$$
\begin{array}{l}
v ::= \\
\quad \mid \ \mathbf{Var}\ x \\
\quad \mid \ v[a] \\
\quad \mid \ v[a_1..a_2] \\
\quad \mid \ v[\ \overline{a_n}\ ]
\end{array}
\qquad
\begin{array}{l}
a ::= \\
\quad \mid \ \mathbf{aevar}\ x \\
\quad \mid \ z \\
\quad \mid \ a_1\ aop\ a_2
\end{array}
\qquad
\begin{array}{l}
e ::= \\
\quad \mid \ z \\
\quad \mid \ \mathbf{evar}\ v \\
\quad \mid \ (\ \overline{e_n}\ ) \\
\quad \mid \ \mathbf{not}\ e \\
\quad \mid \ e_1\ lop\ e_2 \\
\quad \mid \ e_1\ aop\ e_2 \\
\quad \mid \ e\ sop\ a \\
\quad \mid \ f(\ \overline{e_n}\ ) \\
\quad \mid \ f < a > (\ \overline{e_n}\ )
\end{array}
\qquad
\begin{array}{l}
deq ::= \\
\quad \mid \ \overline{v_n}\ = e \\
\quad \mid \ \overline{v_n}\ := e \\
\quad \mid \ \mathbf{for}\ x = a_1\ \mathbf{to}\ a_2\ \mathbf{do}\ \overline{deq_n}\ \mathbf{done}
\end{array}
$$

Figure 1: AST de Usuba

$$
\frac{\Gamma \vdash x : \sigma \in \Gamma}{\Gamma \vdash \mathbf{Var}\ x : (\sigma, 1)}
$$

$$
\frac{\begin{array}{c}\Gamma \vdash v : (\mathbf{Uint}\ dir\ i\ (h\ ::\ form), a) \\ \Gamma \vdash 0 \leqslant a_1 < h \\ \Gamma \vdash 0 \leqslant a_2 < h\end{array}}{\Gamma \vdash v[a_1..a_2] : (\mathbf{Uint}\ dir\ i\ form, a \times (abs(a_1 - a_2) + 1))}
$$

$$
\frac{\begin{array}{c}\Gamma \vdash v : (\mathbf{Uint}\ dir\ i\ (h\ ::\ form), a) \\ \Gamma \vdash 0 \leqslant a < h\end{array}}{\Gamma \vdash v[a] : (\mathbf{Uint}\ dir\ i\ form, a)}
$$

$$
\frac{\begin{array}{c}\Gamma \vdash v : (\mathbf{Uint}\ dir\ i\ (h\ ::\ form), a) \\ \Gamma \vdash 0 \leqslant \overline{a_n} < h\end{array}}{\Gamma \vdash v[\ \overline{a_n}\ ] : (\mathbf{Uint}\ dir\ i\ form, a \times \mathbf{len}\ \overline{a_n})}
$$

Figure 2: Typage variables

NB : Si l'on as $x : u32$ et $y : u16$, alors $(x, y) + (x, y)$ est non typable.

$$\frac{}{\textbf{from}\,(\textbf{Uint}\;\mathit{dir}\;i\;\textbf{nil}, 1) = (\mathit{dir}, i, \textbf{None})}$$

$$\frac{\mathit{form} \neq \textbf{nil} \vee \ell \neq 1}{\textbf{from}\,(\textbf{Uint}\;\mathit{dir}\;i\;\mathit{form}, \ell) = (\mathit{dir}, i, \textbf{Some}\,(\ell \times \textbf{prod}\,\mathit{form}))}$$

Figure 3: Definition de l'opérateur from

$$\frac{}{\textbf{normalize}\,[\,] = [\,]}$$

$$\frac{}{\textbf{normalize}\,[(\mathit{dir}, i, o)] = [(\mathit{dir}, i, o)]}$$

$$\frac{\textbf{normalize}\,\mathit{etypL} = (\mathit{dir}, i, o_1) :: \mathit{etypL}'}{\textbf{normalize}\,((\mathit{dir}, i, o_2) :: \mathit{etypL}) = ((\mathit{dir}, i, o_1 + o_2) :: \mathit{etypL}')}$$

$$\frac{\begin{array}{c}\textbf{normalize}\,\mathit{etypL} = (\mathit{dir}_2, i_2, o_2) :: \mathit{etypL}' \\ \mathit{dir}_1 \neq \mathit{dir}_2 \vee i_1 \neq i_2\end{array}}{\textbf{normalize}\,((\mathit{dir}_1, i_1, o_1) :: \mathit{etypL}) = (\mathit{dir}_1, i_1, o_1) :: (\mathit{dir}_2, i_2, o_2) :: \mathit{etypL}'}$$

Figure 4: Normalisation de type

$$\frac{\Gamma, \mathbf{P}, \mathbf{A} \vdash \overline{e_n \rightsquigarrow e'_n : \mathit{etyp}_n}}{\Gamma, \mathbf{P}, \mathbf{A} \vdash (\,\overline{e_n}\,) \rightsquigarrow (\,\overline{e'_n}\,) : \textbf{normalize}\,(\textbf{flatten}\,[\,\overline{\mathit{etyp}_n}\,])}$$

$$\frac{\Gamma \vdash v : (\textbf{Uint}\;\mathit{dir}\;i\;\mathit{form}, \ell)}{\Gamma, \mathbf{P}, \mathbf{A} \vdash \textbf{evar}\;v \rightsquigarrow \textbf{evar}\;v : [\textbf{from}\,(\textbf{Uint}\;\mathit{dir}\;i\;\mathit{form}, \ell)]}$$

$$\frac{\begin{array}{c}\Gamma, \mathbf{P}, \mathbf{A} \vdash e_1 \rightsquigarrow e'_1 : [(\mathit{dir}, i, \textbf{None})] \\ \Gamma, \mathbf{P}, \mathbf{A} \vdash e_2 \rightsquigarrow e'_2 : [(\mathit{dir}, i, \textbf{None})] \\ \mathbf{A} \vdash \textbf{Arith}\;\mathit{dir}\;i\end{array}}{\Gamma, \mathbf{P}, \mathbf{A} \vdash e_1\;\mathit{aop}\;e_2 \rightsquigarrow e'_1\;\mathit{aop}_{\mathit{dir},i}\;e'_2 : [(\mathit{dir}, i, \textbf{None})]}$$

$$\frac{\begin{array}{c}\Gamma, \mathbf{P}, \mathbf{A} \vdash e_1 \rightsquigarrow e'_1 : [(\mathit{dir}, i, \textbf{Some}\,\ell)] \\ \Gamma, \mathbf{P}, \mathbf{A} \vdash e_2 \rightsquigarrow e'_2 : [(\mathit{dir}, i, \textbf{Some}\,\ell)] \\ \mathbf{A} \vdash \textbf{Arith}\;\mathit{dir}\;i\end{array}}{\Gamma, \mathbf{P}, \mathbf{A} \vdash e_1\;\mathit{aop}\;e_2 \rightsquigarrow e'_1\,\mathit{aop}^{\ell}_{\mathit{dir},i}\,e'_2 : [(\mathit{dir}, i, \textbf{Some}\,\ell)]}$$

$$\frac{\begin{array}{c}\Gamma, \mathbf{P}, \mathbf{A} \vdash e_1 \rightsquigarrow e'_1 : [(\mathit{dir}, i, \textbf{None})] \\ \Gamma, \mathbf{P}, \mathbf{A} \vdash e_2 \rightsquigarrow e'_2 : [(\mathit{dir}, i, \textbf{None})] \\ \mathbf{A} \vdash \textbf{Logic}\;\mathit{dir}\;i\end{array}}{\Gamma, \mathbf{P}, \mathbf{A} \vdash e_1\;\mathit{lop}\;e_2 \rightsquigarrow e'_1\;\mathit{lop}_{\mathit{dir},i}\;e'_2 : [(\mathit{dir}, i, \textbf{None})]}$$

$$\frac{\begin{array}{c}\Gamma, \mathbf{P}, \mathbf{A} \vdash e_1 \rightsquigarrow e'_1 : [(\mathit{dir}, i, \textbf{Some}\,\ell)] \\ \Gamma, \mathbf{P}, \mathbf{A} \vdash e_2 \rightsquigarrow e'_2 : [(\mathit{dir}, i, \textbf{Some}\,\ell)] \\ \mathbf{A} \vdash \textbf{Logic}\;\mathit{dir}\;i\end{array}}{\Gamma, \mathbf{P}, \mathbf{A} \vdash e_1\;\mathit{lop}\;e_2 \rightsquigarrow e'_1\,\mathit{lop}^{\ell}_{\mathit{dir},i}\,e'_2 : [(\mathit{dir}, i, \textbf{Some}\,\ell)]}$$

$$\frac{\begin{array}{c}\Gamma, \mathbf{P}, \mathbf{A} \vdash e \rightsquigarrow e' : [(\mathit{dir}, i, \textbf{None})] \\ \mathbf{A} \vdash \textbf{Logic}\;\mathit{dir}\;i\end{array}}{\Gamma, \mathbf{P}, \mathbf{A} \vdash \textbf{not}\;e \rightsquigarrow \textbf{not}_{\mathit{dir},i}\;e' : [(\mathit{dir}, i, \textbf{None})]}$$

$$\frac{\begin{array}{c}\Gamma, \mathbf{P}, \mathbf{A} \vdash e \rightsquigarrow e' : [(\mathit{dir}, i, \textbf{Some}\,\ell)] \\ \mathbf{A} \vdash \textbf{Logic}\;\mathit{dir}\;i\end{array}}{\Gamma, \mathbf{P}, \mathbf{A} \vdash \textbf{not}\;e \rightsquigarrow \textbf{not}^{\ell}_{\mathit{dir},i}\,e' : [(\mathit{dir}, i, \textbf{Some}\,\ell)]}$$

$$\frac{\begin{array}{c}\mathbf{P} \vdash f : \forall\,\overline{d_n}\,, \forall\,\overline{s_m}\,, \overline{\mathit{typc}_j} \Rightarrow \mathit{etyp}_1 \to \mathit{etyp}_2 \\ \Gamma, \mathbf{P}, \mathbf{A} \vdash (\,\overline{e_n}\,) \rightsquigarrow (\,\overline{e'_n}\,) : \mathit{etyp}_1[\,\overline{d_n \leftarrow d'_n}\;;\;\overline{s_m \leftarrow s'_m}\,] \\ \overline{\mathbf{A} \vdash \mathit{typc}_j[\,\overline{d_n \leftarrow d'_n}\;;\;\overline{s_m \leftarrow s'_m}\,]}\end{array}}{\Gamma, \mathbf{P}, \mathbf{A} \vdash f(\,\overline{e_n}\,) \rightsquigarrow f(\,\overline{e'_n}\,) : \mathit{etyp}_2[\,\overline{d_n \leftarrow d'_n}\;;\;\overline{s_m \leftarrow s'_m}\,]}$$

Figure 5: Règles de typage des expressions

$$\frac{\begin{array}{c}\Gamma, \mathbf{P}, \mathbf{A} \vdash e \leadsto e' : \mathbf{normalize}\,[\,\overline{\mathbf{from}\,(typ_n, a_n)}\,] \\ \Gamma \vdash \overline{v_n : (typ_n, a_n)}\end{array}}{\Gamma, \mathbf{P},\ \mathbf{A} \vdash_D \overline{v_n} := e \leadsto \overline{v_n} := e'}$$

$$\frac{\begin{array}{c}\Gamma, \mathbf{P}, \mathbf{A} \vdash e \leadsto e' : etyp \\ \Gamma, \mathbf{P}, \mathbf{A} \vdash (\,\overline{\mathbf{evar}\, v_n}\,) \leadsto (\,\overline{\mathbf{evar}\, v_n}\,) : etyp\end{array}}{\Gamma, \mathbf{P},\ \mathbf{A} \vdash_D \overline{v_n} = e \leadsto \overline{v_n} = e'}$$

$$\frac{\forall\, \mathbf{c} \in [a_1, a_2].\ \Gamma,\ \mathbf{P}\,,\ \mathbf{A} \vdash_D \overline{deq_n[x \leftarrow \mathbf{c}\,] \leadsto deq'_n[x \leftarrow \mathbf{c}\,]}}{\Gamma, \mathbf{P},\ \mathbf{A} \vdash_D \mathbf{for}\ x = a_1\ \mathbf{to}\ a_2\ \mathbf{do}\ \overline{deq_n}\ \mathbf{done} \leadsto \mathbf{for}\ x = a_1\ \mathbf{to}\ a_2\ \mathbf{do}\ \overline{deq'_n}\ \mathbf{done}}$$

Figure 6: Typage des equations

$$\frac{\begin{array}{c}\overline{x_m : typ_m} + \overline{y_n : typ'_n} + \overline{t_j : typ''_j}, \mathbf{P},\ \mathbf{A} \vdash_D deq \leadsto deq' \\ node = \mathbf{node}\, f\,(\,\overline{x_m : typ_m}\,) \rightarrow (\,\overline{y_n : typ'_n}\,)\, \mathbf{vars}\,(\,\overline{t_j : typ''_j}\,)\, \overline{deq_k}\end{array}}{\mathbf{P} \leftarrow node \vdash f : \forall\, \overline{d_n}\,, \forall\, \overline{s_m}\,, \mathbf{A} \Rightarrow \mathbf{normalize}\,[\,\overline{\mathbf{from}\,(typ_m, 1)}\,] \rightarrow \mathbf{normalize}\,[\,\overline{\mathbf{from}\,(typ'_n, 1)}\,]}$$

Figure 7: Typage d'un noeud