.
- Students might make errors in signs when doing synthetic division and synthetic substitution because values are added rather than subtracted as in long division. Remind them that terms are always added for synthetic substitution and synthetic division.

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Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Arithmetic with Polynomials & Rational Expressions**	Strand: **Perform arithmetic operations on polynomials**

Suggested Student Discourse Questions

- Can you identify a maximum or minimum from a polynomial?
- What can finding a max or min tell you?
- If you add or subtract two rational numbers, will you always get an irrational number?
- Compare strategies for add/ subtracting with multiplying/ dividing? Do they work the same? What are the differences? Are they the same, why or why not the same?
- Given the relationship between the length and width of a rectangular pool, the width and total area of the surrounding walkway, how would you find the dimensions of the pool?

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ASSESSMENT GUIDE

- Perform arithmetic operations on polynomials.
- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems.
- Rewrite rational expressions.

Grade	*CCSS Domain*	*CCSS Strand*
A2	**Arithmetic with Polynomials & Rational Expressions**	**Perform arithmetic operations on polynomials**

Sample Task #1 (Constructed Response)

Source: https://satsuitequestionbank.collegeboard.org/

Question ID 5094624

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■□	Passport to Advanced Mathematics	Equivalent expressions	2. Fluently add, subtract, and multiply polynomials.	No Calculator

Which of the following is equivalent to the sum of $3x^4+2x^3$ and $4x^4+7x^3$?

A. $16x^{14}$

B. $7x^8+9x^6$

C. $12x^4+14x^3$

D. $7x^4+9x^3$

Rationale

Choice D is correct. Adding the two expressions yields $3x^4+2x^3+4x^4+7x^3$. Because the pair of terms $3x^4$ and $4x^4$ and the pair of terms $2x^3$ and $7x^3$ each contain the same variable raised to the same power, they are like terms and can be combined as $7x^4$ and $9x^3$, respectively. The sum of the given expressions therefore simplifies to $7x^4+9x^3$. Choice A is incorrect and may result from adding the coefficients and the exponents in the given expressions. Choice B is incorrect and may result from adding the exponents as well as the coefficients of the like terms in the given expressions. Choice C is incorrect and may result from multiplying, rather than adding, the coefficients of the like terms in the given expressions.

Grade	CCSS Domain	CCSS Strand
A2	Arithmetic with Polynomials & Rational Expressions	Understand the relationship between zeros and factors of polynomials

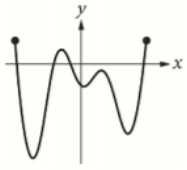
Sample Task #1 (Multiple Choice)

Source: SAT https://satsuitequestionbank.collegeboard.org/results

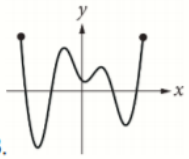
20079	■■□	Passport to Advanced Math	Passport to Advanced Mathematics	Nonlinear functions	3. For a factorable or factored polynomial or simple rational function, b. understand and use the fact that for the graph of y = f(x), the solutions to f(x) = 0 correspond to x-intercepts of the graph and f(0) corresponds to the y-intercept of the graph; interpret these key features in terms of a context;	Calculator

If the function f has five distinct zeros, which of the following could represent the complete graph of f in the xy-plane?

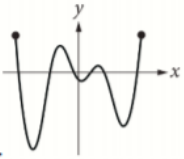
- A.



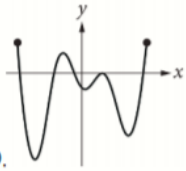
- B.



- C.



- D.



Rationale

- Choice D is correct. A zero of a function corresponds to an x-intercept of the graph of the function in the xy-plane. Therefore, the complete graph of the function f, which has five distinct zeros, must have five x-intercepts. Only the graph in choice D has five x-intercepts, and therefore, this is the only one of the given graphs that could be the complete graph of *f* in the xy-plane.
- Choices A, B, and C are incorrect. The number of x-intercepts of each of these graphs is not equal to five; therefore, none of these graphs could be the complete graph of f, which has five distinct zeros.

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Grade	CCSS Domain	CCSS Strand
A2	Arithmetic with Polynomials & Rational Expressions	Use polynomial identities to solve problems

Sample Task #1 (Constructed Response)

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Illustrative Mathematics

Felicia notices what appears to be an interesting pattern between powers of 11 and powers of x+1:

$11^0 = 1$ $\quad (x+1)^0 = 1$

$11^1 = 11$ $\quad (x+1)_1 = x+1$

$11^2 = 121$ $\quad (x+1)^2 = x^2+2x+1$

- The digits of the number 11n are the same as the coefficients of the polynomial $(x+1)^n$. Is this always true?
- Does this pattern continue for n=3 and n=4?
- What is the answer to Felicia's question?

IM Commentary
This task has students combine polynomial arithmetic with pattern-matching. Students can expand powers of x+1 using either repeated multiplication (A-APR.1) or by the binomial theorem (A-APR.5), and then are asked to analyze the question of whether the similarity of coefficients with the digits of powers of 11 is a coincidence. Identifying patterns, as Felicia has done, is an important part of mathematics. In this case, there is a deep relationship between the numbers and polynomials that Felicia is investigating; on the other hand, further consideration shows that the pattern does not continue. It is important for students not only to identify patterns but also to look more deeply to understand whether or not the patterns are "generalizable" or true because of some essential mathematical structure.

Grade	CCSS Domain	CCSS Strand
A2	Arithmetic with Polynomials & Rational Expressions	Rewrite rational expressions

Sample Task #1 (Constructed Response)

Source: SAT

$$\frac{4x^2+6x}{4x+2}$$

Which of the following is equivalent to ?

- A. x
- B. $x+4$
- C. $x-\frac{2}{4x+2}$
- D. $x+1-\frac{2}{4x+2}$

MLSS AND CLR GUIDE

- Perform arithmetic operations on polynomials.
- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems.
- Rewrite rational expressions.

CCSS Domain	*CCSS Cluster*
Arithmetic with Polynomials and Rational Expressions	Perform arithmetic operations on polynomials

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on performing arithmetic operations on polynomials, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students, in becoming a critical thinker and problem solver. Students take a skill familiar to them (arithmetic operations with integers) and apply it to something new, arithmetic operations on polynomials. This unit practices learning something new from existing knowledge.	
Cross-Curricular Connections	History: The history of exponents dates back many centuries and Euclid is credited with the first known usage of exponents. He used the term 'power' to represent what we know today, how many times a number is multiplying by itself. The ancient Greek mathematicians used, and many other mathematicians added onto the use of exponents as they learned more about their use. Archimedes generalized the same idea of powers and later mathematicians in the Islamic golden age utilized powers of two and three in their work in algebra. In our project, you will see many other mathematicians and their contributions to the development of exponents from the 14th century up to the use of exponents today. https://www.sutori.com/story/history-of-exponents--wNbwYExXdzFNYPh1zFUYhbDc	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the*	• Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it hinders students with strong prior familiarity with school mathematics procedures for solving problems from more methods for solving tasks that occur outside of school mathematics. For example, when performing

	mathematical abilities of students of marginalized cultures and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	arithmetic operations on polynomials, the types of mathematical tasks are critical because they build on prior knowledge of arithmetic operations. Time spent on conceptual understanding of the four basic operations (addition, subtraction, multiplication, division) using integers can bridge to the conceptual understanding of those operations of polynomials. From here, time can be spent on procedural fluency of the mechanics of the operations with polynomials. Students will be expected to demonstrate proficiency on End of Course exams and the SAT in English, an opportunity presents itself to bridge home language to the language of these exams.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to applying the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. **(8.EE.A.1)**	• Connect to using the properties of operations to write expressions in different but equivalent forms. **(HSA.SSE.A.2)**	• Connect to performing operations with rational expressions **(HSA.APR.7)** • Connect to deriving the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems. *For example, calculate mortgage payments.* **(HS.A.SSE.B.4)**

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on arithmetic operations on polynomials because the structure of the four basic operations hold true for arithmetic operations on polynomials.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.EE.A.1: This standard provides a foundation for work with arithmetic operations on polynomials because the student must know and apply the properties of integer exponents. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• Different forms of an expression can be equivalent and are useful in different contexts. • The addition, subtraction, multiplication, or division of rational expressions results in another rational expression. • When a situation and its potential constraints will be represented by all available types of	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form. • Perform the operations of addition, subtraction, multiplication, and division with rational expressions. • Determine reasonable solutions based on	• Build on students' experience with the following skills: ○ Graphing on the coordinate plane (6.NS.C.8) ○ Solving systems of equations / inequalities (8.EE.C.8) ○ Adding / subtracting / multiplying / dividing and simplify fractions ○ Writing and solving one-step and two-step equations (HSA.REI.B.3, HSA.REI.B.4) ○ Modeling linear, exponential, quadratic and absolute value functions (HSF.LE.A, HSF.LE.B) ○ Different forms of linear (linear standard form, point-slope form, slope intercept form) and quadratic equations (quadratic standard form and vertex form)

equations/inequalities, including simple root function, or a system of those equations/inequalities. ● When solving graphically/with a table is more efficient than solving algebraically.	the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities. ● Solve systems using a graph and a table as well as rewrite an equation as two functions (and vice versa).	(HSF.LE.A) ● Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on arithmetic operations on polynomials by providing specific feedback to students on their work through a short mini-lesson because looking at integer rules for arithmetic operations apply directly to arithmetic operations with polynomials.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on arithmetic operations on polynomials by confronting student misconceptions because integer rules concerning positives and negatives are common errors that lead to misconceptions when performing arithmetic operations on polynomials.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to explore links between various topics when studying arithmetic operations on polynomials because addition and subtraction are inverse operations as are multiplication and division.

CCSS Domain	*CCSS Cluster*
Arithmetic with Polynomials and Rational Expressions	Understand the relationship between zeros and factors of polynomials

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on understanding the relationship between zeros and factors of polynomials, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, students practice using mathematical tools to solve new problems. Students learn to identify what tools are at their disposal when connecting the zeros of a polynomial to its factors. This prepares them for life outside the math classroom by providing skills that are lifelong.	
Cross-Curricular Connections	Science: This approach is extended to a spherical body rolling on a curved path. Assuming that a curved path can be approximated by a sequence of many very short inclines, the problem is approached as a body rolling on this sequence of inclines, solving each with the work-energy theorem. Defining the curved path as a differentiable function, the slope of each incline is obtained through the function. Teach Engineering Roller Coaster - Spherical Body rolling Social Studies: Not much is really known about the Pythagoreans or their rather mysterious founder, Pythagoras. Several different accounts of the Pythagoreans have come down to us from antiquity. Plato and Aristotle both reference the Pythagoreans throughout their philosophical writings. Even still, the true nature of the "cult of Pythagoras" is often shrouded in mystery. Pythagoras and his followers were a mystical society that placed great importance on the mathematical relations of the universe. There is no denying that they contributed greatly to the area of mathematics and philosophy. One needs only to reflect on the Pythagorean theorem, a mathematical principle said to have been discovered by Pythagoras himself, to appreciate the profound impact they had on the development of scientific thought. https://classicalwisdom.com/philosophy/cult-of-pythagoras/	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving

	marginalized cultures and languages? ● *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when understanding the relationship between zeros and factors of polynomials the use of mathematical representations within the classroom is critical because graphing technology and area models can be used to factor polynomials and check the zeros of those factors. Mathematics can be designed in a context to connect home culture or interests in a way that a polynomial function could represent a quantity where its solution(s) could represent a critical value within the context. For example, a cubic function could represent the profit of a fundraiser given the cost of a ticket for the fundraiser. The zero(s) of the function would represent the break-even point. The context of the fundraiser could be framed around a cultural interest. A graph could be used to show a representation of the function to support the learner in bridging different representations.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
● Connect to factoring and completing the square and using the Remainder Theorem **(standards A-APR.B.2, F-IF.C.8.a)**	● Connect to calculating the zero in the Remainder Theorem or by factoring to graph the zeros of a polynomial function **(standards A-APR.B.2, A-APR.B.3)**	● Connect to graphing key features of polynomial functions to identifying zeros and sketching a graph **(standards F-IF.C.7)**

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that analyzes common misconceptions regarding the relationship between zeros and factors of polynomials because when using the Remainder Theorem students must use the opposite sign of the factor in the dividend. Students must also know how to factor polynomials when the leading coefficient is equal to 1 and not equal to 1 which can lead to common misconceptions.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	A.SSE.B.3.A and A.APR.B.6: These standards provide a foundation for work with understanding the relationship between zeros and factors of polynomials because students will be producing equivalent forms of polynomials to reveal properties of the expression (in this case the factored form will reveal zeros; the Remainder Theorem can be used instead of long division to check if a factor is a zero of the expression). If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• Different forms of an expression can be equivalent and are useful in different contexts.
• The addition, subtraction, multiplication, or division of rational expressions results in another rational expression.
• When a situation and its potential constraints will be represented by all available types of equations/inequalities, including simple root function, or a system of those equations/inequalities.
• When solving graphically/with a table is more efficient than solving algebraically.
	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form.
	• Perform the operations of addition, subtraction, multiplication, and division with rational expressions.
	• Determine reasonable solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities.
	• Solve systems using a graph and a table as well as rewrite an equation as two functions (and vice versa).
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Solving systems of equations / inequalities (8.EE.C.8)
		  ○ Adding / subtracting / multiplying / dividing and simplify fractions
		  ○ Writing and solving one-step and two-step equations (HSA.REI.B.3, HSA.REI.B.4)
		  ○ Modeling linear, exponential, quadratic and absolute value functions (HSF.LE.A, HSF.LE.B)
		  ○ Different forms of linear (linear standard form, point-slope form, slope intercept form) and quadratic equations (quadratic standard form and vertex form) (HSF.LE.A)
		
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		
		• Encourage students to use alternative tools to better access the grade level content. Examples include:
		  ○ Desmos.com
		  ○ Graphing calculator
		  ○ Sketch a graph
		  ○ Create a table of values
		  ○ Algebra tiles
		  ○ Graphic organizers

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on understanding the relationship between zeros and factors of polynomials by critiquing student approaches/solutions to make connections through a short mini-lesson because zeros of polynomials must match the graph of the polynomial. By critiquing other students' work, the learner can immediately make connections to the correctness of the work by observing a graph.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on understanding the relationship between zeros and factors of polynomials by offering opportunities to understand and explore different strategies because investigating graphs to identify zeros and using area models to factor polynomials can offer structure to the student as a starting point.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students. because some learners may be ready to factor more difficult polynomials.

CCSS Domain	*CCSS Cluster*
Arithmetic with Polynomials and Rational Expressions	Use polynomial identities to solve problems

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on the use of polynomial identities to solve problems, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, the student makes use of structure and decides on a method to expand binomials that is effective and efficient and makes sense to them. The Standards for Mathematical Practice come alive as they use polynomial identities to solve problems as the bridge to perseverance and making use of structure and repeated reasoning.	
Cross-Curricular Connections	In this activity, students relate the graph of a rational function to the graphs of the polynomial functions of its numerator and denominator. Students graph these polynomials one at a time and identify their y-intercepts and zeros. Asymptotes and Zeros of Rational Functions: Algebra 2	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	• Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it hinders students with strong prior familiarity with school mathematics procedures for solving problems from learning more methods for solving tasks that occur outside of school mathematics. For example, when studying the use of polynomial identities to solve problems the types of mathematical tasks are critical because, for example, students' familiarity of structure can support the expansion of binomials. When squaring a binomial, students have a working knowledge of area models and the FOIL method. As the power of a binomial grows, these methods break down and become messy. The student can then decide when it would be more efficient to use the Binomial Theorem and Pascal's Triangle to expand a binomial. Conceptual understanding of expanding binomials will lead to procedural

		fluency as the student decides what method works best for their learning style.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Students are building on their knowledge of zeros and factors of quadratics learned in Algebra 1.	• Students are learning about factoring with polynomials of degrees higher than 2 (perfect cubes, quadratics, factor by grouping, etc). Students are also understanding that not all polynomials are factorable, but still can be divided by another polynomial. Students continue to build their understanding of how factored form relates to zeros on a graph. Later in the year, these skills are used in simplifying rational expressions.	• In 4th year math (Pre-Calculus, Calculus, and college level math) students will build on their factoring skills (with rational expressions and trigonometric expressions). Students will also determine zeros of trigonometric functions in subsequent math courses.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that introduces new representations (e.g., Pascal's Triangle, the Binomial Theorem) when studying the use of polynomial identities to solve problems because there are structures that exist to make expanding binomials more efficient and effective.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	A.SSE.A.2: This standard provides a foundation for work with the use of polynomial identities to solve problems because students have looked at the structure of an expression to identify ways to rewrite it. If students have unfinished learning

		within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• Different forms of an expression can be equivalent and are useful in different contexts. • The addition, subtraction, multiplication, or division of rational expressions results in another rational expression. • When a situation and its potential constraints will be represented by all available types of equations/inequalities, including simple root function, or a system of those equations/inequalities. • When solving graphically/with a table is more efficient than solving algebraically.	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form. • Perform the operations of addition, subtraction, multiplication, and division with rational expressions. • Determine reasonable solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities. • Solve systems using a graph and a table as well as rewrite an equation as two functions (and vice versa).	• Build on students' experience with the following skills: ○ Graphing on the coordinate plane (6.NS.C.8) ○ Solving systems of equations / inequalities (8.EE.C.8) ○ Adding / subtracting / multiplying / dividing and simplify fractions ○ Writing and solving one-step and two-step equations (HSA.REI.B.3, HSA.REI.B.4) ○ Modeling linear, exponential, quadratic and absolute value functions (HSF.LE.A, HSF.LE.B) ○ Different forms of linear (linear standard form, point-slope form, slope intercept form) and quadratic equations (quadratic standard form and vertex form) (HSF.LE.A) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content.

		Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on the use of polynomial identities to solve problems by revisiting student thinking through a short mini-lesson because looking at other students' work can support all learners in understanding the structure of the Binomial Theorem. Learners listening to their peers explain their thinking can benefit all as student thinking is delivered in student friendly language.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on the use of polynomial identities to solve problems by offering opportunities to understand and explore different strategies because some students will insist on using area models or the FOIL method for expanding a binomial regardless of the power of the binomial. As the Binomial Theorem will be more effective and efficient to expand certain binomials, pockets of students are afforded the opportunity to explore and apply previous strategies.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension to explore links between various strategies when studying the use of polynomial identities to solve problems. Some learners can investigate and explain when it would be more appropriate to use

	an area model, the FOIL method, or the Binomial Theorem when expanding binomials.

CCSS Domain	*CCSS Cluster*
Arithmetic with Polynomials and Rational Expressions	Rewrite rational expressions

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on rewriting rational expressions, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. Learners look for and make use of structure and make sense of problems and persevere in solving them. As students rewrite rational expressions in equivalent forms, they are building confidence in taking what they know to apply to problem solving scenarios. These Mathematical Practices skills exist outside of school as the student builds their critical thinking skills through rewriting rational expressions.	
Cross-Curricular Connections	Medicine and Analytical Chemistry: MRI and NMR Spectroscopy involves Fast Fourier Transformation that allows the creation of images from the "ringing" after the atoms are subjected to radio waves in strong magnetic fields. The Fourier series consists of terms of increasing orders. (An Algorithm for the Machine Calculation of Complex Fourier Series by James W. Cooley and John W. Tukey)	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying rewriting rational expressions, the use of mathematical

	mathematical identities as capable mathematicians that can use mathematics within school and society?	representations within the classroom is critical because students are in fact rewriting simple rational expressions as equivalent representations. Students are asked to draw on their mathematical competence by simplifying rational expressions previously learned within this standard domain. For example, a student might have to factor a numerator and/or denominator to simplify a rational expression. Or a student might have to perform synthetic division to rewrite a rational expression as an equivalent representation. These skills could build a bridge for students to position them as competent and capable mathematicians and leverage further study of mathematics.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Students are building on their knowledge of factors of quadratics learned in Algebra 1.	• Students will use the skills learned to factor and divide polynomials to simplify rational expressions.	• Students will be able to perform all operations with rational expressions.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on rewriting rational expressions because arithmetic operations with polynomials and factoring will be used when rewriting rational expressions.
Intensive	*What critical understandings will prepare students to access the mathematics for this*	A.SSE.A.2: This standard provides a foundation for work with rewriting rational expressions because students use the structure of an expression to

	cluster?	identify ways to rewrite it. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• Different forms of an expression can be equivalent and are useful in different contexts. • The addition, subtraction, multiplication, or division of rational expressions results in another rational expression. • When a situation and its potential constraints will be represented by all available types of equations/inequalities, including simple root function, or a system of those equations/inequalities. • When solving graphically/with a table is more efficient than solving algebraically.	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form. • Perform the operations of addition, subtraction, multiplication, and division with rational expressions. • Determine reasonable solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities. • Solve systems using a graph and a table as well as rewrite an equation as two functions (and vice versa).	• Build on students' experience with the following skills: ○ Graphing on the coordinate plane (6.NS.C.8) ○ Solving systems of equations / inequalities (8.EE.C.8) ○ Adding / subtracting / multiplying / dividing and simplify fractions ○ Writing and solving one-step and two-step equations (HSA.REI.B.3, HSA.REI.B.4) ○ Modeling linear, exponential, quadratic and absolute value functions (HSF.LE.A, HSF.LE.B) ○ Different forms of linear (linear standard form, point-slope form, slope intercept form) and quadratic equations (quadratic standard form and vertex form) (HSF.LE.A) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content.

		Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on rewriting rational expressions by providing specific feedback to students on their work through a short mini lesson because an expression might not be fully simplified. Students might not have applied a full set of mathematical properties to rewrite a rational expression and may benefit from focused feedback on where to go next in their work.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit rewriting rational expressions by confronting student misconceptions because the student might rewrite a rational expression incorrectly and simplify a polynomial incorrectly (e.g., (x+ y)2 = x2+ y2) or might have factored a polynomial incorrectly.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension such as the opportunity to understand concepts in greater depth. When rewriting rational expressions some students will be ready for more complex rational expressions with more complex terms than others. Pockets of students can be paired homogeneously by ability to work on more complex rational expressions to explore in greater depth.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

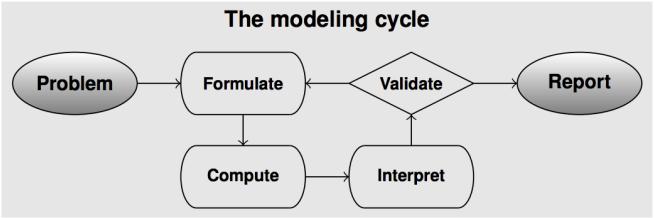
Standards Breakdown

- Create equations that describe numbers or relationships.
  - HSA.CED.A.1
  - HSA.CED.A.2
  - HSA.CED.A.3
  - HSA.CED.A.4

Grade	*CCSS Domain*	*CCSS Cluster*
A2	Creating Equations	Create equations that describe numbers or relationships

Cluster Standard: HSA.CED.A.1

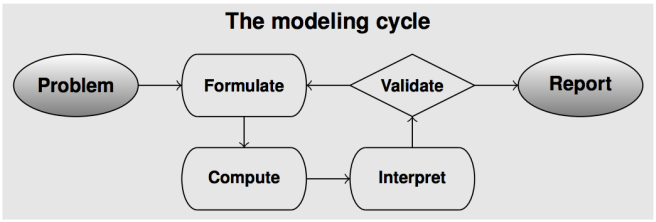
Standard	Standards for Mathematical Practice
Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Equations and inequalities can be created to represent and solve real world and mathematical problems. • Students check their solutions to real-world problems which can be found by modeling them with equations and graphs. • Constraints are necessary to balance a mathematical model with real-world context. Variable quantities may be able to take on only certain values and expressing these restrictions, or constraints, algebraically in an important part of modeling with mathematics. • Formulas are equations with specific meaning that show the relationship between two or more quantities and are written in the same way literal equations are solved for a given variable, by isolating the desired variable on one side of the equation. • All the standards in the Creating Equations group carry a modeling star, denoting their connection with the Modeling category in high school. This connotes not only an increase in the complexity of the equations studied, but an upgrade of the student's ability in every part of the modeling cycle.	• Create equations and inequalities in one variable and use them to solve problems. • Write equations in one variable and use them to solve problems. • Write inequalities in one variable and use them to solve problems.

	
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A2	Creating Equations	Create equations that describe numbers or relationships

Cluster Standard: HSA.CED.A.2

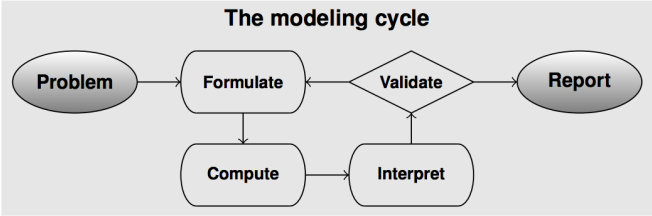
Standard	Standards for Mathematical Practice
HSA.CED.A.2: Create equations in two or more variables to represent relationships between quantities, graph equations on coordinate axes with labels and scales.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Equations and inequalities can be created to represent and solve real world and mathematical problems. • Students check their solutions to real-world problems which can be found by modeling them with equations and graphs. • Constraints are necessary to balance a mathematical model with real-world context. Variable quantities may be able to take on only certain values and expressing these restrictions, or constraints, algebraically is an important part of modeling with mathematics. • Formulas are equations with specific meaning that show the relationship between two or more quantities and are written in the same way literal equations are solved for a given variable, by isolating the desired variable on one side of the equation. • All the standards in the Creating Equations group carry a modeling star, denoting their connection with the Modeling category in high school. This connotes not only an increase in the complexity of the equations studied, but an upgrade of the student's ability in every part of the modeling cycle.	• Create equations in two or more variables based on a given context. • Write equations in two or more variables based on a given context. • Graph equations on coordinate axes with scales clearly labeling the axes, defining what the values on the axes represent and the unit of measure. • Select intervals for the scale that are appropriate for the context and display adequate information about the relationship. • Analyze points on and off a graph and interpret them in context.

	
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	*CCSS Domain*	*CCSS Cluster*
A2	**Creating Equations**	**Create equations that describe numbers or relationships**

Cluster Standard: HSA.CED.A.3

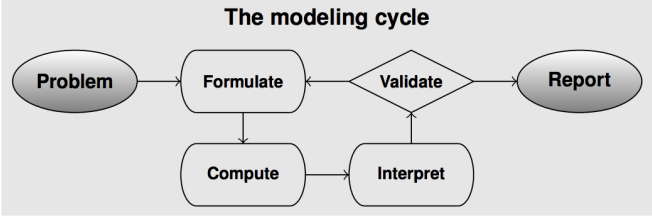
Standard	**Standards for Mathematical Practice**
Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Equations and inequalities can be created to represent and solve real world and mathematical problems. • Students check their solutions to real-world problems which can be found by modeling them with equations and graphs. • Constraints are necessary to balance a mathematical model with real-world context. Variable quantities may be able to take on only certain values and expressing these restrictions, or constraints, algebraically in an important part of modeling with mathematics.	• Identify constraints of equations, inequalities, and systems of equations and inequalities given a context. • Interpret solutions of equations, inequalities, and systems of equations and inequalities as viable or non-viable given a context. • Interpret solutions analytically and graphically to answer questions about the quantities in context.

• Formulas are equations with specific meaning that show the relationship between two or more quantities and are written in the same way literal equations are solved for a given variable, by isolating the desired variable on one side of the equation. • All the standards in the Creating Equations group carry a modeling star, denoting their connection with the Modeling category in high school. This connotes not only an increase in the complexity of the equations studied, but an upgrade of the student's ability in every part of the modeling cycle. 	
DOK	**Blooms**
1-3	Understand, Apply, Analyze, Evaluate

Grade	CCSS Domain	CCSS Cluster
A2	Creating Equations	Create equations that describe numbers or relationships

Cluster Standard: HSA.CED.A.4

Standard	Standards for Mathematical Practice
Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*	• **SMP4:** Reason abstractly and quantitatively. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Equations and inequalities can be created to represent and solve real world and mathematical problems. • Students check their solutions to real-world problems which can be found by modeling them with equations and graphs. • Constraints are necessary to balance a mathematical model with real-world context. Variable quantities may be able to take on only certain values and expressing these restrictions, or constraints, algebraically in an important part of modeling with mathematics. • Formulas are equations with specific meaning that show the relationship between two or more quantities and are written in the same way literal equations are solved for a given variable, by isolating the desired variable on one side of the equation. • All the standards in the Creating Equations group carry a modeling star, denoting their connection with the Modeling category in high school. This connotes not only an increase in the complexity of the equations studied, but an upgrade of the student's ability in every part of the modeling cycle.	• Solve for a specified variable in a literal equation.

	
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Students may believe only linear and quadratic expressions can be used within inequalities.
- Students may believe that ellipses and hyperbolas are the same, but are reversed on the axis
- Students may believe absolute value cannot be inverted and struggle when there is more than one term inside absolute value
- Students may believe that midpoint and distance are the same thing and confuse the formulas.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Creating Equations**	Strand: **Create equations that describe numbers or relationships**

Suggested Student Discourse Questions

- How can you simplify a rational function? Why or why not?
- Create an equation and inequality in one variable How could you create an equation and inequality in one variable? How is your method different from your partner?
- How could you rearrange formulas to highlight a specific quantity? Solve for a specific unknown. For example, A = LW, find width.
- The challenge continues to examine US census data to select and refine a model for the population of the United States over time. Check to see if it is changing at a constant rate or at equal proportional rates. How could the information from the table be used to model a linear or exponential function?

ASSESSMENT GUIDE

- Create equations that describe numbers or relationships

Grade	*CCSS Domain*	*CCSS Cluster*
A2	Creating Equations	Create equations that describe numbers or relationships

Sample Task #1 (Constructed Response)

Source: https://satsuitequestionbank.collegeboard.org/

Question ID 19489

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	■□□	Heart of Algebra	Linear inequalities in one or two variables	1. Create and use linear inequalities in one or two variables to solve problems in a variety of contexts.	Calculator

19489

Wyatt can husk at least 12 dozen ears of corn per hour and at most 18 dozen ears of corn per hour. Based on this information, what is a possible amount of time, in hours, that it could take Wyatt to husk 72 dozen ears of corn?

Rationale

The correct answer is any number between 4 and 6, inclusive. Since Wyatt can husk at least 12 dozen ears of corn per hour, it will take him no more than $\frac{72}{12} = 6$ hours to husk 72 dozen ears of corn. On the other hand, since Wyatt can husk at most 18 dozen ears of corn per hour, it will take him at least $\frac{72}{18} = 4$ hours to husk 72 dozen ears of corn.

Therefore, the possible times it could take Wyatt to husk 72 dozen ears of corn are 4 hours to 6 hours, inclusive. Any number between 4 and 6, inclusive, can be gridded as the correct answer.

MLSS AND CLR GUIDE

- Create equations that describe numbers or relationships

CCSS Domain	*CCSS Cluster*
Creating Equations	Create equations that describe numbers or relationships

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on HS.CED.A: Create equations that describe numbers or relationships cluster, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, how statistics are used to describe how the risk of different cultural and ethnic groups for developing breast cancer and how this might affect medical breast cancer screening frequency recommendations. Example: During a unit focused on creating equations in two variables, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, exploring how changing the structure for an equation is similar to how a sentence can be re-structured to convey different meanings depending on the structure of the words.	
Cross-Curricular Connections	Economics: Linear programming with a system of inequalities is often used to model the constraint of resources for production. Consider providing a connection where students are starting their own business and must maximize profit or production with the possible solutions of the system. Science: There are many formulas in science such as Ohm's Law and the Doppler formulas that may require isolating and solving for a specific variable given certain conditions. Consider providing a connection where students must rearrange the same formulas in multiple ways to highlight different quantities of interest.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and*	• Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it hinders students with

	languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? ● *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	strong prior familiarity with school mathematics procedures for solving problems from learning to build more methods for solving tasks that occur outside of school mathematics. For example, when studying HS.CED.A: Create equations that describe numbers or relationships cluster the types of mathematical tasks are critical because fluency in Algebra is akin to becoming fluent in a spoken or written language. Fluency is essential to obtaining a deep understanding of the function and meaning of any language. Algebra is no different.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
● Connect to the work of Algebra 1 around linear, quadratic, and exponential (integer inputs only) with this cluster. **(HSA.CED.A)** ● Connect to graphing systems of equations and inequalities. **(HSA.REI.7)** ● Connect to solving equations in one variable including those equations with coefficients represented by variables. **(HSA.REI.3-4)**	● Connect to communicating relevant domain and range for linear, exponential and quadratic functions. **(HSF.IF.4)** ● Connect to graphing equations and inequalities. **(HSF.IF.7)** ●	● Connect to extending knowledge to include additional types of functions such as trigonometric, rational, and polynomial. **(HSA.CED.1-4)** ● Connect to communicating relevant domain and range for all types of functions. **(HSF.IF.4)**

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on creating equations that describe numbers or relationships because this cluster requires students to create equations they have already studied from relationships and contexts. A recap of the key features of the families of functions studied can help students more easily apply their prior learnings to these problems.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.F.B.4: This standard provides a foundation for work with creating equations that describe numbers or relationships because this standard called on students to specifically write linear equations from a given relationship and explain the parts of the equation in context of a scenario. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• Different forms of an expression can be equivalent and are useful in different contexts. • The addition, subtraction, multiplication, or division of rational expressions results in another rational expression. • When a situation and its potential constraints will be	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form. • Perform the operations of addition, subtraction, multiplication, and division with rational expressions. • Determine	• Build on students' experience with the following skills: ○ Graphing on the coordinate plane (6.NS.C.8) ○ Solving systems of equations / inequalities (8.EE.C.8) ○ Adding / subtracting / multiplying / dividing and simplify fractions ○ Writing and solving one-step and two-step equations (HSA.REI.B.3, HSA.REI.B.4) ○ Modeling linear, exponential, quadratic and absolute value functions (HSF.LE.A, HSF.LE.B) ○ Different forms of linear (linear

represented by all available types of equations/inequalities, including simple root function, or a system of those equations/inequalities. ● When solving graphically/with a table is more efficient than solving algebraically.	reasonable solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities. ● Solve systems using a graph and a table as well as rewrite an equation as two functions (and vice versa).	standard form, point-slope form, slope intercept form) and quadratic equations (quadratic standard form and vertex form) (HSF.LE.A) ● Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on creating equations that describe numbers or relationships by clarifying mathematical ideas and/or concepts through a short mini-lesson because students may see problems as having one specific solution when infinitely many solutions are appropriate. Students may benefit from revisiting contexts with inequalities and discussing many potential solutions and why they each make sense in context of the problem. Further, students may benefit from discussing why a solution can be found mathematically but why it may not make sense in context of a problem.
Intensive	What assessment data will help identify content needing to be revisited for	For example, some students may benefit from intensive extra time during and after a unit creating equations that describe numbers or relationships by addressing

	intensive interventions?	conceptual understanding because students must have a firm grasp of the features of equations and inequalities before they can model scenarios with them. Students may require support in conceptualizing the different families of functions and/or the difference between an equation and an inequality.
Extension		
Essential Question		***Examples***

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when creating equations that describe numbers or relationships because once students are fluent in applying equations to contexts, they can be challenged by selecting their own problems relating to specific careers or interests.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

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Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
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	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Build a function that models a relationship between two quantities.
  - HSF.BF.A.1
- Build new functions from existing functions.
  - HSF.BF.B.3
  - HSF.BF.B.4.A

Grade	CCSS Domain	CCSS Cluster
A2	Interpreting Functions	Build a function that models a relationship between two quantities

Cluster Standard: HSF.BF.A.1

Standard	Standards for Mathematical Practice
Write a function that describes a relationship between two quantities. * • HSF.BF.A.1.B Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	• **SMP2:** Reason abstractly and quantitatively. • **SMP4**: Model with mathematics. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should write **functions** for given **relationships** between **quantities**. Students can use functions to **model real-life situations** and make **predictions**. Students should be able to use functions to describe relationships between two quantities, usually x and $f(x)$, where $f(x)$ is some **output** value that depends on the **input** value x. Within a context, students should be able to express a given relationship as a function.	• Build a function using different functions and arithmetic operations in context.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students may want to try to use a linear function, specifically the slope-intercept form for every situation.
- Students may tend to focus on the symbolic form of a function and may need additional support in working with other forms.

Grade	CCSS Domain	CCSS Cluster
A2	Interpreting Functions	Build new functions from existing functions

Cluster Standard: HSF.BF.B.3

Standard	Standards for Mathematical Practice
Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	• **SMP5**: Use appropriate tools strategically. • **SMP8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should describe the effect of **stretches, shrinkages**, **vertical and horizontal transformations** on **functions**. They should be able to find the value of the **transformation** when given a **graph** and be able to explain effects of transformations using technology. Students should know that adding a **constant** k to a function will change the graph of the function depending not only on the value of the constant, but on where it is inserted as well. If $y = f(x)$ is changed to $y = f(x) + k$, the curve will **shift** vertically (up for $k > 0$, down if $k < 0$). Adding k to x such that $y = f(x + k)$ will shift the curve horizontally (left for $k > 0$, right for $k < 0$). Multiplying $f(x)$ by a constant k stretches ($k > 1$) or **squishes** ($0 < k < 1$) the graph vertically. If $k < 0$, the graph is also **flipped** over the ***x*-axis**. Multiplying x by k stretches ($k > 0$) or squishes ($k < 0$) the graph horizontally.	• Identify vertical transformations from a function or a graph. • Identify horizontal transformations from a function or a graph. • Identify a shrink or a stretch from a function or a graph. • Write the results from such transformations. • Recognize odd and even functions. • Identify transformations of a function on a graph. • Describe the effects of transformations on parent functions.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A2	Interpreting Functions	Build new functions from existing functions

Cluster Standard: HSF.BF.B.4.A

Standard	Standards for Mathematical Practice
Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x^3 or f(x) = (x+1)/(x-1) for $x \neq 1$.*	• **SMP 6:** Attend to precision. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should be able to find the **inverse** of **functions** and recognize that other functions may not have an inverse unless there are **restrictions** placed on the **domain**. If $f(x) = y$ is a function, the inverse function can be found by switching the place of x and y (f(y) = x), and then solving for y so that f-1(x) = y. For instance, if the function $f(x)$ is $y = 2x^3$, then the inverse function $f^{-1}(x)$ consists of switching the places of x and y ($x = 2y^3$) and then solving for y.	• Write the inverse of a function. • Determine restrictions on the domain to allow for an inverse to exist. • Relate using an inverse as an operation that undoes another operation. • Solve an equation of the form $f(x)=c$ for a function f that has an inverse and write an expression for the inverse.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students often have difficulty determining the direction of the horizontal shifts.
- Students often confuse the notation for the inverse and negative numbers.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Building Functions**	Strand: **Build a function that models a relationship between two quantities**

Suggested Student Discourse Questions

- In what ways are the independent and dependent variables related? How do they differ?
- Think about a relationship between two variables (time spent doing homework vs grade in class) - can you sketch a graph modeling this relationship? (I.e., Which is the independent variable? Which is dependent? Why?) Now pass your sketch to your partner. Does your partner's sketched function make sense in the context they came up with? Why or why not?
- How is the rate of change of this (insert function here - linear, quadratic, exponential, cubic, logarithmic) function represented in a table? How is it represented in the function's graph?
- Can you write an equation or draw a graph to model your monthly cell phone data use? What happens if you don't have unlimited data? Can you write a second equation or draw a second graph to model what happens when you hit your data limit? How could you combine both functions (or graphs) to model your monthly cell data use both before and after it hits the limit?

ASSESSMENT GUIDE

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

Grade	*CCSS Domain*	*CCSS Cluster*
A2	Building Functions	Build a function that models a relationship between two quantities
	Sample Task #1 (Constructed Response)	
	Standards Aligned Instructionally Embedded Formative Assessment Resources: Source: http://tasks.illustrativemathematics.org/content-standards/HSF/BF/A/2/tasks/1695 This type of assessment question requires students to analyze a number pattern described in context and fit both a recursive function to the pattern and use it to answer questions. Students will engage with SMP1 and SMP4 as they persevere to express the pattern mathematically and model the scenario with an equation	

Grade	*CCSS Domain*	*CCSS Cluster*
A2	Building Functions	Build new functions from existing functions
	Sample Task #1 (Constructed Response)	
	Standards Aligned Instructionally Embedded Formative Assessment Resources: http://tasks.illustrativemathematics.org/content-standards/HSF/BF/B/3/tasks/742 This type of assessment question requires students to apply vertical and horizontal translations as well as a reflection to a given graph. Further, students are asked to identify the location of specific coordinates on the new graphs. This will engage students with SMP7 as they use the structure of the graph, the expression of the transformation and/or a table of values to create new graphs and identify the imaged points. https://www.engageny.org/resource/algebra-i-module-3-topic-c-lesson-17	

MLSS AND CLR GUIDE

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

CCSS Domain	*CCSS Cluster*
Building Functions	Build a function that models a relationship between two quantities

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on building a function that models a relationship between two quantities, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning about the various ways that functions relate to quantities can be a great way to connect school and home with making references to those quantities that can be encountered at home and how they relate to the tasks being created in the classroom. Making that connection allows for students to become more comfortable with learning the content and provides evidence of prior knowledge that the student can bring into the lessons.	
Cross-Curricular Connections	"https://www.nextgenscience.org/topic-arrangement/hsinheritance-and-variation-traits"Science: In high school the NGSS students should apply concepts of statistics and probability to explain the variation and distribution of expressed traits in a population. Consider providing a connection for students to examine scientific data and predict the effect of a change in one variable on another. https://www.nextgenscience.org/topic-arrangement/hsinheritance-and-variation-traits	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures*	• Task: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when building a function that models a relationship between two quantities the types of mathematical tasks are critical because when students are able to make connections, it is

	and languages? ● *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	easier for them to learn and store information, like making a connection to background knowledge or prior learning to create an optimal environment for learning, as they bring this knowledge with them to class each day.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
● Connect to Algebra 1 work focusing on linear, exponential, and quadratic within this cluster. ● Connect to recognizing situations that grow by a constant rate or percent. **(HSF.LE.1)**	● Connect to identifying patterns in the function's rate of change, specifying intervals of increase and decrease, and graphing to model functions. **(HSF.IF.4,6)** ● Connect to discussing the relative strengths and weaknesses of each representation and which are most efficient to be able to assist them in making symbolic functions. **(HSF.IF.9)**	● Connect to arithmetic and geometric sequences and use them to model situations. **(HSF.BF.A.2)**

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit (e.g., cell phone plans). When building a function that models a relationship between two quantities, the new contexts will show an alignment to new material that will be covered.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.F.B.4: This standard provides a foundation for work with building a function that makes a relationship between two quantities because prior learning on constructing a function modeling a linear relationship between two quantities is learned and will be expanded on in the current unit. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• What the domain of a function represents in terms of the function and what values are appropriate (i.e., discrete v. continuous values and other restrictions based on the type of function and the given context).
• In functions there is an underlying structure that determines the transformation of any
	• Find and interpret key features of a graph or table of a function, including extreme values, end behavior, and intervals of increase and decrease.
	• identify and graph parent functions and their transformations (i.e. vertical translation, horizontal
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Building and solving proportional relationships (7.RP.A.2)
		  ○ Recognize and use function notation (HSF.IF.A)
		  ○ Recognizing inverse functions (HSF.BF.B.4)
		  ○ Graphing, solving and modeling quadratic, linear, exponential and absolute value functions (HSF.LE.A, HSF.LE.B)
		  ○ Recognize, build and solve trigonometric ratios (HSG.CRT.C)

function, regardless of its type. ● How to use trigonometric ratios, reference angles, and symmetry to find patterns on the unit circle. ● Inverse trigonometric functions have restricted domains and ranges and are one to one.	translation, vertical stretch/shrink, reflect over x-axis, etc.). ● Calculate values of sine, cosine, and tangent for given angles. ● Solve trigonometric equations, including those written in quadratic form and equations containing more than one angle.	● Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers ○ SOH CAH TOA

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on building a function that models a relationship between two quantities by revisiting student thinking through a short mini-lesson because students should be able to recall prior knowledge in the content previously learned and can use that prior knowledge to build on current content.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit building a function that models a relationship between two quantities by helping students move from specific answers to generalizations for certain types of problems because recalling prior knowledge will aid the student with current understanding and show the alignment to prior knowledge and will engage the student in the current content.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying building a function that models a relationship between two quantities because activating prior knowledge will allow for students to take the understanding of the current content to a greater level and will allow for better understanding of the content.

CCSS Domain	*CCSS Cluster*
Building Functions	Build new functions from existing functions

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on building new functions from existing functions, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning about the characteristics of building functions from existing functions allows for making connections to building in any concept which can be done at home such as working on a home project that requires building on to something that already exists. Students can make connections to prior learning (something that already exists) and build onto that knowledge.	
Cross-Curricular Connections	Science: The equation for velocity, $M(v) = 6v^2$, is one where the variable, v, has directions. Therefore, an inverse function of $M(v)$ cannot give back both a positive and negative velocity. Consider providing a connection for students to consider how they will handle this situation.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical*	• Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when building new functions from existing functions goal setting is critical because it allows students to take ownership of the content and what the expectations for learning are as they are clearly identified while making a meaningful connection between the learning and daily lives.

	identities as capable mathematicians that can use mathematics within school and society?	

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to the work in Algebra 1 with linear, exponential, quadratic, and absolute value functions for this cluster. **(HSF.BF.B)**	• Connect to graphing functional relationships. **(HSF.IF.4)** • Connect to trigonometric functions. **(HS.F-TF.B)** •	• Connecting to understanding the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents. **(HS.F-BF.B.5)**

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on building new functions from existing functions because in prior lessons and grade levels, students have been introduced to many aspects and content of functions.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	HS.F-IF.A.1: This standard provides a foundation for work with building new functions from existing functions because this prerequisite has students understanding functions based on domain and range. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• What the domain of a function represents in terms of the function and what values are appropriate (i.e. discrete v. continuous values and other restrictions based on the type of function and the given context). • In functions there is an underlying structure that determines the transformation of any function, regardless of its type. • How to use trigonometric ratios, reference angles, and symmetry to find patterns on the unit circle. • Inverse trigonometric functions have restricted domains and ranges and are one to one.	• Find and interpret key features of a graph or table of a function, including extreme values, end behavior, and intervals of increase and decrease. • identify and graph parent functions and their transformations (i.e. vertical translation, horizontal translation, vertical stretch/shrink, reflect over x-axis, etc.). • Calculate values of sine, cosine, and tangent for given angles. • Solve trigonometric equations, including those written in quadratic form and equations containing more than one angle.	• Build on students' experience with the following skills: ○ Graphing on the coordinate plane (6.NS.C.8) ○ Building and solving proportional relationships (7.RP.A.2) ○ Recognize and use function notation (HSF.IF.A) ○ Recognizing inverse functions (HSF.BF.B.4) ○ Graphing, solving and modeling quadratic, linear, exponential and absolute value functions (HSF.LE.A, HSF.LE.B) ○ Recognize, build and solve trigonometric ratios (HSG.CRT.C) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers ○ SOH CAH TOA

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Level of Intensity	***Essential Question***	***Examples***
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Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit building new functions from existing functions by offering opportunities to understand and explore different strategies because providing students with various strategies allows for further depth in understanding and further delving into the depth of the content.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to explore links between various topics when building new functions from existing functions because their learning potential is increased, and prior knowledge activation is improved upon and built upon.

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Standards Breakdown

- Perform arithmetic operations with complex numbers.
  - HSN.CN.A.1
  - HSN.CN.A.2
- Use complex numbers in polynomial identities and equations.
  - HSN.CN.C.7

Grade	CCSS Domain	CCSS Cluster
A2	The Complex Number System	Perform arithmetic operations with complex numbers

Cluster Standard: HSN.CN.A.1

Standard	Standards for Mathematical Practice
Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP2:** Reason abstractly and quantitatively. • **SMP6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Complex numbers expand the number system to include square roots of negative numbers and allow applications of complex numbers to electronics. Students use the properties of operations as it applies to complex numbers to simplify expressions and to build foundations to solve quadratic equations having complex solutions.	• Identify the real number and the imaginary number of a complex number • Define an imaginary number (i.e. $i^2 = -1$). • Define complex numbers. • Find the complex conjugate. • Describe complex numbers in terms of their real and imaginary parts.
DOK	**Blooms**
1-2	Understand

Grade	CCSS Domain	CCSS Cluster
A2	The Complex Number System	Perform arithmetic operations with complex numbers

Cluster Standard: HSN.CN.A.2

Standard	Standards for Mathematical Practice
Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2:** Reason abstractly and quantitatively. • **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Complex numbers expand the number system to include square roots of negative numbers and allow applications of complex numbers to electronics. Students use the properties of operations as it applies to complex numbers to simplify expressions and to build foundations to solve quadratic equations having complex solutions.	• Recognize that $i^2 = -1$ • Use the properties of operations to add and subtract complex numbers • Use the distributive property and the relation $i^2 = -1$ to multiply complex numbers. • Apply the commutative, associative, and distributive properties to complex numbers in order to add, subtract, and multiply.
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Since most variables are letters and symbols, students may confuse *i* as a variable.
- Students may try to simplify a complex number by combining the real part and the imaginary part.

Grade	CCSS Domain	CCSS Cluster
A2	The Complex Number System	Use complex numbers in polynomial identities and equations

Cluster Standard: HSN.CN.C.7

Standard	Standards for Mathematical Practice
Solve quadratic equations with real coefficients that have complex solutions.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 3**: Construct viable arguments and critique the reasoning of others.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students will be able to use multiple methods to solve quadratic equations with complex solutions.	• Determine the number and nature of quadratic solutions • Solve a quadratic equation using various methods (e.g., factoring, completing the square, quadratic formula)
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Students may confuse non-real, imaginary and irrational numbers.

ASSESSMENT GUIDE

- Perform arithmetic operations with complex numbers.
- Use complex numbers in polynomial identities and equations.

Grade	*CCSS Domain*	*CCSS Strand*
A2	The Complex Number System	Perform arithmetic operations with complex numbers

Sample Task #1 (Constructed Response)

CollegeBoard **Question ID 5344950**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Additional Topics in Math	Hard	Additional Topics in Math	Complex numbers	1. Apply knowledge and understanding of the complex number system to add, subtract, multiply, and divide with complex numbers and solve problems.	No Calculator

$i^2+(-i)^2$

In the complex number system, what is the value of the given expression? (Note: $i=\sqrt{-1}$)

Question Difficulty: Hard

A. -2

B. 0

C. 2

D. $2i$

Choice A is correct. The power of a product property states that $(xy)^a=x^a y^a$. Using this property, the second term of the given expression can be rewritten as $(-1\times i)^2=(-1)^2 i^2$, or i^2. Substituting i^2 in place of $(-i)^2$ in the given expression yields i^2+i^2, or $2i^2$. Since $i=\sqrt{-1}$, $i^2=-1$ and $2i^2=2(-1)$, or -2.

Choice B is incorrect and may result from rewriting $(-i)^2$ as $-i^2$ instead of i^2. Choice C is incorrect and may result from rewriting i^2 as 1 instead of -1. Choice D is incorrect and may result from rewriting i^2 as i instead of -1.

Grade	CCSS Domain	CCSS Cluster
A2	The Complex Number System	Use complex numbers in polynomial identities and equations

Sample Task #1 (Constructed Response)

Renee reasons as follows to solve the equation $x^2+x+1=0$.

First I will rewrite this as a square plus some number.

$$x^2+x+1=(x+12)^2+3/4$$

Now I can subtract 34 from both sides of the equation

$$(x+12)^2=-3/4.$$

But I can't take the square root of a negative number so I can't solve this equation.

a. Show how Renee might have continued to find the complex solutions of $x^2+x+1=0$.
b. Apply Renee's reasoning to find the solutions to $x^2+4x+6=0$.

http://tasks.illustrativemathematics.org/content-standards/HSN/CN/C/7/tasks/1690

MLSS AND CLR GUIDE

- Perform arithmetic operations with complex numbers.
- Use complex numbers in polynomial identities and equations.

CCSS Domain	*CCSS Cluster*
The Complex Number System	Perform arithmetic operations with complex numbers

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on performing arithmetic operations with complex numbers, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students, learning about the history of the complex number systems and how the complex numbers originated and used in other countries. Different families can contribute small history pieces and will eventually turn into a big presentation to the class by the student.	
Cross-Curricular Connections	**Science -** Science and Electrical Engineering use complex numbers, especially when dealing with light and radio wave. http://faculty.wcas.northwestern.edu/~infocom/Ideas/electric.html **History -** The ancient Greeks once believed that all numbers were rational numbers; that is, that every number could be expressed as the ratio of two integers, and they were very disturbed when it was demonstrated that the measure of the hypotenuse of an isosceles right triangle, having arms of unit measure, was not a rational number. http://mathforum.org/library/drmath/view/55747.html	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create*	• Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, when performing arithmetic operations with complex numbers, eliciting and using student thinking is critical because when making mistakes and finding errors, students make adjustments and begin asking questions without any repercussions. They are comfortable and know mistakes are allowed and corrections can be made.

	connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	Mistakes allow students to try instead of leaving questions blank. Challenging questions can also lead to critical thinking and when a task is complete, whether it's right or wrong, students feel the ownership of learning.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In Algebra 1, students solved quadratic equations using a variety of methods. Their solutions however were limited to real solutions.	• Students will learn to solve quadratic and higher-order polynomial equations that have complex answers as those found within this cluster.	• Students will relate this knowledge of complex numbers to solving rational equations, trigonometric equations and trigonometric form in subsequent math courses (Pre-Calculus, AP Calculus, College Algebra, etc).

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that introduces new representations (e.g., number lines) when studying to perform arithmetic operations with complex numbers because students no longer will be using real numbers on both axes in their graphs. The y-axis will be used for the imaginary numbers.

Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.NS.A.1: This standard provides a foundation for work with performing arithmetic operations with complex numbers because all numbers are classified as rational or irrational. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on performing arithmetic operations with complex numbers by revisiting student thinking through a short mini-lesson because one of the students' misconception is that the i is another variable. Check for misconceptions using aggressive monitoring.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit performing arithmetic operations with complex numbers by offering opportunities to understand and explore different strategies because students may make the connection between the properties of equations and the procedures within the complex number operations.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension such as the opportunity to explore links between various topics because students explore how the operations will be used in later lessons by watching a short video.

CCSS Domain	*CCSS Cluster*
The Complex Number System	Use complex numbers in polynomial identities and equations

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on using complex numbers in polynomial identities and equations, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning about the different complex numbers and making reference to how they could be used at home or in the community can be a great way to connect the tasks to their own personal tasks.	
Cross-Curricular Connections	During a unit focused on using complex numbers in polynomial identities and equations, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning about the different complex numbers and making reference to how they could be used at home or in the community can be a great way to connect the tasks to their own personal tasks.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical*	• Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when using complex numbers in polynomial identities and equations, goal setting is critical because students are able to make connections to their learning and prior knowledge can be accessed when goals are clearly identified.

	identities as capable mathematicians that can use mathematics within school and society?	

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In Algebra 1, students solved quadratic equations using a variety of methods. Their solutions were limited however to real solutions.	• Students learn to solve polynomial equations that have complex answers.	• Students will connect this knowledge of complex numbers to solving rational equations, trigonometric equations and trigonometric form in subsequent math courses (Pre-Calculus, AP Calculus, College Algebra, etc).

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that uses complex numbers in polynomial identities and equations because students will have to recall prior knowledge from previous grade levels.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	7.EE.A.1: This standard provides a foundation for work using complex numbers in polynomial identity and equations because students should be able to apply properties of operation strategies. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on using complex numbers in polynomial identities and equations by providing specific feedback to students on their work through a short mini-lesson because students who are having difficulty or who may be struggling will be able to get immediate feedback which will help them to better understand possible misconceptions.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit using complex numbers in polynomial identities and equations by offering opportunities to understand and explore different strategies. Students will be able to visualize different perspectives with the different strategies and may get a better understanding of the content being presented.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when using complex numbers in polynomial identities and equations because students will be able to direct their studying to the specific areas that they need further clarification in.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Summarize, represent, and interpret data on a single count or measurement variable
  - HSS.ID.A.4

Grade	CCSS Domain	CCSS Cluster
A2	Interpreting Categorical & Quantitative Data	Summarize, represent, and interpret data on a single count or measurement variable

Cluster Standard: HSS.ID.A.4

Standard	Standards for Mathematical Practice
Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 7:** Look for and make use of structure. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• At this level, students are not expected to fit **normal curves** to data. (In fact, it is rather complicated to rescale data plots to be **density plots** and then find the best fitting curve.) Instead, the aim is to look for broad **approximations**, with application of the rather rough "**empirical rule**" (also called the 68%–95% Rule) for distributions that are somewhat **bell-shaped**. The better the bell, the better the approximation. Using such approximations is partial justification for the introduction of the **standard deviation**.	• Explain data distributions: when the data is normal, mean and standard deviation are used to represent the data; when data is skewed, median and Interquartile Range (IQR) are used to represent the data. • Explain why normal distribution can be used to estimate population percentages. • Estimate areas under a normal curve. • Calculate mean, median (Q2), standard deviation, IQR (Q1 & Q3). • Calculate z- score based on mean and standard deviation. • Use the empirical rule to understand area under the distribution • Apply this knowledge to estimate population percentages. • Use real world data to determine distributions.
DOK	**Blooms**
1-4	Remember, Understand, Apply, Analyze, Evaluate

Common Misconceptions

- Students may believe all data follows a normal distribution because of prior work with a sampling distribution of means.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Interpreting Categorical & Quantitative Data**	Strand: **Summarize, represent, and interpret data on a single count or measurement variable**

Suggested Student Discourse Questions

- In what ways are the mean, median, and mode represented in the population of _____?
- Which strategy (calculator, spreadsheet, table) are you using? In what ways does it represent the data in the same manner as (student's name) strategy? In what ways is it different?
- How is using a calculator different from a spreadsheet or a table?
- How can this be used to estimate the population of _____? (Elk in the forest, people in the city, high school students in NM, COVID-positive cases in the USA, etc.)

NEW MEXICO
Public Education Department

ASSESSMENT GUIDE

- Summarize, represent, and interpret data on a single count or measurement variable

Grade	*CCSS Domain*	*CCSS Cluster*
A2	Interpreting Categorical & Quantitative Data	Summarize, represent, and interpret data on a single count or measurement variable
	Sample Task #1 (Constructed Response)	
	Standards Aligned Instructionally Embedded Formative Assessment Resources: http://tasks.illustrativemathematics.org/content-standards/HSS/ID/A/4/tasks/1020 This type of assessment requires students to analyze a scenario approximately modeled by a normal distribution with given mean and standard deviation to make and support a statistical claim. Students will engage with SMP3 by making and supporting statistical claims. If students share their solutions with others, SMP3 can be further targeted by critiquing the solutions of peers.	

NEW MEXICO
Public Education Department

MLSS AND CLR GUIDE

- Summarize, represent, and interpret data on a single count or measurement variable.

CCSS Domain	*CCSS Cluster*
Interpreting Categorical Data and Quantitative Data	Summarize, represent, and interpret data on a single count or measurement variable

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on analyzing data, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning about the different data analytical techniques helps provide a robust set of results which provide differing points of view. For example, mean and median of a data set can be used to estimate the symmetry of a curve, providing insight into the number of people that might be affected at the extremes of the population.	
Cross-Curricular Connections	Social Studies: In high school the New Mexico Social Studies Standards state students should "explain how to use technological tools to research data, verify facts and information, and communicate findings." Consider providing a connection for students to write a report describing and analyzing a specific set of data.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home*	• Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying HS.ID.A: Summarize, represent, and interpret data on a single count or measurement variable cluster, goal setting is critical because statistics are often used to help describe how a specific ethnic or cultural group is doing and what needs they may need. The census is the largest statistical tool the United States uses to help with this objective.

	culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to work in previous math courses: when students learned to determine mean, median, mode, range, IQR, minimum, maximum. Students also learned how to graph data distributions (e.g., histograms, box plots).	• Connect students using this information to make inferences and justify conclusions from sample surveys, experiments and observational studies.	• Connect to future work students may do in subsequent statistics courses (AP or college level).

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	• Some learners may benefit from targeted pre-teaching that analyzes common misconceptions when summarizing, representing, and interpreting data on a single count or measurement variable because there are common misconceptions which learners often encounter. Addressing these misconceptions will often resolve issues that would otherwise have to be addressed in more time-consuming one-on-one instruction.

		• For example: Learners often misunderstand the meaning of range and standard deviation. Directly targeting this misconception can often allow students to move forward without time-consuming one-on-one instruction.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	6.SP.B.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.: This standard provides a foundation for work with the Interpreting Categorical And Quantitative Data cluster because learners which don't understand the meaning and use of different types of data graphs will have difficulty interpreting them for meaning when solving problems. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on summarizing, representing, and interpreting data on a single Count or measurement variable by providing specific feedback to students on their work through a short mini-lesson because learners often get hung up on a single misconception or step. Clearing that step will often enable the learner to progress in solving the problem.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit in the Summarize, Represent, And Interpret Data On A Single Count Or Measurement Variable cluster by helping students move from specific answers to generalizations for certain types of problems because learners sometimes have difficulties with recall or have difficulty with the cognitive load of recalling and processing information spread out over one or more days. This is especially true when instruction is spread over weekends or breaks. Intensive Reteaching eases this cognitive load and empowers learners to accomplish the task.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying how to summarize, represent, and interpret data on a single count or measurement variable. The non-Gaussian distribution is an example because there may be learners that need additional challenges beyond the curriculum.

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	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Interpret functions that arise in applications in terms of the context.
  - HSF.IF.B.4
  - HSF.IF.B.5
  - HSF.IF.B.6
- Analyze functions using different representations.
  - HSF.IF.C.7
  - HSF.IF.C.8
  - HSF.IF.C.9

Grade	*CCSS Domain*	*CCSS Cluster*
A2	Interpreting Functions	Interpret functions that arise in applications in terms of the context

Cluster Standard: HSF.IF.B.4

Standard	Standards for Mathematical Practice
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students interpret the **key features** of the different **functions** listed in the standard. When given a **table** or **graph** of a function that models a **real-life situation**, explain the meaning of the characteristics of the table or graph in the **context** of the problem. Key features of a **linear function** are **slope** and **intercepts**; of a **quadratic function** are **intervals of increase/decrease**, **positive/negative**, **maximum/minimum**, **symmetry**, and **intercepts**; of an **exponential function** include ***y*-intercept** and **increasing/decreasing intervals** and of an **absolute value** include ***y*-intercept**, **minimum or maximum**, **increasing or decreasing intervals**, and **symmetry**.	• Identify intercepts of a function. • Identify intervals where the function is increasing. • Identify intervals where the function is decreasing. • Identify intervals where the function is positive. • Identify intervals where the function is negative. • Identify relative maximums of a function. • Identify relative minimums of a function. • Identify symmetries in the functions. • Identify the end behavior of the functions. • Sketch graphs given a list of key features or a verbal model. • Sketch functions that model key feature behavior. • Label intercepts and intervals of a graph. • Interpret where the function is increasing, decreasing, positive, or negative. • Interpret relative maximums and minimums. • Interpret various symmetries, end behaviors, and periodicity.
DOK	**Blooms**
1-2	Understand, Apply and Analyze

Grade	*CCSS Domain*	*CCSS Cluster*
A2	Interpreting Functions	Interpret functions that arise in applications in terms of the context

Cluster Standard: HSF.IF.B.5

Standard	Standards for Mathematical Practice
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*	• **SMP 3:** Construct viable arguments and critique the reasoning of others**.** • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should focus their attention on possible **input and output values**, framing them as the **domain and range of a function**. When given a description of a function that represents a situation, the students should determine reasonable domain and range. Students relate the domain of a function to its graph and, where applicable, to the **quantitative relationship** it describes. Students need to explain the reasonableness of a domain for a given context. • Students should understand that the domain of a function is the set of all possible inputs and the range is the set of all possible outputs. Also looking at if a function is **continuous** (time, amount of liquid filling a container) or discrete (number of people or things) and connecting back to number classifications.	• Make connections between a graph of a function and its domain. • Make connections between the graph of a function and the context it describes. • Identify when the domain of a given context is discrete or continuous and explain why.
DOK	**Blooms**
1-2	Understand, Apply and Analyze

Grade	*CCSS Domain*	*CCSS Cluster*
A2	Interpreting Functions	Interpret functions that arise in applications in terms of the context

Cluster Standard: HSF.IF.B.6

Standard	Standards for Mathematical Practice
Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	• **SMP 4**: Model with mathematics. • **SMP 5:** Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students will calculate and interpret the **average rate of change** of a linear, quadratic, **piecewise linea**r (to include absolute value), and exponential function (presented **symbolically** or as a table) over a specified **interval.** Students will **estimate** the rate of change from a graph. In addition to finding average rates of change from functions given symbolically, graphically, or in a table, students may collect **data** from **experiments** or **simulations** (ex. falling ball, velocity of a car, etc.) and find average rates of change over various intervals.	• Calculate the average rate of change of a function over a specified interval presented symbolically. • Calculate the average rate of change of a function over a specified interval presented in a table. • Interpret the average rate of change of a function over a specified interval presented symbolically for a given context. • Interpret the average rate of change of a function over a specified interval presented in a table for a given context. • Estimate the rate of change of a function from a graph.
DOK	**Blooms**
1-2	Understand, Apply and Analyze

Common Misconceptions

- Students may confuse scatter plots and correlations.
- Students may focus on the y values of the graph instead of the x values of the interval, when identifying key features of a graph.
- Students may have difficulty understanding the domain.
- Students may confuse independent and dependent variables.
- Students may confuse shift with rate of change.

Grade	CCSS Domain	CCSS Cluster
A2	Interpreting Functions	Analyze functions using different representations

Cluster Standard: HSF.IF.C.7

Standard	Standards for Mathematical Practice
Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. • HSF.IF.C.7.B: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. • HSF.IF.C.7.C: Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. • HSF.IF.C.7.E: Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	• **SMP 4:** Model with mathematics • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should be able to describe the significant features of different **functions graphically** and **algebraically**. Students should be able to use the **significant features** to **sketch** the graph of the function. Students should graph **linear** and **quadratic functions** and show **intercepts**, **maxima**, and **minima.** Students should know the **slope-intercept form** of linear functions, $y = mx + b$, and how to extract enough information from the equation to be able to draw it. When graphing **roots**, remember that for $\sqrt[n]{x}$, if n is **even,** the domain includes all **positive integers**. Otherwise, **negative** values are included as well. When graphing roots of the for $y = a\sqrt{x} + b$, remember the ***y*-intercept** is b. Students should remember that roots are	• Graph exponential, logarithmic, and trigonometric functions. • Describe key features of exponential, logarithmic, and trigonometric functions. • Graph functions expressed symbolically showing key features of the graph by hand in simple cases and with technology for more complicated cases. • Compare and contrast functions.

fractional exponents. Students should know to look at the **highest degree of the polynomial** and its **coefficient,** ax^n. If *n* is even, the function will extend either **up** or **down** on both ends (as *x* goes to **positive** or **negative infinity**). If *n* is **odd**, they'll go in **opposite directions**. If *a* is positive, the even powered functions will go up and the odd powered functions will start down and go up. If *a* is negative, the even powered functions will go down, and the odd powered functions will start up and go down.	
DOK	**Blooms**
1-2	Understand, Apply and Analyze

Grade	CCSS Domain	CCSS Cluster
A2	Interpreting Functions	Analyze functions using different representations

Cluster Standard: HSF.IF.C.8

Standard	Standards for Mathematical Practice
Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. • HSF.IF.C.8.A: Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. • HSF.IF.C.8.B: Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay.	• **SMP 4:** Model with mathematics. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should be able to rewrite quadratic and **exponential functions** in different ways to find key features of the expression and interpret those key features in terms of the context they represent. Students should be able to find the ***x*-intercepts** of a quadratic function using both **factoring** and **completing the square**.	• Rewrite a function to find and highlight key features. • Factor a quadratic expression to find zeros, extrema and symmetry • Interpret the meaning of zeros, extrema and symmetry within the context of a problem. • Complete the square for a quadratic function to reveal its key features. • Interpret the key features of a quadratic expression in terms of the context it represents. • Use properties of exponents to relate parts of an exponential function to its context (e.g., describe the initial value, growth/decay rate or factor and the growth period). • Identify how key features of an exponential function relate to characteristics in a real-world context. • Classify real-world problems as an exponential growth or decay.

DOK	Blooms
1-2	Understand, Apply and Analyze

Grade	*CCSS Domain*	*CCSS Cluster*
A2	Interpreting Functions	Analyze functions using different representations

Cluster Standard: HSF.IF.C.9

Standard	Standards for Mathematical Practice
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum*.	• **SMP 5:** Use appropriate tools strategically. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should be able to compare two given functions (linear, exponential, quadratic) whether that be as a function or **equation**, in a **table,** in a **graph,** or by **verbal description**. Students should start by knowing the difference between linear, quadratic and exponential functions, and be able to identify them by equation and by graph. Students should be able to compare two functions even when they're both represented differently. To do this successfully, they must be able to **translate** between an equation, a graph, verbal, and a table of values, and understand how certain aspects of one **representation** impact the rest.	• Make comparisons between functions in different forms using their knowledge of key features.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students may have difficulty identifying the key features needed to sketch the graphs or identifying those features algebraically.
- Students may have difficulty with contextualizing and decontextualizing expressions.
- Students will often confuse functions given in a table as a representation of a finite set of numbers rather than a subset of the entire function. They also may have difficulty with the abstractness of determining what is happening with a function over intervals of the domain that they cannot see.
- Students may not distinguish between the different type of logarithms, i.e., natural logs, when using calculator
- Students may struggle with applying translations, stretches, compressions, and reflections to a parent function.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Interpreting Functions**	Strand: **Interpret functions that arise in applications in terms of the context**

Suggested Student Discourse Questions

- Can you identify (insert key feature here - increasing / decreasing interval, relative maximum / minimum, zeros, positive / negative intervals, symmetries, etc.) using the table? Can you identify it on the graph?
- Turn and talk to your partner - how is their strategy in finding (insert key feature here - increasing / decreasing interval, relative maximum / minimum values, zeros, positive / negative intervals, intercepts, symmetries, etc.) different from yours? Which strategy is more efficient? In what contexts would your strategy work better? In which contexts would your partner's strategy work better?
- How are graphical, analytical, and tabular representations of this function the same? How do they differ? (Guide student thinking towards key features of given functions)
- Which function would be better used to model growth of a tree? Population growth? A football's trajectory across the field? (Extension - use regression to model each)

Domain: **Interpreting Functions**	Strand: **Analyze functions using different representations**
Suggested Student Discourse Questions	
• A sine wave models the height of a rider in a Ferris wheel. What does the amplitude mean in this context? What about midline? Period? • Can you identify your partner's zeros? End behaviors? Intercepts? Asymptotes? How are your sketches and your partner's sketches the same? How are they different? (Switch out the functions - repeat with cubic / quartic functions, or sine and cosine functions).	• Compare graphs polynomial, trigonometric and logarithmic functions - how are the zeros, intercepts, and end behavior similar? How are they different? • Would you use a polynomial, trigonometric, or logarithmic function to model the volume of your water bottle? Would that function also be useful to model sound waves? Why or why not?

ASSESSMENT GUIDE

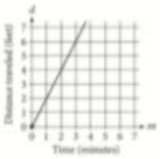
- Interpret functions that arise in applications in terms of the context.
- Analyze functions using different representations.

Grade	*CCSS Domain*	*CCSS Cluster*
A2	Interpreting Functions	Interpret functions that arise in applications in terms of the context

Sample Task #1 (Constructed Response)

CollegeBoard **Question ID 1474415**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	Easy	Heart of Algebra	Linear functions	4. Make connections between verbal, tabular, algebraic, and graphical representations of a linear function, by c. determining how a graph is affected by a change to its equation.	No Calculator



The graph above shows the distance traveled d, in feet, by a product on a conveyor belt m minutes after the product is placed on the belt. Which of the following equations correctly relates d and m ?

Question Difficulty: Easy

A. $d = 2m$

B. $d = \frac{1}{2}m$

C. $d = m + 2$

D. $d = 2m + 2$

Choice A is correct. The line passes through the origin. Therefore, this is a relationship of the form $d = km$, where k is a constant representing the slope of the graph. To find the value of k, choose a point (m,d) on the graph of the line other than the origin and substitute the values of m and d into the equation. For example, if the point $(2,4)$ is chosen, then $4 = k(2)$, and $k = 2$. Therefore, the equation of the line is $d = 2m$.

Choice B is incorrect and may result from calculating the slope of the line as the change in time over the change in distance traveled instead of the change in distance traveled over the change in time. Choices C and D are incorrect because each of these equations represents a line with a d-intercept of 2. However, the graph shows a line with a d-intercept of 0.

Grade	CCSS Domain	CCSS Cluster
A2	Interpreting Functions	Analyze functions using different representations

Sample Task #1 (Constructed Response)

Standards Aligned Instructionally Embedded Formative Assessment Resources:

http://tasks.illustrativemathematics.org/content-standards/HSF/IF/C/9/tasks/1279

This type of assessment question requires students to analyze function and a graph and compare the key features in context of a scenario. Students will engage with SMP 7 as they use the structure of both the equation and the graph to answer questions in context.

Additional assessment:
Analyzing Graphs
https://www.map.mathshell.org/lessons.php?unit=9245&collection=8
https://www.map.mathshell.org/lessons.php?unit=9240&collection=8

MLSS AND CLR GUIDE

- Interpret functions that arise in applications in terms of the context.
- Analyze functions using different representations.

CCSS Domain	*CCSS Cluster*
Interpreting Functions	Interpret functions that arise in applications in terms of the context

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on interpreting functions that arise in applications in terms of the context, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, families create their own story and make the connection with a table and graph and identify key features. Every family will have a different story.	
Cross-Curricular Connections	In this lesson, students use exponential decay and rational functions to understand why addicted patients seek more and stronger opioids to alleviate their pain. Students discuss the role that various parties played in creating the crisis and ways they can help to solve it. House of Pain: A Lesson by Mathalicious	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in*	• Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice leads to more students viewing themselves as capable mathematicians. For example, when interpreting functions that arise in applications in terms of the context the types of mathematical tasks are critical because clearly defined tasks set the routine for interaction and support for students. Interpreting and sketching key characteristics of graphs and tables, students make the connections graphically, verbally, tabularly, and symbolically. Allowing students to explain, think out loud, making conjectures, and communicate with peers to come up with mathematical ideas.

	creating mathematical identities as capable mathematicians that can use mathematics within school and society?	

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to the work of Algebra 1 within this cluster around linear, quadratic, and exponential. **(HSF.IF.B)**	• Connect to discovering features of families of functions. **(HSF.IF.7)** • Connect to finding key features of the entire family of functions. **(HSF.IF.4)**	• Connect to the work with trigonometric functions. **(HST.TF.B)**

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that analyzes common misconceptions when interpreting functions because quantities in graphs and tables should be interpreted in context to the problem and domains should be within the context as well.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.F.A.1: This standard provides a foundation for work with interpreting functions that arise in applications in terms of the context because students need to interpret the ordered pairs on the graph for analyzing or making predictions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• What the domain of a function represents in terms of the function and what values are appropriate (i.e. discrete v. continuous values and other restrictions based on the type of function and the given context).
• In functions there is an underlying structure that determines the transformation of any function, regardless of its type.
• How to use trigonometric ratios, reference angles, and symmetry to find patterns on the unit circle.
• Inverse trigonometric functions have restricted domains and ranges and are one to one.
	• Find and interpret key features of a graph or table of a function, including extreme values, end behavior, and intervals of increase and decrease.
	• identify and graph parent functions and their transformations (i.e. vertical translation, horizontal translation, vertical stretch/shrink, reflect over x-axis, etc.).
	• Calculate values of sine, cosine, and tangent for given angles.
	• Solve trigonometric equations, including those written in quadratic form and equations containing more than one angle.
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Building and solving proportional relationships (7.RP.A.2)
		  ○ Recognize and use function notation (HSF.IF.A)
		  ○ Recognizing inverse functions (HSF.BF.B.4)
		  ○ Graphing, solving and modeling quadratic, linear, exponential and absolute value functions (HSF.LE.A, HSF.LE.B)
		  ○ Recognize, build and solve trigonometric ratios (HSG.CRT.C)
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		• Encourage students to use alternative tools to better access the grade level content. Examples include:
		  ○ Desmos.com
		  ○ Graphing calculator
		  ○ Sketch a graph
		  ○ Create a table of values
		  ○ Algebra tiles
		  ○ Graphic organizers
		  ○ SOH CAH TOA

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	This standard 8.F.A.1 provides a foundation for work with interpret functions that arise in applications in terms of the context because students need to interpret the ordered pairs on the graph for analyzing or making predictions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit interpreting functions that arise in applications in terms of the context by addressing conceptual understanding because students need to make connections about the numbers they choose for their domain and range in context with the problem. They will need to interpret the characteristics of a graph

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when interpreting functions that arise in applications in terms of the context because students can interpret their own graphs and explore how graphs can be integrated according to their interest.

CCSS Domain	*CCSS Cluster*
Interpreting Functions	Analyze functions using different representations

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on analyzing functions using different representations, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, use community data and technology to graph, calculate and analyze the regression between the variables. Data and graphs can be used to make comparisons between the communities.	
Cross-Curricular Connections	Many of the Navajo rug designs you will discover by following the project will be good examples of symmetrical balance. Symmetrical balance is a type of visual balance where the overall composition is arranged to look like it is the same on both sides of the center of the design. In other words, it is a design which could be folded in half, and as the design folds, each part of the design would match up with its symmetrical counterpart on the opposite side of the center. The rug design on the right is symmetrical left-to-right. If a line was drawn vertically down the center of the rug, the arrangement of shapes and colors would appear to be exactly the opposite of each other on both sides of that line. Design a Navajo Rug	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities*	• Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying analyzing functions using different representations the pattern of questions within the classroom is critical because asking open ended questions allows the students to think, answer and have a reason for their answer. Ask probing questions that allow students to elaborate and clarify their different graphs and key

	as capable mathematicians that can use mathematics within school and society?	representations. The explanation of the different family functions from linear to trigonometric functions include all types of learners from the low to the high so everyone feels included.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to the work in Algebra 1 with this cluster which focused on linear, exponential, quadratic, absolute value, step, and piecewise defined by supporting Algebra 2 students to focus on using key features to guide selection of an appropriate type of model function.	• Connect to writing linear, quadratic, and exponential functions to describe relationships between quantities. **(HSA.CED.1-3)** • Connect to analyzing transformations of parent functions for linear, quadratic, and exponential functions. **(HSF.BF.3)**	• Connect to graphing all parent functions by hand and using technology and identifying their key features. **(HSF.IF.7)** • Connect to factoring to complete the square with quadratic functions with complex zeros. **(HSN.CN.7)**

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that introduces new representations (e.g., number lines) when analyzing functions because connections between the different representations allow students exposure to the family of functions (linear, quadratic, polynomial, etc.)
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	This standard 8.F.A.1 provides a foundation for working with analyzing functions using different representations because ordered pairs have a relationship, and the point is represented on the graph contextually. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• What the domain of a function represents in terms of the function and what values are appropriate (i.e. discrete v. continuous values and other restrictions based on the type of function and the given context).
• In functions there is an underlying structure that determines the transformation of any function, regardless of its type.
• How to use trigonometric ratios, reference angles, and symmetry to find patterns on the unit circle.
• Inverse trigonometric functions have restricted domains and ranges and are one to one.
	• Find and interpret key features of a graph or table of a function, including extreme values, end behavior, and intervals of increase and decrease.
	• identify and graph parent functions and their transformations (i.e. vertical translation, horizontal translation, vertical stretch/shrink, reflect over x-axis, etc.).
	• Calculate values of sine, cosine, and tangent for given angles.
	• Solve trigonometric equations, including those written in quadratic form and equations containing more than one angle.
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Building and solving proportional relationships (7.RP.A.2)
		  ○ Recognize and use function notation (HSF.IF.A)
		  ○ Recognizing inverse functions (HSF.BF.B.4)
		  ○ Graphing, solving and modeling quadratic, linear, exponential and absolute value functions (HSF.LE.A, HSF.LE.B)
		  ○ Recognize, build and solve trigonometric ratios (HSG.CRT.C)
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		• Encourage students to use alternative tools to better access the grade level content. Examples include:
		  ○ Desmos.com
		  ○ Graphing calculator
		  ○ Sketch a graph
		  ○ Create a table of values
		  ○ Algebra tiles
		  ○ Graphic organizers
		  ○ SOH CAH TOA

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on analyzing functions using different representations by providing specific feedback to students on their work through a short mini lesson because immediate feedback provides support for learning. There are several family functions with different key features and interpretation.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on analyzing functions using different representations by offering opportunities to understand and explore different strategies because explaining the context of the problem verbally, graphically and writing, students comprehend the different family functions/equations.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when analyzing functions using different representations because making a real-world connection with a choice to select what the learner is interested in will make a deeper connection to the mathematical concept and skill.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Construct and compare linear, quadratic, and exponential models and solve problems.
  - HSF.LE.A.4

Grade	CCSS Domain	CCSS Cluster
A2	Linear, Quadratic, & Exponential Models	Construct and compare linear, quadratic, and exponential models and solve problems

Cluster Standard: HSF.LE.A.4

Standard	Standards for Mathematical Practice
For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	• **SMP 4:** Model with mathematics. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students will be able to go back and forth between an exponential model and logarithmic model and know when one model may be more useful than another one to solve problems in context.	• Use the properties of logs. • Describe the key features of logs. • Use logarithmic form to solve exponential models.
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Students may mix-up direction of term with value of exponents.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Linear, Quadratic & Exponential Models**	Strand: **Construct and compare linear, quadratic, and exponential models and solve problems**

Suggested Student Discourse Questions

- How are rates of change different in linear, quadratic and exponential models?
- Talk to your peer about which math tool (TI calculator, Desmos, etc.) works best for them. How does their strategy compare to yours?
- In what ways can we use technology (TI calculators, Desmos, etc.) to solve this logarithmic equation?
- Which model (linear, quadratic, exponential) would be best used to model _____?

ASSESSMENT GUIDE

- Construct and compare linear, quadratic, and exponential models and solve problems

Grade	*CCSS Domain*	*CCSS Strand*
A2	Linear, Quadratic, & Exponential Models	Construct and compare linear, quadratic, and exponential models and solve problems
	Sample Task #1 (Constructed Response)	
	Standards Aligned Instructionally Embedded Formative Assessment Resources: http://tasks.illustrativemathematics.org/content-standards/HSF/LE/A/tasks/213 This type of assessment question requires students to analyze and compare two exponential functions. Students will engage with SMP7 as they use the structure of the exponential function to analyze a scenario and, if allowed use of technology, SMP5 as they use graphs to reason their solutions.	

NEW MEXICO Public Education Department

MLSS AND CLR GUIDE

- Construct and compare linear, quadratic, and exponential models and solve problems

CCSS Domain	*CCSS Cluster*
Linear, Quadratic, and Exponential Models	Construct and compare linear, quadratic, and exponential models and solve problems

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on constructing and comparing linear, quadratic and exponential models and using them to solve problems, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, collecting/researching a socially relevant data set to fit with a mathematical model. This context allows students to reason with and discuss mathematics but also provides a purpose and drives engagement.	
Cross-Curricular Connections	Many of the Navajo rug designs you will discover by following the project will be good examples of symmetrical balance. Symmetrical balance is a type of visual balance where the overall composition is arranged to look like it is the same on both sides of the center of the design. In other words, it is a design which could be folded in half, and as the design folds, each part of the design would match up with its symmetrical counterpart on the opposite side of the center. The rug design on the right is symmetrical left-to-right. If a line was drawn vertically down the center of the rug, the arrangement of shapes and colors would appear to be exactly the opposite of each other on both sides of that line. http://web.archive.org/web/20080130134231/http:/www.mpsaz.org/arts/elements/balance/page1.html	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities*	• Task: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when constructing and comparing linear, quadratic and exponential models

	of students of marginalized cultures and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	and using them to solve problems, the types of mathematical tasks are critical because this is a golden opportunity to show the relevance and usefulness of mathematics, whether it relates to sports, careers, social data, etc. Every student can find a use for this mathematics given the proper context.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In 8th grade and Algebra 1, students learned about exponential models.	• Students will use this knowledge to solve more complex logarithmic problems that include the use of logarithmic properties	• Students will build on this knowledge of exponents and logarithms in future math courses.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that addresses constructing and comparing linear, quadratic and exponential models and using them to solve problems because these are all function families that should have been previously studied. Students,

		therefore, may be able to spark each other's memory about the shapes of graphs, patterns in numbers and/or forms of equations.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.F.B4/F.BF.A2: This standard provides a foundation for work with constructing and comparing linear, quadratic and exponential models and using them to solve problem because the 8th grade standard addresses students modeling linear data with linear functions and the high school standard addresses when students build a function to model arithmetic and geometric sequences. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework

A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• What the domain of a function represents in terms of the function and what values are appropriate (i.e. discrete v. continuous values and other restrictions based on the type of function and the given context). • In functions there is an underlying structure that determines the transformation of any function, regardless of its type. • How to use trigonometric ratios, reference angles, and symmetry to find patterns on the unit circle.	• Find and interpret key features of a graph or table of a function, including extreme values, end behavior, and intervals of increase and decrease. • identify and graph parent functions and their transformations (i.e. vertical translation, horizontal translation, vertical stretch/shrink, reflect over x-axis, etc.). • Calculate values of sine, cosine, and tangent for given	• Build on students' experience with the following skills: ○ Graphing on the coordinate plane (6.NS.C.8) ○ Building and solving proportional relationships (7.RP.A.2) ○ Recognize and use function notation (HSF.IF.A) ○ Recognizing inverse functions (HSF.BF.B.4) ○ Graphing, solving and modeling quadratic, linear, exponential and absolute value functions (HSF.LE.A, HSF.LE.B) ○ Recognize, build and solve trigonometric ratios (HSG.CRT.C) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies

• Inverse trigonometric functions have restricted domains and ranges and are one to one.	angles. • Solve trigonometric equations, including those written in quadratic form and equations containing more than one angle.	○ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers ○ SOH CAH TOA

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging constructing and comparing linear, quadratic and exponential models and using them to solve problems by critiquing student approaches/solutions to make connections through a short mini lesson because explanations may arise using tables, graphs and function algebra. Building connections between these approaches can illuminate errors as well as push students beyond estimations and toward exact answers.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit constructing and comparing linear, quadratic and exponential models and using them to solve problems by addressing conceptual understanding because the basis of this cluster is in strategically selecting a model based on characteristics provided. If students do not have a firm understanding of the characteristics of linear, quadratic and exponential functions, they will not be able to select or therefore use an appropriate model to solve problems.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when constructing and comparing linear, quadratic and exponential models and using them to solve problems. Challenging students to select a data set to model requires them to reason with the different types of data sets available, strategically explore models and interpret their findings in context. This can also serve to reach the interest of these students on a deeper level.

New Mexico Instructional Scope
Algebra 2 Making Inferences and Justifying Conclusions Guide

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Understand and evaluate random processes underlying statistical experiments
  - HSS.IC.A.1
  - HSS.IC.A.2
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies
  - HSS.IC.B.3
  - HSS.IC.B.4
  - HSS.IC.B.5
  - HSS.IC.B.6

Grade	*CCSS Domain*	*CCSS Cluster*
A2	Making Inferences & Justifying Conclusions	Understand and evaluate random processes underlying statistical experiments

Cluster Standard: HSS.IC.A.1

Standard	Standards for Mathematical Practice
Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students move beyond analyzing data to making sound statistical decisions based on probability models. The reasoning process is as follows: develop a statistical question in the form of a hypothesis (supposition) about a population parameter; choose a probability model for collecting data relevant to that parameter; collect data; compare the results seen in the data with what is expected under the hypothesis.	• Explain the difference between bias and unbiased sampling. • Explain why the statistical measures of a random sample should be roughly the same as the statistical measures of the population. • Make inferences about a population based on a random sample.
DOK	**Blooms**
2-3	Understand, Analyze

Grade	CCSS Domain	CCSS Cluster
A2	Making Inferences & Justifying Conclusions	Understand and evaluate random processes underlying statistical experiments

Cluster Standard: HSS.IC.A.2

Standard	Standards for Mathematical Practice
Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model*?	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 5:** Use appropriate tools strategically. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students move beyond analyzing data to making sound statistical decisions based on probability models. The reasoning process is as follows: develop a statistical question in the form of a hypothesis (supposition) about a population parameter; choose a probability model for collecting data relevant to that parameter; collect data; compare the results seen in the data with what is expected under the hypothesis.	• Explain the difference between a rare event and an ordinary event. • Use tools to analyze results from data-generating processes, i.e., create a simulation. • Make inferences about a model to decide if a model is consistent with the result given.
DOK	**Blooms**
3-4	Analyze, Evaluate

Common Misconceptions

- Students may struggle with the difference between rare and impossible.
- Students may struggle with recognizing the difference between random and non-repeating events. Humans tend to believe random means an outcome will not repeat but in large data sets of random outcomes it's common to have strings of the same outcome (i.e., flipping a coin 100 times and finding a string of 5 heads in a row).

Grade	*CCSS Domain*	*CCSS Cluster*
A2	Making Inferences & Justifying Conclusions	Make inferences and justify conclusions from sample surveys, experiments, and observational studies

Cluster Standard: HSS.IC.B.3

Standard	Standards for Mathematical Practice
Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	• **SMP 3:** Construct viable arguments and critique the reasoning of others.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Once students see how **probability** intertwines with data collection and analysis, students use this knowledge to make **statistical inferences** from data collected in **sample surveys** and in **designed experiments**, aided by **simulation** and the technology that affords it.	• Recognize that randomization is necessary to making accurate statistical inferences. • Compare and contrast the differences between a sample survey, experiment, and observational study and the advantages to their uses. • Explain how to use random sampling techniques and the importance of random sampling.
DOK	**Blooms**
3	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
A2	Making Inferences & Justifying Conclusions	Make inferences and justify conclusions from sample surveys, experiments, and observational studies

Cluster Standard: HSS.IC.B.4

Standard	Standards for Mathematical Practice
Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP5:** Use appropriate tools strategically. • **SMP8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should be able to explain the reasoning in a **statistical decision** and the nature of the error that may have been made.	• Estimate a population mean and calculate margin of error given a simulation. • Use real world data to determine the population mean and margin of error.
DOK	**Blooms**
3	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
A2	Making Inferences & Justifying Conclusions	Make inferences and justify conclusions from sample surveys, experiments, and observational studies

Cluster Standard: HSS.IC.B.5

Standard	Standards for Mathematical Practice
Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP5:** Use appropriate tools strategically. • **SMP8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should be able to explain the reasoning in a **statistical decision** and the nature of the error that may have been made.	• Identify differences between parameters. • Compare two treatment groups in an experiment and decide if the difference in parameters is significant.
DOK	**Blooms**
1-3	Analyze, Evaluate

Grade	CCSS Domain	CCSS Cluster
A2	Making Inferences & Justifying Conclusions	Make inferences and justify conclusions from sample surveys, experiments, and observational studies

Cluster Standard: HSS.IC.B.6

Standard	Standards for Mathematical Practice
Evaluate reports based on data.	• **SMP 6:** Attend to precision. • **SMP 7:** Look for and make use of structure. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Student will look at the quality of a statistical question to be answered, question clarity, quantization of responses and error calculations, as absolute value of difference of the sample values from the mean.	• Explain the parameters of data and their significance • Define the characteristics of experimental design (control, randomization, and replication). • Evaluate the experimental study design, how the data was gathered, what analysis (numerical or graphical) was used. • Draw conclusions based on graphical and numerical summaries. • Evaluate reports based on data.
DOK	**Blooms**
3	Analyze, Evaluate

Common Misconceptions

- Students may struggle with distinguishing between the difference between estimates for means and proportions.
- Students may have difficulty in relating the margin of error from a simulation model to the inference about a population.
- Students may have the misconception that when the margin of error increases that the statistic does not contain the true population parameter.
- Students may struggle with using statistical language that includes the possibility of error in measurement rather than absolute language such as always, never, guaranteed.

NEW MEXICO
Public Education Department

ASSESSMENT GUIDE

- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Grade	*CCSS Domain*	*CCSS Strand*
A2	Making Inferences & Justifying Conclusions	Understand and evaluate random processes underlying statistical experiments

Sample Task #1 (Constructed Response)

CollegeBoard	Question ID 4169753						
Assessment **SAT**	Test **Math**	Cross-Test and Subscore **Problem Solving and Data Analysis**	Difficulty **Medium**	Primary Dimension **Problem Solving and Data Analysis**	Secondary Dimension **Evaluating statistical claims: Observational studies and experiments**	Tertiary Dimension **1. With random samples, describe which population the results can be extended to.**	Calculator **Calculator**

A sample of 40 fourth-grade students was selected at random from a certain school. The 40 students completed a survey about the morning announcements, and 32 thought the announcements were helpful. Which of the following is the largest population to which the results of the survey can be applied?

Question Difficulty: Medium

A. The 40 students who were surveyed

B. All fourth-grade students at the school

C. All students at the school

D. All fourth-grade students in the county in which the school is located

Choice B is correct. Selecting a sample of a reasonable size at random to use for a survey allows the results from that survey to be applied to the population from which the sample was selected, but not beyond this population. In this case, the population from which the sample was selected is all fourth-grade students at a certain school. Therefore, the results of the survey can be applied to all fourth-grade students at the school.

Choice A is incorrect. The results of the survey can be applied to the 40 students who were surveyed. However, this isn't the largest group to which the results of the survey can be applied. Choices C and D are incorrect. Since the sample was selected at random from among the fourth-grade students at a certain school, the results of the survey can't be applied to other students at the school or to other fourth-grade students who weren't represented in the survey results. Students in other grades in the school or other fourth-grade students in the country may feel differently about announcements than the fourth-grade students at the school.

Grade	*CCSS Domain*	*CCSS Strand*
A2	Making Inferences & Justifying Conclusions	Make inferences and justify conclusions from sample surveys, experiments, and observational studies

Sample Task #1 (Constructed Response)

CollegeBoard **Question ID 4789744**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Problem Solving and Data Analysis	Medium	Problem Solving and Data Analysis	Inference from sample statistics and margin of error	1. Use sample mean and sample proportion to estimate population mean and population proportion. Utilize, but do not calculate, margin of error.	Calculator

A bag containing 10,000 beads of assorted colors is purchased from a craft store. To estimate the percent of red beads in the bag, a sample of beads is selected at random. The percent of red beads in the bag was estimated to be 15%, with an associated margin of error of 2%. If r is the actual number of red beads in the bag, which of the following is most plausible?

Question Difficulty: Medium

A. $r > 1,700$

B. $1,300 < r < 1,700$

C. $200 < r < 1,500$

D. $r < 1,300$

Choice B is correct. It was estimated that 15% of the beads in the bag are red. Since the bag contains 10,000 beads, it follows that there are an estimated $10,000 \times 0.15 = 1,500$ red beads. It's given that the margin of error is 2%, or $10,000 \times 0.02 = 200$ beads. If the estimate is too high, there could plausibly be $1,500 - 200 = 1,300$ red beads. If the estimate is too low, there could plausibly be $1,500 + 200 = 1,700$ red beads. Therefore, the most plausible statement of the actual number of red beads in the bag is $1,300 < r < 1,700$.

Choices A and D are incorrect and may result from misinterpreting the margin of error. It's unlikely that more than 1,700 beads or fewer than 1,300 beads in the bag are red. Choice C is incorrect because 200 is the margin of error for the number of red beads, not the lower bound of the range of red beads.

NEW MEXICO
Public Education Department

MLSS AND CLR GUIDE

- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

CCSS Domain	*CCSS Cluster*
Making Inferences and Justifying Conclusions	Understand and evaluate random processes underlying statistical experiments

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on evaluating random processes underlying statistical experiments, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, how statistics are used to describe the risk of different cultural and ethnic groups for developing breast cancer and how this might affect medical breast cancer screening frequency recommendations.	
Cross-Curricular Connections	Social Studies: In high school the New Mexico Social Studies Standards state students should "explain how to use technological tools to research data, verify facts and information, and communicate findings." Consider providing a connection for students to determine the best fit of a function for a set of data and explain their choice.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of*	• Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying HS.IC.B cluster goal setting is critical because statistics are often used to help describe how a specific ethnic or cultural group is doing and what needs they may need. Knowing how to interpret and present data in meaningful ways helps develop a more robust model of the community. The 10-year Census is the largest statistical tool the United States uses to help with this objective.

	your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to the work students have done to determine mean, median, mode, range, IQR, minimum, maximum.	• Connect to work throughout the Statistics and Probability domain around interpreting and making inferences about populations based upon sample quantitative data.	• Connect to future work with Statistics and Probability in college level courses and careers.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that previews new contexts for tasks regarding understanding and evaluating random processes underlying statistical experiments because a preview often piques learner curiosity and knows that there is a reason for learning this material.
Intensive	*What critical understandings will*	7.SP.C.7: This standard focuses on developing a probability model and using it to find probabilities of

	prepare students to access the mathematics for this cluster?	events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. This standard provides a foundation for work with units in Understand and Evaluate Random Processes Underlying Statistical Experiments cluster because misconceptions with probabilities will impact a learner's ability to evaluate random processes. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on understanding and evaluating random processes underlying statistical experiments by critiquing student approaches/solutions to make connections through a short mini-lesson because misconceptions about the nature of random events is common and may lead to incorrect conclusions. For example, many people believe that luck is real and can affect the outcome of random events.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit in the understanding and evaluating random processes underlying statistical experiments by addressing conceptual understanding because small subtle misconceptions about random processes can lead to invalid or inaccurate decisions. For example, many people believe that if they get cancer once, they are less likely to get it again.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when evaluating random processes underlying statistical experiments because many learners want or need to

	know how this cluster will relate to future career or college interests.

CCSS Domain	*CCSS Cluster*
Making Inferences and Justifying Conclusions	Make inferences and justify conclusions from sample surveys, experiments, and observational studies

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on making inferences and justifying conclusions, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, how statistics are used to describe the risk of different cultural and ethnic groups for developing breast cancer and how this might affect medical breast cancer screening frequency recommendations. Another option to consider is students learning about how data might allow us to infer how an infection might be slowed in a large extended family living in one house.	
Cross-Curricular Connections	Social Studies: Connection to the difference between correlation versus causation when reading data.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical*	• Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying HS.IC: Making Inferences & Justifying Conclusions cluster goal setting is critical because interpreting statistical results are often used to infer how a specific ethnic or cultural group is doing and what needs they may need. The census is the largest statistical tool the United States uses to help with this objective.

	identities as capable mathematicians that can use mathematics within school and society?	

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to work in previous math courses, where students have learned to determine mean, median, mode, range, IQR, minimum, maximum. Students have also learned how to graph data distributions (e.g., histograms, box plots).	• Connect to students work with evaluating the randomness of a sample and use this to determine if a specified model is consistent with the results. **(HSS.IC.A)**	• Connect to work in subsequent statistics course (AP or college level).

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when making Inferences and justifying conclusions because learners are often more ready to engage the material at a deeper level when they feel frustrated and confused just enough to have questions that need answered during the upcoming units.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	7.SP.A: This cluster of standards provides a foundation for work with the units in the HS.S-IC.B: Making Inferences And Justifying Conclusions Cluster because the 7th grade standard provides a foundation for valid random sampling techniques, providing validity for conclusions based on the data. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-

		teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on HS.S-IC.B: Making Inferences and Justifying Conclusions cluster by revisiting student thinking through a short mini-lesson because this conversation can serve as a diagnostic tool so the teacher can prescribe the needed review needed to get the learner moving.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit in the HS.S-IC.B: Making Inferences And Justifying Conclusions cluster by helping students move from specific answers to generalizations for certain types of problems because learners often benefit from seeing the work of more experienced problem solvers.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics because advanced or gifted learners often need or want to explore more into how data is used. For example, a learner could look at how Big Data is being used to make life better but not without potential risks.

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Standards Breakdown

- Understand solving equations as a process of reasoning and explain the reasoning.
  - REI.A.2
- Represent and solve equations and inequalities graphically.
  - HSA.REI.D.11

Grade	CCSS Domain	CCSS Cluster
A2	Reasoning with Equations and Inequalities	Understand solving equations as a process of reasoning and explain the reasoning

Cluster Standard: REI.A.2

Standard	Standards for Mathematical Practice
Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	• **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• This cluster builds on the framework of solving equations and extends it to rational and radical equations (and the knowledge of extraneous solutions). • Equations are solved as a process of reasoning using properties of operations and equality, which can justify each step of the process. Students solve simple rational and radical equations using a variety of methods and explain why and where in the solution process the extraneous solution arose.	• Determine the domain of a rational function. • Determine the domain of a radical function. • Solve radical equations in one variable. • Solve rational equations in one variable. • Explain and give examples how extraneous solutions may arise when solving rational and radical equations.
DOK	**Blooms**
1-3	Apply, Analyze

Common Misconceptions

- Students may struggle identifying when there is an extraneous solution.
- Struggle with expression versus equation.
- When students multiply or divide both sides of an inequality by a negative value, they forget to reverse the inequality symbol.
- Students may sometimes forget to consider the cases when the LCD is positive and negative when solving a rational inequality algebraically,

Grade	CCSS Domain	CCSS Cluster
A2	Reasoning with Equations and Inequalities	Represent and solve equations and inequalities graphically

Cluster Standard: HSA.REI.D.11

Standard	Standards for Mathematical Practice
Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 5:** Use appropriate tools strategically. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• HSA.REI.D.11: Just as the **algebraic** work with **equations** can be reduced to a series of algebraic moves unsupported by reasoning, so can the **graphical visualization of solutions**. The simple idea that an equation $f(x) = g(x)$ can be **solved (approximately)** by graphing $y = f(x)$ and $y = g(x)$ and finding the **intersection points** involves a number of pieces of conceptual understanding. [This method] seeks to convert an equation in one variable, $f(x) = g(x)$, to a s**ystem of equations** in two **variables**, $y = f(x)$ and $y = g(x)$, by introducing a second variable *y* and **setting it equal** to each side of the equation. If *x* is a solution to the original equation, then $f(x)$ and $g(x)$ are equal, and thus **(*x*, *y*)** is a solution to the new system.	• Recognize what the solution $y = f(x)$ and $y = g(x)$ means on a graph. • Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. • Find approximate solutions for the system $y = f(x)$ and $y = g(x)$ using graphs and tables. • Find successive approximations and use them to solve the system $y = f(x)$ and $y = g(x)$. • Use paper/pencil or technology to produce a table of values. • Explain what x-coordinate of a common ordered pair represents in the context of the problem.
DOK	**Blooms**
1-3	Understand, Apply, Analyze, Evaluate

Common Misconceptions

- Students often interpret the solutions to an equation or graphical representation of an equation as only integer values.
- Students may believe an estimate of a value between two integer points is sufficient, but the standard states that students should find successive approximations to approximate the solution.
- Students believe the graph of a function is simply a line or curve "connecting the dots," without recognizing that the graph represents all solutions to the equation.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Reasoning with Equations & Inequalities**	Strand: **Understand solving equations as a process of reasoning and explain the reasoning**

Suggested Student Discourse Questions

- Why do we see extraneous solutions to this _____ (rational / radical) equation? Explain your thinking using mathematical vocabulary.
- Turn and talk to your partner and come up with a method to identify extraneous solutions to this _____ (rational / radical) equation. How is your method different from that of another group's? How is it similar?
- How are extraneous solutions to this _____ (rational / radical) equation identified graphically? Algebraically? Using a table?
- Given the area of _____ (classroom, your bedroom, football field, grocery store, lake), and one side length, why must the width be a positive measure? How is this modeled in a radical equation?

Domain: **Reasoning with Equations & Inequalities**	Strand: **Represent and solve equations and inequalities graphically**
Suggested Student Discourse Questions	
• In what ways are inequalities different from equations? In what way are they the same? • Turn and talk with your partner. How did they work this problem? How is that different from your method? Which method do you prefer? Why?	• How does the graph of this _____ (equation / inequality) model the equation? • How does a linear inequality model _____? (Ex: correct combination of drugs to give to a patient, legal speed limit on a highway, data limit on cell phone plans, time needed to walk to school, maximum profit based on materials used)

NEW MEXICO
Public Education Department

ASSESSMENT GUIDE

- Understand solving equations as a process of reasoning and explain the reasoning.
- Represent and solve equations and inequalities graphically.

Grade	*CCSS Domain*	*CCSS Cluster*
A2	Reasoning with Equations and Inequalities	Understand solving equations as a process of reasoning and explain the reasoning

Sample Task #1 (Constructed Response)

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: An Extraneous Solution Illustrative Mathematics

Megan is working solving the equation

$$\frac{2}{x^2-1} - \frac{1}{x-1} = \frac{1}{x+1}.$$

She says

> ***If I clear the denominators I find that the only solution is $x = 1$ but when I substitute in $x = 1$ the equation does not make any sense.***

a. Is Megan's work correct?

b. Why does Megan's method produce an x value that does not solve the equation?

IM Commentary

The goal of this task is to examine how extraneous solutions can arise when solving rational equations. The task presents an operation, "clearing denominators," which appears to lead to a contradiction. To resolve the contradiction, we examine more carefully what is happening when we clear denominators (MP6). One way to describe the process is that we find a common denominator for both sides and set the numerators equal to each other. This gives solutions to the original equation provided the solutions are in the domain of the rational functions on both sides, that is, provided they are not zeros of one or more of the denominators. In this case, the solution x=1 makes the numerators equal to one another but also makes the denominators of two of the expressions zero, and so x=1 is an extraneous solution.

Grade	CCSS Domain	CCSS Cluster
A2	Reasoning with Equations and Inequalities	Represent and solve equations and inequalities graphically

Sample Task #1 (Multiple Choice)

Source: SAT

$$x+1=\frac{2}{x+1}$$

Question 1474935 Answers

In the equation above, which of the following is a possible value of $x+1$?

- A. $1-\sqrt{2}$
- B. $\sqrt{2}$
- C. 2
- D. 4

NEW MEXICO Public Education Department

MLSS AND CLR GUIDE

- Understand solving equations as a process of reasoning and explain the reasoning.
- Represent and solve equations and inequalities graphically.

CCSS Domain	*CCSS Cluster*
Reasoning with Equations and Inequalities	Understand solving equations as a process of reasoning and explain the reasoning

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on understanding solving equations as a process of reasoning and explaining the reasoning, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, some students learn differently when it comes to expressing their ideas, linguistically speaking they learned different languages. Learning about the different structures for the process and explaining it across the languages in the classroom can lead to a more robust understanding of solving equations for all students by making connections to the different structures of understanding solving equations in other languages.	
Cross-Curricular Connections	This lesson uses right triangle trigonometry and a rational function to explore the percent of your visual field that is occupied by the area of a television. Sofa Away From Me: A Lesson by Mathalicious	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying

	school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	understanding solving equations as a process of reasoning and explaining the reasoning the use of mathematical representations within the classroom is critical because linguistically and culturally speaking, experiences of students provide different and varied types of representations for solving mathematical problems varies on different types of exposures they have in their home and communities. Students' abilities in explaining mathematical reasoning varies on their language preferences and representations.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In Algebra 1, students solved linear and quadratic equations.	• In this course, students will extend their solving skills learned in Algebra 1 to rational and radical equations. Students will also relate the solving of other nonlinear equations learned in this course to solving rational and radical equations.	• In future math classes students will solve more challenging nonlinear equations, including trigonometric equations.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on solving equations and explaining each step because students may need to justify the inverse operation used in each step with viable arguments. Students may practice expressing their mathematical thinking verbally and symbolically.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	6.EE.B.5: This standard provides a foundation for work with reasoning and solving one-variable equations because students need to understand each step of

		solving one-variable equations and explain the reason for each step. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• Different forms of an expression can be equivalent and are useful in different contexts. • The addition, subtraction, multiplication, or division of rational expressions results in another rational expression. • When a situation and its potential constraints will be represented by all available types of equations/inequalities, including simple root function, or a system of those equations/inequalities. • When solving graphically/with a table is more	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form. • Perform the operations of addition, subtraction, multiplication, and division with rational expressions. • Determine reasonable solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities. • Solve systems using a graph and a table as well as rewrite an equation as two functions (and vice versa).	• Build on students' experience with the following skills: ○ Graphing on the coordinate plane (6.NS.C.8) ○ Solving systems of equations / inequalities (8.EE.C.8) ○ Adding / subtracting / multiplying / dividing and simplify fractions ○ Writing and solving one-step and two-step equations (HSA.REI.B.3, HSA.REI.B.4) ○ Modeling linear, exponential, quadratic and absolute value functions (HSF.LE.A, HSF.LE.B) ○ Different forms of linear (linear standard form, point-slope form, slope intercept form) and quadratic equations (quadratic standard form and vertex form) (HSF.LE.A) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com

efficient than solving algebraically.		○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on explaining the reason of each step of solving equations by critiquing student approaches/solutions to make connections through a short mini-lesson because students need to understand why the specific inverse operation is used and develop the viable argument using properties of equality.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit explaining the steps of solving equations by offering opportunities to understand and explore different strategies because students need to understand why some steps are interchangeable when solving the equations. Students need to explain the order of applying the inverse operations and how that relates to the order of operation of the equations.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when solving complex equations because students may deepen their understanding of inverse operation (e.g. logarithm as the inverse operation of exponent). Students explore strategies of solving equations with complex operations and justify their reason in cooperative learning groups.

CCSS Domain	*CCSS Cluster*
Reasoning with Equations and Inequalities	Represent and solve equations and inequalities graphically

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on representing and solving equations and inequalities graphically, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, bring in the different languages spoken in the home and connect it to the tools available to translate different languages, i.e. Google translate, closed captions on televisions, etc. make connections that show that in the culture of mathematics, tools are used to translate mathematics and help us make sense of what we are seeing.	
Cross-Curricular Connections	Students will model projectile motion in both function and parametric graphing. This was designed as an in-class modeling activity to be used prior to actually launching air-powered projectile rockets. A set of data is given in a spreadsheet and students create model functions using a variety of methods: vertex form (then using grab and move to fit the curve to the data points), standard form (using matrices), and quadratic regression. Rocket Simulation Activity	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language,*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when representing and solving equations and inequalities graphically the use of mathematical representations within the classroom is critical. When students are given a

	the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	situation in two variables, they must find the value of one variable given the value of the other, create an equation to represent the situation, use technology to create a graph, and interpret each representation. Understanding how lines and tables represent solution sets of linear relationships will help students make sense of graphs of solutions to linear inequalities, and later, to make sense of solutions to systems of linear equations in their Algebra 1 class.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to the work of Algebra 1 in this cluster around linear and exponential. **(HSA.REI.D)**	• Connect this cluster across all of Algebra 2, particularly when each new function is presented.	• Connect this cluster to future work around determining specific solutions to new functions (i.e. zeros). Also, in Calculus, when students discuss the area between two curves and volume with rotation.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that introduces new representations (e.g., number lines) when representing and solving equations and inequalities graphically. Different representations within the same problem allow students to make connections between the graph, table, word problem, and equations. When two expressions are equal to each other, the variable equal to a numerical value is the solution algebraically and graphically.
Intensive	*What critical understandings will*	6.EE.B.5: This standard provides a foundation for work to represent and solve equations and inequalities

	prepare students to access the mathematics for this cluster?	graphically because by substituting numerical values into an equation to determine if the equation is true, the student will comprehend that the answer is a solution. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework

A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• Different forms of an expression can be equivalent and are useful in different contexts. • The addition, subtraction, multiplication, or division of rational expressions results in another rational expression. • When a situation and its potential constraints will be represented by all available types of equations/inequalities, including simple root function, or a system of those equations/inequalities. • When solving graphically/with a table is more efficient than solving algebraically.	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form. • Perform the operations of addition, subtraction, multiplication, and division with rational expressions. • Determine reasonable solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities. • Solve systems using a graph and a table as well as rewrite an equation as two functions (and vice versa).	• Build on students' experience with the following skills: ○ Graphing on the coordinate plane (6.NS.C.8) ○ Solving systems of equations / inequalities (8.EE.C.8) ○ Adding / subtracting / multiplying / dividing and simplify fractions ○ Writing and solving one-step and two-step equations (HSA.REI.B.3, HSA.REI.B.4) ○ Modeling linear, exponential, quadratic and absolute value functions (HSF.LE.A, HSF.LE.B) ○ Different forms of linear (linear standard form, point-slope form, slope intercept form) and quadratic equations (quadratic standard form and vertex form) (HSF.LE.A) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content. Examples

		include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on representing and solving equations and inequalities graphically by critiquing student approaches/solutions to make connections through a short mini-lesson because connections between solution (algebraically) and intersection (graphically) are equivalent. When students compare answers graphically and algebraically, intersections are the solution.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit to represent and solve equations and inequalities graphically by offering opportunities to understand and explore different strategies because interpreting solution using the different representations allows the students to visualize the answer that was only written as a system of equations or two expressions equal to each other. Students can check their work graphically to confirm their answer.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension such as the opportunity to explore links between various topics when representing and solving equations and inequalities graphically because different types of equations (logarithmic, exponential, trigonometric, etc.) can use the graphing method to find solutions to word problems or algebraically.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Extend the domain of trigonometric functions using the unit circle.
  - HSF.TF.A.1
  - HSF.TF.A.2
- Model periodic phenomena with trigonometric functions.
  - HSF.TF.B.5
- Prove and apply trigonometric identities
  - HSF.TF.C.8

Grade	*CCSS Domain*	*CCSS Cluster*
A2	Trigonometric Functions	Extend the domain of trigonometric functions using the unit circle

Cluster Standard: HSF.TF.A.1

Standard	Standards for Mathematical Practice
Understand the radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	• **SMP2:** Reason abstractly and quantitatively. • **SMP5:** Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students will be able to extend their knowledge of circle and trigonometric ratios (sine and cosine) to arc length, evaluating using a unit circle, and graphing trigonometric functions (sine and cosine).	• Find the measures of an angle in standard position and the reference angle. • Find arc length using radian measure on the unit circle. • Convert between degrees and radians.
DOK	**Blooms**
1-2	Understand, Apply

Grade	*CCSS Domain*	*CCSS Cluster*
A2	Trigonometric Functions	Extend the domain of trigonometric functions using the unit circle

Cluster Standard: HSF.TF.A.2

Standard	Standards for Mathematical Practice
Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	• **SMP 6:** Attend to precision. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students will be able to extend their knowledge of circle and trigonometric ratios (sine and cosine) to arc length, evaluating using a unit circle, and graphing trigonometric functions (sine and cosine).	• Identify the relationship between the unit circle and the coordinate plane.
DOK	**Blooms**
1-2	Understand

Common Misconceptions

- Students may confuse the direction of positive and negative angle measures, writing the wrong sign with the measure.
- Students may mix-up radian and degree measures

Grade	CCSS Domain	CCSS Cluster
A2	Trigonometric Functions	Model periodic phenomena with trigonometric functions

Cluster Standard: HSF.TF.B.5

Standard	Standards for Mathematical Practice
Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	• **SMP 4:** Model with mathematics. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students apply the concept of inverse functions to trigonometric functions and use that concept to solve problems.	• Graph sine and cosine functions using radian and degrees. • Identify properties of the sine function. • Model a real-world situation using trigonometric functions. Students can then use inverse trigonometric functions to find solutions.
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Students may mix-up sine and cosine.

Grade	CCSS Domain	CCSS Cluster
A2	Trigonometric Functions	Prove and apply trigonometric identities

Cluster Standard: HSF.TF.C.8

Standard	Standards for Mathematical Practice
Prove the Pythagorean identity $\sin^2(\vartheta) + \cos^2(\vartheta) = 1$ and use it to find $\sin(\vartheta)$, $\cos(\vartheta)$, or $\tan(\vartheta)$ given $\sin(\vartheta)$, $\cos(\vartheta)$, or $\tan(\vartheta)$ and the quadrant of the angle.	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 5:** Use appropriate tools strategically. • **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students will make connections between their knowledge of the Pythagorean Theorem, trigonometric ratios, the unit circle and coordinate plane.	• Use the concepts from the Pythagorean identity to calculate trigonometric ratios in any quadrant on the coordinate plane
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Students may struggle to explain how the identities frame responses.

ASSESSMENT GUIDE

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.
- Prove and apply trigonometric identities.

Grade	*CCSS Domain*	*CCSS Strand*
A2	Trigonometric Functions	Extend the domain of trigonometric functions using the unit circle

Sample Task #1 (Constructed Response)

CollegeBoard **Question ID 423225**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Additional Topics in Math	Hard	Additional Topics in Math	Circles	6. Convert between angle measures in degrees and radians.	No Calculator

The number of radians in a 720-degree angle can be written as $a\pi$, where a is a constant. What is the value of a ?

Question Difficulty: Hard

The correct answer is 4. There are π radians in a $180°$ angle. An angle measure of $720°$ is 4 times greater than an angle measure of $180°$. Therefore, the number of radians in a $720°$ angle is 4π.

This type of assessment question requires students to translate an angle measurement from degrees to radians. Students may use a conversion factor to do so or may use the fact that pi radians is equivalent to 180 degrees,

Grade	CCSS Domain	CCSS Strand
A2	Trigonometric Functions	Model periodic phenomena with trigonometric functions

Sample Task #1 (Constructed Response)

Standards Aligned Instructionally Embedded Formative Assessment Resources:

http://tasks.illustrativemathematics.org/content-standards/HSF/TF/B/5/tasks/816

This type of assessment question requires students to model the population of two species using trigonometric functions given a table of data. Since students may use positive or negative sine and cosine functions to model the data, consider having students work in corporate groups and comparing solutions between groups.

Students will engage with SMP1, SMP4 and if comparing work with other groups, SMP3.

Grade	CCSS Domain	CCSS Strand
A2	Trigonometric Functions	Prove and apply trigonometric identities

Sample Task #1 (Constructed Response)

Standards Aligned Instructionally Embedded Formative Assessment Resources:

http://tasks.illustrativemathematics.org/content-standards/HSF/TF/C/8/tasks/1835

This type of assessment question requires students to take a given ratio for a trigonometric function and use it to exactly state the value of two other trigonometric functions. This will require knowledge of the trigonometric functions as ratios of side lengths and possibly trigonometric identities. Students will engage with SMP 7 as they use the structure of the ratio to determine the remaining trigonometric values.

Additional Assessment:
https://www.map.mathshell.org/lessons.php?unit=9255&collection=8&redir=1

MLSS AND CLR GUIDE

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.
- Prove and apply trigonometric identities.

CCSS Domain	*CCSS Cluster*
Trigonometric Functions	Extend the domain of trigonometric functions using the unit circle

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on extending the domain of trigonometric functions using the unit circle, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, periodicity refers to a pattern that repeats over time. Consider providing examples of numeric and non-numeric patterns that exist (springs bouncing, pendulums swinging, temperature fluctuations throughout the year, hunger growing and decreasing as you approach mealtime) and then ask students to provide some examples of their own.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to*	• Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion, we position them as knowers and doers of mathematics. Using equitable talk moves students in how they talk about who is and isn't capable of mathematics. As a result, we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when extending the domain of trigonometric functions using the unit circle facilitating meaningful mathematical discourse is critical because some students will naturally see and understand the patterns of periodic functions quicker than others.

	support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	Eliciting peer-to-peer explanations can build a cooperative environment in the classroom. Further, by providing sentence frames for students, all students can engage in mathematical conversations, even if they are not sure what the solution is. When students speak about mathematics, they engage meaningfully in the content and deepen their understanding.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In Geometry, students learned the relationship of trigonometric ratios and special right triangles (which leads to evaluating on the unit circle). Students also learned about radian measure and how to convert to degree measure.	• Students will use this knowledge of trigonometric functions (sine and cosine) and apply transformations and identify key features of these trigonometric functions. They will also be able to evaluate trig ratios on a coordinate plane (not on the unit circle).	• In Precalculus and Calculus courses, students will connect this learning cluster to other trigonometric ratios and functions (tangent, cosecant, secant and cotangent) and inverse trigonometric functions.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when extending the domain of trigonometric functions using the unit circle because this is the first time many students will be working with a unit circle so providing that visual at the very beginning and explaining its purpose can be helpful in later parts of the lessons
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	F.IF.A1: This standard provides a foundation for work with extending the domain of trigonometric functions using the unit circle because this is the standard where students gain conceptual understanding of domain and range as sets of inputs and outputs for a given function. If

		students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• What the domain of a function represents in terms of the function and what values are appropriate (i.e. discrete v. continuous values and other restrictions based on the type of function and the given context). • In functions there is an underlying structure that determines the transformation of any function, regardless of its type. • How to use trigonometric ratios, reference angles, and symmetry to find patterns on the unit circle. • Inverse trigonometric functions have restricted domains and ranges and are one to one.	• Find and interpret key features of a graph or table of a function, including extreme values, end behavior, and intervals of increase and decrease. • identify and graph parent functions and their transformations (i.e. vertical translation, horizontal translation, vertical stretch/shrink, reflect over x-axis, etc.). • Calculate values of sine, cosine, and tangent for given angles. • Solve trigonometric equations, including those written in quadratic form and equations containing more than one angle.	• Build on students' experience with the following skills: ○ Graphing on the coordinate plane (6.NS.C.8) ○ Building and solving proportional relationships (7.RP.A.2) ○ Recognize and use function notation (HSF.IF.A) ○ Recognizing inverse functions (HSF.BF.B.4) ○ Graphing, solving and modeling quadratic, linear, exponential and absolute value functions (HSF.LE.A, HSF.LE.B) ○ Recognize, build and solve trigonometric ratios (HSG.CRT.C) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers

		○ SOH CAH TOA
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on extending the domain of trigonometric functions using the unit circle by providing specific feedback to students on their work through a short mini-lesson because students may have a fear of trigonometry that may affect their willingness to interact with the content. Providing student specific feedback, identifying things students are doing correctly and/or effort students are putting forth can encourage students to continue engaging with the material. There are many opportunities for students to make minor errors and if this is the focus, students may shut down. Focused and encouraging feedback can help to counteract any feelings that arise as errors are made.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit extending the domain of trigonometric functions using the unit circle by addressing conceptual understanding because the goal of this cluster is understanding the basis of periodic functions, rather than procedural computations. Students may lack conceptual understanding of domain/range, radian measures of angles and/or the periodicity of functions. Each of these concepts should be explored so students can adequately express their understanding.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension such as extending the domain of trigonometric functions using the unit circle because once students understand the behavior of periodic functions, they can extend the pattern on their own with little guidance from the teacher.

CCSS Domain	*CCSS Cluster*
Trigonometric Functions	Model periodic phenomena with trigonometric functions

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on modelling periodic phenomena with trigonometric functions, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. Have students provide examples of music they enjoy and discuss how music creates sound waves that are modeled by trigonometric functions. Using free online tools, you can create sound wave images (as a class or on an individual basis) and discuss the features of these images as they relate to periodic functions.	
Cross-Curricular Connections	Many of the Navajo rug designs you will discover by following the project will be good examples of symmetrical balance. Symmetrical balance is a type of visual balance where the overall composition is arranged to look like it is the same on both sides of the center of the design. In other words, it is a design which could be folded in half, and as the design folds, each part of the design would match up with its symmetrical counterpart on the opposite side of the center. The rug design on the right is symmetrical left-to-right. If a line was drawn vertically down the center of the rug, the arrangement of shapes and colors would appear to be exactly the opposite of each other on both sides of that line. http://web.archive.org/web/20080130134231/http:/www.mpsaz.org/arts/elements/balance/page1.html	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of*	• Task: When planning with your HQIM, consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when studying modelling periodic phenomena with trigonometric functions the types of mathematical tasks are critical because we often experience patterns in the real-world, whether it be related to science or societal issues. This is an opportunity for students to explore claims like "climate change is not real" or "the violent crime rate always rises in the warmer months" mathematically by attempting to apply features of

	your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	periodic functions to describe them or showing that these features do not exist within the data.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In Geometry and Algebra II, students have defined trigonometric ratios using the acute angles of right triangles	• In Algebra II, students define inverse functions	• Inverse trigonometric functions play a major role in Calculus, when using operations such as differentiation and integration

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that models periodic phenomena with trigonometric functions because there is new vocabulary introduced in this cluster that relates to the graphs of trigonometric functions. These terms will then be applied to contextual scenarios. By rehearsing how to precisely use these terms to describe graphs can support student's later work in using them to describe scenarios.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	F-IF.C.7: This standard provides a foundation for work with modelling periodic phenomena with trigonometric functions because this standard is where students focus on describing key features of other function families. If students have unfinished learning within this standard,

		based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• What the domain of a function represents in terms of the function and what values are appropriate (i.e. discrete v. continuous values and other restrictions based on the type of function and the given context). • In functions there is an underlying structure that determines the transformation of any function, regardless of its type. • How to use trigonometric ratios, reference angles, and symmetry to find patterns on the unit circle. • Inverse trigonometric functions have restricted domains and ranges and are one to one.	• Find and interpret key features of a graph or table of a function, including extreme values, end behavior, and intervals of increase and decrease. • identify and graph parent functions and their transformations (i.e. vertical translation, horizontal translation, vertical stretch/shrink, reflect over x-axis, etc.). • Calculate values of sine, cosine, and tangent for given angles. • Solve trigonometric equations, including those written in quadratic form and equations containing more than one angle.	• Build on students' experience with the following skills: ○ Graphing on the coordinate plane (6.NS.C.8) ○ Building and solving proportional relationships (7.RP.A.2) ○ Recognize and use function notation (HSF.IF.A) ○ Recognizing inverse functions (HSF.BF.B.4) ○ Graphing, solving and modeling quadratic, linear, exponential and absolute value functions (HSF.LE.A, HSF.LE.B) ○ Recognize, build and solve trigonometric ratios (HSG.CRT.C) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers ○ SOH CAH TOA

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on modelling periodic phenomena with trigonometric functions by critiquing student approaches/solutions to make connections through a short mini-lesson because students may formulate solutions from multiple perspectives (table, graph, calculation, logical reasoning). Presenting multiple solution methods allows students to think from a new perspective and analyze the features of these functions in a new way. This may illuminate errors in their own work or in the work of others.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit modelling periodic phenomena with trigonometric functions by offering opportunities to understand and explore different strategies because students may, at times, have an easier time solving a problem using a table rather than a graph or equation and vice versa. Allowing students the opportunity to explore these different strategies and to discuss their usefulness will help students deepen their understanding of the concepts as well as build their skills of using different representations to solve problems.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when modelling periodic phenomena with trigonometric functions because once a student has a conceptual understanding of periodic functions, the applications are easy to see in science and career specific scenarios. Linking these can allow students to explore their interests beyond mathematics.

CCSS Domain	*CCSS Cluster*
Trigonometric Functions	Prove and apply trigonometric identities

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on proving and applying trigonometric identities, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, connecting the process of proofs with the concept of replacing equivalent items can allow students to relate to the process. Students may have experienced this in recipes (replacing butter with oil), in making purchases (choosing one brand over another), etc. Use this as an opportunity to talk about equivalence. In some instances, the outcome may be changed by replacement, but in this mathematical process, equivalence is preserved.	
Cross-Curricular Connections	Economics – Substitution and utility when making purchases	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable*	• Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when proving and applying trigonometric identities, goal setting is critical because these algebraic proofs can be very challenging for students and if they do not have a productive mind frame as a starting point, they will struggle to make any progress. Encouraging students to set a goal to simply start these proofs by choosing to work from left to right or right to left or a goal to accurately rewrite an expression in terms of sin/cos can give students the support they need in engaging with difficult material. The importance is not that they can complete every proof, but rather that they have a goal they can achieve, and they work mathematically toward that goal.

	mathematicians that can use mathematics within school and society?	

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In Geometry, students learned the relationship of trigonometric ratios. In 8th grade, Algebra 1, and Geometry, students also learned the Pythagorean Theorem and how to graph on a coordinate plane	• Students will use their knowledge of the unit circle and relate that to determine trigonometric ratios not on the unit circle.	• In Precalculus and Calculus courses, students will connect this learning cluster to other trigonometric ratios (tangent, cosecant, secant and cotangent).

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when proving and applying trigonometric identities. Algebraic proofs can be very challenging for students and we want to confront that fact by providing extra supports and time for students to engage with the material, whether their work is exactly correct or not. The extra time experiencing the material will build deeper understanding.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.GB.7: This standard provides a foundation for work with proving and applying trigonometric identities because students used the visual structure of right triangles to apply the Pythagorean theorem. This process can be thought of as a concrete version of the algebraic process this cluster calls for. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready

		to access grade level instruction and assignments.

Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• What the domain of a function represents in terms of the function and what values are appropriate (i.e. discrete v. continuous values and other restrictions based on the type of function and the given context). • In functions there is an underlying structure that determines the transformation of any function, regardless of its type. • How to use trigonometric ratios, reference angles, and symmetry to find patterns on the unit circle. • Inverse trigonometric functions have restricted domains and ranges and are one to one.	• Find and interpret key features of a graph or table of a function, including extreme values, end behavior, and intervals of increase and decrease. • identify and graph parent functions and their transformations (i.e. vertical translation, horizontal translation, vertical stretch/shrink, reflect over x-axis, etc.). • Calculate values of sine, cosine, and tangent for given angles. • Solve trigonometric equations, including those written in quadratic form and equations containing more than one angle.	• Build on students' experience with the following skills: ○ Graphing on the coordinate plane (6.NS.C.8) ○ Building and solving proportional relationships (7.RP.A.2) ○ Recognize and use function notation (HSF.IF.A) ○ Recognizing inverse functions (HSF.BF.B.4) ○ Graphing, solving and modeling quadratic, linear, exponential and absolute value functions (HSF.LE.A, HSF.LE.B) ○ Recognize, build and solve trigonometric ratios (HSG.CRT.C) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers ○ SOH CAH TOA

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on proving and applying trigonometric identities by critiquing student approaches/solutions to make connections through a short mini-lesson because students frequently are able to take certain steps in a proof but may find themselves feeling stuck. Whether this is because they have made an error or just cannot see the next step, showing these approaches to their peers can help students make connections between the work they did and that of others, as well as critique steps that are different.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit proving and applying trigonometric identities by confronting student misconceptions because students may expect that each proof will progress in the same format, or that identities will always appear in the same way. Every proof is different and showing students that identities can be applied in a variety of ways may help them feel freed from looking for specific instances of the identities.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students. For example, prove and apply trigonometric identities because when students see the patterns and process clearly, we should allow them to challenge themselves at their own pace to try increasingly more challenging proofs.