$$
\begin{array}{l}
v ::= \\
\quad \mid x \\
\quad \mid v[a] \\
\quad \mid v[a_1..a_2] \\
\quad \mid v[\,\overline{a_n}\,]
\end{array}
$$

$$
\begin{array}{l}
a ::= \\
\quad \mid i \\
\quad \mid z \\
\quad \mid a_1 \mathit{aop}\; a_2
\end{array}
$$

$$
\begin{array}{l}
\mathit{aop} ::= \\
\quad \mid + \\
\quad \mid - \\
\quad \mid / \\
\quad \mid \times
\end{array}
$$

$$
\begin{array}{l}
e ::= \\
\quad \mid z \\
\quad \mid v \\
\quad \mid (\,\overline{e_n}\,) \\
\quad \mid \mathbf{not}_{dir,s}\; e \\
\quad \mid \mathbf{not}^{a}_{dir,s}\; e \\
\quad \mid e_1\; \mathit{lop}_{dir,s}\; e_2 \\
\quad \mid e_1\; \mathit{lop}^{a}_{dir,s}\; e_2 \\
\quad \mid e_1\; \mathit{aop}_{dir,s}\; e_2 \\
\quad \mid e_1\; \mathit{aop}^{a}_{dir,s}\; e_2 \\
\quad \mid e\; \mathit{sop}_{dir,s}\; a_2 \\
\quad \mid e\; \mathit{sop}^{a}_{dir,s}\; a_2 \\
\quad \mid f(\,\overline{e_n}\,) \\
\quad \mid f < a > (\,\overline{e_n}\,)
\end{array}
$$

$i,\ d,\ s$	Static Identifiers: $\in \mathit{Ident}$
$x,\ y,\ t$	Dynamic Identifiers: $\in \mathit{Ident}$
f	Node Identifiers: $\in \mathit{Ident}$
$\ell,\ z$	Integers: $\in \mathbb{N}$
$n,\ m,\ j,\ k$	Index variables

$$
\begin{array}{l}
\mathit{deq} ::= \\
\quad \mid \overline{v_n} = e \\
\quad \mid \overline{v_n} := e \\
\quad \mid \mathbf{for}\; i = a_1\; \mathbf{to}\; a_2\; \mathbf{do}\; \overline{\mathit{deq}_n}\; \mathbf{done}
\end{array}
$$

$$
\begin{array}{l}
\mathit{node} ::= \\
\quad \mid \mathbf{node}\, f(\,\overline{x_m : \sigma_m}\,) \rightarrow (\,\overline{y_n : \sigma'_n}\,)\; \mathbf{vars}\; (\,\overline{t_j : \sigma''_j}\,)\; \mathbf{let}\; \overline{\mathit{deq}_k}\; \mathbf{tel} \\
\quad \mid \mathbf{table}\, f(x : \sigma) \rightarrow (y : \sigma')[\,\overline{a_n}\,]
\end{array}
$$

Figure 1: AST de Usuba

$$
\begin{array}{l}
\mathit{dir} ::= \\
\quad \mid \mathbf{V} \\
\quad \mid \mathbf{H} \\
\quad \mid d
\end{array}
$$

$$
\begin{array}{l}
\mathit{size} ::= \\
\quad \mid s \\
\quad \mid z
\end{array}
$$

$$
\begin{array}{l}
\mathit{typc} ::= \\
\quad \mid \mathbf{Arith}\; \mathit{dir}\; a \\
\quad \mid \mathbf{Logic}\; \mathit{dir}\; a \\
\quad \mid \mathbf{Shift}\; \mathit{dir}\; a_1\; a_2
\end{array}
$$

$$
\begin{array}{l}
A ::= \\
\quad \mid \overline{\mathit{typc}_n}
\end{array}
$$

$$
\begin{array}{l}
P ::= \\
\quad \mid \mathbf{nil} \\
\quad \mid P \leftarrow \mathit{node}
\end{array}
$$

$$
\begin{array}{l}
\Gamma ::= \\
\quad \mid \overline{x_n : \sigma_n}
\end{array}
$$

$$
\begin{array}{l}
\sigma ::= \\
\quad \mid \mathbf{Uint}\; \mathit{dir}\; \mathit{size}\; \mathit{form}
\end{array}
$$

$$
\begin{array}{l}
\mathcal{T} ::= \\
\quad \mid \overline{\sigma_n}
\end{array}
$$

Figure 2: Types et contextes en Usuba

NB : Si l'on as $x : u32$ et $y : u16$, alors $(x, y) + (x, y)$ est non typable.

$\boxed{\Gamma \vdash_V v : \mathcal{T}}$

$$\frac{\Gamma \vdash_I x : \sigma \in \Gamma}{\Gamma \vdash_V x : \sigma}\ \textsc{Ident}$$

$$\frac{\begin{array}{c}\Gamma \vdash_V v : \mathbf{Uint}\ \mathit{dir}\ s\ (\ell :: \mathit{form})^{a'} \\ \vdash 0 \leqslant a < \ell\end{array}}{\Gamma \vdash_V v[a] : \mathbf{Uint}\ \mathit{dir}\ s\ \mathit{form}^{a'}}\ \textsc{Index}$$

$$\frac{\begin{array}{c}\Gamma \vdash_V v : \mathbf{Uint}\ \mathit{dir}\ s\ (\ell :: \mathit{form})^{a} \\ \vdash 0 \leqslant a_1 < \ell \\ \vdash 0 \leqslant a_2 < \ell\end{array}}{\Gamma \vdash_V v[a_1..a_2] : \mathbf{Uint}\ \mathit{dir}\ s\ \mathit{form}^{a \times (abs(a_1 - a_2) + 1)}}\ \textsc{Range}$$

$$\frac{\begin{array}{c}\Gamma \vdash_V v : \mathbf{Uint}\ \mathit{dir}\ s\ (\ell :: \mathit{form})^{a} \\ \vdash 0 \leqslant \overline{a_n} < \ell\end{array}}{\Gamma \vdash_V v[\,\overline{a_n}\,] : \mathbf{Uint}\ \mathit{dir}\ s\ \mathit{form}^{a \times \mathbf{len}\ \overline{a_n}}}\ \textsc{Slice}$$

Figure 3: Typage variables

$\boxed{\mathcal{T}_1 \cong \mathcal{T}_2}$

$$\frac{}{\mathcal{T} \cong \mathcal{T}}\ \textsc{Refl}$$

$$\frac{\mathcal{T}_1 \cong \mathcal{T}_2}{\mathcal{T}_2 \cong \mathcal{T}_1}\ \textsc{Sym}$$

$$\frac{\begin{array}{c}\mathcal{T}_1 \cong \mathcal{T}_2 \\ \mathcal{T}_2 \cong \mathcal{T}_3\end{array}}{\mathcal{T}_1 \cong \mathcal{T}_3}\ \textsc{Trans}$$

$$\frac{\mathcal{T}_1 \cong \mathcal{T}_2}{\sigma :: \mathcal{T}_1 \cong \sigma :: \mathcal{T}_2}\ \textsc{Rec}$$

$$\frac{}{\mathbf{Uint}\ \mathit{dir}\ s\ \mathit{form} :: \mathcal{T} \cong \mathbf{Uint}\ \mathit{dir}\ s\ (\mathbf{prod}\ \mathit{form} :: \mathbf{nil}) :: \mathcal{T}}\ \textsc{SimplForm}$$

$$\frac{}{\mathbf{Uint}\ \mathbf{V}\ 1\ \mathit{form} :: \mathcal{T} \cong \mathbf{Uint}\ \mathbf{H}\ 1\ \mathit{form} :: \mathcal{T}}\ \textsc{Bool}$$

$$\frac{}{(\mathbf{Uint}\ \mathit{dir}\ s\ (\ell_1 :: \mathbf{nil})) :: (\mathbf{Uint}\ \mathit{dir}\ s\ (\ell_2 :: \mathbf{nil})) :: \mathcal{T} \cong (\mathbf{Uint}\ \mathit{dir}\ s\ (\ell_1 + \ell_2 :: \mathbf{nil})) :: \mathcal{T}}\ \textsc{Join}$$

Figure 4: Equivalence de types

$\boxed{\Gamma, P, A \vdash_E e : \mathcal{T}}$

$$\frac{\Gamma \vdash_V v : \mathcal{T}}{\Gamma, P, A \vdash_E v : \mathcal{T}}\ \textsc{Var}$$

$$\frac{\begin{array}{c}\Gamma, P, A \vdash_E e_1 : \textbf{Uint}\ \mathit{dir}\ s\ \textbf{nil} \\ \Gamma, P, A \vdash_E e_2 : \textbf{Uint}\ \mathit{dir}\ s\ \textbf{nil} \\ A \vdash \textbf{Arith}\ \mathit{dir}\ s\end{array}}{\Gamma, P, A \vdash_E e_1\ \mathit{aop}_{\mathit{dir},s}\ e_2 : \textbf{Uint}\ \mathit{dir}\ s\ \textbf{nil}}\ \textsc{Arith}$$

$$\frac{\begin{array}{c}\Gamma, P, A \vdash_E e_1 : \mathcal{T}_1 \\ \Gamma, P, A \vdash_E e_2 : \mathcal{T}_2 \\ A \vdash \textbf{Arith}\ \mathit{dir}\ s \\ \textbf{Uint}\ \mathit{dir}\ s\ (a\ ::\ \textbf{nil})\ \cong\ \mathcal{T}_1 \wedge \forall\, d, \mathcal{T}_1 \neq \textbf{Uint}\ d\ s\ \textbf{nil} \\ \textbf{Uint}\ \mathit{dir}\ s\ (a\ ::\ \textbf{nil})\ \cong\ \mathcal{T}_2 \wedge \forall\, d, \mathcal{T}_2 \neq \textbf{Uint}\ d\ s\ \textbf{nil}\end{array}}{\Gamma, P, A \vdash_E e_1\ \mathit{aop}^{\textbf{prod}\,\mathit{form}}_{\mathit{dir},s}\ e_2 : \textbf{Uint}\ \mathit{dir}\ s\ (a\ ::\ \textbf{nil})}\ \textsc{ArithL}$$

$$\frac{\begin{array}{c}\Gamma, P, A \vdash_E e_1 : \textbf{Uint}\ \mathit{dir}\ s\ \textbf{nil} \\ \Gamma, P, A \vdash_E e_2 : \textbf{Uint}\ \mathit{dir}\ s\ \textbf{nil} \\ A \vdash \textbf{Logic}\ \mathit{dir}\ s\end{array}}{\Gamma, P, A \vdash_E e_1\ \mathit{lop}_{\mathit{dir},s}\ e_2 : \textbf{Uint}\ \mathit{dir}\ s\ \textbf{nil}}\ \textsc{Logic}$$

$$\frac{\begin{array}{c}\Gamma, P, A \vdash_E e_1 : \mathcal{T}_1 \\ \Gamma, P, A \vdash_E e_2 : \mathcal{T}_2 \\ A \vdash \textbf{Logic}\ \mathit{dir}\ s \\ \textbf{Uint}\ \mathit{dir}\ s\ (a\ ::\ \textbf{nil})\ \cong\ \mathcal{T}_1 \wedge \forall\, d, \mathcal{T}_1 \neq \textbf{Uint}\ d\ s\ \textbf{nil} \\ \textbf{Uint}\ \mathit{dir}\ s\ (a\ ::\ \textbf{nil})\ \cong\ \mathcal{T}_2 \wedge \forall\, d, \mathcal{T}_2 \neq \textbf{Uint}\ d\ s\ \textbf{nil}\end{array}}{\Gamma, P, A \vdash_E e_1\ \mathit{lop}^{\textbf{prod}\,\mathit{form}}_{\mathit{dir},s}\ e_2 : \textbf{Uint}\ \mathit{dir}\ s\ (a\ ::\ \textbf{nil})}\ \textsc{LogicL}$$

$$\frac{\begin{array}{c}\Gamma, P, A \vdash_E e : \textbf{Uint}\ \mathit{dir}\ s\ \textbf{nil} \\ A \vdash \textbf{Logic}\ \mathit{dir}\ s\end{array}}{\Gamma, P, A \vdash_E \textbf{not}_{\mathit{dir},s}\ e : \textbf{Uint}\ \mathit{dir}\ s\ \textbf{nil}}\ \textsc{Not}$$

$$\frac{\begin{array}{c}\Gamma, P, A \vdash_E e : \mathcal{T} \\ A \vdash \textbf{Logic}\ \mathit{dir}\ s \\ \textbf{Uint}\ \mathit{dir}\ s\ (a\ ::\ \textbf{nil})\ \cong\ \mathcal{T} \wedge \forall\, d, \textbf{Uint}\ \mathit{dir}\ s\ \textbf{nil} \neq \mathcal{T}\end{array}}{\Gamma, P, A \vdash_E \textbf{not}^{\textbf{prod}\,\mathit{form}}_{\mathit{dir},s}\ e : \textbf{Uint}\ \mathit{dir}\ s\ (a\ ::\ \textbf{nil})}\ \textsc{NotL}$$

$$\frac{\begin{array}{c}\Gamma, P, A \vdash_E e : \textbf{Uint}\ \mathit{dir}\ s\ \textbf{nil} \\ A \vdash \textbf{Shift}\ \mathit{dir}\ s\ a\end{array}}{\Gamma, P, A \vdash_E e\ \mathit{sop}_{\mathit{dir},s}\ a : \textbf{Uint}\ \mathit{dir}\ s\ \textbf{nil}}\ \textsc{Shift}$$

$$\frac{\begin{array}{c}\Gamma, P, A \vdash_E e : \mathcal{T} \\ \vdash 0 \leqslant a < \ell \\ \textbf{Uint}\ \mathit{dir}\ s\ (\ell\ ::\ \textbf{nil})\ \cong\ \mathcal{T} \wedge \forall\, d, \textbf{Uint}\ \mathit{dir}\ s\ \textbf{nil} \neq \mathcal{T}\end{array}}{\Gamma, P, A \vdash_E e\ \mathit{sop}^{\ell}_{\mathit{dir},s}\ a : \mathcal{T}}\ \textsc{ShiftL}$$

$$\frac{\Gamma, P, A \vdash_E \overline{e_n : \mathcal{T}_n}}{\Gamma, P, A \vdash_E (\,\overline{e_n}\,) : \overline{\mathcal{T}_n}}\ \textsc{Tuple}$$

$$\frac{\begin{array}{c}P \vdash f\ :\ \forall\, \overline{d_n}\,,\ \forall\, \overline{s_m}\,,\ \overline{\mathit{typc}_j} \Rightarrow \mathcal{T}_1 \rightarrow \mathcal{T}_2 \\ \Gamma, P, A \vdash_E (\,\overline{e_n}\,) : \mathcal{T}'_1 \\ A \vdash\ \overline{\mathit{typc}_j[\,\overline{d_n \leftarrow d'_n}\ ;\ \overline{s_m \leftarrow s'_m}\,]} \\ \mathcal{T}'_1\ \cong\ \mathcal{T}_1[\,\overline{d_n \leftarrow d'_n}\ ;\ \overline{s_m \leftarrow s'_m}\,]\end{array}}{\Gamma, P, A \vdash_E f(\,\overline{e_n}\,) : \mathcal{T}_2[\,\overline{d_n \leftarrow d'_n}\ ;\ \overline{s_m \leftarrow s'_m}\,]}\ \textsc{Fun}$$

Figure 5: Règles de typage des expressions

$$\boxed{\Gamma, P, A \vdash_D deq}$$

$$\dfrac{\begin{array}{c}\Gamma, P, A \vdash_E e : \mathcal{T} \\ \mathcal{T} \cong \mathcal{T}' \\ \Gamma \vdash_V \overline{v_n} : \mathcal{T}'\end{array}}{\Gamma, P, A \vdash_D \overline{v_n} := e} \quad \textsc{EqnT}$$

$$\dfrac{\begin{array}{c}\Gamma, P, A \vdash_E e : \mathcal{T} \\ \mathcal{T} \cong \mathcal{T}' \\ \Gamma \vdash_V \overline{v_n} : \mathcal{T}'\end{array}}{\Gamma, P, A \vdash_D \overline{v_n} = e} \quad \textsc{EqnF}$$

$$\dfrac{\forall\, i \in [a_1, a_2].\, \Gamma, P, A \vdash_D \overline{deq_n[x \leftarrow i]}}{\Gamma, P, A \vdash_D \textbf{for}\ i = a_1\ \textbf{to}\ a_2\ \textbf{do}\ \overline{deq_n}\ \textbf{done}} \quad \textsc{Loop}$$

Figure 6: Typage des equations

$$\boxed{P \vdash f : \forall\, \overline{d_n}\,, \forall\, \overline{s_m}\,, A \Rightarrow \mathcal{T}_1 \to \mathcal{T}_2}$$

$$\dfrac{\begin{array}{c}\overline{x_m : \sigma_m} + \overline{y_n : \sigma'_n} + \overline{t_j : \sigma''_j}, P, A \vdash_D \overline{deq_k} \\ node = \textbf{node}\, f(\, \overline{x_m : \sigma_m}\,) \to (\, \overline{y_n : \sigma'_n}\,)\, \textbf{vars}\, (\, \overline{t_j : \sigma''_j}\,)\, \textbf{let}\, \overline{deq_k}\, \textbf{tel}\end{array}}{P \leftarrow node \vdash f : \forall\, \overline{d_n}\,, \forall\, \overline{s_m}\,, A \Rightarrow \overline{\sigma_m} \to \overline{\sigma'_n}} \quad \textsc{Node}$$

$$\dfrac{\begin{array}{c}\vdash 0 \leqslant \overline{z_n} < 1 \ll i_2 \\ \textbf{len}\, \overline{z_n} = 1 \ll i_1 \\ node = \textbf{table}\, f(x : \textbf{Uint}\ d\ s\ (i_1 :: \textbf{nil})) \to (y : \textbf{Uint}\ d\ s\ (i_2 :: \textbf{nil}))[\, \overline{z_n}\,]\end{array}}{P \leftarrow node \vdash f : \forall\, d, \forall\, s, \textbf{Logic}\ d\ s \Rightarrow \textbf{Uint}\ dir\ s\ (i_1 :: \textbf{nil}) \to \textbf{Uint}\ dir\ s\ (i_2 :: \textbf{nil})} \quad \textsc{Table}$$

Figure 7: Typage d'un noeud