$$
\begin{array}{lll}
v ::= & & e ::= \\
\quad \mid x & & \quad \mid z \\
\quad \mid v[a] & & \quad \mid v \\
\quad \mid v[a_1..a_2] & & \quad \mid (\,\overline{e_n}\,) \\
\quad \mid v[\,\overline{a_n}\,] & & \quad \mid monop_\tau\ e \\
a ::= & & \quad \mid e_1\ binop_\tau\ e_2 \\
\quad \mid i & & \quad \mid f(\,\overline{e_n}\,) \\
\quad \mid z & & \quad \mid f < a > (\,\overline{e_n}\,) \\
\quad \mid a_1\ aop\ a_2 & & \\
aop ::= & & \\
\quad \mid + & & \\
\quad \mid - & & \\
\quad \mid / & & \\
\quad \mid \times & &
\end{array}
$$

$i,\ d,\ s$	Static Identifiers: $\in Ident$
$x,\ y,\ t$	Dynamic Identifiers: $\in Ident$
f	Node Identifiers: $\in Ident$
$\ell,\ z$	Integers: $\in \mathbb{N}$
$n,\ m,\ j,\ k$	Index variables

$$
\begin{array}{l}
deq ::= \\
\quad \mid \overline{v_n} = e \\
\quad \mid \overline{v_n} := e \\
\quad \mid \textbf{for } i = a_1 \textbf{ to } a_2 \textbf{ do } \overline{deq_n} \textbf{ done} \\
node ::= \\
\quad \mid \textbf{node}\, f(\,\overline{x_m : \tau_m}\,) \rightarrow (\,\overline{y_n : \tau'_n}\,)\, \textbf{vars}\, (\,\overline{t_j : \tau''_j}\,)\, \textbf{let}\, \overline{deq_k}\, \textbf{tel} \\
\quad \mid \textbf{table}\, f(x : \tau) \rightarrow (y : \tau')[\,\overline{a_n}\,]
\end{array}
$$

Figure 1: AST de Usuba

$$
\begin{array}{llll}
dir ::= & typc ::= & P ::= & \sigma ::= \\
\quad \mid \mathbf{V} & \quad \mid \textbf{Arith}\, \tau & \quad \mid \textbf{nil} & \quad \mid \mathbf{U}\ dir\ size \\
\quad \mid \mathbf{H} & \quad \mid \textbf{Logic}\, \tau & \quad \mid P \leftarrow node & \quad \mid \sigma\{a\} \\
\quad \mid d & \quad \mid \textbf{Shift}\, \tau\ a_2 & \Gamma ::= & \tau ::= \\
size ::= & \quad \mid \textbf{ClassOf}\ op\, \tau & \quad \mid \overline{x_n : \tau_n} & \quad \mid \sigma \\
\quad \mid s & A ::= & & \quad \mid \tau[a] \\
\quad \mid z & \quad \mid \overline{typc_n} & & \mathcal{T} ::= \\
 & & & \quad \mid \overline{\tau_n}
\end{array}
$$

Figure 2: Types et contextes en Usuba

NB : Si l'on as $x : u32$ et $y : u16$, alors $(x, y) + (x, y)$ est non typable.

$\boxed{\Gamma \vdash_V v : \tau}$

$$\frac{\Gamma \vdash_I x : \tau \in \Gamma}{\Gamma \vdash_V x : \tau}\ \textsc{Ident}$$

$$\frac{\begin{array}{c}\Gamma \vdash_V v : \sigma[\ell]\,\overline{[d_n]} \\ \vdash 0 \leqslant a < \ell\end{array}}{\Gamma \vdash_V v[a] : \sigma\,\overline{[d_n]}}\ \textsc{Index}$$

$$\frac{\begin{array}{c}\Gamma \vdash_V v : \sigma[\ell]\,\overline{[d_n]} \\ \vdash 0 \leqslant a_1 < \ell \\ \vdash 0 \leqslant a_2 < \ell\end{array}}{\Gamma \vdash_V v[a_1..a_2] : \sigma\{abs(a_1 - a_2) + 1\}\,\overline{[d_n]}}\ \textsc{Range}$$

$$\frac{\begin{array}{c}\Gamma \vdash_V v : \sigma[\ell]\,\overline{[d_n]} \\ \vdash 0 \leqslant \overline{a_n} < \ell\end{array}}{\Gamma \vdash_V v[\,\overline{a_n}\,] : \sigma\{\mathbf{len}\,\overline{a_n}\}\,\overline{[d_n]}}\ \textsc{Slice}$$

Figure 3: Typage variables

$\boxed{\mathcal{T}_1 \cong \mathcal{T}_2}$

$$\frac{}{\mathcal{T} \cong \mathcal{T}}\ \textsc{Refl}$$

$$\frac{\mathcal{T}_1 \cong \mathcal{T}_2}{\mathcal{T}_2 \cong \mathcal{T}_1}\ \textsc{Sym}$$

$$\frac{\begin{array}{c}\mathcal{T}_1 \cong \mathcal{T}_2 \\ \mathcal{T}_2 \cong \mathcal{T}_3\end{array}}{\mathcal{T}_1 \cong \mathcal{T}_3}\ \textsc{Trans}$$

$$\frac{\mathcal{T}_1 \cong \mathcal{T}_2}{\tau :: \mathcal{T}_1 \cong \tau :: \mathcal{T}_2}\ \textsc{Rec}$$

$$\frac{}{\mathbf{U}\,\mathit{dir}\,s\,\overline{\{a_n\}}\,\overline{[a'_m]} :: \mathcal{T} \cong \mathbf{U}\,\mathit{dir}\,s[\mathbf{prod}\,[\,\overline{a_n}\,] \times \mathbf{prod}\,[\,\overline{a'_m}\,]] :: \mathcal{T}}\ \textsc{SimplForm}$$

$$\frac{}{\mathbf{U}\,\mathbf{V}\,1\,\overline{\{a_n\}}\,\overline{[a'_m]} :: \mathcal{T} \cong \mathbf{U}\,\mathbf{H}\,1\,\overline{\{a_n\}}\,\overline{[a'_m]} :: \mathcal{T}}\ \textsc{Bool}$$

$$\frac{}{\mathbf{U}\,\mathit{dir}\,s[\ell_1] :: \mathbf{U}\,\mathit{dir}\,s[\ell_2] :: \mathcal{T} \cong \mathbf{U}\,\mathit{dir}\,s[\ell_1 + \ell_2] :: \mathcal{T}}\ \textsc{Join}$$

Figure 4: Equivalence de types

$\boxed{A \vdash \overline{\mathit{typc}_n}}$

$$\frac{A \vdash \mathbf{Arith}\,\sigma}{A \vdash \mathbf{Arith}\,\sigma\{\ell\}}\ \textsc{ArithLI}$$

$$\frac{A \vdash \mathbf{Arith}\,\tau}{A \vdash \mathbf{Arith}\,\tau[\ell]}\ \textsc{ArithL}$$

$$\frac{A \vdash \mathbf{Logic}\,\sigma}{A \vdash \mathbf{Logic}\,\sigma\{\ell\}}\ \textsc{LogicLI}$$

$$\frac{A \vdash \mathbf{Logic}\,\tau}{A \vdash \mathbf{Logic}\,\tau[\ell]}\ \textsc{LogicL}$$

Figure 5: Inférence des type-class

$\boxed{\Gamma, P, A \vdash_E e : \mathcal{T}}$

$$\frac{\Gamma \vdash_V v : \tau}{\Gamma, P, A \vdash_E v : \tau}\ \textsc{Var}$$

$$\frac{\begin{array}{c}\Gamma, P, A \vdash_E e_1 : \tau\\ \Gamma, P, A \vdash_E e_2 : \tau\\ A \vdash \textbf{ClassOf}\ \mathit{binop}\ \tau\end{array}}{\Gamma, P, A \vdash_E e_1\ \mathit{binop}_\tau\ e_2 : \tau}\ \textsc{Binop}$$

$$\frac{\begin{array}{c}\Gamma, P, A \vdash_E e : \tau\\ A \vdash \textbf{ClassOf}\ \mathit{monop}\ \tau\end{array}}{\Gamma, P, A \vdash_E \mathit{monop}_\tau\ e : \tau}\ \textsc{Monop}$$

$$\frac{\Gamma, P, A \vdash_E \overline{e_n : \mathcal{T}_n}}{\Gamma, P, A \vdash_E (\,\overline{e_n}\,) : \overline{\mathcal{T}_n}}\ \textsc{Tuple}$$

$$\frac{\begin{array}{c}P \vdash f : \forall\, \overline{d_n}\,, \forall\, \overline{s_m}\,, \overline{\mathit{typc}_j} \Rightarrow \mathcal{T}_1 \to \mathcal{T}_2\\ \Gamma, P, A \vdash_E (\,\overline{e_n}\,) : \mathcal{T}_1'\\ A \vdash\ \overline{\mathit{typc}_j[\,\overline{d_n \leftarrow d_n'}\ ;\ \overline{s_m \leftarrow s_m'}\,]}\\ \mathcal{T}_1' \cong \mathcal{T}_1[\,\overline{d_n \leftarrow d_n'}\ ;\ \overline{s_m \leftarrow s_m'}\,]\end{array}}{\Gamma, P, A \vdash_E f(\,\overline{e_n}\,) : \mathcal{T}_2[\,\overline{d_n \leftarrow d_n'}\ ;\ \overline{s_m \leftarrow s_m'}\,]}\ \textsc{Fun}$$

Figure 6: Règles de typage des expressions

$\boxed{\Gamma, P, A \vdash_D \mathit{deq}}$

$$\frac{\begin{array}{c}\Gamma, P, A \vdash_E e : \mathcal{T}\\ \mathcal{T} \cong \mathcal{T}'\\ \Gamma \vdash_V \overline{v_n} : \mathcal{T}'\end{array}}{\Gamma, P, A \vdash_D \overline{v_n} := e}\ \textsc{EqnT}$$

$$\frac{\begin{array}{c}\Gamma, P, A \vdash_E e : \mathcal{T}\\ \mathcal{T} \cong \mathcal{T}'\\ \Gamma \vdash_V \overline{v_n} : \mathcal{T}'\end{array}}{\Gamma, P, A \vdash_D \overline{v_n} = e}\ \textsc{EqnF}$$

$$\frac{\forall\, i \in [a_1, a_2].\ \Gamma, P, A \vdash_D \overline{\mathit{deq}_n[x \leftarrow i]}}{\Gamma, P, A \vdash_D \textbf{for}\ i = a_1\ \textbf{to}\ a_2\ \textbf{do}\ \overline{\mathit{deq}_n}\ \textbf{done}}\ \textsc{Loop}$$

Figure 7: Typage des equations

$\boxed{P \vdash f : \forall\, \overline{d_n}\,, \forall\, \overline{s_m}\,, A \Rightarrow \mathcal{T}_1 \to \mathcal{T}_2}$

$$\frac{\begin{array}{c}\overline{x_m : \tau_m} + \overline{y_n : \tau_n'} + \overline{t_j : \tau_j''}, P, A \vdash_D \overline{\mathit{deq}_k}\\ \mathit{node} = \textbf{node}\, f(\,\overline{x_m : \tau_m}\,) \to (\,\overline{y_n : \tau_n'}\,)\,\textbf{vars}\,(\,\overline{t_j : \tau_j''}\,)\,\textbf{let}\ \overline{\mathit{deq}_k}\ \textbf{tel}\end{array}}{P \leftarrow \mathit{node} \vdash f : \forall\, \overline{d_n}\,, \forall\, \overline{s_m}\,, A \Rightarrow \overline{\tau_m} \to \overline{\tau_n'}}\ \textsc{Node}$$

$$\frac{\begin{array}{c}\vdash 0 \leqslant \overline{z_n} < 1 \ll i_2\\ \textbf{len}\,\overline{z_n} = 1 \ll i_1\\ \mathit{node} = \textbf{table}\, f(x : \mathbf{U}\ d\ s[i_1]) \to (y : \mathbf{U}\ d\ s[i_2])[\,\overline{z_n}\,]\end{array}}{P \leftarrow \mathit{node} \vdash f : \forall\, d, \forall\, s, \textbf{Logic}\,(\mathbf{U}\ d\ s) \Rightarrow \mathbf{U}\ \mathit{dir}\ s[i_1] \to \mathbf{U}\ \mathit{dir}\ s[i_2]}\ \textsc{Table}$$

Figure 8: Typage d'un noeud