NEW MEXICO
Public Education Department

New Mexico Instructional Scope
Geometry Congruence Guide

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Experiment with transformations in the plane
  - G.CO.A.1
  - G.CO.A.2
  - G.CO.A.3
  - G.CO.A.4
  - G.CO.A.5
- Understand congruence in terms of rigid motions
  - HSG.CO.B.6
  - HSG.CO.B.7
  - HSG.CO.B.8
- Prove geometric theorems
  - HSG.CO.C.9
  - HSG.CO.C.10
  - HSG.CO.C.11
- Make geometric constructions
  - HSG.CO.D.12
  - HSG.CO.D.13

Grade	*CCSS Domain*	*CCSS Cluster*
G	Congruence	Experiment with transformations in the plane

Cluster Standard: G.CO.A.1

Standard	Standards for Mathematical Practice
State and apply precise definitions of angle, circle, perpendicular, parallel, ray, line segment, and distance based on the undefined notions of point, line, and plane.	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students formalize their transformation language by building precise definitions based on properties.	• Demonstrate the knowledge of precise definitions of angles, line, point, plane, circles, perpendicular and parallel lines, and line segments • Calculate the linear distance and arc length • Demonstrate the use of proper notation • Make connections with rigid motions in relation to the definitions of words above
DOK	**Blooms**
2-3	Understand, Apply

Grade	*CCSS Domain*	*CCSS Cluster*
G	Congruence	Experiment with transformations in the plane

Cluster Standard: G.CO.A.2

Standard	Standards for Mathematical Practice
Represent transformations in the plane. (e.g., using transparencies and/or geometry software) *a*. Describe transformations as functions that take points in the plane as inputs and give other points as outputs. *b*. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus dilation).	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• They use formal notation and precise descriptions of transformations and sequences of transformations.	• Write a function that maps a preimage to its image or a description of a transformation. • Understand which transformations result in figures with congruent sides and angles and which do not • Represent transformations in the plane • Compare rigid motions that preserve distance and angle measure (translations, reflections, rotations) to transformations that do not preserve both distance and angle measure (e.g. dilations) • Understand that rigid motions produce congruent figures while dilations produce similar figures
DOK	**Blooms**
2-3	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
G	Congruence	Experiment with transformations in the plane

Cluster Standard: G.CO.A.3

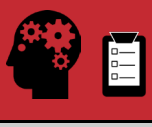
Standard	Standards for Mathematical Practice
Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and/or reflections that map the figure onto itself.	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP4:** Use appropriate tools strategically. • **SMP5:** Use appropriate tools strategically. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Rotational and reflection symmetry are identified with a specific degree of rotation or line(s) of symmetry.	• Identify and describe the different symmetries (line symmetry, rotational symmetry, point symmetry) of a figure • Determine the maximum possible lines of symmetries that exist for a given polygon • Determine the order and angle of a rotational symmetry • Determine the symmetries of a parallelogram, rectangle, rhombus, square, trapezoid and regular polygon • Understand symmetry in terms of transformations" (The Common Core Mathematics Companion) • Explore which shapes are symmetric and what symmetries they will have" (The Common Core Mathematics Companion) • Develop generalizations for the symmetries held by various geometric shapes" (The Common Core Mathematics Companion) • Determine the properties of a shape based on its symmetries" (The Common Core Mathematics Companion)
DOK	**Blooms**
2-3	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
G	Congruence	Experiment with transformations in the plane

Cluster Standard: G.CO.A.4

Standard	Standards for Mathematical Practice
Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
They use formal notation and precise descriptions of transformations and sequences of transformations. Rotational and reflection symmetry are identified with a specific degree of rotation or line(s) of symmetry.	• Describe rotations, reflections and translations • Determine and apply the properties of the isometric transformations • Identify which transformation has taken place based on the properties found between the preimage and image. • Identify the orientation relationship between the preimage and image. • Explore properties of transformations using common geometric relationships (e.g., parallel, perpendicular, and congruence)" (The Common Core Mathematics Companion). • Develop definitions of the transformations in terms of their properties" (The Common Core Mathematics Companion).
DOK	**Blooms**
2-3	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
G	Congruence	Experiment with transformations in the plane



Cluster Standard: G.CO.A.5

Standard	Standards for Mathematical Practice
Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure.	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP5:** Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
They use formal notation and precise descriptions of transformations and sequences of transformations. Rotational and reflection symmetry are identified with a specific degree of rotation or line(s) of symmetry.	• Model a sequence of transformations including reflections, rotations, and translations with a geometric figure. • Determine the sequence of transformations performed between a given preimage and image. • Describe which single transformation is the result of two reflections over parallel lines. • Describe which single transformation is the result of two reflections over intersecting lines. • Identify a transformation by its coordinate rule and then apply it to transform the shape. • Demonstrate how some composite transformations are not commutative.
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Students may confuse transformation and translation and/or not know how to express the differences between the two terms.
- Students may confuse the connection between the terms (angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.) in the standard G.CO.A.1 and their application in this standard.

Grade	CCSS Domain	CCSS Cluster
G	Congruence	Understand congruence in terms of rigid motions

Cluster Standard: HSG.CO.B.6

Standard	Standards for Mathematical Practice
Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP6:** Attend to precision. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students create a definition of triangle congruence in terms of rigid motions. They work to develop a set of criteria for triangle congruence and build a foundation for geometric proofs.	• Demonstrate that two figures are congruent if there is a sequence of rigid motions that map one figure to another. • Express verbally and in writing that two figures are congruent if and only if they have the same shape and size. • Model composite transformations to map one figure onto another. • Recognize and explain the effects of rigid motion on orientation and location of a figure. • Define congruence as a test to see if two figures are congruent.
DOK	**Blooms**
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
G	Congruence	Understand congruence in terms of rigid motions

Cluster Standard: HSG.CO.B.7

Standard	Standards for Mathematical Practice
Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP6:** Attend to precision. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students create a definition of triangle congruence in terms of rigid motions. They work to develop a set of criteria for triangle congruence and build a foundation for geometric proofs.	• Identify corresponding angles and sides based on congruence statements. • Develop and write congruence statements for two congruent triangles. • Determine if two triangles are congruent based on their corresponding parts. • Compare given figures to determine congruence and indicate whether the figure went through a rigid transformation. • Explain, using rigid motions, why in congruent triangles, corresponding parts must be congruent.
DOK	**Blooms**
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
G	Congruence	Understand congruence in terms of rigid motions

Cluster Standard: HSG.CO.B.8

Standard	Standards for Mathematical Practice
Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP6:** Attend to precision. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students create a definition of triangle congruence in terms of rigid motions. They work to develop a set of criteria for triangle congruence and build a foundation for geometric proofs.	• Create a method to determine unknown measurements of congruent triangles. • Explain the approach that was used to determine the congruency of two triangles given limited parts of triangles. • Explain the approach that was used to determine the congruence of two triangles. • Apply the criteria of SSS, SAS, ASA to prove triangle congruency.
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Combinations such as SSA or AAA are also a congruence criterion for triangles. All transformations, including dilations, are rigid motions. Any two figures that have the same area represent a rigid transformation. Students should recognize that the areas remain the same, but preservation of side and angle lengths determine that the transformation is rigid.

Grade	CCSS Domain	CCSS Cluster
G	Congruence	Prove geometric theorems

Cluster Standard: HSG.CO.C.9

Standard	Standards for Mathematical Practice
Prove theorems about lines and angles. Theorems include vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	• **SMP2:** Reason abstractly and quantitatively. • **SMP5:** Use appropriate tools strategically. • **SMP7:** Look for and make use of structure. • **SMP8**:Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students focus on formalizing geometric proof structure and language. They write formal proofs focusing on angle relationships, triangle segment and angle relationships, and parallelogram properties.	• Prove the vertical angle theorem and alternate interior angle theorem. • Prove corresponding angles are congruent. • Prove the converse of the alternate interior angle theorem and the corresponding angle theorem and use it to show that two lines are parallel. • Use perpendicular bisectors to locate the circumcenter of a triangle and to find the center of a circle given three points on the circle. • Express proofs both in writing and orally by using precise mathematical language • Examine and critique proofs produced by other students as well as their own
DOK	**Blooms**
2-3	Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
G	Congruence	Prove geometric theorems

Cluster Standard: HSG.CO.C.10

Standard	Standards for Mathematical Practice
Prove theorems about triangles. Theorems include measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	• **SMP2:** Reason abstractly and quantitatively. • **SMP5:** Use appropriate tools strategically. • **SMP7:** Look for and make use of structure. • **SMP8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students focus on formalizing geometric proof structure and language. They write formal proofs focusing on angle relationships, triangle segment and angle relationships, and parallelogram properties.	• Prove and apply that the sum of the interior angles of a triangle is 180°. • Prove and apply that the base angles of an isosceles triangle are congruent. • Prove and apply the midsegment (midline) of triangle theorem. • Prove that the medians of a triangle meet at a point, a point of concurrency. • Prove and apply the exterior angle theorem. • Determine the conditions for forming a triangle, when given three lengths.
DOK	**Blooms**
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
G	Congruence	Prove geometric theorems

Cluster Standard: HSG.CO.C.11

Standard	Standards for Mathematical Practice
Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.	• **SMP2:** Reason abstractly and quantitatively. • **SMP5:** Use appropriate tools strategically. • **SMP7:** Look for and make use of structure. • **SMP8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students focus on formalizing geometric proof structure and language. They write formal proofs focusing on angle relationships, triangle segment and angle relationships, and parallelogram properties.	• Prove properties of parallelograms and then apply them. • Prove the properties of rectangles and then apply them. • Prove the properties of rhombi and then apply them. • Prove the properties of squares and then apply them. • Classify a quadrilateral by its properties. • Identify the conditions necessary to prove that a quadrilateral is a parallelogram.
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Students may have a hard time generalizing and spend unnecessary time looking for multiple counterexamples to prove or disprove a proof, or, they may assume a conjecture is always true because it worked in all examples that were explored. Additionally, they may assume the converse of a statement is always true.

Grade	*CCSS Domain*	*CCSS Cluster*
G	Congruence	Make geometric constructions

Cluster Standard: HSG.CO.D.12

Standard	Standards for Mathematical Practice
Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.): *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*	• **SMP1**: Make sense of problems and persevere in solving them. • **SMP4:** Model with mathematics. • **SMP5:** Use appropriate tools strategically. • **SMP6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This cluster focuses on hands-on basic constructions. Students use geometric tools (compass, straightedge, software, etc.) to generate foundational pieces of geometry.	• Use construction techniques (compass, straight edge, software, etc.) to create figures. • Perform constructions including copy a segment, copy an angle, bisect segments and angles, construct perpendicular lines/segments, construct parallel lines.
DOK	**Blooms**
1-2	Understand, Apply

Grade	*CCSS Domain*	*CCSS Cluster*
G	Congruence	Make geometric constructions

Cluster Standard: HSG.CO.D.13

Standard	Standards for Mathematical Practice
Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP4:** Model with mathematics. • **SMP5:** Use appropriate tools strategically. • **SMP6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This cluster focuses on hands-on basic constructions. Students use geometric tools (compass, straightedge, software, etc.) to generate foundational pieces of geometry.	• Construct an equilateral triangle inscribed within a circle using construction techniques • Construct a square inscribed within a circle using construction techniques • Construct a regular hexagon inscribed within a circle using construction techniques.
DOK	**Blooms**
3-4	Apply, Create

Common Misconceptions

- Some students may believe that a construction is the same as a sketch or drawing. Emphasize the need for precision and accuracy when doing constructions. Stress the idea that a compass and straightedge are identical to a protractor and ruler. Explain the difference between measurement and construction.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Congruence**	Strand: **Experiment with transformations in the plane**

Suggested Student Discourse Questions

- Share how you used transformations to find lines of symmetry- give feedback to your partner about their strategy.
- How can you determine the size of a field irrigated with a single length of pipe anchored in the center and being rotated around the field?
- Is it possible to use the x-y axis to perform rotations without a protractor or compass? How do you know?
- Is there a relationship between the coordinates of corresponding points in a transformation? Is the relationship always the same? Why or why not?

Domain: **Congruence**	Strand: **Understand congruence in terms of rigid motions**

Suggested Student Discourse Questions

- Share with your group/partner the process you used to determine if one shape maps to another. What feedback can you give to fellow students?
- How could you use rigid transformations to determine if the walls of a room in a house could be the same height?
- Did you follow the same process in making rigid motions as your fellow students? If not, what were the differences?
- How do rigid motions in transformations preserve congruence between corresponding sides and corresponding angles?

Domain: **Congruence**	Strand: **Prove geometric theorems**
Suggested Student Discourse Questions	
● Look at the way fellow students wrote their proofs. Did they follow the same steps as you? What differences and similarities do you see? ● If you knew the length of one object, could you use it to estimate the length of another object based on what you see around them? Why or why not?	● Think of the easiest way to prove one segment is parallel to another segment. Then, change the steps to make the proof different. Do this again as many times as you can. ● Could it be proven that two lines are perpendicular to the segments crossing between them? If not, what else would you need to know in order to write such a proof?

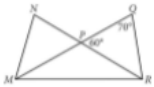
ASSESSMENT GUIDE

Grade	*CCSS Domain*	*CCSS Cluster*
G	Congruence	Experiment with transformations in the plane
	Sample Task #1 (Constructed Response)	
	Source: http://tasks.illustrativemathematics.org/content-standards/HSG/CO/A/tasks/1468 The linked assessment question addresses G-CO.A, specifically the question requires students to look at lines of symmetry using reflections. Two different arguments are presented using triangle congruence and another which uses rotations and reflections. This assessment should be given to students after they've been introduced to the formal definition of reflections. Students will engage in SMP1, SMP7, and potentially SMP3 depending on if students work in groups to share their solutions.	

Grade	*CCSS Domain*	*CCSS Cluster*
G	Congruence	Understand congruence in terms of rigid motions
	Sample Task #2 (Constructed Response)	
	SAT Item #: 422659: The linked assessment question addresses G-CO.B., specifically the question requires students to identify and use congruent angles to find the angle measure in a triangle.	

CollegeBoard Question ID 422659

SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Lines, angles, and triangles	1. Use concepts and theorems relating to congruence and similarity of triangles to solve problems.	No Calculator



In the figure above, $\overline{MQ}$ and $\overline{NR}$ intersect at point P, $NP = QP$, and $MP = PR$. What is the measure, in degrees, of $\angle QMR$? (Disregard the degree symbol when gridding your answer.)

Question Difficulty: Medium

The correct answer is 30. It is given that the measure of $\angle QPR$ is $60°$. Angle MPR and $\angle QPR$ are collinear and therefore are supplementary angles. This means that the sum of the two angle measures is $180°$, and so the measure of $\angle MPR$ is $120°$. The sum of the angles in a triangle is $180°$. Subtracting the measure of $\angle MPR$ from $180°$ yields the sum of the other angles in the triangle MPR. Since $180 - 120 = 60$, the sum of the measures of $\angle QMR$ and $\angle NRM$ is $60°$. It is given that $MP = PR$, so it follows that triangle MPR is isosceles. Therefore $\angle QMR$ and $\angle NRM$ must be congruent. Since the sum of the measure of these two angles is $60°$, it follows that the measure of each angle is $30°$.

An alternate approach would be to use the exterior angle theorem, noting that the measure of $\angle QPR$ is equal to the sum of the measures of $\angle QMR$ and $\angle NRM$. Since both angles are equal, each of them has a measure of $30°$.

Additional Assessment:
Properties of Congruent Triangles: https://tasks.illustrativemathematics.org/content-standards/HSG/CO/B/7/tasks/1637
The linked assessment question addresses G-CO.B, specifically the question requires students to look at two triangles and connect the concept of congruence of corresponding parts to congruence of shape in terms of rigid motion. Two different approaches are prompted: one assuming the triangles can be mapped on each other and asking students to explain which parts are congruent, and another assuming the triangles are congruent and asking students to create the sequence of transformations to map one onto the other. This assessment should be given to students after they've had time to work with rigid motion and have a firm grasp of naming parts of triangles. Students will engage in SMP3, SMP6 and SMP7.

Grade	CCSS Domain	CCSS Cluster
G	Congruence	Prove geometric theorems

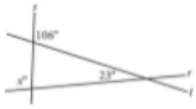
Sample Task #3 (Constructed Response)

SAT Item #: 422005 The linked assessment question addresses G-CO.C., specifically the question requires students to know and apply theorems about exterior and interior angles.

CollegeBoard **Question ID 422005**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Additional Topics in Math	Hard	Additional Topics in Math	Lines, angles, and triangles	4. Know and directly apply relevant theorems such as b. triangle similarity and congruence criteria;	No Calculator

Intersecting lines r, s, and t are shown below.



What is the value of x ?

Question Difficulty: Hard

The correct answer is 97. The intersecting lines form a triangle, and the angle with measure of $x°$ is an exterior angle of this triangle. The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles of the triangle. One of these angles has measure of $23°$ and the other, which is supplementary to the angle with measure $106°$, has measure of $180° - 106° = 74°$. Therefore, the value of x is $23 + 74 = 97$.

Additional Assessment:
Midpoints of the Sides of a Parallelogram: https://tasks.illustrativemathematics.org/content-standards/HSG/CO/C/11/tasks/35
The linked assessment question addresses G-CO.C, specifically the question requires students to prove segments of a parallelogram are congruent. Students use knowledge of corresponding parts of congruent triangles are congruent to form arguments. This assessment should be given to students after they've worked with corresponding parts of triangles. Students will engage in SMP 2 and SMP 7.

Grade	*CCSS Domain*	*CCSS Cluster*
G	Congruence	Make geometric constructions
	Sample Task #4 (Constructed Response)	
	Standards Aligned Instructionally Embedded Formative Assessment Resources: Source: https://tasks.illustrativemathematics.org/content-standards/HSG/CO/D/12/tasks/966 The linked assessment question addresses G-CO.D, specifically the question requires students to prove that a specific segment is a perpendicular bisector to another segment. Students may use knowledge of corresponding parts of congruent triangles are congruent to form arguments or may work through explanation using congruence as it follows from rigid motion transformations. This assessment should be given to students after they've been introduced to geometric construction tools and have a firm grasp of congruence. Students will engage in SMP1, SMP5 and SMP6.	

MLSS AND CLR GUIDE

- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions

CCSS Domain	*CCSS Cluster*
Congruence	Experiment with transformations in the plane

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on Exploring Transformations in the Plane, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, viewing artwork from a variety of cultures or having students select and bring in artwork that shows specific transformations can build the bridge between things they may have seen at home to what they are seeing in school. These cultural connections can be made explicitly or can be incorporated into word problems and other abstract application style questions.	
Cross-Curricular Connections	Art: Support students in making connections between geometric transformation and art, especially in relation to the idea of repetition and perspective in artwork.[2]	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and*	• Equity Based Practices (Tasks): The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying Exploring Transformations in the Plane the types of mathematical tasks are critical because the purpose of the standard is for students to explore and experience the transformations for

	linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	themselves. This requires teachers to select tasks and activities that provide for students to experiment with the transformations in the plane rather than explicitly tell students what steps to take.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	***Current Learning***	***Future Learning***
• This cluster aligns directly to the learning in 8th grade, when students were introduced to congruence and similarity using physical models, transparencies, or geometry software. They have described sequences of rigid motions informally and in terms of coordinates. Learners have verified experimentally the properties of transformations and describe their effects on two-dimensional figures using coordinates.	• Transformation definitions will serve as the basis for theorems that will be proven later in the year.	• In Algebra II, students will connect their knowledge of transformations to functions. They will use their knowledge of transformation language to compare a function to its parent function, identify lines of symmetry, and other characteristics and behavior of functions.

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying transformations in the plane because students are experimenting with transformations and precise language will be important.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	4.GA.1: This standard provides a foundation for work with transformations in the plane because students should have already built a firm grasp of key vocabulary such as angle, point, line, parallel, perpendicular, etc. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• Transformations as functions and symmetry in terms of transformations.
• Two figures are congruent if there is a sequence of transformations that maps one onto another.
• Two figures are similar if they have the same shape with congruent angles and proportional side
	• Carry out rotations, reflections, translations, and dilations using a variety of tools and compare and contrast their effects.
	• Prove two triangles are congruent using ASA, SAS, and SSS or two triangles are similar using
		• Build on students' experience with the following skills:
		  ○ Understand and use the coordinate axis
		  ○ Write and solve linear equations, especially proportions
		  ○ Recognize and draw geometric shapes (square, triangle, trapezoid, etc.)
		  ○ Understand and use the formulas of geometric shapes (Perimeter, Area, Volume, etc.)
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies

lengths. ● The formal language for relationships between angles including vertical angles, angles created when a transversal intersects parallel lines, angles and sides of triangles, the segments and angles of parallelograms, and the trigonometric ratios sine, cosine, and tangent.	AA, SAS, and SSS. ● Build formal justifications (proofs) for the theorems about lines and angles, triangles, and parallelograms. ● Find unknown side lengths and angle measures of right triangles using trigonometric ratios and the Pythagorean Theorem.	○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos graphing calculator ○ Desmos scientific calculator ○ Desmos geometry tool ○ GeoGebra ○ Graphing or scientific calculator ○ Google Drawing ○ Geometric tools (ruler, protractor, compass, etc.) ○ Tracing paper ○ Graph paper and mirror/string/etc. ○ Craft tools (scissors, string, construction paper, etc.) ○ Paper folding

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on transformations in the plane by clarifying mathematical ideas and/or concepts through a short mini lesson because students may struggle with rotations or reflections about a point/line that is not at the origin.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on transformations in the plane by offering opportunities to understand and explore different strategies because there may be several approaches to map one object onto another.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the application and development of abstract thinking skills when studying transformations in the plane. Geometric software can allow students to experiment with combinations of transformations and using such software can allow for abstract application of knowledge of transformations.

CCSS Domain	*CCSS Cluster*
Congruence	Understand congruence in terms of rigid motions

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on Congruence in terms of Rigid Motion, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, teachers can create connections from native language to English by focusing on cognates (Words that sound the same in two different languages). Incorporating the usage of cognates throughout a unit validates and affirms all languages and can encourage students to explore these terms in language other than their native language.	
Cross-Curricular Connections	Computer Science: program to create visual demo of transformations	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures*	• Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying Understanding Congruence in terms of Rigid Motion the types of mathematical tasks are critical because students need the time and experience in

	and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	connecting prior knowledge of congruence to rigid motions in the plane. Tasks should activate knowledge of both congruence and rigid motions and build the bridge between them.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In 7th grade, students focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. In 8th grade, students develop understanding of congruence using physical models, transparencies, or geometry software. Students also understand that figures are congruent if the second can be obtained from the first by a sequence of rotations, reflections, and/or translations. These foundational skills are applied within this standard to continue to explore congruence and the	• Students will use triangle congruence concepts to develop future postulates and theorems. Concepts of triangle congruence serve to build a foundation for work with triangle proofs in future clusters.	• In later courses, students consider triangle congruence and the ambiguous case when working with the Law of Sines and Law of Cosines.

impact of rigid motions on geometric shapes.		

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying understanding congruence in terms of rigid motion because students may incorrectly interchange congruence and similarity.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.GA.2: This standard provides a foundation for work with understanding congruence in terms of rigid motion because this is where the concept of congruence is solidified in terms of one object being able to be moved directly on top of another and match perfectly. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework

A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• Transformations as functions and symmetry in terms of transformations.
• Two figures are congruent if there is a sequence of transformations that maps one onto another.
• Two figures are similar
	• Carry out rotations, reflections, translations, and dilations using a variety of tools and compare and contrast their effects.
	• Prove two triangles are
		• Build on students' experience with the following skills:
		  ○ Understand and use the coordinate axis
		  ○ Write and solve linear equations, especially proportions
		  ○ Recognize and draw geometric shapes (square, triangle, trapezoid, etc.)
		  ○ Understand and use the formulas of geometric shapes (Perimeter, Area, Volume, etc.)

if they have the same shape with congruent angles and proportional side lengths. ● The formal language for relationships between angles including vertical angles, angles created when a transversal intersects parallel lines, angles and sides of triangles, the segments and angles of parallelograms, and the trigonometric ratios sine, cosine, and tangent.	congruent using ASA, SAS, and SSS or two triangles are similar using AA, SAS, and SSS. ● Build formal justifications (proofs) for the theorems about lines and angles, triangles, and parallelograms. ● Find unknown side lengths and angle measures of right triangles using trigonometric ratios and the Pythagorean Theorem.	● Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos graphing calculator ○ Desmos scientific calculator ○ Desmos geometry tool ○ GeoGebra ○ Graphing or scientific calculator ○ Google Drawing ○ Geometric tools (ruler, protractor, compass, etc.) ○ Tracing paper ○ Graph paper and mirror/string/etc. ○ Craft tools (scissors, string, construction paper, etc.) ○ Paper folding

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on understanding congruence in terms of rigid motion by clarifying mathematical ideas and/or concepts through a short mini-lesson because students may incorrectly apply the triangle congruence theorems (ASA, SSS, etc.) and could benefit from clarifying these and connecting them to the concept of congruence.
Intensive	What assessment data will help identify content needing to be revisited for	For example, some students may benefit from intensive extra time during and after a unit understanding congruence in terms of rigid motion by helping students

	intensive interventions?	move from specific answers to generalizations for certain types of problems because this cluster requires students to prove, generally speaking, if two triangles are congruent by applying knowledge of congruence rather than using specific triangles with concrete measurements.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension focused on the application of and development of abstract thinking skills when understanding congruence in terms of rigid motion. Their established knowledge base of congruence could allow them to generate examples and non-examples of congruent shapes.

CCSS Domain	*CCSS Cluster*
Congruence	Prove geometric theorems

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on proving geometric theorems, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, the history of geometry spans across cultures. This history can be incorporated as students learn how to build and create formal proofs from informal diagrams, experimentation and/or oral arguments.	
Cross-Curricular Connections	Links to history can be made by exploring how geometric proofs developed over time in different cultures. Further, links to computer science can be made by discussing and displaying how coding reflects a variety of steps to get from one point to another, as is mirrored in proofs.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students*	• Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion, we position them as knowers and doers of mathematics. Using equitable

	and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	talk moves students and the ways students talk about who is and isn't capable of mathematics. As a result, we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying Proving Geometric Theorems facilitating meaningful mathematical discourse is critical because proofs of theorems can frequently be seen from multiple perspectives. Purposefully sequencing, discussing and validating these different perspectives can help students internalize their worth in the classroom. It can also show students that knowing a piece of the larger puzzle often "unlocks" the door to the next step in a proof. The key is in selecting and sequencing a variety of perspectives to share out and focus on how it helps the process, rather than whether it is complete and correct.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In 7th grade, students use facts about supplementary, complementary, vertical, and adjacent angles. In 8th grade, learners use informal arguments to establish facts about the angle sum and exterior angle of triangles, and about the angles created when parallel lines are cut by a transversal. These angle facts connect this cluster as students apply them when	• The formalized theorems within this cluster will be used to build theorems and proofs for concepts in future clusters within the Geometry course.	• Understanding the logical flow of developing a proof will be used in future courses such as when proving trigonometric identities

creating proofs.		

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., structured proofs) when studying proving geometric theorems because students may be unfamiliar with traditionally structured mathematical proofs.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	G.CO.B6: This standard provides a foundation for work with proving geometric theorems because this is where students have formalized an understanding of congruence in terms of rigid motion, which is required for every aspect of this cluster. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework

A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• Transformations as functions and symmetry in terms of transformations.
• Two figures are congruent if there is a sequence of transformations that maps one onto another.
• Two figures are similar if they have
	• Carry out rotations, reflections, translations, and dilations using a variety of tools and compare and contrast their effects.
	• Prove two triangles are congruent using ASA, SAS, and SSS or two triangles
		• Build on students' experience with the following skills:
		  ○ Understand and use the coordinate axis
		  ○ Write and solve linear equations, especially proportions
		  ○ Recognize and draw geometric shapes (square, triangle, trapezoid, etc.)
		  ○ Understand and use the formulas of geometric shapes (Perimeter, Area, Volume, etc.)
		• Cognitive Strategies

the same shape with congruent angles and proportional side lengths. ● The formal language for relationships between angles including vertical angles, angles created when a transversal intersects parallel lines, angles and sides of triangles, the segments and angles of parallelograms, and the trigonometric ratios sine, cosine, and tangent.	are similar using AA, SAS, and SSS. ● Build formal justifications (proofs) for the theorems about lines and angles, triangles, and parallelograms. ● Find unknown side lengths and angle measures of right triangles using trigonometric ratios and the Pythagorean Theorem.	○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos graphing calculator ○ Desmos scientific calculator ○ Desmos geometry tool ○ GeoGebra ○ Graphing or scientific calculator ○ Google Drawing ○ Geometric tools (ruler, protractor, compass, etc.) ○ Tracing paper ○ Graph paper and mirror/string/etc. ○ Craft tools (scissors, string, construction paper, etc.) ○ Paper folding

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on proving geometric theorems by critiquing student approaches/solutions to make connections through a short mini-lesson because students will often see different methods of progressing through a proof which can benefit all students to see. Further, students may make specific claims or statements that are unsupported or incorrect, and critiquing this reasoning will strengthen all students' understanding of the content.
Intensive	What assessment data will help identify content	For example, some students may benefit from intensive extra time during and after a unit about proving

	needing to be revisited for intensive interventions?	geometric theorems by offering opportunities to understand and explore different strategies because students may struggle with writing their own proofs, particularly in the beginning. It may be helpful to provide many examples and ask students to analyze each statement as to whether it can be mathematically supported or not. This could also be an opportunity to allow students to show their thinking with pictures rather than a formal proof structure.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying proving geometric theorems because giving students the opportunity to select certain theorems to prove can be both challenging and engaging for those with a firm grasp of the process.

CCSS Domain	*CCSS Cluster*
Congruence	Make geometric constructions

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on Making Geometric Constructions, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, architects and other technical careers make regular use of the same tools used in geometric constructions. Connecting the use of mathematical tools with the real world can solidify the importance and relevance of material being learned, as well as encourage students to goal-set for future careers and interests.	
Cross-Curricular Connections	Art: drafting, geometric shape work	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge concepts for students. Mathematical representations are

	the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying Making Geometric Constructions the use of mathematical representations within the classroom is critical because students may have a varied background in reading/writing technical written directions and/or using rulers, straightedges, compasses, geometric software, etc. This background could be established in prior schooling or in specific cultural/home usage. Connecting tools they are familiar with to tools that may be new or uncomfortable to them shows the value of their current knowledge at the same time as expanding that knowledge base.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• This cluster connects to students previously taught skills from 7th grade when they constructed	• Construction connects and adds on to learning from previous clusters within the Geometry course by	• Construction techniques could be applied to unit circle, and conic sections in future courses.

geometric shapes when given certain conditions in the 7.G.A cluster. Additionally, in 8th grade within the 8.G.A cluster, students worked with two-dimensional figures and verified their properties.	building on triangle congruence theorems (SSS, SAS), properties of parallel and perpendicular lines, and polygons and their properties.	

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that rehearses prior learning experiences working with geometric construction tools (straight edge, ruler, compass, etc.) because students may not use them consistently, correctly, or may need a refresher.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	7.GA.2: This standard provides a foundation for work with making geometric constructions because this standard called for students to use a variety of tools to draw shapes that fit given constraints. This is where they should have mastered use of ruler/straight edge and compass. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework

A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• Transformations as functions and symmetry in terms of transformations.
• Two figures are
	• Carry out rotations, reflections, translations, and dilations using a variety of tools and	• Build on students' experience with the following skills:
		  ○ Understand and use the coordinate axis
		  ○ Write and solve linear equations, especially proportions

congruent if there is a sequence of transformations that maps one onto another. ● Two figures are similar if they have the same shape with congruent angles and proportional side lengths. ● The formal language for relationships between angles including vertical angles, angles created when a transversal intersects parallel lines, angles and sides of triangles, the segments and angles of parallelograms, and the trigonometric ratios sine, cosine, and tangent.	compare and contrast their effects. ● Prove two triangles are congruent using ASA, SAS, and SSS or two triangles are similar using AA, SAS, and SSS. ● Build formal justifications (proofs) for the theorems about lines and angles, triangles, and parallelograms. ● Find unknown side lengths and angle measures of right triangles using trigonometric ratios and the Pythagorean Theorem.	○ Recognize and draw geometric shapes (square, triangle, trapezoid, etc.) ○ Understand and use the formulas of geometric shapes (Perimeter, Area, Volume, etc.) ● Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos graphing calculator ○ Desmos scientific calculator ○ Desmos geometry tool ○ GeoGebra ○ Graphing or scientific calculator ○ Google Drawing ○ Geometric tools (ruler, protractor, compass, etc.) ○ Tracing paper ○ Graph paper and mirror/string/etc. ○ Craft tools (scissors, string, construction paper, etc.) ○ Paper folding

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on making geometric constructions by clarifying mathematical ideas and/or concepts through a short mini-lesson because students may have simple errors (e.g. copying a segment without measurement, etc.) that require short and direct instruction.

Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on making geometric constructions by offering opportunities to understand and explore different strategies> because some students may work more efficiently with one consistent tool, while others may benefit from having a variety of tools at their disposal. Further, some students benefit from using geometric software to visualize their constructions rather than pencil and paper.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying making geometric constructions because this can challenge students to construct any shape, explaining or showing their methods for each.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Understand and apply theorems about circles
  - HSG.C.A.1
  - HSG.C.A.2
  - HSG.C.A.3
- Find arc lengths and areas of sectors of circles
  - HSG.C.B.5

Grade	CCSS Domain	CCSS Cluster
G	Circles	Understand and apply theorems about circles

Cluster Standard: HSG.C.A.1

Standard	Standards for Mathematical Practice
Prove that all circles are similar.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP5:**Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Learners will apply concepts of similarity to circles and their related components, explore inscribed and circumscribed circles and their associated polygons through constructions. This cluster builds many of the basic properties for angles, lines, and segments related to circles. Learners explore those properties and form conjectures. They should then be encouraged to create justifications regarding why their conjectures are correct. Learners refer back to their work with transformations to better understand these relationships.	• Show that all circles are similar by proving that the ratio of a circle's circumference to its diameter for different sized circles is a constant • Calculate the circumference of a circle, given the diameter or radius. • Calculate angles inside and outside of a circle. • Prove that circles are similar. • Compare the ratios of the radius and circumference of multiple circles to determine similarity.
DOK	**Blooms**
3-4	Analyze, Evaluate

Grade	CCSS Domain	CCSS Cluster
G	Circles	Understand and apply theorems about circles

Cluster Standard: HSG.C.A.2

Standard	Standards for Mathematical Practice
Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP5:** Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Learners will apply concepts of similarity to circles and their related components, explore inscribed and circumscribed circles and their associated polygons through constructions. This cluster builds many of the basic properties for angles, lines, and segments related to circles. Learners explore those properties and form conjectures. They should then be encouraged to create justifications regarding why their conjectures are correct. Learners refer back to their work with transformations to better understand these relationships.	• Identify central angles, inscribed angles, circumscribed angles, tangent line and chords on a circle from a drawing. • Construct and explain examples of central angles, inscribed angles, circumscribed angles, tangent line and chords on a circle. • Describe the relationship between central angles, inscribed angles, circumscribed angles, tangent lines, and chords.
DOK	**Blooms**
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
G	Circles	Understand and apply theorems about circles

Cluster Standard: HSG.C.A.3

Standard	Standards for Mathematical Practice
Construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP5:** Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Learners will apply concepts of similarity to circles and their related components, explore inscribed and circumscribed circles and their associated polygons through constructions. This cluster builds many of the basic properties for angles, lines, and segments related to circles. Learners explore those properties and form conjectures. They should then be encouraged to create justifications regarding why their conjectures are correct. Learners refer back to their work with transformations to better understand these relationships.	• Construct angle bisectors, perpendicular bisectors, inscribed circle of a triangle and circumscribed circle about a triangle. • Prove that opposite angles of an inscribed quadrilateral are supplementary.
DOK	**Blooms**
1-3	Apply, Analyze

Common Misconceptions

- Students may try to solve by sketching the circles instead of ensuring precision by using a compass or appropriate tool to construct an accurate circle.
- Students might confuse the relationships of central angle, inscribed angles, circumscribe angles, as well as tangent line and chords of a circle.

Grade	CCSS Domain	CCSS Cluster
G	Circles	Find arc lengths and areas of sectors of circles

Cluster Standard: HSG.C.B.5

Standard	Standards for Mathematical Practice
Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	• **SMP2:** Reason abstractly and quantitatively. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• This cluster explores the relationship between the length of an arc and the measure of a central angle. Learners develop a definition for the radian measure of an angle and apply radians to find the area of sectors.	• Calculate the length of an intercepted arc. • Demonstrate that the constant of proportionality between arc length and the radius of the circle is the radian measure of the central angle. • Derive the formula for the area of a sector using similarity. • Calculate the area of a sector.
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Students often struggle with precision while working within this cluster. Small errors in constructions will lead to results that do not work.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Circles**	Strand: Understand and apply theorems about circles

Suggested Student Discourse Questions

- Share the process you used to prove the circles are similar. What feedback do you have for them?
- How do you know that distances on a globe can represent distances in real-life?
- Which technique for determining similarity between circles is easier for you to use- transformations on the plane or manipulation of equations?
- How are the radius and diameter of similar circles related to each other?

ASSESSMENT GUIDE

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circle

Grade	CCSS Domain	CCSS Cluster
G	Circles	Understand and apply theorems about circles

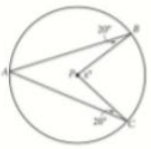
Sample Task #1 (Constructed Response)

Standards Aligned Instructionally Embedded Formative Assessment Resources:

SAT Item # 422459: The linked assessment question addresses G-C.A., specifically the question requires students to use knowledge of inscribed and central angles.

CollegeBoard **Question ID 422459**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Additional Topics in Math	Hard	Additional Topics in Math	Circles	1. Use definitions, properties, and theorems relating to circles and parts of circles, such as radii, diameters, tangents, angles, arcs, arc lengths, and sector areas to solve problems.	Calculator



Point P is the center of the circle in the figure above. What is the value of x ?

Question Difficulty: Hard

The correct answer is 80. If points A and P are joined, then the triangles that will be formed, APB and APC, are isosceles because $PA = PB = PC$. It follows that the base angles on both triangles each measure $20°$. Angle BAC consists of two base angles; therefore, the measure of angle BAC = $40°$. Since the measure of an angle inscribed in a circle is half the measure of the central angle that intercepts the same arc, it follows that the value of x is $80°$.

Additional Assessment:

Additional Assessment:

http://tasks.illustrativemathematics.org/content-standards/HSG/C/A/1/tasks/1368

The linked assessment question addresses G-C.A, specifically the question requires students to make use of visualizing transformations as well as knowledge of equations for a circle. This assessment should be given to students after they've been introduced to these concepts. Students will engage in SMP1 and SMP8.

Grade	CCSS Domain	CCSS Cluster
G	Circles	Find arc lengths and areas of sectors of circles

Sample Task #1 (Constructed Response)

Standards Aligned Instructionally Embedded Formative Assessment Resources:

SAT Item # 5209208: The linked assessment question addresses G-C.B., specifically the question requires students to use knowledge of arc length.

CollegeBoard Question ID 5209208

SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Circles	1. Use definitions, properties, and theorems relating to circles and parts of circles, such as radii, diameters, tangents, angles, arcs, arc lengths, and sector areas to solve problems.	No Calculator

The circle above has center O, the length of arc $\overset{\frown}{ADC}$ is 5π, and $x = 100$. What is the length of arc $\overset{\frown}{ABC}$?

Question Difficulty: Medium

A. 9π

B. 13π

C. 18π

D. $\frac{13}{2}\pi$

Choice B is correct. The ratio of the lengths of two arcs of a circle is equal to the ratio of the measures of the central angles that subtend the arcs. It's given that arc $\overset{\frown}{ADC}$ is subtended by a central angle with measure 100°. Since the sum of the measures of the angles about a point is 360°, it follows that arc $\overset{\frown}{ABC}$ is subtended by a central angle with measure $360° - 100° = 260°$. If s is the length of arc $\overset{\frown}{ABC}$, then s must satisfy the ratio $\frac{s}{5\pi} = \frac{260}{100}$. Reducing the fraction $\frac{260}{100}$ to its simplest form gives $\frac{13}{5}$. Therefore, $\frac{s}{5\pi} = \frac{13}{5}$. Multiplying both sides of $\frac{s}{5\pi} = \frac{13}{5}$ by 5π yields $s = 13\pi$.

Choice A is incorrect. This is the length of an arc consisting of exactly half of the circle, but arc $\overset{\frown}{ABC}$ is greater than half of the circle. Choice C is incorrect. This is the total circumference of the circle. Choice D is incorrect. This is half the length of arc $\overset{\frown}{ABC}$, not its full length.

MLSS AND CLR GUIDE

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circle

CCSS Domain	*CCSS Cluster*
Circles	Understand and apply theorems about circles

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on understanding and applying theorems about circles, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, provide a task that requires students to calculate the dimensions needed for a new restaurant to build a triangular deck (with one side being the restaurant building) will relate school learning to community/home application.	
Cross-Curricular Connections	Art: Consider discussing how inscribed and circumscribed angles may be used in calculating specific designs in landscape, apparel, etc. Designers are often given constraints in which to create an image and may use knowledge of these angles to help design an appropriate image.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home*	• Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice leads to more students viewing themselves as capable mathematicians. For example, when understanding and applying theorems about circles the types of mathematical tasks are critical because they will help students make connections to how the math in this cluster is applicable to real-life context. In some regards, tasks can be designed to highlight various cultures which in turn allows for students to learn about the diversity amongst their peers' cultures. Tasks should be cognizant of culturally responsive academic vocabulary, language, and literacy.

	culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In 8th grade, learners have worked with two-dimensional figures and verified their properties. These skills from 8.G.A. will support students in their ability to work with circles while working within this standard.	• Previous work with similarity will be applied to circles. Construction adds to learning from previous clusters by increasing skills with formal construction, building on angle congruence, perpendicular lines/segments, and properties of polygons. This will lead to work with arcs and areas of sectors, as well as prepare learners for future work in the Geometry course with 3-dimensional geometry and cross-sections.	• Unit circles, their central angles, and reference angles will build on foundational skills learned in this cluster.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this*	Some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas because new concepts (inscribed, circumscribed and central angles and tangent

	cluster within your HQIM?	lines) are connected with prior held concepts (parallel, perpendicular, radii, etc.). Students may need extra time to wrestle with the differences between these types of angles/lines and the work they have already mastered.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	SRTA2/CO.C.9: This standard provides a foundation for understanding and applying theorems about circles because these lay the groundwork for understanding similarity and congruence, both of which are the foundational concepts of this cluster. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on understand and apply theorems about circles by critiquing student approaches/solutions to make connections through a short mini-lesson because proofs and constructions can be seen from a variety of perspectives, some correct and others incorrect but will offer insight into common misconceptions all students may have.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit to understand and apply theorems about circles by confronting student misconceptions because when students do not have a firm grasp of congruence and similarity, progressing forward and applying those to circles and proofs will be impossible.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension understanding and applying theorems about circles because proofs with circles can be challenging and require students to pull facts from a variety of places, some implied and others explicitly stated. This develops and pushes students' abstract thinking skills.

CCSS Domain	*CCSS Cluster*
Circles	Find arc lengths and areas of sectors of circles

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on arc lengths and areas of sectors of circles, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, use every day circular items to show how sectors are part of the entire circle.	
Cross-Curricular Connections	Economics: Connect to a variety of circular foods, talking about maximizing crust or finding the largest slice.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that*	• Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it hinders students with strong prior familiarity from learning more methods for solving tasks that occur outside of school mathematics. For example, when studying arc lengths and areas of sectors of circles the types of mathematical tasks are critical because the struggle to be all-inclusive can be an issue. Where students with strong procedural knowledge will easily follow a process, some students will struggle and need adaptation and accommodations. Some ways to address this would be to adapt procedures into the students spoken language, apply terminology and problems from the student's daily life, use hands-on demonstrations, and use bi-lingual grouping of students

	can use mathematics within school and society?	

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In 7th grade, the formulas for the area and circumference of a circle are learned and then applied to solve problems. They give an informal derivation of the relationship between the circumference and area of a circle.	• Later in the Geometry course when calculating geometric probabilities, students will need to know how to calculate the area of a sector which is taught within this cluster.	• In future courses, students expand on their basic understanding of the radian measure of an angle. They apply radian measures when discovering relationships within the unit circle and while learning trigonometric relationships.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on arc lengths and areas of sectors of circles because understanding here depends on how deeply a student understands earlier concepts such as area of a circle and the terminology involved.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	7.GB.4 and 6.RP.A2: : This standard provides a foundation for work with arc lengths and areas of sectors of circles because it deals with the concept of area of circles and the concept of proportionality. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	Students may benefit from re-engaging with content during a unit on arc lengths and areas of sectors of circles by examining tasks from a different perspective through a short mini-lesson because we are welding concepts together to form a new concept, this process is not automatic and by backing up and looking at the problem from a different point of view.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on arc length and areas of sectors of circles by offering opportunities to understand and explore different strategies because what works for one student may not work for another.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension when studying arc lengths and areas of sectors of circles because linking the concept to something that a student will experience in their own lives will add depth to their experience regarding this concept.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Understand independence and conditional probability and use them to interpret data
  - HSS.CP.A.1
  - HSS.CP.A.2
  - HSS.CP.A.3
  - HSS.CP.A.4
  - HSS.CP.A.5
- Use the rules of probability to compute probabilities of compound events.
  - HSS.CP.B.6
  - HSS.CP.B.7

Grade	CCSS Domain	CCSS Cluster
G	Conditional Probability & The Rules of Probability	Understand independence and conditional probability and use them to interpret data

Cluster Standard: HSS.CP.A.1

Standard	Standards for Mathematical Practice
Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	• **SMP2:** Reason abstractly and quantitatively. • **SMP6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• A probability model may consist of a list or description of possible outcomes (the sample space) each of which is assigned a certain probability. Probability rules can be developed and understood through the use of the sample space. When events are independent, the outcome of the first event does not change the sample space for subsequent events. Independent events, knowing one event has occurred affects the likelihood of another event occurring. Use of two-way frequency tables helps learners develop conceptual understanding of conditional probability. The use of tables, symbols, and real-world scenarios are emphasized. Learners consider the context of situations as they build mathematical models, interpret events, and explain results in terms of a probability model.	• Analyze a sample space to describe an event, union of events, intersection of events and complement of event • Use tree diagrams, organized lists, tables, and/or Venn diagrams to represent sample spaces. • Determine unions of sample spaces. • Determine intersections of sample spaces • Determine complements of sample sets. • Represent unions, intersections, and complements using set notation.
DOK	**Blooms**
2-3	Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
G	Conditional Probability & The Rules of Probability	Understand independence and conditional probability and use them to interpret data

Cluster Standard: HSS.CP.A.2

Standard	Standards for Mathematical Practice
Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	• **SMP2:** Reason abstractly and quantitatively. • **SMP3:** Construct viable arguments and critique the reasoning of others.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• A probability model may consist of a list or description of possible outcomes (the sample space) each of which is assigned a certain probability. Probability rules can be developed and understood through the use of the sample space. When events are independent, the outcome of the first event does not change the sample space for subsequent events. Independent events, knowing one event has occurred affects the likelihood of another event occurring. Use of two-way frequency tables helps learners develop conceptual understanding of conditional probability. The use of tables, symbols, and real-world scenarios are emphasized. Learners consider the context of situations as they build mathematical models, interpret events, and explain results in terms of a probability model.	• Explain and apply the formula to determine if two events are independent. • Test for independence using the definition of independent events. • State problems' independence and dependence contextually.
DOK	**Blooms**
1-3	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
G	Conditional Probability & The Rules of Probability	Understand independence and conditional probability and use them to interpret data

Cluster Standard: HSS.CP.A.3

Standard	Standards for Mathematical Practice
Understand the conditional probability of *A* given *B* as *P(A and B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	• **SMP2:** Reason abstractly and quantitatively. • **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP6**: Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• A probability model may consist of a list or description of possible outcomes (the sample space) each of which is assigned a certain probability. Probability rules can be developed and understood through the use of the sample space. When events are independent, the outcome of the first event does not change the sample space for subsequent events. Independent events, knowing one event has occurred affects the likelihood of another event occurring. Use of two-way frequency tables helps learners develop conceptual understanding of conditional probability. The use of tables, symbols, and real-world scenarios are emphasized. Learners consider the context of situations as they build mathematical models, interpret events, and explain results in terms of a probability model.	• Read and state conditional probabilities of two events and explain how they are different. • Apply conditional probability to argue if two events are independent. • Calculate conditional probabilities. • Relate conditional probability to relative frequency tables and/or tree diagrams. • Use conditional probabilities to determine whether events are independent.
DOK	**Blooms**
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
G	Conditional Probability & The Rules of Probability	Understand independence and conditional probability and use them to interpret data

Cluster Standard: HSS.CP.A.4

Standard	Standards for Mathematical Practice
Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*	• **SMP2**: Reason abstractly and quantitatively. • **SMP3**: Construct viable arguments and critique the reasoning of others. • **SMP4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• A probability model may consist of a list or description of possible outcomes (the sample space) each of which is assigned a certain probability. Probability rules can be developed and understood through the use of the sample space. When events are independent, the outcome of the first event does not change the sample space for subsequent events. Independent events, knowing one event has occurred affects the likelihood of another event occurring. Use of two-way frequency tables helps learners develop conceptual understanding of conditional probability. The use of tables, symbols, and real-world scenarios are emphasized. Learners consider the context of situations as they build mathematical models, interpret events, and explain results in terms of a	• Interpret and organize data to describe events and independence of events using 2-way frequency tables • Collect sample data from a real-world situation in order to examine conditional probabilities and independence of events. • Interpret and make sense of these in context of the situation.

probability model.	
DOK	**Blooms**
2-3	Apply, Analyze

Grade	*CCSS Domain*	*CCSS Cluster*
G	Conditional Probability & The Rules of Probability	Understand independence and conditional probability and use them to interpret data

Cluster Standard: HSS.CP.A.5

Standard	Standards for Mathematical Practice
Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP4**: Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• A probability model may consist of a list or description of possible outcomes (the sample space) each of which is assigned a certain probability. Probability rules can be developed and understood through the use of the sample space. When events are independent, the outcome of the first event does not change the sample space for subsequent events. Independent events, knowing one event has occurred affects the likelihood of another event occurring. Use of two-way frequency tables helps learners develop conceptual understanding of conditional probability. The use of tables, symbols, and real-world scenarios are emphasized. Learners consider the context of situations as they build mathematical models,	• Analyze a scenario to describe conditional probability in terms of a real-life context • Use conditional probability to make decisions and justify claims of relationships to contextual situations. • Interpret conditional probability and independence across a variety of situations. • Distinguish between association and causality.

interpret events, and explain results in terms of a probability model.	
DOK	**Blooms**
2-3	Apply, Analyze

Common Misconceptions

- Students may think that two events occurring is as simple as adding their probabilities.
- Students may fail to check both parts of the algorithm. Students may also assume if p(a)=p(a|b) then p(b) must equal p(b|a).
- Students may also have an incomplete understanding of conditional probability.

Grade	CCSS Domain	CCSS Cluster
G	Conditional Probability & The Rules of Probability	Use the rules of probability to compute probabilities of compound events

Cluster Standard: HSS.CP.B.6

Standard	Standards for Mathematical Practice
Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP2:** Reason abstractly and quantitatively. • **SMP4**: Model with mathematics. • **SMP6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• The development and uses of algorithms are built on conceptual understanding as concepts of sample spaces are explored and deepened. Probabilities are described in terms of the intersections and unions of events. Venn diagrams and two-way frequency tables will be generalized to discover patterns and create algorithms and formulas that can be used in routine fashion. Although learners will use these formulas strategically to determine different values, the use of tree diagrams, organized lists, and other tools will help make sense of these abstractions.	• Describe how to find conditional probabilities • Calculate conditional probabilities • Explain conditional probability in context of a scenario • Interpret a given scenario and relate context to conditional probability, both abstractly and mathematically • Justify reasoning in making conditional probability arguments
DOK	**Blooms**
1-3	Understand, Apply, Evaluate

Grade	CCSS Domain	CCSS Cluster
G	Conditional Probability & The Rules of Probability	Use the rules of probability to compute probabilities of compound events

Cluster Standard: HSS.CP.B.7

Standard	Standards for Mathematical Practice
Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.	• **SMP2:** Reason abstractly and quantitatively. • **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP4:** Model with mathematics. • **SMP6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• The development and uses of algorithms are built on conceptual understanding as concepts of sample spaces are explored and deepened. Probabilities are described in terms of the intersections and unions of events. Venn diagrams and two-way frequency tables will be generalized to discover patterns and create algorithms and formulas that can be used in routine fashion. Although learners will use these formulas strategically to determine different values, the use of tree diagrams, organized lists, and other tools will help make sense of these abstractions.	• Calculate the union of two events • Explain the union of two events in terms of the context of the problem • Given a scenario, interpret what the union of two events represents and calculate the probability
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Students may struggle with determining the correct denominator. They may use the total rather than the specified event.
- Students may struggle to understand the “overlap” in compound events.

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ASSESSMENT GUIDE

- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events

Grade	*CCSS Domain*	*CCSS Cluster*
G	Conditional Probability & The Rules of Probability	Understand independence and conditional probability and use them to interpret data

Sample Task #1 (Constructed Response)

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item #: 1474700 The linked assessment question addresses S-SCPA.A, specifically the question requires students to read a two-way frequency table and state conditional probability in context.

CollegeBoard | Question ID 1474700

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Problem Solving and Data Analysis	Medium	Problem Solving and Data Analysis	Probability and conditional probability	1. Compute and interpret probability and conditional probability in simple contexts.	Calculator

Number of Adults Contracting Colds

	Cold	No cold	Total
Vitamin C	21	129	150
Sugar pill	33	117	150
Total	54	246	300

The table shows the results of a research study that investigated the therapeutic value of vitamin C in preventing colds. A random sample of 300 adults received either a vitamin C pill or a sugar pill each day during a 2-week period, and the adults reported whether they contracted a cold during that time period. What proportion of adults who received a sugar pill reported contracting a cold?

Question Difficulty: Medium

A. $\frac{11}{18}$

B. $\frac{11}{50}$

C. $\frac{9}{50}$

D. $\frac{11}{100}$

Choice B is correct. A total of 150 adults received the sugar pill. Of those, 33 reported contracting a cold. Therefore, $\frac{33}{150}$, or the equivalent $\frac{11}{50}$, is the proportion of adults receiving a sugar pill who reported contracting a cold.

Choice A is incorrect. This is the proportion of adults receiving a sugar pill and contracting a cold to all adults contracting a cold $\left(\frac{33}{54}\right)$. Choice C is incorrect. This is the proportion of adults who reported contracting a cold to all the participants in the study $\left(\frac{54}{300} = \frac{9}{50}\right)$. Choice D is incorrect. This is the proportion of adults who received a sugar pill and reported contracting a cold to all the participants in the study $\left(\frac{33}{300} = \frac{11}{100}\right)$.

Grade	CCSS Domain	CCSS Cluster
G	Conditional Probability & The Rules of Probability	Use the rules of probability to compute probabilities of compound events

Sample Task #2 (Constructed Response)

Standards Aligned Instructionally Embedded Formative Assessment Resources:

SAT Item #: 4168721 The linked assessment question addresses S-SCPA.A, specifically the question requires students to read a two-way frequency table and state compound probability in context.

CollegeBoard **Question ID 4168721**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Problem Solving and Data Analysis	Medium	Problem Solving and Data Analysis	Probability and conditional probability	1. Compute and interpret probability and conditional probability in simple contexts.	Calculator

Observed Matings among Fruit Flies

		Female fruit fly group		
		Female raised on starch	Female raised on maltose	Total
Male fruit fly group	Male raised on starch	22	9	31
	Male raised on maltose	8	20	28
	Total	30	29	59

The table above shows the observed mating frequencies among a group of fruit flies raised on either a starch medium or a maltose medium. What fraction of the observed matings were between fruit flies that were raised on the same medium?

Question Difficulty: Medium

A. $\frac{9}{31}$

B. $\frac{17}{59}$

C. $\frac{31}{59}$

D. $\frac{42}{59}$

NEW MEXICO Public Education Department

MLSS AND CLR GUIDE

- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events

CCSS Domain	*CCSS Cluster*
CONDITIONAL PROBABILITY & THE RULES OF PROBABILITY	Understand independence and conditional probability and use them to interpret data

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on independence and conditional probability, use them to interpret data, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For students, consider what types of conditional probability occur in the students' lives outside of school.	
Cross-Curricular Connections	Social Studies: Connect to census data, voter demographics Forensic Science: Connect to crime scene analysis given suspect characteristics	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language,*	• Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice leads to more students viewing themselves as capable mathematicians. The types of mathematical tasks are critical because the problems presented to students would need to reflect a relevance to the students' life experiences. This would offer a meaning to the student that the math can go beyond the classroom.

	the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In previous years, learners used sample spaces to represent compound events in organized lists, tables, and tree diagrams. Learners are initially introduced to probability in 7th grade. They have investigated chance processes and developed probability models using experimental and theoretical probability.	• Learners will use their knowledge of conditional probability and their skills of determining conditional probability to make decisions for real world situations. They will also expand the knowledge of this cluster to learn specific rules such as the Addition Rule. This knowledge will lead into permutations and combinations.	• Learners will extend their learning to develop and make sense of the Multiplication Rule and Addition Rule. Future learning such as binomial distribution and statistical significance build upon conditional probability. Other applications are found in calculus, statistics, engineering, and the sciences.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on independence and conditional probability and use them to interpret data because students will need to develop an appropriate vocabulary usage for new subjects as well as tying it to previously learned material

Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	7.SPC.7: This standard provides a foundation for work with independence and conditional probability and use them to interpret data because this previously learned standard forms the foundation of understanding probability and probability models. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on independent events and conditional probability and use them to interpret data by clarifying mathematical ideas and/or concepts through a short mini-lesson because precise usage of terms in this cluster is the key to future understanding.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit independent events and conditional probability and use them to interpret data by confronting student misconceptions because there will be much confusion of terminology here and that will lead to errors in calculations later.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics to interpret data because students will be able to analyze experiments and studies of their own choosing to further their understanding of independence and conditional probability.

CCSS Domain	*CCSS Cluster*
CONDITIONAL PROBABILITY & THE RULES OF PROBABILITY	Use the rules of probability to compute probabilities of compound events

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on the rules of probability to compute probabilities of compound events., consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, consider what types of compound probability a student will experience on a daily basis to form a foundation for this concept.	
Cross-Curricular Connections	Social Studies: Connect to voter demographics Science: Connect to crime science investigation/analysis	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical*	• Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying the rules of probability to compute probabilities of compound events, goal setting is critical because in this cluster of Statistics and Probability its necessary to be organized and complete in which procedure must be used at a given time. Helping a student set a piecewise organizational goal will assist. This can be organized linearly, as a graphic organizer, or any method of the students choosing.

	identities as capable mathematicians that can use mathematics within school and society?	

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In grade 7, learners have investigated chance processes, and developed, used, and evaluated probability models. They have learned that probability of a chance event is a number between 0 and 1 (7.SP.5) and found probabilities of compound events (7.SP.8).	• Learners are expanding their understanding and skills explored and learned in the G.SP.A cluster. They are discovering that conditional probability can be found from a narrowed subset of the original sample space.	• Future learning such as binomial distribution and statistical significance build upon conditional probability. Other applications are found in calculus, statistics, engineering, and the sciences.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying the rules of probability to compute probabilities of compound events because this cluster focuses on compound probability with the introduction of combinations and permutations which take practice and perseverance to master
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	7.SPC.8: This standard provides a foundation for work with the rules of probability to compute probabilities of compound events because this older standard introduces the formal definitions of compound events and calls for modeling of the standard to represent its situations. If

		students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on computing probabilities of compound events by providing specific feedback to students on their work through a short mini-lesson because by pinpointing minor errors in a multistep process we can perfect our processes.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on computing probabilities of compound events by addressing conceptual understanding because by sitting down and helping a student analyze their process, we can bring them to a deeper level of understanding of their errors as well as the content.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students. When studying the rules of probability to compute probabilities of compound events students working together opens up new paths of thinking and reasoning for them.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Translate between the geometric description and the equation for a conic section
  - HSG.GPE.A.1
  - HSG.GPE.A.2
- Use coordinates to prove simple geometric theorems algebraically
  - HSG.GPE.B.4
  - HSG.GPE.B.5
  - HSG.GPE.B.6
  - HSG.GPE.B.7

Grade	CCSS Domain	CCSS Cluster
G	Expressing Geometric Properties with Equations	Translate between the geometric description and the equation for a conic section

Cluster Standard: HSG.GPE.A.1

Standard	Standards for Mathematical Practice
Derive the equation of a circle given the center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	• **SMP2:** Reason abstractly and quantitatively. • **SMP3**: Construct viable arguments and critique the reasoning of others. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
The introduction of coordinates into geometry connects geometry and algebra, allowing algebraic proofs of geometric theorems.	• Explain how the Pythagorean Theorem can be used to derive the equation of a circle. • Write the equation of a circle, given the center and radius. • Complete the square within the equation of a circle in order to find the center and radius
DOK	**Blooms**
2-3	Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
G	Expressing Geometric Properties with Equations	Translate between the geometric description and the equation for a conic section

Cluster Standard: HSG.GPE.A.2

Standard	Standards for Mathematical Practice
Derive the equation of a parabola given a focus and directrix.	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
The introduction of coordinates into geometry connects geometry and algebra, allowing algebraic proofs of geometric theorems.	• Describe the characteristics of a parabola given its equation. • Derive the equation for a parabola given the focus and directrix.
DOK	**Blooms**
1-3	Analyze

Common Misconceptions

- Students commonly swap h and k when working with the equations for the circle.
- Students will make similar mistakes with *h* and *k*, when finding the vertex of a parabola

Grade	CCSS Domain	CCSS Cluster
G	Expressing Geometric Properties with Equations	Use coordinates to prove simple geometric theorems algebraically

Cluster Standard: HSG.GPE.B.4

Standard	Standards for Mathematical Practice
Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*	• **SMP2:** Reason abstractly and quantitatively. • **SMP6:** Attend to precision. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
The focus of this cluster is coordinate geometry. Students work with coordinates to find slope, distances, midpoints, and locations that are at a specified ratio from an endpoint. They then use this information to prove geometric relationships such as properties of quadrilaterals or location of a point on a circle. Using slope criteria for parallel and perpendicular lines, students write equations of lines. Using lengths computed from coordinates, students find perimeters and areas of polygons.	• Use the distance formula to find the distance between coordinates. • Find the slope of a line connecting two coordinates. • Determine if a point lies on a specific circle. • Use coordinates to prove that a quadrilateral is, or is not, a parallelogram, rectangle, rhombus, square, or trapezoid. • Use coordinates to prove a triangle's classification by its sides.
DOK	**Blooms**
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
G	Expressing Geometric Properties with Equations	Translate between the geometric description and the equation for a conic section

Cluster Standard: HSG.GPE.B.5

Standard	Standards for Mathematical Practice
Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	• **SMP2:** Reason abstractly and quantitatively. • **SMP5:** Use appropriate tools strategically. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
The focus of this cluster is coordinate geometry. Students work with coordinates to find slope, distances, midpoints, and locations that are at a specified ratio from an endpoint. They then use this information to prove geometric relationships such as properties of quadrilaterals or location of a point on a circle. Using slope criteria for parallel and perpendicular lines, students write equations of lines. Using lengths computed from coordinates, students find perimeters and areas of polygons.	• Prove how parallel lines increase at the same rate of change. • Explain that perpendicular lines intersect at a right angle. • Construct an equation of a line that is parallel or perpendicular to a given line. • Calculate slope from given ordered pairs. • Classify lines or segments as parallel or perpendicular given slopes, graphs, and/or equations of lines. • Write equations for parallel lines and perpendicular lines given a point and an equation of a line.
DOK	**Blooms**
1-3	Understand, Analyze

Grade	CCSS Domain	CCSS Cluster
G	Expressing Geometric Properties with Equations	Translate between the geometric description and the equation for a conic section

Cluster Standard: HSG.GPE.B.6

Standard	Standards for Mathematical Practice
Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
The focus of this cluster is coordinate geometry. Students work with coordinates to find slope, distances, midpoints, and locations that are at a specified ratio from an endpoint. They then use this information to prove geometric relationships such as properties of quadrilaterals or location of a point on a circle. Using slope criteria for parallel and perpendicular lines, students write equations of lines. Using lengths computed from coordinates, students find perimeters and areas of polygons.	• Determine the coordinates of a point of a given partition on a directed segment. • Use the midpoint formula, the section formula, and the distance formula to find the partition point of a given line segment. • Determine the ratio of a partition using the distance formula. • Given two points, find the point on a line segment between the two points that divides the segment into a given ratio
DOK	**Blooms**
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
G	Expressing Geometric Properties with Equations	Translate between the geometric description and the equation for a conic section

Cluster Standard: HSG.GPE.B.7

Standard	Standards for Mathematical Practice
Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	• **SMP2:** Reason abstractly and quantitatively. • **SMP4:** Model with mathematics. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
The focus of this cluster is coordinate geometry. Students work with coordinates to find slope, distances, midpoints, and locations that are at a specified ratio from an endpoint. They then use this information to prove geometric relationships such as properties of quadrilaterals or location of a point on a circle. Using slope criteria for parallel and perpendicular lines, students write equations of lines. Using lengths computed from coordinates, students find perimeters and areas of polygons.	• Use the distance formula to find the length of sides of a polygon. • Choose the appropriate formula for perimeter or area of a given polygon. • Calculate areas and perimeters of polygons. • Use appropriate labels for the areas and perimeters.
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Students may misunderstand the negative reciprocal slope with perpendicular lines.
- Students commonly forget to take the square root of the constant to find the radius in the equation of a circle.

NEW MEXICO
Public Education Department

ASSESSMENT GUIDE

- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically

Grade	*CCSS Domain*	*CCSS Strand*
G	Expressing Geometric Properties with Equations	Translate between the geometric description and the equation for a conic section

Sample Task #1 (Constructed Response)

SAT Item #: 421901 The linked assessment question addresses G-GPE.B., specifically the question requires students to analyze a given equation and determine if a point is within the circle.

CollegeBoard **Question ID 421901**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Additional Topics in Math	Hard	Additional Topics in Math	Circles	1. Use definitions, properties, and theorems relating to circles and parts of circles, such as radii, diameters, tangents, angles, arcs, arc lengths, and sector areas to solve problems.	Calculator

A circle in the xy-plane has equation $(x+3)^2+(y-1)^2=25$. Which of the following points does NOT lie in the interior of the circle?

Question Difficulty: Hard

A. $(-7, 3)$

B. $(-3, 1)$

C. $(0, 0)$

D. $(3, 2)$

Choice D is correct. The circle with equation $(x+3)^2+(y-1)^2=25$ has center $(-3,1)$ and radius 5. For a point to be inside of the circle, the distance from that point to the center must be less than the radius, 5. The distance between $(3,2)$ and $(-3,1)$ is $\sqrt{(-3-3)^2+(1-2)^2}=\sqrt{(-6)^2+(-1)^2}=\sqrt{37}$, which is greater than 5. Therefore, $(3,2)$ does NOT lie in the interior of the circle.

Choice A is incorrect. The distance between $(-7,3)$ and $(-3,1)$ is $\sqrt{(-7+3)^2+(3-1)^2}=\sqrt{(-4)^2+(2)^2}=\sqrt{20}$, which is less than 5, and therefore $(-7,3)$ lies in the interior of the circle. Choice B is incorrect because it is the center of the circle. Choice C is incorrect because the distance between $(0,0)$ and $(-3,1)$ is $\sqrt{(0+3)^2+(0-1)^2}=\sqrt{(3)^2+(1)^2}=\sqrt{8}$, which is less than 5, and therefore $(0,0)$ in the interior of the circle.

Additional Assessment

http://tasks.illustrativemathematics.org/content-standards/HSG/GPE/B/4/tasks/605

The linked assessment question addresses G-GPE.B, specifically the question requires students to create their own quadrilateral and state verbally what observations they can make. Students then apply generic coordinate algebra to prove their observation. This assessment should be given to students after they've had practice applying coordinate math to prove conjectures about geometric figures. Students will engage in SMP2, SMP6 and possibly SMP3 if they are asked to share and critique others' work.

Grade	*CCSS Domain*	*CCSS Strand*
G	Expressing Geometric Properties with Equations	Use coordinates to prove simple geometric theorems algebraically

Sample Task #2 (Constructed Response)

A circle in the xy-plane has equation $(x+3)^2+(y-1)^2=25$. Which of the following points does NOT lie in the interior of the circle?

Sample Task #3 (Multiple Choice)

a.

What are the coordinates of the point on the directed line segment from $K(-5,-4)$ to $L(5,1)$ that partitions the segment into a ratio of 3 to 2?

(1) $(-3,-3)$
(2) $(-1,-2)$
(3) $(0,-\frac{3}{2})$
(4) $(1,-1)$

NEW MEXICO Public Education Department

MLSS AND CLR GUIDE

- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically

CCSS Domain	*CCSS Cluster*
Expressing Geometric Properties with Equations	**Translate between the geometric description and the equation for a conic section**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on translating between the geometric description and the equation for a conic section, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. Relating these standards to real-life context such as finding the distance from one building to another in the community will solidify the school learning to the actual application of standards/skills.	
Cross-Curricular Connections	During a unit focused on the use of coordinates to prove simple geometric theorems algebraically, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, allow students to relate this math to their home and community by plotting points to create an approximate map of a sectioned off area at home or parking lots in the community and find the amount of fencing needed.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?*	• Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying translating between the geometric description and the equation for a conic section, goal setting is critical because students may be at varying levels of academic and language proficiency. To help students

	• *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	identify goals, teachers can use strategies such as writing prompts to gauge their thinking. Teachers can also provide students with a means to track their personal data (this assists in knowing where you currently are in attaining your goal). Teachers should work to build rapport, relationships and respect with their students and amongst each other to create that positive classroom culture in which students are willing to share/monitor their goals with one another without being judged.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Students have worked with coordinates, slope, and the Pythagorean Theorem in 8th grade math. This work exploring facts about right triangles connects to the foundational formulas in analytic geometry. Additionally, in Algebra I, students have been rewriting expressions in different forms (factoring and completing the square) which directly correlates to the work they will complete in this cluster when creating algebraic proofs of the theorems.	• Students will connect the information in this cluster to learning later in the course by extending the precise definitions of circles and polygons to work with coordinates on the plane.	• Learners will continue with graphing quadratic functions, showing vertices, intercepts, and identifying maxima or minima in the Algebra II course.

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching on translating between geometric descriptions and the equations for a conic section because students will need to know the distance formula which is learned in Grade 8. A review of distance between two points on the coordinate grid will help students in this cluster.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.G.B.8: This standard provides a foundation for work with translating between the geometric descriptions and the equation for a conic section because students have to apply the Pythagorean theorem to find the distance between two points. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on translating between the geometric description and the equation for a conic section by critiquing student approaches/solutions to make connections through a short mini-lesson because this cluster requires students to generalize patterns they see through exploration. These patterns may not be the same for every student but connecting the different patterns can reveal opportunities to deepen understanding and/or correct misunderstandings.
Intensive	What assessment data will help identify content needing to be revisited for	For example, some students may benefit from intensive extra time during and after a unit of translating between the geometric description and the equation for a conic

	intensive interventions?	section by helping students move from specific answers to generalizations for certain types of problems because this cluster calls on students to recognize and generalize patterns. Students may need extra support in moving from concrete examples to generic patterns.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to explore concepts in greater depth than other students when translating between the geometric description and the equation for a conic section because some students may see generalizations easier than others. Allowing these students to move faster through the concrete examples to get to the abstract generalizations will allow them to stretch their expression of mathematical reasoning from concrete to abstract.

CCSS Domain	*CCSS Cluster*
Expressing Geometric Properties with Equations	**Use coordinates to prove simple geometric theorems algebraically**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on the use of coordinates to prove simple geometric theorems algebraically, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, allow students to relate this math to their home and community by plotting points to create an approximate map of a sectioned off area at home or parking lots in the community and find the amount of fencing needed.	
Cross-Curricular Connections	Home Economics: Connect to construction and agriculture.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in*	• Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying the use of coordinates to prove simple geometric theorems algebraically the pattern of questions within the classroom is critical because it can allow students to build upon each other's ideas. Further, teachers can tap into student's prior knowledge and use it to promote learning for all students. Teachers can utilize strategic sequencing and questioning to encourage all students to participate in engaging with the content and seeing connections between

	creating mathematical identities as capable mathematicians that can use mathematics within school and society?	multiple representations and solution methods. This can facilitate cross-content connections. When posing purposeful questions to the whole group, teachers should have protocols in place (classroom management) that tend to how students will respond to and discuss questions. Finally, teachers can use activities in which students are able to share (partners or groups) their thoughts and ideas in a judgement-free zone.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In 6th grade, learners find the area of polygons by composing into rectangles or decomposing into triangles and other shapes. They also draw polygons in the coordinate plane given coordinates for the vertices and find the length of horizontal and vertical sides. In 7th grade, learners solve real-world and mathematical problems involving area of triangles, quadrilaterals, and polygons. In 8th grade, learners apply the Pythagorean Theorem to find the distance between two points in a coordinate system. In Algebra I, learners write equations of lines given a slope and point.	• Learners have already had experience with properties of quadrilaterals, equations of circles, and finding area and perimeter earlier in the course. They now apply this knowledge to working with coordinates. Learners will use the concept of distance and midpoint throughout the rest of the geometry course. They apply the concepts later when calculating volumes and surface areas or when proving types of quadrilaterals given the ordered pairs of their vertices. They also use distance and midpoint when writing and deriving the equation of circles.	• Distance is an application important for many future concepts. For example, when writing equations of conic sections or converting between polar and rectangular coordinates or finding the magnitude of vectors.

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on the use of coordinates to prove simple geometric theorems algebraically because students will need to be familiar with discovering geometric properties before they can make sense of how to use coordinates to understand the properties.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	HS.G-GPE.A.1: This standard provides a foundation for using coordinates to prove simple geometric theorems algebraically because students should be able to work with and derive equations of geometric shapes before proving theorems. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on using coordinate to prove simple geometric theorems algebraically by clarifying mathematical ideas and/or concepts through a short mini-lesson because students may struggle to apply knowledge of geometric figures to coordinate algebra or vice-versa.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit of using coordinates to prove simple geometric theorems algebraically by addressing conceptual understanding because revisiting basic geometric shapes in the coordinate plane will assist in discovering geometric properties, which will help them understand how coordinates can help prove theorems

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics because students will have the autonomy to make connections to personalized real-life situations in connection to this standard.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

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Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Explain volume formulas and use them to solve problems
  - HSG.GMD.A.1
  - HSG.GMD.A.3
- Visualize relationships between two-dimensional and three-dimensional objects
  - HSG.GMD.B.4

Grade	CCSS Domain	CCSS Cluster
G	Geometric Measurement & Dimension	Explain volume formulas and use them to solve problems

Cluster Standard: HSG.GMD.A.1

Standard	Standards for Mathematical Practice
Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	• **SMP2:** Reason abstractly and quantitatively. • **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students move from applying volume formulas to justifying them. Students will be exposed to advanced concepts in an informal setting. Learners will deconstruct complex geometric shapes into basic three-dimensional shapes to calculate their surface areas and volumes.	• Demonstrate Cavalieri's Principle concretely. • Give an informal argument for circumference and area formulas for circles. • Give an informal argument for volume formulas of cylinders, pyramids, and cones. • Construct viable arguments to validate the circumference of a circle, volume of a cylinder, volume of a pyramid, and volume of a cube by using Cavalieri's Principle.
DOK	**Blooms**
1-3	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
G	Geometric Measurement & Dimension	Explain volume formulas and use them to solve problems

Cluster Standard: HSG.GMD.A.3

Standard	Standards for Mathematical Practice
Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP4:** Model with mathematics. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students move from applying volume formulas to justifying them. Students will be exposed to advanced concepts in an informal setting. Learners will deconstruct complex geometric shapes into basic three-dimensional shapes to calculate their surface areas and volumes.	• Identify these geometric shapes: cylinders, pyramids, cones, and spheres. • Calculate volume for cylinders, pyramids, cones and spheres. • Use formulas to solve problems involving three-dimensional figures. • Apply volume to real world problems.
DOK	**Blooms**
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
G	Geometric Measurement & Dimension	Visualize relationships between two-dimensional and three-dimensional objects

Cluster Standard: HSG.GMD.B.4

Standard	Standards for Mathematical Practice
Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects	• **SMP1:** Make sense of problems and persevere in solving them.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students move from applying volume formulas to justifying them. Students will be exposed to advanced concepts in an informal setting. Learners will deconstruct complex geometric shapes into basic three-dimensional shapes to calculate their surface areas and volumes.	• Explain why a cross section is a two-dimensional representation of a slice of a three-dimensional object. • Realize that the cross section can be different depending on location and angle where the three-dimensional object is cut. • Identify possible cross sections in a given object • Identify three-dimensional objects generated by the rotations of two-dimensional objects
DOK	**Blooms**
1-2	Apply, Analyze

Common Misconceptions

• Students may mix up which formula to use for a given figure. • Students may have difficulty identifying the base of a figure. • When considering units, students may struggle to interpret inches cubed or forget these units in the final answer	• Students may struggle to visualize cross sections and rotations. • Students may need support with the concept of “slicing” a three-dimensional figure

NEW MEXICO
Public Education Department

ASSESSMENT GUIDE

- Explain volume formulas and use them to solve problems
- Visualize relationships between two-dimensional and three-dimensional objects

Grade	*CCSS Domain*	*CCSS Cluster*
G	Geometric Measurement and Dimension	Explain volume formulas and use them to solve problems

Sample Task #1 (Constructed Response)

Area of a circle

The goal of this task is to explain why the area enclosed by a circle C of radius r is πr^2. Recall that π is the ratio of the circumference of a circle to its diameter and that this ratio is independent of the size of the circle.

a.
Draw a picture of a regular octagon O inscribed in C. Find a formula for the area of the octagon in terms of its perimeter.

b.
Reasoning as in part (a), find a formula for the area of a regular n sided polygon, for $n \geq 3$, inscribed in C: the formula should give the area of the polygon *in terms of* its perimeter.

c. Using your formula from part (b), explain why the area of C is πr^2.

Grade	CCSS Domain	CCSS Cluster
	Geometric Measurement and Dimension	Visualize relationships between two-dimensional and three-dimensional objects

Sample Task #1 (Constructed Response)

Global Positioning System I

The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}.$$

a. For each of the following equations, describe the set of solutions geometrically and sketch this solution set in x-y-z coordinates:

i. $x^2 + y^2 + z^2 = 1.$

ii. $(x - 3)^2 + y^2 + z^2 = 4.$

iii. $x^2 + y^2 + \left(z - \frac{3}{2}\right)^2 = 1.$

NEW MEXICO Public Education Department

MLSS AND CLR GUIDE

- Explain volume formulas and use them to solve problems
- Visualize relationships between two-dimensional and three-dimensional object

CCSS Domain	*CCSS Cluster*
Geometric Measurement and Dimension	Explain volume formulas and use them to solve problems

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on explaining volume formulas and using them to solve problems, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning the Cavalieri's principle is a great entry point and supports students to create arguments and use it to solve for the volume of a given geometric figure.	
Cross-Curricular Connections	Because volume can be found for any given item, this connection can be made to a variety of areas: science-beakers, social studies-coffins, art-paint bottles, music- variety of instruments.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and*	• Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice makes sense of mathematics and perseverance in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of

	linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	the students. For example, when studying, explaining volume formulas and use them to solve problems supporting productive struggle is critical because students may not see the connections of other formulas to volume, and they need those to solve problems involving volume. However, students may see the connection by presenting the actual image which is relevant and relatable to them, such as, car tires, basketball, and any other shape that is visible to them.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In 7th grade, learners worked with area, and circumference which extended to find components needed for surface area and volume. Throughout grades 6, 7, and 8 students calculated the volumes and surface areas of prisms, cones, cylinders, and spheres which will connect to their work within this cluster.	• Students will continue to expand their work to include composite figures. They will also justify volume formulas and other constructions.	• In Calculus, students will apply Cavalieri's principle to calculate volumes for solids of rotation.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with*	Some learners may benefit from targeted pre-teaching that rehearses prior learning when explaining volume formulas and using them to solve problems because

	the mathematics for this cluster within your HQIM?	students have to use different formulas that are necessary to support them on solving problems involving volume of three-dimensional figures, such as, cylinder, pyramid, and cone.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. This standard provides a foundation for work with explaining volume formulas and using them to solve problems because students need to master the different formulas and use them to solve real-world problems. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on GMD.A: Explain Volume Formulas and Use Them to Solve Problems by providing specific feedback to students on their work through a short mini-lesson because teachers need to monitor students on how they use different formulas and use them precisely to answer a given problem. Providing students specific feedback will help students to create strong justifications of their answers.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit GMD.A: Explain Volume Formulas and Use Them to Solve Problems by helping students move from specific answers to generalizations for certain types of problems because students need to master how they justify answers mathematically and base from specific concepts to come up with a general, accurate and precise answers.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to explore links between various topics when explaining volume formulas and using them to solve problems because these topics are interconnected, and it will help students to master the whole concepts. Students need to have a strong foundation of different formulas and let them use it to formulate and come up with justifications on volumes.

CCSS Domain	*CCSS Cluster*
Geometric Measurement and Dimension	Visualize relationships between two-dimensional and three-dimensional objects

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on visualizing relationships between two-dimensional and three-dimensional objects, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, let the students identify two-dimensional and three-dimensional objects and let them discuss the difference.	
Cross-Curricular Connections	Students can produce interesting pieces of art by rotating a two-dimensional object that has been dipped in ink. This can also link to Language Arts and Science by having students verbally describe a conjecture of what they think an end result may be and then experimentally verifying and reflecting upon their initial thoughts. Stressing the importance of precise language in writing descriptions deepens the connection between ELA and math.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge concepts for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when visualizing relationships between two-dimensional and three-dimensional objects the use of mathematical representations within the classroom is critical because students need to critically think about the relationship of two-dimensional and three-

	mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	dimensional objects. We can help our students by presenting them the actual image and image that they can relate to, like a ball, car tires, cellphone and any other object that is visible to them.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In 6th grade, students represented three-dimensional figures using nets made up of rectangles and triangles to calculate surface area. In 7th grade students moved on to describe the two-dimensional figures that result from slicing three-dimensional figures, focusing on right rectangular prisms and pyramids. These skills prepare students to address the content within this cluster.	• Students use cross section dimensions in volume calculations (i.e. the height of the triangle when calculating the volume of the cone)	• In later courses, students study conic sections which can be described as cross sections of a cone. Calculus concepts will build on the volume of solids of rotation.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when visualizing relationships between two-dimensional and three-dimensional objects because students need to have a visual representation of the object. As much as possible, use objects that are familiar

		to students.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	7.G.A.3: Describe the two-dimensional figures that result from slicing three- dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. This standard provides a foundation for work with visualizing relationships between two-dimensional and three-dimensional objects because students need to visualize how it looks when a three-dimensional object is being cut. What are the different products after cutting? If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• Transformations as functions and symmetry in terms of transformations. • Two figures are congruent if there is a sequence of transformations that maps one onto another. • Two figures are similar if they have the same shape with congruent angles and proportional side lengths. • The formal language for relationships between angles including vertical angles, angles created when a	• Carry out rotations, reflections, translations, and dilations using a variety of tools and compare and contrast their effects. • Prove two triangles are congruent using ASA, SAS, and SSS or two triangles are similar using AA, SAS, and SSS. • Build formal justifications (proofs) for the theorems about lines and angles, triangles, and	• Build on students' experience with the following skills: ○ Understand and use the coordinate axis ○ Write and solve linear equations, especially proportions ○ Recognize and draw geomteric shapes (square, triangle, trapezoid, etc.) ○ Understand and use the formulas of geometric shapes (Perimeter, Area, Volume, etc.) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas

transversal intersects parallel lines, angles and sides of triangles, the segments and angles of parallelograms, and the trigonometric ratios sine, cosine, and tangent.	parallelograms. Find unknown side lengths and angle measures of right triangles using trigonometric ratios and the Pythagorean Theorem.	• Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos graphing calculator ○ Desmos scientific calculator ○ Desmos geometry tool ○ GeoGebra ○ Graphing or scientific calculator ○ Google Drawing ○ Geometric tools (ruler, protractor, compass, etc.) ○ Tracing paper ○ Graph paper and mirror/string/etc. ○ Craft tools (scissors, string, construction paper, etc.) ○ Paper folding

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on visualize relationships between two-dimensional and three-dimensional objects by providing specific feedback to students on their work through a short mini-lesson because students have to properly visualize the object, it's movement to the plane so students can accurate use the object to solve problems.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit visualizing relationships between two-dimensional and three-dimensional objects by offering opportunities to understand and explore different strategies because offering students different ways of visualizing two-dimensional and three-dimensional objects will give them the opportunity to express their thinking through illustrations and analysis.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the application of and development of abstract thinking skills when visualizing relationships between two-dimensional and three-dimensional objects because students will deepen their understanding of the topic and they can use this in the real-world. Challenge the students to do a frustum of two-dimensional and three-dimensional objects.

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Standards Breakdown

- Apply geometric concepts in modeling situations
  - HSG.MG.A.1
  - HSG.MG.A.2
  - HSG.MG.A.3

Grade	CCSS Domain	CCSS Cluster
G	MODELING WITH GEOMETRY	Apply geometric concepts in modeling situations

Cluster Standard: HSG.MG.A.1

Standard	Standards for Mathematical Practice
Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP4:** Make sense of problems and persevere in solving them.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Modeling is the process of choosing and using appropriate mathematics to analyze situations, to understand them better, and to improve decisions. Modeling links classroom mathematics to everyday life, work, and decision making. Mathematical objects that represent a situation from outside mathematics can be used to model and solve problems. Modeling often involves making simplifying assumptions and sometimes minimizes or disregards some features of the situation being modeled. Modeling is best interpreted not as a collection of isolated topics, but in relation to other standards as well.	• Recognize the geometric shape that corresponds to a real object. • Utilize geometric shapes, measures, and properties to describe objects.
DOK	**Blooms**
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
G	MODELING WITH GEOMETRY	Apply geometric concepts in modeling situations

Cluster Standard: HSG.MG.A.2

Standard	Standards for Mathematical Practice
Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP4:** Make sense of problems and persevere in solving them.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Modeling is the process of choosing and using appropriate mathematics to analyze situations, to understand them better, and to improve decisions. Modeling links classroom mathematics to everyday life, work, and decision making. Mathematical objects that represent a situation from outside mathematics can be used to model and solve problems. Modeling often involves making simplifying assumptions and sometimes minimizes or disregards some features of the situation being modeled. Modeling is best interpreted not as a collection of isolated topics, but in relation to other standards as well.	• Construct the different volume and area formulas for shapes and figures. • Explain how to find density for different types of information. • Apply formulas to find density for different types of information.
DOK	**Blooms**
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
G	MODELING WITH GEOMETRY	Apply geometric concepts in modeling situations

Cluster Standard: HSG.MG.A.3

Standard	Standards for Mathematical Practice
Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP4:** Make sense of problems and persevere in solving them.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Modeling is the process of choosing and using appropriate mathematics to analyze situations, to understand them better, and to improve decisions. Modeling links classroom mathematics to everyday life, work, and decision making. Mathematical objects that represent a situation from outside mathematics can be used to model and solve problems. Modeling often involves making simplifying assumptions and sometimes minimizes or disregards some features of the situation being modeled. Modeling is best interpreted not as a collection of isolated topics, but in relation to other standards as well.	• Determine which geometric concepts/figures best model a given situation. • Apply an array of formulas to determine the appropriate geometric solutions. • Design a model of a real-life object using geometric figures.
DOK	**Blooms**
1-3	Understand, Apply, Create

Common Misconceptions

• Students may struggle identifying approximate shapes to model scenarios. • Students may struggle breaking complex shapes into a combination of simpler shapes.	• Students may struggle applying concepts like volume and surface area to language and contexts such as "has a capacity of" or "wraps around."

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Modeling with Geometry**	Strand: **Apply geometric concepts in modeling situations**

Suggested Student Discourse Questions

- Share the process you used to solve the problem with your shoulder partner. What feedback do you have for them?
- How can you use basic geometric shapes to represent real-life objects when solving problems? How accurate and precise can your answers be using this technique?
- Compare the strategies used to solve the problem. Are there other shapes that could have been used in the process?
- When finding the area or volume of a specific object, are you limited to the shapes you can use? Why or why not?

ASSESSMENT GUIDE

- Apply geometric concepts in modeling situations

Grade	*CCSS Domain*	*CCSS Cluster*
G	Modeling with Geometry	Apply geometric concepts in modeling situations
	Sample Task #1 (Constructed Response)	

CollegeBoard Question ID 1053899

SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Area and volume	1. Solve real-world and mathematical problems about a geometric figure or an object that can be modeled by a geometric figure using given information such as length, area, surface area, or volume.	No Calculator

A cube has a surface area of 54 square meters. What is the volume, in cubic meters, of the cube?

Question Difficulty: Medium

A. 18

B. 27

C. 36

D. 81

Choice B is correct. The surface area of a cube with side length s is equal to $6s^2$. Since the surface area is given as 54 square meters, the equation $54 = 6s^2$ can be used to solve for s. Dividing both sides of the equation by 6 yields $9 = s^2$. Taking the square root of both sides of this equation yields $3 = s$ and $-3 = s$. Since the side length of a cube must be a positive value, $s = -3$ can be discarded as a possible solution, leaving $s = 3$. The volume of a cube with side length s is equal to s^3. Therefore, the volume of this cube, in cubic meters, is 3^3, or 27.

Choices A, C, and D are incorrect and may result from calculation errors.

Additional Assessment: This question provides a good opportunity for students to address multiple content within one question. Students will need to consider the relationship of the multiple content.

http://tasks.illustrativemathematics.org/content-standards/HSG/MG/A/2/tasks/1146

The linked assessment question addresses G-MG.A, specifically the question requires students to apply the relationship among density, volume and mass to reasonably estimate the number of cells in a human body. In this approach, we assume that a cell is a sphere and use that fact, along with the provided density of a cell to determine the mass of a cell. We then divide an individual's mass by the mass of a single cell. This assessment should be given to students after they've had the opportunity to work with this relationship as well as had time to work with numbers in scientific notation. Students will engage in SMP1 and SMP6.

MLSS AND CLR GUIDE

- Apply geometric concepts in modeling situation

CCSS Domain	*CCSS Cluster*
Modeling with Geometry	Apply geometric concepts in modeling situations

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on applications of geometric concepts in a modeling situation, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning the different geometric figures in school, home, and community can be a great way to connect school task with home task, such as letting the students identify geometric figures around them in school, home, or community. Let them describe the use and how helpful that shape is to the structure or building.	
Cross-Curricular Connections	Business: Connect to minimizing waste, maximizing volume. Social Studies: Connect to census data/population density	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language,*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge concepts for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying applications of geometric concepts in modeling situations the use of mathematical representations within the classroom is critical because of the diverse

	the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	cultural representation of every single student; however, if we let our students draw their own understanding on specific problems, where students can relate and they can justify their claim mathematically then we can say that learning took place by making connections.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In grades 7 and 8, learners worked with formulas for area, perimeter, surface area and volume, solving real world and mathematical problems.	• Students have been modeling throughout the Geometry course with many of the clusters with focus on using skills to model the real-world situations in this cluster.	• More complex modeling will be used in statistics, physics, trigonometry, and calculus when approaching real-world problems analytically.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that introduces new representations when applying geometric concepts in modeling situations because students need a strong foundation on geometric methods. For example, solve for the area and volume and use to solve for the density of a given shape.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. This standard provides a foundation for work with applications of geometric concepts in modeling situation because students need to have a strong foundation on basic formulas and how to properly use it to solve

		problems and use it to justify their answers. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on applications of geometric concepts in modeling situations by critiquing student approaches/solutions to make connections through a short mini-lesson because you want to highlight and model how to decompose a problem and/or image to the apply characteristics of geometric figures. This initial step may be the hardest for students in solving real world problems.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on applications of geometric concepts in modeling situations by offering opportunities to understand and explore different strategies because it is very important that before moving to the next lesson, students must demonstrate understanding on the wide range of application of geometric shape and use in real world, such as solving for volume, area, and density specifically population density of a given area.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension to understand concepts more quickly and explore them in greater depth than other students when studying applications of geometric concepts in modeling situations. Students will have the opportunity to use their own foundation on solving geometric shapes the way they understand it as long as they can justify it mathematically.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards of Mathematical Practice
    - Common Misconceptions
    - Identification of Priority Standards, as identified by NMPED.
    - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Understand similarity in terms of similarity transformation
  - HSG.SRT.A.1
  - HSG.SRT.A.2
  - HSG.SRT.A.3
- Prove theorems involving similarity
  - HSG.SRT.B.4
  - HSG.SRT.B.5
- Define trigonometric ratios and solve problems involving right triangles
  - HSG.SRT.C.6
  - HSG.SRT.C.7
  - HSG.SRT.C.8

Grade	CCSS Domain	CCSS Cluster
G	Similarity, Right Triangles, & Trigonometry	Understand similarity in terms of similarity transformations

Cluster Standard: HSG.SRT.A.1

Standard	Standards for Mathematical Practice
Verify experimentally the properties of dilations given by a center and a scale factor: • HSG.SRT.A.1.A: A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. • HSG.SRT.A.1.B: The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	• **SMP2:** Reason abstractly and quantitatively. • **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP5:**
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This cluster establishes the basic criteria of similarity through an analysis of dilation transformations. Students formalize the similarity theorems and use the theorems to prove pairs of triangles are similar.	• Determine the properties of dilation. Dilate when the center of dilation is in, on and out of the shape. Dilate when given a center of dilation and a scale factor. Determine the center of dilation and the scale factor from a diagram. Dilate using both positive and negative scale factors. • Construct a dilation. • Determine coordinate rules for dilations using any center of dilation. • Construct a dilated image which has corresponding line segments and is transformed along the same line from the center of the dilation. • Verify experimentally that a dilated image is similar to its pre-image by showing congruent, corresponding angles, and proportional sides. • Determine and apply the properties of dilation.
DOK	**Blooms**
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
G	Similarity, Right Triangles, & Trigonometry	Understand similarity in terms of similarity transformations

Cluster Standard: HSG.SRT.A.2

Standard	Standards for Mathematical Practice
Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP5:** Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This cluster establishes the basic criteria of similarity through an analysis of dilation transformations. Students formalize the similarity theorems and use the theorems to prove pairs of triangles are similar.	• Dilate figures using both positive and negative scale factors. • Identify corresponding angles and sides based on similarity statements. • Develop and write similarity statements for two polygons. • Determine if two triangles are similar based on their corresponding parts. • Establish a sequence of similarity transformations between two similar polygons.
DOK	**Blooms**
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
G	Similarity, Right Triangles, & Trigonometry	Understand similarity in terms of similarity transformations

Cluster Standard: HSG.SRT.A.3

Standard	Standards for Mathematical Practice
Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	• **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This cluster establishes the basic criteria of similarity through an analysis of dilation transformations. Students formalize the similarity theorems and use the theorems to prove pairs of triangles are similar.	• Develop the Angle-Angle criteria of similarity by expanding on previously learn properties of angles of Triangles • Use transformations as a tool to discover how AA similarity is derived and to make the process more efficient. • Express the properties of similarity transformations to explain the justification of AA similarity.
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- A common misconception is thinking that the comparison of any pair of angles will be sufficient, when the comparison must be made using corresponding pairs.
- Students may incorrectly apply the scale factor. Some students often do not list the vertices of similar triangles in order. However, the order in which vertices are listed is preferred and especially important for similar triangles so that proportional sides can be correctly identified.

Grade	CCSS Domain	CCSS Cluster
G	Similarity, Right Triangles, & Trigonometry	Prove theorems involving similarity

Cluster Standard: HSG.SRT.B.4

Standard	Standards for Mathematical Practice
Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP5:** Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students continue to develop their ability to create proofs while incorporating similarity. They will prove the Pythagorean Theorem based on similar triangles. They will then apply similarity to a variety of real world situations.	• Prove two triangles are similar using AA (could extend to SAS or SSS) similarity theorem. • Use proportion to understand and justify logical claims. • Analyze a proof that two triangles are similar to determine if the argument is valid. • Prove various theorems about a triangle's properties. • Determine if two lines are parallel. • Set up and solve a proportion. • Apply the Pythagorean Theorem. • Organize and write a mathematical proof, including justification of my argument.
DOK	**Blooms**
1-3	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
G	Similarity, Right Triangles, & Trigonometry	Prove theorems involving similarity

Cluster Standard: HSG.SRT.B.5

Standard	Standards for Mathematical Practice
Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP5:** Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students continue to develop their ability to create proofs while incorporating similarity. They will prove the Pythagorean Theorem based on similar triangles. They will then apply similarity to a variety of real world situations.	• Apply Theorems and postulates of triangle similarity to solve problems and prove relationships within and between geometric figures. • Use similar figures to find missing side lengths and missing angle measures. • Use congruent figures to find missing side lengths and missing angle measures. • Determine if two geometric figures are congruent or similar. • Justify why two figures are congruent or similar using theorems from Geometry.
DOK	**Blooms**
1-3	Understand, Apply, Analyze

Common Misconceptions

- Students may forget the importance of the order of vertices when making similarity statements.

Grade	CCSS Domain	CCSS Cluster
G	Similarity, Right Triangles, & Trigonometry	Define trigonometric ratios and solve problems involving right triangles

Cluster Standard: HSG.SRT.C.6

Standard	Standards for Mathematical Practice
Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP4:** Model with mathematics. • **SMP6:** Attend to precision. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This cluster builds on the concepts of similarity to define the trigonometric ratios. Using Pythagorean Theorem and trigonometric ratios, students solve for unknown side lengths and angle measures in right triangles.	• Use similarity, side ratios, and angles in right triangles to develop and define trigonometric ratios to help in completion of triangles • Identify the side opposite to and adjacent to an acute angle in a right triangle. • Write and simplify ratios using the sides of a right triangle. • Compare side ratios of similar right triangles and identify if they are equivalent. • Use the definition of sine, cosine, tangent, secant, cosecant, and cotangent to write those trigonometric ratios for a given triangle.
DOK	**Blooms**
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
G	Similarity, Right Triangles, & Trigonometry	Define trigonometric ratios and solve problems involving right triangles

Cluster Standard: HSG.SRT.C.7

Standard	Standards for Mathematical Practice
Explain and use the relationship between the sine and cosine of complementary angles.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP4**: Model with mathematics. • **SMP6**: Attend to precision. • **SMP7**: Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This cluster builds on the concepts of similarity to define the trigonometric ratios. Using Pythagorean Theorem and trigonometric ratios, students solve for unknown side lengths and angle measures in right triangles.	• Use the concept of complementary angles to show how sine and cosine are related • Identify the opposite leg, adjacent leg, and hypotenuse with respect to an angle in a right triangle. • Explain the relationship between sine and cosine of complementary angles of right triangles.
DOK	**Blooms**
1	Understand

Grade	CCSS Domain	CCSS Cluster
G	Similarity, Right Triangles, & Trigonometry	Define trigonometric ratios and solve problems involving right triangles

Cluster Standard: HSG.SRT.C.8

Standard	Standards for Mathematical Practice
Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP4:** Model with mathematics. • **SMP6:** Attend to precision. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This cluster builds on the concepts of similarity to define the trigonometric ratios. Using Pythagorean Theorem and trigonometric ratios, students solve for unknown side lengths and angle measures in right triangles.	• Apply the trig ratios and the Pythagorean theorem to solve right triangle models • Identify the unknown parts of a right triangle using the sine/cosine/tangent ratios. • Solve for the unknown angle measures of a right triangle using inverse sine, inverse cosine, and inverse tangent. • Solve for the unknown parts of a right triangle using Pythagorean Theorem. • Solve real world problems using trigonometric ratios and the Pythagorean Theorem.
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Students may confuse the alternate interior angle theorem and its converse as well as the Pythagorean Theorem and its converse.
- Students may confuse side lengths with angle measurements and will place values as the wrong substitutions in the ratios.
- Students may think that right triangles must be oriented a particular way. They may not realize that opposite and adjacent sides need to be identified with reference to a particular acute angle in a right triangle.

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Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Similarity, Right Triangles, & Trigonometry**	Strand: **Understand similarity in terms of similarity transformations**

Suggested Student Discourse Questions

• Examine your partner's sequence of transformations. Would you make changes to their sequence? Why? • Can you identify sequences like these in what you see around you? In architecture? In nature?	• Compare your sequence of transformations with another group. Which sequence seems to be more complete? Why? • How would proving two triangles are similar help us prove that corresponding angles of the triangles have the same measure?

Domain: **Similarity, Right Triangles, & Trigonometry**	Strand: **Prove theorems involving similarity**

Suggested Student Discourse Questions

• Share the steps you used to prove the triangles in the diagram are similar right triangles. What feedback can you give to others about the steps they used? • Where can you find similar right triangles in real life? Why do you believe they are there?	• After you share the steps, you used to prove the triangles in the diagram are similar right triangles, did you think of any other steps you could use? Come up with as many as you can. • How could you use similarity of triangles with other shapes, such as trapezoids and hexagons?

Domain: **Similarity, Right Triangles, & Trigonometry**	Strand: **Define trigonometric ratios and solve problems involving right triangles**

Suggested Student Discourse Questions

- Share the ratios you created to solve the problem. Give feedback to other students- is the ratio correct? If not, what should be changed?
- If you know one dimension of an object, how could you use trigonometric ratios to find other dimensions of that object without using a ruler?
- What is the easiest way to decide upon a trigonometric ratio for a given situation?
- How do trigonometric ratios use similar shapes to find measures of the legs and hypotenuse of right triangles?

ASSESSMENT GUIDE

- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangles

Grade	CCSS Domain	CCSS Cluster
G	Similarity, Right Triangles, & Trigonometry	Understand similarity in terms of similarity transformations

Sample Task #1 (Constructed Response)

CollegeBoard **Question ID 421874**

SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Right triangles and trigonometry	1. Solve problems in a variety of contexts using b. right triangle trigonometry;	Calculator

Triangles ABC and DEF are shown above. Which of the following is equal to the ratio $\frac{BC}{AB}$?

Question Difficulty: Medium

A. $\frac{DE}{DF}$

B. $\frac{DF}{DE}$

C. $\frac{DF}{EF}$

D. $\frac{EF}{DE}$

Choice B is correct. In right triangle ABC, the measure of angle B must be 58° because the sum of the measure of angle A, which is 32°, and the measure of angle B is 90°. Angle D in the right triangle DEF has measure 58°. Hence, triangles ABC and DEF are similar (by angle-angle similarity). Since $\overline{BC}$ is the side opposite to the angle with measure 32° and AB is the hypotenuse in right triangle ABC, the ratio $\frac{BC}{AB}$ is equal to $\frac{DF}{DE}$.

Alternate approach: The trigonometric ratios can be used to answer this question. In right triangle ABC, the ratio $\frac{BC}{AB} = \sin(32°)$. The angle E in triangle DEF has measure 32° because $m(\angle D) + m(\angle E) = 90°$. In triangle DEF, the ratio $\frac{DF}{DE} = \sin(32°)$. Therefore, $\frac{DF}{DE} = \frac{BC}{AB}$.

Choice A is incorrect because $\frac{DE}{DF}$ is the reciprocal of the ratio $\frac{BC}{AB}$. Choice C is incorrect because $\frac{DF}{EF} = \frac{BC}{AC}$, not $\frac{BC}{AB}$. Choice D is incorrect because $\frac{EF}{DE} = \frac{AC}{AB}$, not $\frac{BC}{AB}$.

Additional Assessment: Similar Triangles:

https://achievethecore.org/coherence-map/HS/G/116/611/611

The linked assessment question addresses G-SRT.A, specifically the question requires students to look at two triangles with a given pair of congruent angles and state a series of transformations to map one onto the other. Students will apply rotation, translation and a generic dilation in this example. This assessment should be given to students after they've had time to work with concrete examples of dilations as this more complicated example requires abstract algebra in terms of the scale factor. Students will engage in SMP1, SMP2, and potentially SMP3 depending on if students work in groups to share their solutions.

Grade	*CCSS Domain*	*CCSS Cluster*
G	Similarity, Right Triangles, & Trigonometry	Prove theorems involving similarity

Sample Task #2 (Constructed Response)

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item #: 422453 The linked assessment question addresses G-SRT.B, specifically the question requires students to solve for a side length in a series of composed right triangles.

CollegeBoard	Question ID 422453						
Assessment **SAT**	Test **Math**	Cross-Test and Subscore **Additional Topics in Math**	Difficulty **Hard**	Primary Dimension **Additional Topics in Math**	Secondary Dimension **Right triangles and trigonometry**	Tertiary Dimension **1. Solve problems in a variety of contexts using c. properties of special right triangles.**	Calculator **No Calculator**

In the figure above, $\overline{BD}$ is parallel to $\overline{AE}$. What is the length of $\overline{CE}$?

Sample Task #2

Question Difficulty: Hard

The correct answer is 30. In the figure given, since $\overline{BD}$ is parallel to $\overline{AE}$ and both segments are intersected by $\overline{CE}$, then angle BDC and angle AEC are corresponding angles and therefore congruent. Angle BCD and angle ACE are also congruent because they are the same angle. Triangle BCD and triangle ACE are similar because if two angles of one triangle are congruent to two angles of another triangle, the triangles are similar. Since triangle BCD and triangle ACE are similar, their corresponding sides are proportional. So in triangle BCD and triangle ACE, $\overline{BD}$ corresponds to $\overline{AE}$ and $\overline{CD}$ corresponds to $\overline{CE}$. Therefore, $\frac{BD}{CD} = \frac{AE}{CE}$. Since triangle BCD is a right triangle, the Pythagorean theorem can be used to give the value of CD: $6^2 + 8^2 = CD^2$. Taking the square root of each side gives $CD = 10$. Substituting the values in the proportion $\frac{BD}{CD} = \frac{AE}{CE}$ yields $\frac{6}{10} = \frac{18}{CE}$. Multiplying each side by CE, and then multiplying by $\frac{10}{6}$ yields $CE = 30$. Therefore, the length of $\overline{CE}$ is 30.

http://tasks.illustrativemathematics.org/content-standards/HSG/SRT/B/4/tasks/1568

The linked assessment question addresses G-SRT.B, specifically the question requires students to show two triangles are similar and then use ratios of side lengths to derive the Pythagorean theorem. This assessment should be given to students after they've worked with setting up ratios for similar triangles. Students will engage in SMP3, SMP6 and, if asked to share and critique work of peers, SMP3.

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Grade	CCSS Domain	CCSS Cluster
G	Similarity, Right Triangles, & Trigonometry	Define trigonometric ratios and solve problems involving right triangles

Sample Task #3 (Constructed Response)

CollegeBoard Question ID 4169029

SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Right triangles and trigonometry	4. Solve problems using the relationship between sine and cosine of complementary angles.	Calculator

In a right triangle, the tangent of one of the two acute angles is $\frac{\sqrt{3}}{3}$. What is the tangent of the other acute angle?

Question Difficulty: Medium

A. $-\frac{\sqrt{3}}{3}$

B. $-\frac{3}{\sqrt{3}}$

C. $\frac{\sqrt{3}}{3}$

D. $\frac{3}{\sqrt{3}}$

Choice D is correct. The tangent of a nonright angle in a right triangle is defined as the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle. Using that definition for tangent, in a right triangle with legs that have lengths a and b, the tangent of one acute angle is $\frac{a}{b}$ and the tangent for the other acute angle is $\frac{b}{a}$. It follows that the tangents of the acute angles in a right triangle are reciprocals of each other. Therefore, the tangent of the other acute angle in the given triangle is the reciprocal of $\frac{\sqrt{3}}{3}$ or $\frac{3}{\sqrt{3}}$.

Choice A is incorrect and may result from assuming that the tangent of the other acute angle is the negative of the tangent of the angle described. Choice B is incorrect and may result from assuming that the tangent of the other acute angle is the negative of the reciprocal of the tangent of the angle described. Choice C is incorrect and may result from interpreting the tangent of the other acute angle as equal to the tangent of the angle described.

	Additional Assessment: http://tasks.illustrativemathematics.org/content-standards/HSG/SRT/C/tasks/1316 The linked assessment question addresses G-SRT.C, specifically the question requires students to apply right triangle geometry to the context of points on a map. Students will need to visualize points on a map forming a right triangle and then apply formulas and concepts they are familiar with to solve contextual problems. This assessment should be given to students after they've been introduced to the formal definition of trigonometric ratios and applications of Pythagorean theorem and similar triangles. Students will engage in SMP1, SM 4, and potentially SMP5 if students are required to generate their own maps using tools.

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MLSS AND CLR GUIDE

- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangle

CCSS Domain	*CCSS Cluster*
Similarity, Right Triangles, and Trigonometry	Understand similarity in terms of similarity transformations

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on understanding similarity in terms of similarity transformation, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, when looking at ancient pottery pattern samples, how can Mesopotamian pottery patterns relate to Native American or African pottery patterns displayed throughout various cultures.	
Cross-Curricular Connections	Drafting/Architecture: Connect to trusses, shadow lengths	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language,*	• Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion, we position them as knowers and doers of mathematics. Using equitable talk moves students in the ways they talk about who is and isn't capable of mathematics. As a result, we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when understanding similarity in terms of similarity transformation, facilitating

	the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	meaningful mathematical discourse is critical because instructors should be able to draw from student misconceptions and translate these into learning pieces which will engage students in building on each other's ideas and deepen understanding of similarity transformation.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In 8th grade, students perform transformations, including dilations, in a coordinate plane. They also identify a sequence of transformations that highlights the similarity of two figures.	• In later clusters within the Geometry course, students connect their conceptual understanding of similarity to explore trigonometric relationships including special right triangles and trigonometric ratios.	• Students will continue their work with similar figures in later courses when working with trigonometric ratios and the unit circle. They will use their understanding of dilations when working with functions to determine a stretch/shrink transformation.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when understanding similarity in terms of similarity transformation SRT.A. cluster because it is important for students to understand prior knowledge vocabularies as they are introduced to more complex ones.
Intensive	*What critical understandings will*	Standard 8.G.A.4- Understand that a two-dimensional figure is similar to another if the second can be obtained

	prepare students to access the mathematics for this cluster?	from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. This standard provides a foundation for working with understanding similarity in terms of similarity transformation SRT.A. cluster because when students are not clear on the language structure of the mathematical problem at hand, it allows for a lot of misconceptions when the language has been presented in an advanced manner. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade-level instruction and assignments.

Universal Support Framework

A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• Transformations as functions and symmetry in terms of transformations.
• Two figures are congruent if there is a sequence of transformations that maps one onto another.
• Two figures are similar if they have the same shape with congruent angles and proportional side lengths.
• The formal language for relationships between angles including vertical angles, angles created when a transversal intersects parallel lines, angles and sides of triangles, the
	• Carry out rotations, reflections, translations, and dilations using a variety of tools and compare and contrast their effects.
	• Prove two triangles are congruent using ASA, SAS, and SSS or two triangles are similar using AA, SAS, and SSS.
	• Build formal justifications (proofs) for the theorems about lines and angles, triangles, and parallelograms.
	• Find unknown side lengths and angle measures of right
		• Build on students' experience with the following skills:
		  ○ Understand and use the coordinate axis
		  ○ Write and solve linear equations, especially proportions
		  ○ Recognize and draw geometric shapes (square, triangle, trapezoid, etc.)
		  ○ Understand and use the formulas of geometric shapes (Perimeter, Area, Volume, etc.)
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		• Encourage students to use alternative tools to better access the grade level content. Examples include:
		  ○ Desmos graphing calculator

segments and angles of parallelograms, and the trigonometric ratios sine, cosine, and tangent.	triangles using trigonometric ratios and the Pythagorean Theorem.	○ Desmos scientific calculator ○ Desmos geometry tool ○ GeoGebra ○ **Graphing or scientific calculator** ○ **Google Drawing** ○ **Geometric tools (ruler, protractor, compass, etc.)** ○ **Tracing paper** ○ **Graph paper and mirror/string/etc.** ○ **Craft tools (scissors, string, construction paper, etc.)** ○ **Paper folding**

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on understanding similarity in terms of similarity transformation SRT. A. cluster by examining tasks from a different perspective through a short mini-lesson because allowing students to connect their knowledge of scale and transitions into more complex thought processes such as dilations can help re-shift the reason why this standard is important in this context.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on understanding similarity in terms of similarity transformation SRT. A. cluster by offering opportunities to understand and explore different strategies because students might be able to explore the concept of “same shape” much easier than the concept of congruence.

Extension

Essential Question	***Examples***
What type of extension will offer additional challenges to ‘broaden’ your student’s knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension to explore links between various topics when understanding similarity in terms of similarity transformation SRT.A. cluster. Introducing students to angle measurements and how they aid in the process of transformation as well as congruence will help students avoid any misconceptions

	in the similarity cluster.

CCSS Domain	*CCSS Cluster*
Similarity, Right Triangles, and Trigonometry	Prove theorems involving similarity

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on proving theorems involving similarity, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, in the field of forensics, being able to determine the height of an individual contrasted with a fixed object in a video frame.	
Cross-Curricular Connections	Physics- Connect to Vectors, particularly in resultants and to Dimensional Kinematics Art – Connect to drafting/architecture and to shapes and reflection within works of art	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to*	• Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it hinders students with strong prior familiarity from learning more methods for solving tasks that occur outside of school mathematics. For example, when proving theorems involving similarity, the types of mathematical tasks are critical because the connections that can be utilized from an ELA standpoint (argumentative critical thinking) towards proving theorems in a mathematical world can be of instrumental value prior to introducing the procedural fluency (e.g. the Pythagorean theorem).

	support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In 8th grade, students developed the idea of "same shape" and "scale factor" as a definition of similarity. They will develop and connect these ideas when proving theorems within this cluster.	• Having previously studied dilations, students expand their definition of similarity to include congruence and dilation. These concepts lead to the criteria for triangle similarity. Students use proportional reasoning to approach problems involving similar figures. Trigonometric ratios will be developed using similar right triangles in connection to the work within this cluster.	• The trigonometric ratios (sine, cosine, tangent) will be founded on right triangles and similarity in subsequent learning. The Pythagorean theorem is generalized to non-right triangles by the Law of Cosines and Law of Sines.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when proving theorems involving similarity because students can make connections between right triangles by drawing a perpendicular line to bisect a bigger right triangle to form two smaller ones.
Intensive	*What critical understandings will*	8.G.B.6- Explain a proof of the Pythagorean Theorem and its converse**.** This standard provides a foundation for

	prepare students to access the mathematics for this cluster?	work with proving theorems involving similarity because when students understand the similarity between right triangles and the Pythagorean theorem, they will be able to make trigonometric connections between the two. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework

A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• Transformations as functions and symmetry in terms of transformations. • Two figures are congruent if there is a sequence of transformations that maps one onto another. • Two figures are similar if they have the same shape with congruent angles and proportional side lengths. • The formal language for relationships between angles including vertical angles, angles created when a transversal intersects parallel lines, angles and sides of triangles, the segments and angles of parallelograms, and the trigonometric ratios sine, cosine,	• Carry out rotations, reflections, translations, and dilations using a variety of tools and compare and contrast their effects. • Prove two triangles are congruent using ASA, SAS, and SSS or two triangles are similar using AA, SAS, and SSS. • Build formal justifications (proofs) for the theorems about lines and angles, triangles, and parallelograms. • Find unknown side lengths and angle measures of right triangles using trigonometric ratios and the Pythagorean Theorem.	• Build on students' experience with the following skills: ○ Understand and use the coordinate axis ○ Write and solve linear equations, especially proportions ○ Recognize and draw geometric shapes (square, triangle, trapezoid, etc.) ○ Understand and use the formulas of geometric shapes (Perimeter, Area, Volume, etc.) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos graphing calculator ○ Desmos scientific calculator ○ Desmos geometry tool ○ GeoGebra ○ **Graphing or scientific calculator** ○ **Google Drawing**

and tangent.		○ **Geometric tools (ruler, protractor, compass, etc.)** ○ **Tracing paper** ○ **Graph paper and mirror/string/etc.** ○ **Craft tools (scissors, string, construction paper, etc.)** ○ **Paper folding**

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on proving theorems involving similarity by revisiting student thinking through a short mini-lesson because it is important to understand where students are in terms of vocabulary such as similar and scale factors, so that when tackling the proofs students are not intimidated by the mathematical language expected.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on proving theorems involving similarity by offering opportunities to understand and explore different strategies because it might help clear up different misconceptions when students are allowed to display understanding in different ways. For example: with EL students one strategy would be to pair up individuals with native English-speaking classmates as they explore the task.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying proving theorems involving similarity because students might be able to broaden their knowledge of similarity into real world scenarios such as in the architectural field.

CCSS Domain	*CCSS Cluster*
Similarity, Right Triangles, and Trigonometry	Define trigonometric ratios and solve problems involving right triangles

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on trigonometric ratios and solving problems involving right triangles, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, when looking at trigonometric ratios of right triangles, students can relate the ratios of the triangle if we focus on sports. Shooting a basketball from 5 feet away vs. shooting a basketball from 10 ft away will show you congruence. Scaling down the basket by ½ the height can provide a transition into trigonometric ratios.	
Cross-Curricular Connections	STEM: Connect to engineering and construction use of trigonometry to determine accurate angles and/or missing lengths.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge concepts for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying trigonometric ratios and solving problems involving right triangles the use of mathematical representations within the classroom is critical because students will relate the background knowledge within cross-curricular activities and relate it to the different mathematical representations needed for this cluster.

	mathematicians that can use mathematics within school and society?	

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In 8th grade, students applied the Pythagorean Theorem to find unknown side length in right triangles and distance between two points. They will make connections between Pythagorean Theorem and trigonometric ratios to continue solving right triangles in this cluster.	• Students will continue to use trigonometric ratios throughout the remainder of the course. A strong procedural fluency is necessary for individuals to apply these ratios to items within future clusters. Pythagorean Theorem and the trigonometric ratios are used to find lengths necessary for finding surface areas and volumes. Students use similarity concepts when defining properties of circles, arc lengths, and sector areas.	• In future courses, trigonometric ratios are used to develop more complex concepts such as relationships within the unit circle. Students will graph the trigonometric functions and observe the cyclic patterns that arise from the trigonometric ratio relationships.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that introduces new representations when studying trigonometric ratios because students will be able to draw on prior knowledge of trigonometric ratios by representing proportional relationships between quantities learned prior to this cluster.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	7.RP.A.2- Recognize and represent proportional relationships between quantities: This standard provides a foundation for work with trigonometric ratios because the ratios explored in graphing linear relationships can now be explored by exposing students to trigonometric

		ratios within a right triangle. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework

A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• Transformations as functions and symmetry in terms of transformations.
• Two figures are congruent if there is a sequence of transformations that maps one onto another.
• Two figures are similar if they have the same shape with congruent angles and proportional side lengths.
• The formal language for relationships between angles including vertical angles, angles created when a transversal intersects parallel lines, angles and sides of triangles, the segments and angles of parallelograms, and the trigonometric ratios sine, cosine, and tangent.
	• Carry out rotations, reflections, translations, and dilations using a variety of tools and compare and contrast their effects.
	• Prove two triangles are congruent using ASA, SAS, and SSS or two triangles are similar using AA, SAS, and SSS.
	• Build formal justifications (proofs) for the theorems about lines and angles, triangles, and parallelograms.
	• Find unknown side lengths and angle measures of right triangles using trigonometric ratios and the Pythagorean Theorem.
		• Build on students' experience with the following skills:
		  ○ Understand and use the coordinate axis
		  ○ Write and solve linear equations, especially proportions
		  ○ Recognize and draw geometric shapes (square, triangle, trapezoid, etc.)
		  ○ Understand and use the formulas of geometric shapes (Perimeter, Area, Volume, etc.)
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		• Encourage students to use alternative tools to better access the grade level content. Examples include:
		  ○ Desmos graphing calculator
		  ○ Desmos scientific calculator
		  ○ Desmos geometry tool
		  ○ GeoGebra
		  ○ **Graphing or scientific calculator**
		  ○ **Google Drawing**
		  ○ **Geometric tools (ruler, protractor, compass, etc.)**
		  ○ **Tracing paper**
		  ○ **Graph paper and mirror/string/etc.**

		○ **Craft tools (scissors, string, construction paper, etc.)** ○ **Paper folding**
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on solving problems involving right triangles by clarifying mathematical ideas and/or concepts through a short mini-lesson because polygons other than triangles are not necessarily similar if each pair of corresponding angles is congruent. For example, all rectangles have congruent corresponding angles, but the corresponding sides of all rectangles do not have the same ratio.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on solving problems involving right triangles by offering opportunities to understand and explore different strategies because by investigating patterns of association in bivariate data students can use scatter plots and linear models.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying to define trigonometric ratios because students can make connections between engineering practices such as building electronics such as TVs. Understanding how trigonometric ratios an intricate part of the development of tv screens and will create a real-life extension for students.